Time Series

Homework 2

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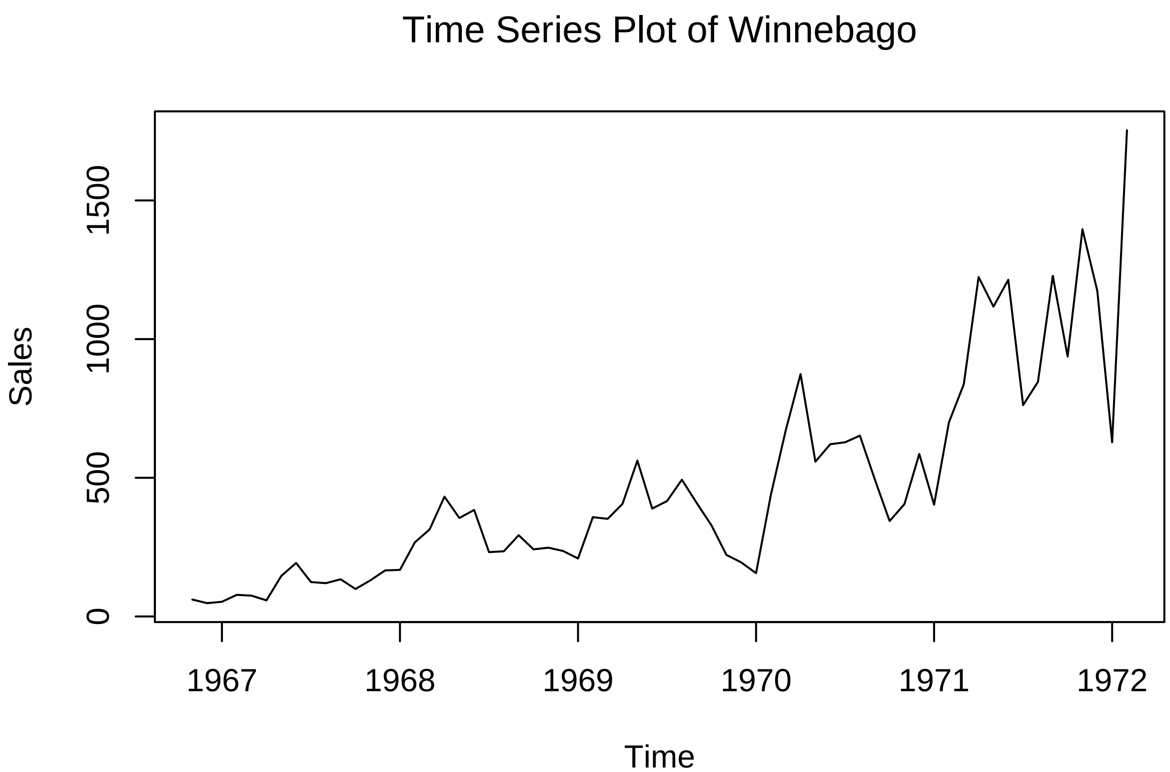
Problem 1

Problem 2

a)

```
library(TSA)
library(tseries)
```

```
data(winnebago)
plot(winnebago, xlab = expression("Time"), ylab = expression("Sales"), main = expression("Time Series Pl
```



Interpretation: - The time series shows an overall upwards trend between the years 1967 and 1972. - Between the years 1970 and 1972 the increase is at its highest.

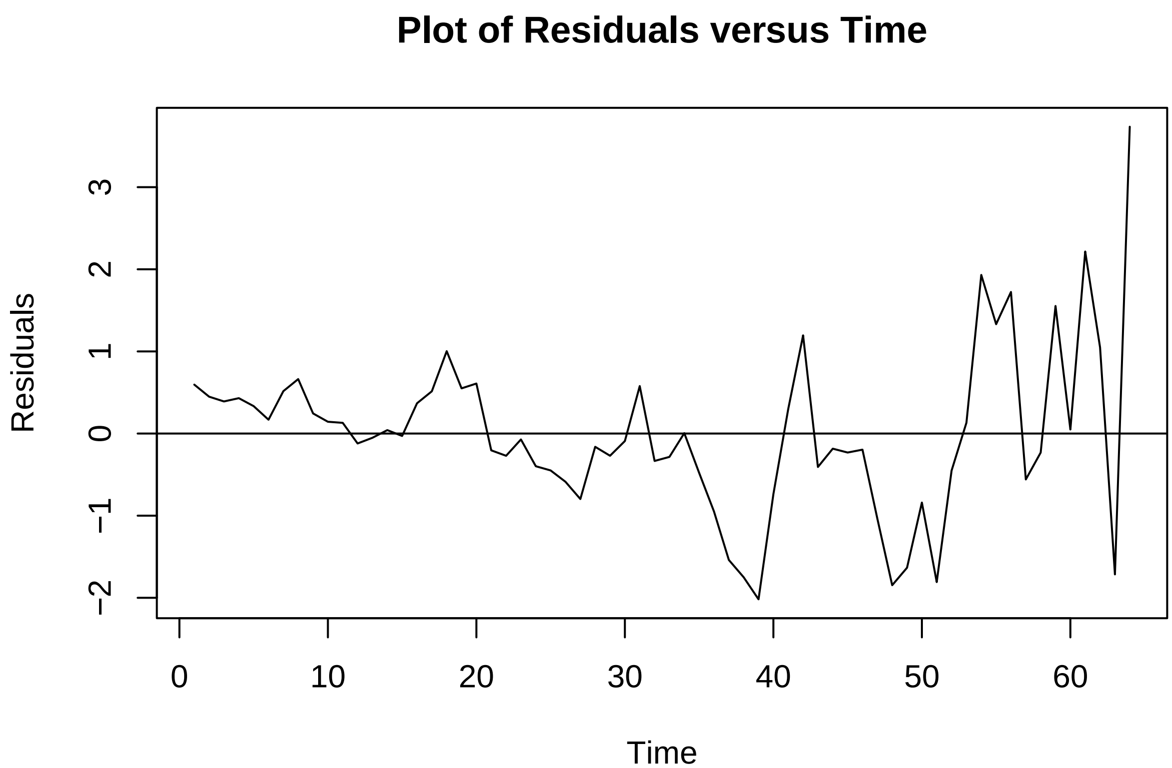
b)

```r
model1 <- lm(winnebago ~ time(winnebago))
summary(model1)
```

```
##
## Call:
## lm(formula = winnebago ~ time(winnebago))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -419.58  -93.13  -12.78   94.96  759.21
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)     -394885.68   33539.77  -11.77   <2e-16 ***
## time(winnebago)     200.74      17.03   11.79   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 209.7 on 62 degrees of freedom
## Multiple R-squared:  0.6915, Adjusted R-squared:  0.6865
## F-statistic: 138.9 on 1 and 62 DF,  p-value: < 2.2e-16
```

- We expect the wages to increase by $200.74 per year
- 69.15 of the variance can be explained by the predictor variables
- the linear trend is significant because the p-value $2.2 \cdot 10^{-16}$ is smaller than 10^{-12}

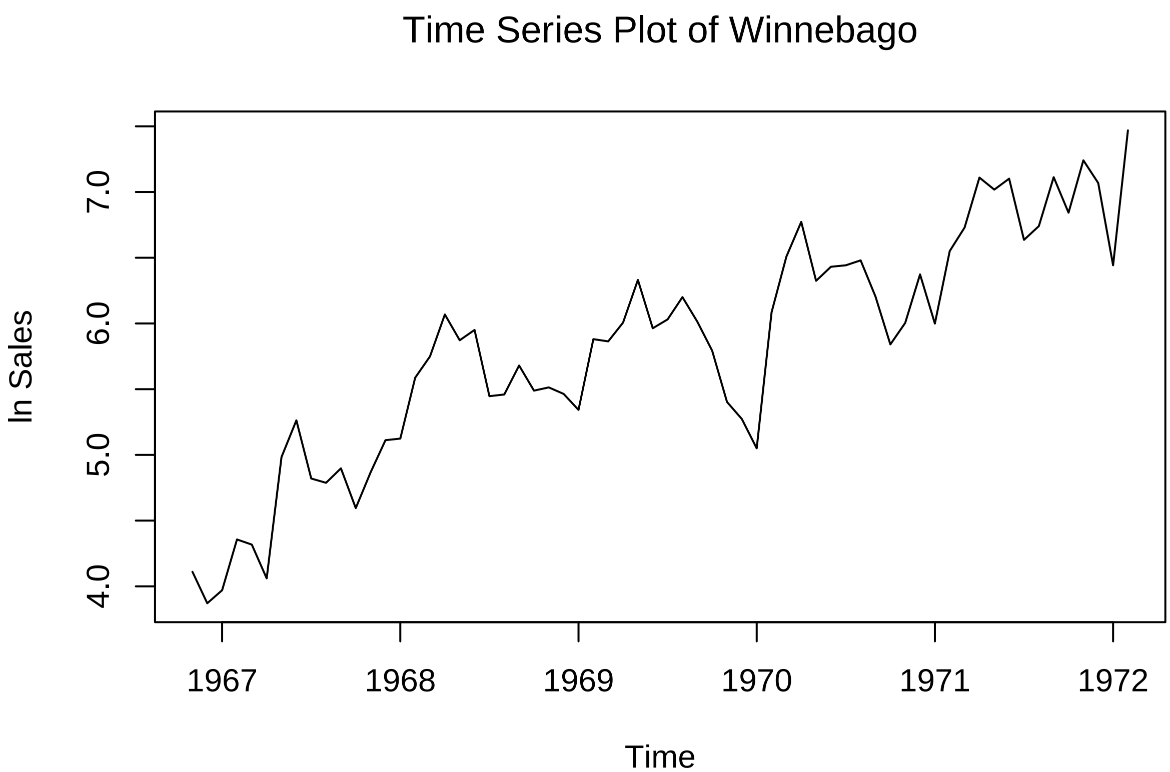
```r
res <- as.ts(rstandard(model1))
plot(res, xlab = expression("Time"), ylab = expression("Residuals"), main = "Plot of Residuals versus Ti
abline(h = 0)
```



Interpretation: - The residuals plot shows somewhat random movement around zero. - More uneven spread between ca. 35 to 65 in comparison to 0 to 35. - There may be a "seasonal" cyclical trend.

c)

```r
ln_winnebago <- log(winnebago)
plot(ln_winnebago, xlab = expression("Time"), ylab = expression("ln Sales"), main = expression("Time Se
```



Interpretation: - The time series plot of the transformed values shows a linear upward trend (which corresponds with the untransformed data) - There is one dip around the year 1970 - The "seasonal" trend seems more pronounced

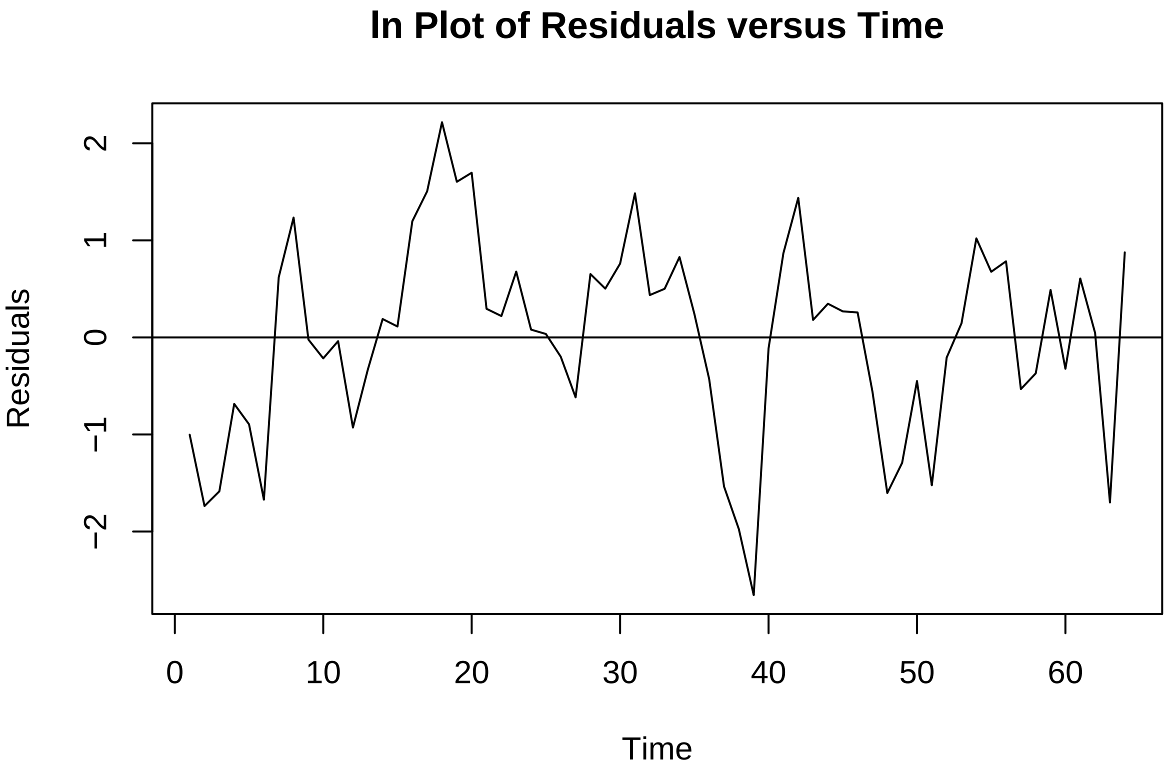
d)

```r
model2 <- lm(ln_winnebago ~ time(ln_winnebago))
summary(model2)
```

```
##
## Call:
## lm(formula = ln_winnebago ~ time(ln_winnebago))
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.03669 -0.20823  0.04995  0.25662  0.86223
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)        -984.93878   62.99472  -15.63   <2e-16 ***
## time(ln_winnebago)    0.50306    0.03199   15.73   <2e-16 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3939 on 62 degrees of freedom
## Multiple R-squared:  0.7996, Adjusted R-squared:  0.7964
## F-statistic: 247.4 on 1 and 62 DF,  p-value: < 2.2e-16
```

```
res2 <- as.ts(rstandard(model2))
plot(res2, xlab = expression("Time"), ylab = expression("Residuals"), main = "ln Plot of Residuals versu
abline(h = 0)
```



Interpretation: - the ln-transformed residuals plot shows random movement around 0 - There seems to be an overall cyclical trend

e) & f)

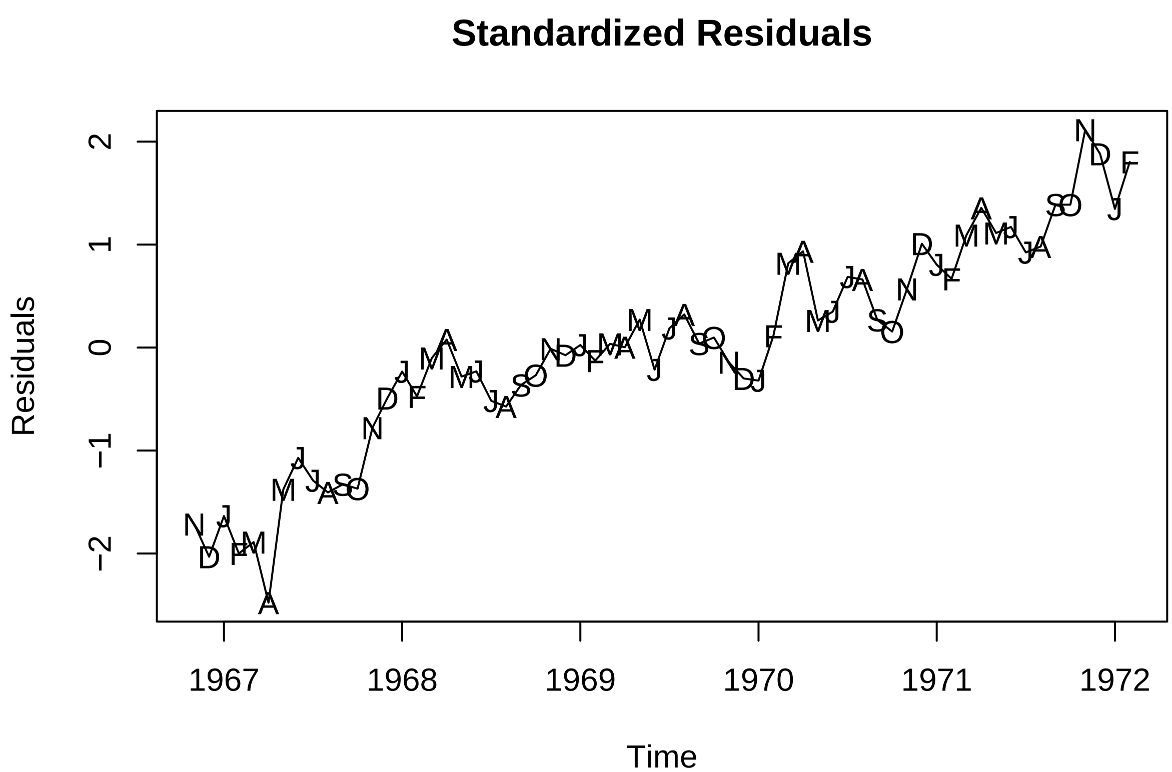
```
months <- season(winnebago)
model3 <- lm(ln_winnebago~months-1)
summary(model3)
```

```
##
## Call:
## lm(formula = ln_winnebago ~ months - 1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.94319 -0.40444  0.02541  0.59421  1.71807
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
```

```
## monthsJanuary      5.3213     0.3752   14.18   <2e-16 ***
## monthsFebruary     5.9882     0.3752   15.96   <2e-16 ***
## monthsMarch        5.8338     0.4111   14.19   <2e-16 ***
## monthsApril        6.0036     0.4111   14.61   <2e-16 ***
## monthsMay          6.1060     0.4111   14.85   <2e-16 ***
## monthsJune         6.1420     0.4111   14.94   <2e-16 ***
## monthsJuly         5.8752     0.4111   14.29   <2e-16 ***
## monthsAugust       5.9336     0.4111   14.44   <2e-16 ***
## monthsSeptember    5.9819     0.4111   14.55   <2e-16 ***
## monthsOctober      5.7121     0.4111   13.90   <2e-16 ***
## monthsNovember     5.5233     0.3752   14.72   <2e-16 ***
## monthsDecember     5.5269     0.3752   14.73   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9192 on 52 degrees of freedom
## Multiple R-squared:  0.9801, Adjusted R-squared:  0.9755
## F-statistic: 213.8 on 12 and 52 DF,  p-value: < 2.2e-16
```

- 98.01 of the variance can be explained by the predictor variables
- the linear trend is significant because the p-value $2.2 \cdot 10^{-16}$ is smaller than 10^{-12}

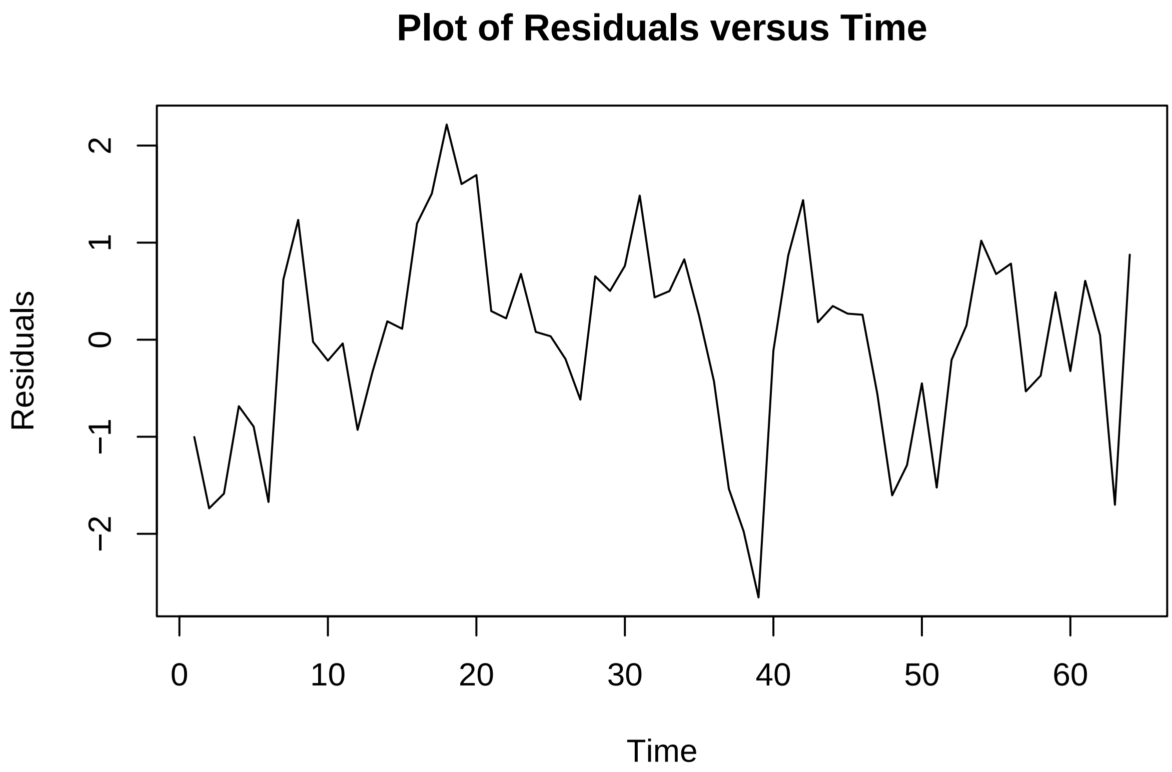
```r
plot(rstudent(model3), x = as.vector(time(winnebago)), xlab="Time", ylab="Residuals", main="Standardized
points(y=rstudent(model3), x=as.vector(time(winnebago)), pch = as.vector(season(winnebago)))
```



Interpretation:

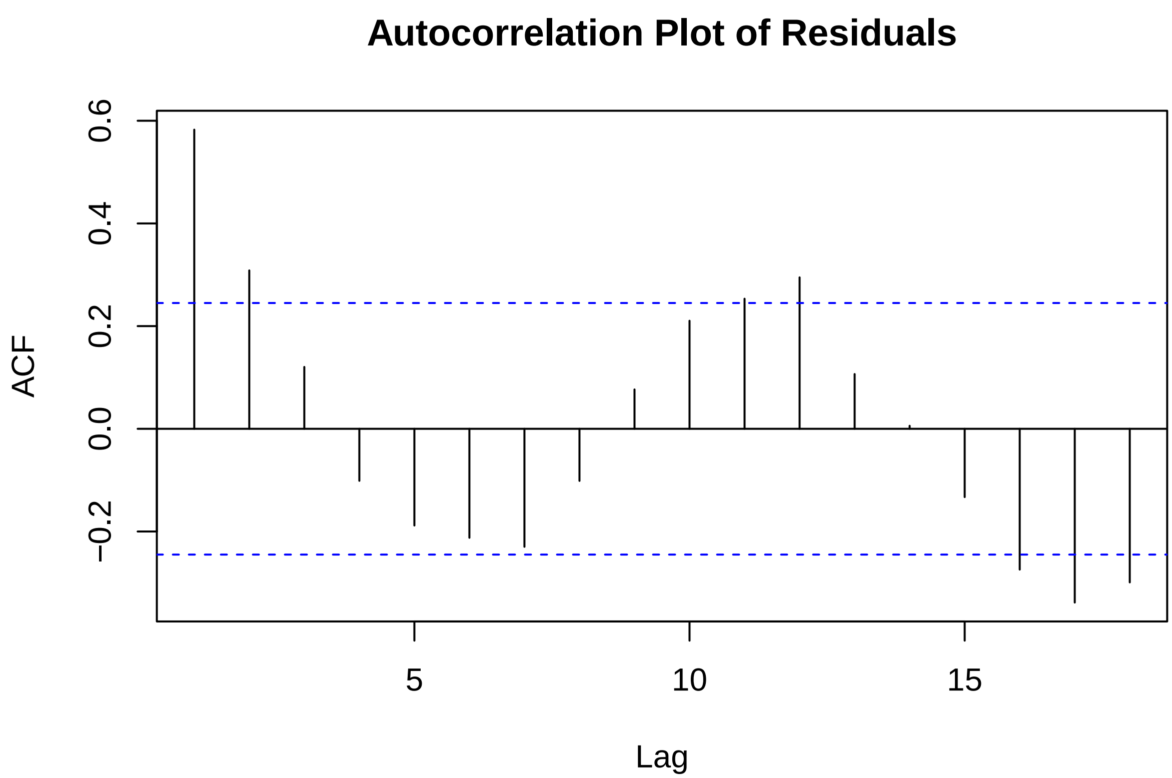
g)

```r
res2 <- as.ts(rstandard(model2))
plot(res2, xlab=expression("Time"), ylab=expression("Residuals"), main="Plot of Residuals versus Time")
```



h)

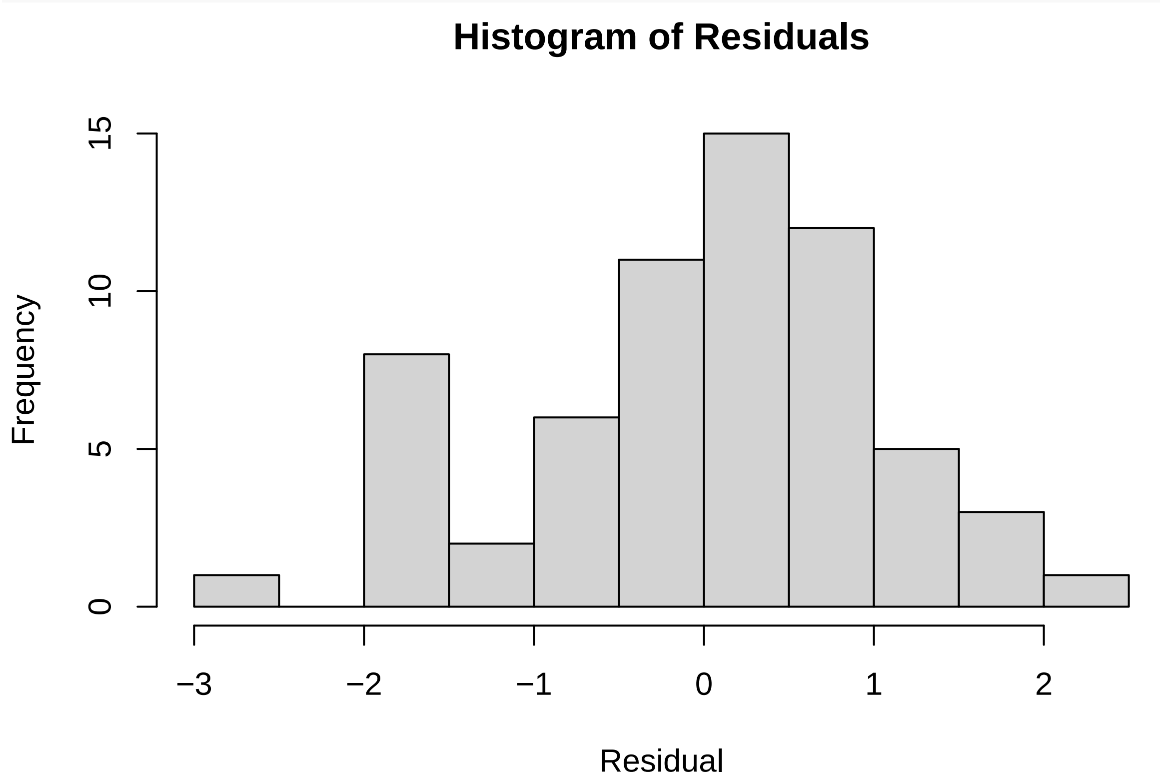
```
acf(res2, main="Autocorrelation Plot of Residuals")$acf
```



```
## , , 1
##
##                [,1]
##  [1,]  0.582621682
##  [2,]  0.308516762
##  [3,]  0.120483855
##  [4,] -0.101296290
##  [5,] -0.188376915
##  [6,] -0.212242495
##  [7,] -0.229810429
##  [8,] -0.101442132
##  [9,]  0.076663936
## [10,]  0.210470866
## [11,]  0.253334338
## [12,]  0.294944901
## [13,]  0.106610155
## [14,]  0.005900711
## [15,] -0.132994388
## [16,] -0.274233631
## [17,] -0.338397733
## [18,] -0.299142743
```

h) i)

```r
hist(res2, xlab="Residual", main="Histogram of Residuals")
```




```r
shapiro.test(res2)
```

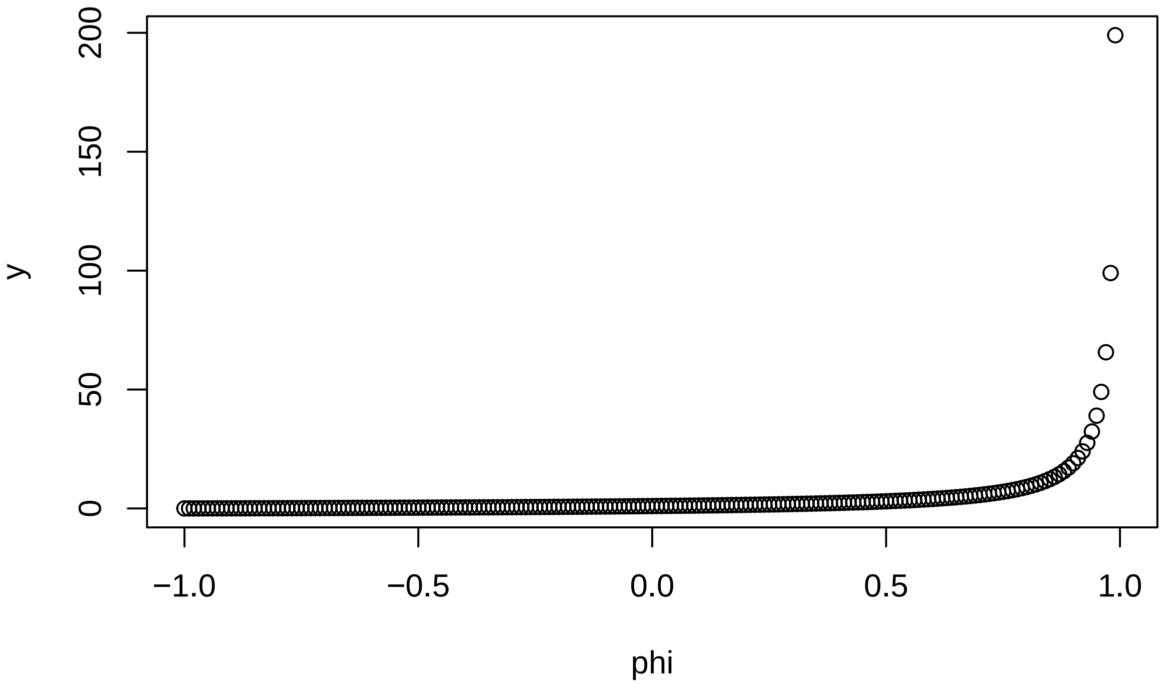
```
##
##  Shapiro-Wilk normality test
```

```
##
## data:  res2
## W = 0.97939, p-value = 0.3603
```

Exercise 3

c)

```
phi <- seq(-1, 1, by = 0.01)
y <- (1 + phi)/(1-phi)
plot(phi, y)
```



The closer ϕ is to -1, the closer the variance is to 0, thus the precision increases. The closer ϕ is to 1, the more the variance increases, and thus the precision decreases.