Cloud parameterization: The assumed PDF method

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Outline

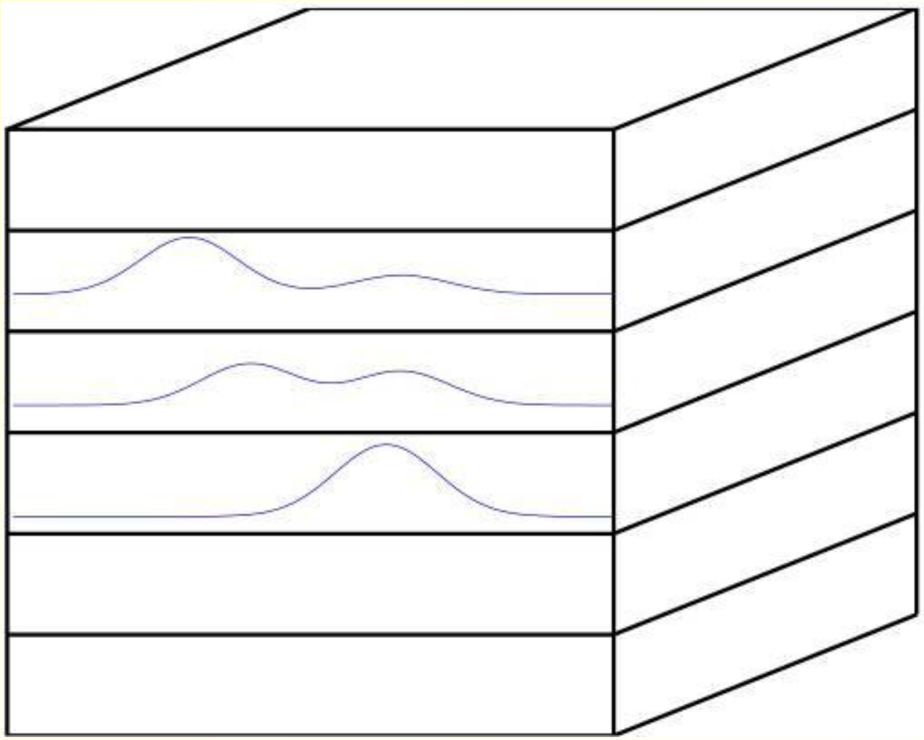
- Motivation
- How does the assumed PDF method work?
- Advantages and disadvantages of the assumed PDF method.
- Results using the assumed PDF method.
- Conclusions.

What is parameterization?

- A 3D large-scale model covers the horizontal domain with a mesh of grid points, that is, a series of adjacent grid columns

- The large-scale model can resolve the large-scale fields, but not the subgrid scales. Many clouds are of subgrid size.

- To represent the effects of the subgrid on the large scales, we build a parameterization.



The parameterization problem

A parameterization needs to supply subgrid-scale fluxes of heat, moisture, and momentum:

$$\frac{\partial \bar{r}_t}{\partial t} = \underbrace{-\bar{w}\frac{\partial \bar{r}_t}{\partial z}}_{\text{Mean Adv}} \underbrace{-\frac{1}{\rho_s}\frac{\partial(\rho_s \overline{w' r_t'})}{\partial z}}_{\text{Turb Adv}} + \overline{\text{Microphys}}$$

$$\frac{\partial \bar{\theta}_l}{\partial t} = \underbrace{-\bar{w}\frac{\partial \bar{\theta}_l}{\partial z}}_{\text{Mean Adv}} \underbrace{-\frac{1}{\rho_s}\frac{\partial(\rho_s \overline{w' \theta_l'})}{\partial z}}_{\text{Turb Adv}} + \overline{\text{Radiation}} + \overline{\text{Microphys}}$$

$$\frac{\partial \bar{u}}{\partial t} = \underbrace{-\bar{w}\frac{\partial \bar{u}}{\partial z}}_{\text{Mean Adv}} \underbrace{-f(v_g - \bar{v})}_{\text{Coriolis/Pressure}} \underbrace{-\frac{1}{\rho_s}\frac{\partial(\rho_s \overline{u' w'})}{\partial z}}_{\text{Turb Adv}}$$

$$\frac{\partial \bar{v}}{\partial t} = \underbrace{-\bar{w}\frac{\partial \bar{v}}{\partial z}}_{\text{Mean Adv}} \underbrace{+f(u_g - \bar{u})}_{\text{Coriolis/Pressure}} \underbrace{-\frac{1}{\rho_s}\frac{\partial(\rho_s \overline{v' w'}}{\partial z})}_{\text{Turb Adv}}$$

Orange = calculated via PDF

Blue = needed by host model

Why are we interested in cloud parameterization?

"Clouds provide the largest source of uncertainty in current model predictions of climate sensitivity." --- Soden and Held (2006)

What type of cloud is parameterized with the most uncertainty? Low clouds

"Marine boundary layer clouds are at the heart of tropical cloud feedback uncertainties in climate models." --- Bony and Dufresne (2005)

Low clouds "are responsible for 59% of the contribution of inter-model differences in the net cloud feedback". --- Webb et al. (2006)

A non-PDF methodology of parameterization: The mass-flux scheme

- Mass-flux schemes directly model cloudy plumes using moist analogs to the equations for dry laboratory plumes.
- In particular, mass-flux schemes estimate entrainment into a plume and then use conservation equations to predict moisture, heat content, and mass flux.
- The plume is usually assumed to be embedded within a uniform environment.

Another non-PDF methodology of parameterization: The eddy-diffusivitity scheme

- Eddy-diffusivity schemes transport scalars down the gradient of the mean using an eddy diffusivity, *K*.
- E.g., the vertical flux of total water is modeled as:

$$\overline{w'r_t'} = -K\frac{d\overline{r_t}}{dz}$$

- Eddy-diffusivity schemes cannot model upgradient fluxes without special treatment

PDF-based parameterizations are, in part, attempts to remove some of the assumptions inherent in mass-flux schemes

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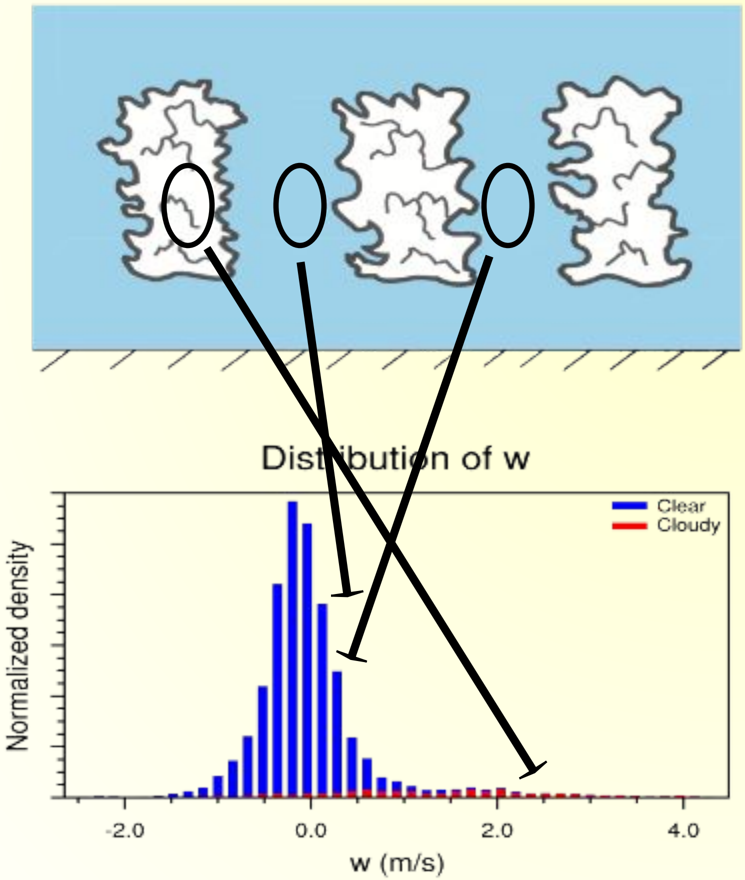
Our PDF-based parameterization

- We have constructed a 1D parameterization called "CLUBB" (Golaz et al. 2002b).
- It parameterizes layer clouds and turbulence in a unified way.
- Initially it was developed for boundary layers. It is being generalized to mid-levels.
- It is based on the Assumed PDF Method.

What is a Probability Density Function (PDF)?

A PDF is a histogram:



Our PDFs represent variability within one grid box at a single timestep.

Caveat: A PDF contains a tremendous amount of information, but none about the horizontal spatial arrangement of air parcels

Mathematically, what is a PDF?

- A PDF is non-negative over the entire domain.
- A PDF is normalized. That is, it integrates to unity.

Example PDFs derived from large-eddy simulations

See slides from Chris Golaz’s Kyoto talk.

Total water PDFs, cloud variability, and cloud fraction are closely related

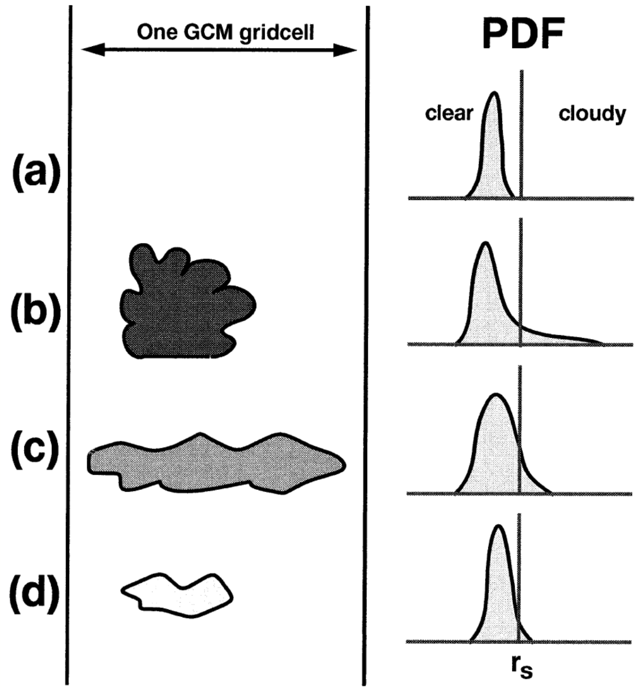


FIG. 6. Schematic of how cloud development is represented in new scheme. (left) An idealized state of the model grid cell; shading indicates cloud thickness. (right) The associated PDF of r_t represented in the scheme, with the saturated value r_s marked as the vertical line.

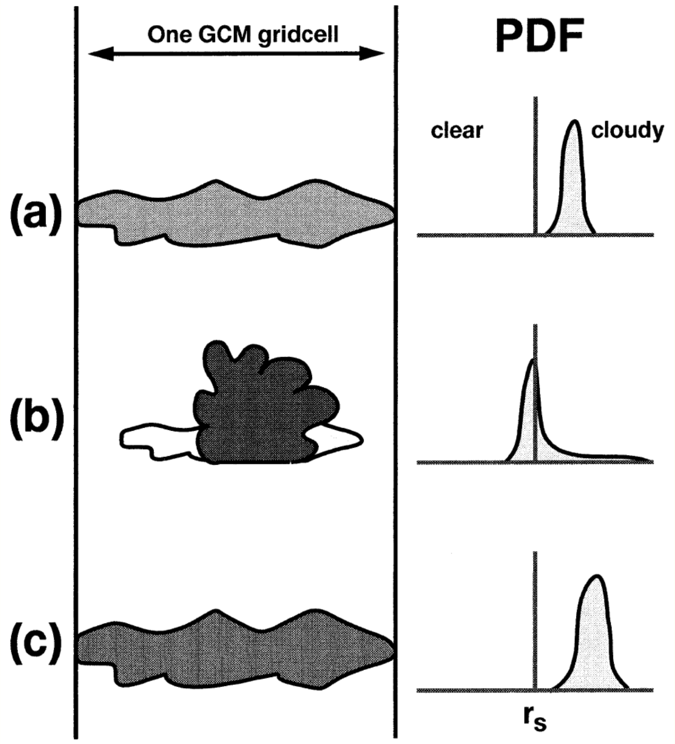


FIG. 7. As for Fig. 6 but for the case where $\overline{r}_t > r_s$.

From Tompkins (2002)

Many quantities can be computed by integrating over the PDF

E.g., cloud fraction is the area under the PDF that is supersaturated. If temperature is constant, then

$$cf = \int H(r_t - r_{sat}) P(r_t) \, dr_t$$

Here, *cf* is cloud fraction, *H* is the Heaviside step function, r_t is total water mixing ratio, and r_{sat} is saturation mixing ratio.

Many quantities can be computed by integrating over the PDF

If temperature is constant, then grid-mean liquid water can be calculated as

$$\overline{r_c} = \int (r_t - r_{\mathrm{sat}}) H(r_t - r_{\mathrm{sat}}) P(r_t)\, dr_t$$

Here, *rc* is liquid water, *H* is the Heaviside step function, r_t is total water mixing ratio, and r_{sat} is saturation mixing ratio.

Some quantities that need to be computed depend on more than one variable

E.g., < w’ theta_v’ > is needed for the TKE equation.

Our PDF includes several variables

We use a three-dimensional PDF of vertical velocity, w, total water (vapor + liquid) mixing ratio, r_t, and liquid water potential temperature, θ_l :

$$P = P(w, r_t, \theta_l)$$

This allows us to couple subgrid interactions of vertical motions and buoyancy.

Randall et al. (1992)

The more moments we predict, the more we know about the PDF

If we know an infinite number of moments, then the PDF is determined completely.

The Assumed PDF Method

Unfortunately, predicting the PDF shape directly is too expensive.

Instead we use the Assumed PDF Method. We *assume* a *functional form* of PDFs. E.g., we could choose a single Gaussian (normal) PDF. (In fact, CLUBB assumes a different PDF shape.)

Then we select a *particular* PDF from within this functional form for each grid box and time step. Therefore, the PDF varies in space and evolves in time.

E.g., Manton and Cotton (1977)

Steps in the Assumed PDF Method

The Assumed PDF Method contains 3 main steps that must be carried out for each grid box and time step:

(1) Prognose means and various higher-order moments.

(2) Use these moments to select a particular PDF member from the assumed functional form.

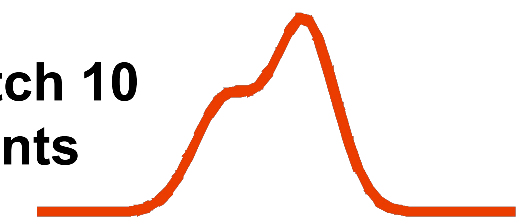
(3) Use the selected PDF to compute many higher-order terms that need to be closed, e.g. buoyancy flux, cloud fraction, etc.

Schematic of the Assumed PDF Method

Advance 10 prognostic equations

$$\overline{w},\ \overline{\theta_l},\ \overline{q_t},\ \overline{w'^2},\ \overline{w'^3},\ \overline{q_t'^2},\ \overline{\theta_l'^2},\ \overline{q_t'\theta_l'},\ \overline{w'q_t'},\ \overline{w'\theta_l'}$$

Select PDF from given family to match 10 moments



$$\Delta t$$

Use PDF to close higher-order moments, buoyancy terms

Diagnose cloud fraction, liquid water from PDF

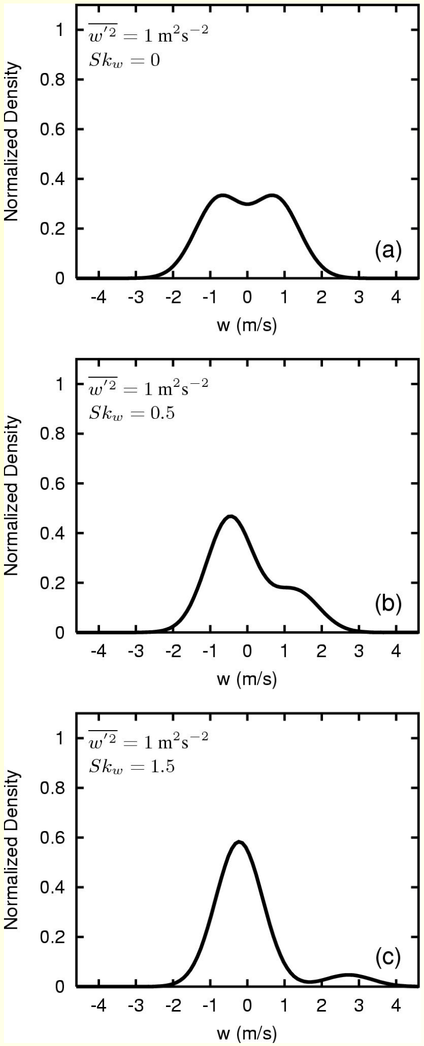
Golaz et al. (2002a)

We use a Normal Mixture PDF Functional Form

A two-component normal mixture PDF is the sum of two Gaussians. It satisfies *three important properties*:

(1) It allows both negative and positive skewness.
(2) It has reasonable-looking tails.
(3) It can be multi-variate.

We do not use a completely general double Gaussian, but instead restrict the family in order to simplify and reduce the number of parameters.



What are the higher-order moment equations?

The complete set of prognosed moments is:

$$\mathrm{Means:} \quad \frac{\partial \overline{u}}{\partial t} = \ldots \quad \frac{\partial \overline{v}}{\partial t} = \ldots \quad \frac{\partial \overline{r_t}}{\partial t} = \ldots \quad \frac{\partial \overline{\theta_l}}{\partial t} = \ldots$$

$$\mathrm{2nd-order:} \quad \frac{\partial \overline{w' r_t'}}{\partial t} = \ldots \quad \frac{\partial \overline{w' \theta_l'}}{\partial t} = \ldots \quad \frac{\partial \overline{w'^2}}{\partial t} = \ldots$$

$$\frac{\partial \overline{r_t'^2}}{\partial t} = \ldots \quad \frac{\partial \overline{\theta_l'^2}}{\partial t} = \ldots \quad \frac{\partial \overline{r_t' \theta_l'}}{\partial t} = \ldots$$

$$\mathrm{3rd-order:} \quad \frac{\partial \overline{w'^3}}{\partial t} = \ldots$$

$w =$ vertical velocity $\qquad r_t =$ total water mixing ratio

$\theta_l =$ liquid water potential temperature

Golaz et al. (2002)

PDF parameterization is based on a different way of thinking

- Other parameterization methods: More phenomenological.
  - E.g., they directly model visible structures such as cloudy plumes.
- PDF parameterization: More analytical.
  - It represents terms in the higher-order moment equations, such as turbulent fluxes.

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What are the advantages of the assumed PDF method over other types of cloud parameterization, e.g. mass-flux or eddy-diffusivity schemes?

Parameterization problem: Ad hoc formulation

Oftentimes, parameterizations are based on conceptual models, e.g. plumes with no subgrid variability, that are only obliquely connected to accepted, analytical theory. Furthermore, the assumptions of these models are often difficult to test using observations or even LES.

The PDF approach: Based on an accepted theoretical foundation

To predict the moments, we use *higher-order equations* that are derived directly from the Navier-Stokes and advection-diffusion equations. These equations are where much of the physics in the Assumed PDF Method resides.

Parameterization problem: Inconsistency

Parameterizations often predict cloud fraction and liquid water using separate formulas, with no guarantee of consistency between them. For instance, such parameterizations may predict "empty clouds", which have non-zero cloud fraction but zero liquid water or ice.

Bretherton (2007)

The PDF approach: Inconsistency

Using a single, joint (3D) PDF allows us to find many closures (e.g. for cloud fraction, liquid water, and buoyancy flux) that are consistent with one another.

Lappen and Randall (2001)

Parameterization problem: Systematic biases

Feeding grid box means into certain microphysical formulas, e.g. autoconversion rate, can lead to systematic biases, because the formulas are non-linear, and in particular, convex.

Larson et al. (2001); Siebesma et al. (2009)

The PDF approach: Systematic biases

Integrating the microphysical formula over the PDF yields an unbiased grid box average.

Larson et al. (2001); Siebesma et al. (2009)

Parameterization problem: Artificial separation of physical processes

Typically, in large-scale models, microphysics and radiation interact directly with the resolved fields of the model, but not directly with each other on the subgrid scales. Instead, Arakawa (2004) argues that we need a parameterization that acts as a “physics coupler.”

(e.g. Arakawa 2004)

The PDF approach: Artificial separation of physical processes

The PDF itself is a physics coupler such as the one that Arakawa (2004) recommends. In CLUBB, the variability embodied in the PDF drives the various physical processes, and the physical processes, in turn, modify the PDF. Since the PDF contains information on subgrid variability, the physical processes interact at the subgrid scale.

Parameterization problem: Artificial separation of cloud types

Although cumulus and stratocumulus clouds are inextricably interconnected, these two cloud types are often treated using distinct parameterizations. Then the consistent interfacing of these two parameterizations with each other (and with the deep convective parameterization) becomes problematic.

(e.g. Arakawa 2004)

The PDF approach: Artificial separation of cloud types

Using “separate schemes for separate regimes” does not reduce overall complexity but instead transfers it to the interactions between schemes (e.g. Bretherton 2007). Instead, CLUBB has one scheme, which contains a single joint PDF that is general enough to model both cumulus and stratocumulus clouds. Therefore no interface between schemes is needed.

Parameterization problem: Lack of memory

Many extant convective parameterizations respond instantly to instability (e.g. CAPE). That is, the parameterizations are diagnostic, rather than prognostic. In nature, however, deep convection evolves gradually from shallow convection. Lack of memory of whether shallow convection has occurred is part of the reason that deep convection onsets prematurely in parameterizations.

(e.g. Arakawa 2004; Gerard and Geleyn 2005)

The PDF approach: Lack of memory

CLUBB is fully prognostic, except for the turbulent length scale and the within-cloud saturation adjustment. All higher-order moment equations contain a time-tendency term. Therefore, CLUBB need not respond instantaneously to changes in CAPE. This permits gradual onset of deep convection.

Parameterization problem: Inappropriate formulation in the terra incognita

At grid spacings between 1 and 10 km, convection is partly resolved and partly subgrid scale. Therefore, these grid spacings are terra incognita for parameterizations. Many parameterizations assume that cloudy updrafts occupy a small fraction of a grid column, but this assumption is no longer true at finer grid spacings.

(Wyngaard 2004; Gerard 2007)

The PDF approach: Inappropriate formulation in the terra incognita

Because CLUBB does not employ a mass-flux scheme, it needs to make no assumption about the fraction of a grid box that is occupied by updrafts or environmental air. From a PDF point of view, the key difference between small and large grid boxes is that in nature, smaller volumes encompass less variance of cloud and turbulence. This can taken into account by making CLUBB's turbulent length scale a function of grid box size.

Parameterization problem: Inconsistent results as the horizontal grid spacing is refined

Ideally, cloud parameterizations would produce similar results when used at different grid spacing. As the grid spacing is progressively refined, the parameterization should progressively contribute less and ultimately reduce to a large-eddy simulation (LES) closure. Insensitivity to resolution is needed especially for simulations with nested grids (Warner and Hsu 2000). In practice, however, cloud parameterizations often work well only over a narrow range of grid spacings because their formulas do not properly take into account the grid spacing.

(e.g. Arakawa 2004)

The PDF approach: Inconsistent results as the horizontal grid spacing is refined

Making the length scale a function of the grid spacing is a simple way of progressively reducing CLUBB's fluxes as resolution increases. As the grid spacing approaches the scale at which convection is resolved, CLUBB largely shuts down, and the model results converge to those of a high-resolution version of the model run without CLUBB.

Parameterization problem: Parameterizability at small horizontal grid spacing

Small grid boxes encompass few clouds. For these small ensembles, the cloud fields may not be related in a one-to-one manner to the grid box mean fields.

To represent this indeterminacy, some models use stochastic parameterizations.

(e.g. Siebesma et al. 2009)

The PDF approach: Parameterizability at small horizontal grid spacing

The PDF method retains a memory of all higher-order moments that it prognoses. Therefore, multiple sets of higher-order moments are allowed to be consistent with the same set of mean profiles. Which higher-order moments are predicted depend on prior time evolution. Therefore, no stochastic element is needed.

(e.g. Siebesma et al. 2009)

Parameterization problem: Accounting for interactions between adjacent grid columns

When grid columns are narrow (< 10 km), there may be significant horizontal interaction between neighboring grid columns. E.g. subgrid convective clouds may be advected downwind to a new grid column.

To represent this effect, some models advect only the mean fields, and re-diagnose the convection in the downwind grid box.

(e.g. Siebesma et al. 2009)

The PDF approach: Accounting for interactions between adjacent grid columns

The PDF method can advect higher-order moments horizontally. These higher-order moments preserve a great deal of information about, e.g. subgrid convection. Therefore, subgrid convection is directly advected downwind, rather than being re-created in the downwind grid box.

(e.g. Siebesma et al. 2009)

What are the *disadvantages* of the assumed PDF method?

Disadvantage 1 of the PDF approach: Complexity

The PDF method does avoid the complexity of interfacing different schemes. However, a PDF parameterization is more complex than a code such as a mass-flux scheme, considered individually.

The problem can be ameliorated by good software engineering.

Disadvantage 2 of the PDF approach: Computational cost

The PDF method needs to prognose many higher-order moments, and is therefore more expensive than other methods. However, it is not as expensive as sophisticated parameterizations of atmospheric chemistry.

Optimization of the code can reduce the runtime of PDF parameterizations to some extent.

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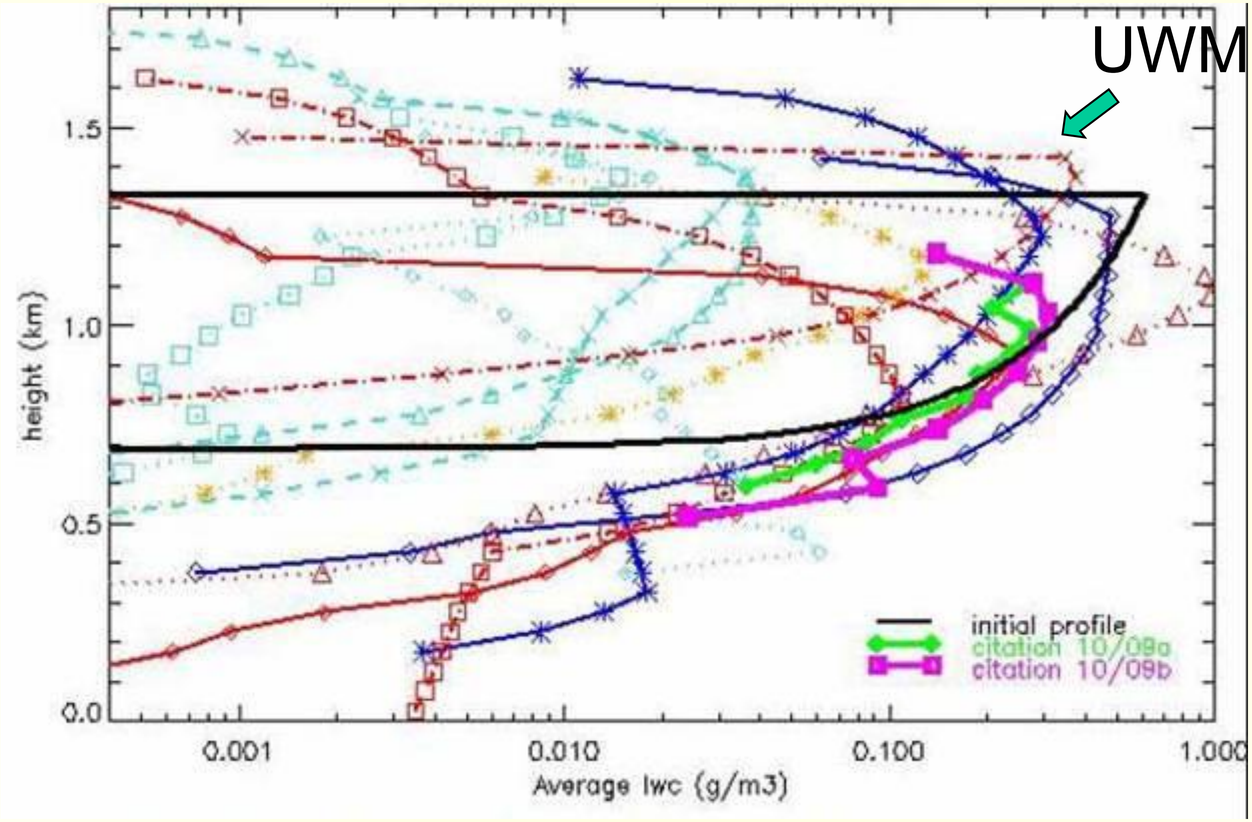
At present, CLUBB successfully simulates 20 benchmark cases in single-column mode

The cases span a wide range of meteorological conditions, including boundary layers, midlevel layers, and a deep convective case.

Results: We have modeled a mixed-phase Arctic stratus cloud observed during MPACE (Case B)

Liquid Water Content of SCMs



Intercomparison led by Steve Klein

Results: Deep convection during the LBA case

As another test, we have simulated the LBA case.

It occurred over the Amazon.

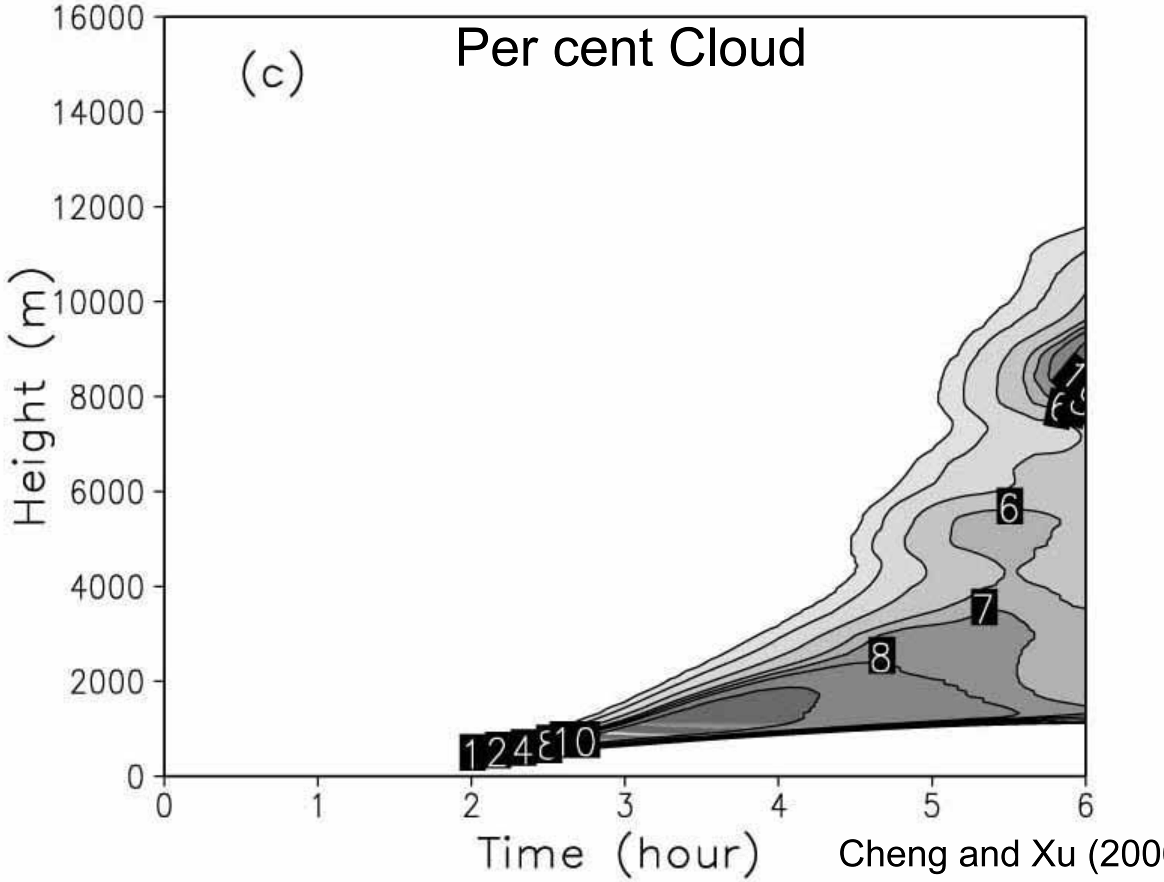
It exhibited shallow cumulus just after sunrise followed by deep convection in the early afternoon.

Results: Deep convection during LBA

As a benchmark, we use a (large) 3D LES of LBA performed by Marat Khairoutdinov.

Then we ran a cloud-resolving model (SAM) with and without our cloud parameterization implemented in it.

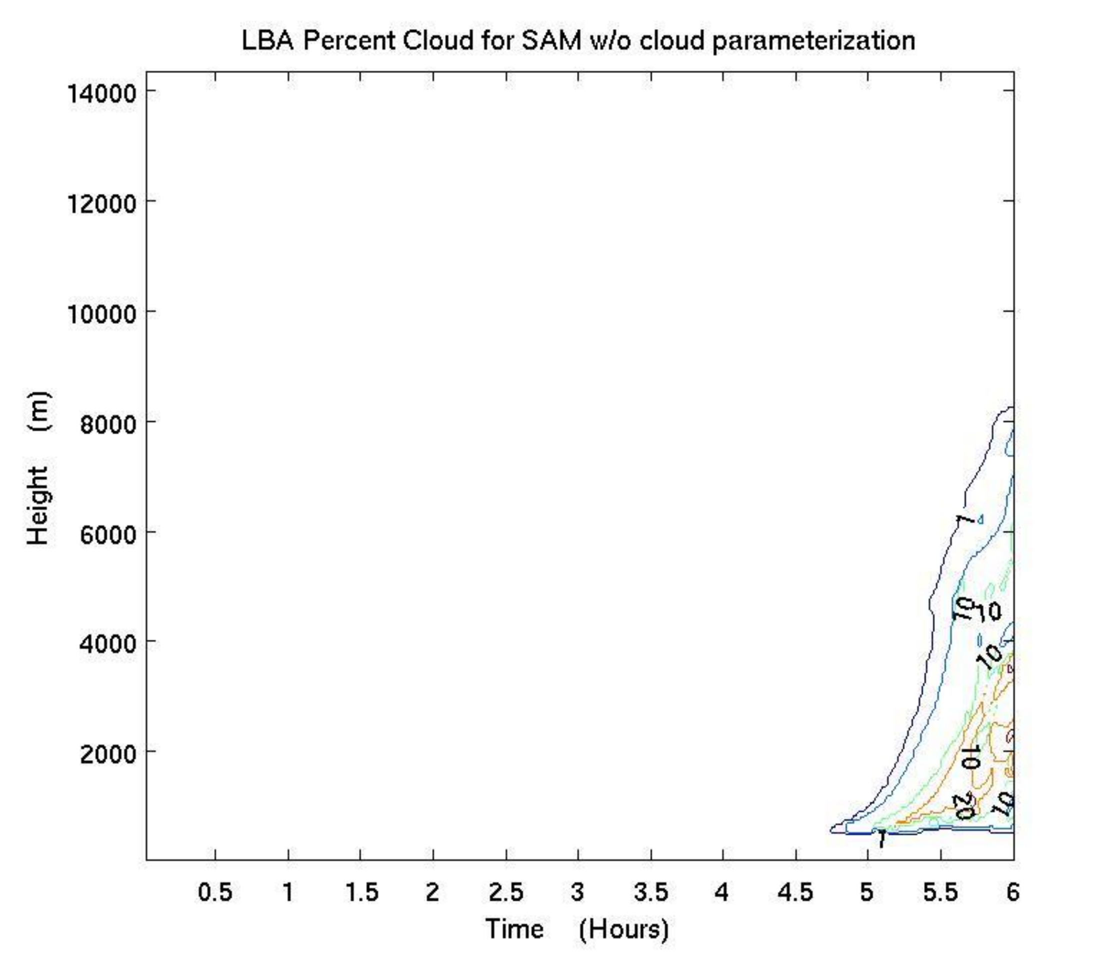
Results: Benchmark LES of LBA



Cheng and Xu (2006)

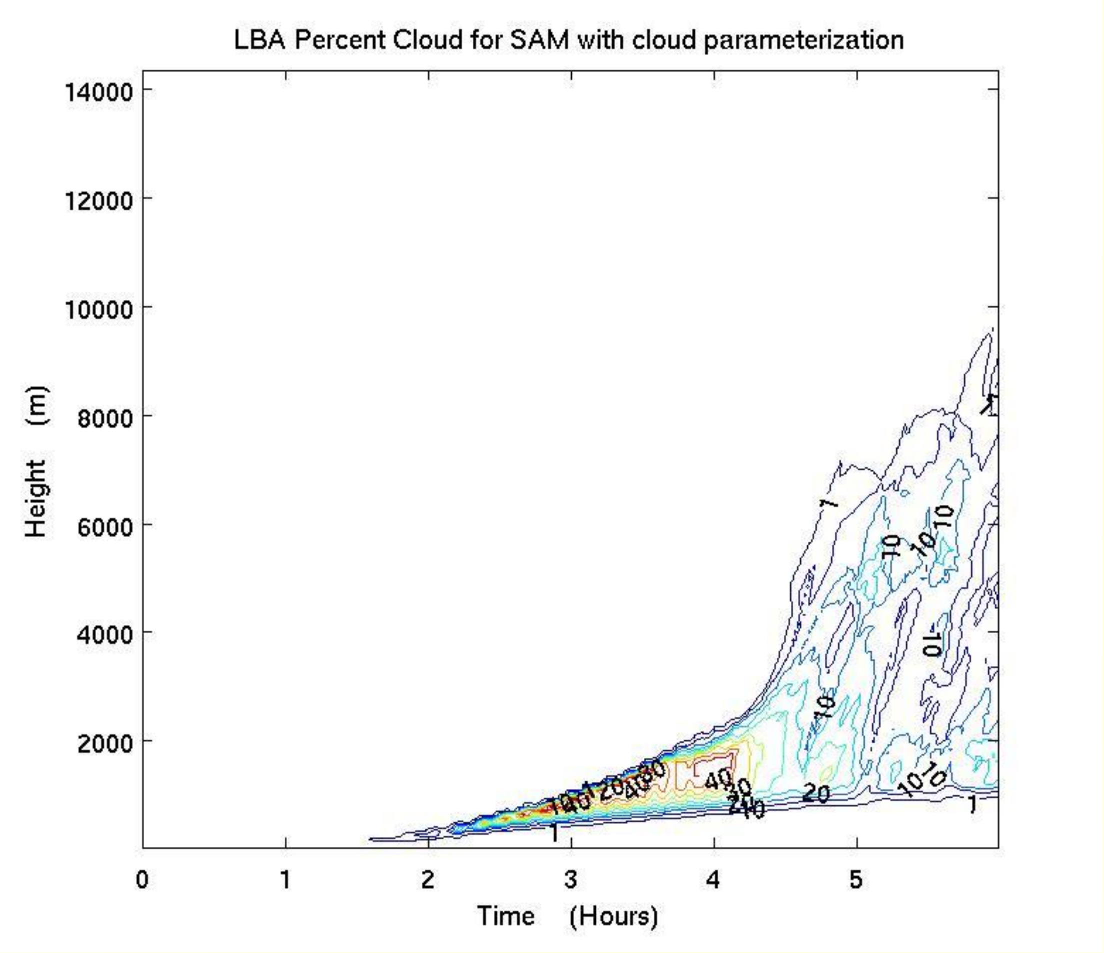
This is a 3D LES of LBA using 100-m horizontal grid spacing and a 154 km x 154 km horizontal domain. It serves as "truth".

Results: Coarse simulation of LBA *without* any cloud parameterization



We run SAM in 2D with 4-km horizontal grid spacing and 32 grid boxes in the horizontal. The early shallow convection is missed.

Results: Coarse simulation of LBA *with* our cloud parameterization



If we run SAM at the same horizontal resolution but implement our cloud parameterization, it restores the low clouds.

Conclusions

1. The assumed PDF method is quite different than other approaches to cloud parameterization.

2. The PDF method can be used to develop a single, unified parameterization that is designed to model all types of clouds and turbulence.

Future vision

The PDF Method is a possible way to handle many aspects of the problem of subgrid variability.

It involves representing the variability by a single multivariate PDF and integrating over it, possibly by Monte Carlo methods.

The Assumed PDF method has possible applications to:

- Radiative transfer, Microphysics, Chemistry
- Boundary layers, Deep convection
- Cloud-resolving, Weather forecast, and General circulation models

A PDF approach could unify the following separate parameterizations in a GCM, without case-specific functions or triggers:

- Boundary layer turbulence
- Stratiform cloud
- Cloud overlap
- Deep convection