MLPM Tutorial 8

December 10, 2019

1. Given a standard HMM and the conditional indpendencies it encodes, how can the following distributions be factorised or simplified:

   (a) $p(\mathbf{x}_{1:T}|z_t)$

   (b) $p(\mathbf{x}_{1:t-1}|\mathbf{x}_t, z_t)$

   (c) $p(\mathbf{x}_{1:t-1}|z_{t-1}, z_t)$

   (d) $p(\mathbf{x}_{t+1:T}|z_t, z_{t+1})$

   (e) $p(\mathbf{x}_{t+1:T}|z_t, \mathbf{x}_t)$

   (f) $p(\mathbf{x}_{1:T}|z_{t-1}, z_t)$

   (g) $p(\mathbf{x}_{t+1}|\mathbf{x}_{1:t}, z_{t+1})$

   (h) $p(z_{t+1}|z_t, \mathbf{x}_{1:t})$

2. Consider a HMM with 3 states, $z_t \in \{1, 2, 3\}$, and 2 output symbols, $x_t \in \{1, 2\}$, with a transition matrix

$$A = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.0 & 0.6 & 0.4 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$$

   where $A_{ij} = p(z_{t+1} = j|z_t = i)$, emission matrix

$$B = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \\ 0.8 & 0.2 \end{bmatrix}$$

   such that $B_{ij} = p(x_t = j|z_t = i)$ and initial state probability vector $a = [0.9 \;\; 0.1 \;\; 0.0]^T$. Given the observed symbol sequence $x_{1:3} = (1, 2, 1)$:

(a) Compute $p(x_{1:3})$

(b) Compute $p(z_1|x_{1:3})$

3. Derive the expected complete data log-likelihood for HMM.

4. Consider an HMM where the observation model is a mixture of Gaussians

$$p(\mathbf{x}_t|z_t = j, \boldsymbol{\theta}) = \sum_k w_{jk}\mathcal{N}(\mathbf{x}_t|\boldsymbol{\mu}_{jk}, \boldsymbol{\Sigma}_{jk})$$

(a) Draw the directed graphical model

(b) Derive the E step.

(c) Derive the M step.