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November 2, 2022

**Problem Set 2**
**Math modeling of the atmosphere**

## Problem 1: Cloud fraction for a "double delta function" PDF

Consider a volume of air that is half occupied by air with $r_t = r_{t1}$ and half occupied by air with $r_t = r_{t2}$. (Let $r_{t1} > r_{t2}$, and let $\Delta r_t \equiv r_{t1} - r_{t2}$.) Further suppose that the volume has uniform temperature everywhere and that the saturation mixing ratio is $r_s$. Assume that any vapor in excess of saturation is immediately converted to liquid, as we assumed earlier in the semester for non-precipitating, ice-free clouds.

(i) Write an expression for cloud fraction, $C$, in terms of $r_{t1}$, $r_{t2}$, $r_s$, and the Heaviside step function.

(ii) Write an expression for average liquid water mixing ratio, $\langle r_l \rangle$, in terms of $r_{t1}$, $r_{t2}$, $r_s$, and the Heaviside step function.

## Problem 2: Cloud fraction for a triangular PDF

Consider now a volume of air that has a symmetric PDF of $r_t$, $P(r_t)$, with the shape of an isosceles triangle. The PDF is a piecewise linear function of $r_t$ that becomes non-zero at $r_t = \overline{r_t} - \Delta r_t$, increases linearly to a peak at $r_t = \overline{r_t}$, and then decreases linearly to zero at $r_t = \overline{r_t} + \Delta r_t$. Assume that temperature is constant and that the saturation mixing ratio is $r_s$. You don't need to write in terms of Heaviside step functions if you don't wish to do so. Instead you can just leave the answer in terms of discrete cases.

(i) Write the functional form of $P(r_t)$ in terms of $\overline{r_t}$, $\Delta r_t$, and $r_t$. Recall that the area under $P(r_t)$ must be 1.

(ii) Find the cloud fraction, $C$, in terms of $\overline{r_t}$, $\Delta r_t$, and $r_s$.

## Problem 3: Sketch cloud fractions for the double delta and triangular PDFs

Overplot the two cloud fraction formulas that you have found in problems 1 and 2 as a function of $\overline{r_t}$ for fixed $r_s$ and $\Delta r_t$. You may sketch by hand, if you wish.