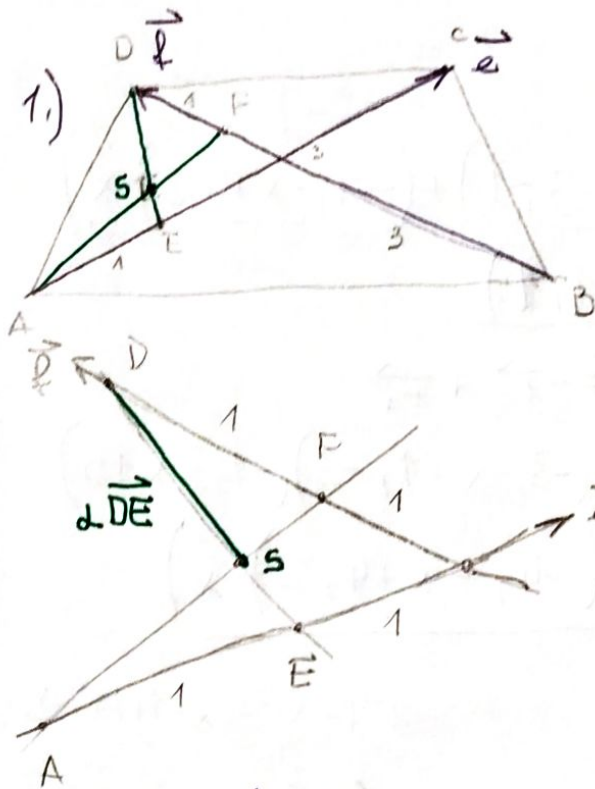


ALGEBRA 1 - 1. KOLOKVIJ

19.11.2020



$$|AE| : |EC| = 1 : 3 \quad |BF| : |FD| = 3 : 1$$

$$\vec{DS} = \alpha (\vec{DE})$$

$$\vec{DS} = \alpha \left(\frac{1}{4} \vec{DB} \right)$$

$$\vec{DS} = \alpha \left(-\frac{1}{2} \vec{f} + \left(-\frac{1}{4}\right) \vec{e} \right)$$

$$\vec{DS} = \alpha \left(-\frac{1}{2} \vec{f} - \frac{1}{4} \vec{e} \right)$$

$$\vec{DS} =$$

$$\begin{aligned} \vec{DS} &= -\frac{1}{2} \vec{f} - \frac{1}{4} \vec{e} + \frac{1}{2} \vec{AF} \\ \vec{DS} &= -\frac{1}{2} \vec{f} - \frac{1}{4} \vec{e} + \frac{1}{2} \left(\frac{1}{2} \vec{e} + \frac{1}{4} \vec{f} \right) \\ \vec{DS} &= -\frac{1}{2} \vec{f} - \frac{1}{4} \vec{e} + \frac{1}{4} \vec{e} + \frac{1}{16} \vec{f} \\ \vec{DS} &= -\frac{8}{16} \vec{f} - \frac{2}{4} \vec{e} + \frac{1}{4} \vec{e} + \frac{1}{16} \vec{f} \end{aligned}$$

$$\vec{DS} = -\frac{1}{2} \vec{f} - \frac{1}{2} \vec{e} + \beta \left(\frac{1}{2} \vec{e} + \frac{1}{2} \vec{f} \right)$$

$$\vec{DS} = -\frac{7}{16} \vec{f}$$

$$\vec{DS} = -\frac{4}{3} \left(-\frac{1}{2} \vec{f} - \frac{1}{4} \vec{e} \right)$$

$$\vec{DS} = \frac{4}{6} \vec{f} + \frac{4}{12} \vec{e}$$

$$\vec{DS} = \frac{2}{3} \vec{f} + \frac{1}{3} \vec{e}$$

$$\vec{DS} = \frac{1}{2} \alpha \vec{f} - \frac{1}{4} \alpha \vec{e} = -\frac{1}{2} \vec{f} - \frac{1}{2} \vec{e} + \frac{1}{2} \beta \vec{e} + \frac{1}{2} \beta \vec{f}$$

$$\text{PRIF: } \frac{1}{2} \alpha = -\frac{1}{2} + \frac{1}{2} \beta / 2$$

$$\alpha = -1 + \beta$$

$$\alpha = -\frac{4}{3}$$

$$\beta = -\frac{1}{3}$$

$$\text{PRI E: } -\frac{1}{4} \alpha = \frac{1}{2} + \frac{1}{2} \beta$$

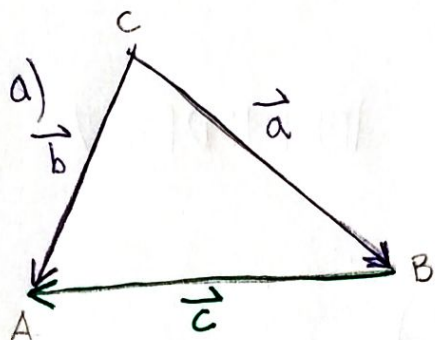
$$-\frac{1}{4} (-1 + \beta) = \frac{1}{2} + \frac{1}{2} \beta$$

$$\frac{1}{4} - \frac{1}{4} \beta = \frac{1}{2} + \frac{1}{2} \beta / 4$$

$$1 - \beta = 2 + 2\beta$$

$$3\beta = -1$$

2.) $A(-1, 3, x+5)$ $B(x, 2, 4)$ $C(3, x-1, 3)$



$$|\vec{c}| = 3 = |\vec{BA}|$$

$$\vec{c} = -\vec{a} + \vec{b}$$

$$\vec{c} = (3-x, x-3, 1) + (-4, -x+4, 2+x)$$

$$\vec{c} = (-x-1, 1, 3+x)$$

$$\vec{a} = -\vec{c} + \vec{b}$$

$$\vec{a} = (-3, -x+1, -3) + (x, 2, 4)$$

$$\vec{a} = (-3+x, -x+3, 1)$$

$$\vec{b} = -\vec{c} + \vec{a}$$

$$\vec{b} = (-3, -x+1, -3) - (-1, 3, x+5)$$

$$\vec{b} = (-4, -x+4, 2+x)$$

$$|\vec{c}| = \sqrt{(-x-1)^2 + 1 + (3+x)^2} = 3 \quad / \text{KUAD.}$$

$$x^2 + 2x + 1 + 2 = 3$$

$$x^2 + 2x + 3 = 3$$

$$x^2 + 2x = 0$$

$$x(x+2) = 0$$

$x_1 = 0$	$x_2 = -2$
$x_1 = 1$	$x_2 = -3$

$$|\vec{c}| = x^2 + 2x + 1 + 1 + 1 + 2x + x^2$$

$$2x^2 + 4x + 3 = 3$$

$$2x^2 + 4x = 0$$

$$x^2$$

b) $S_{\Delta} = \frac{1}{2} S_{\square} = \frac{1}{2} \sqrt{8} = \frac{2\sqrt{2}}{2} = \sqrt{2}$ b) NASL. STRAN \Rightarrow

$$\vec{a} \times \vec{b} = (-3, 3, 1) \times (-4, 4, 2)$$

$$= \left(\begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix}, -\begin{vmatrix} -3 & 1 \\ -4 & 2 \end{vmatrix}, \begin{vmatrix} -3 & 3 \\ -4 & 4 \end{vmatrix} \right)$$

$$= (3 \cdot 2 - 4, -(-6 + 4), (-12 + 12))$$

$$\vec{a} \times \vec{b} = (2, 2, 0)$$

$$|\vec{a} \times \vec{b}| = \sqrt{4+4} = \sqrt{8}$$

$$S_{\Delta} = \sqrt{2}$$

$$(-x-1)^2 + 1^2 + (1+x)^2 = 9$$

$$x^2 + 2x + 1 + 1 + 1 + 2x + x^2 = 9$$

$$2x^2 + 4x + 3 = 9$$

$$2x^2 + 4x - 6 = 0 \quad / :2$$

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

$$b) S_{\Delta} = S_{\square}$$

$$\vec{a} \times \vec{b} = (-2, 2, 1) \times (-4, 3, 3)$$

$$= \left(\begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix}, - \begin{vmatrix} -2 & 1 \\ -4 & 3 \end{vmatrix}, \begin{vmatrix} -2 & 2 \\ -4 & 3 \end{vmatrix} \right)$$

$$= (6-3, -(-6+4), -6+8)$$

$$= (3, 2, 2)$$

$$\sqrt{9+4+4} = \sqrt{17}$$

$$S_{\Delta} = \frac{\sqrt{17}}{2}$$

$$3.) \quad \Pi: x+y+z=2 \quad p=(1,1,0) + \lambda(-1,1,-2) \quad T?$$

a) PREMIKA P

$$x = 1 - \lambda$$

$$y = 1 + \lambda$$

$$z = -2\lambda$$

$$1 - \lambda + 1 + \lambda - 2\lambda = 2$$

$$1 - 2\lambda = 2$$

$$-2\lambda = 1$$

$$\underline{\underline{\lambda = -\frac{1}{2}}}$$

$$(1,1,0) + \frac{1}{2}(-1,1,-2)$$

$$(1,1,0) + \left(\frac{1}{2}, -\frac{1}{2}, 1\right)$$

$$\boxed{T\left(\frac{3}{2}, +\frac{1}{2}, 0\right)}$$

$$p: \frac{3}{2} + \frac{1}{2} + 0 = \frac{4}{2} = \underline{\underline{2}}$$

b) $\angle(\pi, P)$

$$\sin f = \frac{|\vec{P} \cdot \vec{m}_{\pi}|}{|\vec{P}| \cdot |\vec{m}|}$$

$$\sin f = \frac{2}{\sqrt{6} \cdot \sqrt{3}}$$

$$\vec{P} = (-1, 1, -2)$$

$$\vec{m}_{\pi} = (1, 1, 1)$$

$$\begin{aligned}\vec{P} \cdot \vec{m}_{\pi} &= (-1, 1, -2) \cdot (1, 1, 1) \\ &= (-1+1, -2) \\ &= -2\end{aligned}$$

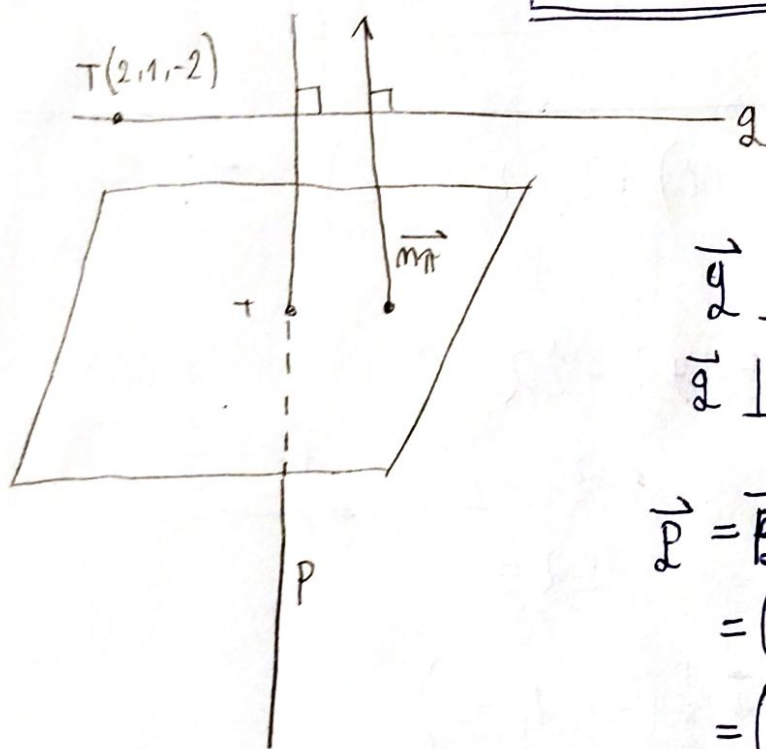
$$|\vec{P}| = \sqrt{1+1+4} = \sqrt{6}$$

$$|\vec{m}_{\pi}| = \sqrt{1+1+1} = \sqrt{3}$$

$$\sin f = \frac{2}{\sqrt{18}} = \frac{\sqrt{2}}{3}$$

$$f = 28,13^{\circ}$$

c)



$$\vec{g} \perp \vec{m}_{\pi} \Rightarrow \vec{g} \cdot \vec{m}_{\pi} = 0$$

$$\vec{g} \perp \vec{P} \Rightarrow \vec{g} \cdot \vec{P} = 0$$

$$\vec{P} = \vec{g} \times \vec{m}$$

$$= (-1, 1, -2) \times (1, 1, 1)$$

$$= \left(\begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix}, -\begin{vmatrix} -1 & -2 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} \right)$$

$$= (1+2, -(-1+2), -1-1)$$

$$\vec{g} = (3, -1, -2)$$

$$g = (2, 1, -2) + \lambda(3, -1, -2)$$

$$4) \ell: \frac{x}{6} = \frac{y-3}{-2} = \frac{-z-5}{1} \quad p = (1, 7, -4) + \lambda(1, -3, 3)$$

$$\ell: (0, 3, -5) + \lambda(\underline{6, -2, -1})$$

a)

$$d(\ell, p) = \frac{|(\vec{\ell} \times \vec{p}) \cdot \vec{LP}|}{|\vec{\ell} \times \vec{p}|}$$

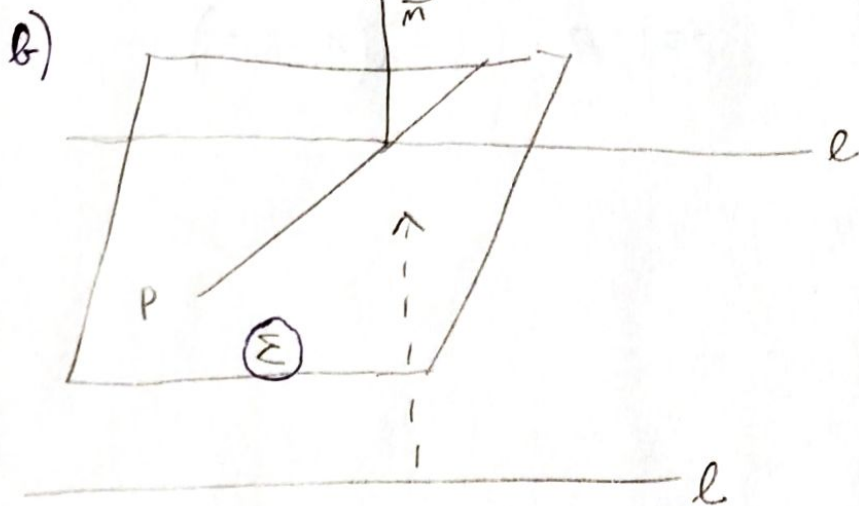
$$\begin{aligned} \vec{\ell} \times \vec{p} &= (1, -3, 3) \times (6, -2, -1) \\ &= \left(\begin{vmatrix} -3 & 3 \\ -2 & -1 \end{vmatrix}, -\begin{vmatrix} 1 & 3 \\ 6 & -1 \end{vmatrix}, \begin{vmatrix} 1 & -3 \\ 6 & -2 \end{vmatrix} \right) \\ &= (3+6, -(-1-18), -2+18) \\ &= \underline{(9, 19, 16)} \end{aligned}$$

$$\begin{aligned} \vec{LP} &= -\vec{\ell} + \vec{p} \\ &= \underline{(-6, 2, 1)} + \underline{(1, 3, 3)} \\ &= (0, -3, 5) + (1, 7, -4) \\ \vec{LP} &= (1, 4, 1) \end{aligned}$$

$$\begin{aligned} (9, 19, 16) \cdot (1, 4, 1) &= \\ &= 9 + 76 + 16 = \\ &= \underline{101} \end{aligned}$$

$$d(\ell, p) = \frac{101}{\sqrt{698}}$$

$$|\vec{\ell} \times \vec{p}| = \sqrt{9^2 + 19^2 + 16^2}$$



$$\vec{m} = \vec{p} \times \vec{l}$$

~~$$\vec{m} = \vec{p} \times \vec{l}$$~~
$$T(1, 7, -4)$$

$$= (1, -3, 3) \times (6, -2, -1)$$

$$= \left(\begin{vmatrix} -3 & 3 \\ -2 & -1 \end{vmatrix}, -\begin{vmatrix} 1 & 3 \\ 6 & -1 \end{vmatrix}, \begin{vmatrix} 1 & -3 \\ 6 & -2 \end{vmatrix} \right)$$

$$= (3 + 6, -(-1 - 18), (-2 + 18))$$

$$\vec{m} = (9, 19, 16)$$

$$\begin{matrix} x & y & z & - \\ 9 \cdot 1 & + 19 \cdot 7 & - 4 \cdot 16 & = 78 \end{matrix}$$

$$\Sigma: 9x + 19y + 16z = 78$$