Analiza I - Temelji analize 2017/2018
1. kolokvij
6. december 2017

1. Pokažite, da za poljubne množice A, B, C in D velja, da je

$$(A \times C) \setminus (B \times D) \subseteq (A \times (C \setminus D)) \cup ((A \setminus B) \times D)$$

Resničnost izjave najprej preverite z ustreznimi diagrami za upodabljanje kartezičnega produkta množic, nato pa zapišite formalen dokaz. Zapišite tudi kdaj natanko med danima množicama velja enačaj. (*9t*)

2. Naj bosta f in g realni funkciji realne spremenljivke, ki sta podani s predpisoma

$$f(x) = \begin{cases} x - \pi/2 & ; \quad x < 0 \\ x - 1 & ; \quad x \geq 0 \end{cases}, \qquad g(x) = \begin{cases} \dfrac{x+1}{x+5} & ; \quad x \geq -1 \\ \sin(2x + \pi/2) & ; \quad x < -1 \end{cases}$$

Zapišite predpis po katerem slika funkcija $g \circ f$. (*8t*)

3. Dokažite, da naslednja enakost velja za vsa naravna števila $n \in \mathbb{N}$.

$$1 \cdot 4 + 2 \cdot 7 + 3 \cdot 10 + \ldots + n(3n + 1) = n(n + 1)^2$$

(*8t*)

Vse odgovore je potrebno ustrezno utemeljiti! Prepisovanje nalog je prepovedano in bo ustrezno kaznovano!

Analysis I 2017/2018
1. Midterm Exam

December 6, 2017

1. Show that the following statement is true for arbitrary sets A, B, C, and D.

$$(A \times C) \setminus (B \times D) \subseteq (A \times (C \setminus D)) \cup ((A \setminus B) \times D)$$

First check the statement using an appropriate diagram for representing the cartesian product of sets. Then state a formal proof. State also the exact condition (the if and only if statement) for the equality of the given sets. (*9t*)

2. Let f and g be real functions of a real variable that are defined by

$$f(x) = \begin{cases} x - \pi/2 & ; \quad x < 0 \\ x - 1 & ; \quad x \geq 0 \end{cases}, \qquad g(x) = \begin{cases} \dfrac{x+1}{x+5} & ; \quad x \geq -1 \\ \sin(2x + \pi/2) & ; \quad x < -1 \end{cases}$$

Write down the rule that definies the function $g \circ f$. (*8t*)

3. Prove that the following equality is true for every positive interger $n \in \mathbb{N}$.

$$1 \cdot 4 + 2 \cdot 7 + 3 \cdot 10 + \ldots + n(3n + 1) = n(n + 1)^2$$

(*8t*)

All answers must be justified! Cheating is prohibited and will be appropriately penalized!