Calc2

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Calc2 is the second writeup of notices in my Master thesis research. This document lists up some formulas and expands some with that higher dimensional things.

Definition of frequently used formulas:

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f^{-1}(r)\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

$$T_H = \left(\frac{1}{4\pi}\frac{\mathrm{d}g_{00}}{\mathrm{d}r}\right)_{r=r_H}$$

$$\gamma(s;x) = \int_0^x t^{s-1}e^{-t}\mathrm{d}t$$

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1 Summary of papers

1.1 NSS 2006

Title Noncommutative geometry inspired Schwarzschild black hole

Keywords NC

Genutzt von Rizzo 2006

Auch eines der wichtigsten, die ich gelesen habe.

$$\rho_\theta(r) = \frac{M}{(4\pi\theta)^{3/2}}e^{-r^2/4\theta} \tag{1}$$

$$f(r) = 1 - \frac{4M}{r\sqrt{\pi}}\gamma(3/2, r^2/4\theta) \tag{2}$$

$$r_H = \frac{4M}{\sqrt{\pi}}\gamma(3/2; r_H^2/4\theta) \tag{3}$$

$$T_H = \frac{1}{4\pi r_H}\left(1 - \frac{r_H^3}{4\theta^{3/2}}\frac{e^{-r_H^2/4\theta}}{\gamma(3/2, r_H^2/4\theta)}\right) \tag{4}$$

1.2 N Aug 2010

Title Entropic force, noncommutative gravity and ungravity

Keywords Emergent gravity, Verlinde

Basic Ideas Newton $F(r)$ herleiten aus $S = k_B \ln N$. Später mit n Raumdimensionen und in $\mathcal{G}$ einige Effekte.

$$f(r) = 1 - \frac{2M}{r^{n-2}c^2}\mathcal{G}(r) \tag{5}$$

$$F = \frac{GMm}{r^2}\left(1 + 4L^2\frac{\partial S}{\partial A}\right) \tag{6}$$

Nicht so passend zum Thema $f(r)$.

1.3 N Feb 2012

Title Nonlocal and generalized uncertainity principle black holes

Keywords EH-Action

Basic Ideas Operator $\mathcal{A}(x-y)$, running $\mathcal{G}(r)$, Length scale l of theory

Nicht passend zum Thema.

$$
\begin{aligned}
S &= \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\mathcal{R}(x) \\
\mathcal{R}(x) &= \int \mathrm{d}^4y\sqrt{-g}\mathcal{A}^2(x-y)R(y) \\
\mathcal{A}^2(x-y) &= \mathcal{A}^2(\Box_x)\delta^4(x-y) \\
\Box_x &= l^2 g_{\mu\nu}\nabla^\mu\nabla^\nu \\
\mathcal{A}(p^2) &= \exp(l^2p^2/2)\dots \\
\mathcal{T}_{\mu\nu} &= \mathcal{A}^{-2}(\Box)T_{\mu\nu} \\
f(r) &= 1-\frac{GM\gamma(2;r/\sqrt{\beta})}{r}
\end{aligned}
$$

1.4 NS Okt 2012

Title Holographic screens in ultraviolet self-complete quantum gravity

Keywords Holography

Das Hauptpaper, was ich als erstes las, darüber geht auch Calc1.

Das Paper umfasst zwei Ansätze, $h_\alpha(r)$ und $h(r)$.

Im ersten Ansatz setzt Bedingung $M_P = M_0$, $M_0 = M(r_0)$ das $\alpha = \alpha_0$, $r_0 = L_P$. Im zweiten Ansatz wird eine der drei Bedigungen an eine Metrik verworfen.

$$f(r) = 1-\frac{2MG}{r}h_{\alpha,\dots}(r) \tag{7}$$

$$h_\alpha(r) = \frac{r^3}{(r^\alpha+(\tilde{r}_0)^\alpha/2)^{3/\alpha}} \tag{8}$$

$$h(r) = \frac{r^2}{r^2+L^2} = 1-\frac{L^2}{r^2+L^2} \tag{9}$$

$$\rho(r) = \frac{M}{2\pi r}\frac{L^2}{(r^2+L^2)^2} \tag{10}$$

$$m(r) = \frac{Mr^2}{L^2+r^2} = M-\frac{LM}{L^2+r^2} \tag{11}$$

1.5 NS 06.11.2013

Title Holographic screens in ultraviolet self-complete quantum gravity

Keywords Holography

Source Elsevier Preprint by Mail am 12.11.13

$$\rho(r) = \frac{M}{4\pi r^2}\delta(r) \tag{12}$$

$$\delta(r) = \frac{\mathrm{d}}{\mathrm{d}r}\Theta(r) \tag{13}$$

$$\Theta(r) \to h(r) \tag{14}$$

$$\rho(r) = \frac{M}{4\pi r^2}\frac{\mathrm{d}}{\mathrm{d}r}h(r) = T_0^0 \tag{15}$$

$$h(r) = 1-L^2/(r^2+L^2) \tag{16}$$

$$\sigma_h = M/(4\pi r_h^2) \tag{17}$$

1.6 NIM 07.11.2013

Title Self-Completeness and the Generalized Uncertainity Principle

Keywords -

Ein neues veröffentlichtes Paper auf ArXiv, parallel zum Preprint. Erstmals hübsche Bilder. Herleitung von $f(r)$ aus Operator $\mathcal{A}$:

$$f(r) = 1 - 2\frac{GM}{c^2 r}\gamma(2; \frac{r}{\sqrt{\beta}}) \tag{18}$$

1.7 Rizzo 2006

Title Noncommutative inspired black holes in extra dimensions

Basiert auf NSS 2006 NC Ansatz (Section 1.1)

$$\rho_\theta(r) = \frac{M}{(4\pi\theta)^{3/2}} e^{-r^2/4\theta} \tag{19}$$

$$\rightarrow \frac{M}{(4\pi\theta)^{(n+3)/2}} e^{-r^2/4\theta} \tag{20}$$

2 Extension von [1.4 NS2012] analog zu [1.7 Rizzo 2006]

In Paper [1.5 NS2013], in 4D, it was like (using $\Sigma := (r^2 + L^2)^2$)

$$\rho(r) = \frac{M}{A_2} h'(r) \propto \frac{1}{r^2}\frac{r}{\Sigma} \quad \Rightarrow \quad \mu(r) = \int_0^r \mathrm{d}r r^2 \rho(r) \quad \Rightarrow \quad \mu(r) = \int_0^r \mathrm{d}r \frac{r}{\Sigma} \propto \left[\frac{1}{\Sigma}\right]_0^r \tag{21}$$

In the holography picture, only the $A_{(n-2)}$-Sphere, which is the surface of an $V_{(n-1)}$ dimensional matter ball in $(n-1)$ spacial dimensions (+1 time dimension makes n space-time dimensions) seems to enter $\rho(r)$. So in n dim, combining [1.5 NS2013] + [1.7 Rizzo 2006]:

$$\rho(r) = \frac{M}{A_{(n-2)}} \frac{\mathrm{d}h(r)}{\mathrm{d}r} \quad \text{Units: } [\rho] = \frac{[M]}{[A_{n-2}]}\left[\frac{\mathrm{d}}{\mathrm{d}r}\right][h] = \frac{E}{L^{n-2}}\frac{1}{L} \cdot 1 = \frac{E}{L^3} = \frac{1}{L^4} = E^4 \tag{22}$$

Formulas to remember for the volume of an n-Ball and its corresponding $(n-1)$-Sphere:

$$V_n = r^n \frac{\pi^{n/2}}{\Gamma\left(\frac{n}{2}+1\right)} \quad \text{and} \quad A_{(n-1)} = \frac{\mathrm{d}V_n}{\mathrm{d}r} = \frac{\pi^{n/2}}{\Gamma\left(\frac{n}{2}+1\right)} n r^{n-1} = 2\frac{\pi^{n/2}}{\Gamma\left(\frac{n}{2}\right)} r^{n-1} \tag{23}$$

$$\begin{matrix} \Gamma(x) = (x-1)! \\ \Gamma(x+1) = x\Gamma(x) \end{matrix} \quad \text{Prefactors:} \quad \begin{matrix} V_n = v_n r^n \\ A_n = a_n r^n \end{matrix} \quad \text{Recursion:} \quad \begin{matrix} v_0 = 1, & v_{n+1} = a_n/(n+1) \\ a_0 = 2, & a_{n+1} = 2\pi v_n \end{matrix} \tag{24}$$

Now we evaluate the $(n-1)$ dimensional integral measure in spherical coordinates $k_\mu = (k_0, \vec{k}) = (k_0, r, \phi, \theta_1, \dots, \theta_{n-3})$, integrating only the spacial components:

$$\int \mathrm{d}^{(n-1)} r = \int_0^\infty \mathrm{d}r \; r^{n-2} \underbrace{\int_0^{2\pi} \mathrm{d}\phi \prod_{j=1}^{n-3} \int_0^\pi \mathrm{d}\theta_j \; \sin^j(\theta_j)}_{=a_{n-2},\text{ since } (n-2)-\text{Surface}} = \frac{2\pi^{(n-1)/2}}{\Gamma\left(\frac{n-1}{2}\right)} \int_0^r \mathrm{d}r \; r^{n-2} \tag{25}$$

The factor r^{n-2} is given by $(n-2)$ angles in n dimensions.
The derivation of eqn. 25 follows Wagner QFT2, not important here:

$$B(x,y) = \int_0^1 \mathrm{d}t t^{x-1}(1-t)^{y-1} = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} \tag{26}$$

$$\Gamma(x) = \int_0^\infty \mathrm{d}t\, t^{x-1}e^{-t} \quad \text{und} \quad \gamma(s,x) = \int_0^x t^{s-1}e^t \mathrm{d}t, \quad \Gamma(1/2) = \sqrt{\pi} \tag{27}$$

$$\int_0^\pi \theta_j \, \sin^j(\theta_j) = \frac{\sqrt{\pi}\,\Gamma\left(\frac{j+1}{2}\right)}{\Gamma\left(\frac{j+2}{2}\right)} \tag{28}$$

So in the end, it's straightforward for general $h(r)$:

$$\mu(r) = a_{n-2}\int_0^r \mathrm{d}r r^{n-2}\rho(r) = a_{n-2}\int_0^r \frac{M}{A_{n-2}} h'(r) r^{n-2}\mathrm{d}r = M\int_0^r \mathrm{d}r \frac{\mathrm{d}h(r)}{\mathrm{d}r} = M\left[h(r) - h(0)\right] \tag{29}$$

By construction of $\rho(r)$, it just kills the $(n-2)$ dimensional sphere. $\mu(r)$ only diverges if $h(0)$ diverges. The $h(r)$ Ansatz from [1.4 NS2012] yields

$$h(r) = \frac{r^2}{r^2+L^2}, \quad \frac{\mathrm{d}h(r)}{\mathrm{d}r} = \frac{2rL^2}{(r^2+L^2)^2} \quad \Rightarrow \quad \mu(r) = \frac{2r\, ML^2}{(r^2+L^2)^2} \tag{30}$$

for *any* dimension n.