Calc15

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Abstract

This document effective Quantum Gravity approaches investigated in the Calc series so far in respect to their divergence curing behaviour.

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1 Divergence of General Relativity

GR exhibits an ultra-violet divergence, that is, expressions typically diverge when $r \to 0 \Leftrightarrow p \to \infty$ in spherical symmetry. Consider the spatial flat integration measure in four Spacetime dimensions:

$$\int \mathrm{d}^3 p \propto \int_{-\infty}^{\infty} p^2 \,\mathrm{d}p = \lim_{\Lambda\to\infty} \int_{-\Lambda}^{\Lambda} p^2 \,\mathrm{d}p \propto \Lambda^3 \tag{1}$$

When examining self-regular black hole solutions, we examine expressions which cure this divergencies. They typically look like

$$\int \frac{\mathrm{d}^3 p}{f(p)} \tag{2}$$

with polynomial functions $f(p)$ that manages a »soft cutoff«. For example, in the GUP principle [Kempf2005], it is $f(p) = 1 + \beta p^2$. The series expansion of $1/f(p)$ at $p \to \infty$ (which corresponds with $\beta \to 0$) is

$$\frac{1}{1+\beta p^2} \approx \frac{1}{\beta p^2} - \mathcal{O}\left(\frac{1}{\beta^2 p^4}\right) \tag{3}$$

Therefore, we can understand the integration modification as

$$\int \frac{\mathrm{d}^3 p}{1+\beta p^2} \propto \int_{-\infty}^{\infty} \frac{p^2 \,\mathrm{d}p}{1+\beta p^2} \overset{(3)}{\approx} \int_{-\infty}^{\infty} \frac{\mathrm{d}p}{\beta} \tag{4}$$

This is good. We like that.

1.1 What $f(p)$ has to archive in higher dimensions

It is obvious that $f(p)$ must scale with the number of extra dimensions, because (1) gets

$$\int \mathrm{d}^{3+n} p \approx \int_{-\infty}^{\infty} p^{2+n} \,\mathrm{d}p \tag{5}$$

Thus the most simple extension of Kempf would be $f(p) = 1 + L^{2+n} p^{2+n}$, with $\beta = L^2$ and L the reduced higher dimensional Planck length. It is easy to show that, using this approach, (4) again gets $\propto \int \mathrm{d}p$.

1.2 How $f(p)$ is archieved with my H-models

This section ties on the formalism I introduced in Calc14 – which is merely the name »H-model« for the approach of talking about the holographic metric ($h(r)$ profile), self-encoding metric ($h_\alpha(r)$ profile) and eventually the Bardeen metric ($h_e(r)$ profile).

In my work, a fourier transformation is typically introduced like

$$\mathcal{A}^{-2}(p^2) = \int \mathrm{d}^{3+n}r \left(\frac{1}{r^{n+2}} \frac{\mathrm{d}H(r)}{\mathrm{d}r} \right) e^{-ipr} \tag{6}$$

The factor r^{n+2} in the denominator is placed there »by design«, as all H-models have a matter density

$$\rho(r) = \frac{M}{\Omega_{n+2} r^{n+2}} H'(r) \tag{7}$$

with the $(n+2)$-surface (spatial surface) in the denominator. When inserting $H(r) = \Theta(r)$, $H'(r) = \delta(r)$, one ends up in the Schwarzschild(-Tangherlini) case. That is, everything is fine in ordinary Schwarzschild:

$$\int \mathrm{d}^{3+n}r \left(\frac{1}{r^{n+2}} \delta(r) \right) \propto \int_{-\infty}^{\infty} \mathrm{d}r\, r^{2+n} \left(\frac{1}{r^{n+2}} \delta(r) \right) = \int \mathrm{d}r\, \delta(r) \tag{8}$$

Caution must be made when performing the $(3+n) \to 1$ dimensional integral rewrite, since an alternating $(-1)^n$ inserts the integrand. This technical detail was first found in Calc13 and discussed in Calc14.

So it looks like in my calculations, $H(r)$ does not need to scale with the number of extradimensions n. This is really weird, I always thought it has to scale. Hm.