MthStat 568/768 – Multivariate Statistical Analysis – Spring 2024

Homework 1

Due Wednesday, February 7

1. Determine which of the following matrices admit a matrix square root, and for the ones that do, find it.

$$\text{(i) } \mathbf{A} = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix}, \quad \text{(ii) } \mathbf{A} = \begin{bmatrix} 3 & 2 & -1 \\ 2 & 3 & 1 \\ -1 & 1 & 2 \end{bmatrix}.$$

2. Let $\mathcal{H}$ be the subspace of $\mathbb{R}^3$ with basis $\{\mathbf{v}_1, \mathbf{v}_2\}$ given by

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ and } \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}.$$

   (a) Find the projecion matrix $\mathbf{P}$ associated with $\mathcal{H}$.

   (b) Find $\hat{\mathbf{x}}$, the best approximation to $\mathbf{x}$ in $\mathcal{H}$, for each of the following:

$$\text{(i) } \mathbf{x} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad \text{(ii) } \mathbf{x} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \quad \text{(iii) } \mathbf{x} = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}.$$

   Cases (ii) and (iii) are somewhat unusual; explain why you get the results you get in these cases.

3. Consider the matrix

$$\mathbf{B} = \begin{bmatrix} 2/3 & 1/3 & 1/3 \\ 1/3 & 2/3 & -1/3 \\ 1/3 & -1/3 & 2/3 \end{bmatrix}.$$

   Is $\mathbf{B}$ a projection matrix for some space $\mathcal{H}$? If so, find a basis for $\mathcal{H}$.