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Praktikum 2, Monte Carlo
Math modeling of the atmosphere

Computing average autoconversion

Autoconversion is the process by which small cloud droplets grow in size and become rain drops. It can be approximated as a function of two variables, χ and N_c:

$$A(\chi, N_c) = H(\chi)\chi^{\alpha} N_c^{\beta}. \tag{1}$$

Here χ is related to the cloud water mixing ratio, and N_c is the number of cloud droplets. The exponent α is often taken to equal 2.47, and the exponent β is often taken to equal -1.79. Therefore, if there is more cloud water or larger drops, rain forms more easily.

If we integrate over this autoconversion expression over a bivariate PDF that is normal in χ and lognormal in N_c, we find

$$\begin{aligned}
&\overline{A(\chi, N_c)} \\
&= \frac{1}{\sqrt{2\pi}} \Big(\sigma_{\chi}\Big)^{\alpha} \exp\left[\mu_{N_{cn}}\beta + \frac{1}{2}\sigma_{N_{cn}}^2\beta^2 - \frac{1}{4}\left(\frac{\mu_{\chi}}{\sigma_{\chi}} + r_{(\chi,N_c)n}\sigma_{N_{cn}}\beta\right)^2\right] \\
&\times \Gamma\left(\alpha+1\right) \quad D_{-(\alpha+1)}\left[-\left(\frac{\mu_{\chi}}{\sigma_{\chi}} + r_{(\chi,N_c)n}\sigma_{N_{cn}}\beta\right)\right].
\end{aligned} \tag{2}$$

Here, a subscript n denotes a moment in the normal space, rather than the lognormal space. For instance, $r_{(\chi,N_c)n}$ is the correlation between χ and N_c in the normal space. Furthermore, Γ denotes the gamma function, and D denotes the parabolic cylinder function. This formula is Eqn. (33) of Larson and Griffin (2013). This article contains other relevant information as well.

1. Write a python function that computes the exact average autoconversion rate given by the formula above. As input, it should take in the means, standard deviations, and correlation of the PDF, along with the exponents α and β. As output, it should produce $\overline{A(\chi, N_c)}$. Use the `scipy.special` function, `pbdv`.

Now compute the same integral using Monte Carlo integration.

2. To represent the χ distribution, create a sample from a univariate uniform distribution and transform it to a standard normal. To represent the N_c distribution, create another, independent

sample from a univariate standard normal. Together, these points form a sample from a bivariate, uncorrelated standard normal distribution.

3. Use a Cholesky decomposition in order to transform the distribution from a 2D uncorrelated standard normal to a 2D correlated normal. Assume a correlation $r_{(\chi,N_c)n}$, a mean N_c of $\mu_{N_{cn}}$, etc.

4. In order to transform to a lognormal distribution in N_c, take the exponential of the N_c component of the sample points.

5. Use Monte Carlo integration in order to find the integral $\overline{A(\chi, N_c)}$.

6. Compute the Monte Carlo estimate for multiple values of N. Check whether your answer converges to the analytic answer as $1/\sqrt{N}$. Make sure that it converges for non-zero values of $r_{(\chi,N_c)n}$. To improve the convergence rate, choose a value of $\sigma_{N_{cn}} < 2$.