MTHSTAT 564/564G/764–Time Series Analysis
Spring 2024
Problem Solving Set 5

Please think about the following problems from the textbook in advance of our problem solving sessions on them:

Problem Solving 5

1. Verify Equation (3.2.6) on Page 29. You will need the fact that

$$\sum_{k=0}^{\infty} \phi^k = \frac{1}{1-\phi}$$

for $-1 < \phi < 1$.

2. Verify Equation 3.2.7 on Page 30. You will need the two sums

$$\begin{aligned} \sum_{t=1}^{n} t &= \frac{n(n+1)}{2} \\ \sum_{t=1}^{n} t^2 &= \frac{n(n+1)(2n+1)}{6}. \end{aligned}$$

3. Use first principles to find the autocorrelation function for the stationary process defined by

$$Y_t = 5 + e_t - \frac{1}{2}e_{t-1} + \frac{1}{4}e_{t-2}$$

where $\{e_t\}$ is mean-zero white noise with variance σ_e^2.

4. Calculate and sketch the autocorrelation functions for each of the following AR(1) models. Plot for sufficient lags that the autocorrelation function has nearly died out.

   (a) $\phi = 0.6$.

   (b) $\phi = -0.6$.

   (c) $\phi = 0.95$ (Do out to lag 20).

   (d) $\phi = 0.3$.

5. Let $\{Y_t\}$ be an AR(2) process of the special form $Y_t = \phi_2 Y_{t-2} + e_t$, where e_t is white noise. Use first principles to find the full range of values of ϕ_2 for which the process is stationary.