MTHSTAT 564/564G/764–Time Series Analysis
Spring 2024
Homework Assignment 2: Due Wednesday, 6 March in Lecture

This homework consists of three problems, two of which require R to some degree. You may feel free to work with classmates, but please be sure to turn in your own work in lecture. I do not need to see your code.

Reading

Chapter 3

Problems

1. Suppose $Y_t = \mu + e_t - e_{t-1}$. Find $\text{Var}(\bar{Y})$. Note any unusual results. In particular, compare your answer to what would have been obtained in $Y_t = \mu + e_t$. (You can avoid Equation 3.2.3 on Page 28 of the textbook if you first do some algebraic simplification on $\sum_{t=1}^{n}(e_t - e_{t-1})$.
2. The data file "winnebago" in the TSA package in R contains monthly unit sales of recreational vehicles from Winnebago, Inc. from November 1966 through February 1972.
   (a) Display and interpret a time series plot for these data.
   (b) Use least squares to fit a line to these data. Interpret the regression output. Plot the standardized residuals from the fit as a time series. Interpret the plot.
   (c) Now take natural logarithms of the monthly sales figures and display and interpret the time series plot of the transformed values.
   (d) Use least squared to fit a line to the data resulting from the log transformation. Display and interpret the time series plot of the standardized residuals from this fit.
   (e) Now use least squares to fit a seasonal-means model plus linear time trend to the log-transformed sales time series and save the standardized residuals for further analysis. Check the statistical significance of each of the regression coefficients in the model.
   (f) Display the time series plot of the standardized residuals obtained in part e). Interpret the plot.
   (g) Calculate the least squares residuals from a seasonal means plus linear time trend model on the logarithms of the sales time series.

(h) Calculate and interpret the sample autocorrelations for the standardized residuals.

(i) Investigate the normality of the standardized residuals (error terms). Consider histograms and normal probability plots. Interpret the plots.

3. Suppose that a stationary time series $\{Y_t\}$ has an autocorrelation function of the form $\rho_k = \phi^k$ for $k > 0$, where ϕ is constant in the range (-1,1).

(a) Show that $\text{Var}(\bar{Y}) = \frac{\gamma_0}{n}\left[\frac{1+\phi}{1-\phi} - \frac{2\phi}{n}\frac{(1-\phi^n)}{(1-\phi)^2}\right]$. (Use Equation (3.2.3) on Page 28, the finite geometric sum and related sum (both below) to help you).

$$\sum_{k=0}^{n} \phi^k = \frac{1-\phi^{n+1}}{1-\phi}$$

$$\sum_{k=0}^{n} k\phi^{k-1} = \frac{d}{d\phi}\left[\sum_{k=0}^{n} \phi^k\right].$$

(b) If n is large, argue that $\text{Var}(\bar{Y}) \approx \frac{\gamma_0}{n}\left[\frac{1+\phi}{1-\phi}\right]$.

(c) (Use R) Plot $\frac{1+\phi}{1-\phi}$ for ϕ over the range (-1,1) Interpret the plot in terms of the precision in estimating the process mean.