MTHSTAT 564/564G/764–Time Series Analysis
Spring 2024
Problem Solving Set 2

Please think about the following problems from the textbook in advance of our problem solving sessions on them:

Problem Solving 2

1. Suppose $\mathbb{E}[X] = 8$, $\text{Var}(X) = 3$, $\mathbb{E}[Y] = 0$, $\text{Var}(Y) = 2$, and $\text{Corr}(X, Y) = 0.5$.
   (a) Find $\text{Var}(X + Y)$.
   (b) Find $\text{Cov}(X, X + Y)$.
   (c) Find $\text{Corr}(X + Y,\, X - Y)$.
2. If X and Y are dependent, but $\text{Var}(X) = \text{Var}(Y)$, find $\text{Cov}(X + Y,\, X - Y)$.
3. Let X have a distribution with mean μ and variance σ^2, and let $Y_t = X$ for all t.
   (a) Show that $\{Y_t\}$ is strictly and weakly stationary.
   (b) Find the autocovariance function for $\{Y_t\}$.
   (c) Sketch a "typical" time series plot of Y_t.
4. Suppose $Y_t = 5 + 2t + X_t$, where $\{X_t\}$ is a zero-mean stationary series with autocovariance function γ_k.
   (a) Find the mean function for $\{Y_t\}$.
   (b) Find the autocovariance function for $\{Y_t\}$.
   (c) Is $\{Y_t\}$ stationary? Why or why not?
5. Let $\{X_t\}$ be a stationary time series, and define

$$Y_t = \begin{cases} X_t & \text{if } t \text{ is odd} \\ X_t + 3 & \text{if } t \text{ is even.} \end{cases}$$

   (a) Show that $\text{Cov}(Y_t, Y_{t-k})$ is free of t for all lags k.
   (b) Is $\{Y_t\}$ stationary? Why or why not?