Multivariate Statistical Analysis

Final work

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```r
library(nnet)
library(caret)
library(kernlab)
```

Problem 1

Consider the `vehicle` dataset. The goal is to identify 3D objects from 2D images captured by cameras at different angles. The objects in this case are four types of vehicles (identified by the variables `class` and `classdigit`), and the other 18 numerical variables are measurements extracted from these 2D images.

```r
vehicles <- read.csv(file = "../Data_csv/vehicle.csv")
# remove the classdigit because we already have a class
# variable
vehicles <- subset(vehicles, select = -classdigit)
vehicles$class <- factor(vehicles$class)
```

Split the data into training and test sets (80/20).

```r
set.seed(42)
inTrain_vehicles <- createDataPartition(vehicles$class, p = 0.8)[[1]]
vehicles_train <- vehicles[inTrain_vehicles, ]
vehicles_test <- vehicles[-inTrain_vehicles, ]
```

On the training set compute:

- a multinomial logistic classifier

```r
vehicles_mlc <- glm(formula = class ~ ., data = vehicles_train,
    family = binomial)
summary(vehicles_mlc)
```

```
##
## Call:
## glm(formula = class ~ ., family = binomial, data = vehicles_train)
##
## Coefficients:
```

```
##                Estimate Std. Error z value Pr(>|z|)
## (Intercept)  -156.90757  106.67026  -1.471 0.141302
## Comp            0.09389    0.14342   0.655 0.512711
## Circ            0.23051    0.51339   0.449 0.653436
## Dcirc           0.34119    0.11270   3.027 0.002466 **
## RR              0.80038    0.20099   3.982 6.83e-05 ***
## PrAxisAR       -2.17440    0.51652  -4.210 2.56e-05 ***
## MaxLAR          0.28915    0.16095   1.797 0.072400 .
## ScatterR        0.53224    0.47430   1.122 0.261789
## Elong           2.77263    0.85024   3.261 0.001110 **
## PrAxisRect      2.91621    1.48835   1.959 0.050070 .
## MaxLRect       -0.14385    0.17987  -0.800 0.423868
## SvarMajAxis    -0.18119    0.12288  -1.475 0.140345
## SvarMinAxis    -0.12203    0.06569  -1.858 0.063214 .
## SradGyration   -0.04661    0.05478  -0.851 0.394857
## SkewMajAxis     0.45840    0.19954   2.297 0.021602 *
## SkewMinAxis     0.38884    0.13223   2.941 0.003275 **
## KurtMinAxis    -0.18557    0.07176  -2.586 0.009705 **
## KurtMajAxis    -1.94070    0.55589  -3.491 0.000481 ***
## Hratio          1.52788    0.44871   3.405 0.000661 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 521.11  on 451  degrees of freedom
## Residual deviance:  61.03  on 433  degrees of freedom
## AIC: 99.03
##
## Number of Fisher Scoring iterations: 10
```

- a single-hidden-layer neural network classifier (with the number of hidden nodes to be determined)

```
vehicles_num_hidden_nodes <- 20
vehicles_nnets <- vector(mode = "list", length = vehicles_num_hidden_nodes)
for (i in 1:vehicles_num_hidden_nodes) {
    vehicles_nnets[[i]] <- nnet(class ~ . - class, data = vehicles_train,
        size = i, trace = F)
}
```

On the test data, find the cross-classification tables and the misclassification rates.

```
vehicles_mlc_pi1_hat <- predict(vehicles_mlc, vehicles_test,
    type = "response")
vehicles_mlc_gr_hat <- ifelse(vehicles_mlc_pi1_hat > 0.5, 2,
    1)
vehicles_mlc_mctable <- table(vehicles_mlc_gr_hat, vehicles_test$class)
vehicles_mlc_mctable
```

```
##
## vehicles_mlc_gr_hat bus opel saab van
##                   1  28    3    0   2
##                   2   1   25   27  26
```

```
1 - sum(diag(vehicles_mlc_mctable))/length(vehicles_test$class)
```

```
## [1] 0.5267857
```

```
vehicles_nnets_cms <- vector(mode = "list", length = vehicles_num_hidden_nodes)
vehicles_nnets_mscrs <- list()
for (i in 1:vehicles_num_hidden_nodes) {
    vehicles_nnets_cms[[i]] <- confusionMatrix(factor(predict(vehicles_nnets[[i]],
        newdata = vehicles_test, type = "class"), levels = levels(vehicles_test$class)),
        vehicles_test$class)
    vehicles_nnets_mscrs[i] <- 1 - vehicles_nnets_cms[[i]]$overall["Accuracy"]
}
```

```
for (i in 1:vehicles_num_hidden_nodes) {
    cat("Misclassification rate for", i, "hidden nodes:", vehicles_nnets_mscrs[[i]],
        "\n")
}
```

```
## Misclassification rate for 1 hidden nodes: 0.7410714
## Misclassification rate for 2 hidden nodes: 0.7410714
## Misclassification rate for 3 hidden nodes: 0.5446429
## Misclassification rate for 4 hidden nodes: 0.7410714
## Misclassification rate for 5 hidden nodes: 0.7410714
## Misclassification rate for 6 hidden nodes: 0.3839286
## Misclassification rate for 7 hidden nodes: 0.7410714
## Misclassification rate for 8 hidden nodes: 0.7410714
## Misclassification rate for 9 hidden nodes: 0.4375
## Misclassification rate for 10 hidden nodes: 0.6607143
## Misclassification rate for 11 hidden nodes: 0.4910714
## Misclassification rate for 12 hidden nodes: 0.3928571
## Misclassification rate for 13 hidden nodes: 0.4375
## Misclassification rate for 14 hidden nodes: 0.7142857
## Misclassification rate for 15 hidden nodes: 0.7410714
## Misclassification rate for 16 hidden nodes: 0.7410714
## Misclassification rate for 17 hidden nodes: 0.5178571
## Misclassification rate for 18 hidden nodes: 0.4196429
## Misclassification rate for 19 hidden nodes: 0.5625
## Misclassification rate for 20 hidden nodes: 0.7410714
```

```
vehicles_nnets_min_mscrs <- which.min(vehicles_nnets_mscrs)
cat("The lowest msr is on the neural network with", vehicles_nnets_min_mscrs,
    "hidden nodes and a rate of", vehicles_nnets_mscrs[[vehicles_nnets_min_mscrs]],
    ".")
```

```
## The lowest msr is on the neural network with 6 hidden nodes and a rate of 0.3839286 .
```

```
vehicles_nnets_cms[[vehicles_nnets_min_mscrs]]
```

```
## Confusion Matrix and Statistics
##
##           Reference
## Prediction bus opel saab van
##       bus   28    2    0   5
##       opel   0   24   23   0
##       saab   0    1    2   8
##       van    1    1    2  15
##
## Overall Statistics
##
##                Accuracy : 0.6161
##                  95% CI : (0.5194, 0.7064)
##     No Information Rate : 0.2589
##     P-Value [Acc > NIR] : 1.987e-15
##
##                   Kappa : 0.4868
##
##  Mcnemar's Test P-Value : NA
##
## Statistics by Class:
##
##                      Class: bus Class: opel Class: saab Class: van
## Sensitivity              0.9655      0.8571     0.07407     0.5357
## Specificity              0.9157      0.7262     0.89412     0.9524
## Pos Pred Value           0.8000      0.5106     0.18182     0.7895
## Neg Pred Value           0.9870      0.9385     0.75248     0.8602
## Prevalence               0.2589      0.2500     0.24107     0.2500
## Detection Rate           0.2500      0.2143     0.01786     0.1339
## Detection Prevalence     0.3125      0.4196     0.09821     0.1696
## Balanced Accuracy        0.9406      0.7917     0.48410     0.7440
```

Which of the above methods is better? Is there any specific type of vehicle that is harder to classify than the others?

Problem 2

The `pendigits` dataset consists of discretized handwritten digits (for a full description, see Section 7.2.10 in the book). From this set, extract the subset corresponding to digits 0, 6, 8 and 9, and scale the variables so that the variances are 1.

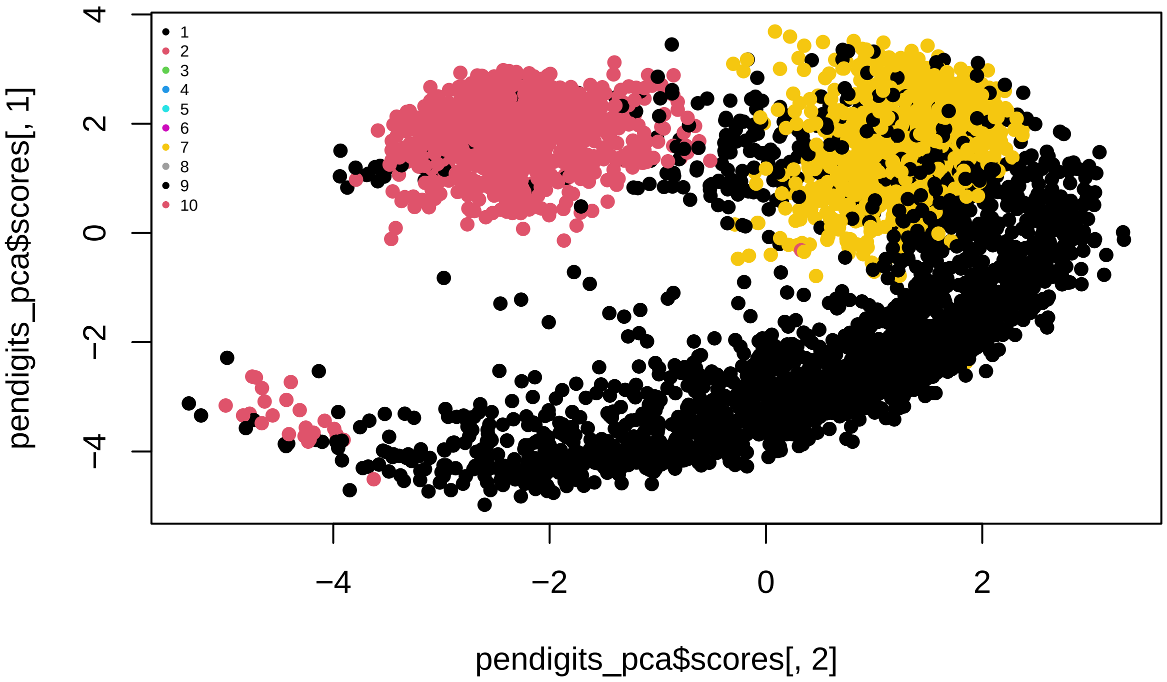
```
pendigits <- read.csv(file = "../Data_csv/pendigits.csv")
pendigits$digit <- factor(pendigits$digit)
pendigits <- pendigits[pendigits$digit %in% c(0, 6, 8, 9), ]
pendigits[, 2:17] <- scale(pendigits[, 2:17])
```

(a)

Compute ordinary principal components and draw a scatterplot of the first two component scores, using different colors (or symbols) for different digits. Are the digits well separated?

```
pendigits_pca <- princomp(~. - digit, data = pendigits)
plot(pendigits_pca$scores[, 1] ~ pendigits_pca$scores[, 2], col = pendigits$digit,
    pch = 16)
```

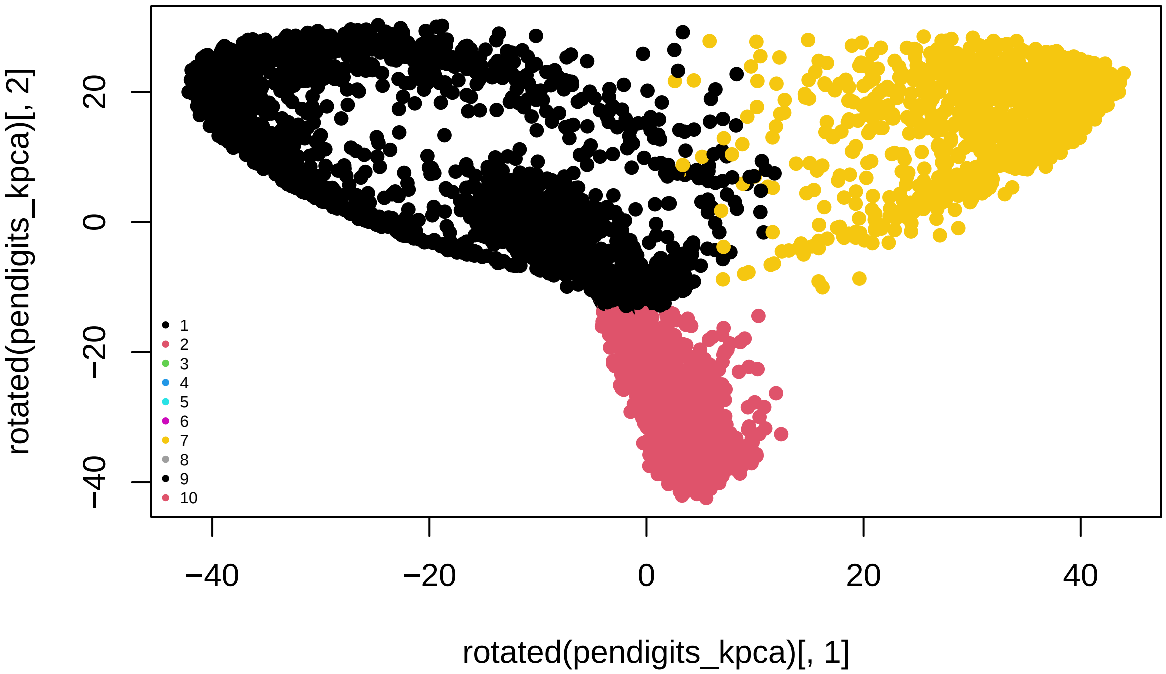
```
legend("topleft", col = 1:10, legend = paste(1:10), pch = 16,
    bty = "n", cex = 0.5)
```



(b)

Compute kernel principal components using Gaussian kernels with various scales, and draw scatterplots of the first two component scores as in (a).

```
pendigits_kpca <- kpca(~. - digit, data = pendigits, kernel = "rbfdot",
    kpar = list(sigma = 0.1), features = 2)
plot(rotated(pendigits_kpca)[, 1], rotated(pendigits_kpca)[,
    2], pch = 16, col = as.numeric(pendigits$digit))
legend("bottomleft", col = 1:10, legend = paste(1:10), pch = 16,
    bty = "n", cex = 0.5)
```



Are the digits now better separated than in (a)?