V. Larson
January 17, 2023

Problem Set 7
Math modeling of the atmosphere

Problem 1: Optimal importance sampling density, $q(x)$

Suppose that our subgrid distribution is given by $P(x) = \mathcal{N}(0, \sigma^2)$ and our autoconversion parameterization is given by $f(x) = H(x)\, x^2$.

i) Find the optimal importance sampling density, $q(x)$. Be sure that $q(x)$ is normalized.

ii) Find the value of x where the optimal $q(x)$ reaches a maximum. Is it greater than or less than the mean, μ, for $P(x)$?

Prof. Larson
January 17, 2023

Problem Set 7, Answer Outline
Math modeling of the atmosphere

i)

$$q(x) \propto |f(x)|P(x) = H(x)x^2 \exp\left(-\frac{x^2}{2\sigma^2}\right) \tag{1}$$

The area under $|f(x)|P(x)$ is $\frac{1}{2}\sqrt{2\pi}\sigma^3$. (To see this, let $\alpha \equiv 1/(2\sigma^2)$ and write $x^2\exp(-\alpha x^2)$ as $-\partial/\partial\alpha(\exp(-\alpha x^2))$.) We must divide by this factor in order to normalize $q(x)$. Therefore,

$$q(x) = 2\frac{1}{\sqrt{2\pi}\sigma^3} H(x)x^2 \exp\left(-\frac{x^2}{2\sigma^2}\right) \tag{2}$$

ii)

The maximum of $q(x)$ occurs where $\partial q(x)/\partial x = 0$. This is at $x = \sqrt{2}\sigma$. This is greater than $\mu = 0$, assuming that $\sigma > 0$.