Calc8

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1 Propagator and modified Einstein Equations

1.1 Einstein modifications with NCBHs

I got plenty of papers following basically modifications $G \to \mathcal{G}$ or $T_{\mu\nu} \to \mathcal{S}_{\mu\nu}$ or $R \to \mathcal{R}$, e.g.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N \mathcal{S}_{\mu\nu} \quad \mathcal{S}_{\mu\nu} = \mathcal{F}^2(\Box(x)/\Lambda_G^2) T_{\mu\nu} \quad \text{[MMN 2010]} \tag{1}$$

See also

- [N Feb 2012]: *Nonlocal and generalized uncertainty principle black holes*
  Introduces the bi-local distribution $\mathcal{A}^2(x-y)$ that modifies R, the Einstein Equations, $T_{\mu\nu}$ and $\delta(\vec{x})$
- [Modesto Moffat N Dec 2010]: *Black holes in an ultraviolet complete quantum gravity*
  Introduces the entire (=holomorphic) function $\mathcal{F}$ that scales like $\mathcal{A}$ before.
- [Isi Nov2013]: *Self-Completeness and the Generalized Uncertainty Principle*
  Follows straightforward the [N Feb 2012] formalism.
- [Isi Feb2014]: *Self-Completeness in Alternative Theories of Gracity*

1.2 A Roadmap

With the approach $\mathcal{T}_{00} \propto M\,\mathcal{A}^{-2}(\Box)\delta(\vec{x})$, it's all about finding a differential operator that modifies the Dirac Delta to the smeared functions $\partial_r h$ or $\partial_r h_\alpha$. Since our approach always was $T_{00} \propto M/\Omega\,\delta(r) \to M/\Omega \frac{\mathrm{d}h}{\mathrm{d}r}$, with

$$h(r) = \frac{r^2}{r^2 + L^2} \tag{2}$$

$$h'(r) = \frac{2rL^2}{(r^2+L^2)^2} \tag{3}$$

$$h_\alpha(r) = \frac{r^{3+n}}{(r^\alpha + L^\alpha/2)^{\frac{3+n}{\alpha}}} \tag{4}$$

$$h'_\alpha(r) = \frac{(n+3)L^\alpha r^{n+2}\left(\frac{L^\alpha}{2} + r^\alpha\right)^{-\frac{n+3}{\alpha}}}{L^\alpha + 2r^\alpha} \tag{5}$$

the propagator $\mathcal{A}^{-2}(\Box)$ really must be a complex one to get $\mathcal{A}^{-2}\delta \to h'_\alpha$ (eq. 5)!

1.3 Note: Spherical Fourier transformation in $3+n$ dimensions

From Felix Karbstein: Performing the Fourier transform of a generic position space potential $V(|\vec{r}|)$ in $d=3$ dimensions to momentum space, we obtain

$$\hat{V}(p) = \int \mathrm{d}^3 r \, e^{-i\vec{r}\vec{p}} \, V(r) = 2\pi \int_{-1}^{+1} \mathrm{d}\cos\theta \int_0^\infty \mathrm{d}r \, r^2 \, e^{-irp\cos\theta} V(r) \tag{6}$$

$$= \frac{2\pi i}{p} \int_0^\infty \mathrm{d}r \, r \, V(r) \left(e^{-irp} - e^{+irp}\right) = \frac{2\pi i}{p} \int_{-\infty}^\infty \mathrm{d}r \, e^{-irp} \, r \, \left[V(r)\Theta(r) + V(-r)\Theta(-r)\right] \tag{7}$$

with $r = |\vec{r}|$, $p = |\vec{p}|$. Note that this effectively amounts to an one dimensional Fourier transform

$$\hat{v}(p) = \int_{-\infty}^\infty \mathrm{d}r \, e^{-irp} \, v(r) \tag{8}$$

with

$$v(r) = r \, \left[V(r)\Theta(r) + V(-r)\Theta(-r)\right] \quad \text{and} \quad \hat{V}(p) = \frac{2\pi i}{p}\hat{v}(p) \tag{9}$$

Now proceed with going from $\mathrm{d}^3 r \to \mathrm{d}^{3+n} r$