Holographic and self-encoding regular Black Holes

my Master's project

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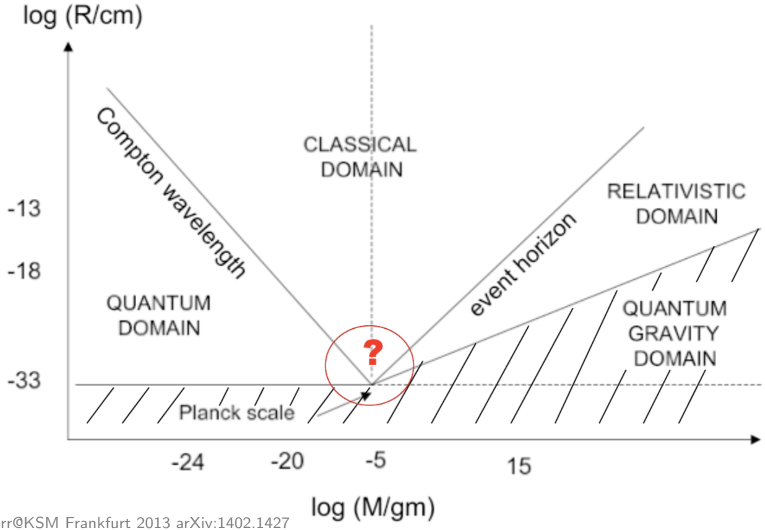
The holy grail



Source: Carr@KSM Frankfurt 2013 arXiv:1402.1427

A wishlist

1. Regular (No curvature singularity at origin)

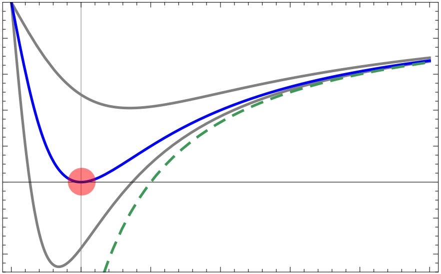
$$\lim_{r\to 0} g_{00}(r) < \infty$$

2. Classical Limit (Schwarzschild)

$$g_{00}(r) = \frac{2Gm}{r} \quad \text{for} \quad r > l_0$$

3. Self-encoding. $r_0 = l_0$

↓ Generic g_{00} that fulfills all requirements.



Metric candidates

- NCBHs: 1 + 2
- Self-Encoding: 1 + 2 + 3
- Holographic: 2 + 3

Approach for static matter density ρ

- Start with Schwarzschild $\rho(r) = \frac{M}{4\pi r^2}\delta(r) = \frac{M}{4\pi r^2}\frac{\mathrm{d}\Theta(r)}{\mathrm{d}r}$
- Smear the distribution $\Theta(r) \to H(r)$
- Make use of extradimensions ($4 + n$ total dimensions):

$$\rho(r) = \frac{M}{\Omega_{2+n} r^{2+n}} \frac{\mathrm{d}H(r)}{\mathrm{d}r} \quad \text{with} \quad \Omega_{n+2} = \frac{2\pi^{(n+3)/2}}{\Gamma\left[(n+3)/2\right]}$$

- Make an educated guess for $H(r)$.

Get the Metric

- $\rho(r) \equiv T^0_0$. $\nabla_\mu T^{\mu\nu} = 0$ gives remaining $T_{\mu\nu}$
- The Mass is *arbitrarily* fixed:

$$m(r) = \Omega_{2+n} \int^r \mathrm{d}x\, x^{2+n} \rho(x) = M \int^r \mathrm{d}x\, H'(x) = M\, H(r) + \text{const}$$

- Choose to match Self-Encoding: $m(r_0) = M_*$

Reduced Planck Constants

$$M_P^2 = V_n M_*^{2+n}$$

with $V_n = (2\pi R_c)^n$ volume of compactified dimensions as tori with radius R_c.

Details (if needed)

$$\mathrm{d}s^2 = -\left(1 - V(r)\right)\mathrm{d}t^2 + \left(1 - V(r)\right)^{-1}\mathrm{d}r^2 + r^{2+n}\mathrm{d}\Omega_{2+n} \quad (1)$$

$$V(r) = \frac{2}{2+n}\frac{M}{M_*^{2+n}}\frac{1}{\Omega_{2+n}}\boxed{\frac{H(r)}{r^{1+n}}} \quad (2)$$

$$M(r_H) = \frac{2+n}{2}\frac{\Omega_{2+n}}{H(r_H)}\left(\frac{r_H}{L_*}\right)^{1+n}\boxed{M_*} \quad (3)$$

Modifying the $H(r)$ profiles for n LXDs

Choices for $H(r)$ are:

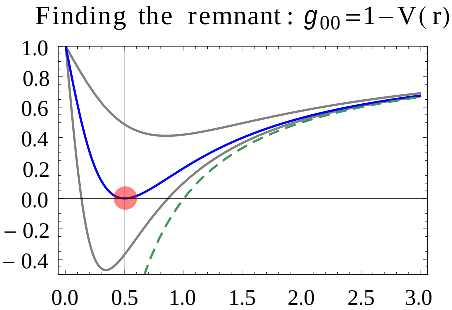
The self-encoding metric

$$h_\alpha(r) = \frac{r^{3+n}}{(r^\alpha + L^\alpha/2)^{\frac{3+n}{\alpha}}}$$

The holographic metric

$$h(r) = \frac{r^{2+n}}{r^{2+n} + L^{2+n}}$$

Results: Self-Encoding Remnant



Extremal Radius remnant equations:

$$\begin{cases} \partial_r\big|_{r=r_0} g_{00}(r) & = 0 \\ g_{00}(r_0) & = 0 \end{cases}$$

Remnant radii:

$$r_0 = L \left(\frac{1}{1+n}\right)^{\frac{1}{2+n}}$$

$$r_{0,\alpha} = L \left(\frac{1}{1+n}\right)^{\frac{1}{\alpha}}$$

Self encoding $M(r_0) = M_*$ fixes α:

$$\alpha_0 = \frac{3+n}{\ln(2+n)} \ln \frac{3+n}{2}$$

Thermodynamical properties

I calculated the Hawking-Temperature $T_H \equiv \frac{1}{4\pi}\, \partial_r g_{00}\big|_{r=r_H}$, Heat Capacity $C = \frac{\partial M}{\partial T_H}$ and Entropy $S(r) = \int \frac{\mathrm{d}M}{T}$. See blackboard for discussion.

Remarkable result: Entropy for holographic model exhibits *log* corrections in any number of LXDs:

$$S(r) = \sharp \left(r_+^{2+n} - L_*^{2+n} \right) + \natural \ln \left(\frac{r_+}{L_*} \right)$$

$\Rightarrow$ quantization in units of area $\mathcal{A} \equiv \Omega_{2+n} r_+^{2+n}$

Modified Field Equations

It is possible modify the Einstein-Hilbert-Action that it gets *intrinsically non-local* by $\delta(x-y) \to \mathcal{A}^2(x-y)$

$$\mathcal{R}(x) = \int \mathrm{d}y \mathcal{A}^2(x-y) R(x)$$

$$\mathcal{T}_0^0 = -M\mathcal{A}^{-2}(\Box)\delta(\vec{x}) \equiv \frac{M}{\Omega_{2+n} r^{2+n}} \frac{\mathrm{d}H(r)}{\mathrm{d}r}$$

The smearing operator $\mathcal{A}$ is given basically by a FT of $H'(r)$:

$$\mathcal{A}^{-2}(p^2) = \int \mathrm{d}^{3+n} r \left\{ \frac{1}{r^{2+n}} \frac{\mathrm{d}H(r)}{\mathrm{d}r} \right\} e^{i\vec{p}\cdot\vec{r}}$$