



Analiza I - Temelji analize

1. KOLOKVIJ - PRAKTIČNI DEL
– 30. NOVEMBER, 2018 –

Čas: 60 minut. Največje možno število točk: 25. Pišite razločno. Vse odgovore je potrebno utemeljiti.

- (1) [9 t] V množici realnih števil \mathbb{R} rešite neenačbo

$$|x - |x - 1|| \geq x - \frac{1}{3}.$$

Rešitev zapišite s pomočjo intervalov, nato pa tej množici rešitev poiščite supremum, infimum, maksimum in minimum, če obstajajo.

- (2) [9 t] Naj bosta f in g realni funkciji realne spremenljivke, ki sta podani s predpisoma

$$f(x) = \begin{cases} -\frac{x^3+1}{x+1} & ; \quad x < 0 \\ e^{2x} - 1 & ; \quad x \geq 0 \end{cases}, \quad g(x) = \begin{cases} -\frac{x}{x+1} & ; \quad x < -1 \\ 0 & ; \quad -1 \leq x \leq 0 \\ \arctan(x+1) & ; \quad x > 0 \end{cases}$$

Zapišite predpis po katerem slika funkcija $g \circ f$ in utemeljite, ali je ta funkcija injektivna, surjektivna oz. bijektivna.

- (3) [7 t] Naj bo $a \neq 1$. Dokažite, da naslednja enakost velja za vsa naravna števila $n \in \mathbb{N}$.

$$1 + a + a^2 + a^3 + \dots + a^n = \frac{a^{n+1} - 1}{a - 1}$$

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function quicksort(array)
  var list less, equal, greater
  if length(array) ≤ 1
    return array
  select a pivot value pivot from array
  for each x in array
    if x < pivot then append to less
    if x = pivot then append to equal
    if x > pivot then append to greater
  return concatenate(quicksort(list), equal, quicksort(greater))

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University of Primorska

FAMNIT

Academic Year 2018/2019

Analysis I

PARTIAL EXAM 1 - PRACTICAL PART

– NOVEMBER 30, 2018 –

Time: 60 minutes. Maximum number of points: 25. Please write clearly, and justify all your answers.

- (1) [9 p] Find all real solutions of the inequality

$$|x - |x - 1|| \geq x - \frac{1}{3}.$$

The final solution must be stated using intervals. For this set of solutions also find supremum, infimum, maximum and minimum if they exist.

- (2) [9 p] Let f and g be real functions of a real variable that are defined by

$$f(x) = \begin{cases} -\frac{x^3+1}{x+1} & ; \quad x < 0 \\ e^{2x} - 1 & ; \quad x \geq 0 \end{cases}, \quad g(x) = \begin{cases} -\frac{x}{x+1} & ; \quad x < -1 \\ 0 & ; \quad -1 \leq x \leq 0 \\ \arctan(x+1) & ; \quad x > 0 \end{cases}$$

Write down the rule that defines the function $g \circ f$ and explain if this function is injective, surjective or, bijective.

- (3) [7 p] Suppose $a \neq 1$. Prove that the following equality is true for every positive integer $n \in \mathbb{N}$.

$$1 + a + a^2 + a^3 + \dots + a^n = \frac{a^{n+1} - 1}{a - 1}$$