Calc13

Sven Köppel

koeppel@fias.uni-frankfurt.de

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Abstract

Calc13, a correction of Calc12.

Contents

1 The Scheme to get $\mathcal{A}$

At April 04. I presented my derivation of the modified Action by the bilocal distribution $\mathcal{A}^2(x-y)$ and it's derivation in momentum space with a $n+3$ dimensional Fourier transform (Calc10). By expressing the Ricci Scalar R as Integral identity $R(x) = \int \mathrm{d}y\, \delta(x-y)R(x)$ one can replace δ by it's smeared version $\delta \to \mathcal{A}^2(x-y)\,\delta$ and produces a smeared Ricci scalar $\mathcal{R}(x)$, smeared Einstein Equations and finally the smeared Energy-density tensor $\mathcal{T}_\mu^\nu$: Todo: ist der Abschnitt hierdrüber alt oder neu? Also Section 1.

2 Holography, closed form result

Consider again the holographic integral (ignoring $r \to z$ transformation)

$$\mathcal{A}^{-2}(p^2) = \frac{1}{\Omega_{n+2}} \int \mathrm{d}^{3+n}\vec{z} z^{-(2+n)} \frac{\mathrm{d}h}{\mathrm{d}z} e^{-i\vec{p}\vec{z}} \tag{1}$$

Using Appendix A which contains the improved and with signs corrected version of the effective 1d Fourier integral,

$$\mathcal{I} = \int \mathrm{d}^{3+n}\vec{z}\; V(|z|)e^{-i\vec{p}\vec{z}} = \frac{1}{2\Omega_{n+2}} \frac{2\pi i}{p} \int_{-\infty}^{\infty} \mathrm{d}z\; v(z) e^{-ipz} \tag{2a}$$

with the Fourier kernel

$$v(z) = z^{1+n}\left(V(z)\Theta(z) + (-1)^n V(-z)\Theta(-z)\right) \tag{2b}$$

Here, $V(z) := z^{-(2+n)} \frac{\mathrm{d}h}{\mathrm{d}z}$ and therefore

$$v(z) = z^{1+n}\left(\frac{1}{z^{n+2}} \frac{\mathrm{d}h}{\mathrm{d}z}\Theta(z) + \frac{(-1)^n}{(-z)^{n+2}} \frac{\mathrm{d}h}{\mathrm{d}z}(-z)\Theta(-z)\right) \tag{3}$$

As told in the appendix, by construction $v(z)$ is always odd.

The integral value is now given by

$$\begin{aligned}
\mathcal{A}^{-2}(p^2) &= \frac{\Omega_{n+2}}{2\Omega_{n+2}} \frac{2\pi i}{p} \int_{-\infty}^{\infty} \mathrm{d}z\; v(r) e^{-ipr} && (4a)\\
&= \frac{\pi i}{2p}\left[(2\pi i)(-1)\sum_{z_0}\left(\operatorname*{Res}_{z\to z_0}\; v(z) + \operatorname*{Res}_{z\to -\overline{z_0}}\; v(z)\right)\right] && (4b)
\end{aligned}$$

Where z_0 is given by the poles of $v(z)$, which means basically of the poles of $\frac{\mathrm{d}h}{\mathrm{d}z}$ or where the it's denominator is zero: $(-1) = z^{2+n}$, as proposed in eq. (31) in Calc10. So

$$z_0 = \exp\frac{2\pi i(1/2 + k)}{2+n}, \quad k \in \mathbb{N} \tag{5}$$

Actually there are $|z_0| = 2+n$ solutions, so basically $k = 0, 1, \ldots, 2+n$. In my pole summation rule I proposed in Calc10 and more verbosely in Calc12, I only consider poles where $\operatorname{Re} z_0 > 0 \wedge \operatorname{Im} z_0 < 0$ for reasons of the integration path choice. This corresponds to a special range of k that can be derived when performing Eulers formula $e^{i\varphi} = \cos\varphi + i\sin\varphi$ and searching where $\cos > 0 \wedge \sin < 0 \Leftrightarrow 3/2\pi \leq \varphi \leq 2\pi \Leftrightarrow$

$$\begin{aligned}
k_{\min} &= \lceil 1 + 3/4n \rceil \quad \text{(ceiling, aufrunden)} && (6a)\\
k_{\max} &= \lfloor 3/2 + n \rfloor \quad \text{(floor, abrunden)} && (6b)
\end{aligned}$$

Caution must also be made that one does not count poles two times. I did those summations Semi-automated since two month and the problem is, there are always alternating complex results which should not be there according to the Appendix.

n	# poles	Wert für A(p)
0	2	$\frac{2i\pi e^{-p}(p+1)}{p}$
1	2	$\frac{i\left(-\frac{2}{3}\pi e^{(-1)^{5/6}p}\left(p+\sqrt[6]{-1}\right)-\frac{2}{3}(-1)^{5/6}\pi e^{-\sqrt[6]{-1}p}\left(\sqrt[6]{-1}p+1\right)\right)}{p}$
2	2	$\frac{i\left(\frac{1}{2}i\pi e^{(-1)^{3/4}p}\left((-1)^{3/4}+ip\right)-\frac{1}{2}i\pi e^{-\sqrt[4]{-1}p}\left(\sqrt[4]{-1}+ip\right)\right)}{p}$
3	2	$\frac{i\left(-\frac{2}{5}\pi e^{(-1)^{7/10}p}\left(p+(-1)^{3/10}\right)-\frac{2}{5}(-1)^{7/10}\pi e^{-(-1)^{3/10}p}\left((-1)^{3/10}p+1\right)\right)}{p}$
4	4	$\frac{i\left(\frac{2}{3}\pi e^{-p}(p+1)+\frac{1}{3}(-1)^{5/6}\pi e^{(-1)^{2/3}p}\left(\sqrt[6]{-1}p+i\right)-\frac{1}{3}(-1)^{2/3}\pi e^{-\sqrt[3]{-1}p}\left(\sqrt[3]{-1}p+1\right)\right)}{p}$
5	4	$\frac{i\left(-\frac{2}{7}\pi e^{(-1)^{13/14}p}\left(p+\sqrt[14]{-1}\right)-\frac{2}{7}\pi e^{(-1)^{9/14}p}\left(p+(-1)^{5/14}\right)-\frac{2}{7}(-1)^{13/14}\pi e^{-\sqrt[14]{-1}p}\left(\sqrt[14]{-1}p+1\right)-\frac{2}{7}(-1)^{9/14}\pi e^{-(-1)^{5/14}p}\left((-1)^{5/14}\right.\right.}{p}$
6	4	$\frac{i\left(-\frac{1}{4}\pi e^{(-1)^{7/8}p}\left(p+\sqrt[8]{-1}\right)-\frac{1}{4}\pi e^{(-1)^{5/8}p}\left(p+(-1)^{3/8}\right)-\frac{1}{4}(-1)^{7/8}\pi e^{-\sqrt[8]{-1}p}\left(\sqrt[8]{-1}p+1\right)-\frac{1}{4}(-1)^{5/8}\pi e^{-(-1)^{3/8}p}\left((-1)^{3/8}p+1\right)\right)}{p}$
7	4	$\frac{i\left(-\frac{2}{9}\pi e^{(-1)^{5/6}p}\left(p+\sqrt[6]{-1}\right)-\frac{2}{9}\pi e^{(-1)^{11/18}p}\left(p+(-1)^{7/18}\right)-\frac{2}{9}(-1)^{5/6}\pi e^{-\sqrt[6]{-1}p}\left(\sqrt[6]{-1}p+1\right)-\frac{2}{9}(-1)^{11/18}\pi e^{-(-1)^{7/18}p}\left((-1)^{7/18}p+1\right.\right.}{p}$

(7)

Appendices

A Improvement of the radial symmetric $(n+3)$-dimensional FT

These calculations were performed already in

- Calc10, Section 1.1.1: No detailed derivation, just the use of the 3-dimensional Karbstein approach
- Calc12, Appendix A: Derivation only for 3 dimensions, the generalization from $3 \to (3+n)$ was just wrong.

This section will review the calculation of Calc12, but insert the $d = 3 + n$ in every step *and* take care of the prefactors.

Let's define the Fourier transform $\mathcal{F}_d$ in d dimensions ($\vec{x} \in \mathbb{R}^d$) once again as

$$\mathcal{F}_d\left\{f\right\}(\vec{p}) = \tilde{f}(\vec{p}) = \frac{1}{(2\pi)^d} \int \mathrm{d}^d x \, e^{-i\vec{p}\cdot\vec{x}} f(\vec{x}) \tag{8a}$$

$$\mathcal{F}_d^{-1}\left\{\tilde{f}\right\}(\vec{x}) = f(\vec{x}) = \int d^d p \, e^{+i\vec{p}\cdot\vec{x}} \tilde{f}(\vec{p}) \tag{8b}$$

As this section wants to discuss how the integrals in (8a,8b) are computed, lets consider only the integral of the forward transformation $\mathcal{F}_d$ in the next part (That is, supressing the leading $(2\pi)^{-d}$ in the following equations).

Consider the $d = 3 + n$ dimensional spherical integral measure, like introduced in Calc10, eqs (15,16):

$$\int \mathrm{d}^d r = \int_0^\infty \mathrm{d}r \, r^{d-1} \int_0^{2\pi} \mathrm{d}\phi \prod_{i=1}^{d-2} \int_0^\pi \mathrm{d}\theta_i \sin^i(\theta_i) := \int_0^\infty \mathrm{d}r \, \Omega_{d-1} r^{d-1} \tag{9a}$$

$$= \frac{\Omega_{d-1}}{2} \underbrace{\int_0^\pi \mathrm{d}\theta_1 \sin(\theta_1)}_{=2} \int_0^\infty \mathrm{d}r \, r^{d-1} \qquad \text{with} \quad \Omega_{n+2} = 2\frac{\pi^{\frac{n+3}{2}}}{\Gamma\left(\frac{n+3}{2}\right)} \tag{9b}$$

The θ_1 integral in (9b) can only be evaluated to $\int \mathrm{d}\theta_1 \cdots = 2$ if the integrand (which is ommitted in these equations) is not dependend of θ_1. In our calculation, this is not the case.

I won't retrace the full Calc12 calculations here. We end up with

$$\hat{V}(p) = \frac{\Omega_{2+n}}{2} \frac{2\pi i}{p} \int_{-\infty}^\infty \mathrm{d}r \, r^{1+n} \Big(V(r)\Theta(r) + (-1)^{2+n} V(-r)\Theta(-r) \Big) \tag{10}$$

Pay attention the toggling minus $(-1)^{2+n}$, this does **not** allow writing the effective integrand function $v(r) \neq r^{1+n} V(|r|)$ as supposed in Calc12. Why is it not $(-1)^{1+n}$? Because when substituting $r \to -r'$ and $\mathrm{d}r \to -\mathrm{d}r'$, it is

$$r^{1+n}\mathrm{d}r = (-1)^{1+n}(r')^{1+n}(-1)\mathrm{d}r' = (-1)^n r'\mathrm{d}r' = (-1)^{2+n} r'\mathrm{d}r' \tag{11}$$

So opposed as stated in Calc12, $v(r)$ is **always odd** for all n, therefore $\forall n$:

$$\int \mathrm{d}r \, v(r) \in \mathbb{C} \setminus \mathbb{R} \tag{12}$$

$$\mathcal{F}_{n+3}\left\{V(|\vec{r}|)\right\} \in \mathbb{R} \tag{13}$$