

Problem Set 4
Math modeling of the atmosphere

Problem 1: Conditional distribution of a 2D normal distribution

Consider a normal distribution with two variates, x_1 and x_2 . Assume that the distribution has mean $\mu = 0$ and standard deviation $\sigma = 1$. Therefore, we can call the distribution $\mathcal{N}(x_1, x_2|0, 1)$.

- (I) Assume that the covariance, Σ_{12} , between x_1 and x_2 is 0.
 - (a) Sketch contours of $\mathcal{N}(x_1, x_2|0, 1)$.
 - (b) What is the conditional average $\mathbb{E}(X_1|X_2 = 1)$? Is $\mathbb{E}(X_1|X_2 = 1) > \mu_1$, or is $\mathbb{E}(X_1|X_2 = 1) < \mu_1$, or is $\mathbb{E}(X_1|X_2 = 1) = \mu_1$?
 - (c) What is the standard deviation σ of the conditional distribution? Is it less than or greater than the standard deviation of \mathcal{N} , which is 1? Sketch the conditional distribution.
- (II) Now repeat (a), (b), and (c), but assume that $\Sigma_{12} = 0.5$.
- (III) Repeat (a), (b), and (c), but assume that $\Sigma_{12} = 0.99$.
- (IV) Which is larger, the conditional average in (II) or that in (III)? Which is larger, the standard deviation of the conditional distribution in (II) or that in (III)?
- (V) Repeat (a), (b), and (c), but assume that $\Sigma_{12} = -0.5$.