Some notes on calculating GUP/NCBHs in LXDs

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Calc-15b

The Fourier integral, 3 dimensions

$$
\begin{aligned}
\int \mathrm{d}^3 p \; f(p) \; e^{i\vec{x}\vec{p}} &= \int_0^{2\pi} \mathrm{d}\varphi \quad \int_0^{\pi} \mathrm{d}\theta \sin\theta \quad \int_0^{\infty} \mathrm{d}p \; p^2 \; f(p) \; e^{i\vec{x}\cdot\vec{p}} \\
&= 2\pi \quad \int_{-1}^{+1} \mathrm{d}\cos\theta \quad \int_0^{\infty} \mathrm{d}p \; p^2 \; f(p) \; e^{ixp\cos\theta} \\
&= 2\pi \quad \int_0^{\infty} \frac{1}{ixp} \quad \mathrm{d}p \; p^2 \; f(p) \; \left(e^{+ixp} - e^{-ixp}\right)
\end{aligned}
$$

The 3d Fourier integral $\to$ effective 1d

$$\ldots = \frac{2\pi}{ix}\int_0^\infty \mathrm{d}p\ p\ f(p)\ \left(e^{+ixp} - e^{-ixp}\right)$$

$$= \frac{2\pi}{ix}\left(\int_0^\infty \mathrm{d}p\ p\ f(p)\ e^{+ixp} - \int_0^\infty \mathrm{d}p\ p\ f(p)\ e^{-ixp}\right)$$

$$= \frac{2\pi}{ix}\left(\int_0^\infty \mathrm{d}p\ p\ f(-p)\ e^{+ixp} + \overbrace{\int_{-\infty}^0 \mathrm{d}p'\ p'\ f(-p')\ e^{+ixp'}}^{\int_a^b = -\int_b^a \quad \text{and} \quad p' = -p}\right)$$

$$= \frac{2\pi}{ix}\int_{-\infty}^{+\infty} \mathrm{d}p\ p\ \underbrace{\Big[f(p)\Theta(p) + f(-p)\Theta(-p)\Big]}_{=f(|p|)} e^{+ixp}$$

And $\int \mathrm{d}p\ p\ f(|p|)e^{ixp} \in \mathbb{C}\setminus\mathbb{R} \quad \Rightarrow \quad i\int \cdots \in \mathbb{R}$ ✓

Does the same hold also in $3+n$ dimensions? $n \in \mathbb{N}_0$

$$\int \mathrm{d}^{3+n} f(p) e^{i\vec{x}\vec{p}} \propto \int_0^\infty \frac{1}{ixp} \mathrm{d}p \; p^{2+n} \; f(p) \; \left(e^{+ixp} - e^{-ixp}\right)$$

$$\propto \int_{-\infty}^{\infty} \mathrm{d}p \; \underbrace{p^{1+n}\Big[f(p)\Theta(p) + (-1)^n f(-p)\Theta(-p)\Big]}_{:=v(p)} e^{+ixp}$$

Spherical coordinates: ✓
Effective 1d notation: ✓
Issue with Holomorphy? Perhaps
$\mathbb{R}$eal result? ✓

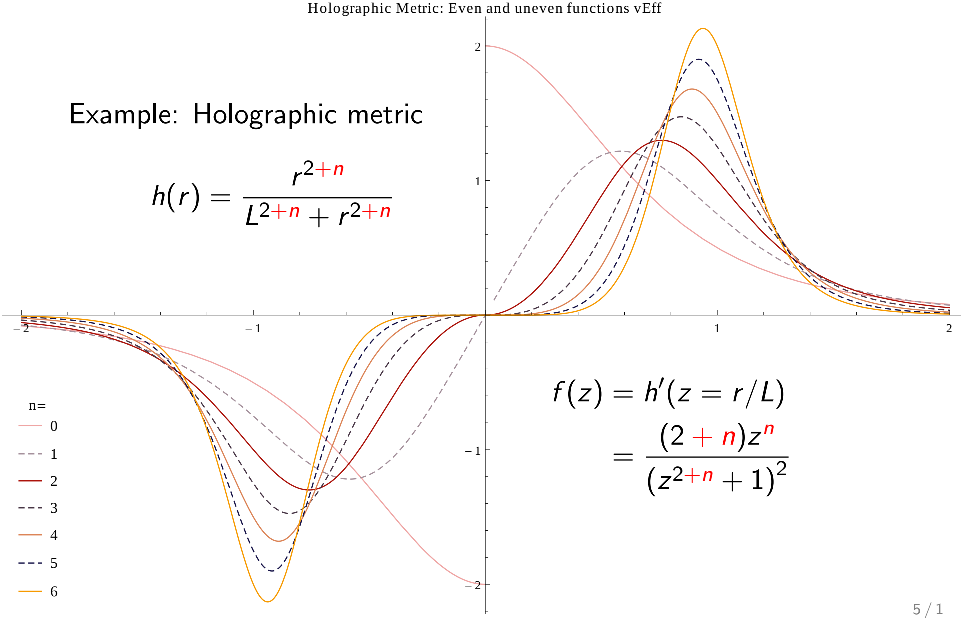
- Even $n = 0, 2, 4$: $v(p) = p^{odd} f(|p|) \Rightarrow \int \in \mathbb{C} \setminus \mathbb{R} \Rightarrow i \int \in \mathbb{R}$
- Odd $n = 1, 3, 5$: $v(p) = p^{even}\Big[odd\Big] \Rightarrow \int \in \mathbb{C} \setminus \mathbb{R} \Rightarrow i \int \in \mathbb{R}$

The Θ composition issue

Example: Holographic metric

$$h(r) = \frac{r^{2+n}}{L^{2+n} + r^{2+n}}$$

$$f(z) = h'(z = r/L) = \frac{(2+n)z^{n}}{(z^{2+n}+1)^2}$$



Ignoring facts: Applying Cauchy theorem

All $f(p)$ choices look roughly like

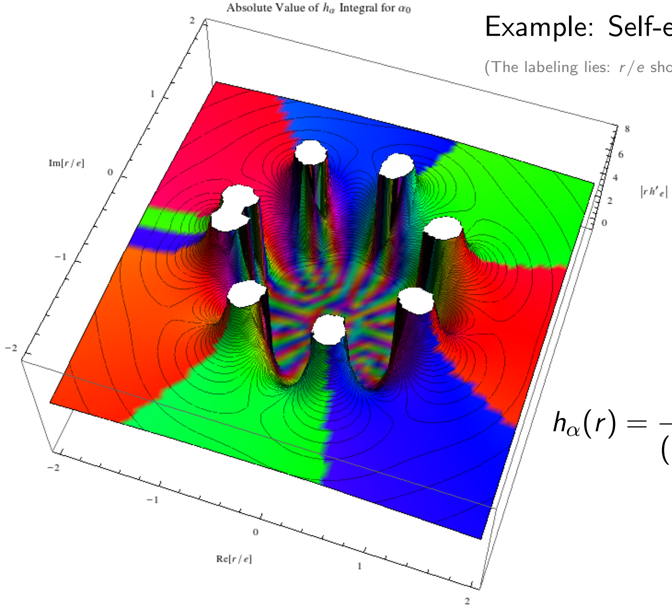
$$f(p) \approx \frac{1}{(1 + L^{\bullet} p^{\bullet})^{\bullet}} = \frac{1}{(1 + q^{\bullet})^{\bullet}}$$

All $(\bullet - 1)$ Poles are on the unit circle $(1 = e^{i\varphi})$:

$$q = (-1)^{\frac{1}{\bullet}} = \exp\left\{\frac{i\pi + 2\pi i k}{\bullet}\right\} \forall k \in \mathbb{N}$$

$\Rightarrow$ It's simple to sketch the setting!

From the complex plane...

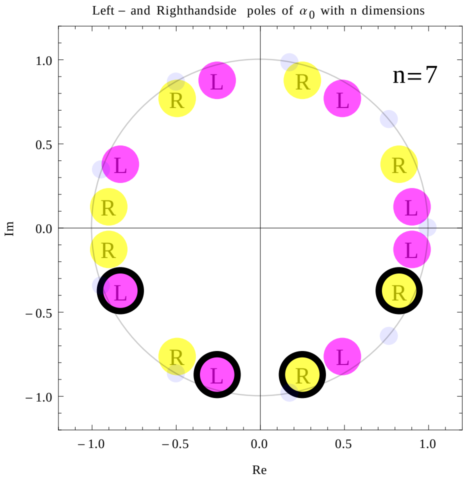


Example: Self-encoding metric

(The labeling lies: r/e should be r/L)

$$h_\alpha(r) = \frac{r^{3+n}}{(r^\alpha + L^\alpha/2)^{3+n/\alpha}}$$

...to the pole diagram



Complex Points

-  Poles of $f(-p)$
-  Poles of $f(p)$
- Chosen poles
- Roots of unity

Only work left:

$$\int \mathrm{d}z = \sum \mathrm{Res}_{z \to z_0}$$

Results: Work must be checked

- Correct for no extra dimensions ($n = 0$).
- Complex values do not vanish $\rightarrow$ Complex masses.