Competitive Sudoku: Rules

1 Normal (General) Sudoku

Traditional sudoku is a single player puzzle game that is played on a square grid containing N^2 cells, with $N = nm$ and $n, m \in \mathbb{N}$. The most common version is played with $n = m = 3$. The grid is divided into a regular sub-grid of rectangular *blocks*, each with size $n \times m$ cells. The game makes reference to three distinct types of *regions*: the *rows*, *columns*, and *blocks* of the grid. There are exactly N of each of these regions, and every cell of the grid belongs to exactly one row, one column, and one block. Every cell of the grid can be either empty, or filled with a number in the range $1, \ldots, N$.

At the beginning of the game, the grid starts out with some empty cells, and some cells filled with numbers (called *clues*). The goal of the game is to fill out every cell of the grid, subject to the following constraint:

  C0 The entries of non-empty cells within each region must be unique.

The *solution* is thus a completely filled-out grid, for which every region—every row, column, and block—contains the numbers $1, \ldots, N$ exactly once. A puzzle is called *proper* if there is exactly one valid solution for the given initial clues. An example puzzle and solution are shown in Figure 1.

Figure 1: Left: Example of a sudoku puzzle with $n = m = 3$. Right: The solved puzzle; original clues are in black, with the values to be entered shown in red.

2   Competitive Sudoku

Competitive sudoku is an adversarial two-player game, in which players take turns filling out cells of a given sudoku puzzle. Players score points by completing regions; when the puzzle is completed, the player with the most points wins. Because the puzzle-aspect of the sudoku is non-trivial—it is in general non-obvious which value should be entered in which cell—the game is played under the supervision of an *oracle* that ensures the puzzle remains solvable after every move. Note that the game may be played with *im*proper puzzles—in particular, the game may be started with a completely empty grid. We now present the full rules in detail.

Start   Both players start with 0 points. The starting player is chosen using some unspecified criterion; the choice may be random or based on external circumstance such as tournament rules. Beginning with the starting player, the players consecutively take *turns* until the game is finished.

Turns   On their turn, the active player proposes to fill out a single (empty) cell of the grid with a chosen value, such that

- entering the chosen value in the chosen cell respects C0;

- the chosen value has not been declared *taboo* for the chosen cell.

This proposed *move* is subsequently judged by the *oracle*. This leads to one of two outcomes:

- if, after playing the proposed move, the sudoku puzzle remains solvable, the move is played, and the active player potentially scores points;

- if, after playing the move, the sudoku puzzle would become *unsolvable*, then the oracle declares the chosen value to be *taboo* for the chosen cell— the move is *not* played, and the active player does not score points for the move.

Regardless of which case happened, the next player then takes their turn.

Scoring   If, during their turn, a player fills out a cell that *completes* a region— the cell was the last empty cell of a region that contains it—they add points to their current score. A single move may complete up to three regions at once; the number of points scored depends on this, according to the following table:

Regions completed	Points scored
0	0
1	1
2	3
3	7

Game End   The game ends when either of the following events occur:

- The active player proposes a value for a cell such that either (i) the cell is non-empty, (ii) the value is not in the range $1, \ldots, N$, (iii) entering the value in the cell would violate C0, or (iv) the cell-value combination has previously been declared taboo by the oracle; in any such case this player loses the game.

- The sudoku grid is successfully solved; the player with the highest score wins the game (or the game is tied when both players have the same score).

Time Controls   To ensure that games are finished in a timely manner, a maximum duration per turn is imposed. For the sake of simplicity, players do not know how much time they have each turn; hence, they need not worry about developing optimal strategies to budget their time. Rather, players can propose moves as often as they like during their turn; once the time for that turn runs out, only the *most recent* proposal is considered as definitive and processed as explained above.