MTHSTAT 564/564G/764–Time Series Analysis
Spring 2024
Problem Solving Set 6

Please think about the following problems from the textbook in advance of our problem solving sessions on them:

Problem Solving 6

1. For the ARMA(1,2) model $Y_t = 0.8Y_{t-1} + e_t + 0.7e_{t-1} + 0.6e_{t-2}$, show that
   (a) $\rho_k = 0.8\rho_{k-1}$ for $k > 2$.
   (b) $\rho_2 = 0.8\rho_1 + 0.6\frac{\sigma_e^2}{\gamma_0}$.

2. Let $\{Y_t\}$ be a stationary process with $\rho_k = 0$ for $k > 1$. Show that we must have $|\rho_1| < \frac{1}{2}$. (Consider $\text{Var}(Y_{n+1} + Y_n + \ldots + Y_1)$ and then $\text{Var}(Y_{n+1} - Y_n + Y_{n-1} - \ldots \pm Y_1)$. Use the fact that both of these must be nonnegative for all n.)

3. Consider a process that satisfies the AR(1) equation $Y_t = \frac{1}{2}Y_{t-1} + e_t$.
   (a) Show that $Y_t = 10\left(\frac{1}{2}\right)^t + e_t + \frac{1}{2}e_{t-1} + \left(\frac{1}{2}\right)^2 e_{t-2} + \ldots$ is a solution of the AR(1) equation.
   (b) Is the solution given in part a) stationary?

4. Consider a process that satisfies the zero-mean, "stationary" AR(1) equation $Y_t = \phi Y_{t-1} + e_t$ with $-1 < \phi < 1$. Let c be any nonzero constant, and define $W_t = Y_t + c\phi^t$.
   (a) Show that $\mathbb{E}[W_t] = c\phi^t$.
   (b) Show that $\{W_t\}$ satisfies the "stationary" AR(1) equation $W_t = \phi W_{t-1} + e_t$.
   (c) Is $\{W_t\}$ stationary?

5. Consider an MA(6) model with $\theta_1 = 0.5$, $\theta_2 = -0.25$, $\theta_3 = 0.125$, $\theta_4 = -0.0625$, $\theta_5 = 0.03125$, and $\theta_6 = -0.015625$. Find a much simpler model that has nearly the same ψ weights.

6. Suppose that $\{Y_t\}$ is an AR(1) process with $\rho_1 = \phi$. Define the sequence $\{b_t\}$ as $b_t = Y_t - \phi Y_{t-1}$.
   (a) Show that $\text{Cov}(b_t, b_{t-k}) = 0$ for all t and k.
   (b) Show that $\text{Cov}(b_t, Y_{t+k}) = 0$ for all t and k.