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Praktikum 0
Math modeling of the atmosphere

Integrating simple closure equations for moisture

Consider the following set of 1D equations for moisture variables. The domain is $z = [0, 1]$.

First, the grid-mean total water mixing ratio, $\overline{r_t}(t, z)$, depends on the vertical flux of r_t, $\overline{w'r_t'}(t, z)$:

$$\frac{\partial \overline{r_t}}{\partial t} = -\frac{\partial}{\partial z} \overline{w'r_t'}. \tag{1}$$

Second, the flux $\overline{w'r_t'}$ is assumed to be related to $\overline{r_t}$ by

$$\overline{w'r_t'} = -K \frac{\partial \overline{r_t}}{\partial z}. \tag{2}$$

That is, the flux is determined by down-gradient diffusion. The boundary condition at the boundaries $z = [0, 1]$ is $\overline{w'r_t'} = 0$.

Finally, the sub-grid variance of total water, $\overline{r_t'^2}(t, z)$, is assumed to be adequately diagnosed by the algebraic expression

$$\overline{r_t'^2} = \tau K \left(\frac{\partial \overline{r_t}}{\partial z} \right)^2, \tag{3}$$

where $\tau = 1$ is a prescribed eddy turnover time, and $K = K(z)$ is a prescribed eddy diffusivity:

$$K(z) = \begin{cases} 1, & \text{if } z \leq 1/2 \\ z + 1/2, & 1/2 < z \leq 1. \end{cases} \tag{4}$$

The initial profile of total water diminishes with height above the mixed layer:

$$\overline{r_t}(t = 0, z) = \begin{cases} r_{t,sfc}, & \text{if } z \leq 1/2 \\ -r_{t,sfc}\,(z - 1/2) + r_{t,sfc}, & 1/2 < z \leq 1, \end{cases} \tag{5}$$

where $r_{t,sfc} = 10$.

- Integrate the equations forward in time using a discretization method of your choice.
- Plot the profiles of $\overline{r_t}$ and $\overline{r_t'^2}$ for later times. Can you explain their behavior in physical terms? What do you expect the behavior will be as $t \to \infty$?