Problem Set 6 Math modeling of the atmosphere

Problem 1: Unbiasedness of sample variance

An estimate of the variance is given by

$$s^{2} = \frac{1}{N-1} \sum_{i=1}^{N} (X_{i} - \hat{\mu})^{2}, \tag{1}$$

where

$$\hat{\mu} = \frac{1}{N} \sum_{i=1}^{N} X_i. \tag{2}$$

Show that s^2 is an unbiased estimate of the variance of the distribution, σ^2 . That is, show that $\mathbb{E}(s^2) = \sigma^2$.