Univerza na Primorskem
Fakulteta za matematiko, naravoslovje
in informacijske tehnologije
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IME: Safet

VPISNA ŠTEVILKA: partdock

PRIIMEK: Penjić

PODPIS: X

Analiza I, izpit - praktični del

1. (a) V množici kompleksnih števil $\mathbb{C}$ poiščite vsa kompleksna števila $z = a + ib$, ki zadoščajo naslednji enačbi

$$|z| + z = 2 + i.$$

(b) Določi množico točk (x, y) v ravnini, ki zadoščajo enačbi

$$yi + (5i - x^2)i + 5 = 0.$$

Re.

(a) $z = a + ib$

$|z| = \sqrt{a^2+b^2}$

$|z| + z = 2 + i$

$\underline{\sqrt{a^2+b^2} + a} + \underline{ib} = \underline{2} + \underline{i}$

$\sqrt{a^2+b^2} + a = 2$

$b = 1 \quad \dots(*)$

$\Downarrow (*)$

$\sqrt{a^2+1} + a = 2$

$\sqrt{a^2+1} = 2 - a \quad /^2$

$a^2 + 1 = (2-a)^2$

$\cancel{a^2} + 1 = 4 - 4a + \cancel{a^2} \Rightarrow$

$4 - 4a = 1$

$4 - 1 = 4a$

$3 = 4a \Rightarrow a = \frac{3}{4} \quad \dots(**)$

$\underbrace{(*) \text{ in } (**)}$

$\Downarrow$

$z = \frac{3}{4} + i$

rešitva enačbe

(b)

$$yi + (5i - x^2)i + 5 = 0$$

$$yi + 5i^2 - x^2 i + 5 = 0$$

$$yi \cancel{-5} - x^2 i \cancel{+5} = 0$$

$$yi - x^2 i = 0$$

$$(y - x^2)i = 0 \implies y - x^2 = 0$$

$$y = x^2$$

$$S = \{(x,y) \in \mathbb{R}^2 \mid y = x^2\}$$

množica točk v ravnini $\mathbb{R}^2$, ki zadoščajo enačbi

2. Naj bosta f in g realni funkciji realne spremenljivke, ki sta podani s predpisoma

$$f(x) = \begin{cases} x^2 - 1, & x \geq 0 \\ e^{-x}, & x < 0 \end{cases} \qquad \text{in} \qquad g(x) = \begin{cases} 1, & x < 1 \\ x - 2, & x \geq 1 \end{cases}.$$

Določite kompozitum $f \circ g$.

Re.

$$(f \circ g)(x) = f(g(x)) = \begin{cases} g(x)^2 - 1, & g(x) \geq 0 \\ e^{-g(x)}, & g(x) < 0 \end{cases} \qquad \ldots (\square)$$

$$g(x) = \begin{cases} 1, & x < 1 \\ x-2, & x \geq 1 \end{cases} \qquad \ldots (*)$$

$$x < 1 \Rightarrow g(x) = 1 > 0 \qquad \ldots (1)$$

$$x - 2 \geq 0$$
$$x \geq 2 \qquad \ldots (2)$$

$$(1) \text{ in } (2) \Rightarrow \begin{array}{l} g(x) \geq 0 \iff x \in (-\infty, 1) \cup [2, +\infty) \\ g(x) < 0 \iff x \in [1, 2) \end{array} \qquad \ldots (\square\square)$$

$$\left.\begin{array}{llll} x \in (-\infty, 1) & \overset{(*)}{\Rightarrow} & g(x) = 1 & (g(x) \geq 0) \\ x \in [1, 2) & \overset{(*)}{\Rightarrow} & g(x) = x - 2 & (g(x) < 0) \\ x \in [2, +\infty) & \overset{(*)}{\Rightarrow} & g(x) = x - 2 & (g(x) \geq 0) \end{array}\right\} \qquad \ldots (\square\square\square)$$

$$(\square), (\square\square) \text{ in } (\square\square\square) \Rightarrow (f \circ g)(x) = \begin{cases} \overset{1^2-1}{0}, & x \in (-\infty, 1) \\ e^{-(x-2)}, & x \in [1, 2) \\ \underset{x^2-4x+3}{\underset{\parallel}{(x-2)^2 - 1}}, & x \in [2, +\infty) \end{cases}$$

3. Izračunaj limito funkcije

$$\lim_{x \to 4} \frac{3x^2 - 13x + 4}{2x^2 - 7x - 4}.$$

Navodila:

$$3x^2 - 13x + 4 = 3(x-4)\left(x - \frac{1}{3}\right) = (x-4)(3x-1)$$

$$2x^2 - 7x - 4 = 2(x-4)\left(x - \frac{1}{2}\right) = (x-4)(2x+1)$$

$$\lim_{x \to 4} \frac{3x^2 - 13x + 4}{2x^2 - 7x - 4} = \lim_{x \to 4} \frac{\cancel{(x-4)}(3x-1)}{\cancel{(x-4)}(2x+1)} = \lim_{x \to 4} \frac{3x-1}{2x+1}$$

$$= \frac{3 \cdot 4 - 1}{2 \cdot 4 + 1} = \frac{11}{9}$$

4. Ugotovi, ali podana vrsta konvergira in izračunaj vsoto

$$\sum_{n=1}^{\infty} \frac{1}{(3n-2)(3n+1)}$$

Re.

$$\frac{1}{(3n-2)(3n+1)} = \frac{A}{3n-2} + \frac{B}{3n+1} \quad / \cdot (3n-2)(3n+1)$$

$$1 = A(3n+1) + B(3n-2)$$

$$3An + A + 3Bn - 2B = 1$$

$$(3A+3B)n + A - 2B = 1$$

$$3A + 3B = 0$$
$$A - 2B = 1 \quad / \cdot 3$$

$$3A + 3B = 0$$
$$-\ 3A - 6B = 3$$

$$9B = -3$$

$$B = -\frac{1}{3}$$

$$A - 2B = 1$$
$$A + \frac{2}{3} = 1$$
$$A = \frac{1}{3}$$

$$a_n = \frac{1}{(3n-2)(3n+1)} = \frac{\frac{1}{3}}{3n-2} + \frac{-\frac{1}{3}}{3n+1}$$

$$a_n = \frac{\frac{1}{3}}{3n-2} - \frac{\frac{1}{3}}{3n+1}$$

Opazimo

$$a_1 = \frac{1}{3} - \frac{\frac{1}{3}}{4}, \qquad a_3 = \frac{\frac{1}{3}}{7} - \frac{\frac{1}{3}}{10},$$

$$a_2 = \frac{\frac{1}{3}}{4} - \frac{\frac{1}{3}}{7}, \qquad a_4 = \frac{\frac{1}{3}}{10} - \frac{\frac{1}{3}}{13}, \ldots$$

$$S_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + \dots + a_{n-1} + a_n =$$

$$= \frac{1}{3} - \cancel{\frac{\frac{1}{3}}{4}} + \cancel{\frac{\frac{1}{3}}{4}} - \cancel{\frac{\frac{1}{3}}{7}} + \dots +$$

$$+ \cancel{\frac{\frac{1}{3}}{3(n-1)-2}} - \cancel{\frac{\frac{1}{3}}{\underbrace{3(n-1)+1}_{3n-2}}} + \cancel{\frac{\frac{1}{3}}{3n-2}} - \frac{\frac{1}{3}}{3n+1}$$

$$S_n = \frac{1}{3} - \frac{\frac{1}{3}}{3n+1}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{1}{3} - \underbrace{\frac{\frac{1}{3}}{3n+1}}_{0}\right) = \frac{1}{3}$$

Vrsta konvergira, in

$$\sum_{n=1}^{\infty} \frac{1}{(3n-2)(3n+1)} = \frac{1}{3}.$$

II način

Če želimo določiti ali vrsta KV, lahko uporabimo Raabejev kriterij.

$$\lim_{n\to\infty} n\left(\left|\frac{a_n}{a_{n+1}} - 1\right|\right) = \lim_{n\to\infty} \frac{6n}{3n-2} = 2 > 0 \Rightarrow \text{vrsta KV.}$$