KOTI

- Kot med 2 vektorjema: $\cos\varphi = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$

- kot med 2 premicama:

Definicija: Kot med premicama p, q, ki nista vzporedni, enak <u>OSTREMU KOTU</u> med njunima smernima vektorjema.

$$\sphericalangle(p,q): \cos\varphi = \frac{|\vec{s}_p\cdot\vec{s}_q|}{|\vec{s}_p|\cdot|\vec{s}_q|}$$

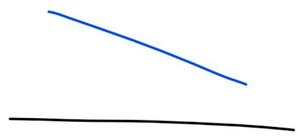
Primer: $p = (2,7,-1) + \lambda(1,0,1)$

$q = (2,0,-1) + \lambda(1,1,2)$

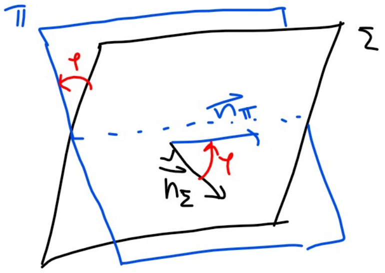
$$\sphericalangle(p,q): \quad \cos\varphi = \frac{|(1,0,1)\cdot(1,1,2)|}{\sqrt{1+0+1}\cdot\sqrt{1+1+4}} = \frac{1+0+2}{\sqrt{2}\cdot\sqrt{6}} = \frac{3}{\sqrt{12}} = \frac{3}{\sqrt{4\cdot 3}}$$

$$= \frac{3}{2\sqrt{3}} \Big/ \cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\cancel{3}\sqrt{3}}{2\cdot\cancel{3}} = \frac{\sqrt{3}}{2}$$

$$\underline{\underline{\varphi = 30^\circ}}$$



- med dvema ravninama



$$\sphericalangle(\Sigma,\Pi): \cos\varphi = \frac{\vec{n}_\Pi\cdot\vec{n}_\Sigma}{|\vec{n}_\Pi||\vec{n}_\Sigma|}$$

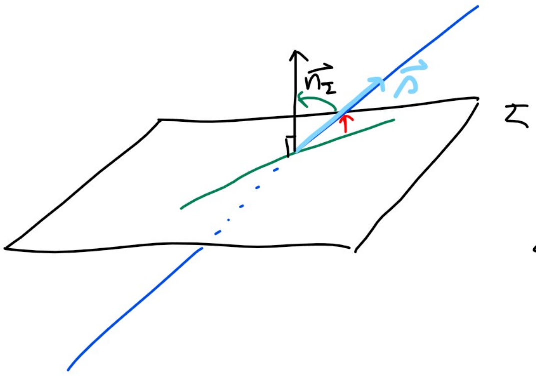
Primer: $\Sigma: x - z = 7 \Rightarrow \vec{n}_\Sigma = (1,0,-1)$

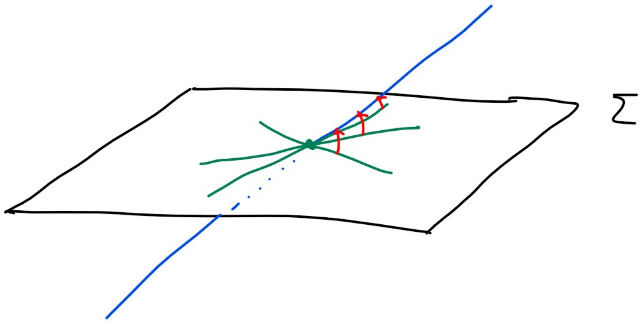
$\Pi: x - y - 2z = 25 \Rightarrow \vec{n}_\Pi = (1,-1,-2)$

$$\cos\varphi = \frac{(1,0,-1)\cdot(1,-1,-2)}{\sqrt{1+0+1}\cdot\sqrt{1+1+4}} = \frac{1+0+2}{\sqrt{2}\cdot\sqrt{6}} = \frac{3}{\sqrt{12}} = \dots = \frac{\sqrt{3}}{2}$$

$$\underline{\underline{\varphi = 30^\circ}}$$

- med premico in ravnino

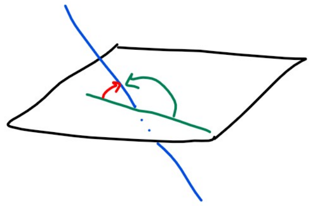




$$\sphericalangle(p,\Sigma) = 90 - \sphericalangle(\vec{s},\vec{n})$$

OSTRI KOT!

$$\Rightarrow \sin\varphi = \frac{|\vec{s}\cdot\vec{n}|}{|\vec{s}|\cdot|\vec{n}|}$$



Primer:

$$p = (2,0,3) + \lambda(0,1,1) \Rightarrow \vec{s}_p = (0,1,1)$$

$$\Sigma = x + y + 2z = \sqrt{13} \Rightarrow \vec{n}_\Sigma = (1,1,2)$$

I. možnost:

$$\sin\varphi = \frac{|(0,1,1)\cdot(1,1,2)|}{\sqrt{0+1+1}\cdot\sqrt{1+1+4}} = \frac{0+1+2}{\sqrt{2}\cdot\sqrt{6}} = \cdots = \frac{\sqrt{3}}{2}$$

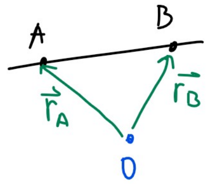
$$\underline{\underline{\varphi = 60^\circ}}$$

II. možnost:

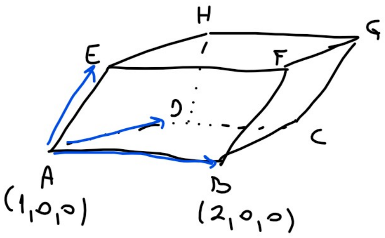
$$\cos\varphi = \frac{\sqrt{3}}{2} \Rightarrow \varphi = 30^\circ$$

$$\sphericalangle(p,\Sigma) = 90^\circ - 30^\circ = 60^\circ$$

1. kolokvij (24. 11. 2017)



1.)



$\vec{AB} = (1,0,0)$

$\vec{AD} = (2,-1,0)$

$\vec{AE} = (0,2,1)$

a) B, C, D, G ?

$\vec{AB} = -\vec{r}_A + \vec{r}_B$

$\vec{AB} + \vec{r}_A = \vec{r}_B$

$\vec{r}_B = (1,0,0) + (1,0,0)$
$= (2,0,0)$

$\vec{AD} = -\vec{r}_A + \vec{r}_D$

$\vec{AD} + \vec{r}_A = \vec{r}_D$

$\vec{r}_D = (2,-1,0) + (1,0,0)$
$= (3,-1,0)$

$\vec{r}_C = \vec{r}_A + \vec{AB} + \vec{BC}$
$= \vec{r}_A + \vec{AB} + \vec{AD}$
$= (1,0,0) + (1,0,0) + (2,-1,0)$
$= (4,-1,0)$

$= \vec{r}_C + \vec{CG} = \vec{r}_C + \vec{AE}$

$\vec{r}_G = \vec{r}_A + \vec{AB} + \vec{BC} + \vec{CG}$

$= \vec{r}_A + \vec{AB} + \vec{AD} + \vec{AE}$

$= (1,0,0) + (1,0,0) + (2,-1,0) + (0,2,1)$

$= (4,1,1)$

b) $\measuredangle(\vec{AB}, \vec{AE})$

$\vec{AB} = (1,0,0)$

$\vec{AE} = (0,2,1)$

$$\cos\varphi = \frac{(1,0,0)\cdot(0,2,1)}{\sqrt{1+0+0}\cdot\sqrt{0+4+1}} = \frac{0+0+0}{\sqrt{1}\cdot\sqrt{5}} = 0 \Rightarrow \underline{\underline{\varphi = 90^\circ}}$$

c)

$V = |(\vec{AB}, \vec{AD}, \vec{AE})| =$

$= |(0,0,-1)\cdot(0,2,1)|$

$= |0+0-1| = \underline{\underline{1}}$

$\vec{AB} \times \vec{AD}$

$\begin{matrix} 1 & 0 & 0 \\ 2 & -1 & 0 \end{matrix}$

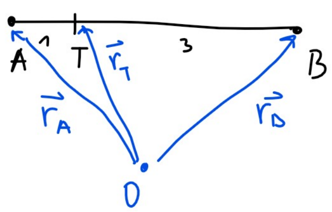
$= \left(\begin{vmatrix} 0 & 0 \\ -1 & 0 \end{vmatrix}, -\begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} \right)$

$= (0-0, -(0-0), -1-0)$

$= (0,0,-1)$

2.) $A(1,2,-1)$

$B(7,2,2)$



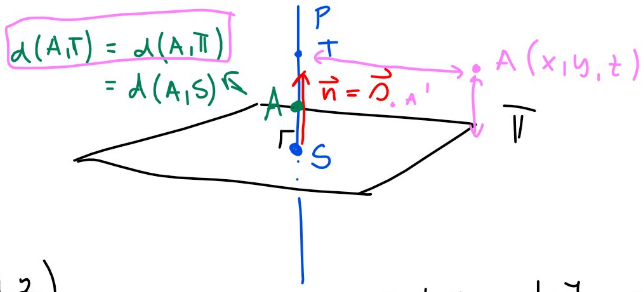
a) T, $|AT| : |TB| = 1:3$

$\vec{AB} = -\vec{r}_A + \vec{r}_B = (-1,-2,1) + (7,2,2) = (6,0,3)$

$\vec{r}_T = \vec{r}_A + \frac{1}{4}\vec{AB} = (1,2,-1) + \frac{1}{4}(6,0,3) = \left(1+\frac{6}{4}, 2, -1+\frac{3}{4}\right) = \left(\frac{10}{4}, 2, -\frac{1}{4}\right)$

b) $|\vec{AB}| = \sqrt{36+0+9} = \sqrt{45}$

3.) $\Pi: \; x - y + 2z = 0$



a) p?

$T(4,0,4)$

$p \perp \Pi$ $\qquad \vec{n} = (1,-1,2)$

$$p = (4,0,4) + \lambda(1,-1,2)$$

splošna točka na premici p

$P(4+1\lambda, -\lambda, 4+2\lambda)$

b)
$$4+\lambda-(-\lambda)+2(4+2\lambda) = 0$$
$$4+\lambda+\lambda+8+4\lambda = 0$$
$$6\lambda = -12 \;/:6$$
$$\underline{\underline{\lambda = -2}} \longrightarrow S(2,2,0)$$

I. možnost:

c) $\underline{A \text{ je razpolovišče daljice } TS.}$

II. možnost:
$$d(A,T) = d(A,\Pi) = d(A,S)$$

$A(4+\lambda, -\lambda, 4+2\lambda)$

$$|\vec{AT}| = d(A,T) = \sqrt{(x_A - x_T)^2 + (y_A - y_T)^2 + (z_A - z_T)^2} =$$
$$= \sqrt{(4+\lambda-4)^2 + (-\lambda-0)^2 + (4+2\lambda-4)^2}$$
$$= \sqrt{\lambda^2 + \lambda^2 + 4\lambda^2} = \sqrt{6\lambda^2}$$

$$d(A,S) = \sqrt{\underbrace{(4+\lambda-2)^2}_{(2+\lambda)^2} + (-\lambda-2)^2 + \underbrace{(4+2\lambda-0)^2}_{(4+2\lambda)^2}}$$
$$= \sqrt{4+4\lambda+\lambda^2+\lambda^2+4\lambda+4+16+\cancel{8}^{16}\lambda+4\lambda^2}$$
$$= \sqrt{6\lambda^2+\cancel{16}^{24}\lambda+24}$$

$$\sqrt{6\lambda^2} = \sqrt{6\lambda^2+24\lambda+24} \;/^2$$
$$\cancel{6\lambda^2} = \cancel{6\lambda^2}+24\lambda+24 \;/-6\lambda^2$$
$$-24\lambda = 24 \;/:(-24)$$
$$\underline{\underline{\lambda = -1}} \longrightarrow A(3,1,2)$$

$$d(T,A) = \sqrt{6\cdot(-1)^2} = \underline{\underline{\sqrt{6}}}$$

4.) $p: \frac{x+2}{5} = -\frac{y}{2} = \frac{z-2}{5}$ $\quad \vec{s}_p = (5,-2,5)$

$\Sigma: 3x + 5y - 1z = 4$

a) $q: \frac{x+1}{2} = \frac{y-1}{3} = z$ $\quad d(p,q) = ?$

$\vec{s}_q = (2,3,1) \Rightarrow$ p in q nista vzporedni

p in q vzporedna → ali je $\vec{s}_q = t \cdot \vec{s}_p$ za nek $t \in \mathbb{R}$

→ ali je $\vec{s}_q \times \vec{s}_p = 0$

$$d(p,q) = \frac{|(\vec{s}_p \times \vec{s}_q) \cdot \vec{PQ}|}{|\vec{s}_p \times \vec{s}_q|}$$

$$\vec{PQ} = -\vec{r}_P + \vec{r}_Q = -(-2,0,+2) + (-1,1,0) = (2,0,-2) + (-1,1,0) = (1,1,-2)$$

$$= \frac{|(-17,5,19) \cdot (1,1,-2)|}{\sqrt{289+25+361}} = \frac{|-17+5-38|}{\sqrt{675}} = \frac{50}{\sqrt{675}} \approx 1,92$$

$$\vec{s}_p \times \vec{s}_q = \begin{matrix} 5 & -2 & 5 \\ 2 & 3 & 1 \end{matrix}$$

$$= \left(\begin{vmatrix} -2 & 5 \\ 3 & 1 \end{vmatrix}, -\begin{vmatrix} 5 & 5 \\ 2 & 1 \end{vmatrix}, \begin{vmatrix} 5 & -2 \\ 2 & 3 \end{vmatrix} \right)$$

$$= (-2-15, -(5-10), 15-(-4))$$

$$= (-17, 5, 19)$$

b) $d(p,\Sigma)$

$\|$

$$d(P,\Sigma) = \frac{|ax+by+cz-d|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{|3\cdot(-2)+5\cdot 0 - 1\cdot 2 - 4|}{\sqrt{9+25+1}}$$

$$= \frac{|-6-2-4|}{\sqrt{35}} = \frac{12}{\sqrt{35}} \approx 2,03$$

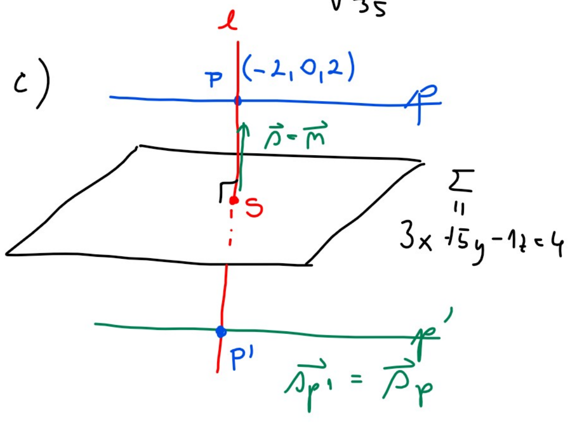
$p \parallel \Sigma \Leftrightarrow \vec{s}_p \perp \vec{m} \Leftrightarrow \vec{s}_p \cdot \vec{m} = 0$

$(5,-2,5)\cdot(3,5,-1) = 15 - 10 - 5 = 0 \Rightarrow p \parallel \Sigma$

$P(\overset{x}{-2}, \overset{y}{0}, \overset{z}{2})$

$\vec{m} = (\overset{a}{3}, \overset{b}{5}, \overset{c}{-1})$ $\quad d = 4$

c)



$l = (-2,0,2) + \lambda(3,5,-1) \Rightarrow (-2+3\lambda, 5\lambda, 2-\lambda)$

$3(-2+3\lambda) + 5\cdot 5\lambda - 1(2-\lambda) = 4$

$-6 + 9\lambda + 25\lambda - 2 + \lambda = 4$

$35\lambda = 4 + 8$

$35\lambda = 12 \quad /:35$

$\lambda = \frac{12}{35} \longrightarrow S\left(-\frac{34}{35}, \frac{12}{7}, \frac{58}{35}\right)$

$$\vec{r}_{P'} = \vec{r}_S + \underbrace{\overset{= -\vec{r}_P + \vec{r}_S}{\vec{PS}}}_{\overrightarrow{SP'}} = \left(-\frac{34}{35}, \frac{12}{7}, \frac{58}{35}\right) - (-2, 0, 2) + \left(-\frac{34}{35}, \frac{12}{7}, \frac{58}{35}\right)$$

$$= \left(\frac{2}{35}, \frac{24}{7}, \frac{46}{35}\right)$$

$$p' = \left(\frac{2}{35}, \frac{24}{7}, \frac{46}{35}\right) + \lambda (5, -2, 5)$$