Negative rates in QuantLib

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Quaternion Risk Management

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History of negative fixings

- it started with negative EONIA fixings end of 2014
- then we had negative Euribor 1m, later 3m, even 6m fixings
- as of 27-Oct-2015 we have a negative CMS2Y fixing (at -3.5 bp)

Implications of negative fixings

- interest compounding on collateral accounts,
  ISDA negative rates protocol, DRV (?)
- payment reversal in swaps under ISDA and DRV (?)
- floored coupons for bonds, schuldscheindarlehen, loans, ... (?)

Implications on pricing

- rate curves should allow for negative forwards
- lognormal models can not reproduce market prices for zero (or negative strike) floors
- lognormal models can even fail to produce high enough prices for boring forward levels like $F = 1\%$ or 2%, because e.g. for shifted lognormal models with shift $d \geq 0$, $c(K)/N(0) \to F + d$ if $\sigma \to \infty$.
- you could actually observe this recently by first exploding, then missing implied lognormal volatility quotes for EUR swaptions with long option tenor ("two holes" in the quoted matrix)

Implications on pricing

- shifted Black76 and normal Black76 models were established as market models for low and negative rates
- shifting is generic, e.g. the shifted SABR model has also become part of the new basic standard of market models
- with a different motivation (produce skew) a shift was introduced in Libor forward models a long time ago
- new models / model variants are discovered to handle negative rates in a more sophisticated way (free boundary SABR, mixed SABR)
- other models need adjustments as well (cms replication coupon pricers, Markov functional model)

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Negative rates switch

- QL_NEGATIVE_RATES
- allows for negative zero yields, forwards, increasing discount factors
- +2012-07-31 14:11 Ferdinando Ametrano
  +
  + * [r18305] ql/userconfig.hpp, test-suite/piecewiseyieldcurve.cpp:
  +
  +   defaulted to allow negative rates (define QL_NEGATIVE_RATES) as this
  +   is happening for EUR OIS, CHF and German treasury yields, etc.

Volatility type

- `ql/termstructures/volatility/volatilitytype.hpp`
- distinguishes between normal and (shifted) lognormal volatilities

```
enum VolatilityType { ShiftedLognormal, Normal };
```

Cap Floor Volatilities

- market quotes normal or shifted lognormal volatilities, with a constant shift across strikes and tenors

```
OptionletStripper(const boost::shared_ptr<CapFloorTermVolSurface>&,
                  const boost::shared_ptr<IborIndex>& iborIndex_,
                  const Handle<YieldTermStructure>& discount =
                                          Handle<YieldTermStructure>(),
                  const VolatilityType type = ShiftedLognormal,
                  const Real displacement = 0.0);
```

Swaption Volatilities

- market quotes normal or shifted lognormal volatilities, with different shifts per underlying
- swaption cubes inherit the shift structure from their embedded atm matrix
- swaption volatility cube 1 uses shifted SABR models
- the shift is bilinearly interpolated in (option, underlying) space

```
SwaptionVolatilityMatrix(
            const Calendar& calendar,
            BusinessDayConvention bdc,
            ...
            const VolatilityType type = ShiftedLognormal,
            const std::vector<std::vector<Real> >& shifts
                        = std::vector<std::vector<Real> >());
```

Libor in arrears adjustments

- convexity adjustment is amended in a straightforward way for shifted lognormal or normal volatilities
- timing adjustment is generalized at the same time for arbitrary non-natural fixing times[1]

```
enum TimingAdjustment { Black76,
                        BivariateLognormal };
BlackIborCouponPricer(const Handle<OptionletVolatilityStructure> &v =
                      Handle<OptionletVolatilityStructure>(),
                      const TimingAdjustment timingAdjustment = Black76,
                      const Handle<Quote> correlation =
                      Handle<Quote>(boost::make_shared<SimpleQuote>(1.0)))
```

[1] see http://ssrn.com/abstract=2170721

Linear TSR pricer for CMS coupons

- volatility type is recognized through the abstraction of `SmileSection`
- the replication range is shifted appropriately (e.g. user bounds set to $[0, 200\%]$ are transformed to $[-1\%, 199\%]$ automatically if the applicable shift is 1% to keep the user input universal under changing shifts in market quotations)
- for a normal model, the replication domain extends to $(-\infty, \infty)$

CMS spread option pricer

- swap rate adjustments use shifted lognormal or normal smiles to determine the drifts of the single swap rate models
- the bivariate model for the swap rates is still purely lognormal currently, which works technically as long as the underlying forward levels are still positive
- with negative 2Y fixings, we will neeed to extend this pricer as well!
- PR #264 allow for shifts in the single rate models or for normal single rate models[2]

[2] see `http://ssrn.com/abstract=2686998`

Calibration helpers

- can be set up with normal and shifted lognormal volatilities
- cooperative with `HullWhite`, `Gsr`, `Lgm`, `MarkovFunctional` models

```
SwaptionHelper(const Period& maturity,
               const Period& length,
               const Handle<Quote>& volatility,
               ...
               const VolatilityType type = ShiftedLognormal,
               const Real shift = 0.0);
```

Markov functional model

- replicates a market smile / density per expiry via the numeraire calibration
- therefore also replicates the density for negative strike ranges
- currently, only shifted lognormal smile input allowed
- todo: allow normal smile input for numeraire calibration

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Questions / Discussion

Thank you for your attention