

1 Event History Analyses for psychological time-to-event data: A tutorial in R with examples
2 in Bayesian and frequentist workflows

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14

Abstract

15 Time-to-event data such as response times, saccade latencies, and fixation durations form a
16 cornerstone of experimental psychology, and have had a widespread impact on our
17 understanding of human cognition. However, the orthodox method for analysing such data
18 – comparing means between conditions – is known to conceal valuable information about
19 the timeline of psychological effects, such as their onset time and duration. The ability to
20 reveal finer-grained, “temporal states” of cognitive processes can have important
21 consequences for theory development by qualitatively changing the key inferences that are
22 drawn from psychological data. Moreover, well-established analytical approaches, such as
23 event history analysis, are able to evaluate the detailed shape of time-to-event distributions,
24 and thus characterise the timeline of psychological states. One barrier to wider use of event
25 history analysis, however, is that the analytical workflow is typically more time-consuming
26 and complex than orthodox approaches. To help achieve broader uptake, in this paper we
27 outline a set of tutorials that detail how to implement one distributional method known as
28 discrete-time event history analysis. We illustrate how to wrangle raw data files and
29 calculate descriptive statistics, as well as how to calculate inferential statistics via Bayesian
30 and frequentist multi-level regression modelling. We touch upon several key aspects of the
31 workflow, such as how to specify regression models, the implications for experimental
32 design, as well as how to manage inter-individual differences. We finish the article by
33 considering the benefits of the approach for understanding psychological states, as well as
34 the limitations and future directions of this work. Finally, the project is written in R and
35 freely available, which means the general approach can easily be adapted to other data
36 sets, and all of the tutorials are available as .html files to widen access beyond R-users.

37 *Keywords:* response times, event history analysis, Bayesian multi-level regression
38 models, experimental psychology, cognitive psychology

39 Word count: X

40 Event History Analyses for psychological time-to-event data: A tutorial in R with examples
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42 **1. Introduction**

43 **1.1 Motivation and background context: Comparing means versus
44 distributional shapes**

45 In experimental psychology, it is standard practice to analyse response times (RTs),
46 saccade latencies, and fixation durations by calculating average performance across a series
47 of trials. Such mean-average comparisons have been the workhorse of experimental
48 psychology over the last century, and have had a substantial impact of theory development
49 and our understanding of the structure of cognition and brain function. However,
50 differences in mean RT conceal when an experimental effect starts, how long it lasts, how it
51 evolves with increasing waiting time, and whether its onset is time-locked to other events
52 (insert REF). Such information is useful not only for interpretation of the effects, but also
53 for cognitive psychophysiology and computational model selection (Panis, Schmidt,
54 Wolkersdorfer, & Schmidt, 2020).

55 As a simple illustration, Figure 1 shows the results of several simulated RT datasets,
56 which show how mean-average comparisons between two conditions can conceal the shape
57 of the underlying RT distributions. For instance, in examples 1-3, mean RT is always
58 comparable between two conditions, while the distribution differs (Figure 1, top row). In
59 contrast, in examples 4-6, mean RT is lower in condition 2 compared to condition 1, but
60 the RT distribution differs in each case (Figure 1, bottom row). Therefore, a comparison of
61 means would lead to a similar conclusion in examples 1-3, as well as examples 4-6, whereas
62 a comparison of the distribution would lead to a different conclusion in every case.

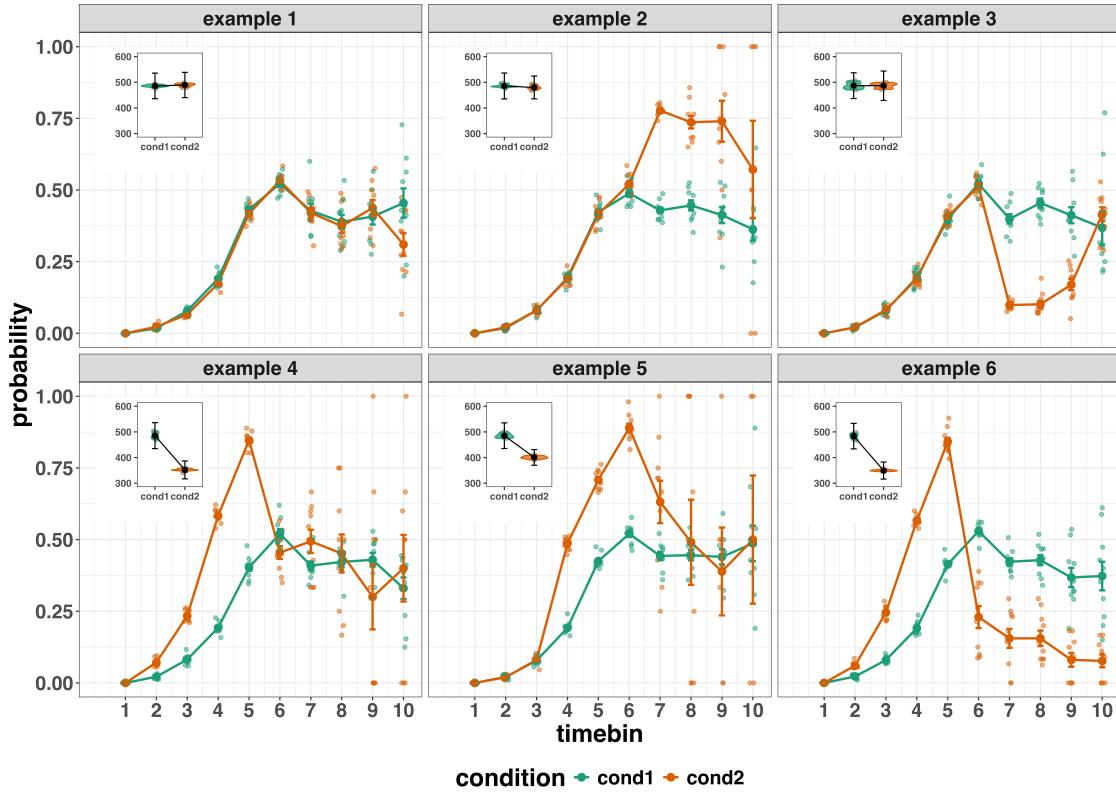


Figure 1. Means versus distributional shapes for six different simulated dataset examples. For our purposes here, it is enough to know that the distributions plotted represent the probability of an event occurring in that timebin, given that it has not yet occurred. Insets show mean reaction time per condition.

63 Why does this matter for research in psychology? Compared to the aggregation of
 64 data across trials, a distributional approach offers the possibility to reveal the timecourse of
 65 psychological states. As such, the approach permits different kinds of questions to be
 66 asked, different inferences to be made, and it holds the potential to discriminate between
 67 different theoretical accounts of psychological and/or brain-based processes. For example,
 68 the distributions in Example 4 show that the effect starts around 200 ms and is gone by
 69 600 ms. In contrast, in Example 5, the effect starts around 400 ms and is gone by 800 ms.
 70 And in the Example 6, the effect reverses around between 500 and 600 ms. What kind of
 71 theory or set of theories could account for such effects? Are there new auxiliary

72 assumptions that theories need to adopt? And are there new experiments that need to be
73 run to test the novel predictions that follow from these analyses? As we show later using
74 concrete examples from past experimental data, for many psychological questions this
75 “temporal states” information can be theoretically meaningful by leading to more
76 fine-grained understanding of psychological processes as well as adding a relatively
77 under-used dimension to our theory building toolkit.

78 From a historical perspective, it is worth noting that the development of analytical
79 tools that can estimate or predict when events will occur is not a new endeavour. Indeed,
80 hundreds of years ago, analytical methods were developed to predict time to death (REFs).
81 The same logic has been applied to psychological time-to-event data, as previously
82 demonstrated (Panis et al., 2020). Here, in the paper, we hope to show the value of EHA
83 for knowledge and theory building in cognitive psychology and related areas of research,
84 such as cognitive neuroscience, as well as provide practical tutorials that provide
85 step-by-step code and instructions in the hope that we can enable others to use EHA in a
86 more routine, efficient and effective manner.

87 1.2 Aims and structure of the paper

88 In this paper, we focus on a distributional method known as discrete-time event
89 history analysis, a.k.a. hazard analysis, duration analysis, failure-time analysis, survival
90 analysis, and transition analysis. We first provide a brief overview of hazard analysis to
91 orient the reader to the basic concepts that we will use throughout the paper. However,
92 this will remain relatively short, as this has been covered in detail before Singer and
93 Willett (2003), Allison (1982), and Allison (2010), and our primary aim here is to
94 introduce a set of tutorials, which explain **how** to do such analyses, rather than repeat in
95 any detail **why** you should do them.

96 We then provide four different tutorials, each of which is written in the R

97 programming language and publicly available on our Github and the Open Science
98 Framework (OSF) pages, along with all of the other code and material associated with the
99 project. The tutorials provide hands-on, concrete examples of key parts of the analytical
100 process, so that others can apply the analyses to their own time-to-event data sets. Each
101 tutorial is provided as an RMarkdown file, so that others can download and adapt the code
102 to fit their own purposes. Additionally, each tutorial is made available as .html file, so that
103 it can be viewed by any web browser, and thus available to those that do not use R.

104 In Tutorial 1, we illustrate how to process or “wrangle” a previously published RT
105 dataset to calculate descriptive statistics when there is one independent variable. The
106 descriptive statistics are plotted, and we comment on their interpretation. In Tutorial 2, we
107 illustrate how one can fit Bayesian multi-level regression hazard models to the data using
108 the R package brms. We discuss possible link functions, and plot the model-based effects of
109 our predictors of interest. In Tutorial 3, we illustrate how to fit the same type of regression
110 hazard models in a frequentist framework using the R package lme4. We then briefly
111 compare and contrast these inferential frameworks when applied to EHA. In Tutorial 4, we
112 provide a generalisation of the approach to illustrate one might describe statistics when
113 using a more complex design, such as when there are two independent variables.

114 In summary, even though event history analyses is a widely used statistical tool and
115 there already exist many excellent reviews (REFs) and tutorials (Allison, 2010) on its
116 general use-cases, we are not aware of any tutorials that are aimed at psychological
117 time-to-event data, and which provide worked examples of the key data processing and
118 multi-level regression modelling steps. Therefore, our ultimate goal is twofold: first, we
119 want to convince readers of the many benefits of using hazard analysis when dealing with
120 time-to-event data with a focus on psychological time-to-event data, and second, we want
121 to provide a set of practical tutorials, which provide step-by-step instructions on how you
122 actually perform hazard analysis.

123 **2. A brief introduction to hazard analysis**

124 For a comprehensive background context to hazard analysis, we recommend several

125 excellent textbooks (Singer & Willett, 2003). Likewise, for general introduction to

126 understanding regression equations, we recommend several introductory level textbooks

127 (REFs). Our focus here is not on providing a detailed account of the underlying regression

128 equations, since this topics has been comprehensively covered many times before. Instead,

129 we want to provide an intuition to how EHA works in general as well as in the context of

130 experimental psychology. As such, we only supply regression equations in supplementary

131 materials and then refer to them in the text whenever relevant.

132 **2.1 Basic features of hazard analysis**

133 To apply event history analysis (EHA), one must be able to:

134 1. define an event of interest that represents a qualitative change that can be situated in
135 time (e.g., a button press, a saccade onset, a fixation offset, etc.)

136 2. define time point zero (e.g., target stimulus onset, fixation onset)

137 3. measure the passage of time between time point zero and event occurrence in discrete
138 or continuous time units.

139 The definition of hazard and the type of models employed depend on whether one is

140 using continuous or discrete time units. Since our focus here is on hazard models that use

141 discrete time units, we describe that approach. After dividing time in discrete, contiguous

142 time bins indexed by t (e.g., $t = 1:10$ timebins), let RT be a discrete random variable

143 denoting the rank of the time bin in which a particular person's response occurs in a

144 particular experimental condition. For example, the first response could occur at 546 ms

145 and it would be in timebin 6 (any RTs from 501 ms to 600). Continuous RT data is treated

146 here as interval-censored data.

147 Discrete-time EHA focuses on the discrete-time hazard function and the discrete-time
148 survivor function (Figure X). The equations that define both of these functions are
149 reported in supplementary materials (part A). The discrete-time hazard probability gives
150 you the probability that the event occurs (sometime) in bin t, given that the event does
151 not occur in previous bins. In other words, it reflects the instantaneous likelihood that the
152 event occurs in the current bin, given that it has not yet occurred in the past, i.e., in one of
153 the prior bins. This conditionality in the definition of hazard is what makes the hazard
154 function so diagnostic for studying event occurrence, as an event can physically not occur
155 when it has already occurred before. In contrast, the discrete-time survivor function
156 cumulates the bin-by-bin risks of event *nonoccurrence* to obtain the probability that the
157 event occurs after bin t. In other words, the survivor function reflects the likelihood that
158 the event occurs in the future, i.e., in one of the subsequent timebins.

159 The survivor function can help to qualify or provide context to the interpretation of
160 the hazard function. For example, it can give a sense of how many trials may contribute to
161 that part of the distribution. If each participant completes 100 trials in an experiment, and
162 the survivor function prob of 0.03, then only 3% of trials remain beyond this point, which
163 in this case would amount to 3 trials. Therefore, the error bars in this part of the
164 distribution would be wider and less precise compared to other parts.

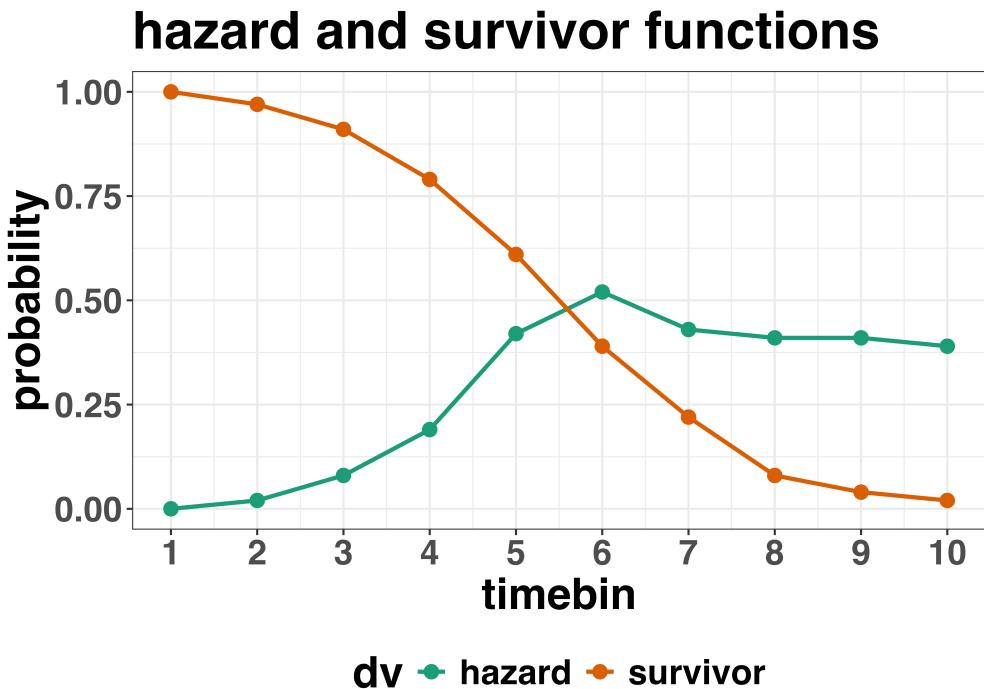


Figure 2. Hazard and survivor functions

¹⁶⁵ **2.2 Hazard analysis in the context of experimental psychology**

¹⁶⁶ **2.2.1 A worked example.** In the context of experimental psychology, it is
¹⁶⁷ common for participants to be presented with a task that has a right and a wrong answer.
¹⁶⁸ For example, a task may involve choosing between two response options with only one of
¹⁶⁹ them being correct. For such two-choice RT data, the discrete-time hazard function can be
¹⁷⁰ extended with the discrete-time conditional accuracy function (see equ. X in Supps), which
¹⁷¹ gives you the probability that a response is correct given that it is emitted in time bin t
¹⁷² (Allison, 2010; Kantowitz & Pachella, 2021; Wickelgren, 1977).

¹⁷³ Integrating results between hazard and conditional accuracy functions can be
¹⁷⁴ informative for understanding psychological processes. To illustrate, we consider a
¹⁷⁵ hypothetical example that is inspired by real data (Panis et al., 2016), but simplified to
¹⁷⁶ make the main point clearer (Figure 3). In a standard response priming paradigm, there is
¹⁷⁷ a prime stimulus (e.g., an arrow pointing left or right) followed by a target stimulus

178 (another arrow pointing left or right). The prime can then be congruent or incongruent
179 with the target. Figure 3 shows that the early upswing in hazard is equal for both
180 conditions, and that early responses are always correct in .. and always incorrect in the
181 incongruent condition. Taken together, the results show that for early responses (< bin 6),
182 responses always follow the prime (and not the target, as instructed). And then for later
183 bins, response hazard is lower in incongruent compared to congruent trials, asthe
184 prime can be overridden, as both conditions are now always correct. This is interesting
185 because mean-average RT would only represent the overall ability of cognition to overcome
186 interference, on average, across trials. And such a conclusion is not supported when the
187 effects are explored over a timeline. Instead, the psychological conclusion is much more
188 nuanced and suggests that multiple states start, stop and possibly interact over a
189 particular temporal window.

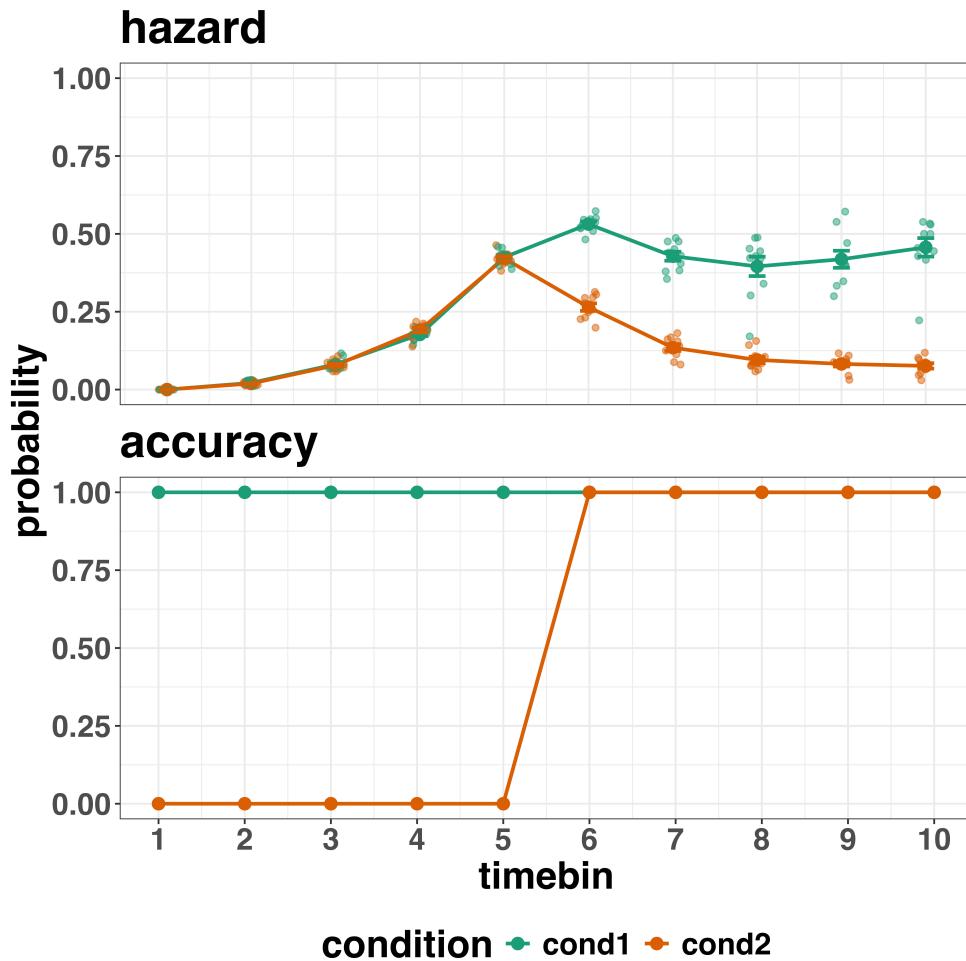


Figure 3. Hazard and conditional accuracy

190 Unlocking the temporal states of cognitive processes can be revealing in and of itself
 191 for theory development and the understanding of basic psychological processes. Possibly
 192 more importantly, however, is that it simultaneously opens the door to address many new
 193 and previously unanswered questions. Do all participants show similar temporal states or
 194 are there individual differences? Do such individual differences extend to those individuals
 195 that have been diagnosed with some form of psychopathology? How do temporal states
 196 relate to brain-based mechanisms that might be studied using other methods from cognitive
 197 neuroscience? And how much of theory in cognitive psychology would be in need of
 198 revision if mean-average comparisons were supplemented with a temporal states approach?

2.2.2 Implications for designing experiments.

Performing hazard analyses in

experimental psychology has implications for how experiments are designed. Indeed, if trials are categorised as a function of when they occur, then each timebin will only include a subset of the total number of trials. For example, let's consider an experiment where each participant performs 2 conditions and there are 100 trial repetitions per condition. Those 100 trials must be distributed in some manner across the chosen number of bins.

In such experimental designs, since the number of trials per condition are spread

across bins, it is important to have a relatively large number of trial repetitions per participant and per condition. Accordingly, experiment designs using this approach typically focus on factorial, within-subject designs, in which a large number of observations are made on a relatively small number of participants (so-called small- N designs). This approach emphasizes the precision and reproducibility of data patterns at the individual participant level to increase the inferential validity of the design (Baker et al., 2021; Smith & Little, 2018).

In contrast to the large- N design that typically average across many participants

without being able to scrutinize individual data patterns, small- N designs retain crucial information about the data patterns of individual observers. This can be advantageous whenever participants differ systematically in their strategies or in the time-courses of their effects, so that averaging them would lead to misleading data patterns. Note that because statistical power derives both from the number of participants and from the number of repeated measures per participant and condition, small- N designs can still achieve what are generally considered acceptable levels of statistical power, if they have have a sufficient amount of data overall (Baker et al., 2021; Smith & Little, 2018).

We used R (Version 4.4.0; R Core Team, 2024)¹ for all reported analyses. The

content of the tutorials is mainly based on Allison (2010), Singer and Willett (2003),

¹ We, furthermore, used the R-packages *bayesplot* (Version 1.11.1; Gabry, Simpson, Vehtari, Betancourt, & Gelman, 2019), *brms* (Version 2.21.0; Bürkner, 2017, 2018, 2021), *citr* (Version 0.3.2; Aust, 2019), *cmdstanr*

²²⁴ McElreath (2018), Kurz (2023a), and Kurz (2023b).

²²⁵ **3. An overview of the general analytical workflow**

²²⁶ Although the focus is on EHA, we also want to briefly comment on broader aspects of
²²⁷ our general analytical workflow, which relate more to data science and data analysis
²²⁸ workflows.

²²⁹ **3.1 Data science workflow and descriptive statistics**

²³⁰ Descriptive, data science workflow. Data wrangling via tidyverse principles and a
²³¹ functional programming approach (cite R4DS textbook here). Functional programming
²³² basically means you don't write your own loops but instead use functions that have been
²³³ built and tested by others. [[more here, as necessary]].

²³⁴ **3.2 Inferential statistical approach**

²³⁵ Our lab adopts a estimation approach to multi-level regression (Kruschke & Liddel,
²³⁶ 2018; Winter, 2019), which is heavily influenced by Bayesian approach as suggested by
²³⁷ Richard McElreath (McElreath, 2020; Kurz, 202?). We also use a “keep it maximal”

(Version 0.8.1.9000; Gabry, Češnovar, Johnson, & Broder, 2024), *dplyr* (Version 1.1.4; Wickham, François, Henry, Müller, & Vaughan, 2023), *forcats* (Version 1.0.0; Wickham, 2023a), *ggplot2* (Version 3.5.1; Wickham, 2016), *lme4* (Version 1.1.35.5; Bates, Mächler, Bolker, & Walker, 2015), *lubridate* (Version 1.9.3; Grolemund & Wickham, 2011), *Matrix* (Version 1.7.0; Bates, Maechler, & Jagan, 2024), *nlme* (Version 3.1.166; Pinheiro & Bates, 2000), *papaja* (Version 0.1.2.9000; Aust & Barth, 2023), *patchwork* (Version 1.2.0; Pedersen, 2024), *purrr* (Version 1.0.2; Wickham & Henry, 2023), *RColorBrewer* (Version 1.1.3; Neuwirth, 2022), *Rcpp* (Eddelbuettel & Balamuta, 2018; Version 1.0.12; Eddelbuettel & François, 2011), *readr* (Version 2.1.5; Wickham, Hester, & Bryan, 2024), *RJ-2021-048* (Bengtsson, 2021), *standist* (Version 0.0.0.9000; Girard, 2024), *stringr* (Version 1.5.1; Wickham, 2023b), *tibble* (Version 3.2.1; Müller & Wickham, 2023), *tidybayes* (Version 3.0.6; Kay, 2023), *tidyxr* (Version 1.3.1; Wickham, Vaughan, & Girlich, 2024), *tidyverse* (Version 2.0.0; Wickham et al., 2019), and *tinylabels* (Version 0.2.4; Barth, 2023).

238 approach to specifying varying (or random) effects (Barr et al., 2013). This means that
239 wherever possible we include varying intercepts and slopes per participant To make
240 inferences, we use two main approaches. We compare models of different complexity, using
241 information criteria, such as WAIC or LOO, to evaluate out-of-sample predictive accuracy.
242 We also take the most complex model and evaluate key parameters of interest using point
243 and interval estimates.

244 **4. Tutorials**

245 Tutorials 1a and 1b show how to calculate and plot the descriptive statistics when
246 there are one and two independent variables, respectively. Tutorials 2a and 2b illustrate
247 how to use Bayesian multilevel modeling to fit hazard and conditional accuracy models,
248 respectively. Tutorials 3a and 3b show how to implement, respectively, multilevel models
249 for hazard and conditional accuracy in the frequentist framework. Additionally, to further
250 simplify the process for other users, the tutorials rely on a set of our own custom functions
251 that make sub-processes easier to automate, such as data wrangling and plotting functions.

252 Our list of tutorials is as follows:

- 253 1a. Wrangle raw data and descriptive stats for one independent variable. 1b.
254 Wrangle raw data and descriptive stats for two independent variables.
255 2a. Bayesian multilevel modeling for $h(t)$ 2b. Bayesian multilevel modeling for $ca(t)$
256 3a. Frequentist multilevel modeling for $h(t)$ 3b. Frequentist multilevel modeling for
257 $ca(t)$

258 Generalisation (T4). Should this be online in Supps?? It would make the main text
259 shorter and simpler, but make it no less available. We could just have a sentence at the end
260 of T1, which says that we provide a generalisation and extension in T4, which is in Supps.

261 Planning (T5) - if we get a simulation and power analysis script working, which we
262 are happy with then we could include it here.

263 **4.1 Tutorial 1a: Calculating descriptive statistics using a life table**

264 **4.1.1 Data wrangling aims.** Our data wrangling procedures serve two related
265 purposes. First, we want to summarise and visualise descriptive statistics that relate to our
266 main research questions. Second, we want to produce two different datasets that can each
267 be submitted to different types of inferential modelling approaches. The two types of data
268 structure we label as ‘person-trial’ data (Table 1) and ‘person-trial-bin’ data (Table 2).
269 The ‘person-trial’ data will be familiar to most researchers who record behavioural
270 responses from participants, as it represents the measured RT and accuracy per trial within
271 an experiment. In contrast, the ‘person-trial-bin’ data has a different, more extended
272 structure, which indicates in which bin a response occurred, if at all, in each trial.
273 Therefore, the ‘person-trial-bin’ dataset generates a 0 in each bin until an event occurs and
274 then it generates a 1 to signal an event has occurred. It is worth pointing out that there is
275 no requirement for an event to occur at all (in any bin), as maybe there was no response on
276 that trial or the event occurred after the timewindow of interest. Likewise, when the event
277 occurs in bin 1 there would only be one row of data for that trial.

Table 1

Data structure for ‘person-trial’ data

| pid | trial | condition | rt | accuracy |
|-----|-------|-------------|--------|----------|
| 1 | 1 | congruent | 373.49 | 1 |
| 1 | 2 | incongruent | 431.31 | 1 |
| 1 | 3 | congruent | 455.43 | 0 |
| 1 | 4 | incongruent | 622.41 | 1 |
| 1 | 5 | incongruent | 535.98 | 1 |
| 1 | 6 | incongruent | 540.08 | 1 |
| 1 | 7 | congruent | 511.07 | 1 |
| 1 | 8 | incongruent | 444.42 | 1 |
| 1 | 9 | congruent | 678.69 | 1 |
| 1 | 10 | congruent | 549.79 | 1 |

Note. The first 10 trials for participant 1 are shown. These data are simulated and for illustrative purposes only.

Table 2
Data structure for ‘person-trial-bin’ data

| pid | trial | condition | timebin | event |
|-----|-------|-------------|---------|-------|
| 1 | 1 | congruent | 1 | 0 |
| 1 | 1 | congruent | 2 | 0 |
| 1 | 1 | congruent | 3 | 0 |
| 1 | 1 | congruent | 4 | 1 |
| 1 | 2 | incongruent | 1 | 0 |
| 1 | 2 | incongruent | 2 | 0 |
| 1 | 2 | incongruent | 3 | 0 |
| 1 | 2 | incongruent | 4 | 0 |
| 1 | 2 | incongruent | 5 | 1 |

Note. The first 2 trials for participant 1 are shown. These data are simulated and for illustrative purposes only.

278 **4.1.2 A real data wrangling example.** To illustrate how to quickly set up life
 279 tables for calculating the descriptive statistics (functions of discrete time), we use a
 280 published data set on masked response priming from Panis and Schmidt (2016). In their
 281 first experiment, Panis and Schmidt (2016) presented a double arrow for 94 ms that
 282 pointed left or right as the target stimulus with an onset at time point zero in each trial.
 283 Participants had to indicate the direction in which the double arrow pointed using their
 284 corresponding index finger, within 800 ms after target onset. Response time and accuracy
 285 were recorded on each trial. Prime type (blank, congruent, incongruent) and mask type
 286 were manipulated. Here we focus on the subset of trials in which no mask was presented.
 287 The 13-ms prime stimulus was a double arrow with onset at -187 ms for the congruent

288 (same direction as target) and incongruent (opposite direction as target) prime conditions.

289 There are several data wrangling steps to be taken. First, we need to load the data
 290 before we (a) supply required column names, and (b) specify the factor condition with the
 291 correct levels and labels.

292 The required column names are as follows:

- 293 • “pid”, indicating unique participant IDs;
- 294 • “trial”, indicating each unique trial per participant;
- 295 • “condition”, a factor indicating the levels of the independent variable (1, 2, ...) and
 296 the corresponding labels;
- 297 • “rt”, indicating the response times in ms;
- 298 • “acc”, indicating the accuracies (1/0).

299 In the code of Tutorial 1a, this is accomplished as follows.

```
data_wr <- read_csv("../Tutorial_1_descriptive_stats/data/DataExp1_6subjects_wrangled.csv")
colnames(data_wr) <- c("pid", "bl", "tr", "condition", "resp", "acc", "rt", "trial")
data_wr <- data_wr %>%
  mutate(condition = condition + 1, # original levels were 0, 1, 2.
        condition = factor(condition, levels=c(1,2,3), labels=c("blank", "congruent", "incongruent")))
```

300 Next, we can set up the life tables and plots of the discrete-time functions $h(t)$, $S(t)$
 301 and $ca(t)$. To do so using a functional programming approach, one has to nest the data
 302 within participants using the `group_nest()` function, and supply a user-defined censoring
 303 time and bin width to our function “`censor()`”, as follows.

```
data_nested <- data_wr %>% group_nest(pid)
data_final <- data_nested %>%
  mutate(censored = map(data, censor, 600, 40)) %>% # ! user input: censoring time, and bin width
  mutate(ptb_data = map(censored, ptb)) %>% # create person-trial-bin dataset
  mutate(lifetable = map(ptb_data, setup_lt)) %>% # create life tables without ca(t)
  mutate(condacc = map(censored, calc_ca)) %>% # calculate ca(t)
  mutate(lifetable_ca = map2(lifetable, condacc, join_lt_ca)) %>% # create life tables with ca(t)
  mutate(plot = map2(.x = lifetable_ca, .y = pid, plot_eha, 1)) # create plots
```

304 Note that the censoring time should be a multiple of the bin width (both in ms). The
305 censoring time should be a time point after which no informative responses are expected
306 anymore. In experiments that implement a response deadline in each trial the censoring
307 time can equal that deadline time point. Trials with a RT larger than the censoring time,
308 or trials in which no response is emitted during the data collection period, are treated as
309 right-censored observations in EHA. In other words, these trials are not discarded, because
310 they contain the information that the event did not occur before the censoring time.

311 Removing such trials before calculating the mean event time can introduce a sampling bias
312 (REFs). The person-trial-bin oriented dataset has one row for each time bin of each trial
313 that is at risk for event occurrence. The variable “event” in the person-trial-bin oriented
314 data set indicates whether a response occurs (1) or not (0) for each bin.

315 The next step is to plot the data using our custom made plotting tool `plot_eha()`.

316 When creating the plots, some warning messages will likely be generated, like these:

- 317 • Removed 2 rows containing missing values or values outside the scale range
318 (`geom_line()`).
- 319 • Removed 2 rows containing missing values or values outside the scale range
320 (`geom_point()`).
- 321 • Removed 2 rows containing missing values or values outside the scale range
322 (`geom_segment()`).

323 The warning messages are generated because some bins have no hazard and $ca(t)$
324 estimates, and no error bars. They can thus safely be ignored. One can now inspect
325 different aspects, including the life table for a particular condition of a particular subject,
326 and a plot of the different functions for a particular participant. It is important to visually
327 inspect the functions first for each participant, in order to identify possible cheaters (e.g., a
328 flat conditional accuracy function at .5 indicates (s)he was only guessing), outlying
329 individulas, and/or different groups with qualitatively different behavior.

330 Table 3 shows the life table for condition “blank” (no prime stimulus presented) for

331 participant 6 - compare to Figure 1. A life table includes for each time bin, the risk set

332 (number of trials that are event-free at the start of the bin), the number of observed

333 events, and the estimates of $h(t)$, $S(t)$, possibly $ca(t)$, and their estimated standard errors

334 (se). At time point zero, no events can occur and therefore $h(t)$ and $ca(t)$ are undefined.

335 Figure 4 displays the discrete-time hazard, survivor, and conditional accuracy

336 functions for each prime condition for participant 6. By using discrete-time $h(t)$ functions

337 of event occurrence - in combination with $ca(t)$ functions for two-choice tasks - one can

338 provide an unbiased, time-varying, and probabilistic description of the latency and

339 accuracy of responses based on all trials of any data set.

340 For example, for participant 6, the estimated hazard values in bin (240,280] are 0.03,

341 0.17, and 0.20 for the blank, congruent, and incongruent prime conditions, respectively. In

342 other words, when the waiting time has increased until *240 ms* after target onset, then the

343 conditional probability of response occurrence in the next 40 ms is more than five times

344 larger for both prime-present conditions, compared to the blank prime condition.

345 Furthermore, the estimated conditional accuracy values in bin (240,280] are 0.29, 1,

346 and 0 for the blank, congruent, and incongruent prime conditions, respectively. In other

347 words, if a response is emitted in bin (240,280], then the probability that it is correct is

348 estimated to be 0.29, 1, and 0 for the blank, congruent, and incongruent prime conditions,

349 respectively.

Table 3

The life table for the blank prime condition of participant 6.

| bin | risk_set | events | hazard | se_haz | survival | se_surv | ca | se_ca |
|--------|----------|--------|--------|--------|----------|---------|------|-------|
| 0.00 | 220.00 | NA | NA | NA | 1.00 | 0.00 | NA | NA |
| 40.00 | 220.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | NA | NA |
| 80.00 | 220.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | NA | NA |
| 120.00 | 220.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | NA | NA |
| 160.00 | 220.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | NA | NA |
| 200.00 | 220.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | NA | NA |
| 240.00 | 220.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | NA | NA |
| 280.00 | 220.00 | 7.00 | 0.03 | 0.01 | 0.97 | 0.01 | 0.29 | 0.17 |
| 320.00 | 213.00 | 13.00 | 0.06 | 0.02 | 0.91 | 0.02 | 0.77 | 0.12 |
| 360.00 | 200.00 | 26.00 | 0.13 | 0.02 | 0.79 | 0.03 | 0.92 | 0.05 |
| 400.00 | 174.00 | 40.00 | 0.23 | 0.03 | 0.61 | 0.03 | 1.00 | 0.00 |
| 440.00 | 134.00 | 48.00 | 0.36 | 0.04 | 0.39 | 0.03 | 0.98 | 0.02 |
| 480.00 | 86.00 | 37.00 | 0.43 | 0.05 | 0.22 | 0.03 | 1.00 | 0.00 |
| 520.00 | 49.00 | 32.00 | 0.65 | 0.07 | 0.08 | 0.02 | 1.00 | 0.00 |
| 560.00 | 17.00 | 9.00 | 0.53 | 0.12 | 0.04 | 0.01 | 1.00 | 0.00 |
| 600.00 | 8.00 | 4.00 | 0.50 | 0.18 | 0.02 | 0.01 | 1.00 | 0.00 |

Note. The column named “bin” indicates the endpoint of each time bin (in ms), and includes time point zero. For example the first bin is (0,40] with the starting point excluded and the endpoint included. se = standard error. ca = conditional accuracy.

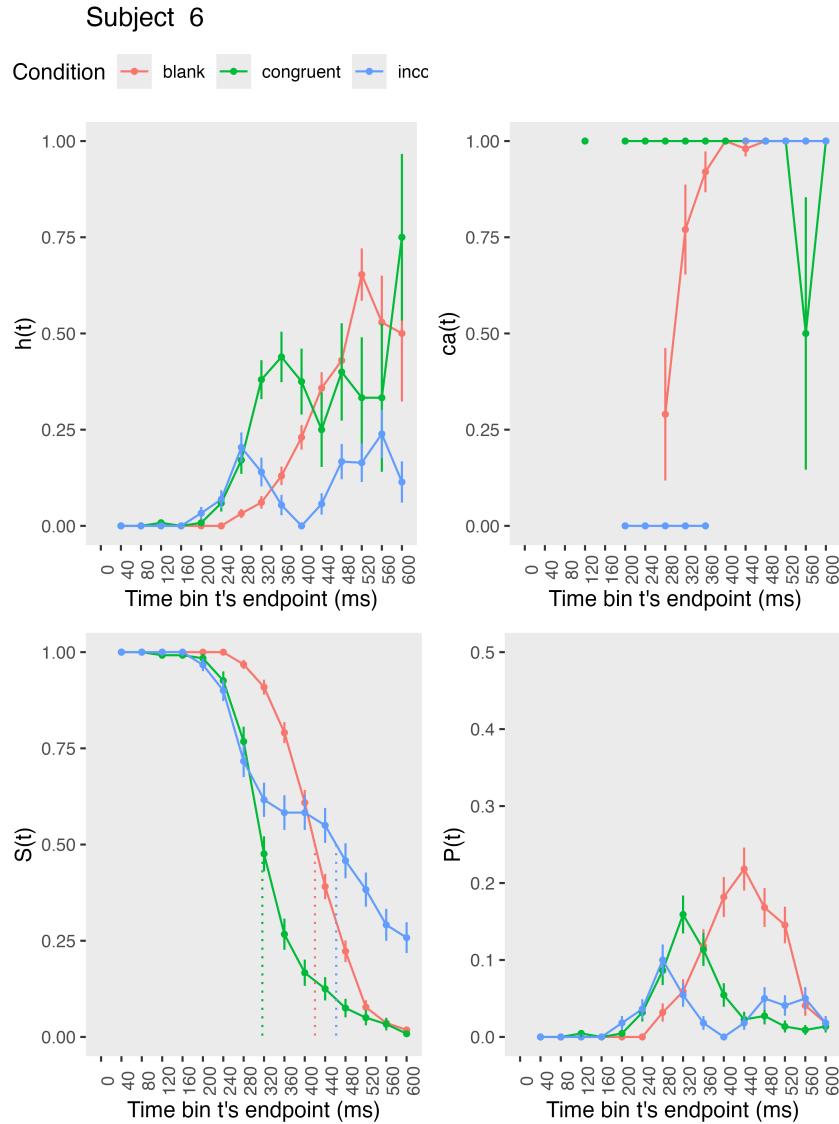


Figure 4. Estimated discrete-time hazard, survivor, and conditional accuracy functions for participant 6, as a function of the passage of discrete waiting time.

350 However, when the waiting time has increased until 400 ms after target onset, then
 351 the conditional probability of response occurrence in the next 40 ms is estimated to be
 352 0.36, 0.25, and 0.06 for the blank, congruent, and incongruent prime conditions,
 353 respectively. And when a response does occur in bin (400,440], then the probability that it
 354 is correct is estimated to be 0.98, 1, and 1 for the blank, congruent, and incongruent prime

355 conditions, respectively.

356 When participants show qualitatively the same distributional patterns, one might
357 consider to aggregate their data and make one plot (see Tutorial_1a.Rmd).

358 These results suggest that the participant is initially responding to the prime even
359 though (s)he was instructed to only respond to the target, that response competition
360 emerges in the incongruent prime condition around 300 ms, and that only later response
361 are fully controlled by the target stimulus. Qualitatively similar results were obtained for
362 the other five participants. These results go against the (often implicit) assumption that all
363 observed responses are primed responses to the target stimulus.

364 At this point, we have calculated, summarised and plotted descriptive statistics for
365 the key variables in EHA. As we will show in Tutorials 2 and 3, statistical models for $h(t)$
366 and $ca(t)$ can be implemented as generalized linear mixed regression models predicting
367 event occurrence (1/0) and accuracy (1/0) in each bin of a selected timewindow for
368 analysis. As such, multi-level regression is what we turn to in those tutorials. But first we
369 consider the descriptive stats for two independent variables.

370 **4.2 Tutorial 1b: Generalising to a more complex design**

371 So far in this paper, we have used a simple experimental design, which involved one
372 condition with two levels. But psychological experiments are often more complex, with
373 crossed factorial designs with more conditions and more than two levels. The purpose of
374 Tutorial 1b, therefore, is to provide a generalisation of the basic approach, which extends
375 to a more complicated design. We felt that this might be useful for researchers in
376 experimental psychology that typically use crossed factorial designs.

377 To this end, Tutorial 1b illustrates how to calculate and plot the descriptive statistics
378 for the full data set of Experiment 1 of Panis and Schmidt (2016), which includes two
379 independent variables: mask type and prime type. As we use the same functional

380 programming approach as in Tutorial 1a, we simply present the sample-based functions for
 381 participant 6 in Figure 8.

382 Compared to the no-mask condition (column 1 in Figure 8), there is a negative
 383 compatibility effect in the hazard and conditional accuracy functions when a (relevant,
 384 irrelevant, or lines) mask is present. T...

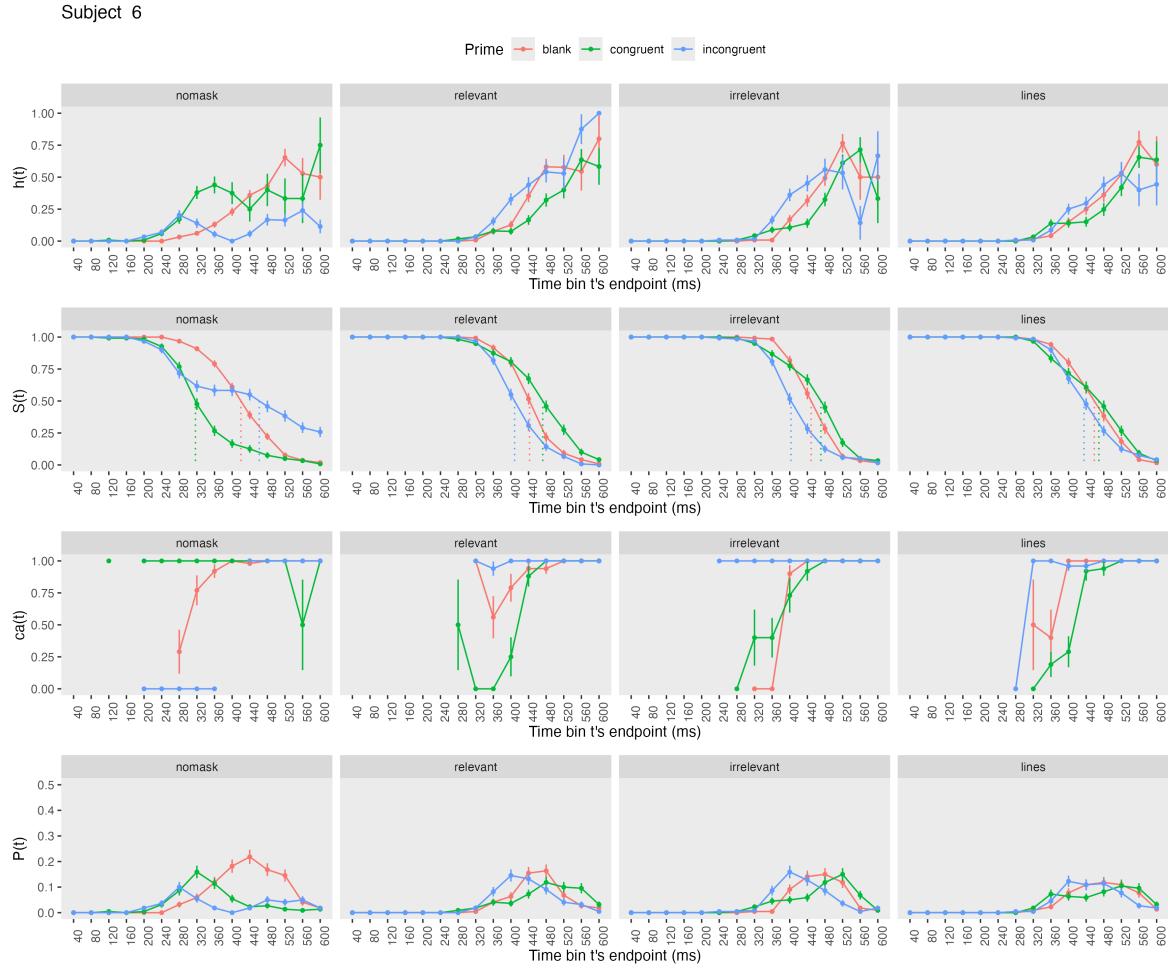


Figure 5. Sample-based discrete-time hazard, survivor, and conditional accuracy functions for participant 6, as a function of prime type (blank, congruent, incongruent) and mask type (no mask, relevant, irrelevant, lines).

385 **4.3 Tutorial 2a: Fitting Bayesian hazard models**

386 In this third tutorial, we illustrate how to fit Bayesian hazard regression models to
387 the masked response priming data set used in the first tutorial. Fitting (Bayesian or
388 non-Bayesian) regression models to the data is important when you want to study how the
389 shape of the hazard function depends on various predictors (Singer & Willett, 2003).

390 **4.3.1 Hazard model considerations.** There are several analytic decisions one
391 has to make when fitting a hazard model. First, one has to select an analysis time window,
392 i.e., a contiguous set of bins for which there is enough data for each participant. Second,
393 given that the dependent variable is binary, one has to select a link function (see Supps).
394 The cloglog link is preferred over the logit link when events can occur in principle at any
395 time point within a bin, which is the case for RT data (Singer & Willett, 2003). Third, one
396 has to choose a specification of the effect of discrete TIME (i.e., the time bin index t). One
397 can choose a general specification (one intercept per bin) or a functional specification, such
398 as a polynomial one (compare model 1 with models 2, 3, and 4 below). We provide relevant
399 example regression formulas in supplementary materials.

400 In the case of a large- N design without repeated measurements, the parameters of a
401 discrete-time hazard model can be estimated using standard logistic regression software
402 after expanding the typical person-trial-oriented data set into a person-trial-bin-oriented
403 data set (Allison, 2010). When there is clustering in the data, as in the case of a small- N
404 design with repeated measurements, the parameters of a discrete-time hazard model can be
405 estimated using population-averaged methods (e.g., Generalized Estimating Equations),
406 and Bayesian or frequentist generalized linear mixed models (Allison, 2010).

407 In general, there are three assumptions one can make or relax when adding
408 experimental predictor variables and other covariates: The linearity assumption for
409 continuous predictors (the effect of a 1 unit change is the same anywhere on the scale), the
410 additivity assumption (predictors do not interact), and the proportionality assumption

411 (predictors do not interact with TIME).

412 In this tutorial we will fit four Bayesian multilevel models (i.e., generalized linear
 413 mixed models) to the person-trial-bin oriented data set that we created in Tutorial 1a. We
 414 select the analysis range (200,600] and the cloglog link. The data is prepared as follows.

```
# load person-trial-bin oriented data set
ptb_data <- read_csv("../Tutorial_1_descriptive_stats/data/inputfile_hazard_modeling.csv")

# select analysis time range: (200,600] with 10 bins (time bin ranks 6 to 15)
ptb_data <- ptb_data %>% filter(period > 5)

# create factor condition, with "blank" as the reference level
ptb_data <- ptb_data %>% mutate(condition = factor(condition, labels = c("blank", "congruent", "incongruent")))

# center TIME (period) on bin 9, and trial on trial 1000 and rescale; Add dummy variables for each bin.
ptb_data <- ptb_data %>%
  mutate(period_9 = period - 9,
         trial_c = (trial - 1000)/1000,
         d6 = if_else(period == 6, 1, 0),
         d7 = if_else(period == 7, 1, 0),
         d8 = if_else(period == 8, 1, 0),
         d9 = if_else(period == 9, 1, 0),
         d10 = if_else(period == 10, 1, 0),
         d11 = if_else(period == 11, 1, 0),
         d12 = if_else(period == 12, 1, 0),
         d13 = if_else(period == 13, 1, 0),
         d14 = if_else(period == 14, 1, 0),
         d15 = if_else(period == 15, 1, 0))
```

415 **4.3.2 Prior distributions.** To get the posterior distribution of each parameters
 416 given the data, we need to specify a prior distribution for each parameter. The middle
 417 column of Supplementary Figure 4 shows seven examples of prior distributions on the logit
 418 and/or cloglog scales.

419 While a normal distribution with relatively large variance is often used as a weakly
 420 informative prior for continuous dependent variables, rows A and B in Figure 3 show that
 421 specifying such distributions on the logit and cloglog scales leads to rather informative

422 distributions on the original probability (i.e., discrete-time hazard) scale, as most mass is
 423 pushed to probabilities of 0 and 1.

424 **4.3.3 Model 1: A general specification of TIME, and main effects of**
 425 **congruency and trial number.** [[Here let's give some intuition on why we would want
 426 to setup the model like this]]

427 For the first model, we use a general specification of TIME (i.e., one intercept per
 428 time bin) for the baseline condition (blank prime), and assume that the effects of
 429 prime-target congruency and trial number are proportional and additive, and that the
 430 effect of trial number is linear. Before we fit model 1, we remove unnecessary columns from
 431 the data, and specify our priors. In the code of Tutorial 2, this is accomplished as follows.

```
# remove unnecessary columns before fitting a model
M1_data <- ptb_data %>% select(-c(bl,tr,trial,period, period_9,d9))

# Specify priors
priors_M1 <- c(
  set_prior("skew_normal(-0.2,0.71,-2.2)", class = "b"),
  set_prior("skew_normal(-1,1,-2)", class = "b", coef = "d6"),
  set_prior("skew_normal(-1,1,-2)", class = "b", coef = "d7"),
  set_prior("skew_normal(-1,1,-2)", class = "b", coef = "d8"),
  set_prior("skew_normal(-1,1,-2)", class = "b", coef = "d10"),
  set_prior("skew_normal(-1,1,-2)", class = "b", coef = "d11"),
  set_prior("skew_normal(-1,1,-2)", class = "b", coef = "d12"),
  set_prior("skew_normal(-1,1,-2)", class = "b", coef = "d13"),
  set_prior("skew_normal(-1,1,-2)", class = "b", coef = "d14"),
  set_prior("skew_normal(-1,1,-2)", class = "b", coef = "d15"),
  set_prior("skew_normal(-1,1,-2)", class = "b", coef = "Intercept"),
  set_prior("normal(0, 1)", class = "sd"),
  set_prior("lkj(2)", class = "cor")
)
```

432 We can now estimate our first Bayesion regression model, as follows.

```
plan(multicore)

model_M1 <-
```

```

brm(data = M1_data,
  family = binomial(link="cloglog"),
  event | trials(1) ~ 0 + d6 + d7 + d8 + Intercept + d10 + d11 + d12 + d13 + d14 + d15 +
  condition + trial_c +
  (d6 + d7 + d8 + 1 + d10 + d11 + d12 + d13 + d14 + d15 + condition + trial_c | pid),
  prior = priors_M1,
  chains = 4, cores = 4, iter = 3000, warmup = 1000,
  control = list(adapt_delta = 0.999, step_size = 0.04, max_treedepth = 12),
  seed = 12, init = "O",
  file = "Tutorial_2_Bayesian/models/model_M1")

```

433 Estimating model M1 took about 70 minutes on a MacBook Pro (Sonoma 14.6.1 OS,
 434 18GB Memory, M3 Pro Chip).

435 **4.3.4 Model 2: A polynomial specification of TIME, and main effects of
 436 congruency and trial number.** [[Here let's give some intuition on why we would want
 437 to modify the formula and model features]]

438 For the second model, we use a third-order polynomial specification of TIME for the
 439 baseline condition (blank prime), and again assume that the effects of prime-target
 440 congruency and trial number are proportional and additive, and that the effect of trial
 441 number is linear. We first remove unnecessary columns and specify our priors.

442 Estimating model M2 took about 144 minutes.

443 **4.3.5 Model 3: A polynomial specification of TIME, and relaxing the
 444 proportionality assumption.** [[Here let's give some intuition on why we would want to
 445 modify the formula and model features]]

446 For the third model, we use a third-order polynomial specification of TIME for the
 447 baseline condition (blank prime), and relax the proportionality assumption for the
 448 predictor variables congruency (variable "condition") and trial number (variable "trial_c").
 449 We use the same data set and priors as for model 2.

450 Estimating model M3 took about 268 minutes.

451 4.3.6 Model 4: A polynomial specification of TIME, and relaxing all three

452 **assumptions.** Based on previous work (Panis, 2020; Panis, Moran, Wolkersdorfer, &
453 Schmidt, 2020; Panis & Schmidt, 2022; Panis, Torfs, Gillebert, Wagemans, & Humphreys,
454 2017; Panis & Wagemans, 2009), we relax all three assumptions in model 4. We use the
455 same data set and priors as for model 2.

456 Estimating model M4 took about 8 hours.

457 4.3.7 Compare the models. We can compare the four models using the Widely

458 Applicable Information Criterion (WAIC) and Leave-One-Out (LOO) cross-validation, and
459 look at model weights (Kurz, 2023a; McElreath, 2018).

460 Clearly, both weighting schemes prefer model M4.

461 4.3.8 Evaluate parameter estimates. Figure 5 shows the effects of congruent

462 and incongruent primes relative to neutral primes, for each time bin in trial numbers 500,
463 1000, and 1500 for the selected model.

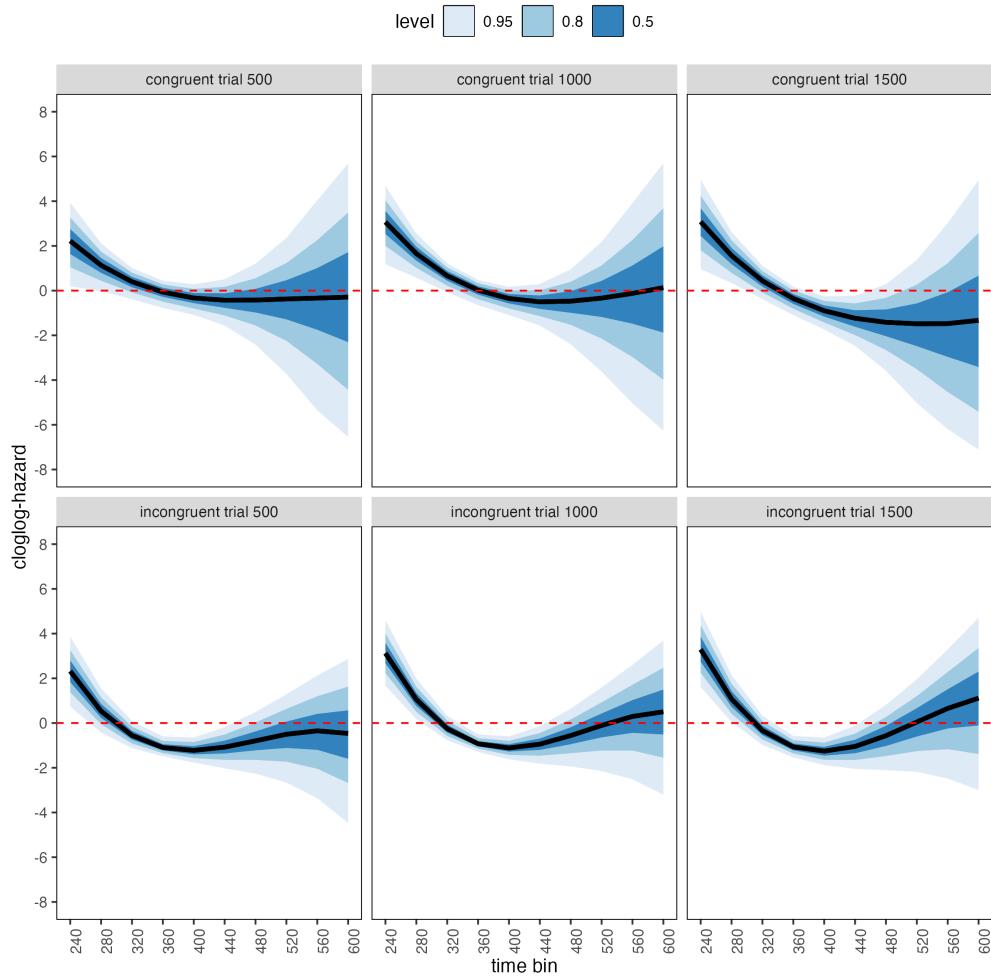


Figure 6. 50/80/95 percentile intervals of the draws from the posterior distributions representing the effect of congruent and incongruent primes relative to neutral primes in trial number 1000.

⁴⁶⁴ Tabel X shows the mean point estimates and the upper and lower limits of the 95%
⁴⁶⁵ highest density intervals for each bin

Table 4

Point and interval estimates.

| bin_endpoint | condition | mean | .lower | .upper | .width | hazard ratio |
|--------------|-----------|------|--------|--------|--------|--------------|
| 240.00 | c500 | 2.18 | 0.33 | 4.05 | 0.95 | 8.82 |
| 280.00 | c500 | 1.11 | -0.02 | 2.11 | 0.95 | 3.03 |

Table 4 continued

| bin_endpoint | condition | mean | .lower | .upper | .width | hazard ratio |
|--------------|-----------|-------|--------|--------|--------|--------------|
| 320.00 | c500 | 0.37 | -0.34 | 1.04 | 0.95 | 1.45 |
| 360.00 | c500 | -0.09 | -0.70 | 0.48 | 0.95 | 0.91 |
| 400.00 | c500 | -0.35 | -1.02 | 0.34 | 0.95 | 0.71 |
| 440.00 | c500 | -0.45 | -1.50 | 0.56 | 0.95 | 0.64 |
| 480.00 | c500 | -0.48 | -2.32 | 1.27 | 0.95 | 0.62 |
| 520.00 | c500 | -0.48 | -3.57 | 2.52 | 0.95 | 0.62 |
| 560.00 | c500 | -0.52 | -5.69 | 4.27 | 0.95 | 0.60 |
| 600.00 | c500 | -0.66 | -8.56 | 6.99 | 0.95 | 0.52 |
| 240.00 | c1000 | 3.03 | 1.37 | 4.82 | 0.95 | 20.63 |
| 280.00 | c1000 | 1.63 | 0.68 | 2.63 | 0.95 | 5.13 |
| 320.00 | c1000 | 0.64 | -0.02 | 1.24 | 0.95 | 1.90 |
| 360.00 | c1000 | -0.01 | -0.57 | 0.52 | 0.95 | 0.99 |
| 400.00 | c1000 | -0.38 | -1.01 | 0.22 | 0.95 | 0.68 |
| 440.00 | c1000 | -0.54 | -1.52 | 0.32 | 0.95 | 0.58 |
| 480.00 | c1000 | -0.54 | -2.20 | 1.11 | 0.95 | 0.58 |
| 520.00 | c1000 | -0.45 | -3.40 | 2.35 | 0.95 | 0.64 |
| 560.00 | c1000 | -0.34 | -5.78 | 3.90 | 0.95 | 0.71 |
| 600.00 | c1000 | -0.25 | -8.34 | 6.73 | 0.95 | 0.78 |
| 240.00 | c1500 | 3.05 | 1.07 | 5.02 | 0.95 | 21.02 |
| 280.00 | c1500 | 1.54 | 0.40 | 2.65 | 0.95 | 4.66 |
| 320.00 | c1500 | 0.42 | -0.36 | 1.13 | 0.95 | 1.52 |
| 360.00 | c1500 | -0.38 | -1.05 | 0.21 | 0.95 | 0.68 |
| 400.00 | c1500 | -0.92 | -1.70 | -0.24 | 0.95 | 0.40 |
| 440.00 | c1500 | -1.26 | -2.41 | -0.18 | 0.95 | 0.28 |
| 480.00 | c1500 | -1.47 | -3.36 | 0.43 | 0.95 | 0.23 |
| 520.00 | c1500 | -1.60 | -4.86 | 1.58 | 0.95 | 0.20 |
| 560.00 | c1500 | -1.71 | -7.01 | 3.37 | 0.95 | 0.18 |
| 600.00 | c1500 | -1.88 | -10.07 | 5.98 | 0.95 | 0.15 |
| 240.00 | i500 | 2.31 | 0.79 | 3.93 | 0.95 | 10.10 |
| 280.00 | i500 | 0.55 | -0.46 | 1.52 | 0.95 | 1.72 |
| 320.00 | i500 | -0.54 | -1.13 | 0.08 | 0.95 | 0.58 |
| 360.00 | i500 | -1.08 | -1.50 | -0.61 | 0.95 | 0.34 |
| 400.00 | i500 | -1.22 | -1.78 | -0.65 | 0.95 | 0.30 |
| 440.00 | i500 | -1.08 | -2.03 | -0.19 | 0.95 | 0.34 |
| 480.00 | i500 | -0.81 | -2.16 | 0.59 | 0.95 | 0.44 |
| 520.00 | i500 | -0.55 | -2.50 | 1.42 | 0.95 | 0.58 |
| 560.00 | i500 | -0.42 | -3.16 | 2.28 | 0.95 | 0.65 |
| 600.00 | i500 | -0.58 | -4.35 | 3.10 | 0.95 | 0.56 |

Table 4 continued

| bin_endpoint | condition | mean | .lower | .upper | .width | hazard ratio |
|--------------|-----------|-------|--------|--------|--------|--------------|
| 240.00 | i1000 | 3.12 | 1.66 | 4.58 | 0.95 | 22.68 |
| 280.00 | i1000 | 1.06 | 0.15 | 1.95 | 0.95 | 2.88 |
| 320.00 | i1000 | -0.24 | -0.78 | 0.31 | 0.95 | 0.78 |
| 360.00 | i1000 | -0.92 | -1.30 | -0.52 | 0.95 | 0.40 |
| 400.00 | i1000 | -1.11 | -1.61 | -0.59 | 0.95 | 0.33 |
| 440.00 | i1000 | -0.95 | -1.80 | -0.12 | 0.95 | 0.39 |
| 480.00 | i1000 | -0.58 | -1.86 | 0.70 | 0.95 | 0.56 |
| 520.00 | i1000 | -0.14 | -1.90 | 1.77 | 0.95 | 0.87 |
| 560.00 | i1000 | 0.24 | -2.33 | 2.75 | 0.95 | 1.27 |
| 600.00 | i1000 | 0.42 | -3.17 | 3.85 | 0.95 | 1.52 |
| 240.00 | i1500 | 3.30 | 1.63 | 4.98 | 0.95 | 27.07 |
| 280.00 | i1500 | 1.08 | 0.05 | 2.14 | 0.95 | 2.94 |
| 320.00 | i1500 | -0.33 | -0.94 | 0.36 | 0.95 | 0.72 |
| 360.00 | i1500 | -1.06 | -1.52 | -0.57 | 0.95 | 0.35 |
| 400.00 | i1500 | -1.26 | -1.88 | -0.65 | 0.95 | 0.28 |
| 440.00 | i1500 | -1.06 | -1.99 | -0.09 | 0.95 | 0.35 |
| 480.00 | i1500 | -0.59 | -2.01 | 0.88 | 0.95 | 0.55 |
| 520.00 | i1500 | 0.00 | -2.05 | 2.09 | 0.95 | 1.00 |
| 560.00 | i1500 | 0.58 | -2.23 | 3.54 | 0.95 | 1.79 |
| 600.00 | i1500 | 1.01 | -3.02 | 4.86 | 0.95 | 2.75 |

Note. The posterior distributions of the effects of congruent and incongruent primes relative to no primes are summarized by point estimates and the lower and upper

...

466

467 Figure 6 shows the model-based hazard functions for each prime type for participant
 468 6, in trial 500, 1000, and 1500.

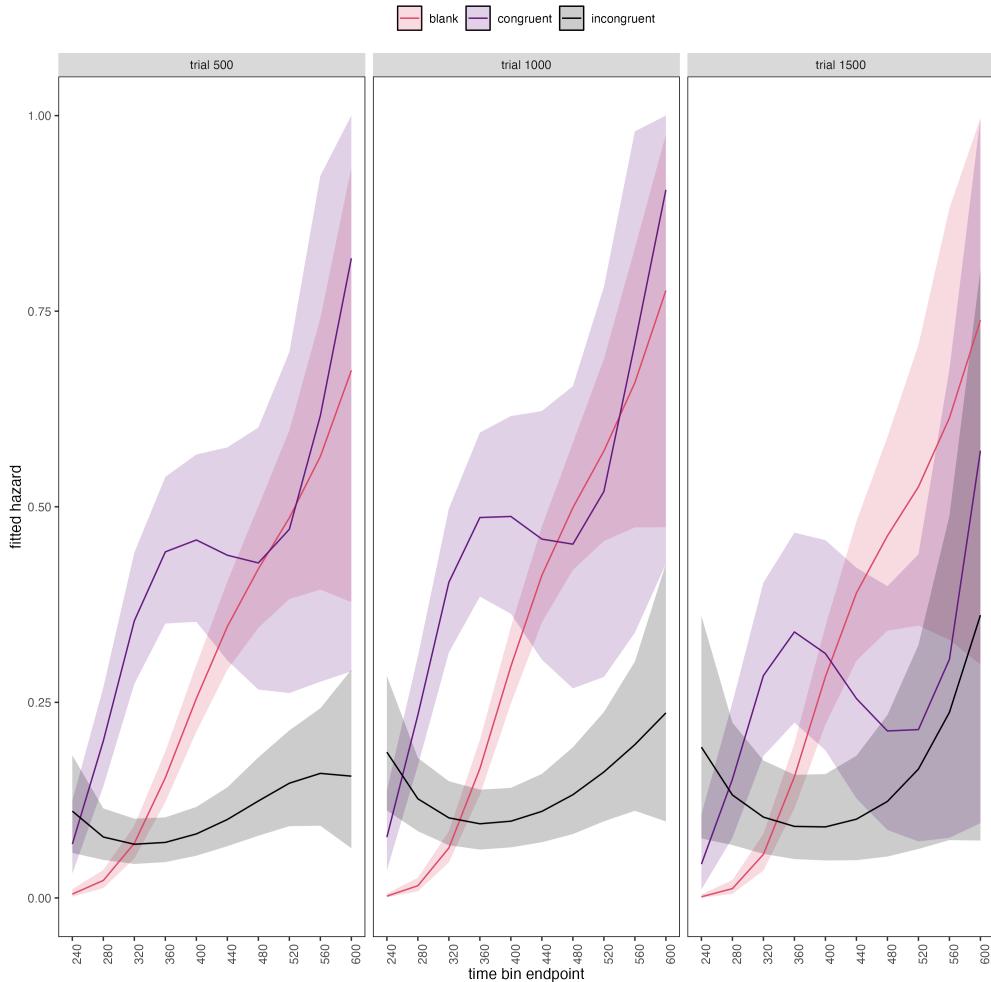


Figure 7. Model-based hazard functions for participant 6 in trial 500, 1000, and 1500.

469

[[let's have a paragraph on how we might interpret these plots.]]

470

4.4 Tutorial 2b: Fitting Bayesian conditional accuracy models

471

4.5 Tutorial 3a: Fitting Frequentist hazard models

472

In this third tutorial we illustrate how to fit a multilevel hazard regression model in

473

the frequentist framework, for the data set used in the first tutorial. For illustration

474

purposes, we only fitted model M3 using the function `glmer()` from the package `lme4`.

475 In Figure 7 we compare the parameter estimates of model M3 from `brm()` with those

476 of `glmer()`.

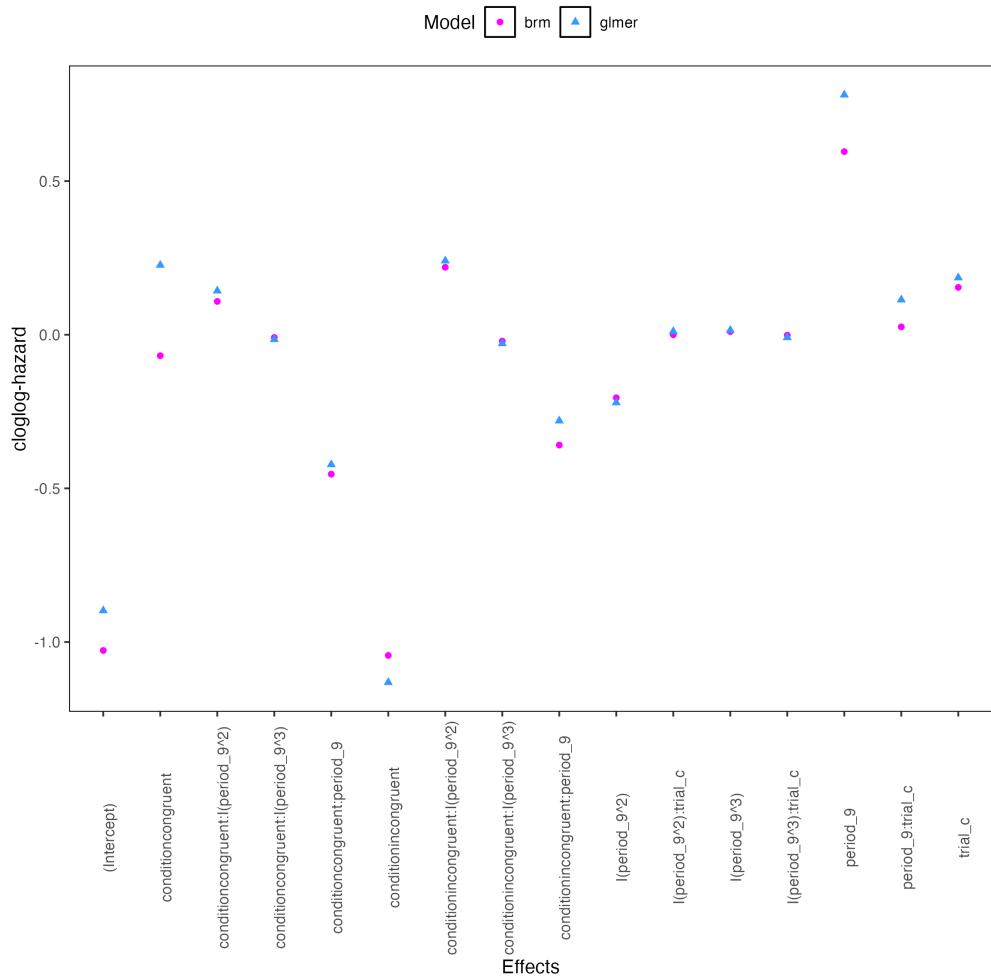


Figure 8. Parameter estimates for model M3 from `brm()` and `glmer()`.

477 Figure 7 confirms that the parameter estimates from both Bayesian and frequentist

478 models are pretty similar. However, the random effects structure of model M3 was already

479 too complex for the frequentist model as it did not converge and resulted in a singular fit.

480 This is of course one of the reasons why Bayesian modeling has become so popular in

481 recent years. But the price you pay for being able to fit more complex models in a

482 Bayesian framework is computation time. In other words, as we have noted throughout,

483 some of the Bayesian models in Tutorial 2 took several hours to build.

484 4.6 Tutorial 3b: Fitting Frequentist conditional accuracy models**485 5. Discussion**

486 This main motivation for writing this paper is the observation that event history
487 analysis remains under-used in psychological research, which means the field of research is
488 not taking full advantage of the many benefits EHA provides compared to more
489 conventional analyses. By providing a freely available set of tutorials, which provide
490 step-by-step guidelines and ready-to-use R code, we hope that researchers will feel more
491 comfortable using EHA in the future. Indeed, we hope that our tutorials may help to
492 overcome a barrier to entry with EHA, which is the increase in analytical complexity
493 compared to mean-average comparisons. While we have focused here on within-subject,
494 factorial, small- N designs, it is important to realize that event history analysis can be
495 applied to other designs as well (large- N designs with only one measurement per subject,
496 between-subject designs, etc.). As such, the general workflow and associated code can be
497 modified and applied more broadly to other contexts and research questions. In the
498 following, we discuss issues relating to individual differences, limitations of the approach,
499 and future extensions.

500 5.1 Advantages of hazard analysis

501 Statisticians and mathematical psychologists recommend focusing on the hazard
502 function when analyzing time-to-event data for various reasons. First, as discussed by
503 Holden, Van Orden, and Turvey (2009), “probability density functions can appear nearly
504 identical, both statistically and to the naked eye, and yet are clearly different on the basis
505 of their hazard functions (but not vice versa). Hazard functions are thus more diagnostic
506 than density functions” (p. 331) when one is interested in studying the detailed shape of a
507 RT distribution (see also Figure 1 in Panis, Schmidt, et al., 2020). Therefore, [[why should
508 people care? What is the functional relevance for exp psych and researchers?]]

509 [[This para needs to be way shorter and easier to read or we get rid of it]] Second,

510 because RT distributions may differ from one another in multiple ways, Townsend (1990)

511 developed a dominance hierarchy of statistical differences between two arbitrary

512 distributions A and B. For example, if $F_A(t) > F_B(t)$ for all t , then both cumulative

513 distribution functions are said to show a complete ordering. Townsend (1990) showed that

514 a complete ordering on the hazard functions — $\lambda_A(t) > \lambda_B(t)$ for all t — implies a complete

515 ordering on both the cumulative distribution and survivor functions — $F_A(t) > F_B(t)$ and

516 $S_A(t) < S_B(t)$ — which in turn implies an ordering on the mean latencies —mean A <

517 mean B. In contrast, an ordering on two means does *not* imply a complete ordering on the

518 corresponding $F(t)$ and $S(t)$ functions, and a complete ordering on these latter functions

519 does *not* imply a complete ordering on the corresponding hazard functions. This means

520 that stronger conclusions can be drawn from data when comparing the hazard functions

521 using EHA. For example, when mean A < mean B, the hazard functions might show a

522 complete ordering (i.e., for all t), a partial ordering (e.g., only for $t > 300$ ms, or only for t

523 < 500 ms), or they may cross each other one or more times. As a result, instead of using

524 delta-plots for RT – differences in quantiles from $F(t)^{-1}$ – one can simply plot delta-h(t)

525 functions (see Panis, 2020).

526 Third, EHA does not discard right-censored observations when estimating hazard

527 functions, that is, trials for which we do not observe a response during the data collection

528 period in a trial so that we only know that the RT must be larger than some value (i.e., the

529 response deadline). This is important because although a few right-censored observations

530 are inevitable in most RT tasks, a lot of right-censored observations are expected in

531 experiments on masking, the attentional blink, and so forth. In other words, by using EHA

532 you can analyze RT data from experiments that typically do not measure response times.

533 As a result, EHA can also deal with long RTs in experiments without a response deadline,

534 which are typically treated as outliers and are discarded before calculating a mean. This

535 orthodox procedure can lead to a sampling bias, however, which results in underestimation

536 of the mean. By introducing a fixed censoring time for all trials at the end of the analysis
537 time window, trials with long RTs are not discarded but contribute to the risk set of each
538 bin.

539 Fourth, hazard modeling allows incorporating time-varying explanatory covariates
540 such as heart rate, electroencephalogram (EEG) signal amplitude, gaze location, etc.
541 (Allison, 2010). This is useful for linking physiological effects with behavioral effects when
542 performing cognitive psychophysiology (Meyer, Osman, Irwin, & Yantis, 1988).

543 Finally, as explained by Kelso, Dumas, and Tognoli (2013), it is crucial to first have a
544 precise description of the macroscopic behavior of a system (here: $h(t)$ and $ca(t)$ functions)
545 in order to know what to derive on the microscopic level. EHA can thus solve the problem
546 of model mimicry, i.e., the fact that different computational models can often predict the
547 same mean RTs as observed in the empirical data, but not necessarily the detailed shapes
548 of the empirical RT hazard distributions. Also, fitting parametric functions or
549 computational models to data without studying the shape of the empirical discrete-time
550 $h(t)$ and $ca(t)$ functions can miss important features in the data (Panis, Moran, et al.,
551 2020; Panis & Schmidt, 2016).

552 5.2 Individual differences

553 One important issue is that of possible individual differences in the overall location of
554 the distribution, and the time course of psychological effects. For example, when you wait
555 for a response of the participant on each trial, you allow the participant to have control
556 over the trial duration, and some participants might respond only when they are confident
557 that their emitted response will be correct. These issues can be avoided by introducing a
558 (relatively short) response deadline in each trial, e.g., 600 ms for simple detection tasks,
559 1000 ms for more difficult discrimination tasks, or 2 s for tasks requiring extended
560 high-level processing. Because EHA can deal in a straightforward fashion with

561 right-censored observations (i.e., trials without an observed response), introducing a
562 response deadline is recommended when designing RT experiments. Furthermore,
563 introducing a response deadline and asking participants to respond before the deadline as
564 much as possible, will also lead to individual distributions that overlap in time, which is
565 important when selecting a common analysis time window when fitting hazard models.

566 But even when using a response deadline, participants can differ qualitatively in the
567 effects they display (see Panis, 2020). One way to deal with this is to describe and
568 interpret the different patterns. Another way is to run a clustering algorithm on the
569 individual hazard estimates across all conditions. The obtained dendrogram can then be
570 used to identify a (hopefully big) cluster of participants that behave similarly, and to
571 identify a (hopefully small) cluster of participants with outlying behavioral patterns. One
572 might then exclude the outlying participants before fitting a hazard model.

573 **5.3 Limitation(s)**

574 Compared to the orthodox method – comparing mean-averages between conditions –
575 the most important limitation of multilevel hazard modeling is that it might take a long
576 time to estimate the parameters using Bayesian methods or the model might have to be
577 simplified significantly to use frequentist methods.

578 Another issue is that you need a relatively large number of trials per condition to
579 estimate the hazard function with high temporal resolution. Indeed, in general, there is a
580 trade-off between the number of trials per condition and the temporal resolution (i.e., bin
581 width) of the hazard function. Therefore, we recommend researchers to collect as many
582 trials as possible per experimental condition, given the available resources and considering
583 the participant experience (e.g., fatigue and boredom). For instance, if the maximum
584 session length deemed reasonable is between 1 and 2 hours, what is the maximum number
585 of trials per condition that you could reasonably collect? After consideration, it might be

586 worth conducting multiple testing sessions per participant and/or reducing the number of
587 experimental conditions. Finally, there is a user-friendly online tool for calculating
588 statistical power as a function of the number of trials as well as the number of participants,
589 and this might be worth consulting to guide the research design process (Baker et al., 2021).

590 We did not discuss continuous-time hazard analysis. As indicated by –allison –
591 learning discrete-time methods first will help in learning continuous-time methods. Given
592 that RT is typically treated as a continuous variable, it is possible that continuous-time
593 methods will ultimately prevail. However, they require much more data to estimate the
594 continuous-time hazard (rate) function well. Thus, by trading a bit of temporal resolution
595 for a lower number of trials, discrete-time methods seem ideal for dealing with typical
596 psychological time-to-event data sets for which there are less than ~200 trials per condition
597 per experiment.

598 **5.4 Extensions**

599 The hazard models in this tutorial assume that there is one event of interest. For RT
600 data, this event constitutes a single transition between an “idle” state and a “responded”
601 state. However, in certain situations, more than one event of interest might exist. For
602 example, in a medical or health-related context, an individual might transition back and
603 forth between a “healthy” state and a “depressed” state, before a final “death” state.
604 When you have data on the timing of these transitions, one can apply multi-state models,
605 which generalize survival analysis to transitions between three or more states (Steele,
606 Goldstein, & Browne, 2004). Also, the predictor variables in this tutorial are
607 time-invariant, i.e., their value did not change over the course of a trial. Thus, another
608 extension is to include time-varying predictors, i.e., predictors whose value can change
609 across the time bins within a trial (REF). [[give a concrete example for this latter point]]

610

6. Conclusions

611 RT and accuracy distributions are a rich source of information on the time course of
612 cognitive processing, which have been largely undervalued in the history of experimental
613 psychology and cognitive neuroscience. We hope that by providing a set of hands-on,
614 step-by-step tutorials, which come with custom-built and freely available code, researchers
615 will feel more comfortable embracing event history analysis and investigating the temporal
616 profile of cognitive states. On a broader level, we think that wider adoption of such
617 approaches will have a meaningful impact on the inferences drawn from data, as well as the
618 development of theories regarding the structure of cognition.

619

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752

Supplementary material

753 **A. Definitions of discrete-time hazard, survivor, and conditional accuracy
functions**

755 The shape of a distribution of waiting times can be described in multiple ways (Luce,
 756 1991). After dividing time in discrete, contiguous time bins indexed by t , let RT be a
 757 discrete random variable denoting the rank of the time bin in which a particular person's
 758 response occurs in a particular experimental condition. Discrete-time EHA focuses on the
 759 discrete-time hazard function

760
$$h(t) = P(RT = t | RT \geq t) \quad (1)$$

761 and the discrete-time survivor function

762
$$S(t) = P(RT > t) = [1-h(t)].[1-h(t-1)].[1-h(t-2)] \dots [1-h(1)] \quad (2)$$

763 and not on the probability mass function

764
$$P(t) = P(RT = t) = h(t).S(t-1) \quad (3)$$

765 nor the cumulative distribution function

766
$$F(t) = P(RT \leq t) = 1-S(t) \quad (4)$$

767 The discrete-time hazard function of event occurrence gives you the probability that
 768 the event occurs (sometime) in bin t , given that the event has not occurred yet in previous
 769 bins. While the discrete-time hazard function assesses the unique risk of event occurrence
 770 associated with each time bin, the discrete-time survivor function cumulates the bin-by-bin
 771 risks of event *nonoccurrence* to obtain the probability that the event occurs after bin t . The
 772 probability mass function cumulates the risk of event occurrence in bin t with the risks of
 773 event nonoccurrence in bins 1 to $t-1$. From equation 3 we find that hazard in bin t is equal
 774 to $P(t)/S(t-1)$.

775 For two-choice RT data, the discrete-time hazard function can be extended with the

776 discrete-time conditional accuracy function

777 $ca(t) = P(\text{correct} \mid RT = t)$ (5)

778 which gives you the probability that a response is correct given that it is emitted in

779 time bin t (Allison, 2010; Kantowitz & Pachella, 2021; Wickelgren, 1977). This latter

780 function is also known as the micro-level speed-accuracy tradeoff (SAT) function.

781 The survivor function provides a context for the hazard function, as $S(t-1) = P(RT >$

782 $t-1) = P(RT \geq t)$ tells you on how many percent of the trials the estimate $h(t) = ..$ is

783 based. The probability mass function provides a context for the conditional accuracy

784 function, as $P(t)$ tells you on how many percent of the trials the estimate $ca(t)$ is based.

785 When time is treated as a continuous variable, let RT be a continuous random variable

786 denoting a particular person's response time in a particular experimental condition.

787 Because waiting times can only increase, continuous-time EHA does not focus on the

788 cumulative distribution function $F(t) = P(RT \leq t)$ and its derivative, the probability

789 density function $f(t) = F(t)'$, but on the survivor function $S(t) = P(RT > t)$ and the

790 hazard rate function $\lambda(t) = f(t)/S(t)$. The hazard rate function gives you the instantaneous

791 *rate* of event occurrence at time point t , given that the event has not occurred yet.

792 B. Custom functions for descriptive discrete-time hazard analysis

793 We defined 13 custom functions that we list here.

- 794 • `censor(df,timeout,bin_width)` : divide the time segment $(0,timeout]$ in bins, identify
any right-censored observations, and determine the discrete RT (time bin rank)
- 796 • `ptb(df)` : transform the person-trial data set to the person-trial-bin data set
- 797 • `setup_lt(ptb)` : set up a life table for each level of 1 independent variable
- 798 • `setup_lt_2IV(ptb)` : set up a life table for each combination of levels of 2
independent variables

- 800 • calc_ca(df) : estimate the conditinal accuracies when there is 1 independent variable
- 801 • calc_ca_2IV(df) : estimate the conditional accuraiies when there are 2 independent
- 802 variables
- 803 • join_lt_ca(df1,df2) : add the ca(t) estimates to the life tables (1 independent
- 804 variable)
- 805 • join_lt_ca_2IV(df1,df2) : add the ca(t) estimates to the life tables (2 independent
- 806 variables)
- 807 • extract_median(df) : estimate quantiles S(t).50 (1 independent variable)
- 808 • extract_median_2IV(df) : estimate quantiles S(t).50 (2 independent variables)
- 809 • plot_oha(df,subj,haz_yaxis) : create plots of the discrete-time functions (1
- 810 independent variable)
- 811 • plot_oha_2IV(df,subj,haz_yaxis) : create plots of the discrete-time functions (2
- 812 independent variables)
- 813 • plot_oha_agg(df,subj,haz_yaxis) : create 1 plot for aggregated data (1 independent
- 814 variable)

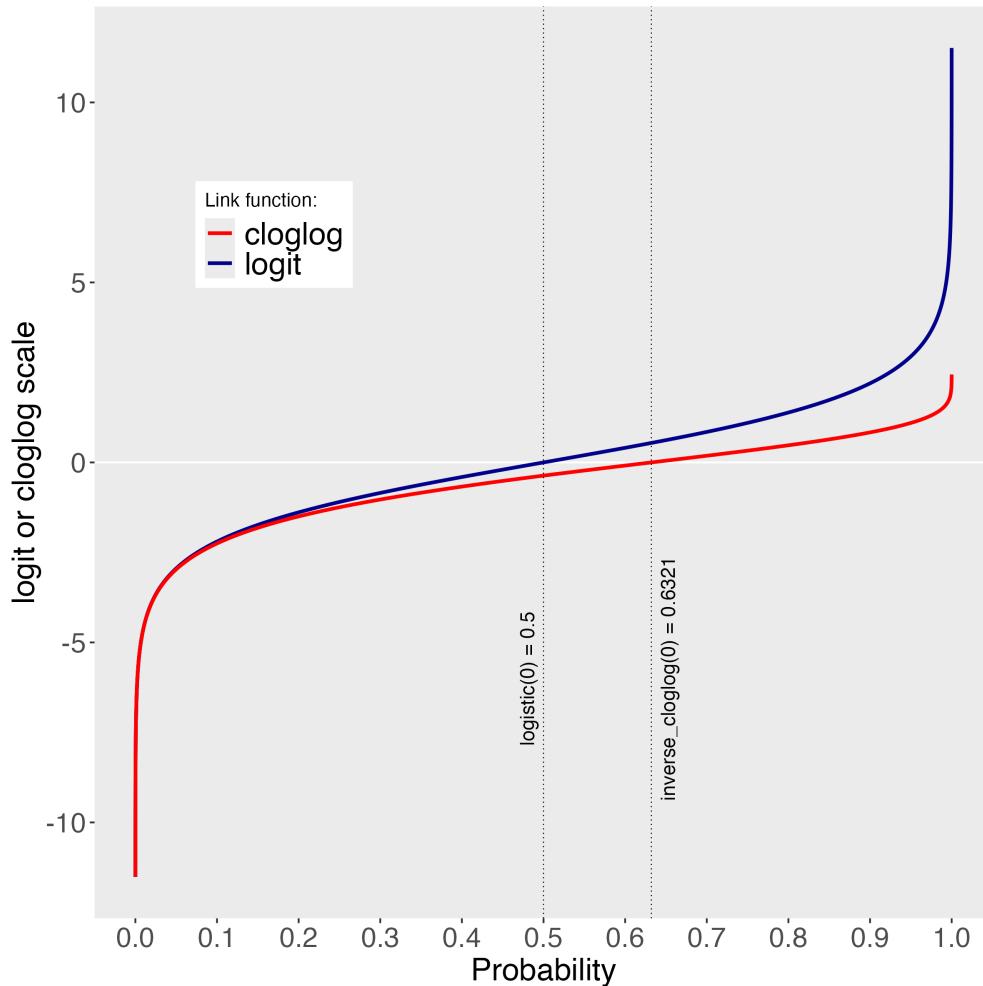
815 **C. Link functions**

Figure 9. The logit and cloglog link functions.

816 **D. Regression equations**

817 An example (single-level) discrete-time hazard model with three predictors (TIME,
 818 X₁, X₂), the cloglog link function, and a third-order polynomial specification for TIME can
 819 be written as follows:

820 $cloglog[h(t)] = \ln(-\ln[1-h(t)]) = [\alpha_1 \text{ONE} + \alpha_2(\text{TIME}-9) + \alpha_3(\text{TIME}-9)^2] + [\beta_1 X_1 +$
 821 $\beta_2 X_2 + \beta_3 X_2(\text{TIME}-9)]$

822 The main predictor variable TIME is the time bin index t that is centered on value 9

823 in this example. The first set of terms within brackets, the alpha parameters multiplied by

824 their polynomial specifications of (centered) time, represents the shape of the baseline

825 cloglog-hazard function (i.e., when all predictors X_i take on a value of zero). The second set

826 of terms (the beta parameters) represents the vertical shift in the baseline cloglog-hazard

827 for a 1 unit increase in the respective predictor variable. Predictors can be discrete,

828 continuous, and time-varying or time-invariant. For example, the effect of a 1 unit increase

829 in X_1 is to vertically shift the whole baseline cloglog-hazard function by β_1 cloglog-hazard

830 units. However, if the predictor interacts linearly with TIME (see X_2 in the example), then

831 the effect of a 1 unit increase in X_2 is to vertically shift the predicted cloglog-hazard in bin

832 9 by β_2 cloglog-hazard units (when $\text{TIME}-9 = 0$), in bin 10 by $\beta_2 + \beta_3$ cloglog-hazard

833 units (when $\text{TIME}-9 = 1$), and so forth. To interpret the effects of a predictor, its β

834 parameter is exponentiated, resulting in a hazard ratio (due to the use of the cloglog link).

835 When using the logit link, exponentiating a β parameter results in an odds ratio.

836 An example (single-level) discrete-time hazard model with a general specification for

837 TIME (separate intercepts for each of six bins, where D1 to D6 are binary variables

838 identifying each bin) and a single predictor (X_1) can be written as follows:

$$839 \text{cloglog}[h(t)] = \ln(-\ln[1-h(t)]) = [\alpha_1 D1 + \alpha_2 D2 + \alpha_3 D3 + \alpha_4 D4 + \alpha_5 D5 + \alpha_6 D6] +$$

$$840 [\beta_1 X_1]$$

841 E. Prior distributions

842 To gain a sense of what prior *logit* values would approximate a uniform distribution

843 on the probability scale, Kurz (2023a) simulated a large number of draws from the

844 Uniform(0,1) distribution, converted those draws to the log-odds metric, and fitted a

845 Student's t distribution. Row C in Figure 4 shows that using a t-distribution with 7.61

846 degrees of freedom and a scale parameter of 1.57 as a prior on the logit scale, approximates

847 a uniform distribution on the probability scale. According to Kurz (2023a), such a prior
 848 might be a good prior for the intercept(s) in a logit-hazard model, while the $N(0,1)$ prior in
 849 row D might be a good prior for the non-intercept parameters in a logit-hazard model, as it
 850 gently regularizes p towards .5 (i.e., a zero effect on the logit scale).

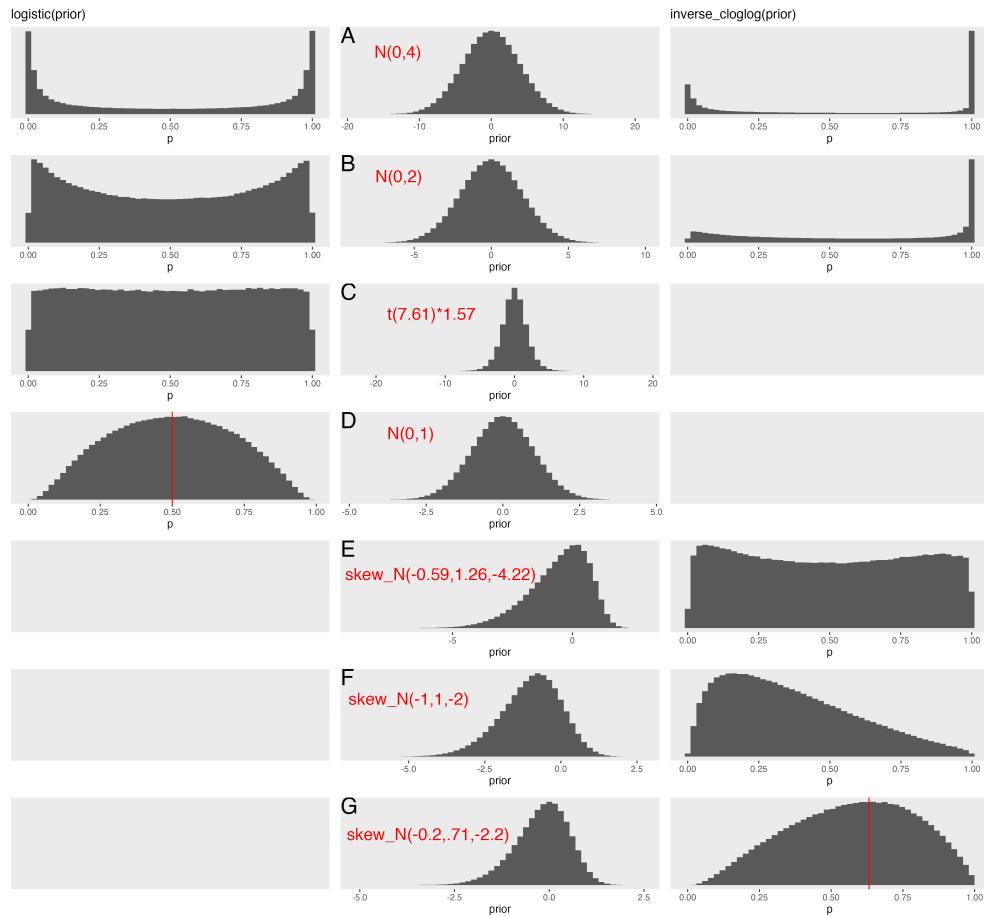


Figure 10. Prior distributions on the logit and/or cloglog scales (middle column), and their implications on the probability scale after applying the inverse-logit (or logistic) transformation (left column), and the inverse-cloglog transformation (right column).

851 To gain a sense of what prior *cloglog* values would approximate a uniform distribution
 852 on the hazard probability scale, we followed Kurz's approach and simulated a large number
 853 of draws from the Uniform(0,1) distribution, converted them to the cloglog metric, and
 854 fitted a skew-normal model (due to the asymmetry of the cloglog link function). Row E

855 shows that using a skew-normal distribution with a mean of -0.59, a standard deviation of
856 1.26, and a skewness of -4.22 as a prior on the cloglog scale, approximates a uniform
857 distribution on the probability scale. However, because hazard values below .5 are more
858 likely in RT studies, using a skew-normal distribution with a mean of -1, a standard
859 deviation of 1, and a skewness of -2 as a prior on the cloglog scale (row F), might be a good
860 weakly informative prior for the intercept(s) in a cloglog-hazard model. A skew-normal
861 distribution with a mean of -0.2, a standard deviation of 0.71, and a skewness of -2.2 might
862 be a good weakly informative prior for the non-intercept parameters in a cloglog-hazard
863 model as it gently regularizes p towards .6321 (i.e., a zero effect on the cloglog scale).