Preliminary mathematics

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1 Mathematical Induction

Mathematical induction, a proof technique widely used in computer science, is a method of proving mathematical results based on the principle of mathematical induction.

- An assertion $A(x)$, depending on a natural number x, is regarded as proved if $A(1)$ has been proved and if for any natural number k the assumption that $A(k)$ is true implies that $A(k+1)$ is also true.
- The proof of $A(1)$ is the first step (or base) of the induction and the proof of $A(k+1)$ from the assumed truth of $A(k)$ is called the induction step. Here k is called the induction parameter and the assumption of $A(k)$ for the proof of $A(k+1)$ is called the induction assumption or induction hypothesis [1].

Example. Prove that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, $\forall\ n \in \mathbb{N}$.

Base case. Let $n = 1$. It is straightforward to see that $\sum_{i=1}^{1} i = 1 = \frac{1(1+1)}{2}$.

Induction hypothesis. Let us suppose that $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$.

Statement to prove. $\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$.

Proof.

$$\begin{aligned}
\sum_{i=1}^{k+1} i &= (\sum_{i=1}^{k} i) + (k+1) \\
&= \frac{k(k+1)}{2} + (k+1) \\
&= \frac{k(k+1)+2(k+1)}{2} \\
&= \frac{k^2+k+2k+2}{2} \\
&= \frac{k^2+3k+2}{2} \\
&= \frac{(k+1)(k+2)}{2}
\end{aligned}$$

Exercises-Homework.
Instructions.
a) Prove the following statements (from 1 to 6).
b) Please, either type or write down your answers and send them by e-mail, as a pdf file, to Professor Venegas-Andraca (svenegas@itesm.mx).
c) If you choose to write down your answers, please make sure that your homework is legible.
5) Deadline: Tuesday 14 August 2018.

1. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

2. $\sum_{i=1}^{n} i^3 = [\frac{n(n+1)}{2}]^2$

3. $n! \geq 2^{n-1}$, $\forall\ n \in \mathbb{N}$

4. $5^n - 1$ is divisible by 4, $\forall\ n \in \mathbb{N}$

5. Let A_i be sets $\Rightarrow (\bigcap_{i=1}^{n} A_i)^c = \bigcup_{i=1}^{n} A_i^c$, $\forall\ n \in \mathbb{N}$

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References

[1] Encyclopedia of Mathematics. https://www.encyclopediaofmath.org