The Relation Between Wellblock and Wellbore Pressures in Numerical Simulation of Horizontal Wells

D.K. Babu, SPE, Mobil R&D Corp.; **A.S. Odeh**, SPE, consultant; **A.J. Al-Khalifa**, SPE, Aramco; and **R.C. McCann**, SPE, Mobil R&D Corp.

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Summary. An analytical equation for calculating the effective radius, r_o, in a wellblock is derived. The radius r_o is needed to relate wellblock pressure to well pressure. The equation is general and valid for vertical and horizontal wells and for any well location, aspect ratio of the well's drainage area, and anisotropy. The equation is used to show when Peaceman's formulations are adequate and when they require modification. Generally, if the drainage area of the well is a rectangle with sides of large aspect ratio and/or the formation is highly anisotropic, Peaceman's equations need modifications. Simplified forms of the equation applicable to special cases are reported. Furthermore, an easy-to-use approximate equation for calculating r_o values for nearly centered horizontal wells is given. The formulas of this work, as well as those of Peaceman, are relevant only to simulations on uniform grids in homogeneous media.

Introduction

Reservoir simulators require a functional relation between wellblock and wellbore pressures to calculate the wellbore pressure when the flow rate is assigned or the flow rate when the wellbore pressure is given. Peaceman[1] published his first paper on the subject in 1978. Using a repeated five-spot pattern and square gridblocks, he showed that for an isotropic, square medium containing a producer and injector, the wellblock pressure, p_o, is related to the wellbore pressure, p_{wf}, by

$$q=\frac{7.08\times10^{-3}k\Delta z(p_o-p_{wf})}{B\mu\ \ln(r_o/r_w)}.$$

This equation is applicable to a vertical well where p_o is considered the steady-state flowing pressure located at a radius $r_o=0.2\Delta x$. Here, the grid is a square and Δx is the grid dimension. This relation has been accepted almost universally and has replaced the many equations used before Peaceman's publications. In 1983, Peaceman[2] published a second paper that provided equations for calculating r_o values when the wellblock is a rectangle and/or the formation is anisotropic.

To the best of our knowledge, no method was available in the literature to test the applicability of Peaceman's formulas to horizontal wells until recently. To do this, one needs an independent approach for calculating a reliable value for p_{wf} for an assigned set of parameters. The solution given in Refs. 3 and 4 provided a means to do so. In this work, we detail a procedure for calculating the wellblock radius, r_o.

In our formulation of the problem, the well's drainage volume is idealized as a rectangular box-shaped region with all six faces closed to crossflow. The flow domain is homogeneous but anisotropic. The well axis is idealized as a constant-rate, uniform line sink. The physical wellbore is assumed to coincide with a cylindrical surface at a radial distance of r_w from this line sink.

Peaceman studied a steady-state flow problem with producers and injectors placed at the corners of a square pattern. The effects of rectangular patterns and rectangular drainage areas on r_o were not studied. By isolating the well, Peaceman eliminated the influence of boundaries on the flow near the well. The assumption was that r_o values were a function of only the properties of the wellblocks. Our analysis shows that the aspect ratio of the drainage area of the well has a strong effect on r_o values. The aspect ratio is defined as the ratio of the scaled dimensions of the area perpendicular to the well direction. A scaled dimension is defined as $\ell_i/\sqrt{k_i}$, where i may be x, y, or z, and ℓ_i is a typical physical length in the i direction. Because the aspect ratio for a horizontal well is considerably different from unity, it became necessary to investigate the deviations from Peaceman's formulas for horizontal wells. Peaceman solved the isotropic reservoir case by specifying uniform (and constant-rate) flux q at the wellbore. We found, however, that Peaceman's equation for an anisotropic medium, regardless of the aspect ratio, is based on the assumption of constant pressure at the wellbore. Because p_{wf} is a function of r_o, an error in r_o translates to an error in p_{wf}. Kim[5] also examined the effect of partial penetration on the r_o values calculated by Peaceman when $k_V\neq k_H$. He found that Peaceman's r_o should be modified to obtain the correct p_{wf}. Kim's work needs to be extended to the horizontal-well environment.

In this work, we derive an accurate analytical equation for calculating r_o. This equation is general and valid for vertical and horizontal wells, for any well location, and for isotropic and anisotropic formations. We use the equation to derive simplified forms applicable to special cases and show the conditions under which Peaceman's formulation is valid. We also report an easy-to-use approximate equation for calculating r_o applicable to horizontal wells that are basically centrally located in the drainage volumes.

Throughout this work, gravity is neglected. To account for gravity, we simply select a reference datum and replace "pressure head $(p/g\rho)$" with the "hydraulic head $(p/g\rho+z)$," paying proper attention to the units of gravitational acceleration, g, density, ρ, and elevation, z.

Calculation of the Correct Effective Radius

We present two methods for calculating r_o. The first is analytical, and the second is graphical. Both methods start with the general solution[3] relating pressure and flow rate for any well of arbitrary location in a box-shaped drainage volume.

Analytical Method. Fig. 1 indicates a finite-difference grid $(n_x\times n_z)$ in the vertical cross sections of the drainage area of a horizontal well. The following steps lead to an analytical formula for r_o. We assume that the well is located at (x_w,z_w). The nodes are represented by (i,j), with $i=0,1\ldots(n_x-1)$, and $j=0,1\ldots(n_z-1)$. If (i_w,j_w) are the well node coordinates, then $x_w=\Delta x(i_w+½)$ and $z_w=\Delta z(j_w+½)$. If a is the length and h the thickness, it follows that $a\equiv n_x\Delta x$ and $h\equiv n_z\Delta z$.

The following system of finite-difference equations for the pressure is obtained on the grid of Fig. 1.

$$\left(\frac{k_x}{\Delta x^2}\right)(p_{i-1,j}-2p_{i,j}+p_{i+1,j})+\left(\frac{k_z}{\Delta z^2}\right)(p_{i,j-1}-2p_{i,j}$$

$$+p_{i,j+1})=\left(\frac{887q\mu B}{b\Delta x\Delta z}\right)\left(\delta_{ij}-\frac{\Delta x\Delta z}{ah}\right),$$

where $p_{i,j}$ = the pressure at Node (i,j); $i=0,1\ldots(n_x-1)$; $j=0,1\ldots(n_z-1)$; and $\delta_{ij}=1$ for the gridblock with the well and $\delta_{ij}=0$

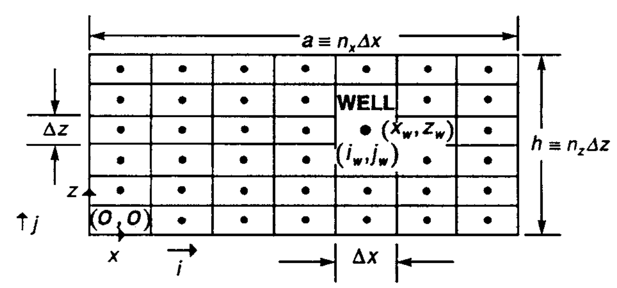

Fig. 1—Finite-difference grid ($n_x \times n_z$) in the vertical (x,z) plane. Well location at $x_w = \Delta x(i_w + ½)$, $z_w = \Delta z(j_w + ½)$. Length of drainage area in x direction $= a = n_x \Delta x$. Length of drainage area in z direction $= h = n_z \Delta z$.

otherwise. Impermeable boundaries are described by the reflection conditions

$$p_{i-1,j} = p_{i,j} \text{ for } i=0,n_x;\ j=0,1\ldots(n_z-1)$$

and $p_{i,j-1} = p_{i,j}$ for $j=0,n_z;\ i=0,1\ldots(n_x-1)$.

Step 1. Obtain an exact, closed-form, analytical solution to the above finite-difference system of equations on the $(n_x \times n_z)$ grid (Eqs. B-54, B-57, and B-27a) of Ref. 6. This analytical solution relates the wellblock pressure, p_o, to the average pressure, $\bar{p}$, in the drainage volume of the well.

$$\bar{p} - p_o = \left(\frac{887 q\mu B}{2\pi b\sqrt{k_x k_z}}\right)\left[\frac{2\pi a}{h}\sqrt{\frac{k_z}{k_x}}\left(\frac{1}{3} - \frac{x_w}{a} + \frac{x_w^2}{a^2} - \frac{1}{12 n_x^2}\right) + (S_{xz})\right], \quad \ldots (1)$$

where the single series sum, S_{xz}, is given by

$$S_{xz} \equiv \left(\frac{\pi}{n_z}\right)\sum_{n=1}^{n_z-1} \cos^2\left(\frac{\pi n\lambda}{2n_z}\right)(1+x_n^{-\nu})(1+x_n^{\nu-2n_x}) \Big/ \left[\sin\left(\frac{\pi n}{2n_z}\right)\sqrt{1+\alpha_n^2}\,(1-x_n^{-2n_x})\right]. \quad \ldots (2)$$

$$\text{Here, } \alpha \equiv \left(\frac{\Delta x}{\Delta z}\sqrt{\frac{k_z}{k_x}}\right) \equiv \left(\frac{a}{h}\sqrt{\frac{k_z}{k_x}}\right)\left(\frac{n_z}{n_x}\right), \quad \ldots (3)$$

$$\alpha_n \equiv \alpha \ \sin(\pi n/2n_z), \quad \ldots (4)$$

$$x_n \equiv (\alpha_n + \sqrt{1+\alpha_n^2}\,)^2, \quad \ldots (5)$$

$$\nu \equiv (2i_w+1),\ i_w = 0,1\ldots(n_x-1), \quad \ldots (6a)$$

$$\text{and } \lambda \equiv (2j_w+1),\ j_w = 0,1\ldots(n_z-1), \quad \ldots (6b)$$

with the well location given by

$$(x_w, z_w) \equiv ½(\nu\Delta x, \lambda\Delta z). \quad \ldots (6c)$$

Several runs of the simulator on a cross-sectional 2D model reproduced quite accurately the block pressures p_o predicted by Eq. 1.

Step 2. Obtain an accurate analytical formula that relates p_{wf} with $\bar{p}$ from the work of Babu and Odeh (Ref. 3, Eqs. 2 and 4; Ref. 4, Eqs. B-16 and B-19).

For $(a/\sqrt{k_x}) \geq (0.75\ h/\sqrt{k_z})$,

$$\bar{p} - p_{wf} = \left(\frac{887 q\mu B}{2\pi b\sqrt{k_x k_z}}\right)\left[\ln\frac{h}{r_w} + \frac{2\pi a}{h}\sqrt{\frac{k_z}{k_x}}\left(\frac{1}{3} - \frac{x_w}{a} + \frac{x_w^2}{a^2}\right) + 0.25\ \ln\left(\frac{k_x}{k_z}\right) - \ln\left(\sin\frac{\pi z_w}{h}\right) - 1.84\right] - (B_E). \quad \ldots (7)$$

The boundary term, B_E, was viewed as negligible in Refs. 3 and 4 (we set $r_w = 0$ in the terms containing x_o in Eq. B-13b of Ref. 4) and is given by

$$B_E \equiv \sum_{\ell=1}^{\infty}\left[\frac{1+\cos(2\pi\ell z_w/h)}{\ell}\right]\exp\left(-\frac{2\pi\ell x_o}{h}\sqrt{\frac{k_z}{k_x}}\right). \quad \ldots (8a)$$

Use of Eq. A-16 in Ref. 4 yields a closed-form expression ($x_w \equiv x_o$):

$$(B_E) \equiv \ln(1-E_1) + 0.5\ \ln\left[1 - 2\ \cos\left(\frac{2\pi z_w}{h}\right)E_1 + E_1^2\right], \quad \ldots (8b)$$

$$\text{where } E_1 \equiv \exp\left[-\frac{2\pi\ \min(x_w, a-x_w)}{h}\sqrt{\frac{k_z}{k_x}}\right]. \quad \ldots (8c)$$

B_E is generally dropped unless the well is located too close to the $x=0$ or $x=a$ boundary.

Step 3. Use the definition of the wellblock radius r_o in the form

$$p_o - p_{wf} \equiv \left(\frac{887 q\mu B}{2\pi b\sqrt{k_x k_z}}\right)\ln\left(\frac{r_o}{r_w}\right). \quad \ldots (9)$$

Step 4. Solve for $(p_o - p_{wf})$ with Eqs. 1 and 7 and substitute Eq. 9 to obtain a formula for the wellblock radius, r_o. Thus, for $(a/\sqrt{k_x}) \geq 0.75(h/\sqrt{k_z})$,

$$\ln\left(\frac{r_o}{h}\right) = \left(\frac{\pi a}{6hn_x^2}\sqrt{\frac{k_z}{k_x}}\right) + 0.25\ \ln\left(\frac{k_x}{k_z}\right) - \ln\left(\sin\frac{\pi z_w}{h}\right) - 1.84 - B_E - (S_{xz}). \quad \ldots (10)$$

Eq. 10 is the general analytical formula. In the Appendix, we present several simplified forms of Eq. 10 relevant to most practical situations.

Eq. 10 is valid when $a/\sqrt{k_x} \geq 0.75(h/\sqrt{k_z})$. When this condition is violated, all we need to do is interchange k_x and k_z, a and h, x_w and z_w, n_x and n_z, and ν and λ in Eqs. 1 through 10.

Graphical Method. For assigned input parameters, we calculate p_{wf} under pseudo-steady-state flow, numerically evaluating the exact infinite-series solution given in Ref. 4. Alternatively, we may use the following equation[3] to compute p_{wf} by use of a highly accurate analytical approximation for ($\ln C_H$) and s_t:

$$q = \frac{7.08\times 10^{-3} b\sqrt{k_x k_z}\,(\bar{p} - p_{wf})}{B\mu\{\ln[(A)^{½}/r_w] + \ln C_H - 0.75 + s_t\}}. \quad \ldots (11a)$$

Eq. 11a is applicable to a horizontal well. For a vertical well, k_y replaces k_z and h replaces b. The average pressure used in Eq. 11a is calculated from total production (qt) after t days:

$$\bar{p} \equiv p_i - 5.61 Bqt/(abh\phi c_t). \quad \ldots (11b)$$

The next steps are to grid the drainage volume, to run the simulator, and to obtain p_o for the same flow time as in Eq. 11a. The simulation t should be large enough for pseudo-steady-state flow to develop; i.e., (dp/dt) should reach a constant value throughout the drainage volume. According to Peaceman, the inflow equation relating the wellblock pressure to the well pressure is

$$q = \frac{7.08\times 10^{-3}\Delta y\sqrt{k_x k_z}\,(p_o - p_{wf})}{\mu B\ \ln(r_o/r_w)}. \quad \ldots (12)$$

For the sake of illustration, we assume in Eq. 11a that $s_t = 0$. Eq. 12 is applicable to a horizontal well. For a vertical well, Δz replaces Δy and k_y replaces k_z.

The only unknown in Eq. 12 is r_o because p_o and p_{wf} are obtained independently and the rest of the parameters are entered. Thus,

$$\ln\frac{r_o}{r_w} = \frac{7.08\times 10^{-3}\Delta y\sqrt{k_x k_z}\,(p_o - p_{wf})}{\mu Bq}. \quad \ldots (13)$$

We used the above-described procedure and calculated r_o values for a variety of input data and gridding systems. Graphical methods with approximate curve-fitting techniques allowed us to arrive at the following easy-to-use equation for calculating r_o:

$$r_g=(0.14)(k_x k_z)^{1/4}\left(\frac{\Delta x^2}{k_x}+\frac{\Delta z^2}{k_z}\right)^{1/2}$$

$$\left[\frac{1+\exp(2.215-3.88n_x n_z/\alpha)}{1+0.533(\alpha/n_z)}\right]. \qquad (14)$$

Eq. 14 is suitable for wells located near the center of the drainage area. For $\alpha \leq (0.2n_z)$ and isotropic cases, Eq. 14 approximates the Peaceman formula to within 10% error. In the range $0.2n_z \leq \alpha \leq n_x n_z$ and for $n_x \leq 20$, Eq. 14 yields wellblock radii to within 13% of the correct analytically calculated values. For $n_x > 20$, the graphical approximation (Eq. 14) is not suitable.

Results

Table 1 compares values of r_o, r_a, r_g, and r_p. Here, r_o is the correct value obtained from Eq. 10 or by the simulator procedure described in Eq. 11a, 11b, 12, or 13. Both methods gave the same r_o values. Also, r_a, r_g, and r_p indicate the values from approximate analytical equations (from the Appendix), graphical methods (Eq. 14), and Peaceman's formula for isotropic media ($r_p \equiv 0.14\sqrt{\Delta x^2+\Delta z^2}$), respectively.

Fig. 2 illustrates graphically the deviation of r_o from Peaceman's formula caused by large aspect ratios of grids and of well drainage areas. For the cases considered in Table 1 and Fig. 2, it is seen that Peaceman's formula needs modification whenever $(a/h) \geq 5.0$ and $(\Delta x/h) \geq 0.63$.

Discussion

Eq. 10 provides an accurate analytical formula for the wellblock radius. As mentioned in the Introduction, a constant-rate, uniform flux, q, is maintained in the immediate vicinity of the wellbore in our calculations.[3] The wellbore pressure, p_{wf}, is identified with the pressure midway between the ends of the horizontal well and at a point on the well perimeter located at an angle of $[\tan^{-1}(k_z/k_x)^{1/4}]$ with the x axis in the vertical (x,z) plane. For isotropic reservoirs, the wellbore becomes a surface of constant pressure.

In contrast, Peaceman's formula for anisotropic reservoirs is based on the assumption of constant p_{wf}. Therefore, a fair comparison between the results of this paper and Peaceman's formulas must be confined to the isotropic cases.

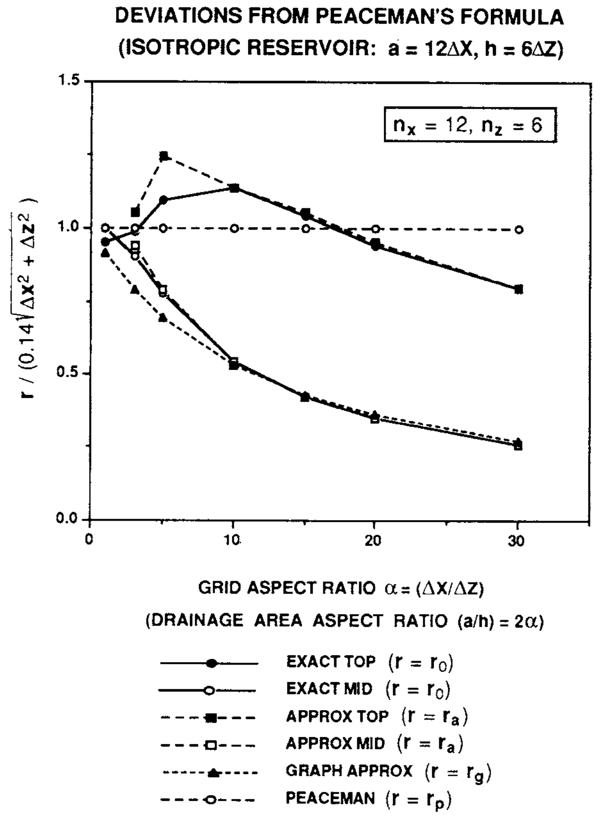

Fig. 2—Comparison of various wellblock radii at top center and middle center of drainage area. Peaceman ($r=r_p$); exact analytical ($r=r_o$, Eq. 10); approximate analytical ($r=r_a$, Eqs. A-3 through A-5); and graphical ($r=r_g$, Eq. 14).

Conclusions and Comments

Detailed analysis of Eqs. 1 through 10 leads to the following conclusions.

1. It is possible to derive a closed-form, analytical formula for wellblock radius, r_o, and wellblock pressure, p_o, for uniform grids on homogeneous reservoirs.
2. Peaceman's formula for r_o for isotropic drainage areas can be viewed as a specialized and simplified version of the general formulas from the present work.

TABLE 1—COMPARISON OF WELL BLOCK RADII, ISOTROPIC RESERVOIR

					Ratio of r_o, r_a, and r_g to r_p		
Grid Size, $n_x \times n_z$	Wellblock Location	Grid Aspect Ratio, $\alpha \equiv \Delta x/\Delta z$	Drainage Area Aspect Ratio $a/h \equiv n_x\alpha/n_z$	Grid vs. Thickness, $\Delta x/h \equiv \alpha/n_z$	Graphical, Eq. 14 (r_g/r_p)	Analytical, Eq. A-3, A-4, or A-5 (r_a/r_p)	Exact, Eq. 10 (r_o/r_p)
12×6	Corner	1.0	2.0	0.17	—	1.07	0.94
	Center				0.92	1.07	1.00
20×20	Corner	5.0	5.0	0.25	—	1.31	1.30
	Center				0.88	1.04	0.98
20×8	Corner	5.0	12.5	0.63	—	1.16	1.16
	Center				0.75	0.92	0.85
20×4	Corner	5.0	25.0	1.25	—	0.91	0.88
	Center				0.60	0.64	0.64
27×12	Corner	10.0	22.5	0.83	—	1.88	1.63
	Center				0.69	0.76	0.76
12×2	Corner	10.0	10.0	0.83	—	1.92	1.66
	Center					0.78	0.77
12×6	Corner	10.0	20.0	1.70	—	1.14	1.12
	Center				0.53	0.53	0.54
12×6	Corner	30.0	60.0	5.0	—	0.79	0.79
	Center				0.28	0.26	0.26

3. For isotropic drainage areas, Peaceman's formula is valid for three cases: nearly square drainage areas ($a/h \sim 1.0$) with arbitrary grid aspect ratios ($\Delta x/\Delta z$); a large number ($n_x, n_z \geq 5$) of nearly square grids ($\Delta x/\Delta z \sim 1.0$) with arbitrary drainage area aspect ratios; and the near-center interior well locations in drainage areas on which a large number of gridblocks (fine meshes) are constructed $[(1/n_z) \leq (\Delta x/h) < 0.65]$.

4. For drainage areas with very high aspect ratios $[(a/h) > 0.65 n_x]$, or equivalently the cases with large grid aspect ratios $(\Delta x/\Delta z) \geq 0.65 n_z$, Peaceman's formula needs modifications.

By its very structure, the Peaceman formula portrays a steady-state radial flow pattern in wellblocks in isotropic media. Therefore, Peaceman's formula remains valid in all physical situations that ensure the existence of essentially radial flows—either in regions very close to the wells effectively covered by small gridblocks or in regions that are centrally located and interior to large drainage areas.

High aspect ratios of well drainage areas generate predominantly linear flow patterns, except in regions that are very close to the well. Modifications to Peaceman's formula are therefore dictated by the contrast in the magnitudes of the aspect ratios of grids and of drainage areas.

Nomenclature

a = extension of drainage volume of well in x direction, ft
A = drainage area of horizontal well $= ah$, ft^2
b = extension of drainage volume of well in y direction, ft
B = FVF, RB/STB
c_t = total compressibility, psi^{-1}
C_H = geometric factor defined in Ref. 3
h = extension of drainage volume of well in z direction, ft
(i,j) = general node location (block centered) in vertical cross sections
(i_w,j_w) = nodal coordinates of horizontal well in vertical cross section
k = permeability in horizontal (xy) plane, md
k_H = $k = \sqrt{k_x k_y}$, horizontal permeability, md
k_V = k_z = vertical permeability, md
k_x = permeability in x direction, md
k_y = permeability in y direction, md
k_z = permeability in vertical z direction, md
n_x = number of gridblocks in x direction
n_z = number of gridblocks in z direction
$\bar{p}$ = average pressure in drainage volume, psi
p_i = initial reservoir pressure, psi
$p_{i,j}$ = simulator-calculated pressure at Node (i,j), psi
p_o = simulator-calculated wellblock pressure at Node (i_w,j_w), psi
p_{wf} = flowing bottomhole pressure, psi
q = flow rate, B/D
r_a = analytical approximation to block radius, ft
r_g = graphical approximation to block radius, ft
r_o = exact block radius, ft
r_p = Peaceman formula-based block radius, ft
r_w = wellbore radius, ft
s_t = total skin
t = time, days
(x_w,z_w) = well coordinates (ft,ft)
$(\Delta x,\Delta z)$ = grid size (ft,ft)
α = $(\Delta x/\Delta z)\sqrt{k_x/k_z} \geq 1$, scaled grid aspect ratio
β = $1/\alpha \leq 1$, scaled grid aspect ratio
λ = integers
μ = viscosity, cp
ν = integers
ρ = density, lbm/ft^3
ϕ = porosity

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Appendix—Analytical Formulas for Special Cases

We list below formulas relevant to most practical situations. Details of the derivations of these formulas are given in Ref. 6.

Single-Layer-Grid Models, $n_z = 1$. In Eq. 10, we set $z_w/h = 0.5$ and $S_{xz} = 0.0$. The wellblock radius, r_o, is given by

$$\ln\left(\frac{r_o}{h}\right) = \frac{\pi a}{6hn_x^2}\sqrt{\frac{k_z}{k_x}} + 0.25\ \ln\left(\frac{k_x}{k_z}\right) - 1.84$$
$$+ \ln\left[1 - \exp\left(-\frac{4\pi x'_w}{h}\sqrt{\frac{k_z}{k_x}}\right)\right], \quad \text{(A-1)}$$

where $x'_w \equiv \min(x_w, a - x_w)$.

Models With $2 \leq n_z \leq 5$. We recommend the use of Eq. 10 along with Eqs. 8 and 2. If n_z is large (>5), several analytical approximations to S_{xz} can be derived with integration techniques. Two parameters play a key role in the derivation of these approximations: the (scaled) grid aspect ratio

$\alpha \equiv (\Delta x/\Delta z)\sqrt{k_z/k_x}$

and the well drainage area aspect ratio (scaled)

$(a/h)\sqrt{k_z/k_x} \equiv (n_x/n_z)\alpha$.

The following approximations are valid for $n_z > 5$.

Nearly Square (Scaled) Drainage Areas, $0.75 \leq (a/h)\sqrt{k_z/k_x} \leq 1.33$. From Eq. B-79,[6]

$$r_o = (k_x k_z)^{1/4}(0.150)\left(\frac{\Delta x^2}{k_x} + \frac{\Delta z^2}{k_z}\right)^{1/2}. \quad \text{(A-2)}$$

For isotropic reservoirs, Eq. A-2 is equivalent to the Peaceman formula.

Nearly Square (Scaled) Grids, $\alpha \sim 1.0$, and Arbitrary Aspect Ratios of Drainage Area. From Eqs. B-64a through B-68, and B-21,[6] we get a formula for r_o identical to Eq. A-2. Again, the results are equivalent to the Peaceman formula for isotropic cases.

Authors

Babu

Odeh

Al-Khalifa

McCann

D. Krishna Babu is a research associate in the Reservoir Engineering Analysis Group at Mobil R&D Corp. in Dallas. His interests are in the application of quasianalytical and numerical techniques to problems in porous media, with special reference to horizontal wells. He previously was a research engineer in the Civil Engineering Dept. at Princeton U. and on the faculty of the City College of New York. He holds an MA degree in applied mathematics from Osmania U., India, and a PhD degree in applied analysis from the State U. of New York at Stony Brook. **Aziz S. Odeh** is a reservoir engineering consultant in Dallas. He retired from Mobil R&D Corp. in 1989 as a senior scientist. He holds a BS degree in petroleum engineering from the U. of California, Berkeley, and a PhD degree in engineering from the U. of California, Los Angeles. A Distinguished Member, Odeh received the 1984 John Franklin Carll Award. He chaired the 1971-72 Editorial Review Committee and the 1976-77 Monograph Committee. He was on the program committee for three Middle East oil shows and several annual meetings. Odeh was inducted into the Natl. Academy of Engineering in 1987 and was the recipient of the outstanding achievement award of the SPE Dallas Chapter in 1989. **A.J. Al-Khalifa,** currently a lead reservoir engineer with Saudi Aramco, worked at Mobil R&D Corp. for 6 months. He holds BS and MS degrees from the King Fahd U. of Petroleum and Minerals and a PhD degree from Stanford U., all in petroleum engineering. Al-Khalifa is a member of the Editorial Review Committee. **Roger C. McCann** is an associate at Mobil R&D Corp. His primary research interest is the mathematical modeling of drilling-related phenomena. He holds a BA degree from Pomona C. and a PhD degree from Case Western Reserve U., both in mathematics.

Moderate to High Scaled Grid Aspect Ratios, $1 \le \alpha \le 0.65 n_z$. For these cases, the analytical expressions in Eqs. B-76a through B-76g[6] that give r_o have a complicated structure. Therefore, we recommend use of the original formula (Eq. 10) whenever possible. However, two special locations yield relatively simple expressions.

Near-Center Interior Locations, $(x_w, z_w) \sim 0.5(a,h)$. From Eq. B-82,[6] we have

$$r_o=(0.142)(k_xk_z)^{1/4}\left(\frac{\Delta x^2}{k_x}+\frac{\Delta z^2}{k_z}\right)^{1/2}$$

$$\div\{0.5[1+\sqrt{1+2.47(\alpha/n_z)^2}\,]\}. \quad \text{(A-3)}$$

For large grids ($n_z \ge 5\alpha$) and isotropic reservoirs, this is equivalent to the Peaceman formula.

Near-Boundary Well Locations, Extreme Off-Centering, $(z_w/h) \sim 0.0$ or 1.0. From B-85,[6] we have

$$r_o=\left[0.067\ \mathrm{cosec}\left(\frac{\pi z_w}{h}\right)\right](k_xk_z)^{1/4}\left(\frac{\Delta x^2}{k_x}+\frac{\Delta z^2}{k_z}\right)^{1/2}K, \quad \text{(A-4a)}$$

$$\text{with } K\equiv\exp\left[\frac{6.28 z_w}{\Delta x}\sqrt{\frac{k_x}{k_z}}+0.62(\alpha/n_z)^2-1/\sqrt{1+2.47(\alpha/n_z)^2}\right]$$

$$\div 0.5[1+\sqrt{1+2.47(\alpha/n_z)^2}\,]. \quad \text{(A-4b)}$$

High Grid Aspect Ratios, $\alpha > 0.65 n_z$. This situation is especially relevant to the horizontal-well geometries and merits special attention. Because the vertical cross sections of the reservoir (drainage areas) generally have very high aspect ratios, it is possible that even the grid aspect ratio, α, is large enough that $\Delta x/\sqrt{k_x} \ge 0.65h/\sqrt{k_z}$.

From Eqs. B-86 through B-86c,[6] we have

$$\ln\left(\frac{h}{r_o}\right)+0.25\ln\left(\frac{k_x}{k_z}\right)-\ln\left(\sin\frac{\pi z_w}{h}\right)-1.84$$

$$=-\left(\frac{\pi a}{6hn_x^2}\sqrt{\frac{k_z}{k_x}}\right)+\left(\frac{2\pi n_x h}{a}\sqrt{\frac{k_x}{k_z}}\right)(T_1-T_2), \quad \text{(A-5)}$$

$$\text{with } T_1\equiv\frac{1}{3}-\frac{z_w}{h}+\frac{z_w^2}{h^2}-\frac{1}{12n_z^2} \quad \text{(A-6a)}$$

$$\text{and } T_2\equiv\frac{2}{3}\left(\frac{n_xh}{a}\right)^2\left(\frac{k_x}{k_z}\right)\left\{\frac{1}{15}\left(1-\frac{1}{n_z^2}\right)\left(1-\frac{1}{4n_z^2}\right)\right.$$

$$\left.-\left(\frac{z_w^2}{h^2}-\frac{1}{4n_z^2}\right)\left[\left(1-\frac{z_w}{h}\right)^2-\frac{1}{4n_z^2}\right]\right\}. \quad \text{(A-6b)}$$

Vertically Centered Wells, $z_w/h \sim 0.5$. From Eqs. A-6a and A-6b we have $T_1=1/12$ and $T_2 \sim 0.0$. From Eq. B-87,[6]

$$r_o=h\left(\frac{k_z}{k_x}\right)^{1/4}(0.16)\exp\left(\frac{\pi a}{6n_x^2h}\sqrt{\frac{k_z}{k_x}}-\frac{\pi n_x h}{6a}\sqrt{\frac{k_x}{k_z}}\right). \quad \text{(A-7)}$$

Near-Boundary Wells, $z_w/h \sim 0.0$ or 1.0. From Eq. B-88,[6]

$$r_o=\left[\frac{h}{\sin(\pi z_w/h)}\right]\left(\frac{k_z}{k_x}\right)^{1/4}(0.16)\exp\left\{-\frac{2\pi n_x h}{3a}\sqrt{\frac{k_x}{k_z}}\right.$$

$$\left.\times\left[1-\frac{2}{15}\left(\frac{n_xh}{a}\right)^2\frac{k_x}{k_z}\right]\right\}. \quad \text{(A-8)}$$

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