Problem Solving Set 8

Sven Bergmann

March 25, 2024

```
library(TSA)
library(tseries)
```

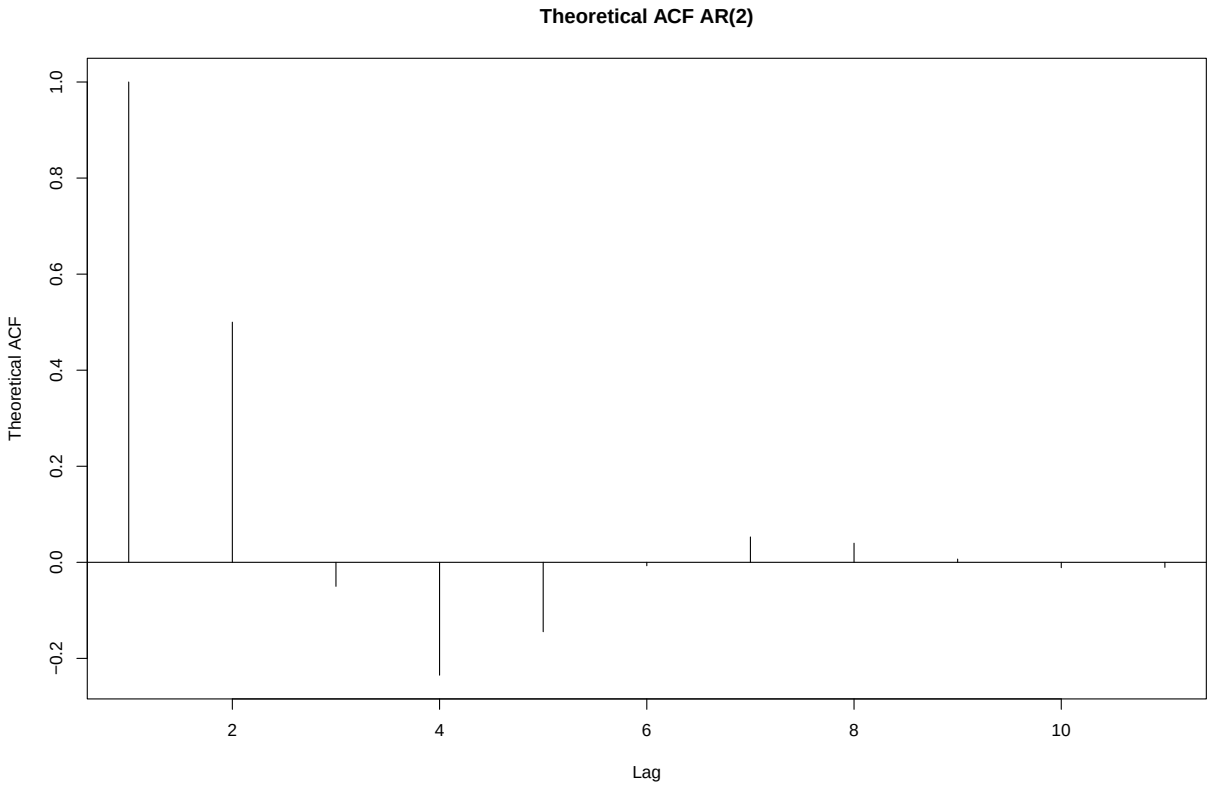
```
set.seed(1172)
```

Problem 7

```
phi <- c(0.7, -0.4)
ar2.series <- arima.sim(n = 72, list(ar = phi, sd = 1))
```

Part a

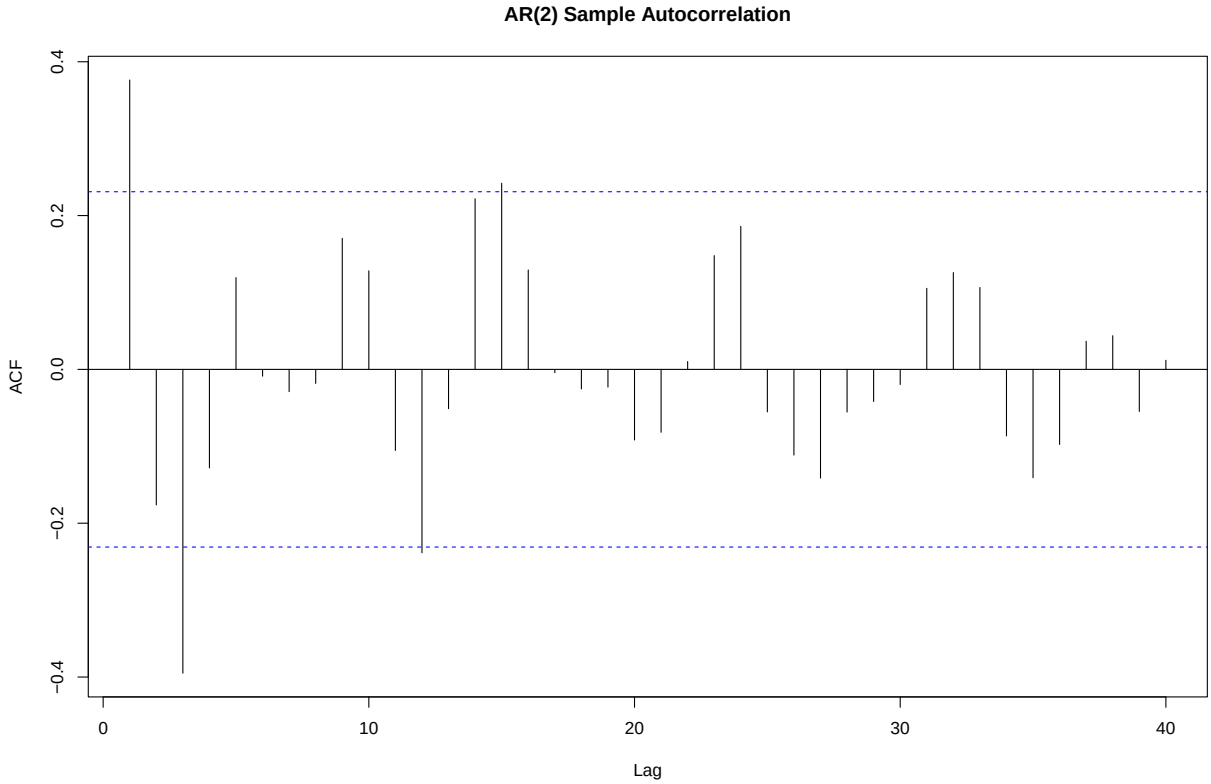
```
ar2acf <- ARMAacf(ar = c(0.7, -0.4), lag.max = 10, pacf = FALSE)
plot(ar2acf, xlab = "Lag", ylab = "Theoretical ACF", main = "Theoretical ACF AR(2)",
    type = "h")  # type = 'h' gives the vertical lines
abline(h = 0)
```



- dies out beyond lag 5

Part b

```
acf.samp <- acf(ar2.series, main = "AR(2) Sample Autocorrelation",
    lag.max = 40)
```




```
acf.samp$acf
```

```
## , , 1
##
##               [,1]
##  [1,]  0.376267052
##  [2,] -0.176240247
##  [3,] -0.395018754
##  [4,] -0.128121967
##  [5,]  0.119350052
##  [6,] -0.008797377
##  [7,] -0.028968715
##  [8,] -0.018252717
##  [9,]  0.170357498
## [10,]  0.128176562
## [11,] -0.105461926
## [12,] -0.238476150
## [13,] -0.051144870
## [14,]  0.221817603
## [15,]  0.242047495
## [16,]  0.129295090
## [17,] -0.004325803
## [18,] -0.025417193
## [19,] -0.023029172
## [20,] -0.091834888
## [21,] -0.081772240
```

```
## [22,]  0.010182142
## [23,]  0.147924465
## [24,]  0.186034643
## [25,] -0.055295383
## [26,] -0.111444236
## [27,] -0.141318072
## [28,] -0.055509945
## [29,] -0.041740479
## [30,] -0.019626443
## [31,]  0.105432853
## [32,]  0.125879280
## [33,]  0.106506811
## [34,] -0.086518645
## [35,] -0.140912832
## [36,] -0.097623296
## [37,]  0.036558353
## [38,]  0.043789925
## [39,] -0.054671199
## [40,]  0.011847240
```

- "cuts off" after lag 1
- dies out way slower than the theoretical autocorrelation
- due to short series / small sample size
- meaning if we take a longer series, e.g. 7200 it dies out way faster

Part c

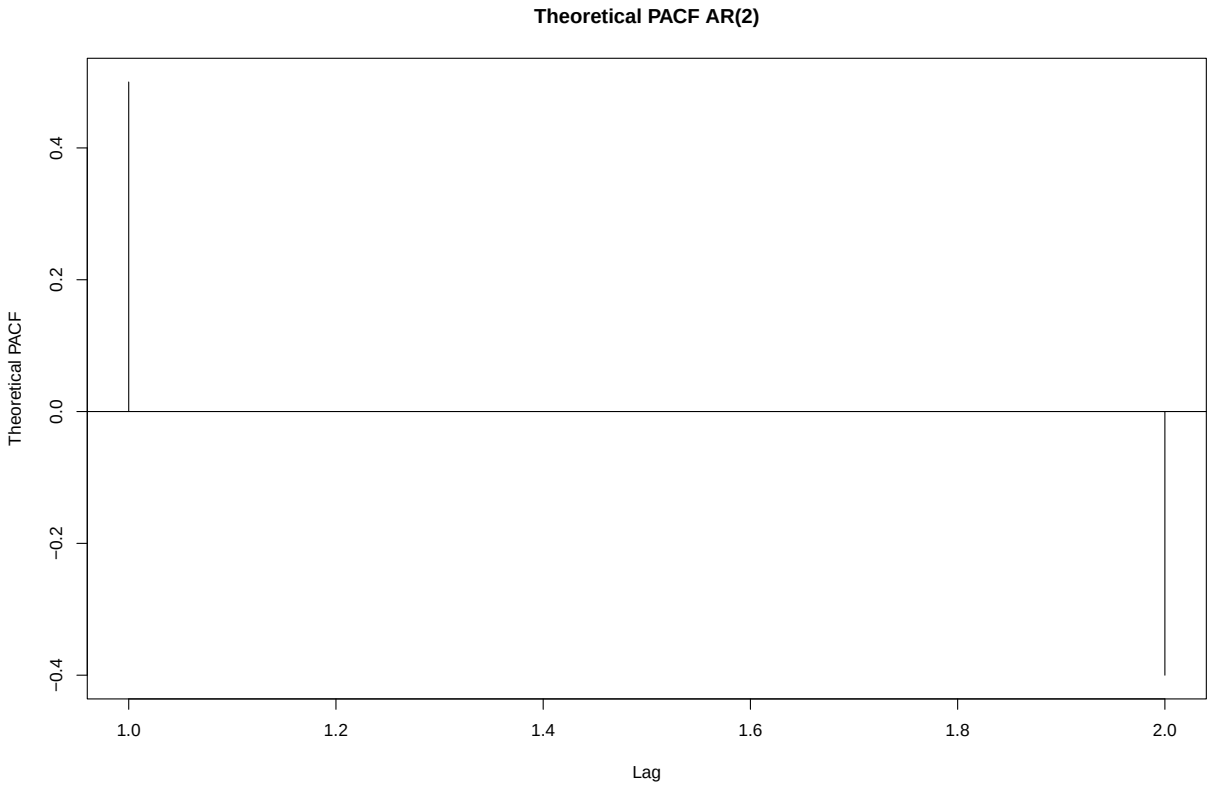
```
rho12 <- ar2acf[2:3]
rho1 <- phi[1]/(1 - phi[2])
rho2 <- (rho12[2] - rho12[1]^2)/(1 - rho12[1]^2)
rho1
```

```
## [1] 0.5
```

```
rho2
```

```
##    2
## -0.4
```

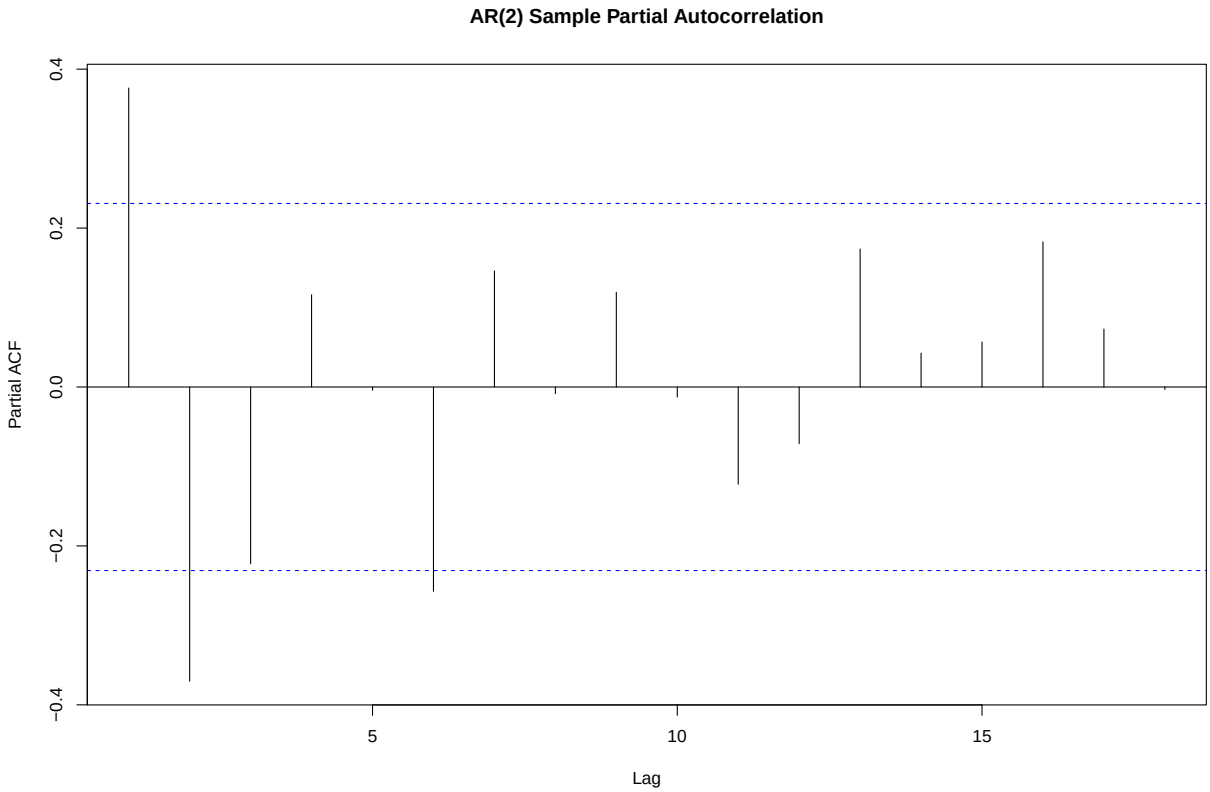
```
ar2pacf <- ARMAacf(ar = c(0.7, -0.4), pacf = TRUE)
plot(ar2pacf, xlab = "Lag", ylab = "Theoretical PACF", main = "Theoretical PACF AR(2)",
    type = "h")
abline(h = 0)
```



- PACF is denoted in the book also as ϕ but with to subscripts
- $\phi_{1,1} = \frac{\phi_1}{1-\phi_2}$
- $\phi_{2,2} = \frac{\rho_2 - \rho_1^2}{1-\rho_1^2}$
- cuts off after lag 2 in theory

Part d

```
samp.pacf <- pacf(ar2.series, main = "AR(2) Sample Partial Autocorrelation")
```



- does not cut off after lag 2
- significant autocorrelation at lag 6
- does not match theoretical PACF well

```
eacf(ar2.series)
```

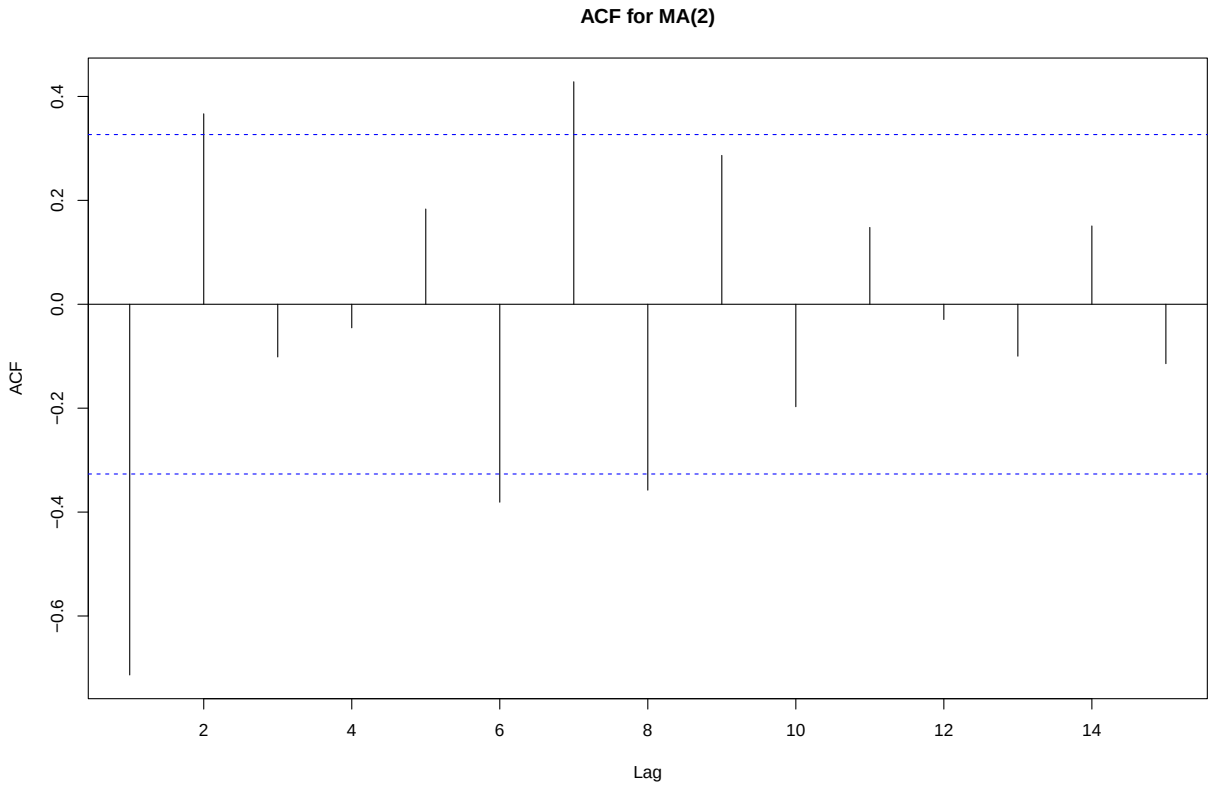
```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x o x o o o o o o o o  o  o  o
## 1 x o x o o o o o o o o  o  o  o
## 2 x x o o o o o o o o o  o  o  o
## 3 x o o o o o o o o o o  o  o  o
## 4 o o o o o o o o o o o  o  o  o
## 5 o x x o o o o o o o o  o  o  o
## 6 x x x x o o o o o o o  o  o  o
## 7 o x o o o o o o o o o  o  o  o
```

Problem 8

```
theta <- c(-0.7, 0.4)
ma2 <- arima.sim(n = 36, list(ma = theta, sd = 1))
ma2acf <- ARMAacf(ma = theta, lag.max = 4, pacf = FALSE)
ma2acf
```

```
##           0           1           2           3           4
##  1.0000000 -0.5939394  0.2424242  0.0000000  0.0000000
```

```r
acfs <- acf(ma2, main = "ACF for MA(2)")
```




```r
acfs$acf
```

```
## , , 1
##
##              [,1]
##  [1,] -0.71348894
##  [2,]  0.36657301
##  [3,] -0.10135395
##  [4,] -0.04529777
##  [5,]  0.18330605
##  [6,] -0.38093757
##  [7,]  0.42825899
##  [8,] -0.35773618
##  [9,]  0.28651235
## [10,] -0.19716301
## [11,]  0.14788039
## [12,] -0.02933188
## [13,] -0.09989288
## [14,]  0.15071381
## [15,] -0.11428146
```

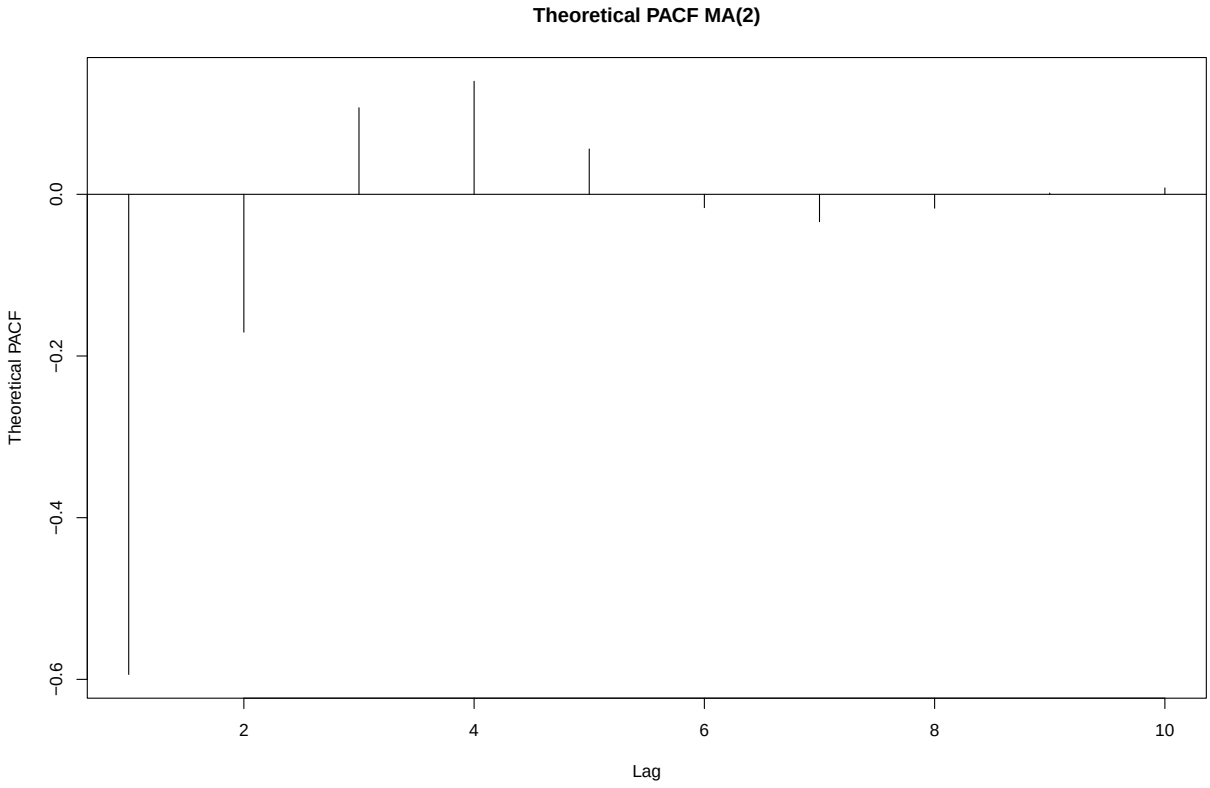
- matches theoretical ACF really well.

```r
acfma2 <- acfs$acf
```

```r
pacfMA2 <- ARMAacf(ma = theta, lag.max = 10, pacf = TRUE)
pacfMA2
```

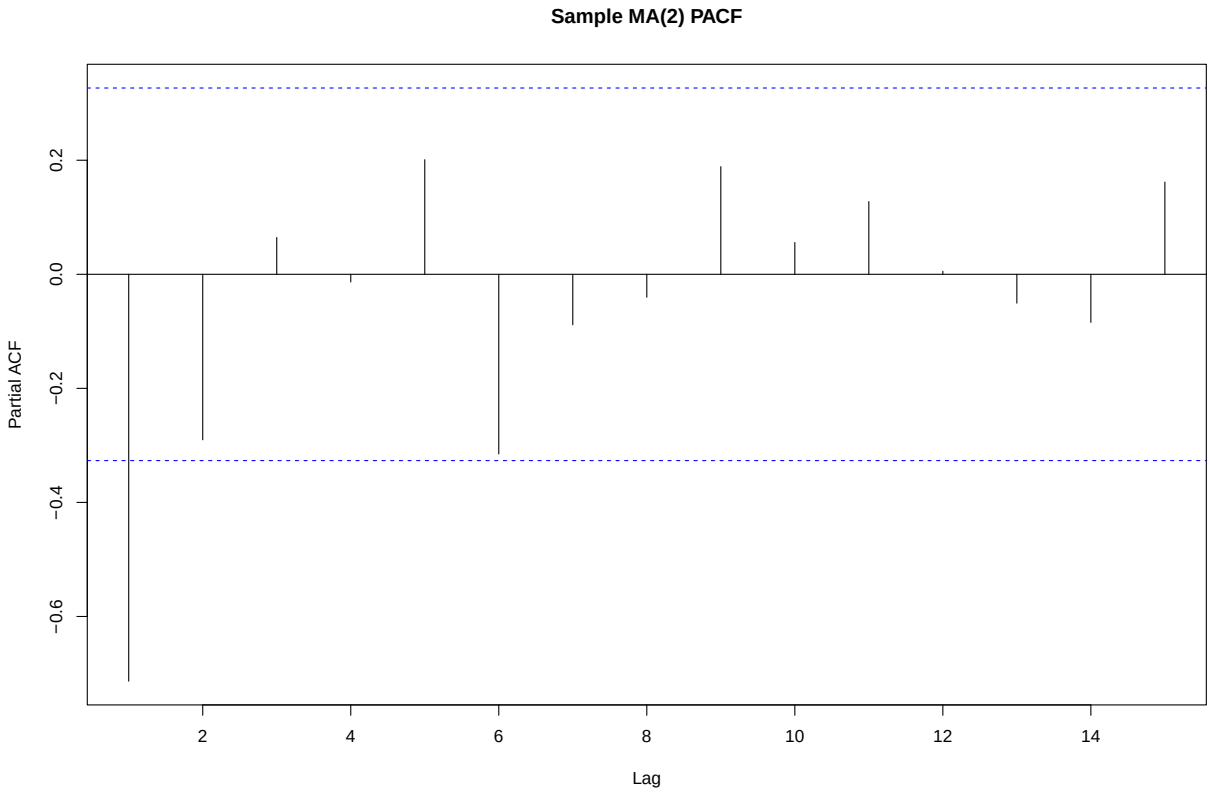
```
##  [1] -0.593939394 -0.170478406  0.107057809  0.139839218  0.056075197
##  [6] -0.016471790 -0.033909675 -0.017157223  0.001549476  0.007946801
```

```r
plot(pacfMA2, xlab = "Lag", ylab = "Theoretical PACF", main = "Theoretical PACF MA(2)",
    type = "h")
abline(h = 0)
```



- dies out

```r
pacf.samp <- pacf(ma2, main = "Sample MA(2) PACF")
```




```
pacfs <- pacf.samp$acf
pacfs
```

```
## , , 1
##
##                [,1]
##  [1,] -0.713488941
##  [2,] -0.290250005
##  [3,]  0.064539513
##  [4,] -0.013484181
##  [5,]  0.200975272
##  [6,] -0.315034338
##  [7,] -0.088599581
##  [8,] -0.040230700
##  [9,]  0.188764959
## [10,]  0.055740058
## [11,]  0.127552437
## [12,]  0.005394857
## [13,] -0.050644177
## [14,] -0.084388074
## [15,]  0.161770225
```

- match up really well with the theoretical PACF

```
# eacf(ma2)
```

Specification

- fit MA(1), AR(1), ARMA(1,1)
- see what works best

Problem 9

Part a

```
theor.ac <- ARMAacf(ar = c(0.7), lag.max = 5, pacf = FALSE)
theor.pac <- ARMAacf(ar = c(0.7), lag.max = 5, pacf = TRUE)
set.seed(2)
ar.series <- arima.sim(n = 48, list(ar = 0.7, sd = 1))
```

Part b

```
acs <- acf(ar.series, plot = F)$acf[c(1, 5)]
acs
```

```
## [1]  0.64589662 -0.01197047
```

Part c

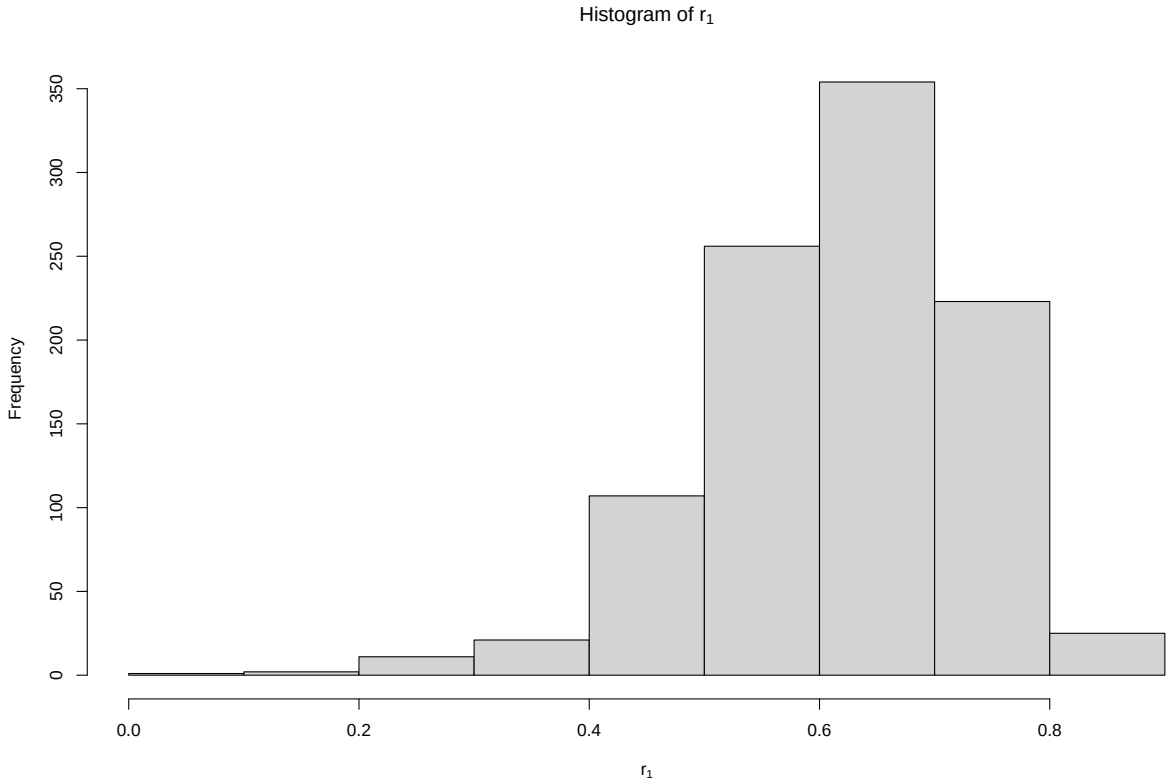
```
ar.series2 <- arima.sim(n = 48, list(ar = 0.7, sd = 1))
acs2 <- acf(ar.series2, plot = F)$acf[c(1, 5)]
acs2
```

```
## [1] 0.6258431 0.1324742
```

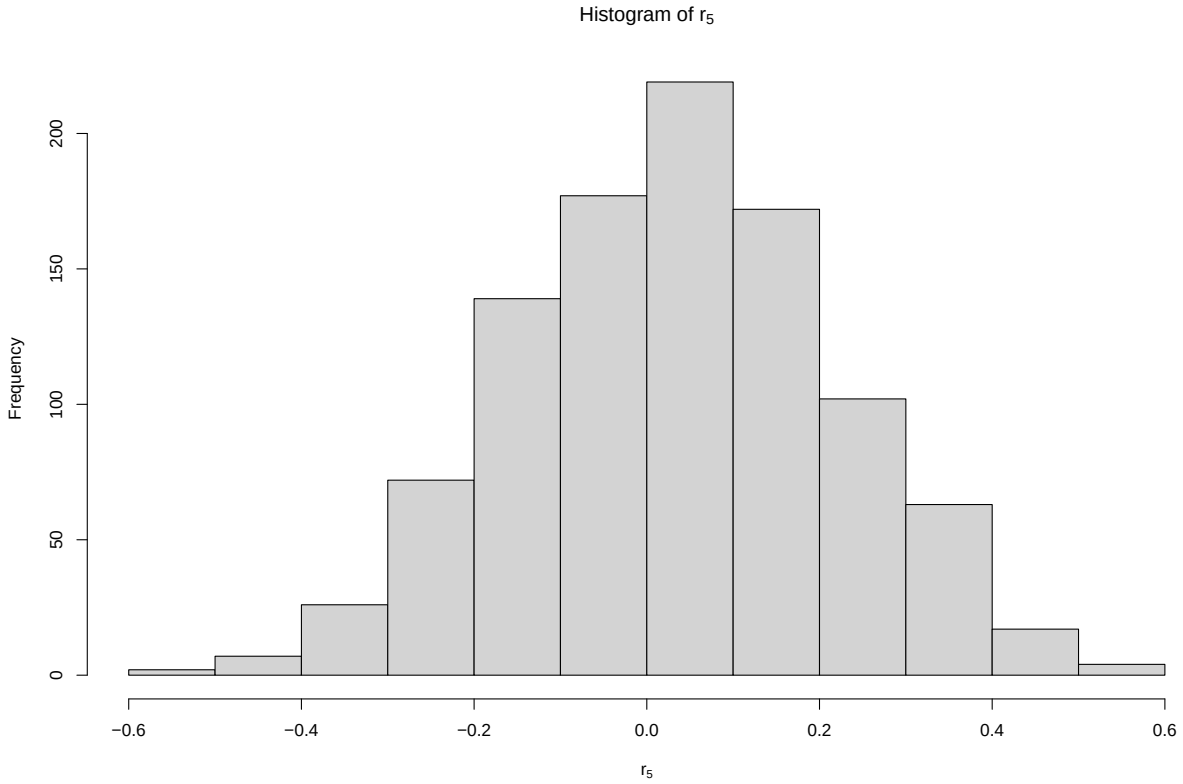
Part d

```
m <- 1000
ac1 <- NULL
ac5 <- NULL
for (i in 1:m) {
    ar.s <- arima.sim(n = 48, list(ar = 0.7, sd = 1))
    ac1 <- c(ac1, acf(ar.s, plot = F)$acf[1])
    ac5 <- c(ac5, acf(ar.s, plot = F)$acf[5])

}
par(mfrow = c(1, 1))
hist(ac1, xlab = expression(r[1]), main = expression(paste("Histogram of ",
    r[1])))
```




```
hist(ac5, xlab = expression(r[5]), main = expression(paste("Histogram of ",
    r[5])))
```




```
var(ac1)
```

```
## [1] 0.01301911
```

```
var(ac5)
```

```
## [1] 0.03445817
```