Time Series

Midterm project

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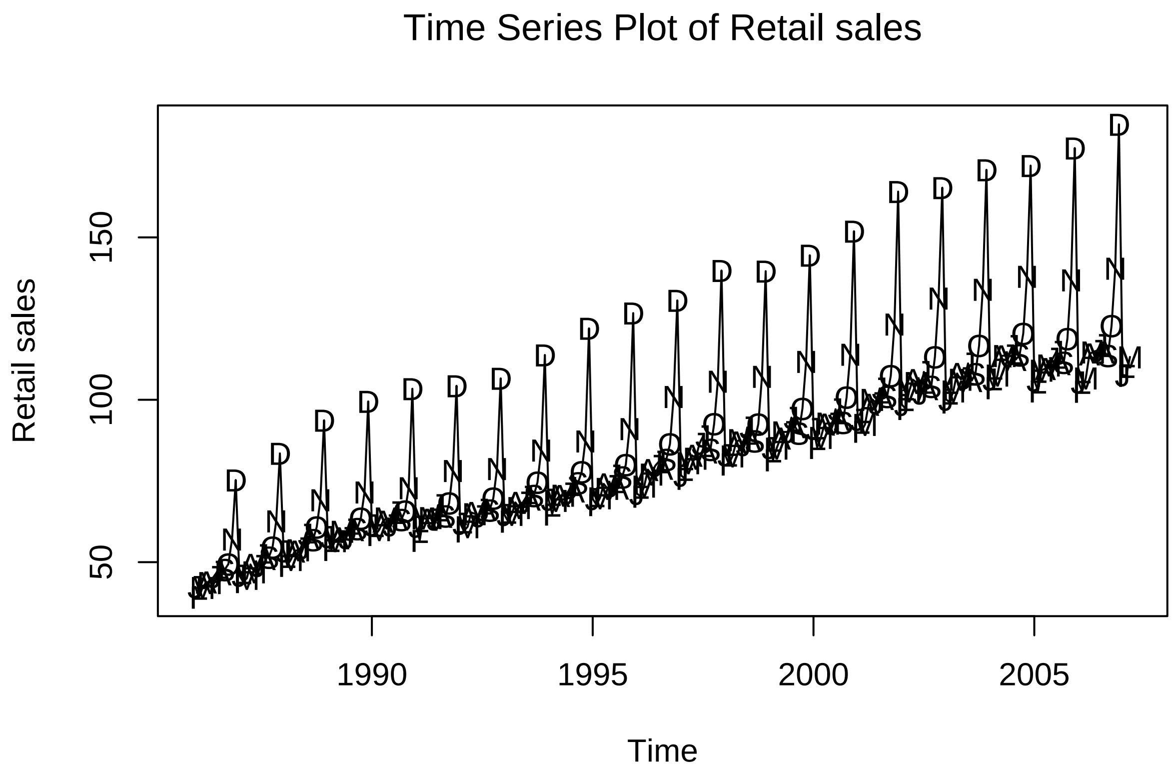
Part 1

1.

I am expecting a somewhat clear upward trend due to the increase in the population. Also, there could be a seasonal trend, such that the retail sales are higher on special holidays. Since the Brexit was later than 2007, I am not expecting a big sink somewhere in the graph.

```
library(TSA)
library(tseries)
```

```
data(retail)
plot(retail, xlab = expression("Time"), ylab = expression("Retail sales"),
    main = expression("Time Series Plot of Retail sales"))
points(retail, pch = as.vector(season(retail)))
```



Seeing the actual graph, I can confirm the upward trend and also a very strong seasonal behaviour. I clearly expected more holidays to be that present as christmas. There surely is a really strong seasonal behaviour due to christmas. This means that the sales are going up starting November each year and reach the peak in December before dropping again.

2.

I would try a seasonal ARIMA model, given the upward trend and the potential seasonality.

```
model <- forecast::auto.arima(retail, seasonal = TRUE)
summary(model)
```

```
## Series: retail
## ARIMA(1,0,4)(0,1,1)[12] with drift
##
## Coefficients:
##          ar1      ma1     ma2     ma3     ma4     sma1   drift
##       0.8469  -0.5939  0.0504  0.0878  0.1742  -0.0966  0.3083
## s.e.  0.0533   0.0788  0.0740  0.0772  0.0756   0.0631  0.0405
##
## sigma^2 = 3.413:  log likelihood = -491.04
## AIC=998.07   AICc=998.69   BIC=1026.02
##
## Training set error measures:
##                          ME     RMSE      MAE         MPE     MAPE      MASE
```

```
## Training set -0.0003999956 1.777353 1.287341 -0.1007814 1.485438 0.3327206
##                      ACF1
## Training set 0.00339952
```

Model type and coefficients

Looking at the summary, we are given an ARIMA model with $p = 1$, $d = 0$, $q = 4$, where q is the autoregressive order with the coefficient $ar_1 = 0.8469$, also known as the lag order. d is the number of differences we take, which is 0, so that indicates that the data is already stationary. And q is the order of the moving average process, where the coefficients are $ma_1 = -0.5939$, $ma_2 = 0.0504$, $ma_3 = 0.0878$, $ma_4 = 0.1742$. Speaking in a sentence, we have a first order degree autoregressive model with zero order of differencing and a fourth order moving average model. That was for the non-seasonal part. For the seasonal part, we have $p = 0$, $d = 1$, $q = 1$, where p, d and q are exactly as above. The number 12 in brackets just states the number of seasons (here: months).

Fit

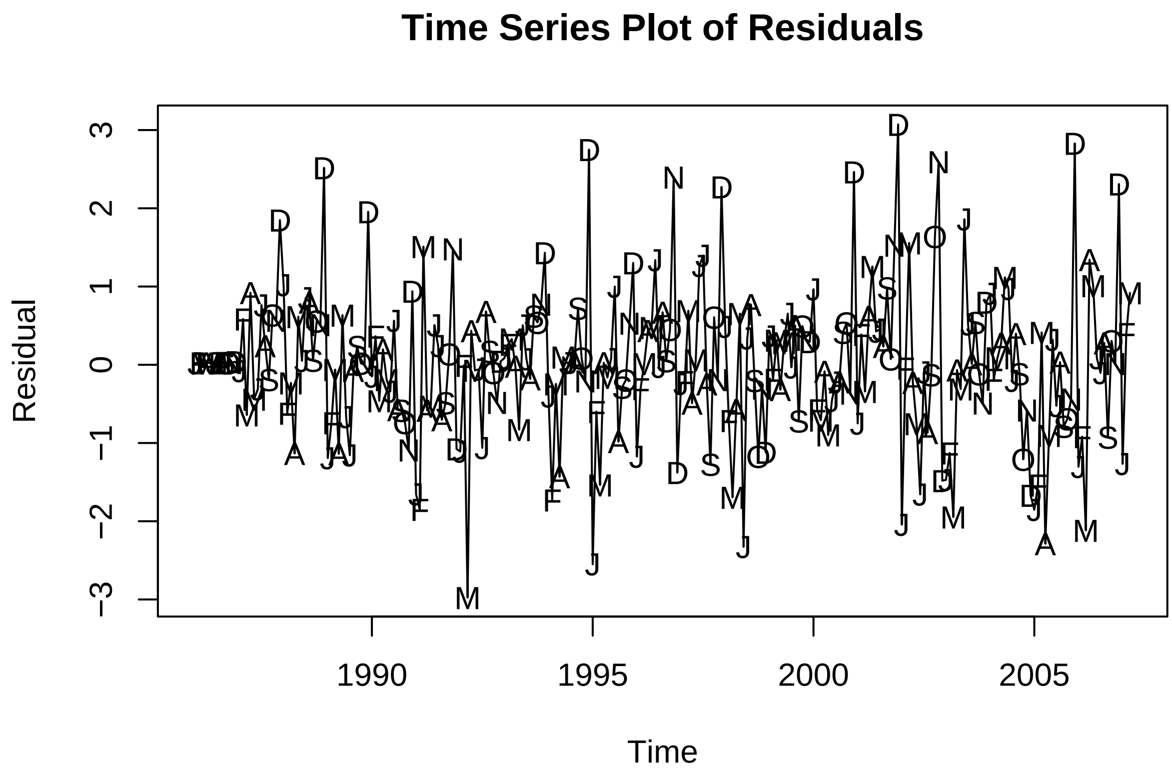
We have some values to rate the model fit, which are

$$\begin{aligned} \sigma^2 &= 3.413, \\ \log \text{likelihood} &= -491.04, \\ AIC &= 998.07, \\ AICc &= 998.69, \\ BIC &= 1026.02. \end{aligned}$$

Here, we can just look at AIC and BIC which suggest a reasonable choice based on model complexity vs fit. The other values are basically for forecasting reasons.

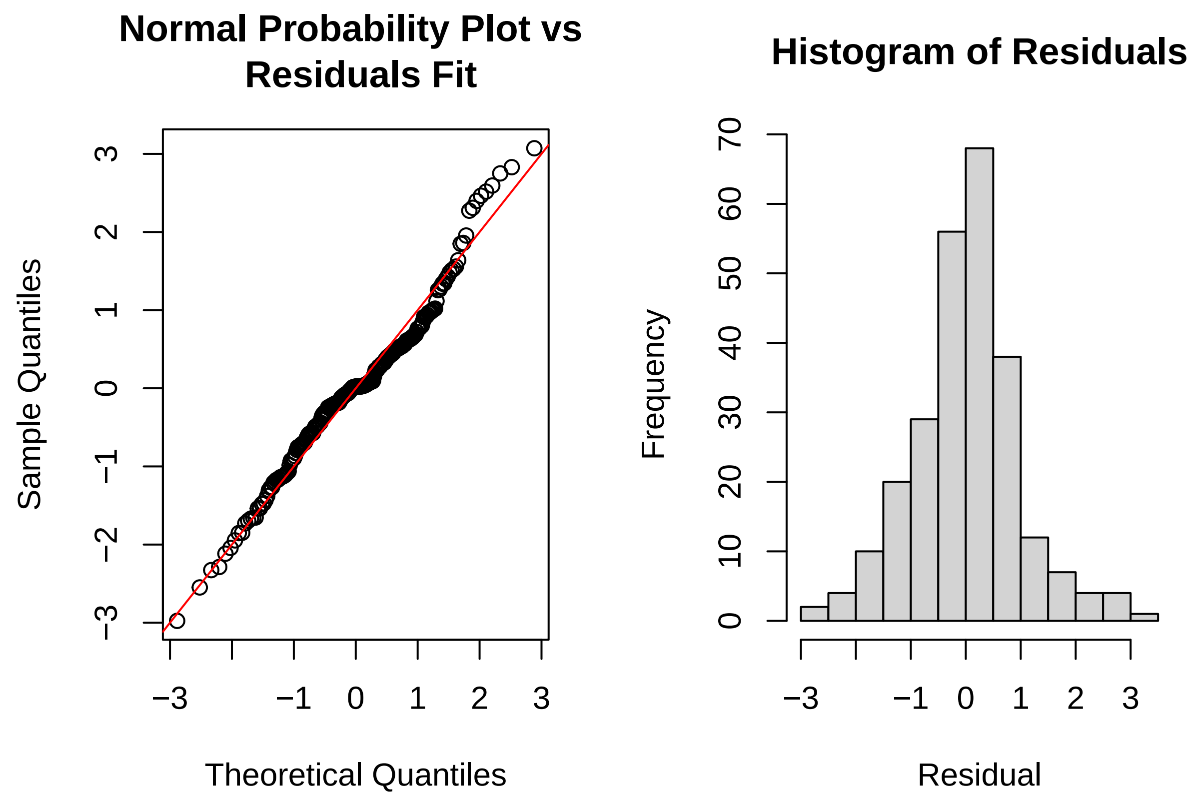
3.

```r
res <- ts(rstandard(model), start = c(1986, 1), end = c(2007,
    3), frequency = 12)
plot(res, xlab = "Time", ylab = "Residual", main = "Time Series Plot of Residuals")
points(y = res, x = time(res), pch = as.vector(season(res)))
```



The plot of the residuals is kind of displaying the seasonal/periodic behaviour.

```
par(mfrow = c(1, 2))
qqnorm(res, main = "Normal Probability Plot vs \n Residuals Fit")
abline(a = 0, b = 1, col = "red")
hist(res, xlab = "Residual", main = "Histogram of Residuals")
```



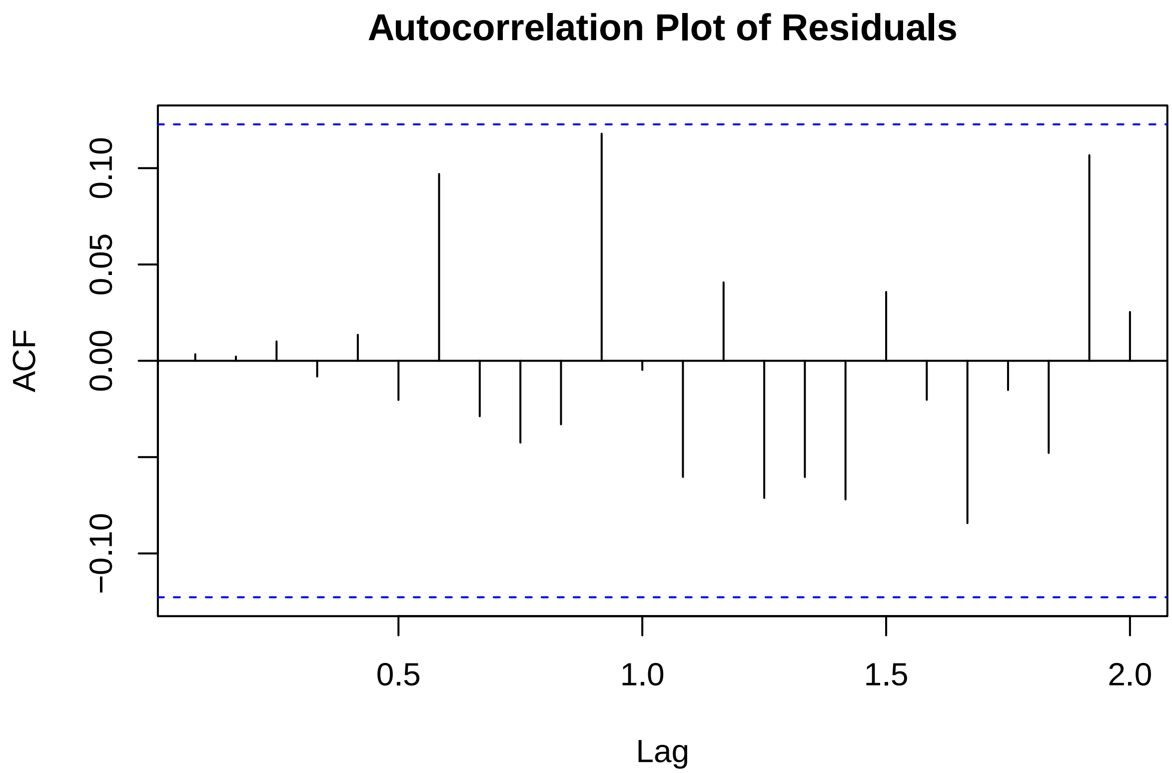
The residuals look like they are normally distributed, indicating a good model fit.

```
shapiro.test(res)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res
## W = 0.97597, p-value = 0.0002613
```

Since the p-value of the Shapiro-Wilk test is also really low, we can say that they are normally distributed.

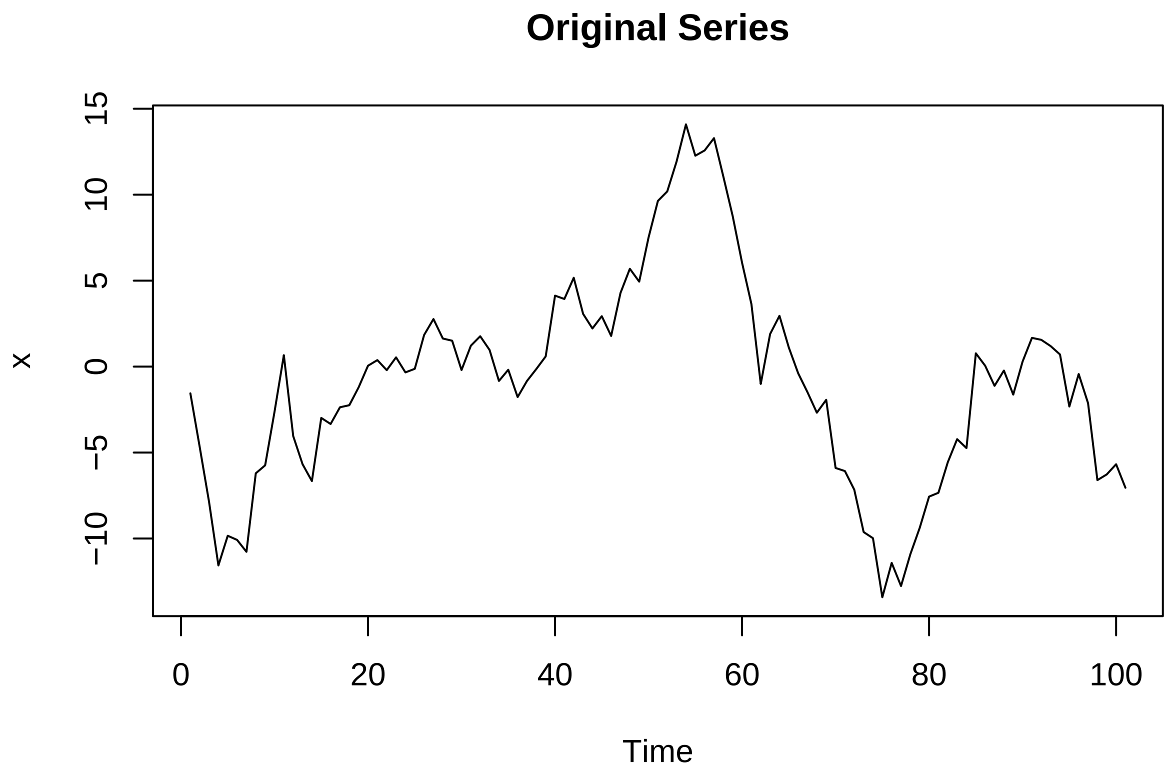
```
acf(res, main = "Autocorrelation Plot of Residuals")
```



Since there are just some "spikes" which are not at all big, the chosen model seems to capture the time-series pretty well. Also, no value is outside of the confidence interval.

Part 2

```
data <- as.ts(read.delim(file = "../homework/MidtermPt2.txt",
    header = TRUE, sep = " "))
plot(data, main = "Original Series")
```

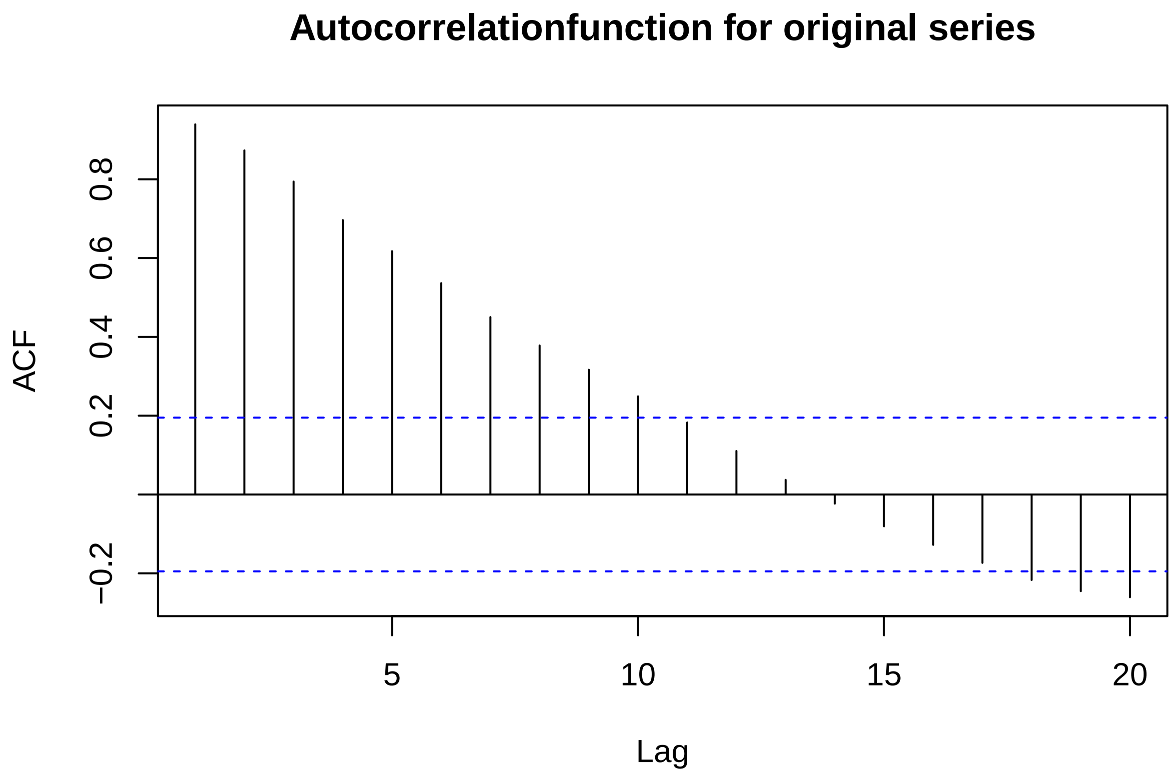



```r
adf.test(data)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  data
## Dickey-Fuller = -2.2725, Lag order = 4, p-value = 0.4637
## alternative hypothesis: stationary
```

Just looking at the plot, we cannot really say if the series is stationary or no. Looking at the Dickey-Fuller Test and seeing that we have a p-value of 0.4637 we cannot reject the null hypothesis, so this indicates that this series is non-stationary.

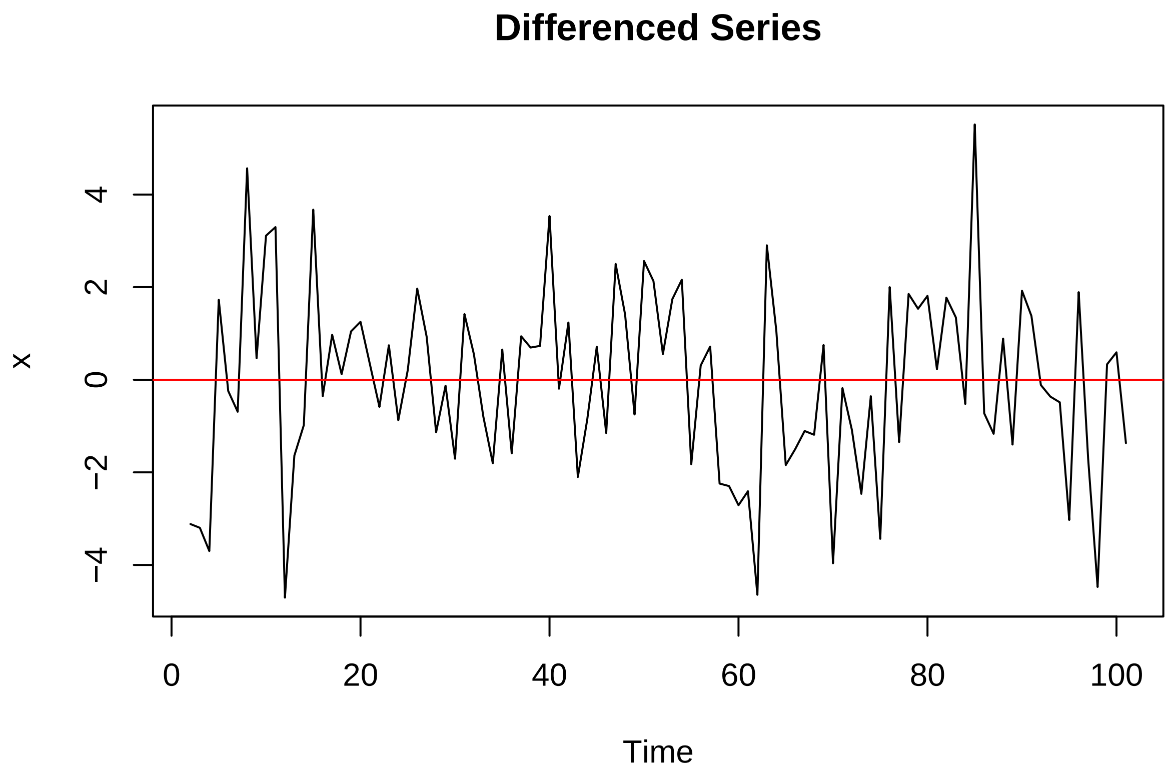
```r
acf(data, main = "Autocorrelationfunction for original series")
```



Looking at the plot for the autocorrelationfunction, we see a decreasing over time and therefore a dependence on time. This is another indicator for a non-stationary series.

Let us consider the first difference as an approach to make the series stationary.

```
d_data <- diff(data)
plot(d_data, main = "Differenced Series")
abline(a = 0, b = 0, col = "red")
```



We see a fairly constant "behaviour" around 0 and an evenly spread, which is an indicator for stationarity.

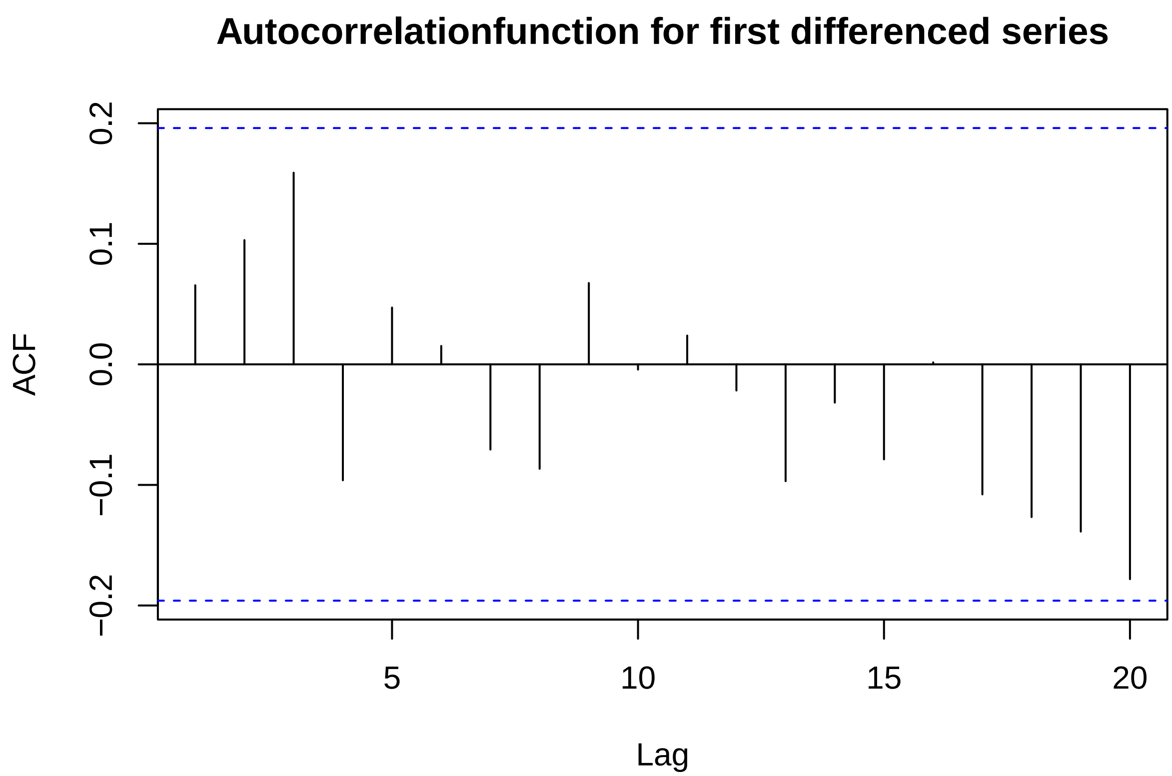
```r
adf.test(d_data)
```

```
## Warning in adf.test(d_data): p-value smaller than printed p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  d_data
## Dickey-Fuller = -4.0875, Lag order = 4, p-value = 0.01
## alternative hypothesis: stationary
```

Looking at the Dickey-Fuller Test again, the p-value is even smaller than the printed value which is 0.01, meaning that we can reject the null-hypothesis and assume that the series is stationary.

```r
acf(d_data, main = "Autocorrelationfunction for first differenced series")
```



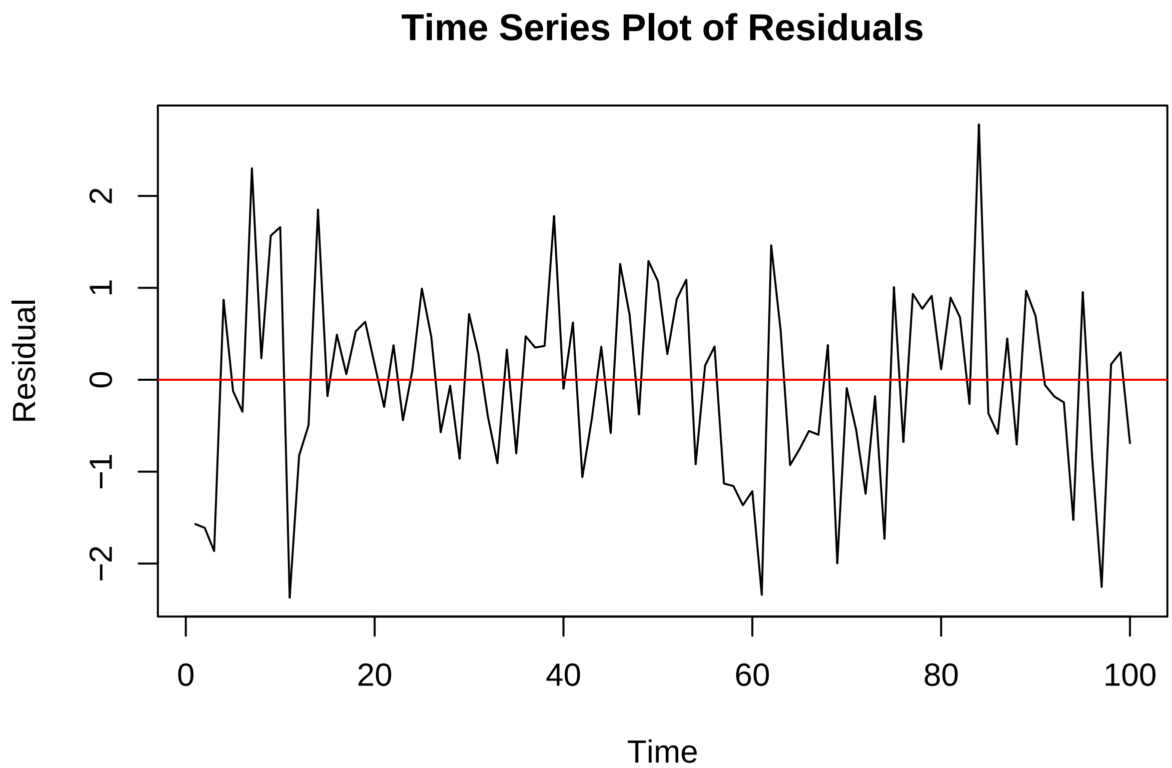
Also, looking at the autocorrelationfunction for the first difference, we can see a strong "improvement". We cannot see any periodic behaviour, or any significant spike and the values are really small, which is an indicator for stationarity.

```r
model <- forecast::auto.arima(d_data)
summary(model)
```

```
## Series: d_data
## ARIMA(0,0,0) with zero mean
##
## sigma^2 = 3.94:  log likelihood = -210.45
## AIC=422.91   AICc=422.95   BIC=425.51
##
## Training set error measures:
##                       ME     RMSE      MAE MPE MAPE      MASE       ACF1
## Training set -0.05495792 1.984958 1.589698 100  100 0.7621968 0.06560549
```

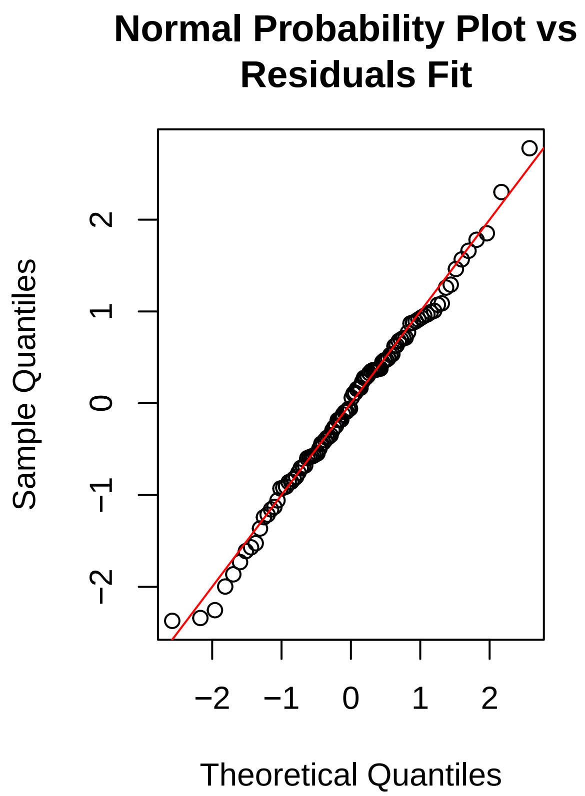
If we are just fitting an ARIMA model to the first difference, we get $p = d = q = 0$, indicating, that we have a stationary model by taking the first difference.

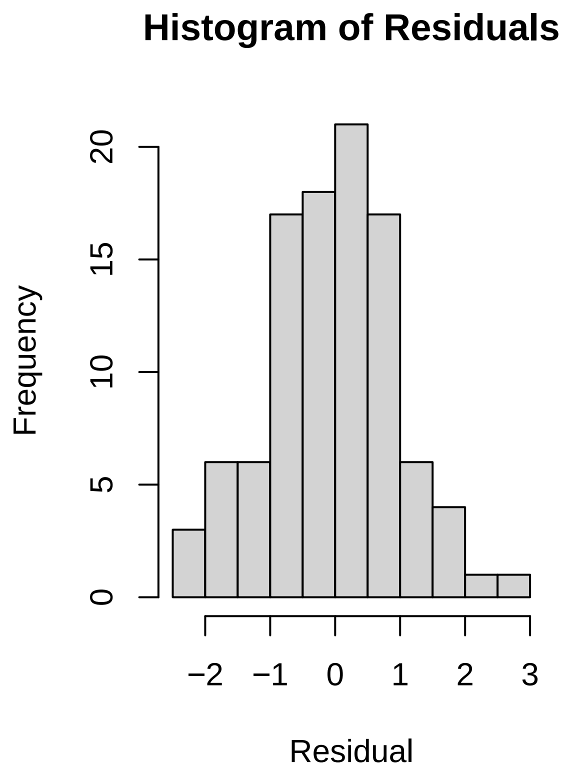
```r
res <- ts(rstandard(model))
plot(res, xlab = "Time", ylab = "Residual", main = "Time Series Plot of Residuals")
abline(a = 0, b = 0, col = "red")
```



Plotting the residuals, we see an evenly spread of the variance and the residuals of the first difference stay around 0.

```
par(mfrow = c(1, 2))
qqnorm(res, main = "Normal Probability Plot vs \n Residuals Fit")
abline(a = 0, b = 1, col = "red")
hist(res, xlab = "Residual", main = "Histogram of Residuals")
```





Looking at the distribution of the residuals, we see that they are normally distributed, which indicates a good model fit and therefore also stationarity.