Problem Solving Set 7

Sven Bergmann

March 11, 2024

```
library(TSA)
```

```
##
## Attaching package: 'TSA'
```

```
## The following objects are masked from 'package:stats':
##
##     acf, arima
```

```
## The following object is masked from 'package:utils':
##
##     tar
```

```
library(MASS)
```

Problem 2

```
wn <- c(0.63, -1.25, 1.8, 1.51, 1.56, 0.62, 0.64, -0.98)
Y <- wn[1]
for (i in 2:length(wn)) {
    Y <- c(Y, 3 * Y[i - 1] + wn[i])
    print(Y)
}
```

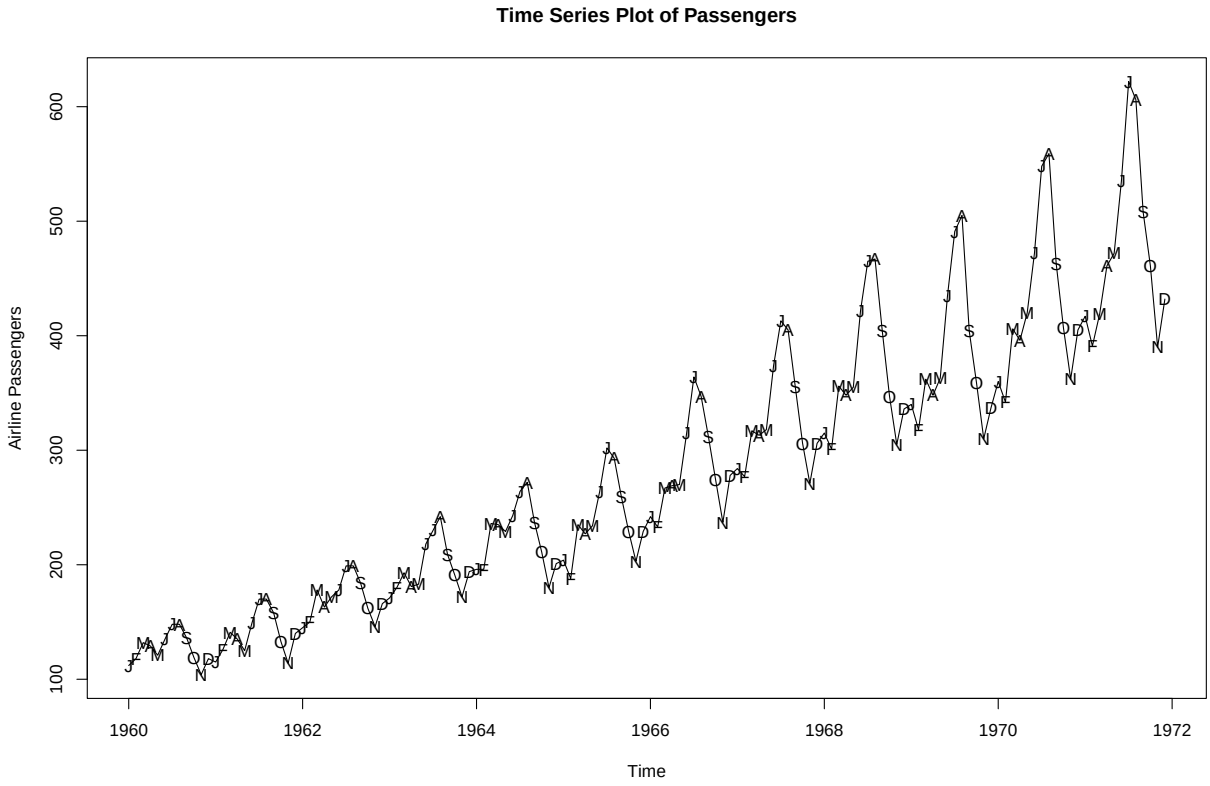
```
## [1] 0.63 0.64
## [1] 0.63 0.64 3.72
## [1]  0.63  0.64  3.72 12.67
## [1]  0.63  0.64  3.72 12.67 39.57
## [1]   0.63   0.64   3.72  12.67  39.57 119.33
## [1]   0.63   0.64   3.72  12.67  39.57 119.33 358.63
## [1]    0.63    0.64    3.72   12.67   39.57  119.33  358.63 1074.91
```

Problem 5

```r
data(airpass)
```

a)

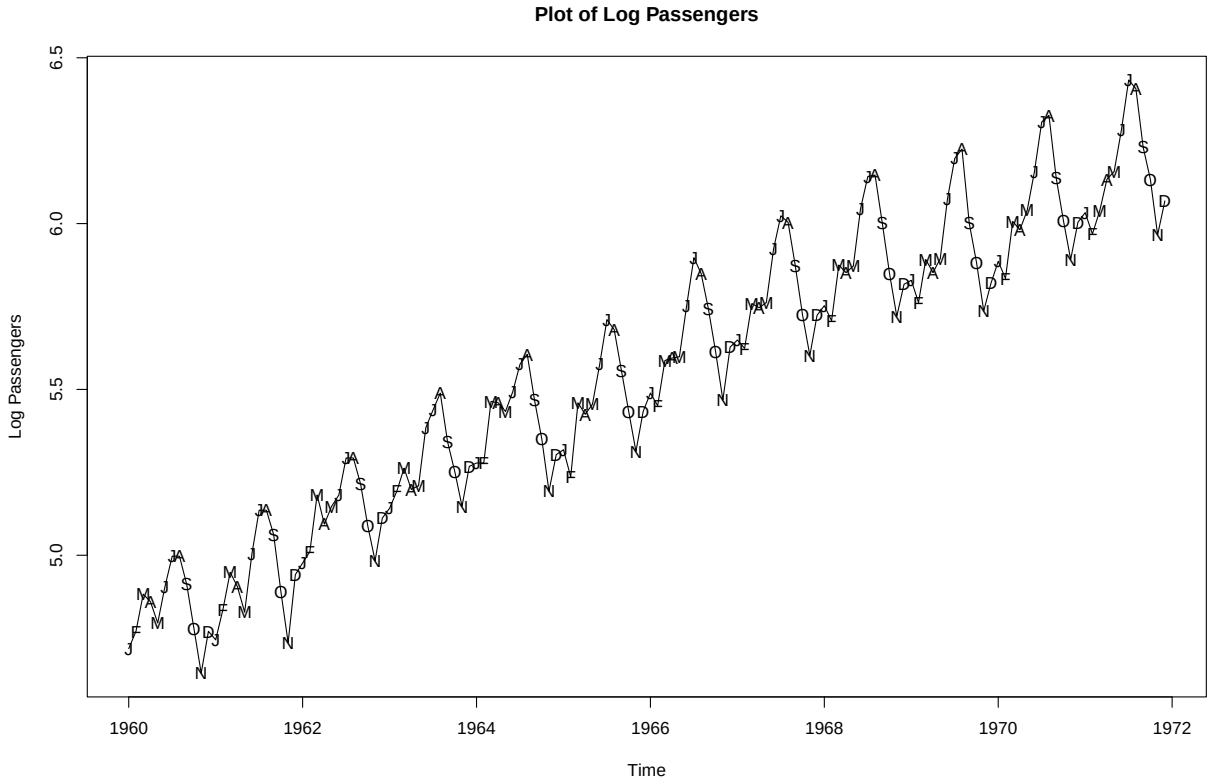
```r
plot(airpass, xlab = "Time", ylab = "Airline Passengers", main = "Time Series Plot of Passengers")
points(y = airpass, x = time(airpass), pch = as.vector(season(airpass)))
```



- seasonality (peaks in warmer months)
- upward linear trend
- increasing spread over time -> not stationary

b)

```r
logs <- as.ts(log(airpass))
plot(logs, xlab = "Time", ylab = "Log Passengers", main = "Plot of Log Passengers")
points(y = logs, x = time(logs), pch = as.vector(season(logs)))
```

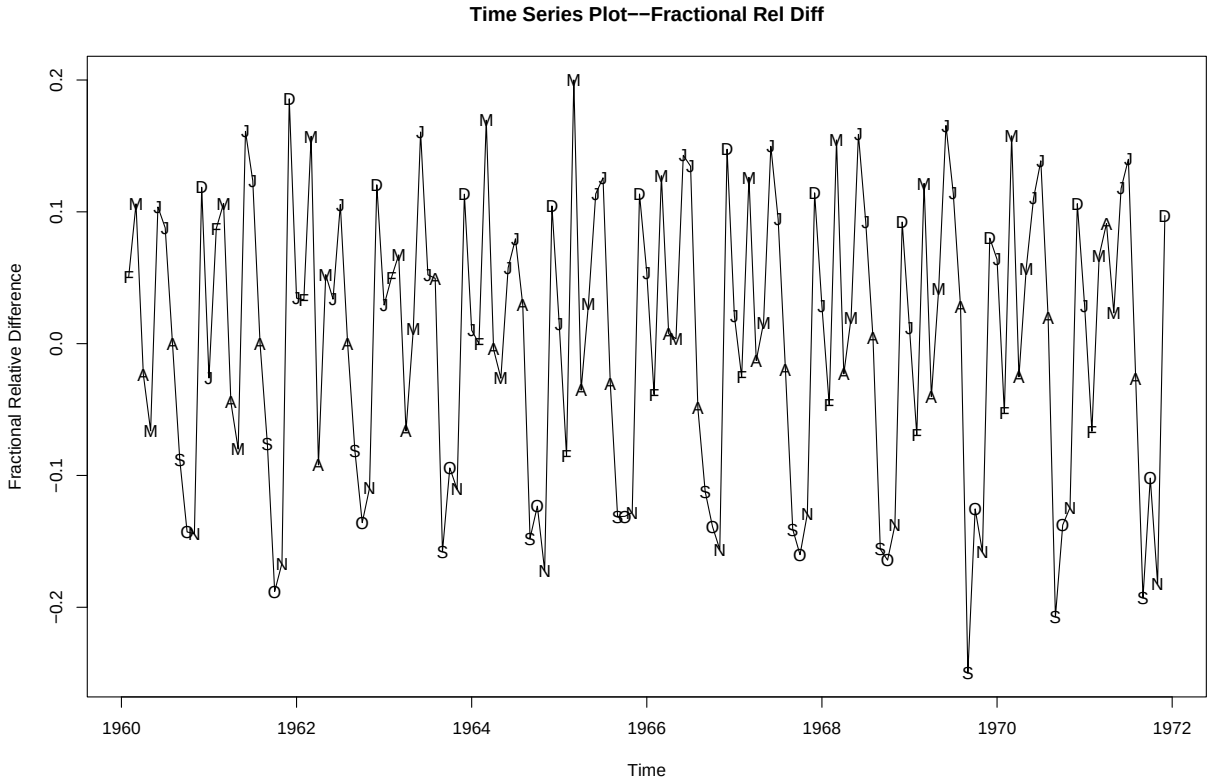


- positive trend
- seasonality
- spread fairly even now

c)

```
diffs <- diff(airpass, 1)
airpass.2 <- ts(airpass[-1], start = c(1960, 2), end = c(1971,
    12), frequency = 12)
frac.rel <- ts(diffs/airpass.2, start = c(1960, 2), end = c(1971,
    12), frequency = 12)
```

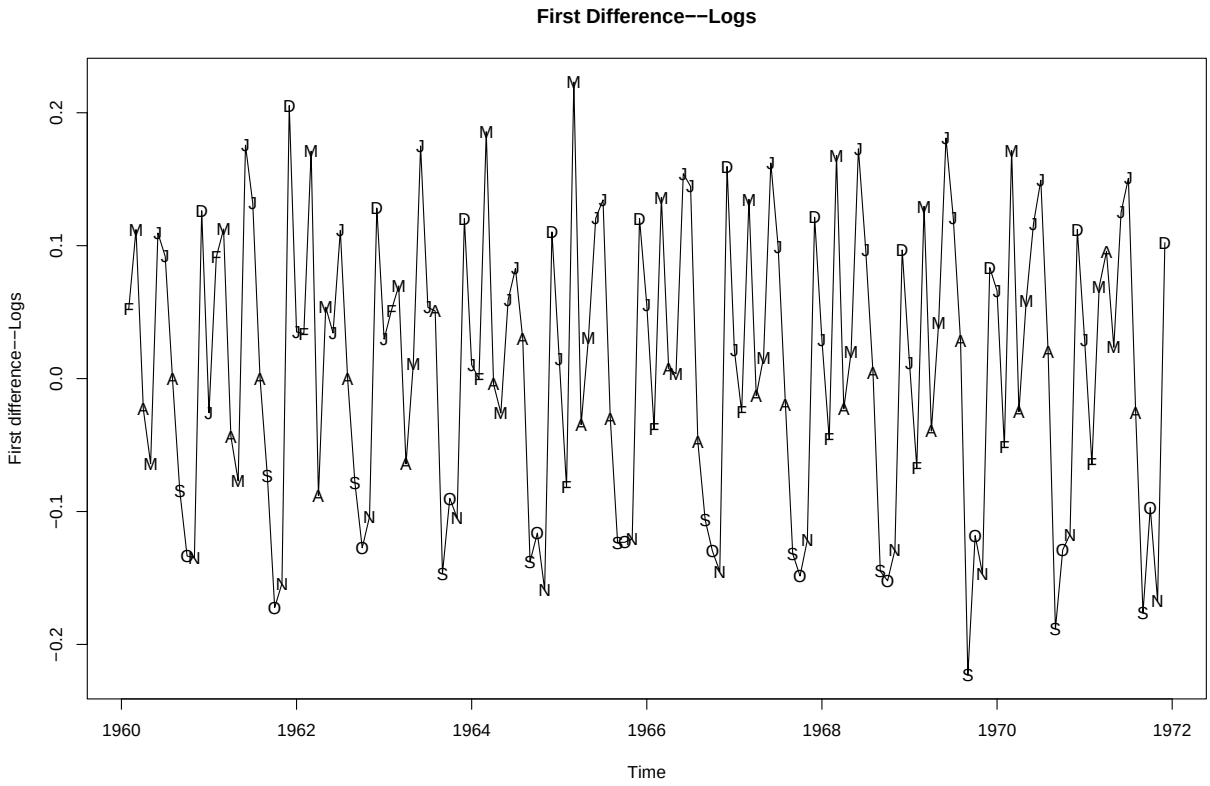
```
plot(frac.rel, xlab = "Time", ylab = "Fractional Relative Difference",
    main = "Time Series Plot--Fractional Rel Diff")
points(y = frac.rel, x = time(frac.rel), pch = as.vector(season(frac.rel)))
```



$\frac{Y_t - Y_{t-1}}{Y_{t-1}}$

- seasonality
- "random" scatter about 0
- fairly even spread

```
plot(diff(logs), xlab = "Time", ylab = "First difference--Logs",
    main = "First Difference--Logs")
points(y = diff(logs), x = time(diff(logs)), pch = as.vector(season(diff(logs))))
```



$\bigtriangledown \log(Y_t)$

- nearly indistinguishable
- from fractional relative difference

Fit seasonal means model to transformed series

```
dif <- diff(logs)
months <- season(diff(logs))
model <- lm(dif ~ months)
```

Modeling Y_t

1. Use $\log(Y_t)$, remove linear trend

- fit seasonal means or use cosine trend

2. Use frac.rel.diff or $\bigtriangledown \log(Y_t)$

- fit seasonal means or use cosine trend

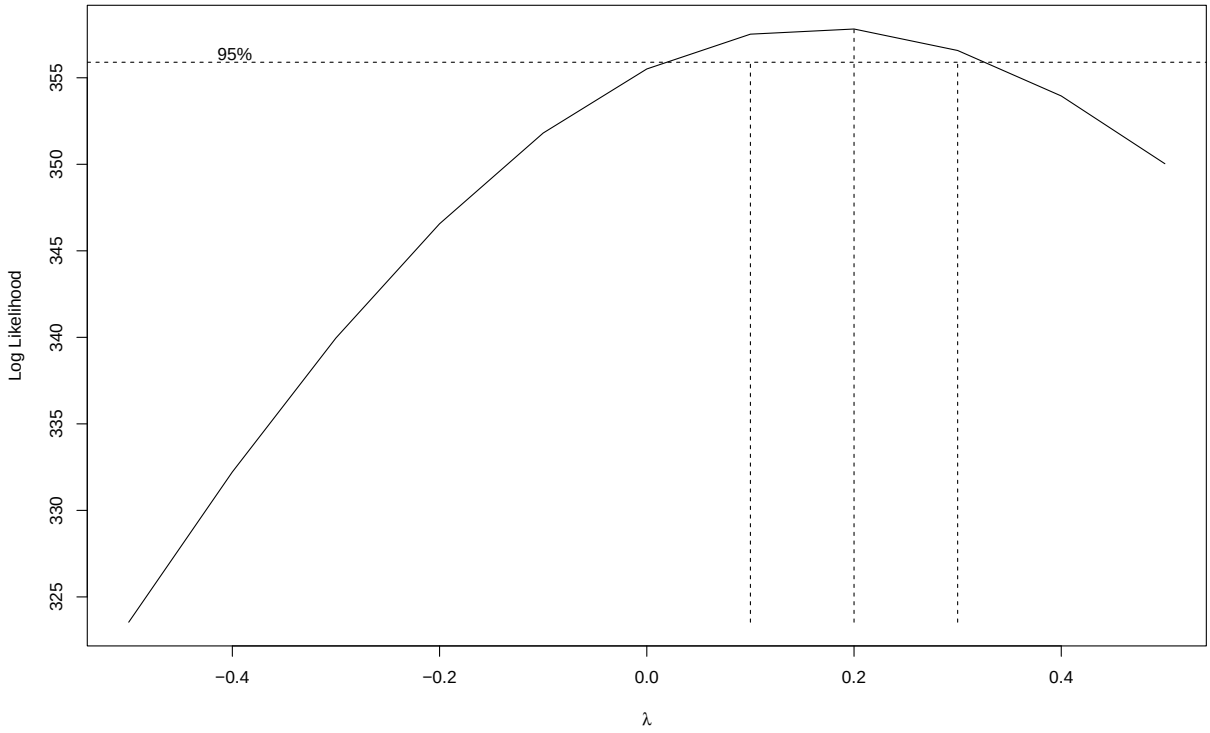
Problem 6

```
data(JJ)
```

```
plot(JJ, xlab = "Time", ylab = "Earnings Per Share", main = "Time Series Plot of Quarterly Earnings Per
points(y = JJ, x = time(JJ), pch = as.vector(season(JJ)))
```

- exponential or quadratic positive trend
- seasonality (peak in Q3, dips in Q4)
- variance increases over time: -> not stationary

```
lam <- seq(-0.5, 0.5, 0.1)
m <- BoxCox.ar(JJ, lambda = lam)$loglike
```



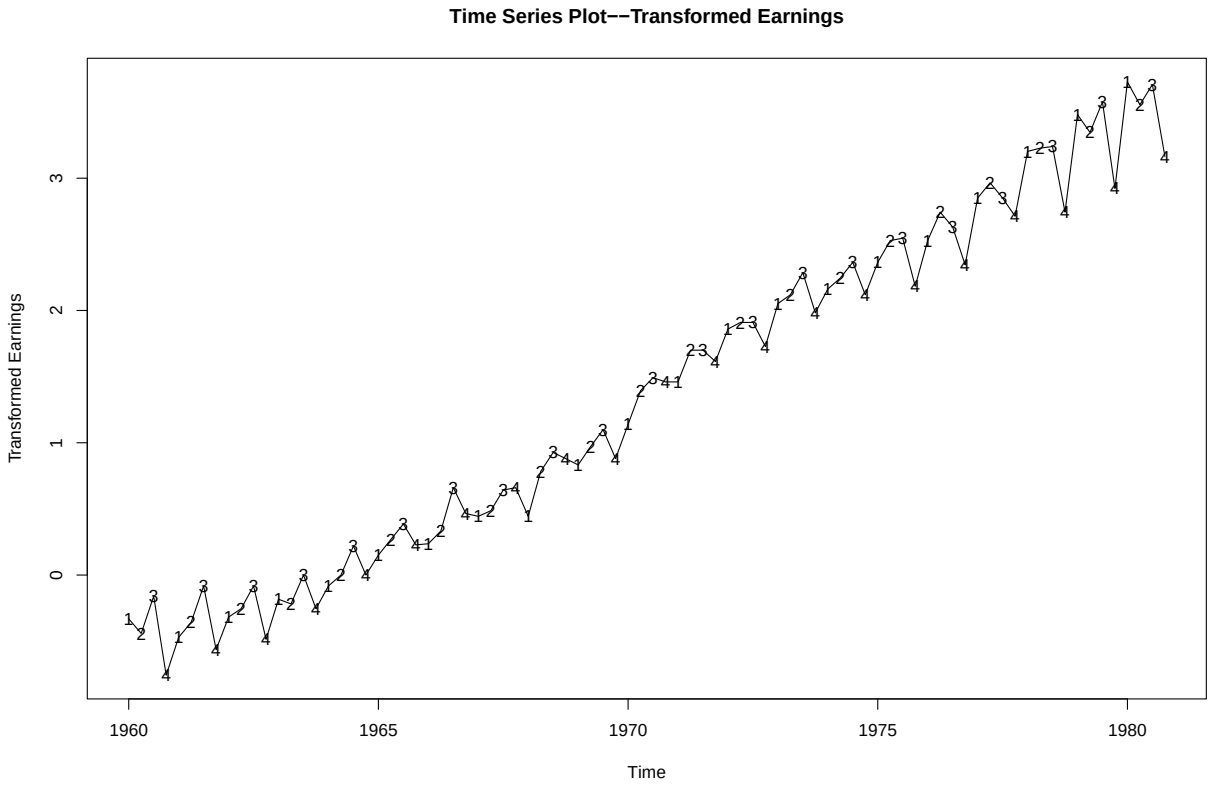
Box-Cox transformation

$y^* = \frac{y^\lambda - 1}{\lambda}$

```
lambda <- lam[which(m == max(m))]
transform <- (JJ^lambda - 1)/lambda
```

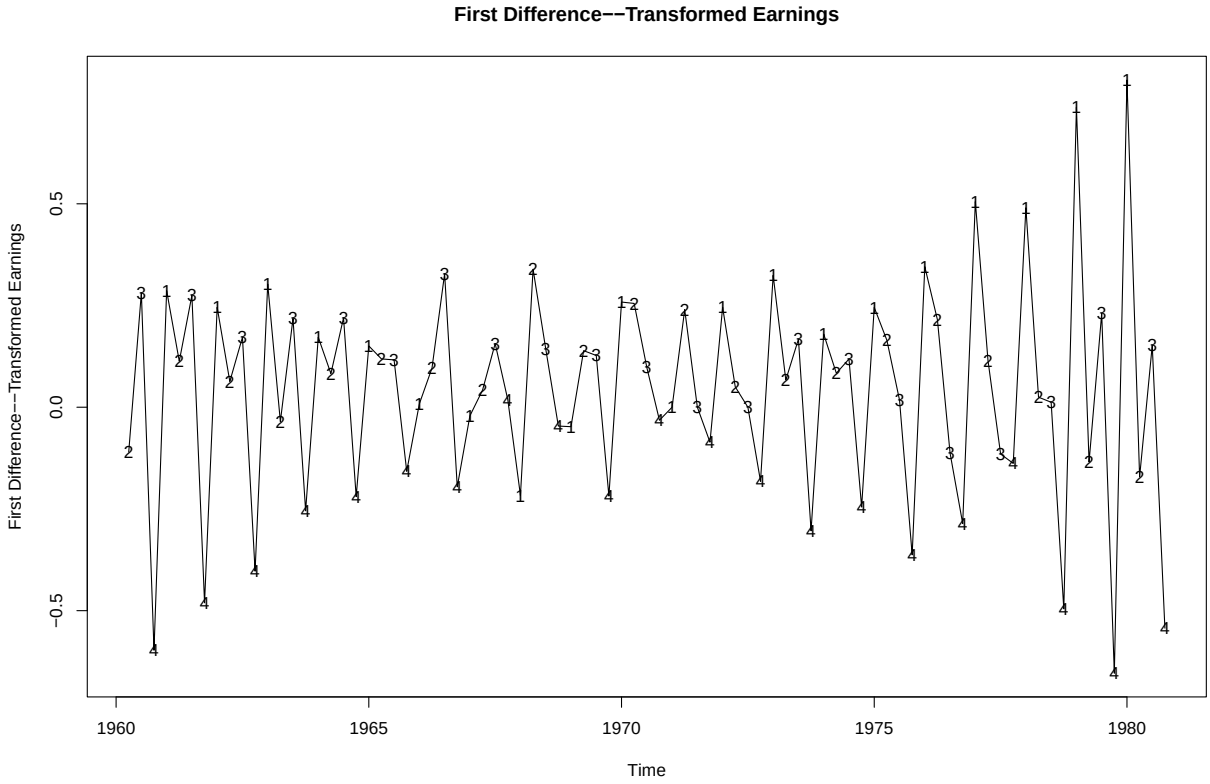
For $\hat{\lambda} = 0.2$: $y^* = \frac{y^{0.2} - 1}{0.2}$

```
plot(transform, xlab = "Time", ylab = "Transformed Earnings",
    main = "Time Series Plot--Transformed Earnings")
points(y = transform, x = time(transform), pch = as.vector(season(transform)))
```



- trend is now linear
- variance is a little more stable, still a bit uneven
- still seasonality

```
diffs <- diff(transform, 1)
plot(diffs, xlab = "Time", ylab = "First Difference--Transformed Earnings",
    main = "First Difference--Transformed Earnings")
points(y = diffs, x = time(diffs), pch = as.vector(season(diffs)))
```



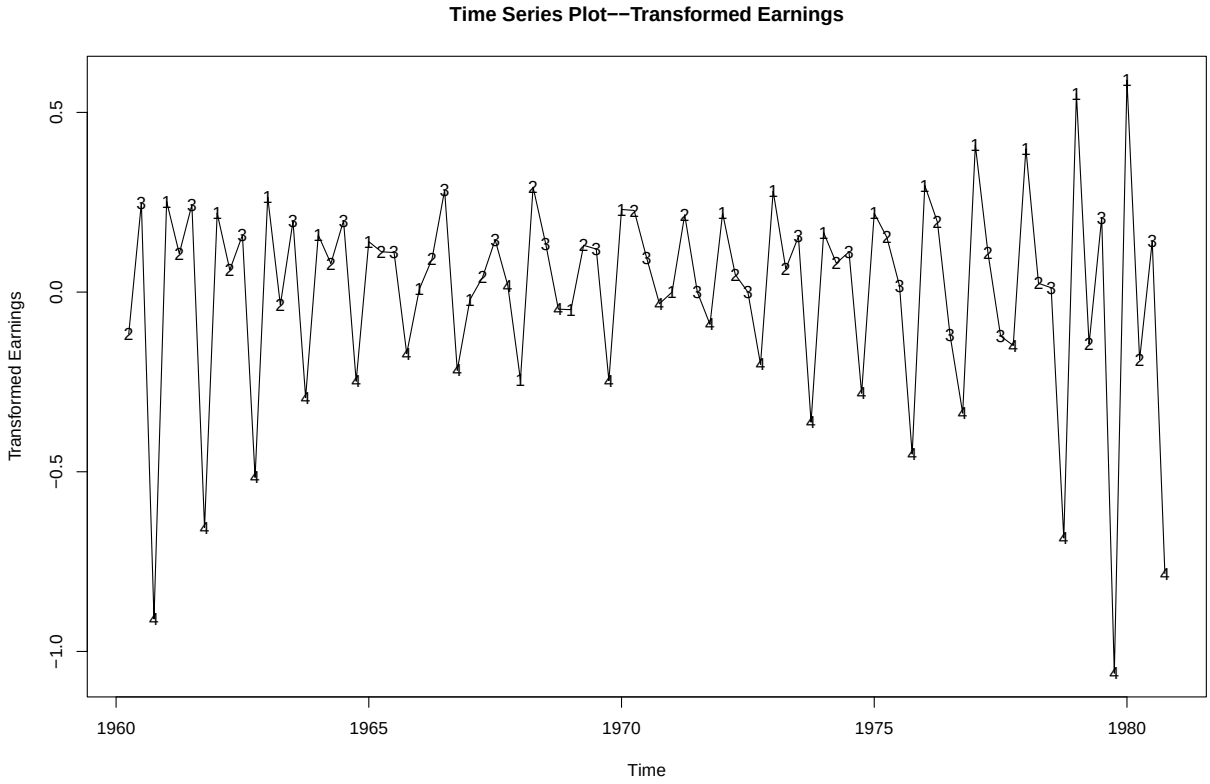
Possible fixes

- take difference to remove the linear trend
- cosine trend or seasonal means to address seasonality

What we actually see

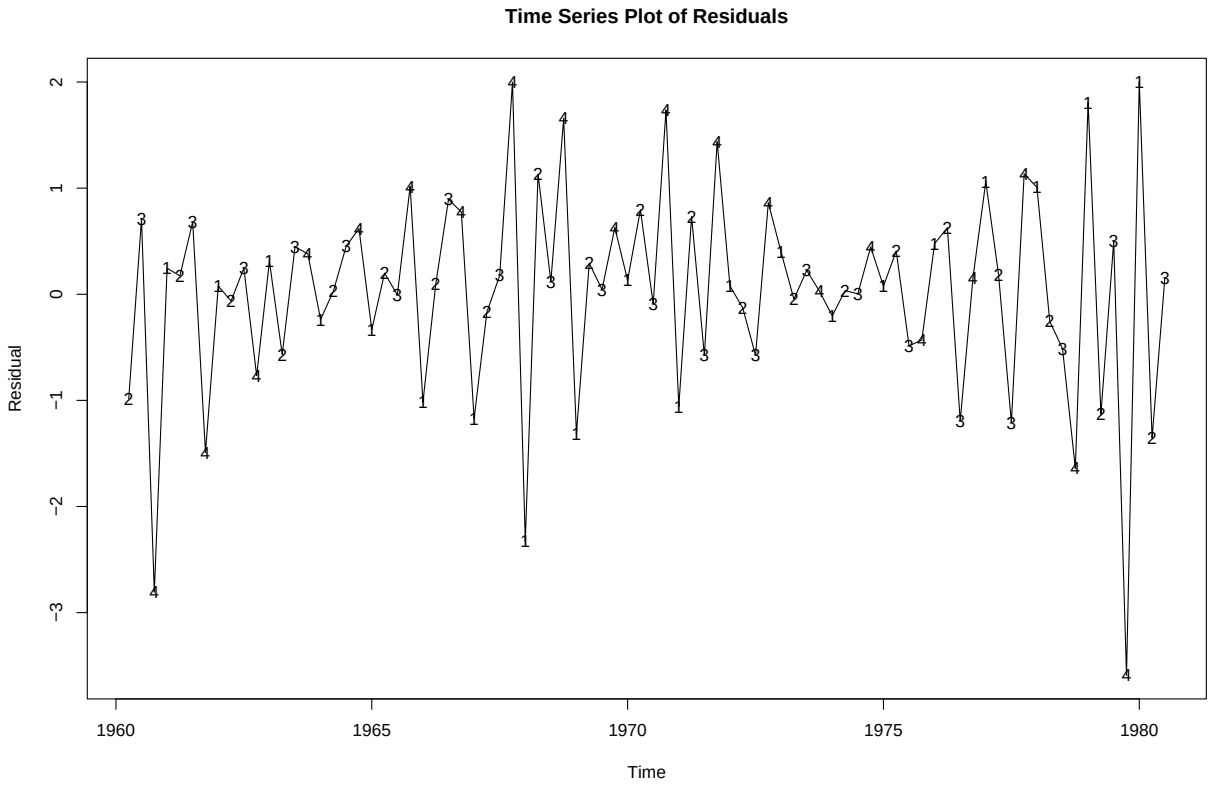
- difference took out the linear trend

```
diffs2 <- diffs + 1  # get rid of negative values
logs2 <- log(diffs2)
months <- season(logs2)
plot(logs2, xlab = "Time", ylab = "Transformed Earnings", main = "Time Series Plot--Transformed Earnings
points(y = logs2, x = time(logs2), pch = as.vector(season(logs2)))
```



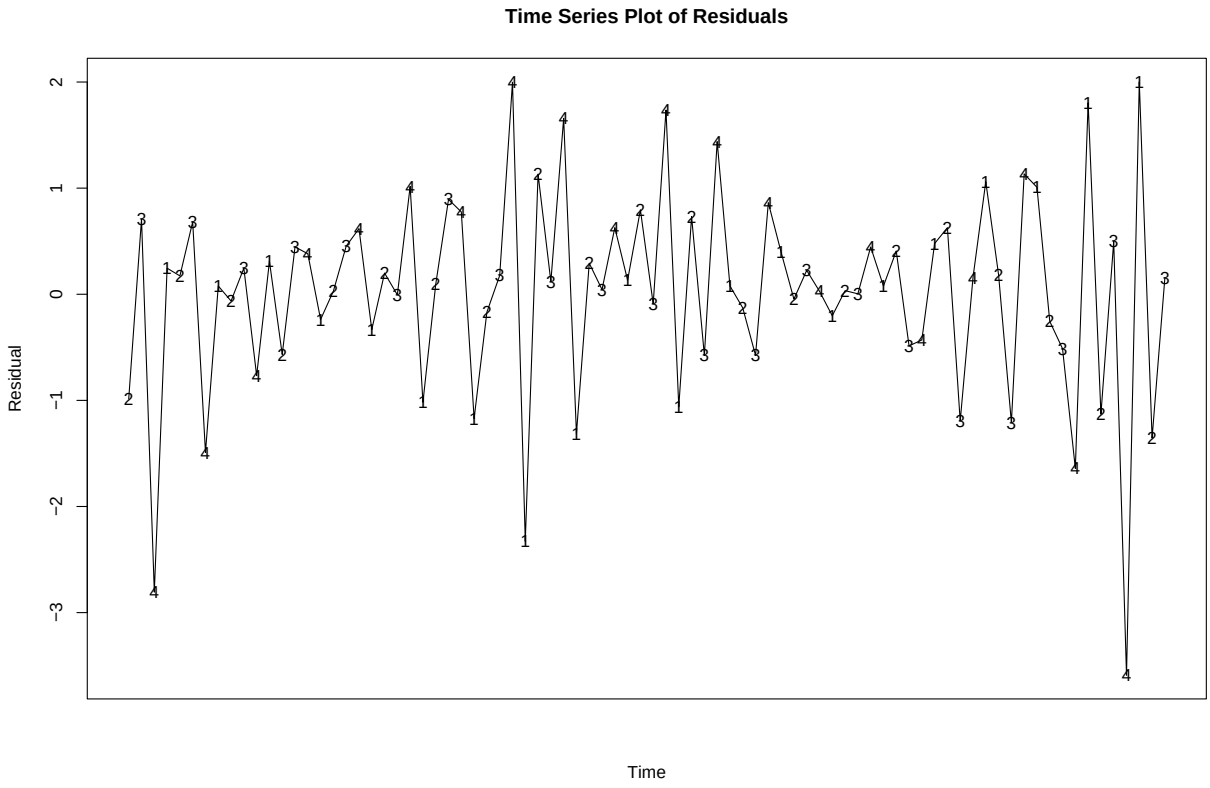
- log transform fared no better
- still have the bowtie pattern
- still have the seasonality

```
model2 <- lm(logs2 ~ months)
res <- ts(rstandard(model2), start = c(1960, 2), end = c(1980,
    3), frequency = 4)
plot(res, xlab = "Time", ylab = "Residual", main = "Time Series Plot of Residuals")
points(y = res, x = time(res), pch = as.vector(season(res)))
```

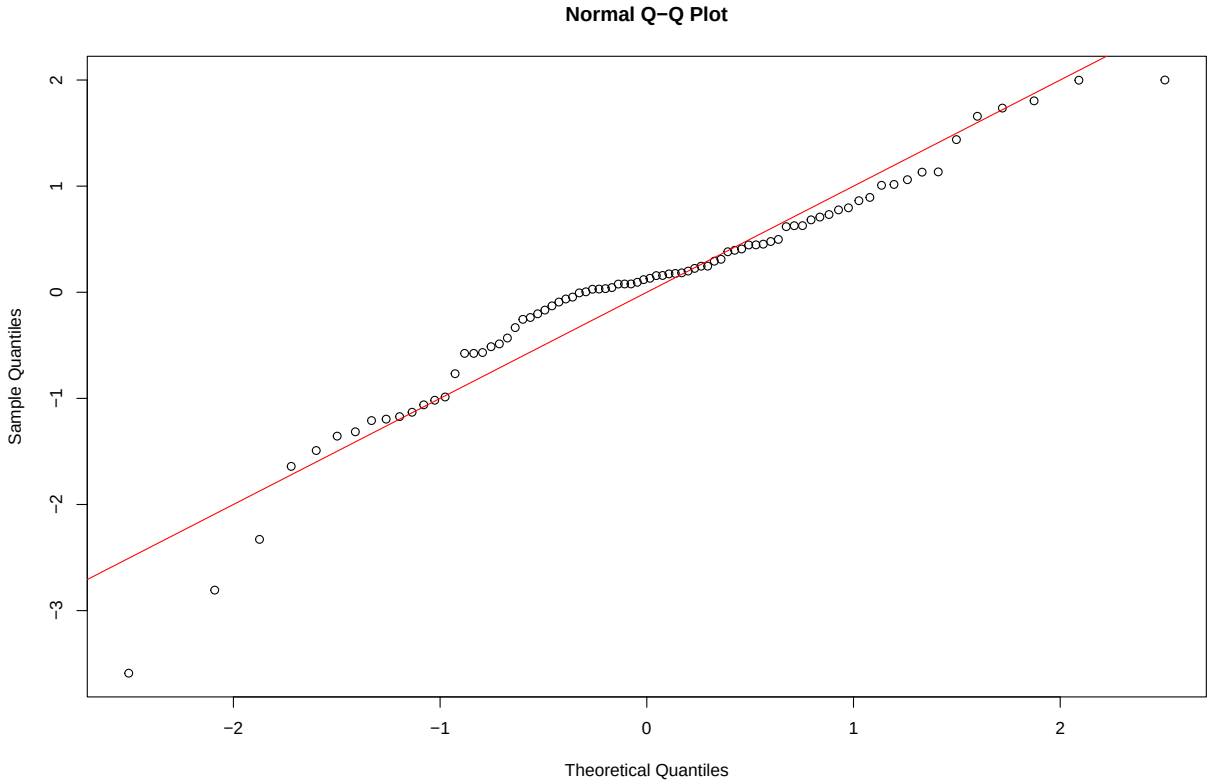


- fitting seasonal means fixes seasonality
- variance problem remains

```
plot(res, xaxt = "n", xlab = "Time", ylab = "Residual", main = "Time Series Plot of Residuals")
points(y = res, x = time(res), pch = as.vector(season(res)))
```




```r
qqnorm(res)
abline(a = 0, b = 1, col = "red")
```


Normal Q−Q Plot
Sample Quantiles
Theoretical Quantiles
2
1
0
−1
−2
−3
−2
−1
0
1
2