MTHSTAT 564/564G/764–Time Series Analysis
Spring 2024
Problem Solving Set 4

Please think about the following problems from the textbook in advance of our problem solving sessions on them:

Problem Solving 4

1. Verify Equation (3.3.2) on page 30, for the least squares estimates of β_0 and of β_1 when the model $Y_t = \beta_0 + \beta_1 t + X_t$ is considered.

2. Suppose $Y_t = \mu + e_t + e_{t-1}$. Find $\text{Var}(\bar{Y})$. Compare your answer to what would have been obtained if $Y_t = \mu + e_t$. Describe the effect that the autocorrelation in $\{Y_t\}$ has on $\text{Var}(\bar{Y})$.

3. The data file "wages" contains monthly values of the average hourly wages (in dollars) for workers in the U.S. apparel and textile produces for July 1981 through June 1987.

   (a) Display and interpret the time series plot for these data.

   (b) Use least squares to fit a linear time trend to this time series. Interpret the regression output and save the standardized residuals from the fit for further analysis.

   (c) Constrict and interpret the time series plot of the standardized residuals from part (b).

   (d) Use least squares to fit a quadratic time trend to the wages time series. Interpret the regression output and save the standardized residuals from the fit for further analysis.

   (e) Construct and interpret the time series plot of the standardized residuals from part (d).

   (f) Calculate and interpret the sample autocorrelations for the standardized residuals from the quadratic trend fit.

   (g) Investigate the normality of the standardized residuals. Consider histograms and normal probability plots. Interpret the plots.

   (h) Plot the standardized residuals versus the fits from the quadratic trend. Interpret the plot and decide whether the specified model is adequate.

4. The data file "beersales" contains monthly U.S. beer sales (in millions of barrels) for the period January 1975 through December 1990.

(a) Display and interpret the plot of the time series for these data.

(b) Now construct a time series plot that uses separate plotting symbols for the various months. Does your interpretation change from that of part (a)?

(c) Use least squares to fit a seasonal means trend to this time series. Interpret the regression output and save the standardized residuals for further analysis.

(d) Construct and interpret the time series plot of the standardized residuals from part (c). Be sure to use proper plotting symbols to check on seasonality in the standardized residuals.

(e) Use least squares to fit a seasonal means plus quadratic time trend to the beer sales time series. Interpret the regression output. Save the standardized residuals from the fit for further analysis.

(f) Construct and interpret the time series plot of the standardized residuals from part (e). Again, use plotting symbols to check for any remaining seasonality in the residuals.

(g) Using the residuals from part (e), calculate and interpret the sample autocorrelations in the standardized residuals.

(h) Construct and interpret a plot of the standardized residuals in part (e) versus the fitted values from the seasonal means plus quadratic trend.

(i) Investigate the normality of the standardized residuals. Consider histograms and normal probability plots. Interpret the plots.

5. The data file "prescrip" gives monthly U.S. prescription costs for the months August 1986 to March 1992. These data are from the state of New Jersey's Prescription Drug Program and are the cost per prescription claim.

(a) Display and interpret the time series plot for these data. Use plotting symbols that permit you to look for seasonality.

(b) Calculate and plot the sequence of month-to-month percentage changes in the prescription costs. Again, use plotting symbols that permit you to look for seasonality.

(c) Use least squares to fit a cosine trend with fundamental frequency of 1/12 to the percentage change series. Interpret the regression output and save the standardized residuals.

(d) Plot the sequence of standardized residuals from part (c) to investigate the adequacy of the cosine trend model.

(e) Calculate and interpret the sample autocorrelations of the standardized residuals from part (c).

(f) Investigate the normality of the standardized residuals from part (c) using a histogram and a normal probability plot.