Panel on Teaching Logic as Tool

Moderators: **David Gries** and **Fred B. Schneider**, Computer Science, Cornell University

Panelists: **Joan Krone**, Mathematics and Computer Science, Denison University
Stan Warford, Computer Science, Pepperdine University
J. Peter Weston, Computer Science, Daniel Webster College

The panelists will discuss their experiences teaching logic and discrete mathematics using a new, rather revolutionary approach. They will describe advantages and disadvantages of the approach, what students think of it, and so forth. The audience will have a chance to question the panelists and to discuss their own experiences.

Instead of teaching logic as one more isolated topic of discrete math, logic can be used as the fundamental tool underlying all other topics of the course. In such a course, six to seven weeks are spent learning logic and how to develop formal proofs. The other topics of discrete math are then taught in a more rigorous fashion than usual, using logic as the foundation. This approach, outlined in more detail on the next page, is the basis for the text *A Logical Approach to Discrete Math* (Gries and Schneider, Springer Verlag, 1993). The approach has been employed by all the panelists.

The panelists are not from high-powered research universities, so the audience will get a chance to see how the approach works with the typical college student in America:

- Joan Krone is using this approach to teach a course for sophomores in Fall 1994 at Denison University in Granville Ohio. The students are taking the course concurrently with their second year computer science course in data structures and algorithm analysis.
- Stan Warford has taught a course twice thus far, to sophomores at Pepperdine in Malibu, California.
- Peter Weston is teaching a course in Fall 1994 to incoming freshmen, in parallel with an introductory course on programming, at Daniel Webster College in Nashua, New Hampshire.

Two of the panelists had not taught the course when this panel was organized. They were in the middle of teaching it for the first time when this description was written. Thus, at the time of this writing, their opinions on the approach were not fully formed.

Come and hear firsthand how this approach works with typical students!

Teaching logic as a tool

We base this new approach to discrete math on the thesis:

> Logic is the glue that binds together methods of reasoning, in all domains.

The traditional proof methods —e.g. proof by assumption, contradiction, and induction— are rooted in formal logic. Thus, whether proofs are to be presented formally or informally, a study of logic can provide understanding into proof techniques. But being skilled in logical manipulation allows one to go beyond mere understanding of techniques; the skill can help one discover proofs systematically and may enable one to derive truths about any domain of interest.

The traditional approach to teaching logic uses natural deduction and the Hilbert style of proof. A proof consists of a sequence of formulas, each of which is an axiom or is proved true using an inference rule and previously occurring formulas. Such proofs convey little about structure, are hard to read, and are harder to develop.

The new approach uses an *equational* logic. It has different inference rules and a different proof style than traditional logic. We don't show how to mimic proofs written in informal English; we provide a completely different, complementary, proof style.

Here are the four inference rules of equational propositional logic:

Leibniz: $\dfrac{P = Q}{E[r := P] = E[r := Q]}$

Transitivity: $\dfrac{P = Q,\ Q = R}{P = R}$

Substitution: $\dfrac{P}{P[r := Q]}$

Equanimity: $\dfrac{P,\ P \equiv Q}{Q}$

The proof style is reminiscent of how proofs are written in high school algebra, with Leibniz (substitution of equals for equals) being the dominant inference rule. For example, here is a proof of $p \Rightarrow \mathit{false} \equiv \neg p$:

$$\begin{array}{ll} & p \Rightarrow \mathit{false} \\ = & \quad \langle \text{Implication, } X \Rightarrow Y \ \equiv\ \neg X \vee Y \rangle \\ & \neg p \vee \mathit{false} \\ = & \quad \langle \text{Identity of } \vee,\ X \vee \mathit{false} \equiv X \rangle \\ & \neg p \end{array}$$

The inference rules used in the proof are not mentioned explicitly because their use is implicit in the proof format. For example, Leibniz justifies each equality and Transitivity justifies the conclusion that the first formula equals the last.

This proof style (with a few extensions) can be used *formally* not only for logic but for all other topics of discrete math —set theory, sequence theory, induction, theory of integers, solving recurrence relations, etc.— without complexity overwhelming.

For example, below is a proof, from set theory, of $A \subseteq C \wedge B \subseteq C \ \equiv\ A \cup B \subseteq C$. Using natural deduction, the proof requires several levels of nesting of proofs within proofs, as well as case analysis. Here, the proof is developed using the following strategy (one of several that we teach):

> To prove a theorem about an operator, first eliminate it from the theorem using its definition, then manipulate, and finally (if necessary) reintroduce the operator.

$$\begin{array}{ll} & A \subseteq C \wedge B \subseteq C \\ = & \quad \langle \text{Definition of } \subseteq \text{, twice} \rangle \\ & (\forall x \mid x \in A : x \in C) \wedge (\forall x \mid x \in B : x \in C) \\ = & \quad \langle \text{Split range} \rangle \\ & (\forall x \mid x \in A \vee x \in B : x \in C) \\ = & \quad \langle \text{Definition of } \cup \rangle \\ & (\forall x \mid x \in A \cup B : x \in C) \\ = & \quad \langle \text{Definition of } \subseteq \rangle \\ & A \cup B \subseteq C \end{array}$$