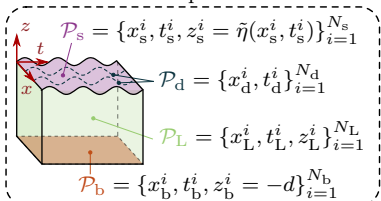
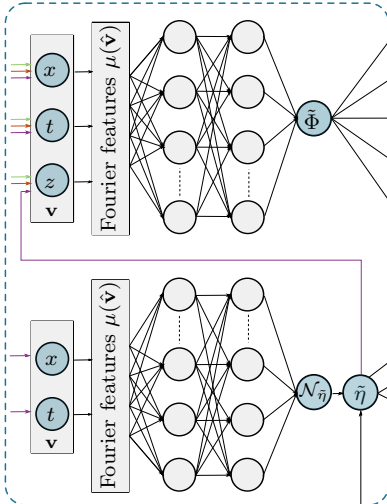


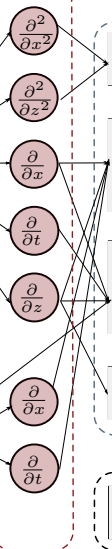
computational domain  $\Omega$   
& collocation points  $\mathcal{P}$



neural networks



AD



physics-informed loss components

$$\mathcal{L}_{\text{Lap}} = \frac{1}{N_L} \sum_{i=1}^{N_L} \left| \tilde{\Phi}_{xx}(x_L^i, t_L^i, z_L^i) + \tilde{\Phi}_{zz}(x_L^i, t_L^i, z_L^i) \right|^2$$

$$\mathcal{L}_{\text{BC,kin}} = \frac{1}{N_s} \sum_{i=1}^{N_s} \left| \tilde{\eta}_t(x_s^i, t_s^i) + \tilde{\eta}_x(x_s^i, t_s^i) \cdot \tilde{\Phi}_x(x_s^i, t_s^i, z_s^i = \tilde{\eta}(x_s^i, t_s^i)) - \tilde{\Phi}_z(x_s^i, t_s^i, z_s^i = \tilde{\eta}(x_s^i, t_s^i)) \right|^2$$

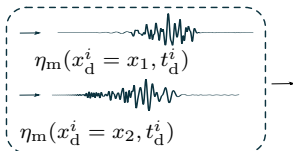
$$\mathcal{L}_{\text{BC,dyn}} = \frac{1}{N_s} \sum_{i=1}^{N_s} \left| \tilde{\Phi}_t(x_s^i, t_s^i, z_s^i = \tilde{\eta}(x_s^i, t_s^i)) + g \cdot \tilde{\eta}(x_s^i, t_s^i) + \frac{1}{2} \left( \tilde{\Phi}_x^2(x_s^i, t_s^i, z_s^i = \tilde{\eta}(x_s^i, t_s^i)) + \tilde{\Phi}_z^2(x_s^i, t_s^i, z_s^i = \tilde{\eta}(x_s^i, t_s^i)) \right) \right|^2$$

$$\mathcal{L}_{\text{BC,bot}} = \frac{1}{N_b} \sum_{i=1}^{N_b} \left| \tilde{\Phi}_z(x_b^i, t_b^i, z_b^i = -d) \right|^2$$

total PINN loss

$$\mathcal{L} = \lambda_{\text{Lap}} \mathcal{L}_{\text{Lap}} + \lambda_{\text{BC,kin}} \mathcal{L}_{\text{BC,kin}} + \lambda_{\text{BC,dyn}} \mathcal{L}_{\text{BC,dyn}} + \lambda_{\text{BC,bot}} \mathcal{L}_{\text{BC,bot}}$$

measurement data



hard constraints

$$\tilde{\eta}(x, t) = M(x, t) + R(x, t) \mathcal{N}_{\tilde{\eta}}(x, t)$$

$$M(x, t): \text{approx. of measurements by } \mathcal{C}_M = \frac{1}{N_d} \sum_{i=1}^{N_d} |M(x_d^i, t_d^i) - \eta_m(x_d^i, t_d^i)|^2$$

$$R(x, t): \text{approx. of distance function by } \mathcal{C}_R = \frac{1}{N_d + N_s} \sum_{i=1}^{N_d + N_s} |R(x_s^i, t_s^i) - r(x_s^i, t_s^i)|^2$$