- 2. (a) 240 ounces = 15 pounds (b) 20 ounces.
 - (c) 3 ounces of nitrogen yields 18 lb 12 oz of rice, 4 ounces of nitrogen yields 20 pounds of rice.
 - (d) Controlled.
 - (e) Yes. The line fits quite well (r = 0.95), and 3 ounces is close to a value that was used.
 - (f) No. That's too far away from the amounts used.
- 3. (a) Predicted son's height $= 0.5 \times \text{father's height} + 35 \text{ inches.}$
 - (b) Predicted father's height $= 0.5 \times \text{son's height} + 33.5 \text{ inches.}$

Comment. There are two regression lines, one predicts son's height from father's height, the other predicts father's height from son's height (section 5 of chapter 10).

4. This testimony is overstatement. Associations in the data may be due to confounding. Without doing the experiment, or working very hard at the observational data, you can't be sure what the impact of interventions will be.

Set B, page 210

- With 12 years of education, height is predicted as 69.75 inches; with 16 years, height is predicted as 70.75 inches. Going to college clearly has no effect on height. This observational study picked up a correlation between height and education due to some third factor in family background.
- 2. 439.16 cm, 439.26 cm. Hanging a bigger weight on the wire makes it stretch more. You can trust the regression line in exercise 2 because it is based on an experiment. In exercise 1, the line was fitted to data from an observational study.
- 3. (a) 540 + 110 = 650
- (b) 540
- (c) Greater than (p. 208).

- 4. (a) 540
- (b) 540
- (c) Greater than (p. 208).

Comment. if you use the average value of y to predict y, the r.m.s. error is the SD of y; see p. 183.

5. The regression line makes the smallest r.m.s. error (p. 208).

Part IV. Probability

Chapter 13. What Are the Chances?

Set A, page 225

- ...
 - (b) (iii)
- (c) (iv)
- (d) (i)

- (e) (ii)
- (f) (v)
- (g) (vi)
- 2. About 500.

1. (a) (vi)

- 3. About 1,000.
- 4. About 14.
- 5. Box (ii), because [3] pays more than [2], and the other ticket is the same.

Set B, page 227

1. (a) The question is about the second ticket, not the first: see part (a) of example 2. The answer is 1/4.

- (b) 1/3; there are 3 tickets left after 2 is drawn.
- 2. (a) 1/4 (b) 1/4

With replacement, the box stays the same.

3. (a) 1/2 (b) 1/2

The chances for the 5th toss of the penny do not depend on the results of the first 4 tosses.

4. (a) 1/52 (b) 1/48

This is like example 2 on p. 226.

Set C, page 229

- 1. (a) 12/51 (b) $13/52 \times 12/51 = 1/17 \approx 6\%$.
- 2. (a) 1/6 (b) $1/6 \times 1/6 \times 1/6 = 1/216 \approx 1/2$ of 1%.
- 3. (a) 4/52 (b) $4/52 \times 4/51 \times 4/50 \approx 5/10,000$.

Comment. In this exercise, the cards are dependent; in exercise 2, the rolls were independent.

- 4. "At least one ace" is the better option: you would choose an exam in which you had to get at least one question right out of six, over an exam in which you had to get all six right.
- 5. This is fine, it's the multiplication rule.
- 6. The coin has to land "tails, heads"; the chance is 1/4.
- 7. (a) 1/8
 - (b) 1 1/8 = 7/8
 - (c) 7/8; you get at least one tail when you don't get three heads: so (b) and (c) are the same.
 - (d) 7/8; just switch heads and tails in (c).

Set D, page 232

- 1. (a) independent: if you get a white ticket, there is 1 chance in 3 to get "1" and 2 chances in 3 to get "2"; if you get the black ticket, the chances for the numbers stay the same.
 - (b) independent
 - (c) dependent: with the white tickets, there is only 1 chance in 3 to get "2"; with the black tickets, there are 2 chances in 3.
- 2. (a,b) independent (c) dependent

Comment. This kind of box will come up again in chapter 27. Here is the argument for (a). Suppose you draw a ticket, and see the first number is 4 but don't see the second number: the chance that the second number will be 3 is 1/2. Likewise if the first number is 1. That is independence.

- 3. Ten years is 520 weeks, so the chance is $(999,999/1,000,000)^{520} \approx 0.9995$. *Comment.* In the New York State Lotto, your chance of winning something is about 1/12,000,000.
- 4. This is false. It's like saying someone doesn't have a temperature because you can't find the thermometer. To figure out whether two things are independent or not, you pretend to know how the first one turned out, and then see if the chances for the second change. The emphasis is on the word "pretend."

5. (a) 5% (b) 20%

To figure (a) out, suppose you have 80 men and 20 women in the class. You also have 15 cards marked "freshman" and 85 cards marked "sophomore." You want to give out a card to each student, so that as few women as possible get "sophomore." The strategy is to give a sophomore card to each man; you are left with 5, which have to go to 5 women. The 15 freshman cards go to the other 15 women.

Comment. If year and sex are independent, the percentage of sophomore women would be 85% of 20% - 17%, between the two extremes.

- 6. Same as previous exercise: the chance of getting a sophomore woman equals the percentage of sophomore women in the class.
- 7. False. The calculation assumes that the percentage of women is the same across all age groups, and it isn't: women live longer than men. (Actually, women age 85 and over accounted for nearly 1.1% of the U.S. population in 2002.)
- 8. If the subject draws the ace of spades from the small pile, he has 13 chances in 52 to draw a spade from the big deck, and win the prize. Likewise if he draws the deuce of clubs. Or any other card. So the answer is 13/52 = 1/4.

Chapter 14. More about Chance

Set A, page 240

- 2. There are 25 possible results; for 5 of them, the sum is 6. So the chance is 5/25. (The figure is not shown.)
- 3. Most often, 7; least often, 2, 12. (Use figure 1 to get the chance of each total, as in exercise 1.)
- 4. (a) 2/4 (b) 2/6 (c) 3/6

Set B, page 242

 False. The question is about the number of children who had either cookies or ice cream, including the gluttons who had both. The number depends on the choices made by the children, and two possibilities are shown in the table.

| Cookies only | Ice cream only | Both | Neither |
|--------------|----------------|------|---------|
| 12 | 17 | 0 | 21 |
| 3 | 8 | 9 | 30 |

In the first case, 12 children had cookies only, 17 children had ice cream only, 0 had both, and 21 had neither. So 12+17=29 had cookies or ice cream. The second line shows another possibility, where 9 children had both cookies and ice cream. In this situation, the number with cookies or ice cream is 3+8+9=20. Just as a check: the number with cookies is 3+9=12, and the number with ice cream is 8+9=17, as given in the problem. But the number with cookies or ice cream is not 12+17, because the addition double counts the 9 gluttons. The number who had cookies or ice cream depends on the number of gluttons who had both.



- 2. (a) 4/20 (b) 8/20 (c) 12/20 (d) 14/20

 Comment. (4 + 8 + 12)/20 gives the wrong answer to (d)—by double-counting some dots and triple-counting others.
- 3. They are the same.
- 4. False. Simply adding the two chances double counts the chance of •. See example 5 on p. 242.
- 5. False. There is 1 chance in 10 of getting 7 on any particular draw, but these events are not mutually exclusive.
- 6. True. 100% (10% + 20%) = 70%. Use the addition rule, and p. 223 for the subtraction.

Set C, page 246

- 1. (a) 1/52 of the contestants step forward.
 - (b) 1/52 of the contestants step forward; example 2 in chapter 13.
 - (c) The ones who got both the ace of hearts on the first card and the king of hearts on the second card step forward twice. (In terms of getting the weekend, that's overkill.) The fraction who step forward twice is $1/52 \times 1/51$.
 - (d) False; as (c) shows, the events aren't mutually exclusive, so addition double counts the chance that both occur.

Comment. The chance in (d) is

$$1/52 + 1/52 - 1/52 \times 1/51$$
.

- 2. (a) 1/52 of the contestants step forward.
 - (b) 1/52 of the contestants step forward.
 - (c) If you get the ace of hearts on the first card, you can't get it on the second card; nobody steps forward twice.
 - (d) True; as (c) shows, the events are mutually exclusive, so addition is legitimate. *Comment.* In exercise 2, the two ways to win are mutually exclusive; not so in exercise 1. Addition is legitimate in exercise 2, not in 1.
- 3. (a,b) True; see example 2 in chapter 13.
 - (c) False. "Top card is the jack of clubs" and "bottom card is the jack of diamonds" aren't mutually exclusive, so you can't add the chances.
 - (d) True. "Top card is the jack of clubs" and "bottom card is the jack of clubs" are mutually exclusive.
 - (e,f) False; these events aren't independent, you need the conditional chances.

- 4. (a) False; $1/2 \times 1/3 = 1/6$, but A and B may be dependent: you need the conditional chance of B given A.
 - (b) True; see section 4 of chapter 13.
 - (c) False. ("Mutually exclusive" implies dependence, and the chance is actually 0.)
 - (d) False; 1/2 + 1/3 = 5/6, but you can't add the chances because A and B may not be mutually exclusive.
 - (e) False; if they're independent, they have some chance of happening together, so they can't be mutually exclusive: don't add the chances.
 - (f) True.

Comment. If you have trouble with exercises 3 and 4, look at example 6, p. 244.

- 5. See example 2 in chapter 13.
 - (a) 4/52
- (b) 4/51
- (c) $4/52 \times 4/51$

Set D, page 250

- 1. (a) (i)
- (b) (i) (ii)
- (c) (iii) (d) (ii) (iii)
- (e) (i) (ii) (f) (i)
- 2. Bets (a) and (f) say the same thing in different language. So do (b) and (e). Bet (d) is better than (c).
- 3. (a) 3/4
- (b) 3/4
- (c) 9/16
- (d) 9/16
- (e) 1 9/16 = 7/16
- 4. (a) Chance of no aces = $(5/6)^3 \approx 58\%$, so chance of at least one ace $\approx 42\%$. Like de Méré, with 3 rolls instead of 4.
 - (b) 67%
- (c) 89%
- 5. $1 (35/36)^{36} \approx 64\%$
- 6. The chance that the point 17 will not come up in 22 throws is $(31/32)^{22} \approx 49.7\%$. The chance that it will come up in 22 throws is therefore 100% 49.7% = 50.3%. So this wager (laid at even money) was also favorable to the Master of the Ball. Poor Adventurers.
- 7. The chance of surviving 50 missions is $(0.98)^{50} \approx 36\%$. Deighton is adding chances for events that are not mutually exclusive.

Chapter 15. The Binomial Coefficients

Set A, page 258

- 1. The number is 4.
- 2. The number is 6.
- 3. (a) $(5/6)^4 = 625/1,296 \approx 48\%$
 - (b) $4(1/6)(5/6)^3 = 500/1,296 \approx 39\%$
 - (c) $6(1/6)^2(5/6)^2 = 150/1,296 \approx 12\%$
 - (d) $4(1/6)^3(5/6) = 20/1,296 \approx 1.5\%$
 - (e) $(1/6)^4 = 1/1,296 \approx 0.08$ of 1%
 - (f) Addition rule: $(150 + 20 + 1)/1,296 \approx 13\%$.
- 4. This is the same as exercise 3(a-c). Rolling an ace is like drawing a red marble, while 2 through 6 correspond to green. To see why, imagine two people, A and B, performing different chance experiments: