Holographic and self-encoding regular Black Holes

my Master's project

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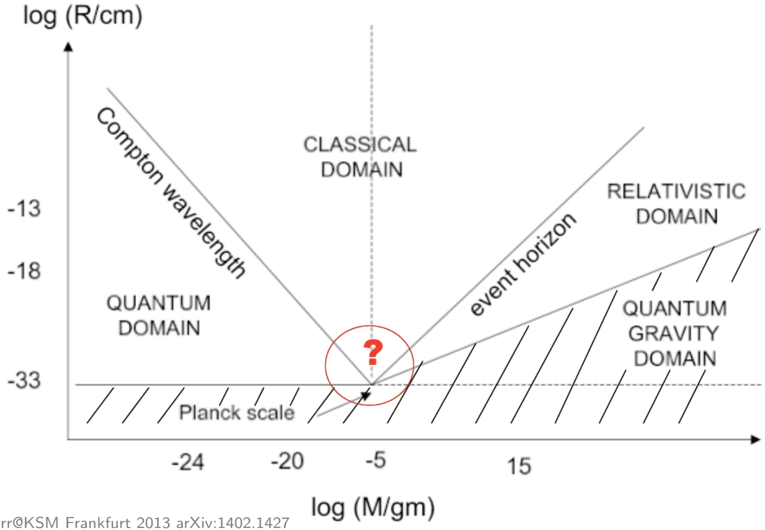
The holy grail



Source: Carr@KSM Frankfurt 2013 arXiv:1402.1427

A wishlist

1. Regular (No curvature singularity at origin)
$$\lim_{r\to 0} g_{00}(r) < \infty$$

2. Classical Limit (Schwarzschild)
$$g_{00}(r) = \frac{2Gm}{r} \quad \text{for} \quad r > l_0$$

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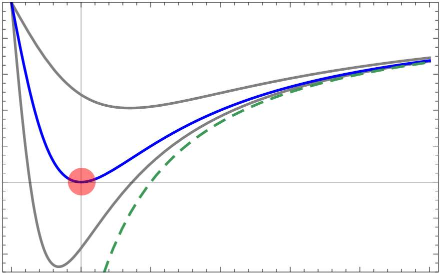

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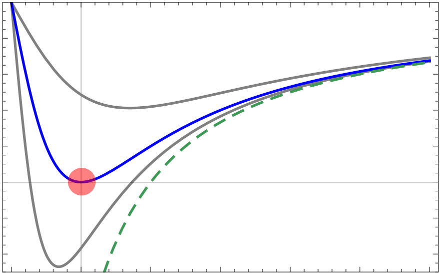

Metric candidates

- NCBHs: 1 + 2
- Self-Encoding: 1 + 2 + 3
- Holographic: 2 + 3

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- Make an educated guess for $H(r)$.

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Reduced Planck Constants

$$M_P^2 = V_n M_*^{2+n}$$

with $V_n = (2\pi R_c)^n$ volume of compactified dimensions as tori with radius R_c.

Details (if needed)

$$\mathrm{d}s^2 = -\left(1 - V(r)\right)\mathrm{d}t^2 + \left(1 - V(r)\right)^{-1}\mathrm{d}r^2 + r^{2+n}\mathrm{d}\Omega_{2+n} \quad (1)$$

$$V(r) = \frac{2}{2+n}\frac{M}{M_*^{2+n}}\frac{1}{\Omega_{2+n}}\boxed{\frac{H(r)}{r^{1+n}}} \quad (2)$$

$$M(r_H) = \frac{2+n}{2}\frac{\Omega_{2+n}}{H(r_H)}\left(\frac{r_H}{L_*}\right)^{1+n}\boxed{M_*} \quad (3)$$

Modifying the $H(r)$ profiles for n LXDs

Choices for $H(r)$ are:

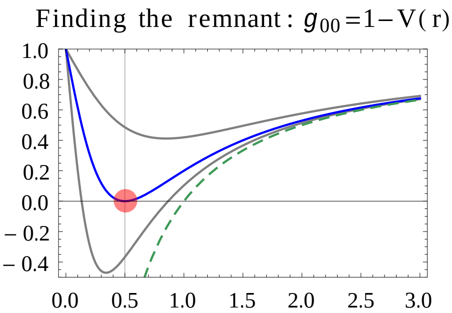
The self-encoding metric

$$h_\alpha(r) = \frac{r^{3+n}}{(r^\alpha + L^\alpha/2)^{\frac{3+n}{\alpha}}}$$

The holographic metric

$$h(r) = \frac{r^{2+n}}{r^{2+n} + L^{2+n}}$$

Results: Self-Encoding Remnant



Extremal Radius remnant equations:

$$\begin{cases} \partial_r\big|_{r=r_0} g_{00}(r) & = 0 \\ g_{00}(r_0) & = 0 \end{cases}$$

Remnant radii:

$$r_0 = L \left(\frac{1}{1+n}\right)^{\frac{1}{2+n}}$$

$$r_{0,\alpha} = L \left(\frac{1}{1+n}\right)^{\frac{1}{\alpha}}$$

Self encoding $M(r_0) = M_*$ fixes α:

$$\alpha_0 = \frac{3+n}{\ln(2+n)} \ln \frac{3+n}{2}$$

Thermodynamical properties

I calculated the Hawking-Temperature $T_H \equiv \frac{1}{4\pi} \partial_r g_{00}\big|_{r=r_H}$, Heat Capacity $C = \frac{\partial M}{\partial T_H}$ and Entropy $S(r) = \int \frac{\mathrm{d}M}{T}$. See blackboard for discussion.

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Remarkable result: Entropy for holographic model exhibits *log* corrections in any number of LXDs:

$$S(r) = \sharp \left(r_+^{2+n} - L_*^{2+n}\right) + \natural \ln\left(\frac{r_+}{L_*}\right)$$

$\Rightarrow$ quantization in units of area $\mathcal{A} \equiv \Omega_{2+n} r_+^{2+n}$

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The smearing operator $\mathcal{A}$ is given basically by a FT of $H'(r)$:

$$\mathcal{A}^{-2}(p^2) = \int \mathrm{d}^{3+n} r \left\{ \frac{1}{r^{2+n}} \frac{\mathrm{d}H(r)}{\mathrm{d}r} \right\} e^{i\vec{p}\cdot\vec{r}}$$