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Problem Set 1
Math modeling of the atmosphere

Reading: Germano, M., 1992: Turbulence: The filtering approach. *J. Fluid Mech.*, **238**, 325–336..

Problem 1: Spatial filtering

Consider a 1D field

$$f(x'') = \frac{1}{2}\left(1 - \cos(x'')\right) \tag{1}$$

that extends indefinitely to the left and the right. In this problem, we'll see how running-mean filtering of f affects the perturbations of f, namely f'', from the filtered version of f, namely $\overline{f}$.

Assume that the filter used is a simple box (uniform) filter, extending from $x'' = x - \pi/2$ to $x'' = x + \pi/2$.

Throughout this problem, when you are asked to plot functions, you may sketch the functions by hand or plot them using a computer.

i) Calculate $\overline{f}(x)$. Show your work.

ii) Overplot the original function $f(x'')$ and the filtered version $\overline{f}$.

iii) Suppose we define $f'' \equiv f(x'') - \overline{f}(x)$. (Note that the definition here uses $\overline{f}(x)$ rather than $\overline{f}(x'')$.) Calculate $\overline{f''}(x)$, showing your work.

iv) Now suppose we define $f'' \equiv f(x'') - \overline{f}(x'')$. (Note that this time the definition uses $\overline{f}(x'')$.) Calculate $\overline{f''}(x)$ using this new definition, showing your work. Plot $\overline{f''}(x)$.

v) Explain in physical terms why $\overline{f''}(x)$ from part iii) is different from $\overline{f''}(x)$ from part iv). Which part obeys Reynolds' rules?