V. Larson
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Praktikum 3, Monte Carlo
Math modeling of the atmosphere

Control variates

A control variate is a function, $h(\mathbf{x})$, that can be integrated analytically and is similar to the function, $f(\mathbf{x})$, that we want to integrate by a Monte Carlo method. We can test the method of control variates by using the autoconversion function studied earlier, $f(\mathbf{x}|\alpha, \beta)$. For pedagogical purposes, suppose that the control variate is $h(\mathbf{x}) = f(\mathbf{x}|\alpha', \beta')$, where $\alpha' \approx \alpha$ and $\beta' \approx \beta$. In other words, let's pretend that we can't integrate $f(\mathbf{x}|\alpha, \beta)$ analytically, but that we can integrate $h(\mathbf{x}) = f(\mathbf{x}|\alpha', \beta')$ analytically. Then let's use h as a control variate.

The method of control variates is described in Section 8.9 of "Monte Carlo theory, methods and examples" by Art Owen. Following that reference, implement the method of control variates and test whether it produces less noise than basic Monte Carlo.

1. Implement h. Let h be the autoconversion function, but instead of using the standard values, $\alpha = 2.47$ and $\beta = -1.79$, use $\alpha' = 2.2$ and $\beta' = -1.5$.
2. Overplot scatterplots of f and h. Do they look correlated? To see better, overplot the line $y = x$. Calculate the sample correlation, ρ, between f and h.
3. Write code to find the optimal value of the coefficient of the regression estimator, β_{opt}. To do so, calculate a sample estimate, $\hat{\beta}_{\text{opt}}$.
4. Use the regression estimator $\hat{\mu}_\beta$ to estimate $\overline{f}$.
5. How much does $\hat{\mu}_\beta$ improve upon the basic Monte Carlo estimator, $\hat{\mu}$? Is the RMSE better by a factor of $\sqrt{1-\rho^2}$? Try different values of α' and β', and see how much the method of control variates improves upon basic Monte Carlo.