Algebra I

2. KOLOKVIJ

– 21. JANUAR 2019 –

Čas pisanja: 90 minut. Maksimalno število točk: 50. Dovoljena je uporaba pisala, kalkulatorja in enega ročno napisanega lista formata A4 z definicijami in formulami (brez rešenih primerov). Pišite razločno in utemeljite vsak odgovor. Srečno!

1. Dane so matrike $A = \begin{bmatrix} 2 & -3 & 1 \\ 4 & -5 & 2 \\ 5 & -7 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 7 & 6 \\ 1 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix}$ in $C = \begin{bmatrix} 2 & 0 & -2 \\ 18 & 12 & 9 \\ 23 & 15 & 11 \end{bmatrix}$.

   Poiščite matriko X iz enačbe $AXB + AX = C$. *(14 točk)*

2. Dan je sistem linearnih enačb

$$\begin{array}{rcrcrcrcl} x & - & 3y & + & z & + & w & = & 2 \\ -x & + & 2y & + & 3z & - & w & = & \alpha \\ 6x & - & 17y & + & 2z & + & \beta w & = & 3 \end{array}$$

   (a) Za katere vrednosti $\alpha, \beta \in \mathbb{R}$ bo sistem protisloven? *(6 točk)*

   (b) Poiščite vse rešitve sistema, če je $\alpha = 6$ in $\beta = 8$. *(7 točk)*

3. Izračunajte determinanto dane matrike A. *(10 točk)*

$$A = \begin{bmatrix} 4 & 0 & 0 & 1 & 0 \\ 3 & 3 & 3 & -1 & 0 \\ 1 & 2 & 4 & 2 & 3 \\ 9 & 4 & 6 & 2 & 3 \\ 2 & 2 & 4 & 2 & 3 \end{bmatrix}$$

4. Določite rang matrike

$$A = \begin{bmatrix} 1 & 1 & a & a \\ a & a & 1 & 1 \\ 1 & a & 1 & a \end{bmatrix}$$

   v odvisnosti od števila $a \in \mathbb{R}$. *(13 točk)*

Algebra I

MIDTERM 2

– JANUARY 21, 2019 –

Time: 90 minutes. Maximum number of points: 50. You are allowed to use a pen, a calculator and one A4 hand-written piece of paper with definitions and formulas (and with no solved exercises). Write clearly, and justify all your answers. Good luck!

1. For matrices $A = \begin{bmatrix} 2 & -3 & 1 \\ 4 & -5 & 2 \\ 5 & -7 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 7 & 6 \\ 1 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 18 & 12 & 9 \\ 23 & 15 & 11 \end{bmatrix}$.

   find the matrix X from the equation $AXB + AX = C$. *(14 points)*

2. We are given the following system of linear equations:

$$\begin{array}{rcrcrcrcl} x & - & 3y & + & z & + & w & = & 2 \\ -x & + & 2y & + & 3z & - & w & = & \alpha \\ 6x & - & 17y & + & 2z & + & \beta w & = & 3 \end{array}$$

   (a) For which values $\alpha, \beta \in \mathbb{R}$ will the system be inconsistent? *(6 points)*

   (b) Find all the solutions of the system, if $\alpha = 6$ and $\beta = 8$. *(7 points)*

3. Compute the determinant of the given matrix A. *(10 points)*

$$A = \begin{bmatrix} 4 & 0 & 0 & 1 & 0 \\ 3 & 3 & 3 & -1 & 0 \\ 1 & 2 & 4 & 2 & 3 \\ 9 & 4 & 6 & 2 & 3 \\ 2 & 2 & 4 & 2 & 3 \end{bmatrix}$$

4. Determine the rank of the matrix

$$A = \begin{bmatrix} 1 & 1 & a & a \\ a & a & 1 & 1 \\ 1 & a & 1 & a \end{bmatrix}$$

   depending on the value of $a \in \mathbb{R}$. *(13 points)*