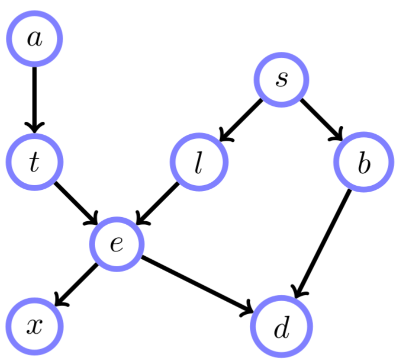
MLPM Tutorial 6

November 19, 2019



x = Positive X-ray
d = Dyspnea (Shortness of breath)
e = Either Tuberculosis or Lung Cancer
t = Tuberculosis
l = Lung Cancer
b = Bronchitis
a = Visited Asia
s = Smoker

Figure 1: Chest clinic belief network.

1. The chest clinic network, see Fig. 1 concerns the diagnosis of lung disease (tuberculosis, cancer, both or neither). In this model a trip to Asia is assumed to increase the probability of tuberculosis. State if the following conditional independence relationships are true or false

   (a) tuberculosis $\perp$ smoking $|$ shortness of breath

   (b) lung cancer $\perp$ bronchitis $|$ smoking

   (c) visit to Asia $\perp$ smoking $|$ lung cancer

   (d) visit to Asia $\perp$ smoking $|$ lung cancer, shortness of breath

2. The variables in the chest clinic belief network are binary and can take the value of true (`tr`) or false (`fa`). Given the conditional probability tables (CPTs) in Fig. 2 calculate $p(d)$, $p(d|s = \texttt{tr})$ and $p(d|s = \texttt{fa})$.

$$
\begin{array}{llll}
p(a = \textsf{tr}) & = 0.01 & p(s = \textsf{tr}) & = 0.5 \\
p(t = \textsf{tr}|a = \textsf{tr}) & = 0.05 & p(t = \textsf{tr}|a = \textsf{fa}) & = 0.01 \\
p(l = \textsf{tr}|s = \textsf{tr}) & = 0.1 & p(l = \textsf{tr}|s = \textsf{fa}) & = 0.01 \\
p(b = \textsf{tr}|s = \textsf{tr}) & = 0.6 & p(b = \textsf{tr}|s = \textsf{fa}) & = 0.3 \\
p(x = \textsf{tr}|e = \textsf{tr}) & = 0.98 & p(x = \textsf{tr}|e = \textsf{fa}) & = 0.05 \\
p(d = \textsf{tr}|e = \textsf{tr}, b = \textsf{tr}) & = 0.9 & p(d = \textsf{tr}|e = \textsf{tr}, b = \textsf{fa}) & = 0.7 \\
p(d = \textsf{tr}|e = \textsf{fa}, b = \textsf{tr}) & = 0.8 & p(d = \textsf{tr}|e = \textsf{fa}, b = \textsf{fa}) & = 0.1 \\
\multicolumn{4}{l}{p(e = \textsf{tr}|t, l) = 0 \textit{ only if both } t \textit{ and } l \textit{ are } \textsf{fa}\textit{, 1 otherwise.}}
\end{array}
$$

Figure 2: Chest clinic CPTs.

3. The RockPaperScissors is played by Player 1 and Player 2. The sequence of moves $x^1_{1:T}$, $x^2_{1:T}$, where $x_i \in \{r, p, s\}$, played by both players is

$$
\begin{aligned}
x^1_{1:T} &= [r\ p\ r\ p\ r\ s\ p\ r\ s\ p\ p\ r\ r\ r\ r\ p\ r\ s\ r\ p\ p\ s\ r] \\
x^2_{1:T} &= [s\ p\ r\ s\ p\ s\ p\ s\ r\ p\ s\ r\ p\ p\ r\ r\ s\ p\ r\ s\ s\ p\ r]
\end{aligned}
$$

Assume that player 1 plays according to a first order Markov chain defined by

$$
p(x^1_{1:T}|x^2_{1:T-1}) = p(x^1_1) \prod_{t=1}^{T} p(x^1_t|x^1_{t-1}, x^2_{t-1})
$$

where the distribution over the initial move is uniform.

(a) Draw the directed graphical model for the described setup.

(b) Use maximum likelihood estimation and the data above to find the conditional probability table (CPT) for $p(x^1_t|x^1_{t-1}, x^2_{t-1})$.

(c) Calculate $p(x^1_{1:T}|x^2_{1:T-1})$.

(d) Is player 1 playing according to the assumed strategy or are they just making random moves?

4. Consider the Bayes net shown in Fig. 3. Here the nodes represent the season $S \in \{winter, spring, summer, autumn\}$, the type of fish $F \in \{salmon, sea\ bass\}$, the lightness of the fish $L \in \{light, medium, dark\}$

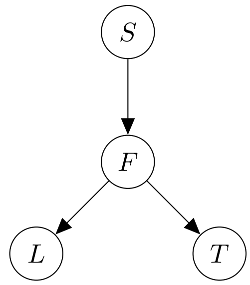


Figure 3: Fish problem DAG.

and the thickness of the fish $T \in \{wide, thin\}$. The corresponding CPTs are:

$$p(S) = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \end{bmatrix} \qquad p(F|S) = \begin{bmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \\ 0.4 & 0.6 \\ 0.8 & 0.2 \end{bmatrix}$$

$$p(L|F) = \begin{bmatrix} 0.33 & 0.33 & 0.34 \\ 0.8 & 0.1 & 0.1 \end{bmatrix} \qquad p(T|F) = \begin{bmatrix} 0.4 & 0.6 \\ 0.95 & 0.05 \end{bmatrix}$$

(a) Suppose the fish was caught on December 20 – the end of autumn and the beginning of winter – and thus let $p(S) = [0.5\ 0\ 0\ 0.5]$ instead of the above prior. (This is called soft evidence, since we do not know the exact value of S, but we have a distribution over it). Suppose the lightness has not been measured but it is known that the fish is thin. Classify the fish as salmon or sea bass.

(b) Suppose all we know is that the fish is thin and medium lightness. What season is it now, most likely? Use $p(S) = [0.25\ 0.25\ 0.25\ 0.25]$.