Calc4

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1 Holography in D dim, corrected

This writeup contains the corrected calculations from Calc3 and then follows more strictly the Rizzo approach for calculation Black Hole properties.

1.1 Framework: Rizzo

From Rizzo2006 we have a generic solution for a Schwarzschild-like Metric in D dimensions (so $n = D - 4$ extra dimensions),

$$\mathrm{d}s^2 = (1 - f(r))\,\mathrm{d}t^2\,(1 - f(r))^{-1}\,\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_{D-2} \tag{1}$$

It is the ODE

$$f'(r) + \frac{n+1}{r} f(r) = \frac{1}{M^\star}\frac{2r\rho(r)}{n+2} \tag{2}$$

with M_* the reduced fundamental mass scale of the theory (shortcut $M^\star = M_*^{n+2}$). It is easy to solve this for any $\rho(r)$ to

$$f(r) = \frac{1}{r^{n+1}}\left(\frac{2}{(n+2)M^\star}\int_{c_1}^{r}(r')^{n+2}\rho(r')\,\mathrm{d}r' + c_2\right) \quad \text{with } c_1, c_2 = \text{const} \tag{3}$$

like already done in Calc3.

1.2 Holography in D dim

With the NS 2011 generalized density $\rho(r)$ to D dimensions,

$$\rho(r) = \frac{M}{\Omega(r)}\frac{\mathrm{d}h(r)}{\mathrm{d}r}, \quad \Omega(r) = \Omega_{D-2}r^{D-2} = \Omega_{n+2}r^{n+2} \tag{4}$$

the integral in $f(r)$ is evaluated in a trivial manner (this was done wrong in Calc3). That is, it reads

$$f(r) = \frac{1}{r^{n+1}}\left(\frac{2}{(n+2)M^\star}\int^{r}\frac{M}{\Omega_{D-2}}h'(r')\mathrm{d}r' + \text{const}\right) \tag{5}$$

$$= \frac{2}{n+2}\frac{M}{M_*^{n+2}}\frac{1}{\Omega_{n+2}}\boxed{\frac{h(r)}{r^{n+1}}} \tag{6}$$

We note that the units are correct. With $h(r) = \theta(r)$ (in 5, $h(r) = 1$ in 6), the result gets the proper Schwarzschild-Tangherlini result $f(r) \propto 1/r^{D-3} = 1/r^{n+1}$.

1.3 Getting r_H

Rizzo already made a lot of effort to calculate r_H at $g_{00} = 0$, that is, $f(r_H) = 1$. The bottom line is that there are no more closed form solutions in the models he explored (NC, Lorentzian). Rizzo writes the horizon equation $f(r_H) = 1$ as

$$\begin{array}{lll} m = M/M_* & y = M_*\sqrt{\theta} & c_n \approx (n+2)\Omega_{n+2} \\ x = M_* R_H & z = x/y = R_H/\sqrt{\theta} & x^{n+1} = \dfrac{m}{c_n} F_n(z) \end{array} \tag{7}$$

He lists possible $\delta(r)$ modeling expressions $\rho(r)$ and the functions $F_n(z)$ to be discussed. I added the two holography ones.

Label	$\rho(r)$	$F_n(z)$
D dim NC (Rizzo2006)	$\rho = \dfrac{M}{(4\pi\theta)^{(n+3)/2}} e^{-r^2/4\theta}$	$F_n(z) = \dfrac{1}{\Gamma\left(\frac{n+3}{2}\right)} \gamma\left(\dfrac{n+3}{2}; \dfrac{z^2}{4}\right)$
Lorentzian (Rizzo2006)	$\rho \sim \dfrac{1}{(r^2+L^2)^{\frac{n+4}{2}}}$	$G_n(z) = \dfrac{2}{\pi} \dfrac{(n+2)!!}{(n+1)!!} \int_0^z \mathrm{d}t \dfrac{t^{n+2}}{(1+t^2)^{(n/2+2)}}$
D dim Holography	$\rho = \dfrac{M}{\Omega_{n+2} r^{n+2}} h'(r)$	$H_n(r) = h(r)$
D dim NS2011 $h = \frac{r^2}{r^2+L^2}$	$\rho = \dfrac{M}{\Omega_{n+2}} \dfrac{1}{r^{n+1}} \dfrac{L^2}{(L^2+r^2)^2}$	$H_n(z) = \dfrac{z^2}{z^2+1}$ with $\sqrt{\theta} = L$

Rizzo claims that all ρ models behave quite similary. I wonder if his Lorentzian approach would be the right D dimensional extension to NS2011. All my holography functions lack a dependence of n.

1.4 Open Questions

- How to choose $\rho(r)$? Toy model or physical motivation? Where is the motivation?
- How much degrees of freedom in $\rho(r)$ choice?
  How to match $r_0 = l_*$, $M_0 = M_*$, $G = M_*^{1-m}$?
- *What* to do with the calculated quantities *Horizons*, *Hawking Temperature*, *surface energy density*, *heat capacity*?