Calc6

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1 The surface issue

These calculations point out a missing surface term when doing the plausibility check for the holographic approach done in [NS11.2013].

I work in $D = 4 + n$ dimensions, but any equation must hold for $n = 0$, too. I follow the Rizzo2006 deviation (also performed in Calc1) for deriving an ODE for the potential $V(r)$ in $g_{00} = 1 - V(r)$ SMM like metrics. Having given only $\rho(r)$, the continuity equation $T^{AB}_{;B} = 0$ gives

$$T^i_i = \rho + \frac{r}{n+2}\partial_r \rho. \tag{1}$$

Which then lead to the first order differential equation

$$V' + \frac{n+1}{r} V = \frac{1}{M_*^{n+2}} \frac{2r\rho}{n+2}. \tag{2}$$

The Ansatz $V(r) = r^{-(n+1)} \left(C \int^r x^2 \rho(x) \mathrm{d}x + D\right)$ solves equation 2. It is simple to derive, as done in Calc3 and Calc4,

$$V(r) = \frac{1}{r^{n+1}} \left(\frac{2}{(n+2)M_*^{n+2}} \int_{c_1}^r x^{n+2} \rho(x) \mathrm{d}x + c_2 \right). \tag{3}$$

It is important to remark that the integral in 3 only looks like the radial part of an partially performed spherical integration, but *there is no surface term*, as there would be if the integral really would be $m(r) = \int \mathrm{d}^{n+3}\, \vec{r}\, \rho(\vec{r})$. That is,

$$\int \mathrm{d}^{n+3}\vec{r}\, \rho(\vec{r}) = \int \mathrm{d}r \, \left(\Omega_{n+2} r^{n+2}\right) \, \rho(r), \tag{4}$$

With $\Omega_{n+2} r^{n+2}$ being the $(n+2)$ dimensional surface (of an $n+3$ dimensionall sphere)

$$\Omega_{n+2} = 2 \frac{\pi^{\frac{n+3}{2}}}{\Gamma\left(\frac{n+3}{2}\right)} \tag{5}$$

The missing Ω_{n+2} in eq. 3 compared to 4 stands out. This is important, because the holographic approach depends on that property of 4.

1.1 NC in D dim

Rizzo introduces the reduced Planck scale M_* by $M_P^2 = V_n M_*^{n+2}$, with $v_n = (2\pi R_c)^n$ the volume of the compacted dimensions as tori with radius R_c. Thus the $n \to 0$ limit gives $M_*^2 = M_P^2 = 1/G$.

Using the gaussian $\rho(r)$, Rizzo (and I in Calc3) got

$$V(r) = \frac{M}{M_*^{n+2}} \frac{1}{(n+2)\pi^{(n+3)/2}} \frac{1}{r} \Gamma\left(\frac{3+n}{2}; \frac{r^2}{4\theta}\right) \tag{6}$$

In the $\theta, n \to 0$ limit, $\Gamma(\frac{3}{2}; \infty) = \sqrt{\pi}/2$ and therefore we end with

$$V(r) = \frac{GM}{4\pi r} \tag{7}$$

1.2 $h(r)$ Profile

In [NS 07.11.2013], the $\theta \to h(r)$ smearing function is introduced, so $\partial_r \theta = \delta \to \partial_r h$ enters a smeared density:

$$\rho(r) = \frac{M}{4\pi r^2}\frac{\mathrm{d}h}{\mathrm{d}r} \xrightarrow{\text{D=n+2 dimensions}} \rho(r) = \frac{M}{\Omega_{n+2} r^{n+2}}\frac{\mathrm{d}h}{\mathrm{d}r} \tag{8}$$

Since $\Omega_2 = 4\pi$, this seems to be true. I showed already in Calc2 that an integration (like in eq 4) over that class of $\rho(r)$ gets trivial in *any* dimension.

Lets apply the solution for $V(r)$ at this density. Since that integration is not a *full* one, it allows the surface constant Ω_{n+2} to enter the metric. We end up with (already showed in Calc4)

$$V(r) = \frac{2}{n+2}\frac{M}{M_*^{n+2}}\frac{1}{\Omega_{n+2}}\frac{h(r)}{r^{n+1}}. \tag{9}$$

This equation cannot produce the SMM value $V(r) = \frac{2GM}{r}$ any more, because nothing kills the $\Omega_2 = 4\pi$. Indeed, if we use the Schwarzschild-Tangherlini density $\rho(r) = M/(\Omega_{n+2})\delta(r)$ and apply it to eq. 3, [Reall-Review Section 3.2]

$$V(r) = \frac{1}{r^{n+1}}\left(\frac{2}{(n+2)M_*^{n+2}}\frac{M}{\Omega_{n+2}}\int \mathrm{d}r\delta(r)\right) = \frac{\mu}{r^{n+2}}, \quad \mu = \frac{16\pi GM}{(n+2)\Omega_{n+2}} \tag{10}$$

TODO: Why μ?. If we now send $n \to 0$, this does not reproduce SMM at all.

The Planck length $M_P^2 = V_n M_*^{n+2}$ is equal to M_* in $n = 0$ dimensions. Since $M_P = 1/\sqrt{G}$ Newtons constant $G = 1/M_*^2$ is restored. Thus, from eq. we get for $n = 0$

$$V(r) = \frac{GM}{r}\frac{1}{4\pi} \tag{11}$$

Conclusion: There seems always the factor 8π to be missing. There seems to be some $G \leftrightarrow 8\pi G$ issue.