ALGEBRA 1 - 2. KOLOKVIJ Andrej Erjavec

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1.) $A = \begin{bmatrix} 2 & 0 & -3 \\ 7 & 5 & 1 \\ 1 & -2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 1 & -1 \\ 0 & -4 & 1 \\ -2 & 3 & 2 \end{bmatrix}$

$$A + X = BX + B^T$$

$$X - BX = B^T - A$$

$$X(I_d - B) = B^T - A \,/\cdot (I_d - B)^{-1}$$ z DESNE

$$X = (B^T - A)(I_d - B)^{-1}$$

$B^T = \begin{bmatrix} 3 & 0 & -2 \\ 1 & -4 & 3 \\ -1 & 1 & 2 \end{bmatrix}$ $B^T - A = \begin{bmatrix} 3 & 0 & -2 \\ 1 & -4 & 3 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 & 3 \\ -7 & -5 & -1 \\ -1 & 2 & -1 \end{bmatrix} =$

$$B^T - A = \begin{bmatrix} 1 & 0 & 1 \\ -6 & -9 & 2 \\ -2 & 3 & 1 \end{bmatrix}$$

$$I_d - B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} -3 & -1 & 1 \\ 0 & 4 & -1 \\ 2 & -3 & -2 \end{bmatrix} = \begin{bmatrix} -2 & -1 & 1 \\ 0 & 5 & -1 \\ 2 & -3 & -1 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 0 & 1 \\ -6 & -9 & 2 \\ -2 & 3 & 1 \end{bmatrix} \cdot \Big[$$

$$(I_d - B)^{-1} = \left[\begin{array}{ccc|ccc} -2 & -1 & 1 & 1 & 0 & 0 \\ 0 & 5 & -1 & 0 & 1 & 0 \\ 2 & -3 & -1 & 0 & 0 & 1 \end{array}\right] V_3 + V_1 \sim$$

$V_2 + V$ ~~$V_3 - V_2$~~ $+4V$ $\sim \begin{bmatrix} -2 & -1 & 1 \\ 0 & 5 & -1 \\ 0 & -9 & 1 \end{bmatrix} V$

$$(Id-B)^{-1}=\left[\begin{array}{ccc|ccc}-2&-1&1&1&0&0\\0&5&-1&0&1&0\\2&-3&-1&0&0&1\end{array}\right]V_3+V_1\sim\left[\begin{array}{ccc|ccc}-2&-1&1&1&0&0\\0&5&-1&0&1&0\\0&-4&0&1&0&1\end{array}\right]$$

V_2+V_3

$$\sim\left[\begin{array}{ccc|ccc}-2&-1&1&1&0&0\\0&1&-1&1&1&1\\0&-4&0&1&0&1\end{array}\right]V_3+4V_2\sim\left[\begin{array}{ccc|ccc}2&-1&1&1&0&0\\0&1&-1&1&1&1\\0&0&-4&5&4&5\end{array}\right]$$

V_1+V_2

$$\sim\left[\begin{array}{ccc|ccc}2&0&0&1&0&0\\0&1&-1&1&1&1\\0&0&-4&5&4&5\end{array}\right]4V_2-V_3\sim\left[\begin{array}{ccc|ccc}2&0&0&1&0&0\\0&4&\cancel{-0}&-1&0&-1\\0&0&-4&5&4&5\end{array}\right]$$

$$(Id)-B)^{-1}=\left[\begin{array}{ccc|ccc}-2&-1&1&1&0&0\\0&5&-1&0&1&0\\2&-3&-1&0&0&1\end{array}\right]\begin{array}{l}V_2+5V_1\\V_3\end{array}\sim\left[\begin{array}{ccc}-2&-1&1\end{array}\right.$$

2.) ~~(1-α) x - y~~

a) NESK. MNOGO REŠ.

b) $\alpha = 1$

$$\begin{bmatrix} 1-\alpha & -1 & 2 & \vert & 1 \\ -3 & -1 & \alpha & \vert & -1 \\ \alpha & 2 & -3 & \vert & -1 \end{bmatrix} V_3 + 2V_2$$

$$\sim \begin{bmatrix} 1-\alpha & -1 & 2 & \vert & 1 \\ -3 & -1 & \alpha & \vert & -1 \\ \alpha-6 & 0 & -3+2\alpha & \vert & \end{bmatrix}$$

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2) $$\begin{bmatrix} 1-\alpha & -1 & 2 & \vert & 1 \\ -3 & -1 & \alpha & \vert & -1 \\ \alpha & 2 & -3 & \vert & -1 \end{bmatrix} \begin{array}{l} \\ V_2+V_1 \\ \cancel{V_3+V_1} \end{array} \sim \begin{bmatrix} 1-\alpha & -1 & 2 & \vert & 1 \\ -3 & -1 & \alpha & \vert & -1 \\ & & & \vert & \end{bmatrix}$$

$$\sim \begin{bmatrix} 1-\alpha & -1 & 2 & \vert & 1 \\ -2-\alpha & -2 & 2+\alpha & \vert & 0 \\ \alpha & 2 & -3 & \vert & -1 \end{bmatrix} \begin{array}{l} \\ V \\ V_3+V_2 \end{array} \sim \begin{bmatrix} 1-\alpha & -1 & 2 & \vert & 1 \\ -2-\alpha & -2 & 2+\alpha & \vert & 0 \\ 1 & 1 & -1 & \vert & 0 \end{bmatrix} V_2 + 2V_3$$

$$\sim \begin{bmatrix} 1-\alpha & -1 & 2 & \vert & 1 \\ -\alpha & 0 & \alpha & \vert & 0 \\ 1 & 1 & -1 & \vert & 0 \end{bmatrix} \Rightarrow \boxed{\alpha = 0 \text{ NESKONČNO MNOGO REŠITEV}}$$

b) $$\begin{bmatrix} 0 & -1 & 2 & \vert & 1 \\ -3 & -1 & 1 & \vert & -1 \\ 1 & 2 & 3 & \vert & -1 \end{bmatrix} 3V_3 + V_2 \sim \begin{bmatrix} 0 & -1 & 2 & \vert & 1 \\ -3 & -1 & 1 & \vert & -1 \\ 0 & 5 & 10 & \vert & -4 \end{bmatrix} V_3 + 5V_1$$

$$\sim \begin{bmatrix} 0 & -1 & 2 & \vert & 1 \\ -3 & -1 & 1 & \vert & -1 \\ 0 & 0 & 20 & \vert & 1 \end{bmatrix} \begin{array}{l} \\ 2V_2 - V_1 \\ \end{array} \Rightarrow 20z = 1$$

$$\boxed{z = \frac{1}{20}}$$

$$\sim \begin{bmatrix} 0 & -1 & 2 & \vert & 1 \\ -6 & -3 & 0 & \vert & -2 \\ 0 & 0 & 20 & \vert & 1 \end{bmatrix}$$

b) $\left[\begin{array}{ccc|c} 0 & -1 & 2 & 1 \\ -3 & -1 & 1 & -1 \\ 1 & 2 & 3 & 1 \end{array}\right] V_2 + 3V_3 \sim \left[\begin{array}{ccc|c} 0 & -1 & 2 & 1 \\ 0 & 5 & 10 & 2 \\ 1 & 2 & 3 & 1 \end{array}\right] V_1 + V_3$

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 5 & 2 \\ 0 & 5 & 10 & 2 \\ 1 & 2 & 3 & 1 \end{array}\right] V_3 - V_1 \sim \left[\begin{array}{ccc|c} 1 & 1 & 5 & 2 \\ 0 & 5 & 10 & 2 \\ 0 & 1 & -2 & -1 \end{array}\right] 5V_3 - V_2 \sim$

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 5 & 2 \\ 0 & 5 & 10 & 2 \\ 0 & 0 & -20 & -1 \end{array}\right] V_2 + \frac{1}{2} V_3 \sim \left[\begin{array}{ccc|c} 1 & 1 & 5 & 2 \\ 0 & 5 & 0 & \frac{3}{2} \\ 0 & 0 & -20 & -1 \end{array}\right] 5V_1 - V_2$

$\boxed{z = \frac{1}{20}}$

$5y = \frac{3}{2}$

$\boxed{y = \frac{3}{10}}$

$\sim \left[\begin{array}{ccc|c} 5 & 0 & 25 & 10 - \frac{3}{2} \\ 0 & 5 & 0 & \frac{3}{2} \\ 0 & 0 & -20 & -1 \end{array}\right]$

3.) $$M=\begin{bmatrix}1&1&1&1&1\\1&x&1&2&1\\1&1&x&1&1\\1&2&1&x&1\\1&1&1&1&2\end{bmatrix}\begin{matrix}\\V_2-V_1\\V_3-V_1\\V_4-V_1\\V_5-V_1\end{matrix}\sim\begin{bmatrix}1&1&1&1&1\\0&x-1&0&1&0\\0&0&x-1&0&0\\0&1&0&x-1&0\\0&0&0&0&1\end{bmatrix}\sim$$

$$\sim 1\cdot\begin{bmatrix}x-1&0&1&0\\0&x-1&0&0\\1&0&x-1&0\\0&0&0&1\end{bmatrix}\sim 1\cdot 1\cdot\begin{bmatrix}x-1&0&1\\0&x-1&0\\1&0&x-1\end{bmatrix}$$

$$(x-1)\begin{vmatrix}x-1&1\\1&x-1\end{vmatrix}$$

$$\det(M)=(x-1)\left((x-1)^2-1\right)$$
$$=(x-1)(x^2-2x\cancel{+1}\cancel{-1})$$
$$=(x-1)(x^2-2x)$$
$$=(x-1)x(x-2)$$

$\det(M)\neq 0 \iff$ OBRNLJIVA

$x_1=1 \quad x_2=0 \quad x_3=2$

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$\boxed{\text{OBRNLJIVA}: x\in\mathbb{R}\setminus\{0,1,2\}}$

4.) $$A=\begin{bmatrix}b&b&b-a\\a-b&-b&a\\a+b&b&0\end{bmatrix}$$

$r(A)=1$ – NI MOGOČE

– 1. IN 2. VRSTICA STA LINEARNO NEODVISNI.

$2\leq r(A)\leq 3$

$$\begin{bmatrix}b&b&b-a\\a-b&-b&a\\a+b&b&0\end{bmatrix}\begin{matrix}\\\cancel{V_2+V_3}\\V_3-V_2\end{matrix}\sim\cancel{\begin{bmatrix}b&b&b-a\\2a&0&a\\a+b&b&0\end{bmatrix}}$$

$$\sim\begin{bmatrix}b&b&b-a\\a-b&-b&a\\2b&2b&0\end{bmatrix}\qquad\begin{bmatrix}b&b&b-a\\a-b&-b&a\\a+b&b&0\end{bmatrix}$$

$$r(A)=\begin{bmatrix} b & b & b-a \\ a-b & -b & a \\ a+b & b & 0 \end{bmatrix}\begin{matrix} b & b \\ a-b & -b \\ a+b & b \end{matrix} = a\cdot b(a+b)+(b-a)(a-b)\cdot b + (a+b)(b-a)b - ab^2 =$$

$\det(A)=0 \Rightarrow r(A)=2$

$$= a^2\cdot b + \cancel{ab^2} + (ab-b^2-a^2+ab)\cdot b + (ab-a^2+b^2-ab)\cdot b - \cancel{ab^2} =$$

$$= \cancel{a^2b} + \cancel{ab^2} - \cancel{b^3} - \cancel{a^2b} + \cancel{ab^2} + \cancel{ab^2} - \cancel{a^2b} + \cancel{b^3} - \cancel{ab^2} + \cancel{ab^2} = 0$$

$$\begin{bmatrix} b & b & b-a \\ a-b & -b & a \\ a+b & b & 0 \end{bmatrix} V_3+V_2 \sim \begin{bmatrix} b & b & b-a \\ a-b & -b & a \\ 2a & 0 & a \end{bmatrix} V_2-V_3 \sim$$

$$\sim \cancel{\begin{bmatrix} b & b & b-a \\ a-b & -b & \end{bmatrix}} \sim \begin{bmatrix} b & b & b-a \\ -a-b & -b & 0 \\ 2a & 0 & a \end{bmatrix}$$