DOMS | Page No.
Date / /

ASSIGNMENT-4

Q. Turing machine to move rightmost symbol to left end. Alphabet : Σ 0, 1, #3 where 0 = blank
Initial head posⁿ : Leftmost non-blank symbol

Logic:
This TM scans right to find the rightmost symbol, erases it, moves to the leftmost symbol, and shifts each symbol one posⁿ right while placing the erased symbol at the leftmost end.

Example: 000#1#1000 → 0001##1000

7-tuple
M = (Q, Σ, Γ, δ, q0, 0, F) where
Σ = Σ1, #, Γ, 3 = Σ0, 1, #3
Q = Σq0, qR, qL1, qL#, qS1, qS#, qacc3
Start state - q0, Blank = 0, Final = Σqacc3

Transition function (δ):

Current state	Read	Write	Move	Next state	Meaning
q0	1/#	Same	R	qR	Move right to find end
qR	1/#	Same	R	qR	Continue Right
qR	0	0	L	qL?	Step back to last symbol
qL1	1/#	Same	L	qL1	Move to leftmost non-blank

Date / /

q_1'	0	0	R	q_{S1}	start shifting (carrying 1)
$q_{L\#}$	1/#	Same	L	$q_{L\#}$	Move left
$q_{L\#}$	0	0	R	$q_{S\#}$	start shifting (carrying #)
q_{S1}	1	1	R	q_{S1}	Continue shift
q_{S1}	#	1	R	$q_{S\#}$	Swap carried symbol
q_{S1}	0	1	R	q_{acc}	finish
$q_{S\#}$	#	#	R	$q_{S\#}$	Continue shift
$q_{S\#}$	1	#	R	q_{S1}	swap
$q_{S\#}$	0	#	R	q_{acc}	finish

Outmost: Rightmost Symbol successfully moved to the left end

Eg:

I/P → 000# 1# 1000

O/P → 0001 # 1 # 000

Q2 $A = (w_1, w_2 \dots w_n)$

$B = (x_1, x_2 \dots x_n)$

$w_{i_1}, w_{i_2}, w_{i_k} = x_{i_1}, x_{i_2} \dots x_{i_k}$

PCP

i	A(w_i)	B(x_i)
1	1	111
2	10111	10
3	10	0

- Start with tile 3
  A = 10
  B = 0 - A is longer → mismatch
- Try tile 1:
  A = 1, B = 111 → B is longer - Mismatch
- Try tile 2:
  A = 10111, B = 10 → Mismatch

No PCP solution exists

Q3. Given pairs
(10, 101)
(01, 100)
(0, 10)
(100, 0)
(1, 010)

Now,

- Start with tile 4 → (100, 0)
  A → 100
  B = 0 → A longer, mismatch
- Start with tile 3 → (0, 10)
  A → 0
  B = 10 → B longer
- Start with tile 5 → (1, 010)
  A = 1, B = 010 → Mismatch
- Tile 1 → (10, 101)
  10 vs 101 → mismatch
- Tile 2 → (01, 100)
  01 vs 100 → mismatch

No prefix matches & adding more tiles increases mismatch

PCP also^N exists for this set

Q4. PCP over unary alphabets Σ{3} is decidable because matching strings is equivalent to checking simple integer sums, which is computationally decidable

Q8. Σ = Σ{0, 1}
A = Σ{10, 0, 01}

B = {11, 100, 003}

1 (110, 11)
2 (0, 100)
3 (01, 00)

Try tile 1
A = 110 B = 11 → A longer → mismatch

Try tile 2
A = 0, B = 100 → B longer → mismatch

Try tile 3
A = 01, B = 00 → mismatch

Now
Tile (1+3)
A = 11001
B = 1100
Mismatch

Tile 3 + Tile 1
A = 01110
B = 0011
Mismatch

No soln exists for this PCP instance

Q9.

Tiles	A	B
1	00	0
2	001	11
3	1000	011

- Tile 1 00 vs 0 → mismatch
- Tile 2 001 vs 11 → 21
- Tile 3 1000 vs 011 → 4

Try Combinations

Tile 1 + Tile 1

A = 0000

B = 00

Mismatch

Tile 2 + Tile 3

A → 00110000

B → 1100

Mismatch

Tile 3 + Tile 1

A = 100000

B = 0110

Mismatch

→

NO PCP SOLN