Time Series

Homework 2

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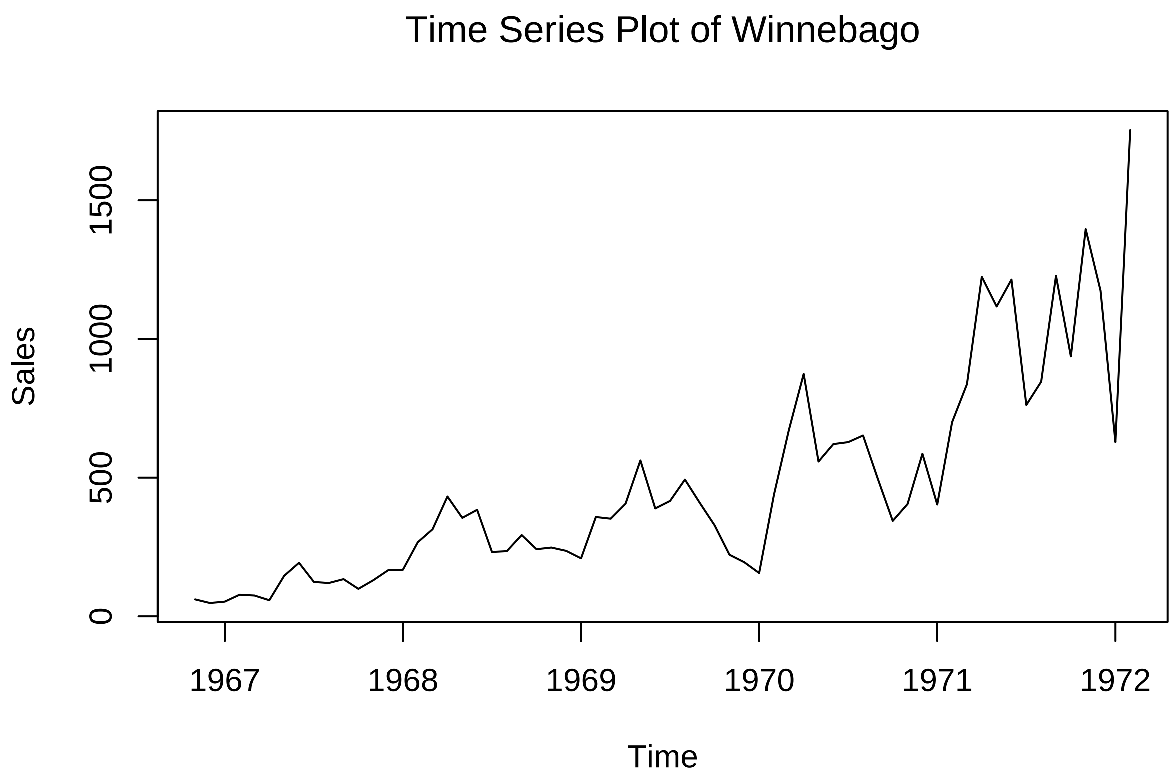
02/28/24

Problem 2

a)

```r
library(TSA)
library(tseries)
```

```r
data(winnebago)
plot(winnebago, xlab = expression("Time"), ylab = expression("Sales"),
    main = expression("Time Series Plot of Winnebago"))
```



Interpretation:

- The time series shows an overall upwards trend between the years 1967 and 1972.
- Between the years 1970 and 1972 the increase is at its highest.

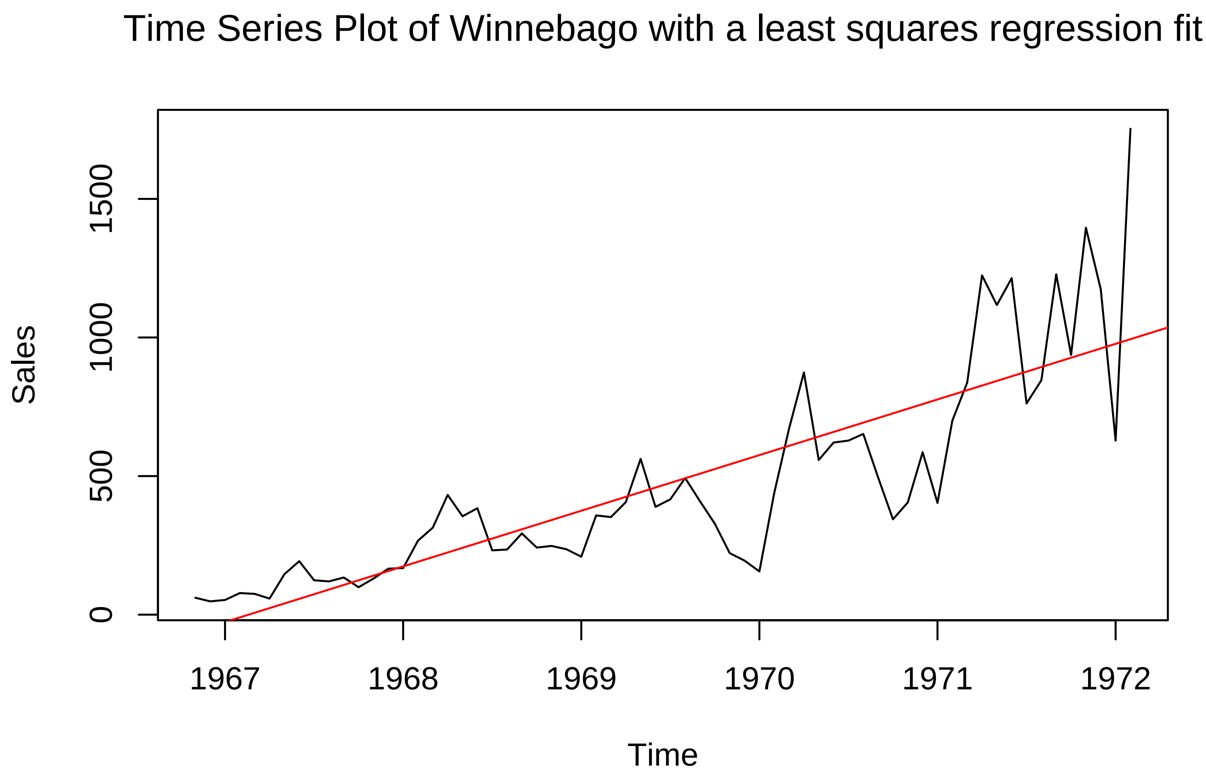
b)

```
model1 <- lm(winnebago ~ time(winnebago))
summary(model1)
```

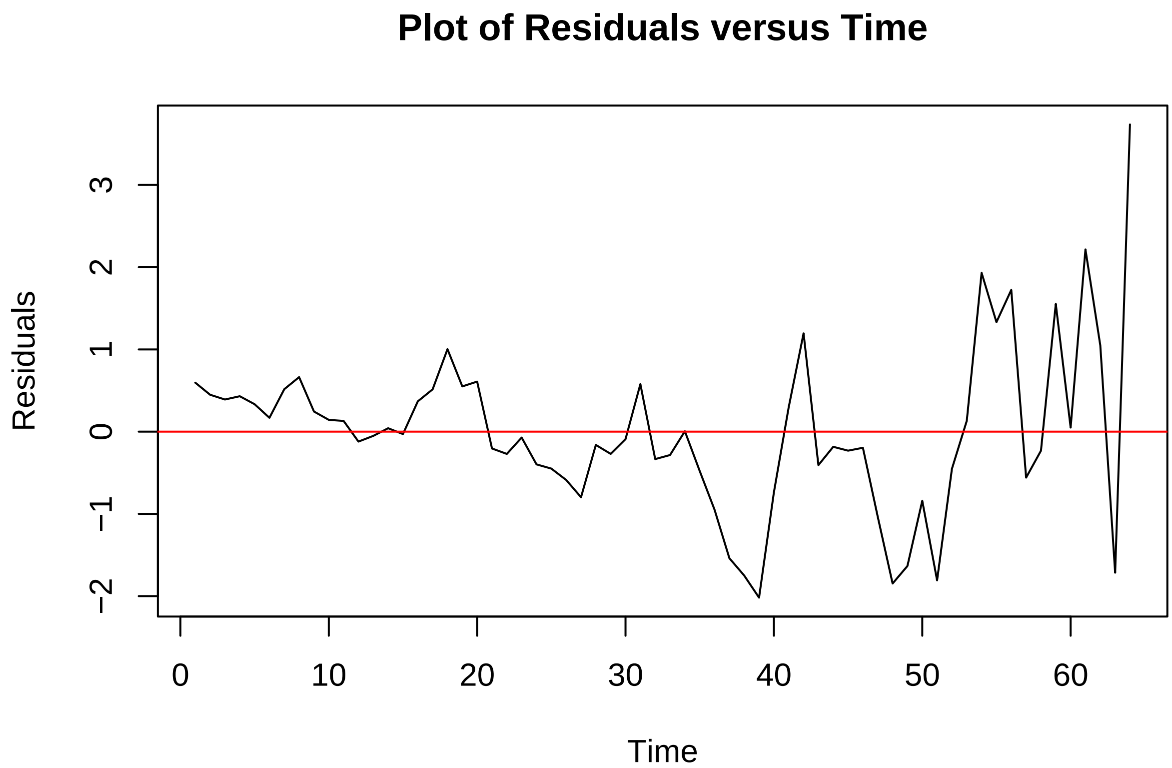
```
##
## Call:
## lm(formula = winnebago ~ time(winnebago))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -419.58  -93.13  -12.78   94.96  759.21
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)     -394885.68   33539.77  -11.77   <2e-16 ***
## time(winnebago)     200.74      17.03   11.79   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 209.7 on 62 degrees of freedom
## Multiple R-squared:  0.6915, Adjusted R-squared:  0.6865
## F-statistic: 138.9 on 1 and 62 DF,  p-value: < 2.2e-16
```

- We expect the wages to increase by \$200.74 per year
- 69.15% of the variance can be explained by the predictor variables
- the linear trend is significant because the p-value $2.2 \cdot 10^{-16}$ is smaller than 10^{-12}

```
plot(winnebago, xlab = expression("Time"), ylab = expression("Sales"),
    main = expression("Time Series Plot of Winnebago with a least squares regression fit"))
abline(model1, col = "red")
```




```r
res1 <- as.ts(rstandard(model1))
plot(res1, xlab = expression("Time"), ylab = expression("Residuals"),
    main = "Plot of Residuals versus Time")
abline(h = 0, col = "red")
```

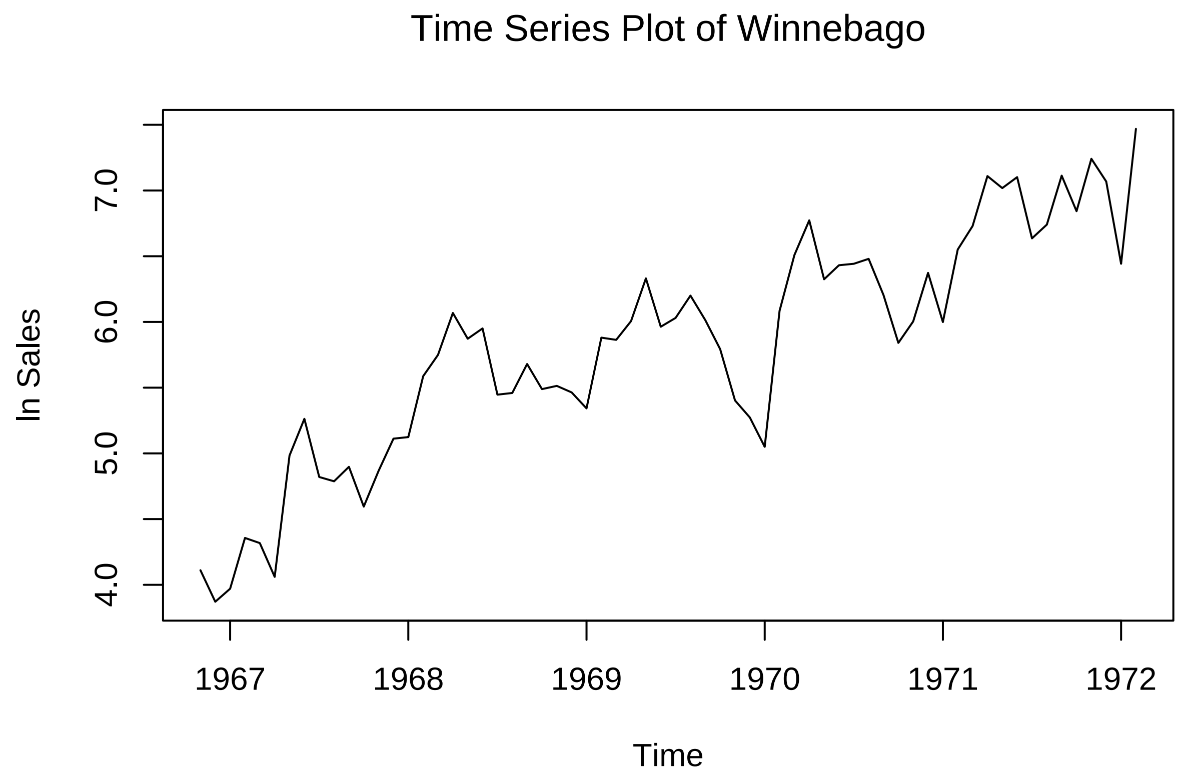


Interpretation:

- The residuals plot shows somewhat random movement around zero.
- More uneven spread between ca. 35 to 65 in comparison to 0 to 35.
- There may be a "seasonal" cyclical trend.

c)

```
ln_winnebago <- log(winnebago)
plot(ln_winnebago, xlab = expression("Time"), ylab = expression("ln Sales"),
    main = expression("Time Series Plot of Winnebago"))
```



Interpretation:

- The time series plot of the transformed values shows a linear upward trend (which corresponds with the untransformed data)
- There is one downward spike around the year 1970
- The "seasonal" trend seems more pronounced

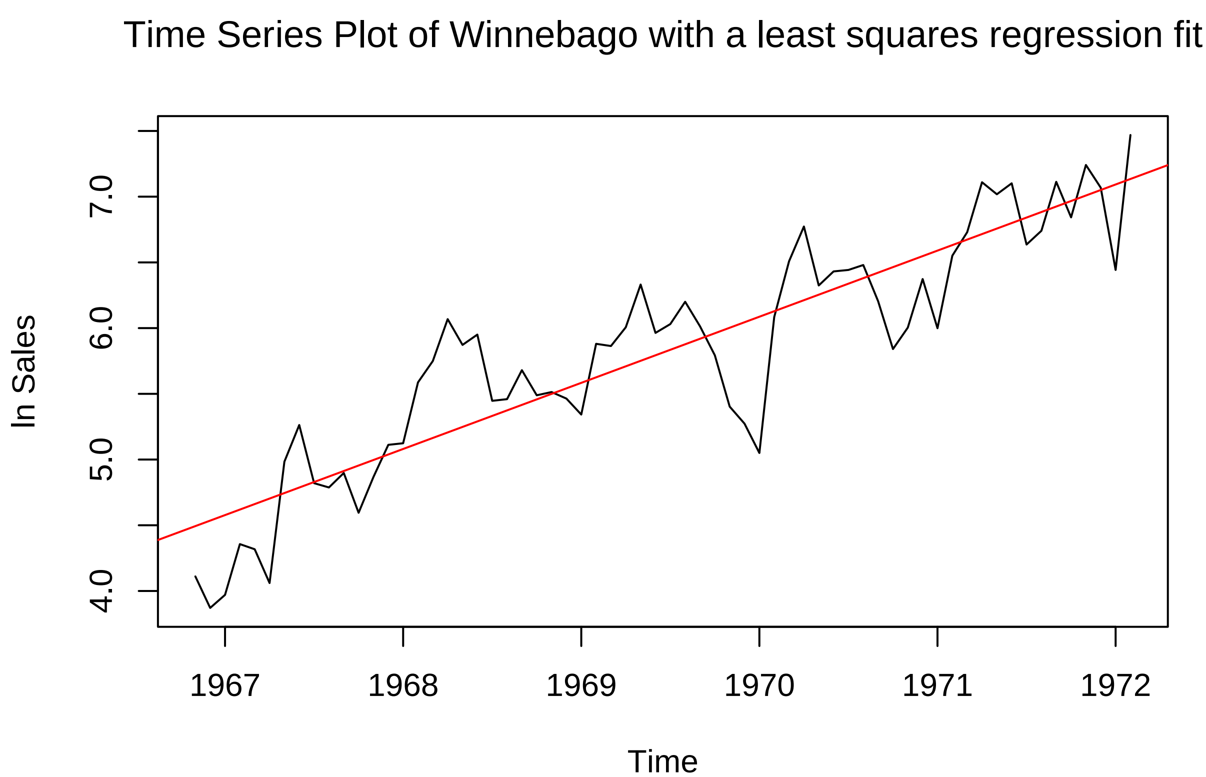
d)

```
model2 <- lm(ln_winnebago ~ time(ln_winnebago))
summary(model2)
```

```
##
## Call:
## lm(formula = ln_winnebago ~ time(ln_winnebago))
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.03669 -0.20823  0.04995  0.25662  0.86223
##
## Coefficients:
##                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)        -984.93878   62.99472  -15.63   <2e-16 ***
## time(ln_winnebago)    0.50306    0.03199   15.73   <2e-16 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3939 on 62 degrees of freedom
## Multiple R-squared:  0.7996, Adjusted R-squared:  0.7964
## F-statistic: 247.4 on 1 and 62 DF,  p-value: < 2.2e-16
```

```r
plot(ln_winnebago, xlab = expression("Time"), ylab = expression("ln Sales"),
    main = expression("Time Series Plot of Winnebago with a least squares regression fit"))
abline(model2, col = "red")
```




```r
res2 <- as.ts(rstandard(model2))
plot(res2, xlab = expression("Time"), ylab = expression("Residuals"),
    main = "ln Plot of Residuals versus Time")
abline(h = 0, col = "red")
```

Interpretation:

- the ln-transformed residuals plot shows random movement around 0
- There seems to be an overall cyclical trend

e)

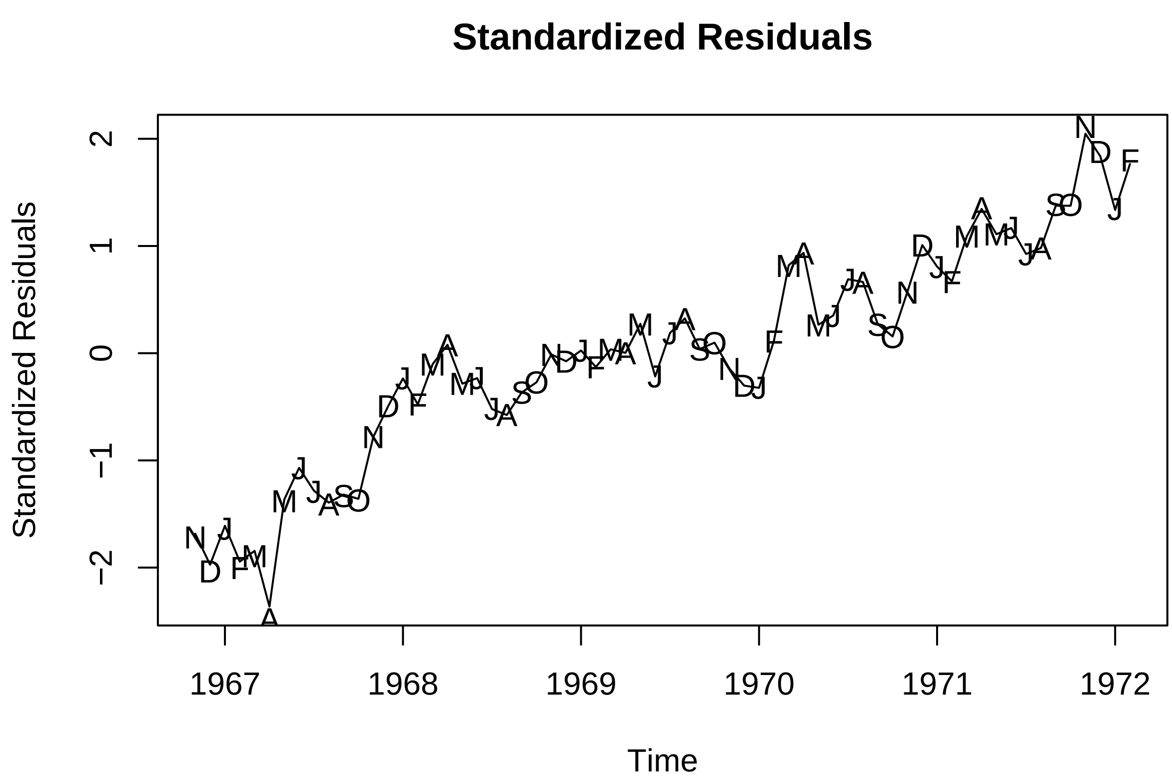
```
months <- season(winnebago)
model3 <- lm(ln_winnebago ~ months - 1)
summary(model3)
```

```
##
## Call:
## lm(formula = ln_winnebago ~ months - 1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.94319 -0.40444  0.02541  0.59421  1.71807
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## monthsJanuary     5.3213     0.3752   14.18   <2e-16 ***
## monthsFebruary    5.9882     0.3752   15.96   <2e-16 ***
## monthsMarch       5.8338     0.4111   14.19   <2e-16 ***
```

```
## monthsApril       6.0036     0.4111   14.61   <2e-16 ***
## monthsMay         6.1060     0.4111   14.85   <2e-16 ***
## monthsJune        6.1420     0.4111   14.94   <2e-16 ***
## monthsJuly        5.8752     0.4111   14.29   <2e-16 ***
## monthsAugust      5.9336     0.4111   14.44   <2e-16 ***
## monthsSeptember   5.9819     0.4111   14.55   <2e-16 ***
## monthsOctober     5.7121     0.4111   13.90   <2e-16 ***
## monthsNovember    5.5233     0.3752   14.72   <2e-16 ***
## monthsDecember    5.5269     0.3752   14.73   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9192 on 52 degrees of freedom
## Multiple R-squared:  0.9801, Adjusted R-squared:  0.9755
## F-statistic: 213.8 on 12 and 52 DF,  p-value: < 2.2e-16
```

- 98.01% of the variance can be explained by the predictor variables
- the linear trend is significant because the p-value $2.2 \cdot 10^{-16}$ is smaller than 10^{-12}

```
res3 <- as.ts(rstandard(model3))
plot(res3, x = as.vector(time(winnebago)), xlab = "Time", ylab = "Standardized Residuals",
    main = "Standardized Residuals", type = "l")
points(y = rstudent(model3), x = as.vector(time(winnebago)),
    pch = as.vector(season(winnebago)))
```

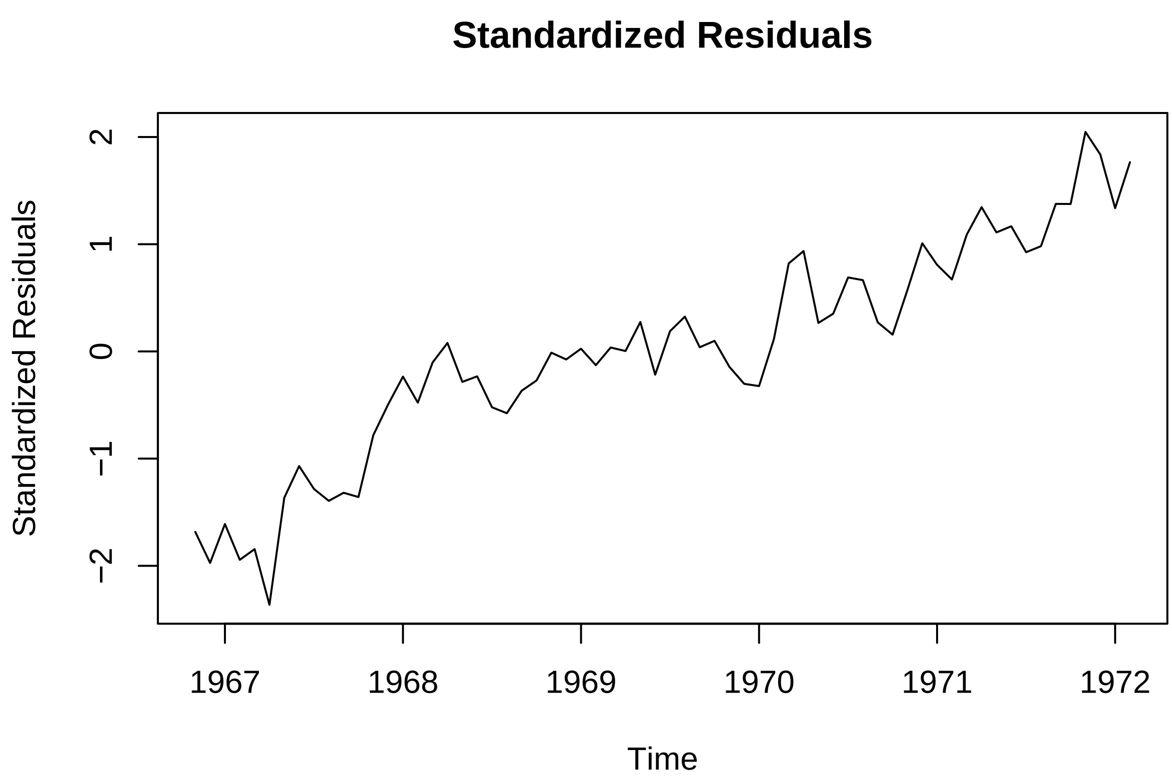


f)

- looking at the residuals, we can revert our assumption of a seasonal trend
- we do not see an obvious pattern of the months at the highs and lows

g)

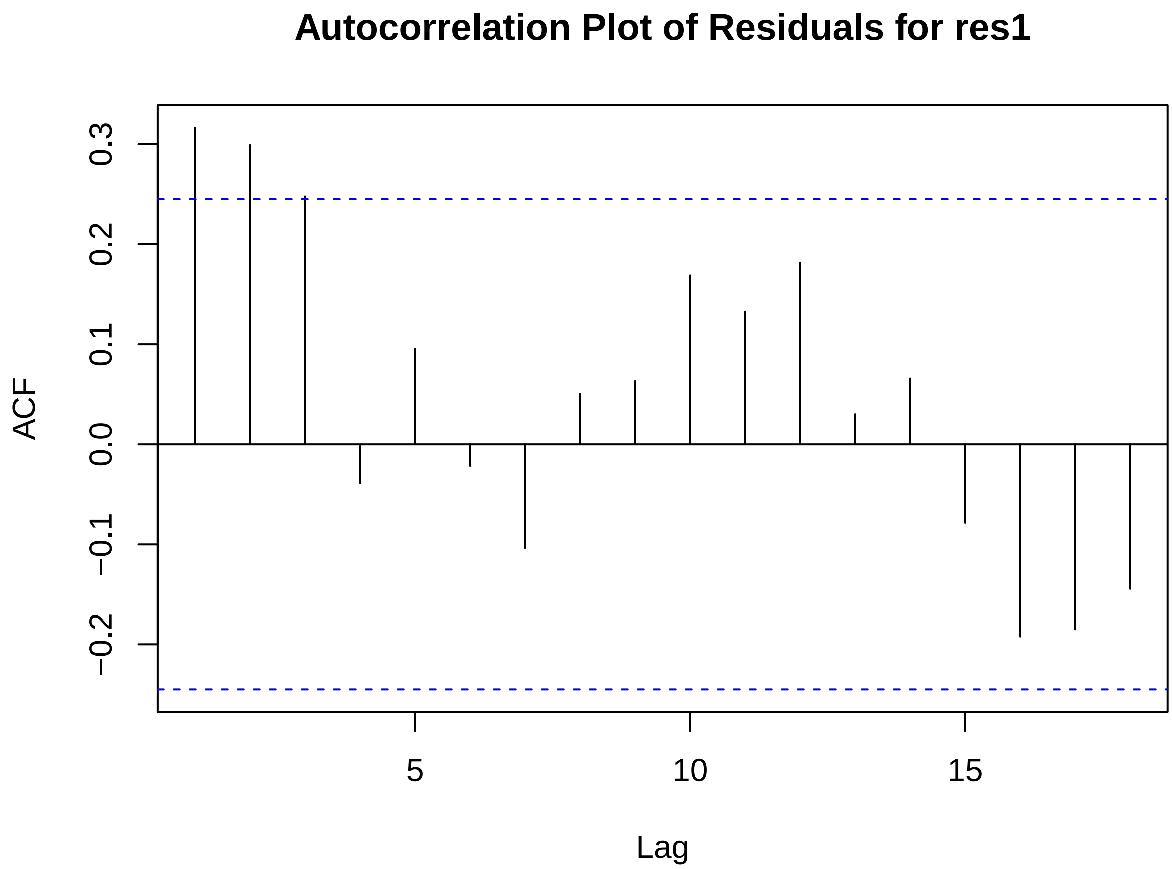
```
plot(res3, x = as.vector(time(winnebago)), xlab = "Time", ylab = "Standardized Residuals",
    main = "Standardized Residuals", type = "l")
```



- We plotted the exact same graph as in e) because we did not see any difference in the exercise.

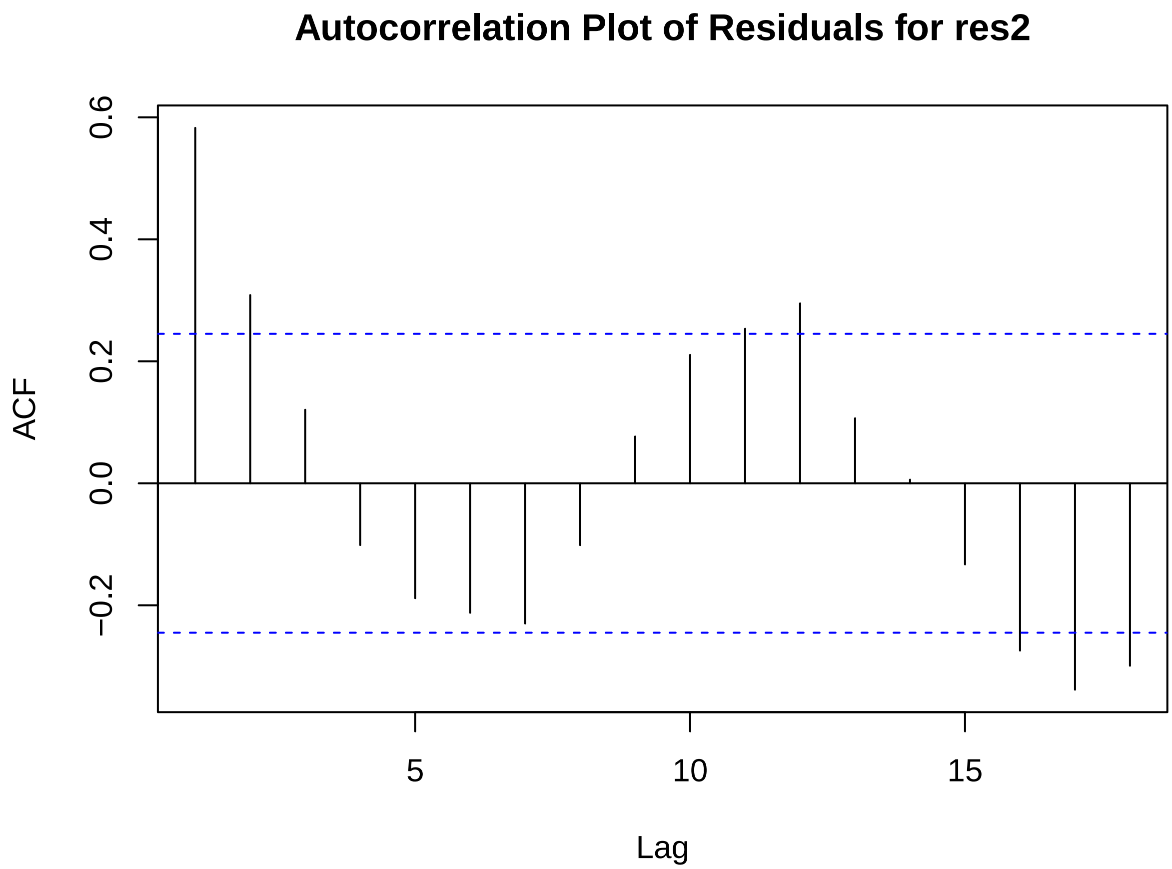
h)

```
acf(res1, main = "Autocorrelation Plot of Residuals for res1")
```



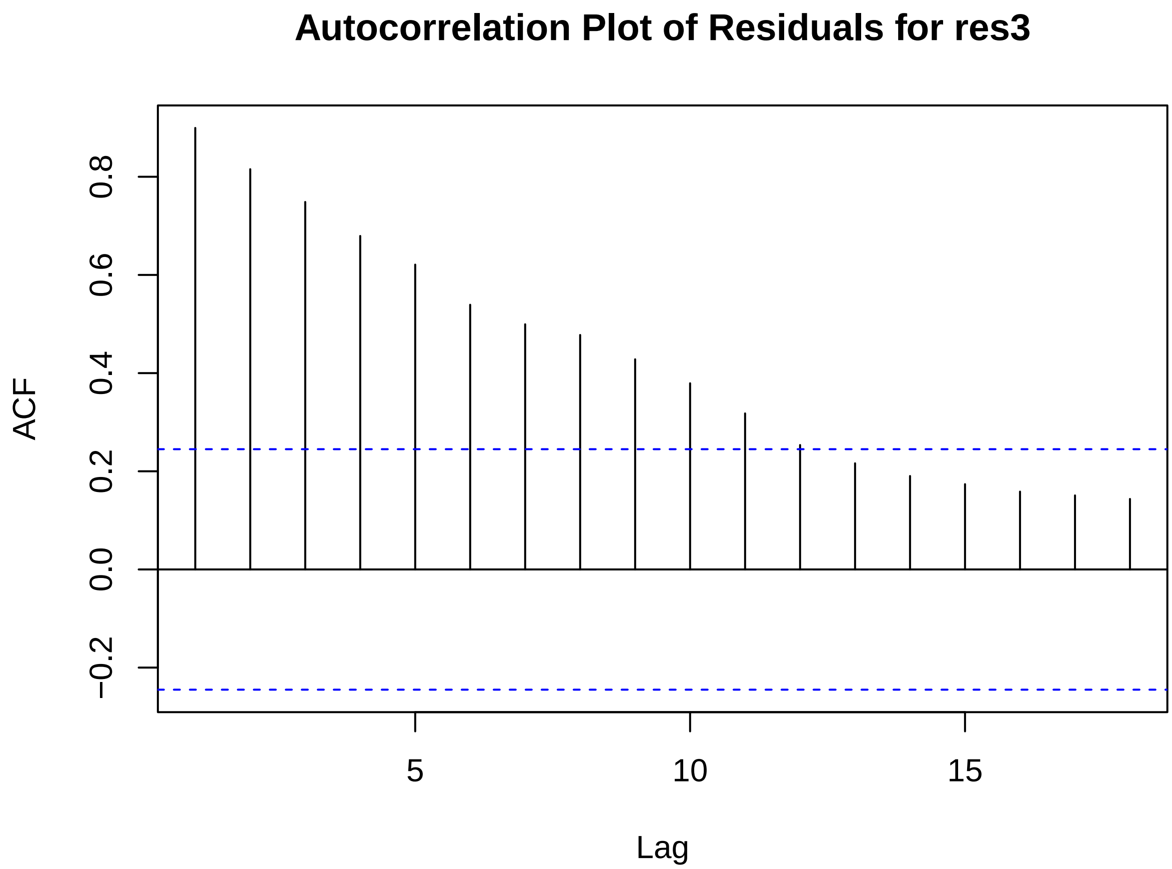
- significant autocorrelation at lags 1, 2, 3
- somewhat periodic behaviour starting at lag 7
- small magnitude with ca. .3

```
acf(res2, main = "Autocorrelation Plot of Residuals for res2")
```



- significant autocorrelation at lags 1, 2, 11, 12, 16, 17, 18
- periodic behaviour

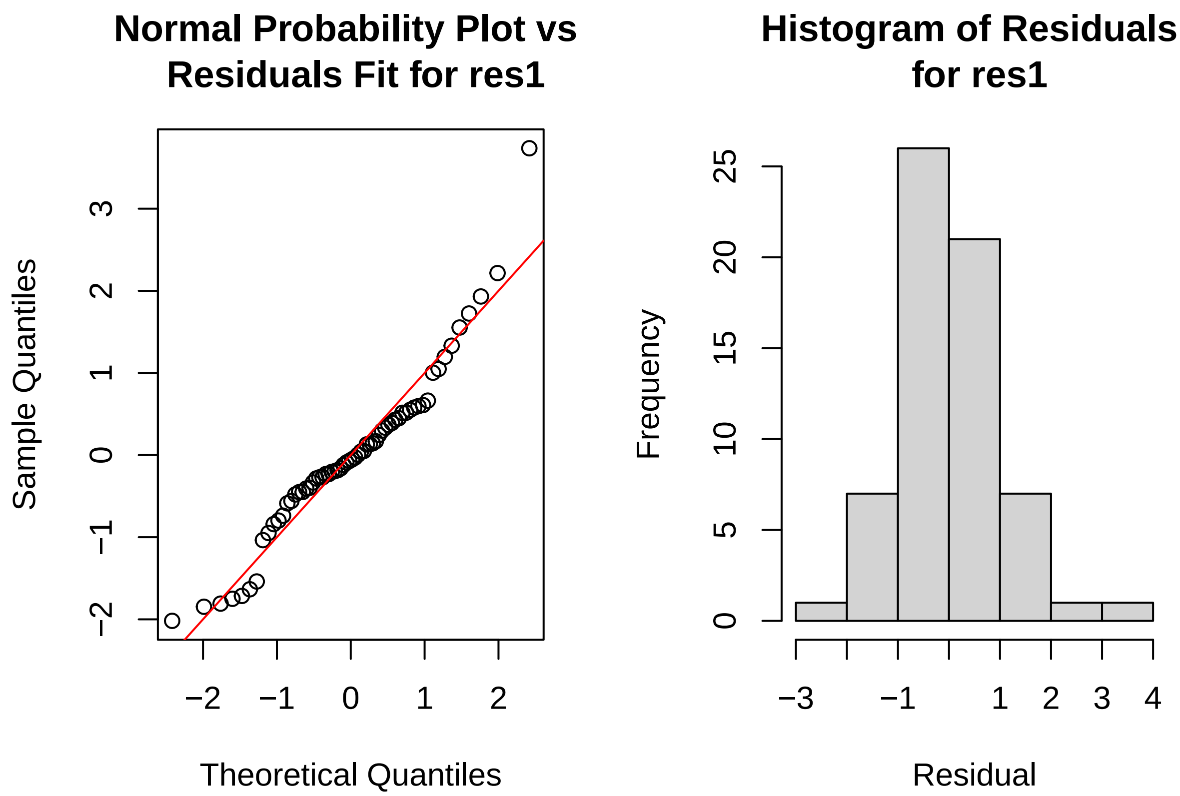
```
acf(res3, main = "Autocorrelation Plot of Residuals for res3")
```



- significant autocorrelation at lags 1 through 12
- clear downward trend (seems like e^{-x} behaviour) with no indication of seasonal trend
- big magnitude with about .9
- only positive values

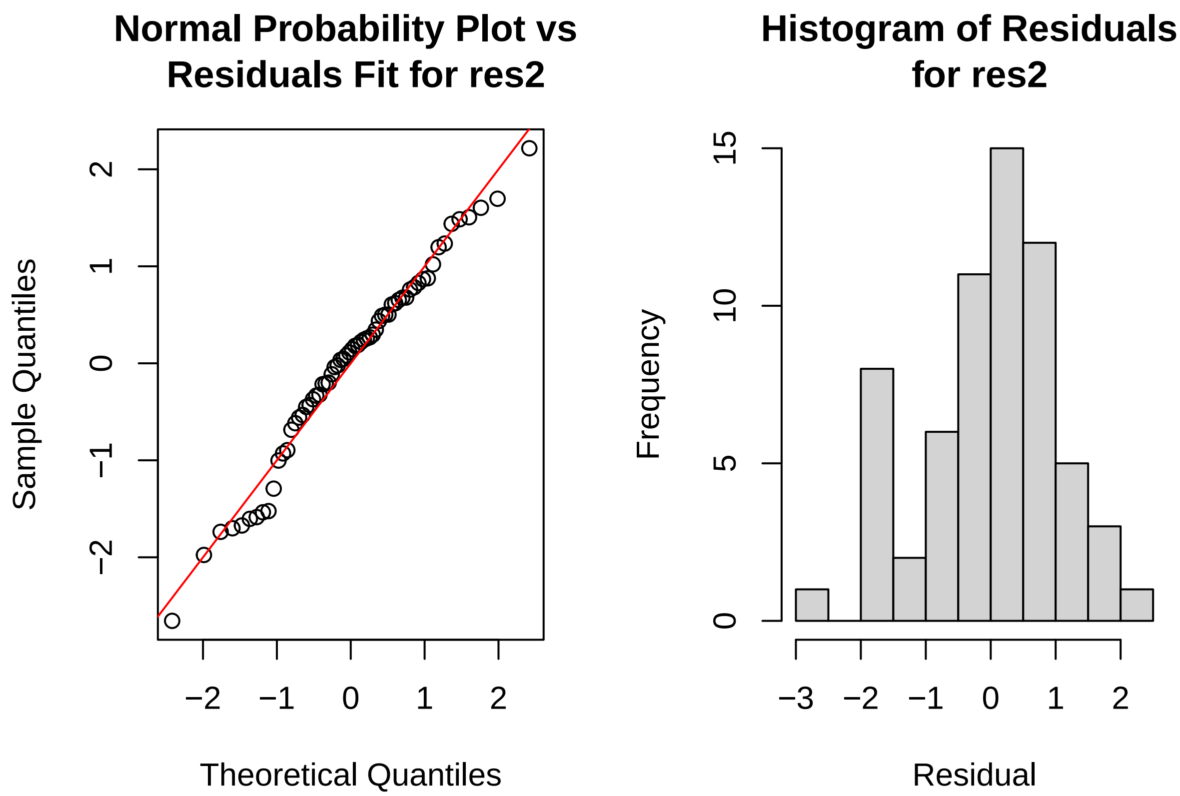
i)

```r
par(mfrow = c(1, 2))
qqnorm(res1, main = "Normal Probability Plot vs \n Residuals Fit for res1")
abline(a = 0, b = 1, col = "red")
hist(res1, xlab = "Residual", main = "Histogram of Residuals \n for res1")
```



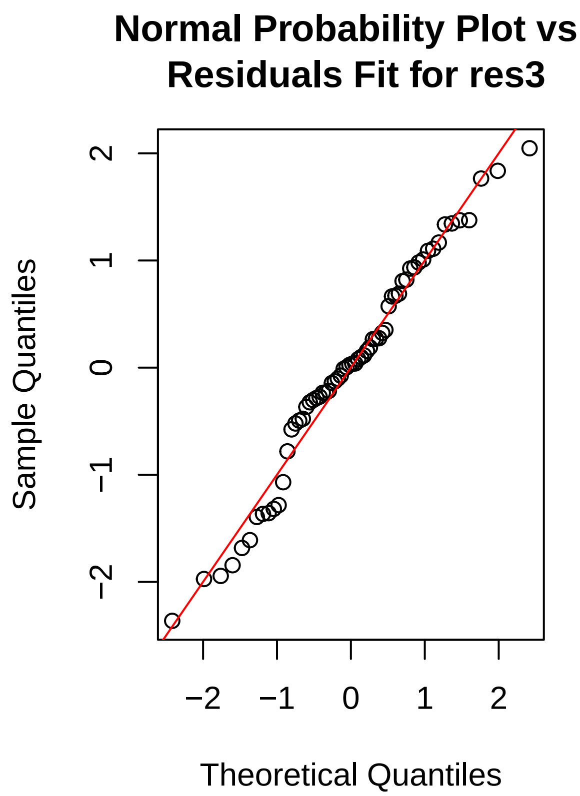
- The quantiles of res1 are a little bit off at about -1 and we have one outlier at about 4.
- Data looks somewhat normally distributed based on the histogram.

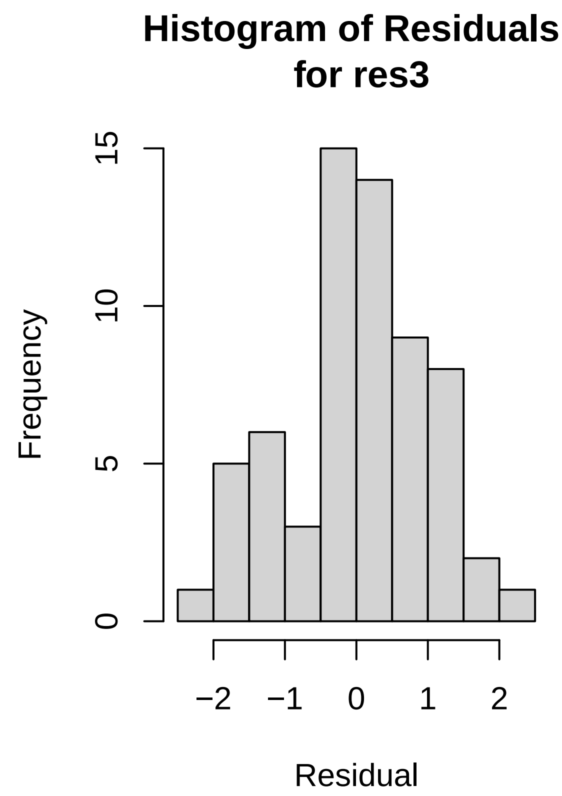
```
par(mfrow = c(1, 2))
qqnorm(res2, main = "Normal Probability Plot vs \n Residuals Fit for res2")
abline(a = 0, b = 1, col = "red")
hist(res2, xlab = "Residual", main = "Histogram of Residuals \n for res2")
```



- Again, we have outliers at -3 and 2 but not as bad as in res1.
- Looking at the histogram, we have normal distributed data if we do not take the values at -2 into account.

```
par(mfrow = c(1, 2))
qqnorm(res3, main = "Normal Probability Plot vs \n Residuals Fit for res3")
abline(a = 0, b = 1, col = "red")
hist(res3, xlab = "Residual", main = "Histogram of Residuals \n for res3")
```



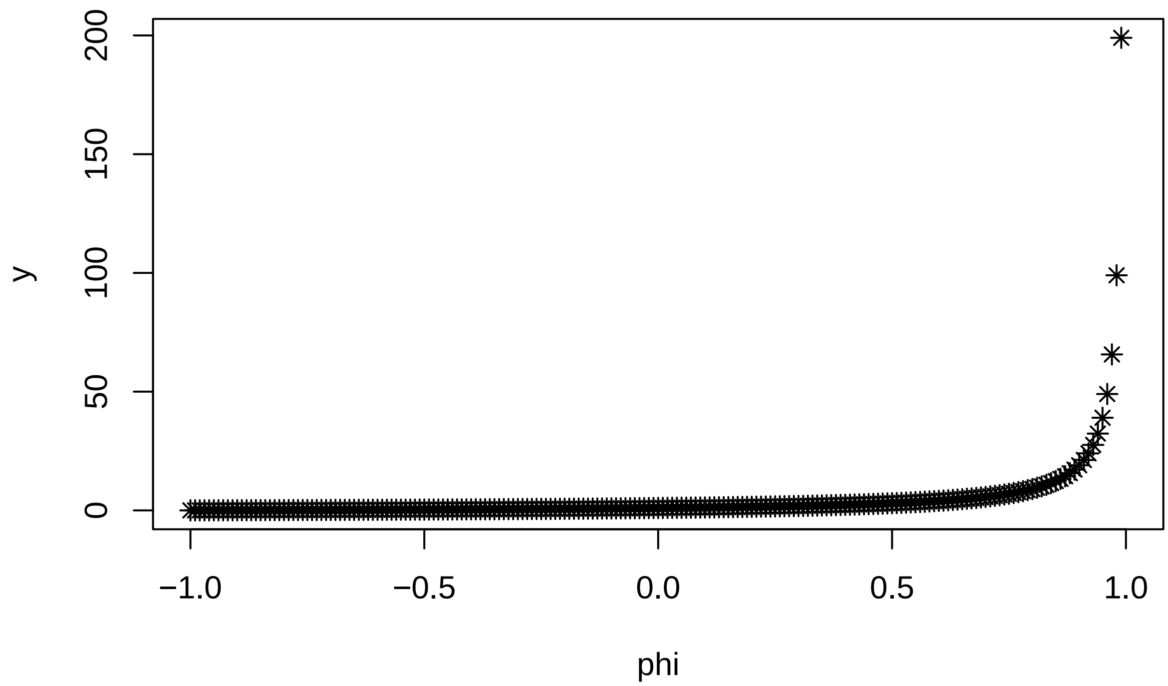


- The quantiles are a bit more off this time but all in all they are also somewhat normally distributed.
- The histogram shows a nearly normal distribution without the values from -.5 to -1.

Problem 3

c)

```
phi <- seq(from = -1, to = 1, by = 0.01)
y <- (1 + phi)/(1 - phi)
plot(phi, y, pch = 8)
```



The closer ϕ is to -1, the closer the variance is to 0, thus the precision increases. The closer ϕ is to 1, the more the variance increases, and thus the precision decreases.