# Modeling the EPEX Intraday Market: A Step Towards Automated Intraday Trading

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# Preface

This project report is written within the field of Applied Economics and Operations Research at the Department of Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU). The report is motivated by the increased demand for short term optimization in power markets, and the successful implementation of trading algorithms in other financial markets.

The quality of this report is significantly higher thanks to the help of both supervisors. We would like to thank our supervisor Asgeir Thomasgard for making the work with this report possible, and our co-supervisor Post Doc Gro Klæboe for her valuable guidance. In particular, she has shown genuine interest, invested significant time and given valuable feedback. We are grateful that her employers, Powel AS and Trønderenergi AS, have let her cooperate with us. The helpfulness of Powel AS has been an essential contribution.

## Abstract

As the German Intraday power market has grown steadily over the last seven years, companies are looking into the prospect of improving and facilitating trading with mathematical programming and machine learning. In this report, optimal Intraday Trading Problem is decomposed into three subproblems, the existing literature on optimization of trading in Intraday Continuous Auction markets is reviewed, a mathematical model for the automation of trading in the market is proposed and solution methods for the model are discussed.

To better define the scope of this report, the optimal Intraday Trading Problem is decomposed into a Price Forecasting Problem, a Cost Estimation Problem and a Strategy Formulation Problem. It is found that the existing literature for the Strategy Formulation Problem is insufficient to automate trading on the EPEX Intraday, and this is therefore modeled the most extensively. It is found that dynamics in both trading time and production time are relevant, and that the Strategy Formulation Problem is simultaneously an Order Execution Problem and a Dynamic Resource Allocation Problem. For the first time, a model handling these *double dynamics* is presented. The model also accounts for all relevant aspects of the market microstructure and uses fundamental parameters to forecast the price, unlike former papers. Finally, the model is applicable for any type of trader and allows the trader to determine all decision variables herself, whereas former papers typically model one type of trader and allows flexibility in only a subset of the relevant decision variables. It is proposed that the model is solved using an ADP algorithm, and several techniques to improve the algorithm's computational complexity are proposed.

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# Nomenclature

**Sets**

$\mathcal{T}$ The set of delivery products.

$D_{t\tau}$ The set of revealed demand bids for delivery product $\tau$ in timestep $t$.

$D_{t_b t\tau}$ The set of accessible demand bids for bid $b_{t_b\tau}$ in timestep $t$.

$T_\tau$ The set of stages when delivery product $\tau$ is tradeable.

**Indices**

$\tau$ Delivery product ($\tau \in \mathcal{T}$).

$i$ Demand bid ($i \in D_{t_b t\tau}$).

$t$ Timeslot. Indicates trading time ($t \in \bigcup_\tau^{\mathcal{T}} T_\tau$).

$t_b$ Timeslot. Indicates time of bid placement ($t_b \in \bigcup_\tau^{\mathcal{T}} T_\tau$).

**Decision Variables**

$\delta^K_{t_b t\tau}$ 1 if the bid for delivery product $\tau$ placed in timeslot $t_b$ has been killed at time $t$, 0 otherwise.

$b_{t_b\tau}$ A sell bid for delivery product $\tau$, placed at time $t_b$, with the features price $b^P_{t_b\tau}$ and volume $b^V_{t_b\tau}$. NB: This is equal to the two-tuple including the decision variables $(b^P_{t_b\tau}, b^V_{t_b\tau})$.

$b^P_{t_b\tau}$ The price of the trader's sell bid for delivery product $\tau$ placed in timeslot $t_b$.

$b^V_{t_b\tau}$ The volume of the trader's sell bid for delivery product $\tau$ placed in timeslot $t_b$.

$q_\tau$ Actual production for delivery product $\tau$.

**Support Variables**

$\delta^C_\tau$ 1 if none of the trader's bids for delivery product $\tau$ has been matched.

$\delta^B_{t_b t\tau}$ 1 if the bid for delivery product $\tau$ placed in timeslot $t_b$ has been placed in or before timeslot $t$, 0 otherwise.

$\delta^C_{t_b t\tau}$ 1 if the bid for delivery product $\tau$ placed in timeslot $t_b$ has been cleared in or before timeslot $t$, 0 otherwise.

$\widetilde{b}^V_{t_b t\tau}$ Remaining volume of the bid for delivery product $\tau$ placed in timeslot $t_b$ in timeslot $t$.

$p^C_{t_b\tau}$ Average transaction price of the bid placed in timeslot $t_b$ for delivery product $\tau$.

$p^C_{t_b t\tau}$ Average transaction price for transactions created in timeslot $t$ involving the bid placed in timeslot $t_b$ for delivery product $\tau$.

$s_\tau$ Stored energy at time $t^D_\tau$.

$v^{BM+}_\tau$ Volume traded in the Up-regulating balancing market for delivery product $\tau$.

$v^{BM-}_\tau$ Volume traded in the Down-regulating balancing market for delivery product $\tau$.

$v^D_{t_b t\tau i}$ The matched volume in timeslot $t$ for delivery product $\tau$ in the transaction containing bid $b_{t_b\tau}$ and bid $d_{t_b t\tau i}$.

$v^C_{t_b t\tau}$ The total cleared volume in timeslot $t$ of the bid for delivery product $\tau$ placed in timeslot $t_b$.

**Functions**

$\mathbf{E}(\cdot)$ Expected value.

$\mathbf{E}_t(\cdot)$ Expected value at time $t$.

$\Pr(\cdot)$ Probability.

$C^P_\tau(\cdot)$ The production cost function of delivery product $\tau$.

**Operators**

$\underline{(\cdot)}$ Lower bound.

$\bar{(\cdot)}$ Upper bound.

$\Delta(\cdot)_{t-1}$ The change in a variable between time step $t-2$ and $t-1$.

$\hat{(\cdot)}$ Optimal decision.

$\vec{(\cdot)}$ Vector.

$\widetilde{(\cdot)}$ Counter or residual.

$|(\cdot)|$ Cardinality of a set.

$||(\cdot)||_n$ n-norm of a vector.

**Parameters**

$\underline{Q}_\tau$ Lower bound on the total production volume during $\tau$, determined at time $\bar{t}_\tau^{GC}$.

$\bar{Q}_\tau$ Upper bound on the total production volume during $\tau$, determined at time $\bar{t}_\tau^{GC}$.

$\bar{S}_\tau$ Storage capacity at time $\bar{t}_\tau^{GC}$.

$\bar{V}$ Maximal bid volume size.

$C^C$ Transaction cost, including a market premium for renewables.

$R^{BM}$ Upper bound on the total volume traded in the balancing markets.

$R^{OTR}$ Order-to-Trade-Ratio limit. The number of permitted uncleared bids per cleared bid for each delivery product.

$V_\tau^{DA}$ Production volume cleared in Day-Ahead market for delivery product $\tau$.

**Stochastic parameters**

$d_{t_b t\tau i}$ Bid of demand bid $i$ at time $t$ included in the accessible residual demand of the trader's bid for delivery product $\tau$ placed in timestep $t_b$. NB1: Lower-case letter to resemble the decision variables of the trader. NB2: This is equal to the two-tuple including the stochastic parameters $(d_{t_b\tau}^P, d_{t_b\tau}^V)$.

$d_{t_b t\tau i}^P$ Bid price of demand bid $i$ at time $t$ included in the accessible residual demand of the trader's bid for delivery product $\tau$ placed in timestep $t_b$. NB: Lower-case letter to resemble the decision variables of the trader.

$d_{t_b t\tau i}^V$ Bid price of demand bid $i$ at time $t$ included in the accessible residual demand of the trader's bid for delivery product $\tau$ placed in timestep $t_b$. NB: Lower-case letter to resemble the decision variables of the trader.

$F_\tau$ Inflow (MWh) of energy resource at time $\bar{t}_\tau^D$.

$P_\tau^{BM+}$ Balancing Up-price for delivery product $\tau$.

$P_\tau^{BM-}$ Balancing Down-price for delivery product $\tau$.

**Time parameters**

$\underline{t}$ Time of gate opening.

$\bar{t}_\tau^D$ Delivery start time of delivery product $\tau$.

$\bar{t}_\tau^{GC}$ Gate closure time of delivery product $\tau$.

# 1 Introduction

In recent years, the share of renewable energy production compared to consumption in Germany has grown from 5.8% in 2004 to 14.6% in 2015 (Eurostat, 2017). As renewable intermittent production facilities must deal with stochastic production, the need for more short-time optimization is growing (Hassler, 2017). The German Intraday market has potential to cover this need, and it has grown from 5.6TWh in 2009 (EPEX, 2009) to 41TWh in 2016 (EPEX, 2016), an impressive 33% YoY growth rate. While volumes are still small enough that a single large trader can disrupt the price trajectory for a given day, this may not hold true for long. The market is still small compared to the Day-Ahead market (EPEX, 2016), but falling Day-Ahead margins have power producers on the lookout for new market opportunities (Klaboe and Fosso, 2013). Garnier and Madlener (2015) states that *"The more liquid and competitive the intraday market is, the more efficient it is to balance forecast errors intra-daily"*.

However, optimizing trading in the Intraday market is hard and costly. The trading happens continuously, both during normal working hours and throughout the night. Skilled traders are short in supply, and expensive to keep up at night. Prices are volatile, more so than for the Day-Ahead market (Garnier and Madlener, 2014) and subject to unpredictable Urgent Market Messages and changes in weather forecasts. As the market still is young and changing, little theory exists on how to optimize the trading strategy in it.

In comparison, trading in the Day-Ahead market requires only the preparation of one bid curve for each hour for each day. The bids are aggregated to a supply curve, and each producer receives the clearing price for the entire market; this reduces the incentive to bid above marginal cost for a price taking producer, in contrast with how the pay-as-bid Intraday market forces suppliers to bid strategically. This makes optimal bidding more complicated in the Intraday market than in the Day-Ahead market. Large volumes are traded on the Day-Ahead market, and a producer with competitive marginal cost can be confident that their bid will clear. Significant theory exists on how to optimize bids in the Day-Ahead market.

To summarize so far, trading in the Intraday market is currently too unreliable and expensive to serve as a main revenue source for a large producer, but persistent trends are gradually making the Intraday market more attractive to trade in. At the same time, algorithms are automating the trading practices of classic financial markets, driving down costs and improving decision making.

In light of this, it is not surprising that Powel, a Norwegian vendor of optimization software for power producers, are looking into improved decision making in the Intraday market (Powel, 2017b). The scope of this report was therefore determined in cooperation with analysts at Powel.

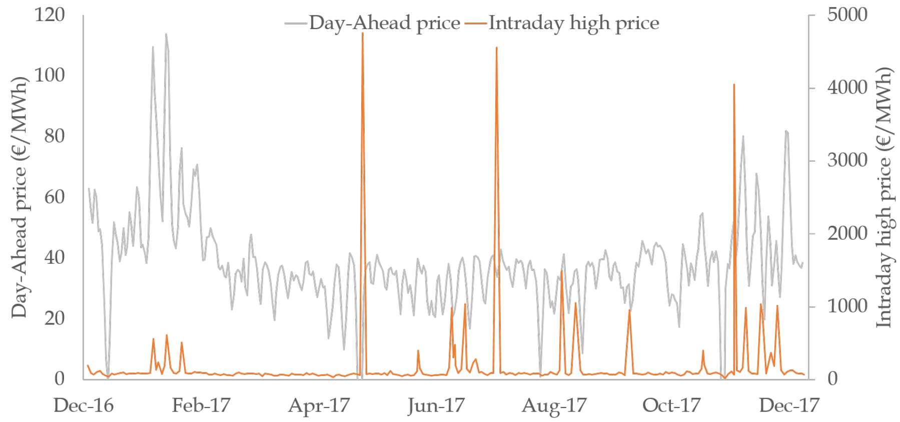



Figure 1.1: Comparison of Intraday and Day-Ahead price development in the period 2016-12-08 - 2017-12-07. Data from (EPEX, 2017f) and (EPEX, 2017e).

The following paragraphs provide a quick overview of the structure of this report. A presentation of the relevant components of the EPEX Intraday market is provided in chapter 2. Unlike previous papers, this report includes a detailed description of the *market microstructure* - specific market features that complicates trading on the EPEX Intraday, such as the Order-to-Trade-Ratio fine. This is vital for the long term goal of developing an automated algorithm, whereas it may have been less relevant for former papers aiming to discover more general insights.

An exhaustive literature research is performed in chapter 3, focusing on the limitations of existing papers and classifying them by which parts of the automated trading problem they consider. It is found that former papers discover relevant features of what optimal decision making looks like for specific market participants in narrow situations. However, the findings don't generalize to other types of traders, or a broader set of trading strategies. No paper accounts for the *double dynamics* in the Intraday continuous market, where decisions are dynamic in both the trading time and the production time (see figure 3.1). Finally, several parts of the market microstructure are largely unaccounted for in all former papers.

In this report, the problem of automating Intraday trading is structured in a more holistic way than before. Specifically, a list of requirements to an autonomous trading algorithm is proposed, four approaches to achieving such an algorithm are suggested, and one of the given approaches is explored in detail. A decomposition of the Intraday Trading Problem into three smaller parts is outlined, viewing the Price Forecasting Problem, the Cost Estimation Problem and the Strategy Formulation Problem as separate optimization problems. These three are elaborated in detail in chapter 4. The latter problem takes the former two as inputs, and this is the problem that is explored the most thoroughly.

A price forecasting model with fundamental parameters, as well as a generalized model for the Strategy Formulation Problem is presented in chapter 5. The proposed model is applicable for any type of trader in the Intraday market considering any strategy. "Any type of trader" includes both demand aggregators, renewable-, hydro-, thermal- and multi-asset producers as well as retailers and purely financial traders. "Any strategy" means that the trader in practice may place and withdraw as many bids as desirable during the day, with whatever price and volume she finds wise. This is a large improvement over much of the existing literature, which usually leaves out significant parts of the decision space. In contrast to former papers, clearing of orders (henceforth referred to as *bids*) is modelled as stochastic. A dynamic approach is adopted throughout the trading windows of the delivery products, and decisions for separate trading products are coordinated. Thus, both an Order Execution Problem and a Dynamic Resource Allocation Problem is modeled, and for the first time dynamics in both trading time and production time is accounted for. Compared to previous models, the model proposed in this report incorporates significantly more of the real-world market microstructure. Also, several model extensions are presented, potentially increasing model performance or flexibility further.

The main goal has been to develop an as realistic model as possible, for two reasons; firstly, so the model may work as a goalpost to reach for in future research, helping researchers be aware of and explicit about the simplifications they are making; and secondly, because it may make it easier to develop suitable heuristics that approximate the exact model.

In chapter 6, two potential methods for solving the Strategy Formulation Problem are suggested and discussed, as well as multiple potentially performance-increasing measures.

Finally, the remaining research required to achieve fully automated superhuman trading is outlined.

# 2 Background

In this section, key aspects of trading in power markets in general, and EPEX Intraday in particular, are covered. Section 2.1 provides a rationale for the current power market structure, as well as a brief introduction to three of the physical power markets. In section 2.2, peculiarities of the prices in the power markets are outlined. section 2.3 covers the main actors in the EPEX Spot markets. In section 2.4, legal requirements to the traders in the EPEX Intraday market are covered, and section 2.5 describe the technical details of trading in the EPEX Intraday market. Finally, section 2.6 outlines the requirements to an automated EPEX Intraday trading algorithm and the approaches to developing such an algorithm. Henceforth, the problem of trading optimally in the EPEX Intraday market is referred to as the Intraday Trading Problem (ITP). This differs from the Automated Intraday Trading Problem (AITP) which is the problem of automating the process of solving ITP.

## 2.1 Introduction to power markets

Electrical power is a very special commodity for several reasons, for instance:

- Production must at all times equal demand; it isn't possible to store in large scale. Even momentary imbalances will affect the frequency and voltage of the power supply, potentially damaging large amounts of expensive electrical equipment. Larger imbalances may cut the power supply entirely, disconnecting large customer groups through no fault of their own.
- Demand is highly seasonal, with both annual, weekly and daily cyclic patterns (see figure 2.1). Typically, demand is high in winter, on weekdays and in the morning and afternoon, whereas it is low in summer, in weekends and during the night. With increasing penetration of solar power, supply is also becoming more cyclic.

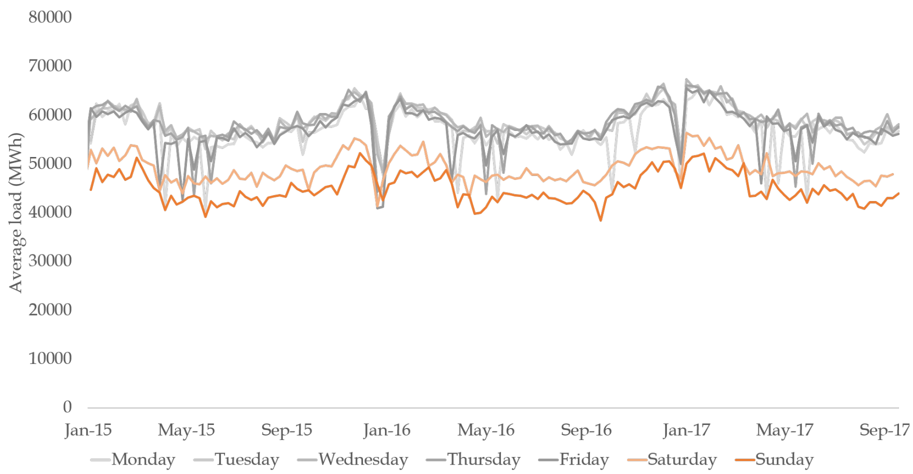



Figure 2.1: Average power load per day in the period 2015-01-01 - 2017-09-29 (ENTSO-e, 2017b)

- The power grid is the only infrastructure that is well suited to transmit electrical power across large distances. Once a production (or consumption) unit is connected to the grid, the production of that unit becomes physically indistinguishable from the production of other grid connected units; the power flow is subject to the laws of physics governing the power grid. Thus, consumers will experience the exact same power quality regardless of which vendor they purchase their power from. This makes it harder for consumers to discriminate between producers and vice versa, making electrical power a near-perfect commodity.

- Finally, consumption is nearly completely inelastic for low volumes (Malic, 2017), creating a strong incentive for price making producers in an oligopoly to artificially withhold production. In current power markets the short-term elasticity is low for all volumes as customers usually are shielded from short term fluctuations in the power price. This may change as adoption of smart meters increases so that consumption can be measured over arbitrarily short time intervals, potentially creating an incentive for demand response that is absent today.

Due to the above-mentioned complications, a complex market structure has been developed to provide power to the customers with the following features:

- High security of supply

- High power quality

- Relatively low prices

In the following sections, we will describe the three power markets that are considered most in this report. While there also exist other, more long term financial markets, they aren't physical markets - that is, markets where the underlying is a certificate to produce one unit of energy. All of the following markets are physical energy-only markets, in the sense that only the sale of energy is rewarded. Other desiderata like available production capacity in the case of demand spikes are not rewarded in such markets. An overview of the markets can be seen in figure 2.2. Also note that the Over-the-Counter (OTC) market has been awarded a spot in this figure. In the OTC market, market participants can trade power bilaterally off-exchange. OTC trade will not be outlined further here.

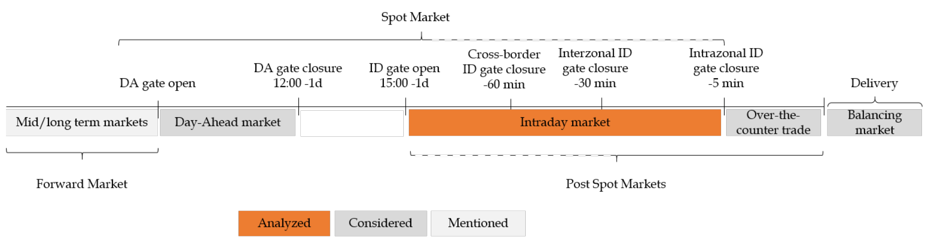



Figure 2.2: Power trading timeline

### 2.1.1 The Day-Ahead Market

The purpose of the Day-Ahead (DA) Market is to assign sufficient production capacity to cover demand for each hour of the following day. In this market, producers are asked to place their bids for power certificates for the next day. Each power certificate gives the right and obligation to produce one unit of energy (MWh), constantly distributed over one specific hour on the next day. The period of validity for the power certificate is referred to as the *delivery product*, and there's a separate auction for each delivery product. The prices of the bids for each delivery product are generally expressed as a function of the cleared volume, $p = p(v)$. After gate closure, the bids are sorted in a non-decreasing aggregated supply curve, and the cheapest bids with a total volume equaling the predicted consumption volume for the given hour are cleared. The producers with the cleared bids will receive a compensation equal to the asking price of the "marginal producer" - that is, the most expensive bid that cleared. This is denominated a "pay as cleared"-market, and removes the incentive to bid strategically for a producer without market power; the optimal bid for the individual producer will equal their marginal cost. The clearing price is also called the "spot price", as the DA market is also referred to as the spot market. Consecutive markets are collectively referred to as "post-spot" markets, although the Intraday market is sometimes included in the "spot markets". In 2016, 235 MWh were sold in the Day Ahead market EPEX (2016).

### 2.1.2 The Intraday Market

In contrast to the DA market, the EPEX Intraday (ID) market is a "pay as bid" market - that is, bidders receive their asking price, plus an eventual spread (see section 2.5.1). The ID market is a continuous auction, similarly to conventional stock markets, starting shortly after the production plan based on the Day-Ahead bids has been published, and currently closing 5 minutes before the time of delivery for each delivery product; the latter is referred to as "gate closure". In the last 25 minutes of the Intraday trading window, referred to as the intrazonal Intraday market, only bids placed in the same control area can be matched. Note that the gate closure times have been changed recently. The pay as bid feature allows each trade to clear independently of other trades, speeding up clearing in the ID market. This is suitable for a market that is designed to allow producers to adjust and reoptimize their production plans over shorter time horizons than the DA market, but comes at a price; it incentivizes bidding above marginal cost, and opens for inefficient allocation of production. In the Intraday market, delivery products with both hourly and quarterly duration are traded. 41 MWh were traded in the Intraday market in 2016 EPEX (2016).

### 2.1.3 The Balancing Market

On a systems level, the rationale for the Balancing Market (BM) is to close any real-time discrepancies between load and production. In practice, this implemented by charging market participants in the DA and ID markets for deviations between the individual net production and the commitment made in the former markets (see section 2.4 for further explanation). Such deviations may arise if the traders were unable to close their open position in the EPEX Intraday, if forecasted residual demand changes near gate closure, et cetera. Production capacity is allocated for the BM one week in advance. The price of power in the BM is denominated *imbalance price* (alternatively "reBAP") is volatile compared to the prices of the DA and ID markets, as figure 2.3 suggest. It is calculated as the total net cost of balancing energy divided by the net balancing energy provided (Regelleistung, 2017). There are two imbalance regulations. The Up-regulation arises if the power consumption is greater than the power generation, and the Down-regulation operates if the power consumption is less than the power generation. The imbalance price is very volatile and high on expectation, incentivizing producers to close their position before the BM opens. This cost is usually symmetrical for the Up and Down regulating balance markets (ENTSO-e, 2017a). The imbalance prices are published with a 20 day lag the 20th of every month (ENTSO-e, 2017a). In addition to the economic incentive to avoid the BM, "*The German regulator legally requires Balancing Responsible Parties* [see section 2.4] *to minimize their use of the imbalance market to the largest possible degree*" (Malic, 2017).

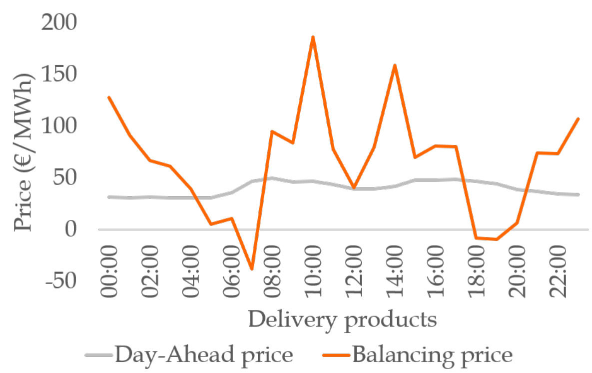



Figure 2.3: Comparison of Day-Ahead and balancing prices 2016-12-21. Data from (EPEX, 2017d) and (ENTSO-e, 2017a).

## 2.2 Power spot market features

For the reasons outlined in section 2.1, power spot prices have several special features:

- **Seasonality:** As there's low storage capacity, price fluctuations are highly correlated with how the demand curve relative fluctuates relative to the supply curve. As both of those have strong seasonal components, the same applies to the price.

- **Mean-reverting price spikes:** combined with strong inelasticity, the seasonality argument explains why periods with high demand and low supply may cause extraordinarily large price spikes, to the point where the highest prices may be more than a 100 times the average throughout the year.

- **Negative prices:** If demand falls steeply for short periods of time, time-interdependent marginal costs of production like ramp up/down-costs or start up/shut down costs may cause utilities to produce above the market equilibrium. As the surplus production cannot be stored, negative prices creates an incentive for consumers to ramp up their consumption and utilities to temporarily ramp down, effectively maintaining the balance between production and consumption. This is exemplified in figure 2.5. Here, the low price is clearly below zero several times, indicating that negative-price transactions have occurred for several delivery products.

- **Correlations between sequential markets:** For a producer with the opportunity of trading in the DA, ID and BM markets, the price at gate closure of one market should reflect the expectation of the price in the next market, as that will be the alternative cost to trading in the first market. Therefore, initial ID prices are likely to start out close to the

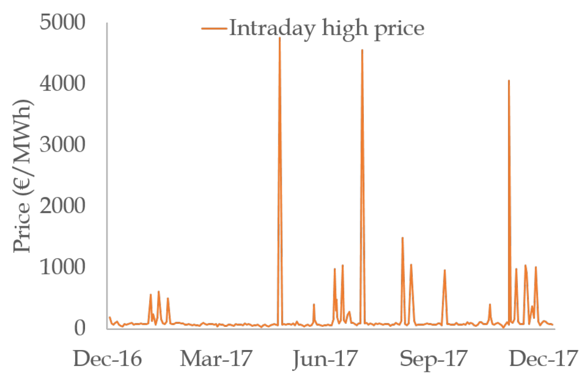



Figure 2.4: The prices of the highest clearing bid of the German Intraday market for each day in the period 2016-11-14 - 2017-11-13 (EPEX, 2017f)


Figure 2.5: The lowest transaction prices of each delivery product on the EPEX Intraday market for the period 2017-10-27 - 2017-10-28 (EPEX, 2017b)

DA price for the same delivery product. A similar argument could be used to argue that the prices of the last trades in the ID market should reflect the expectation of the imbalance price. However, imbalance prices are much more volatile than the other power prices, and a report from Wärtsilä (2014) states that "*it it is near impossible for market participants to manage their imbalance exposure in the spot markets on an informed basis*". Therefore, "*As the delivery horizon draws close, volatility increases and bid-ask spreads widen*" (Garnier and Madlener, 2015), in line with "*The conventional view put forward by market participants and experts is that there is limited visibility on the direction or magnitude of balancing costs and hence the surest way to controlling costs is to minimise the imbalance*"(Ruhnau et al., 2015). For a producer with flexibility, it may be more profitable on expectation to wait until inflexible producers become stressed by the approaching Gate Closure, but it also entails higher risk. These trends are further illustrated in figure 2.6.

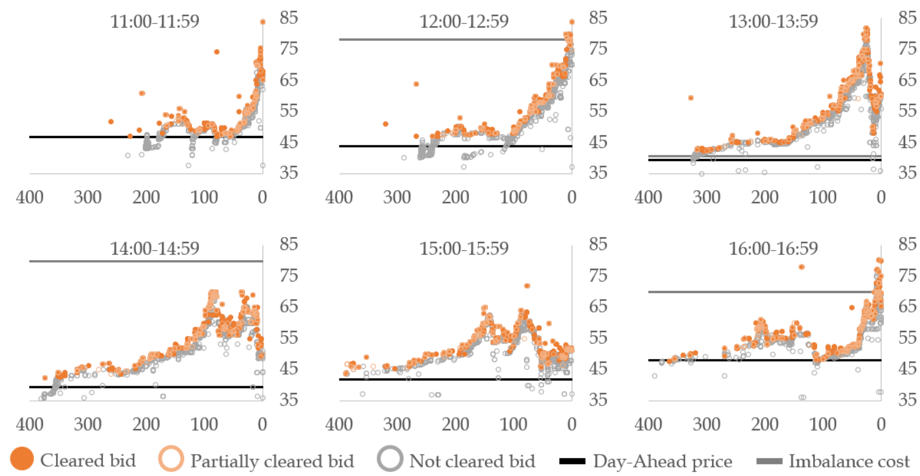



Figure 2.6: The prices of bids for selected delivery products in the EPEX Intraday market 2017-12-21, with minutes until gate closure on the x-axis and prices in €/MWh on the y-axis. The imbalance prices at 11:00 and 15:00 are off the charts, at 187€/MWh and 159€/MWh respectively.

Panagiotelis and Smith (2008) has a slightly more exhaustive list of price features, along with a list of references to papers that cover each topic more closely.

Some statistical features based on orderbook data received from EPEX are presented in this paragraph. The data contains the complete EPEX Intraday orderbook for 2016-12-21. It should be stressed that the empirical foundation of these analyses is rather weak and that the findings should be treated thereafter. However, as the results are relatively unambiguous, they are expected to provide some useful insights. From figure 2.7, one can infer that bid volumes are centered around fives. 19% of Intraday transactions are created during the last 15 minutes before gate closure, according to figure 2.8. From figure 2.9, we see that the most transactions are created within short time after bids were placed on this day. As can be seen from figure 2.10, the volatility of the transaction prices increase as gate closure approaches, in line with the findings of Garnier and Madlener (2015). Also, there are significant *autoregressive properties* along the trading time; if the price increased between the two last time steps, it is likely to increase again. At some time steps (e.g. 11:00 and 12:00) the direction of the change in the price of all delivery products suddenly flips. Presumably, this is due to Urgent Market Messages or updated weather forecasts shifting the forecasted residual demand for all traders in the market. When considering figure 2.10, please note that gate closure was 90 minutes before delivery on 2016-12-21. These findings will have profound consequences for later modelling choices.

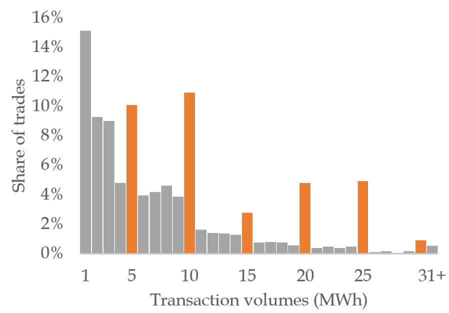



Figure 2.7: Distribution of bid volumes 2016-12-21.

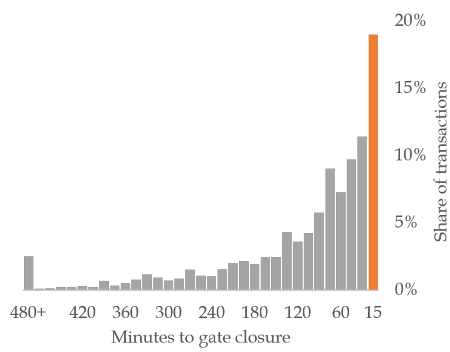



Figure 2.8: Distribution of transaction creation times relative to gate closure 2016-12-21.

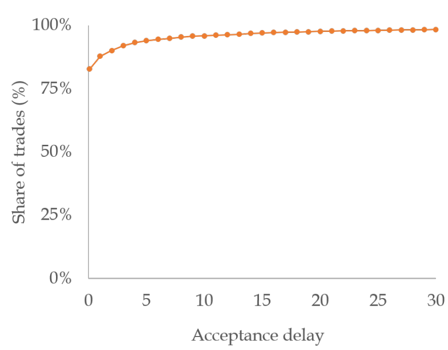



Figure 2.9: Aggregated distribution of transaction creation times relative to time of bid placement 2016-12-21.

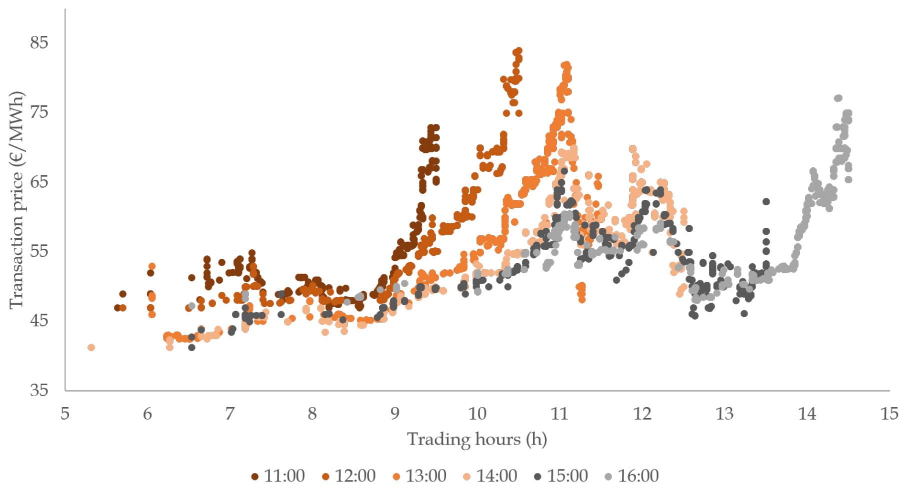



Figure 2.10: Transaction prices for six consecutive delivery products, 2016-12-21.

## 2.3 Power market participants

On the EPEX webpage there's an overview of the main actors (EPEX, 2017j) in the spot power markets, as well as a list of requirements for becoming a member (EPEX, 2017k). In the following paragraphs, all of the main groups of market participants are briefly covered.

**TSOs**, or transmission system operators, are responsible for the transmission of high-voltage power in a certain region, referred to as a control area or control zone. There are four such control areas in Germany, operated by TransnetBW, Tennet TSO, Amprion and 50 Hertz Transmission. One of the responsibilities of the TSOs is to prevent congestions in the grid, so the control areas preferrably have few internal bottlenecks. If there's no available transmission capacity between two control areas, a producer in one zone cannot sell production to a producer in another zone.

**Utilities and Aggregators** represent the major sources and sinks in the power system, respectively; utilities own and operate power plants based on either intermittent, hydropower or thermal production; whereas aggregators represent a group of consumers with flexible demand that allows for load shedding. One example of a utility that trades in the EPEX Spot markets is the Swiss hydropower producer Alpiq.

**Local suppliers** are retailers of power who purchase from wholesalers such as power utilities and sell subscriptions to individual or groups of consumers. In

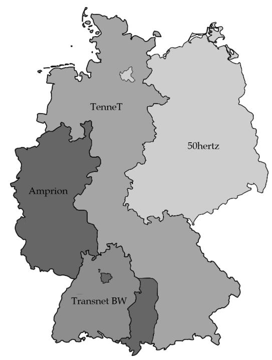



Figure 2.11: The control areas of Germany (Robinius et al., 2017)

contrast to utilities and aggregators the suppliers don't represent major sources or sinks of power; rather, they mainly concern themselves with the financial transactions and allocation of risk between other market participants.

**Purely financial traders**, similarly to traders in pure financial markets, are trying to profit from being able to predict price movements and exploit temporary arbitrage opportunities. By placing informed bids on future price developments their main function is to make the markets more efficient.

Utilities, aggregators, local suppliers and purely financial traders will for the remainder of this report be referred to collectively as *traders* in the ID market.

**Clearing Members** are responsible for the calculation of the physical and financial settlements as well as other tasks related to defaults and transactional risks in the power markets. Examples include banks like UBS, Santander and Goldman Sachs.

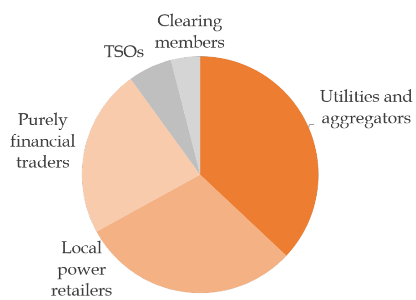



Figure 2.12: The members of the EPEX Spot markets.

## 2.4 Power market regulations

In this section, some of the more relevant regulations that traders in the EPEX Intraday Market needs to comply with are discussed. Specifically, market manipulation, insider trading in the light of *Urgent Market Messages* (UMMs), *Balance Responsible Parties* (BRPs), the *Order-to-Trade Ratio* (OTR) fine and *Mandatory Direct Marketing* (MDM) for renewable producers are discussed. These topics have been chosen for their relevance to the mathematical model presented in chapter 5.

According to the EPEX Code of Conduct (EPEX, 2017c), "*Any engagement in, or attempt to engage in, market manipulation on a Physical Power Contract is prohibited.*". Market manipulation includes, but is not limited to, both cooperative collusion and price fixing behavior. The second part is the most relevant for this report. In the model presented in chapter 5, this is implemented with the assumption that bids don't have long-term price impact.

In the same document, inside information is defined as non-public information that "*would be likely to significantly affect prices*". One example of this is if the availability status of a generator unexpectedly changes - for instance, if the generator breaks down. In this case, an Urgent Market Message is created, and no trading can be performed until the UMM has been published.

In the EPEX Operational Rules (EPEX, 2017h), Balance Responsible Parties are responsible for the delivery of physical power. As stated in Doorman et al. (2011), "*All market participants interact with the market through a (...) BRP, or are a BRP themselves (...) The BRP is generally obliged to try to (...) comply with this balance in real time*". "This balance" refers to that the *Balancing Group* (BG) that the BRP manages should have a total production equal to its total commitment from the DA and ID markets. Furthermore, the paper explains that it is BRPs, and not market participants directly, that pay the imbalance price if the BG as a whole is imbalanced. The BRP will usually organize in such a way that these costs are transferred to the individual market participants in it, however.

The Operational Rules also define the Order-to-Trade Ratio, which is a fine for placing an illegally high number of unmatched bids, to prevent traders from spamming the market with bids. If the number of bids per transaction for a given delivery product over the entire trading window is larger than 50, the trader is alerted. If no bids clear, the OTR is defined as the number of bids. The first four alerts every month are free of charge, after which each OTR costs 100€. While not saying so explicitly, the Operational Rules give the impression that persistently extreme OTRs may cause loss of market access.

The *Erneuerbare-Energien-Gesetz* (EEG) 2014 (Eng: German Renewable Energy Sources Act 2014) (BMWI, 2014) "(...) *made direct marketing mandatory*

(...) *establishing the sliding market premium* (...)" (Purkus et al., 2015). Thus, subsidies are granted to renewable producers for marketing their energy in the spot markets directly. In contrast, the older feed-in tariff was a function of the energy production, regardless of how it was marketed. Thus, incentives for renewable producers are largely similar to incentives for other producers; they simply receive an additional premium for the energy they sell in the DA and ID market. According to the EEG 2014, hydropower (including wave, tidal, salinity gradient and current energy), wind energy, solar radiation energy, geothermal energy or energy from biomass is counted as renewable energy BMWI (2014). This market premium is later referred to as subsidy costs.

## 2.5 Trading in the EPEX Intraday market

The decisions of a trading actor in the EPEX Intraday market include determining the number of bids to open and close at every step in time and some characteristics of every bid including price and volume. In the following section, the most relevant decisions of placing a bid are presented, as well as an overview of the costs related to Intraday trading and description of how the bid clearing process operates. Finally, a decomposition of the Intraday Trading Problem is presented.

### 2.5.1 Placing a bid in the EPEX Intraday market

When placing a bid in the EPEX Intraday market, there are several things to consider (EPEX, 2017g):

- **Volume.** The smallest volume increment is 0.1 MWh. Volumes
- **Price.** The smallest price increment is 0.1€ per MWh. Prices are bounded from below at −9999€ and from above at 9999€.
- **Timing.** As the orderbook prices are rather volatile, the timing of the bid is a critical factor to determine at what price a certain bid will be cleared.
- **Blocks.** It is possible to place multiple bids (blocks) that an eventual buyer will have to buy combined.
- **Trading window and access to regulatory areas.** Gate opening is at 15:00 the day before the power delivery. From this point in time until one hour before the time of delivery, any bid can be matched against any other bid from countries with coupled borders (EPEX, 2017a). That is, all bids placed in a German control area can be matched against any other bid from Germany, Switzerland, Austria or France within this timeframe. After this, bids can still be matched against other bids placed in the German market until 30 minutes before time of delivery. Bids from the same control area can be matched until 5 minutes before delivery. This is summarized in figure 2.13. Note that in this figure, the time of delivery is denoted $\bar{t}_{\tau}^{D}$ with $\tau$ describing a delivery product.


Figure 2.13: Trading windows and access to regulatory areas

- **Spread.** In the case of bid matching, if the ask price is higher than the buy price, then the transaction price will be set equal to the price of the earliest-placed bid among the two. Thus, depending on the order of bid placement, the seller or the buyer will be rewarded the spread.

- **Execution restrictions.** There are two axes of execution restrictions: forcing immediate cancel and allowing partial filling. This results in a total of four different execution restriction combinations. Among the combinations mentioned in table 2.1 the regular bids are by far the most used, measured in number of bids and total bid volume. Note that AON-bids only apply to block bids, and not bids for single delivery products.

Table 2.1: Bid execution restrictions

| | Partial filling | Complete filling |
|---|---|---|
| Instant matching only | Immediate-Or-Cancel (IOC) | Fill-Or-Kill (FOK) |
| Non-instant matching | Regular bids | All-Or-None (AON) |

### 2.5.2 Costs of trading in the EPEX Intraday market

When trading in the EPEX Intraday market, a diverse set of relevant costs are present. In this section, a guide to the most relevant costs is provided. The cost types and their approximate magnitudes are summarized in table 2.2. It should be stressed here that the cost types will differ in magnitude from trader to trader, so the magnitudes of the table may not be accurate for one specific trader. However, it is included here to give an approximate overview.The imbalance cost is the cost of trading in the Balancing market. The production cost is the cost of producing energy. The production cost is considered high for thermal producers and lower for producers of intermittent energy and hydropower. Ramping costs related to the changes in production volume from one time period to another are included in the production cost and are in general of a higher magnitude for thermal producers than intermittent producers (where they are absent).The volume cost and the market saturation cost are both related to the market impact of placing a bid. The volume cost is the short term market impact and arises due to the negative slope of the revealed residual demand curve. It represents the negative effect a transaction has on the revealed residual demand curve. The market saturation cost is the long term market impact, and is caused by strategic adaptation by traders who observe additional demand. The magnitudes of these cost types depend on the size of the total trading volume of the trader and the size of the bid. As explained in section 2.4, the OTR fine is a fine for placing an illegal number of unmatched bids. The transaction cost and subsidy cost are both connected to transactions. Whereas transaction cost is a fixed fee for matching bids, the subsidy cost is actually a premium for intermittent energy producers occurring when their bids are matched. Thus, the subsidy cost can be understood as a negative cost (having positive profit impact), operating as a shift of the transaction cost of intermittent producers in the opposite direction.

Table 2.2: The costs of a trading power producer

| **Type** | **Magnitude** |
|---|---|
| Imbalance cost | High |
| Production cost | High/Low |
| Volume cost | High/Low |
| Market saturation cost | High/Low |
| Order-to-Trade-Ratio fine | Low |
| Transaction cost | Low |
| Subsidy cost | Low/- |

#### 2.5.2.1 Production costs

Depending on e.g. the choice of technology (intermittent, hydropower, thermal) and the number of assets (single-asset vs. multi-asset), the production cost drivers will vary. A brief overview of the cost drivers associated with different technologies are presented in table 2.3:

Table 2.3: The main production cost drivers for different classes of power producers.

| **Production cost drivers** | **Intermittent** | **Hydro** | **Thermal** |
|---|---|---|---|
| Value of Storage | (No) | (Yes) | No |
| Start up/shut down | No | Yes | Yes |
| Ramp up/ramp down | No | (No) | Yes |
| Fuel | No | No | Yes |

In the table above, parentheses are used to indicate that something is usually but not necessarily true; hydropower plants often have a co-located reservoir; most hydropower plants don't have significant ramping costs; intermittent production frequently comes without co-located storage (though that may change if prices of batteries fall significantly).

Note that the Value of Storage contains assumptions about the short term production. As illustrated by Javanainen et al. (2005), the expected returns to extra storage are decreasing on the margin - or conversely, if more energy is sold in the short term than originally planned, the value per unit of the remaining storage increases.

### 2.5.3 Bid clearing

Whether an opened bid will be cleared or not and eventually what the corresponding transaction price will be like is non-trivial using mathematical terms only. This information can be obtained rather easily through examination and processing of the ID order book. There are several conditions needed to be met for a sell bid to be cleared. In general, one or more open buy bids must meet the following requirements for a transaction to occur. This logic clearly also resolves the problem of clearing buy bids.

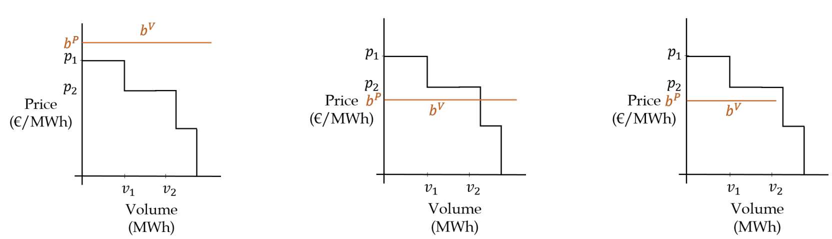
1. The control areas of the buy bid and that of the sell bid must be coupled at the time of an eventual transaction.

2. The buy bid price must not be smaller than the sell bid price

3. Among the buy bids meeting the above-mentioned requirements, the buy bid with the highest buy price will clear the sell bid. In general, if the buy bid volume is not as large as the sell bid volume, multiple buy bids will be considered to attempt to clear the sell bid.

4. The corresponding transaction price is equal to the price of the bid that was placed first. Thus, if multiple buy bids are needed to clear a sell bid, the clearing price of that bid will equal a weighted average of the transaction prices of the transactions needed to clear the sell bid.

Also, recall that bids with the execution restrictions of table 2.1 do not comply to the bid clearing process outlined above directly. In the following paragraph, a more precise description of how transaction price and volume are determined is provided.




(i) No matching (ii) Partial matching (iii) Complete matching

Figure 2.14: Potential market situations when a bid is placed.

Table 2.4: Section specific symbols

| Symbol | Interpretation |
|---|---|
| $b^V$ | Bid volume of the trader's sell bid |
| $b^P$ | Bid price of the trader's sell bid |
| $v_1, v_2$ | Volumes of demand bid 1 and 2 accessible to the trader |
| $p_1, p_2$ | Prices of demand bid 1 and 2 accessible to the trader |
| $v^C$ | Cleared volume of the trader's sell bid |
| $p^C$ | Cleared volume of the trader's sell bid |
| $\widetilde{b}^V$ | Residual demand after matching of trader's sell bid |

Here, a simplified version of the notation described in the Nomenclature (with less subscripts) is used in order to increase readability. The symbols are described in 2.4. Figure 2.14 shows the three potential residual demand situations when placing a sell bid (assuming no execution restriction). Here, the black line is the residual demand for a specific delivery product in a specific timeslot (buy bids) and the orange line is the trader's sell bid. $p_1, p_2, v_1$ and $v_2$ are prices and volumes of open buy bids at the time of bid placement. Here, the trader's bid is represented by a two-tuple $(b^P, b^V)$ indicating the bid's price and volume. In situation (i), the price of the trader's sell bid is greater than the price of all the revealed buy bids. Thus, no transaction will occur. In situation (ii), there exist indeed buy bids with price greater than the trader's sell bid. However, the total demanded volume at price equal to the sell bid's is smaller than the sell bid volume. Thus, only a share $v^C = v_1 + v_2$ of the sell bid will be matched $(v^C < b^V))$. The residual bid volume after having created the initial matching is thus $\widetilde{b}^V = b^V - v^C$. The price of the transactions involving the trader's bid will be $p^C = p_1 v_1 + p_2(v_2 - v_1)$. In situation (iii), the total buy bid volume for the price $b^V$ is large enough to match the entire bid volume of the trader's bid. Thus, $v^C = b^V$ and $p^C = p_1 v_1 + p_2(b^V - v_1) = p_1 v_1 + p_2(v_2 - v_1)$ as in the second situation. In situation (ii) and (iii), in general, more than two buy bids can be required for the sell bid to clear. When new buy bids appear in later timeslots, the price of eventual transactions involving the sell bid, the transaction price will equal $b^P$. The resulting transactions of such situations can easily be generalized from the formulas above. Also, similar reasoning as above can be applied to model bid clearing of buy bids. This will not be covered here.

## 2.6 Automated trading in the Intraday power market

For the reasons outlined in the previous sections, it is both expensive and hard to trade near-optimally in the Intraday power market. To ameliorate this, important market actors are investigating opportunities for algorithmic trading. In the following sections, both requirements to autonomous trading algorithms and some potential approaches to solving the Automated Intraday Trading Problem are presented.

### 2.6.1 Requirements to an autonomous trading algorithm

Below, the requirements that an autonomous trading algorithm would have to satisfy are outlined:

- The decisions of the algorithm...
    - Must on average be as good as that of an average human trader.
    - Must in the worst case have some minimum boundary, relative to either the performance of an average human trader, how the rest of the market performs or the theoretic optimum.
    - Must adapt to a market with changing dynamics over time.
    - Must not be hackable or manipulable by adversaries.
    - Should scale to a stable equilibrium if adopted by a significant portion of the market.
    - Should improve as the algorithm receives more and better data.
    - Should be possible to improve through modular updates of the optimization algorithm.
    - Should be backed up with a clear rationale, provided by the algorithm automatically.
- Customization for individual...
    - cost curves must be provided.
    - risk preferences should be provided.
- Operationally, the algorithm...
    - Must take at most a few minutes to run per time step, preferably less than a minute. Recall that the majority of trades happen shortly after each of the involved bids are placed, as suggested by figure 2.9.
    - Should be accurate for all delivery products throughout the entire year or at least be able to alert the user if it is unusually uncertain about a proposed solution.
    - Should require only moderate amounts of computing power.
    - Should be able to execute the output solutions independently of a human trader.
- Compliance-wise, the algorithm should follow the rules of the EPEX Intraday market.

### 2.6.2 Approaches to automate trading

When addressing a large and complicated problem, the first step is to explore which high-level approaches may provide an answer, and what the state of the art is:

- What are the available approaches to solving the Automated Intraday Trading Problem?
- For the chosen approach, to what degree can the problem be decomposed into well defined optimization problems?
- For each of these problems, what are the best state-of-the-art algorithms and how well do they perform?
- Would the best state-of-the-art optimization algorithms for the different problems be able to communicate and coordinate in order to solve the AITP, or not?
- What is the nature of the open research questions that needs to be solved in order to create better working optimization algorithms for each of the parts of the AITP, and what progress can be expected in that regard?

To answer the first question immediately, there are at least four approaches that potentially could lead to the long term goal described above:

1. **The Optimization Modelling Approach:** Firstly, the market can be modelled based on theory and empirical data, combining techniques from machine learning, operations research and microeconomics to produce a classic optimization model. This is the approach that is considered further throughout the rest of this report.
2. **The Predefined Strategies Approach:** Alternatively, rule-based predefined strategies could be developed by domain experts. Later, these strategies could be incrementally improved using known heuristics to explore the decision space around the given strategies. A toy example would be "if the ID price is 10% above the DA price for a given delivery product, place a bid with 15% of your open position". For the toy example, the local search algorithm would fine-tune the decision boundaries (10%) and amplitudes (15%). While this approach has several similarities to the first approach, the major difference is that no explicit (empirical or theoretical) optimization model is needed to construct the initial predefined strategies. It is uncertain if this approach would be able to effectively explore sufficient regions of the decision space to eventually arrive at a near-optimal solution, as the set of predefined strategies would necessarily be smaller and less flexible than the policies a dynamic programming approach could yield. Powel is currently working on such a solution, separate from this report (Powel, 2017a).

3. **The Self-Playing Algorithm Approach:** In principle, an algorithm playing against different versions of itself in a simulation of the ID market could learn to trade at superhuman level, even without the explicit modelling of price forecasts from approach 1 or the initial predefined strategies of approach 2. This is currently being demonstrated for other games with similarly demanding game structures, for instance Go (DeepMind, 2017). As these breakthroughs are very recent, and the ID market has some difficulties that Go does not, it is unlikely that anyone are attempting this approach for the ID market yet.

4. **The Imitation Game Approach:** Finally, a prediction algorithm could be programmed to guess the actions that one or several human traders would take in a given situation, given access to the same information. This approach assumes that the pattern of how (some average of) one or several human traders act is near-optimal for a machine trader, or at least sufficient as a starting point for incremental improvement heuristics. However, this may not be the case if there are conceptual differences in how humans and machines trade well; for instance, in traditional financial markets high-frequency trading is a form of exploiting very short-term arbitrage opportunities that is infeasible for humans but common for trader bots. To the best knowledge of the authors, this approach is not currently being pursued.

In the following sections, approach 1 is explored further. This is chosen for the following reasons: most of the existing scientific literature related to trading in the Intraday market focuses on the same approach, as covered in chapter 3. In this literature, the approach has a track record of working fairly well, unlike approaches 3 and 4 which to the best knowledge of the authors have never been applied to Intraday power markets. The approach allows for more flexible and dynamic strategies than approach 2, and is therefore likely to improve with more research. Finally, it is more relevant to the study program of the authors of this report than approach 3 and 4. In the end, a version of approach 4 is proposed as a possible augmentation of approach one, providing substantial speedup in the trading velocity.

### 2.6.3 Decomposing the Intraday Trading Problem

For the rest of the report, the ITP is decomposed into three parts:

1. *The Price Forecasting Problem* (PFP) is about predicting the future aspects of the price.

2. *The Cost Estimation Problem* (CEP) considers the marginal cost of production for any power producer.

3. *The Strategy Formulation Problem* (SFP) takes the marginal cost curve and the price forecast as input parameters, and formulates an optimal bidding strategy dynamically.

As the price the trader can expect to get in the market will be a function of the desired trade volume, the Price Forecasting Problem could also be thought of as a residual demand forecasting problem. The alternative formulation makes it obvious how the decomposition follows the logic of standard microeconomics, where the goal of the Price Forecasting Problem is to predict a demand curve, the Cost Estimation Problem attempts to establish a supply curve, and the Strategy Formulation Problem is concerned with finding a policy for converging to the equilibrium. Going forward, in order to find a solution to the ITP, most emphasis is placed on the SFP in the report. The SFP is considered the core of the ITP, with PFP and CEP playing important supporting roles, providing necessary input parameters to the SFP.

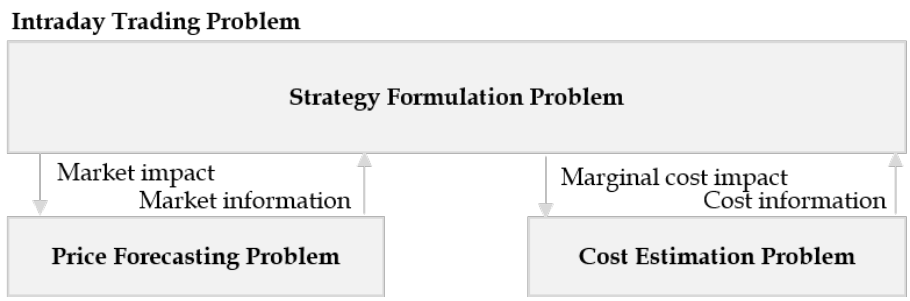



Figure 2.15: Intraday Trading Problem breakdown

Figure 2.15 visualizes the interaction between the abovementioned components of the problem. This interaction should be pseudo-continuous throughout the trading windows for the algorithm to function properly. The flow of information between the three components of the problem is thoroughly discovered in chapter 4. As seen in the figure, the Strategy Formulation Problem is the key issue, and this is covered more in-depth than the two other problems for the remainder of the report.

# 3 Related literature

This chapter contains a review of the most relevant literature of the different components of the Intraday Trading Problem. In section 3.1, papers focusing on the prediction of different price features are covered. There is a rich literature on estimating the variable costs of production, and section 3.2 lists some of these papers without going into the details of each paper. The main causes for complexity in this domain are also described. As the Strategy Formulation Problem is covered the most in-depth later in this report, section 3.3 is a more thorough investigation, and the limitations of the existing papers are explored further.

It is observed that each of the individual SFP papers solve only parts of the AITP, or focus on specific niche cases. No paper solves the AITP in its generality yet, even heuristically.

## 3.1 The Price Forecasting Problem

Most of the literature on electricity price forecasting focuses on the DA market - see for instance Weron (2014) and Aggarwal et al. (2008) for good overviews covering a range of statistical, fundamental, cognitive, game-theoretical and closed-form methods. Also, Nowotarski et al. (2014) focus on combining different DA price forecasts into one improved DA price forecast, which may (partly) translate to the ID market. Of the papers that focus on the ID market, some are concerned solely about predicting or describing features of the price, whereas others also formulate a strategy for trading in the ID market. The latter category is covered in section 3.3. Of the papers that focus only on price forecasts and/or descriptions in the ID market, some focus on very specific situations. For instance, Lazarczyk (2016) explores the impact of UMMs on the ID price, and finds that production outages have the largest impact on the ID premium versus the DA price. Kim (2013) focuses on the effect of special days such as the Easter holidays on the ID price, and develops a SARMA model that achieve a mean absolute percentage error (MAPE) of 2%. For later versions of the model proposed in chapter 5 these aspects may be relevant, but they will not be discussed further here.

Other papers take a more holistic view, and ask which approach is the best suited for ID price forecasting in general. Pape et al. (2016) attempt to forecast the Intraday price solely based on market fundamentals such as transmission congestions or changes in load or production capacity, and shows that while fundamentals explain most of the variance in the Intraday price, forecasting based on fundamentals only struggles to compete with alternative models. Malic (2017) builds on this, citing residual demand as a main driver of Intraday price fluctuations. Using a similar logic, Kiesel and Paraschiv (2017) find that reduced form-equilibrium models perform even better than statistical forecasts, but this contrasts with the earlier advice of Weron (2006). Notably, Weron is mostly focused on the DA market, and there may be fundamental reasons for the different performance of different models in the two markets. One reason may be that Kiesel and Paraschiv (2017) focuses on the prediction of the *price of one delivery product in the next time increment* based on the price history of the same delivery product, whereas DA price forecasting must forecast the prices of the 24 next delivery products based on the prices of other delivery products.

Another (older) example of the application of microeconomics in power markets forecasting is Ugedo et al. (2003), which uses decision trees to build stochastic residual demand curves for the Spanish spot market. Engmark and Sandven (2017) follows a resembling logic, in the sense that they consider the market price to be a function of the desired volume of a bid. This thesis is described in more detail in section 3.3.

## 3.2 The Cost Estimation Problem

The Cost Estimation Problem considers the variable cost of production for any power producer, as well as forecasting imbalance prices. The goal is to accurately evaluate the variable costs of production for each delivery product, as a function of the production decisions. As the Cost Estimation Problem is relevant in all kinds of power markets, it has already been substantially researched. Also, the specific cost curve will depend on the production asset mix of the individual producer, making it hard to generalize to something that is relevant for the entire market. It is therefore not the main focus of this report. However, the state of the art techniques for solving the abovementioned problems are outlined in this section.

Please note that of the costs outlined in table 2.2, volume cost is a function of the trading strategy and therefore it is more naturally handled in the Strategy Formulation Problem. Market saturation costs refer to changes in price forecasts and is therefore most relevant for the Price Forecasting Problem. The OTR fine, transaction costs and renewable premium are all well defined, and estimating them are therefore not an issue. Imbalance price forecasting is included here, as market actors typically see the balancing market as a source of costs; for instance Wärtsilä (2014) uses the formulation "*market participants who avoid imbalance should not incur any costs of balancing*" when describing incentives in the design of imbalance prices. Thus, only production costs and imbalance cost are considered here.

The Cost Estimation Problem complicates the overall ITP in at least three ways:

- The alternative costs of production must be found. For intermittents, this boils down to forecasting the available production capacity and the imbalance prices; errors in the production capacity forecast may incur imbalance costs as well as lost profits from the ID market. For production with storage capacity, such as hydropower or intermittent production with batteries, the value of stored energy (called Water Value for hydropower producers) must be considered. For thermal producers, the future cost of purchasing fuel is the most relevant.

- For multi-asset producers, the cheapest combination of committed units must be found for any volume. Assets typically have non-linear efficiency curves, making the unit commitment problem non-convex.

- Some assets have *intertemporal costs* like start up/shut down costs and ramp up/-down costs. These costs create path dependencies (Wikipedia, 2017p) between production decisions for different delivery products. The path dependencies become especially troublesome when future production is stochastic, for instance due to potential loss of generators or late trades in the ID market when ID trade volumes are price dependent (as they should be (Rudlang et al., 2014)).

### 3.2.1 Alternative costs of production

Note that while imbalance costs technically aren't production cost, it is highly relevant for producers with low flexibility. As was explained in section 2.2, this is very hard to forecast in practice. As the ID price and imbalance price are different, the forecasting error cost for inflexible traders is asymmetrical; they are facing a *Newsvendor problem* as originally described and solved in (Arrow et al., 1951). Since then, the Newsvendor problem has been extensively studied and solved, even for significantly harder problems (such as when the distribution is unknown (Negoescu et al., 2017), and references therein).

For producers with co-located storage, the storage provides an opportunity to arbitrage differences in power prices across delivery products; short term arbitrage is possible for producers with low storage capacity such as solar producers with battery farms, and arbitrage over multiple time lines applies for producers with high storage capacity such as large hydropower reservoirs. As the alternative cost to producing now will equal the best expected value of production at a later point in a stochastic future, finding the optimal alternative cost of stored energy is typically solved with stochastic dynamic programming. The value of storage is then "*the shadow price (...) i.e. the Lagrange multipliers corresponding to the reservoir constraints*" (Javanainen et al., 2005). Rao et al. (2015) solves the problem of determining the optimal Value of Storage for intermittent production with a co-located battery farm, citing Jiang and Powell (2015a) and a handful of other papers on the same topic. Efthymoglou (1987), Javanainen et al. (2005), Abgottspon et al. (2014) and many others solve the equivalent problem for short-term hydropower optimization, and the same techniques have been used successfully for longer planning horizons too.

For fuel-based power production, the short-term volatility in the fuel commodity markets are lower than for electricity (Malic, 2017), and the existence of long term fuel supply contracts and hedging opportunities reduces the uncertainty in the short term. The calculation of fuel costs for Intraday trading purposes are therefore quite straightforward.

### 3.2.2 Stochastic multi-asset unit commitment with intertemporal costs

The issues related to intertemporal costs and non-linear efficiency curves are often solved together in what is called the Unit Commitment Problem. The Unit Commitment Problem considers the optimal production schedule on a per-generator basis for a given aggregated generation schedule, and different versions apply for producers in regulated and deregulated markets. As binary variables are necessary to keep track of which generators are on or off, and non-linear efficiency curves cause non-convexity even in the continuous relaxation of the problem, large instances of the problem are slow to converge to optimality. Current research is therefore based on either simplified model formulations that makes it possible to solve the model to optimality, and/or heuristic-based solution methods. Sample papers covering these issues for hydropower producers include Skjelbred et al. (2017), Belsnes et al. (2016), Löhndorf et al. (2013) and Faghih et al. (2011), while Pang and Chen (1976), Ostrowski et al. (2012), Trivedi et al. (2012) and Bertsimas et al. (2013) focus on different versions (robust, multiobjective, ramping constrained etc.) for thermal producers. Tahanan et al. (2015) - and again several others - solves the Unit Commitment Problem for multi-asset producers with a combination of hydropower and thermal power.

To conclude, for all the parts of the Cost Estimation Problem outlined in this chapter, a significant literature exists on how to tackle them. Also, cost drivers for different production technologies are heterogeneous, reducing the potential for generalization across the entire market. While the Cost Estimation Problem hasn't been solved to optimality yet, it should not be the main focus of this report, as other issues related to Intraday trading are relatively more underexplored, generalizable, and relevant. Specifically, the Strategy Formulation Problem covered in the following section is covered more in depth in this report.

## 3.3 The Strategy Formulation Problem

As mentioned in chapter 1, there are two axes of time that needs to be considered in the ITP: the *trading time* and the *production time*. Figure 2.6 shows the price time series for six delivery products. The time on the x-axis of these charts is the trading time. However, each of the six charts is also labeled with the production time slot for the given delivery product. If production decisions for consecutive delivery products are interconnected, as the intertemporal costs and Value of Storage in the Cost Estimation Problem shows that they are, then the dynamics in production time are relevant too. Figure 3.1 shows the trading time on the x-axis and production time on the y-axis.

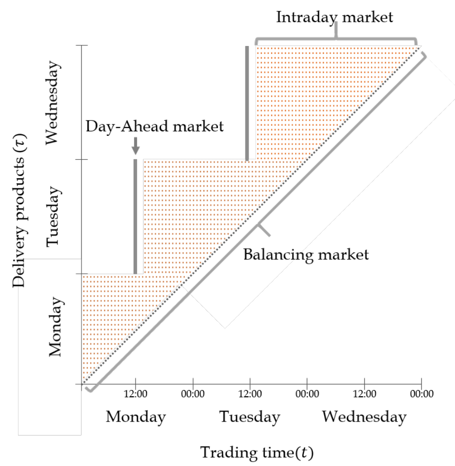



Figure 3.1: Trading time and production time of 72 delivery products inspired by Belsnes et al. (2016)

In this section, papers are categorized by whether they consider dynamics in trading time or production time. The papers are referred to as either $t$-dynamic or $\tau$-dynamic, respectively. Three categories of aspects with the papers are considered; how they model the decisions of the trader, the assumptions they make about prices and trading, and how they model production constraints and costs. Finally, the key aspects in this section are summarized in table 3.1a and 3.1b.

The $t$-dynamic papers include Garnier and Madlener (2015), Aid et al. (2015), Tan and Tankov (2016) and Edoli et al. (2016). These papers solve an Order Execution Problem as described by Almgren and Chriss (2001). While the findings don't include all the necessary information to trade optimally in practice, they can serve as heuristics for traders in the ID market.

Gönsch and Hassler (2016), Hassler (2017), Löhndorf et al. (2013), Farinelli and Tibiletti (2017) and Engmark and Sandven (2017) are among the more relevant $\tau$-dynamic papers. Zhou et al. (2016) and Jiang and Powell (2015a) also follow some of the same logic. At the core, each of these papers address versions of the Dynamic Resource Allocation Problem desribed in Powell et al. (2005). Again the observations in the papers fail to translate into implementable trading strategies directly, but the proposed algorithms may provide priors for how to distribute production resources throughout the day.

### 3.3.1 Decision making

The $\tau$-dynamic papers typically model the decision as a *dispatch plan*. That is, the question is how much power to produce for each delivery product. The reason for this is that they all focus on production with co-located storage, and the stored energy must be allocated to the delivery slots with the most demand. How to translate the dispatch plan into bid timing, size and price is rarely considered. Instead, only one decision is made per delivery product. The problem with this approach is that the production volume either needs to be set before the price trajectory of the Intraday Continuous auction is revealed, discarding relevant price information and contrasting the advice from Rudlang et al. (2014); or alternatively, a single bid can be placed near gate closure (as e.g. Hassler (2017) and Gönsch and Hassler (2016) do). The latter strategy comes with considerable risk as volatility increases near gate closure (Garnier and Madlener, 2015), and if the single bid is large it may have adverse market impacts.

Another weakness with the wait-for-others-to-bid-strategy is that *doesn't scale to a Nash equilibrium* (Wikipedia, 2017l); that is, if a significant proportion of the market adopted the strategy of waiting for trades to happen in the Intraday market before placing their bids, no bids are placed. If no bids are placed, an obvious way to increase profits is to deviate from the strategy and place a bid (with a preposterous price) - since there's no competition for demand as everyone else is on the fence waiting for price signals. As it is obviously profitable to not adopt the strategy if others adopt it, few will end up adopting that strategy.

The exception in this case is Engmark and Sandven (2017) which models the decision as the quantities and prices of several bids, although the timing of the bidding is still left out. Engmark and Sandven (2017) also state explicitly that the model should be run several times throughout the day to be able to make increasingly informed decisions, partly accounting for the double time dynamic. Only information about the BM is updated however; the original ID scenarios are used throughout the entire day.

Almost all of the $t$-dynamic papers model the decision as the optimal *trading rate* - that is, volume traded per time step - for one given delivery product, typically assuming that the dispatch decision for the delivery product has already been made. To reiterate; trading volumes should be price dependent (Rudlang et al., 2014). One exception to the predetermined dispatch plan is Aid et al. (2015), which states that "*the optimal strategy consists in making at each time the forecast marginal cost equal to the forecast Intraday price*", per classical microeconomics. An "open position" is thus not a deviation between committed production and estimated production, but rather a deviation from the equilibrium between marginal costs and -revenues, which can be closed through trading. The same paper also argues that for *martingale prices* (a martingale is a time series for which the average change cannot be predicted; see (Wikipedia, 2017k)), the optimal trading rate is also a martingale. This is a very interesting finding, which is applied as a prior in the proposed solution methods of section 6.3.2.1. Garnier and Madlener (2015) is an exception when it comes to the use of trading rate, as the decision is modeled as the quantity and timing of discrete bids instead of a continuous rate.

All $t$-dynamic papers make the simplification that decisions for separate delivery products are independent. For Garnier and Madlener (2015) and Tan and Tankov (2016) this assumption may hold, as they consider how to balance forecasting errors for intermittent production without storage. However, the assumption does not generalize to producers with storage, intertemporal costs and ramping constraints.

Both Hassler (2017) and Garnier and Madlener (2015) attempt to avoid the added complexity of allowing for active use of balancing markets. The former claims that this doesn't scale to a Nash Equilibrium as it would destabilize the grid. Incremental increase in the demand for balancing services would however increase the price of using them, making it less profitable to rely on the balancing market. It is therefore not clear to the authors of this report why such a strategy not would converge to an equilibrium. Garnier and Madlener (2015) on the other hand claims that an over-adjustment to an open position of the market amounts to balancing market speculation, whereas failing to close the open position does not. Therefore, the residual for the balancing market should always be in the same direction as the original forecast error. It is unclear why the use of the BM in one direction is acceptable, whereas use in the other direction amounts to illegal speculation, ceteris paribus. To summarize, both papers fail to convincingly argue why use of the BM cannot simply be limited by placing a premium on the imbalance cost in the goal function, and cap the maximum volume traded there.

### 3.3.2 Price dynamics and trading

Of the $t$-dynamic papers, all but Aid et al. (2015) assume some form of Gaussian process; Garnier and Madlener (2015) and Tan and Tankov (2016) assume a Geometric Brownian Motion (GBM) (Wikipedia, 2017a), whereas Edoli et al. (2016) uses an Ornstein-Uhlenbeck-dynamic (OU-dynamic) (Wikipedia, 2017m) to capture the speed of the mean reversion. All of these modeling choices are convenient choices that capture the skewness (Wikipedia, 2017q) and kurtosis (Wikipedia, 2017i) of the Intraday price (Malic, 2017). On the other hand, the lack of variation in price assumptions raises the question of how the models perform for other price dynamics. Aid et al. (2015) make fewer assumptions about the price dynamics, and focus on deriving more high-level insights in the cases where the price is a martingale or sub-/super-martingale.

Similarly to the difference between Kiesel and Paraschiv (2017) and Weron (2006), the $\tau$-dynamic papers focus on forecasting the prices of some delivery products based on the prices of other delivery products, unlike the $t$-dynamic papers that focus on the price dynamics of a single delivery product. For this purpose, a larger variety of techniques are applied; some assume Gaussian processes (Wikipedia, 2017g) ((Gönsch and Hassler, 2016), (Farinelli and Tibiletti, 2017)), others use autoregressive techniques ((Hassler, 2017), (Engmark and Sandven, 2017)) and Löhndorf et al. (2013) uses a fundamental model that takes weather forecasts as input parameters.

Almost all of the papers in both categories assume that bids are certain to clear, or avoid the topic of bid clearing altogether. This is equivalent to only matching existing bids, and never placing a bid above the revealed residual demand curve; if a bid is placed above the current price of the residual demand, it is uncertain if the bid will clear (Skajaa et al. (2015) is the only paper that explicitly only matches existing bids). The problem with this approach is that access to the residual demand that has not yet been revealed in the form of bids is lost - if a potential counterparty is following the same strategy, no trade will take place even if the parties could have agreed on a mutually beneficial price, since none will bid first. It is therefore obvious that this strategy doesn't scale to a Nash Equilibrium - if both sides are waiting for bids to match, no bids are placed in the market. Engmark and Sandven (2017) and Garnier and Madlener (2015) present a large improvement in this regard, as both papers explicitly consider stochastic bid clearing. However, neither of the papers follow the logical conclusion from this that it should be possible to kill unmatched bids. Therefore, unmatched bids present a risk as they may be matched at a later point, causing an unintentional over-adjustment to an open position. For this reason, bid killing is a necessary option in a model with stochastic bid clearing.

### 3.3.3 Production and cost structure

As most papers in both categories focus on renewable production, the marginal production costs are near-zero. Garnier and Madlener (2015), Tan and Tankov (2016) and Edoli et al. (2016) therefore don't mention costs altogether, whereas the $\tau$-dynamic papers focus on the alternative cost of the VoS. Aid et al. (2015) is the only paper with thermal production, and quadratic production cost is assumed. The same paper also accounts for the time lag between commitment and production of energy, along with Gönsch and Hassler (2016) and Hassler (2017). Aid et al. (2015) states that the problem with the time lag restriction can be transformed into one without it, though.

No paper accounts for renewable premiums, intertemporal costs, non-convexities in the production cost function or OTR fines. Some papers add explicit costs for large bids ((Garnier and Madlener, 2015), (Gönsch and Hassler, 2016), (Engmark and Sandven, 2017)) and even long term price impacts ((Aid et al., 2015), (Tan and Tankov, 2016)), as well as imbalance costs and transaction costs. The former two are highly relevant in illiquid markets, but may decrease in importance if the market becomes more liquid. In sum, the cost assumptions simplify the models to the point where they are possible to solve without sacrificing much accuracy, but for each paper the model only applies to one or two types of producers. Therefore, no single model is relevant for all producers. If the goal is to automate trading in practice rather than deriving interesting theoretical insights, the neglect of the OTR may be troublesome as an algorithm that spammed the market with bids would likely cause repercussions from the regulator. However, this is not a large issue as none of the former papers aspire to automate trading.

### 3.3.4 Conclusion

The key information of the papers described above have been summarized in tables 3.1a and 3.1b. All of the papers make significant contributions to the goal of deriving useful insights about optimized trading in the Intraday markets, but neither of the papers are sufficient if the goal is to automate Intraday trading. Therefore, the goal of this report is to construct an as realistic model of the trading process as possible.

Note that other papers than the abovementioned may initially look like they solve similar problems, while they in fact model quite different problems. For instance, this may be because they focus on Intraday Single Auction markets rather than the Continuous Auction, or because they consider multi-market resource allocation before the Day-Ahead Auction, with the Intraday market as one stage in a two stage- or multistage-problem. For this reason, they have not been covered here. Examples of such papers are Braun (2016), Braun and Hoffmann (2016), Tang et al. (2017), Ding et al. (2017) and Ayón et al. (2017).

Table 3.1a: Key features of the most relevant strategy formulation papers

| **Paper** | **Trader** | **Output variable** | **Decisions** | $\|\mathcal{T}\| > 1$**?** |
|---|---|---|---|---|
| Garnier & Madlener (2014) | Intermittent | Partial bid strategy | Sequential | No |
| Aïd et al. (2015) | Thermal+Wind | Trading rate | Sequential | No |
| Tan & Tankov (2016) | Wind | Trading rate | Sequential | No |
| Edoli et al. (2016) | [unspecified] | Trading rate | Sequential | No |
| Skajaa et al. (2017) | Wind | Matched bids | Sequential | No |
| Gönsch & Hassler (2016) | Intermittent+ | Dispatch plan | Single-stage | Yes |
| Hassler (2017) | Intermittent+ | Dispatch plan | Single-stage | Yes |
| Löhndorf et al. (2013) | Hydro | Dispatch plan | Single-stage | Yes |
| Farinelli & Tibiletti (2017) | Hydro | Dispatch plan | Single-stage | Yes |
| Engmark & Sandven (2017) | Hydro | Partial bid strategy | Hybrid | Yes |
| **This report** | **All** | **Bid strategy** | **Sequential** | **Yes** |

Table 3.1b: Key features of the most relevant strategy formulation papers

| **Paper** | **Solution** | **Market Power?** | **Price dynamics assumption** |
|---|---|---|---|
| Garnier & Madlener (2014) | Analytic | Short term only | Geometric Brownian Motion |
| Aïd et al. (2015) | Analytic | Yes | Martingale, (super- and sub-) |
| Tan & Tankov (2016) | Analytic | Yes | Geometric Brownian Motion |
| Edoli et al. (2016) | Analytic | No | Ornstein-Uhlenbeck (OU) dynamic |
| Skajaa et al. (2017) | Logic-based | Short term only | N/A |
| Gönsch & Hassler (2016) | Dynamic: ADP | No | Discrete OU-dynamic |
| Hassler (2017) | Dynamic: ADP | No | ARIMA |
| Löhndorf et al. (2013) | Dynamic: ADDP | No | Fundamental linear |
| Farinelli & Tibiletti (2017) | Dynamic w/ IPM | No | Standard Brownian Motion |
| Engmark & Sandven (2017) | SMIP | Short term only | ARMA*, RDC random draw |
| **This report** | **Dynamic: ADP** | **Short term only** | **Fundamental non-parametric** |

Here, table 3.1a and 3.1b are described more thoroughly. The "Trader" column describes which kinds of traders the research is relevant for. Intermittent+ means that the solar and/or wind power plant has co-located storage. Edoli et al. (2016) don't elaborate on which traders their algorithm is relevant for, but rather refers to a master thesis by M. Gallana for the details. The thesis did not show up in online searches and repeated emails to the University of Padova didn't provide a response, so the question remains unresolved. The "Output variable" column describes the output argument from the optimization model. Bid strategies that account for either quantity, timing and/or the price of the bids without accounting for all three are referred to as "partial". The "Decision" column describes whether a decision *sequential,* which means that it is distributed in trading time and updates to new information. This is commonly referred to as a "rolling horizon" approach in OR literature. Engmark and Sandven (2017) only partly do this, since only the BM scenarios and not the ID scenarios are updated for each run of the model. The "$|\mathcal{T}| > 1$?" column describes whether the model optimizes for several delivery products, and thus considers interconnections between the production hours, or not.

The "Solution" column describes the nature of the solution algorithm recommended in the paper. *Analytic* means that a closed-form solution is derived for the Hamilton-Jacobi-Bellman Partial Differential Equation (the *HJB PDE* is the continuous-time version of the infamous Bellman equation; see (Wikipedia, 2017h)). *Logic-based* means that no optimization problem is solved at all. *ADP* is short for Approximate Dynamic Programming, *ADDP* is short for Approximate Dual Dynamic Programming, *IPM* is short for Interior Points Method and *SMIP* is short for Stochastic Mixed Integer Program. ADP, IPM and a related version of ADDP called Stochastic Dual Dynamic Programming (SDDP) are further elaborated on in chapter 6. The "Market Power?" column distinguishes between papers that assume the producer has market power and those that don't. Some papers models the slope of the curve of currently open bids without considering that market actors adjust their bid strategy when large bids are placed, and they have been marked with *short term only*. Finally, the "Price dynamics assumption" column describes the assumptions made for the price dynamics. It is worth noting that the papers that consider one delivery product only focus on the price dynamics of a single delivery product, whereas the papers with a single decision stage assume only one price for each delivery product and then consider how the price changes between consecutive delivery products.

# 4 Problem description

In this chapter, the Intraday Trading Problem is further outlined. Following the same decomposition as before, each optimization problem is described in detail.

The overall objective is to maximize profit generated from trading in the EPEX Intraday market on a specific day. This report will focus on optimizing sell bids for delivery products with hourly duration. It is fairly simple to change the model into a model that focuses only on delivery products with quarterly duration, whereas optimizing for delivery products with overlapping time slots (block bids, or quarterly and hourly durations at the same time) is a harder issue.

Note that this report focuses exclusively on short-term optimization of post-spot decisions. Specifically, the power infrastructure (grid, production facitilies) is taken for granted; there are no investment decisions. Also, the Day-Ahead production schedule is fixed; no optimization of allocation of resources between the Day-Ahead and Intraday markets is performed. Thus the focus is on how to trade optimally in the ID market, for given variable costs of production, for a fixed schedule from the DA market and a fixed power infrastructure.

## 4.1 The Price Forecasting Problem

The goal of the Price Forecasting Problem is to predict the future aspects of the orderbook that are decision relevant in the Strategy Formulation Problem. The desired output from this optimization problem will therefore depend on how the Strategy Formulation Problem is expressed. In this section, the available information relevant for the price forecasting, as well as which price features are the relevant ones to forecast, is discussed.

### 4.1.1 Available information

There are several parameters besides the current price that are relevant for the prediction of later Intraday prices, as documented by Lazarczyk (2016), Löhndorf et al. (2013) and others. Choosing the correct input parameters is an optimization problem itself, but the literature suggests that at least some of the following contain relevant information about the future prices:

- Weather-based forecasting of residual demand (Löhndorf et al., 2013). In particular, the load and intermittent production can be forecasted based on temperature, solar forecasts and wind forecasts.
- Urgent Market Messages (Lazarczyk, 2016). Again, the impact on the residual demand is the key aspect.
- Properties of the Intraday orderbook, such as net buy bid volume less sell bid volume; if the data for weather-based forecasting is too expensive,

it is possible that some information about residual demand may instead be gleaned through the bids placed in the Intraday market. The order book may also contain other relevant information, for instance about the strategic bidding behavior of other traders.

- The Day-Ahead price for the similar delivery product (Farinelli and Tibiletti, 2017).
- The Intraday price of consecutive delivery products. Hassler (2017) demonstrates that there is significant correlation between the weighted average prices of consecutive delivery products, which may to some degree also apply to real time correlations (as seen in figure 2.10).
- The time to gate closure. As noted by Garnier and Madlener (2015), volatility grows as gate closure approaches.

### 4.1.2 Price features to forecast

Some examples of price features that could be interesting to forecast include:

- The price of different order depths - i.e if buy bids are sorted by from highest to lowest price, what is the price of the 1st, 30th and 100th MWh in the orderbook? Particularly, the highest price of a buy bid in a certain timeslot is interesting, as this is the highest price that a sell bid can have if 100% clearing certainty is required.
- The order depth for different price intervals. This is similar to the approach above, but instead of choosing specific volumes along the demand curve, the demand curve is categorized into price levels.
- Statistical parameters like max, min, average and median clearing prices of trades in an upcoming time step.

In order to decide which of the suggestions above should be selected, it is assumed that the trader is going to use the forecasts to determine the probability that a sell bid at a given price will (at least partially) clear (see section 4.3). If a bid (partially) clears, a transaction occurs. As the cheapest sell bids in the market are chosen first, a transaction price above the bid price means that the transaction occurred. If buy bids with higher prices than the given sell bid are placed in the market, that provides no guarantee that the bid clears, as there may be sell bids with lower prices that have priority. Transaction prices (given that they can be observed directly) are the only prices that contain information about both the supply and the demand in the market, whereas buy bid forecasting only provides information about the demand.

The maximum transaction price in a given time step is sufficient in order to determine if a sell bid cleared or not; thus, the estimated *exceedance probability* for the maximum transaction price with the bid price as the critical value is an *unbiased estimator* of the bid clearing probability. An unbiased estimator is an estimator which is neither too positive nor negative on average (Wikipedia, 2017c). The exeedance probability is the probability that the value of a time series will exceed a given critical value within a time interval (Wikipedia, 2017e).

However, the maximum price may be quite volatile, so the forecast could have low accuracy. The weighted average of the transactions in a time step is more stable, but will be negatively biased. These arguments are illustrated in figure 4.1, where the two transaction price parameters can be seen along the accuracy/precision Pareto frontier. The Pareto frontier is the curve where one desideratum can only be improved by reducing another (Wikipedia, 2017o). In this case, it is assumed that high accuracy (low bias) and high precision (low unexplained variance; (Wikipedia, 2017b)) are the relevant desiderata.

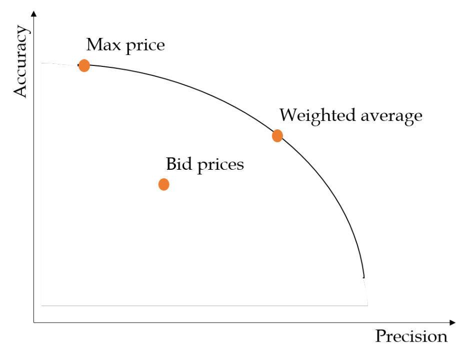



Figure 4.1: Properties of transaction price features. The figure is illustrative and axes are not to scale.

To recap, five assumptions are made to determine which features of the price to forecast; that the trader cares about an accurate and precise estimate of the probability that a sell bid will clear; that transaction prices can be observed directly; that bid prices are biased and noisy signals of transaction prices; that maximum transaction prices are unbiased estimators of whether the sell bid clears; and that the weighted average has lower variance but higher bias than the maximum of the price forecast. Given these five assumptions, the maximum transaction price and/or the weighted average transaction price in each time step should be forecasted.

Without analyzing the added bias and the reduced variance in the weighted average price compared to the maximum clearing price, as well as setting a preference for precision versus accuracy, one can not be claimed to be strictly better than the other. Going forward, the *forecasted price* can be assumed to refer to the weighted average clearing price.

## 4.2 The Cost Estimation Problem

As formerly mentioned, the goal of the Cost Estimation Problem is to accurately evaluate the variable cost of production for a specific power producer for each delivery product, in addition to forecasting the imbalance price. The variable costs of production depend on the estimated Value of Storage, forecasted fuel prices, forecasted generator availability, forecasted intermittent production and intertemporal costs. All of these inputs must be computed if they are unknown. Then, the Cost Estimation Problem takes in a set of possible dispatch plans, and outputs an estimate of the variable costs of production for each delivery product for all dispatch plans. Because of intertemporal costs, the production decisions for consecutive delivery products are functions of the current and former production decision. The estimated marginal cost of production can later be compared to the forecasted imbalance price, to determine the optimal production volume for a given delivery product. It is assumed that computation of the variable cost of production based on the abovementioned parameters will be provided by the producer that wishes to deploy the model. However, a simplified mathematical representation of the Cost Estimation Problem is presented in section 5.2.

## 4.3 The Strategy Formulation Problem

The Strategy Formulation Problem takes the variable costs of production and the price forecast as input parameters, and formulates an optimal bidding strategy. The goal is to maximize the profits of the trader over the decision horizon. The revenue source is cleared bids in the intraday market, as well as subsidies for renewable producers. Cost drivers are outlined in section 2.5.2 and include transaction costs, production costs, OTR fines and imbalance costs. Large bids or many consecutive bids may have short- or long-term price impact, reducing the revenue source from cleared bids; thus the market is assumed to be weakly inefficient.

Only hourly delivery products are assumed to be tradeable, and only regular bids without the execution restrictions IoC, FoK or AoN are allowed. Bid placement and matching is done continuously within the frame of the Intraday trading windows for each delivery product. The decision horizon is the time from Intraday gate opening until the time of delivery of the last delivery product of the day.

The decisions the trader makes include when to bid, bid prices and volumes, when to kill bids and how much to produce for each delivery product given the final commitment from the ID and DA markets. If parts of the commitment is not produced, the imbalance is traded in the BM. The magnitude of the bid price and volume must comply to the bid rules mentioned in section 2.5. The production decision is constrained by the upper and lower bounds on the production capacity for a given trader. The produced energy either stems from an inflow of energy, or stored energy. Storage of energy is limited to the storage capacity, and changes in storage levels are subject to conservation of energy.

Only cleared bids will generate revenues. The clearing process, including the transaction prices corresponding to cleared bids, is described in section 2.5.3. Only open bids with non-zero residual volume that have already been placed can only be matched, and the volume of a transaction cannot be greater than the residual volume of its involved bids. The decisions of the trader are summed up in figure 4.2.

In order to retain the licence to trade in the ID market, traders must comply with the relevant regulations. The regulations are described in section 2.4.

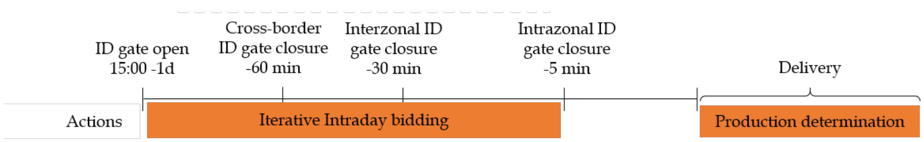



Figure 4.2: The timing of the decisions in the ITP.

# 5 Mathematical models

In this chapter, a mathematical formulation of the Intraday Trading Problem is provided. Before the mathematical formulation of the Strategy Formulation Problem is presented in section 5.3, the focus is on modeling the problems of determining the necessary input quantities. Therefore, mathematical models representing the problems of forecasting Intraday price and estimating costs are covered in section 5.1 and 5.2 respectively, before a mathematical model representing the Strategy Formulation Problem is presented in section 5.3. Finally, in section 5.4 it is explained how the model parameters are different for each type of trader. As the majority of the notation of the Price Forecasting Problem and the Cost Estimation Problem is used only in their respective sections of this chapter, a description of the notation is included in table 5.1 and 5.2 rather than in the Nomenclature. In the rest of this report, trading time is discretized into short time steps denoted $t$.

## 5.1 Modeling the Price Forecasting Problem

In this section a fundamental non-parametric model is proposed to forecast the features required by the Strategy Formulation Problem to operate properly. In order to discuss this topic accurately, the different features of the demand that are used throughout the entire chapter are introduced here. Recall from section 4.1 that in this section, readers may assume that mentions of the "price" in a timestep refers to the weighted average of the transaction prices in that time step.

Table 5.1: Section-specific symbols (Price Forecasting Problem)

| Symbol | Interpretation |
|---|---|
| $R^n$ | The set of real numbers of dimension $n$. |
| $\Omega$ | Set of historical observations. |
| $\omega$ | Historical observation ($\omega \in \Omega$). |
| $t_\tau^R$ | Remaining time until gate closure of delivery product $\tau$. |
| $p_{t\tau}^C$ | Weighted-average transaction price for delivery product $\tau$ computed at time $t$. |
| $\Delta\widetilde{d}(\cdot)$ | Forecasted residual demand function. |
| $F_{t\tau}^P(\cdot)$ | Price distribution forecast for delivery product $\tau$ at time $t$. |
| $F_{t\tau}^L(\cdot)$ | Most recent load distribution forecast for delivery product $\tau$ at time $t$. |
| $F_{t\tau}^W(\cdot)$ | Most recent wind distribution production for delivery product $\tau$ forecast at time $t$. |
| $F_{t\tau}^S(\cdot)$ | Most recent solar distribution production for delivery product $\tau$ forecast at time $t$. |
| $F_{t\tau}^{\text{UMM}}(\cdot)$ | Distribution of most recent forecasted production impact for $\tau$ of UMMs at $t$. |
| $\Phi(\cdot)$ | Kernel Density Estimator. |
| $\phi(\cdot)$ | 1 if the two input vectors have a smaller Manhattan distance than 0.5, 0 otherwise. |
| $\alpha$ | Decay rate. |
| $\Delta\widetilde{d}$ | Forecasted residual demand. |
| $M_\omega$ | Maturity of a historical observation $\omega$. |
| $P_\tau^{DA}$ | Day-Ahead price of delivery product $\tau$ |

The demand features that are relevant for chapter 5 are:

- The *revealed residual demand* ($D_{t\tau}$) is equal to the set of open demand bids for delivery product $\tau$ in the beginning of time step $t$. A given buy bid that is open at time $t$ is referred to as $d_{t\tau i}$, and has the attributes price $d_{t\tau i}^P$ and residual volume $d_{t\tau i}^V$. When these bids are sorted in non-increasing order by price and the volumes are aggregated, it is referred to as the *revealed residual demand curve.*

- The *forecasted residual demand* ($\Delta\widetilde{d}_\tau$) is the expectation of the net sum of the open positions of all actors in the market; that is, how much the market as a whole hopes to buy before gate closure. Note that the forecasted residual demand is used in this section only.

- The *accessible residual demand* ($D_{t_b t\tau}$) is defined on a per-sell bid basis, and is a set of buy bids accessible for the trader's bid $b_{t_b\tau}$ in the beginning of time step $t$. All buy bids $d_{t_b t\tau i} \in D_{t_b t\tau}$ with bid attributes ($d_{t_b t\tau i}^P, d_{t_b t\tau i}^V$) in this set are accessible in the control area of the sell bid $b_{t_b\tau}$ at time $t$ and have higher prices than the price of the sell bid $b_{t_b\tau}^P$. If there are other sell bids with a lower price than $b_{t_b\tau}^P$, those sell bids will have priority before $b_{t_b\tau}$. The highest priced demand bids in $D_{t\tau}$ will therefore not be in $D_{t_b t\tau}$, until all the sell bids with higher priority have cleared completely. If $D_{t_b t\tau}$ is nonempty, at least part of $b_{t_b\tau}$ will clear at time $t$. Only open sell bids have nonempty accessible residual demand.

Note that the revealed and the accessible residual demand is updated by an external subroutine separately from the optimization model. Thus, constraints on how they change over time is superfluous.

In section 3.3, it is observed that papers with dynamics in trading time $t$ focus on estimating the probability distribution for the price in the next trading timestep, $t+1$ (henceforth nominated price transition function). That way, it is possible to make draws from the probability distribution in a forward pass in a dynamic programming framework. All of the $t$-dynamic papers assume some form of Gaussian process (GBM, SBM, OU-dynamic) for the price dynamic. The fact that the variation in price assumptions is so low, begs the question if this is due to convenience or superior performance. To the best knowledge of the authors of this report, neither fundamentals-based nor non-parametric price forecasts have been attempted in this setting. The information gain of exploring these opportunities may therefore be large.

All of the common price assumptions capture the price dynamics imperfectly. The GBM is able to capture price spikes well, but cannot handle negative prices; the SBM (Wikipedia, 2017r) on the other hand, can handle negative prices but disregards the skewness and kurtosis of the power price. The OU-dynamics have the additional feature of capturing mean-reversion, but otherwise come with the same modeling risks as the GBMs and SBMs. None of the Gaussian processes are able to capture autoregressive dynamics. However, with the abundant amounts of bid data from several years of trading on EPEX Intraday there may be no reason to assume a price distribution a priori; a non-parametric price transition probability distribution may capture the empirical transition function even better. A model based on fundamental parameters may also, to some extent answer questions such as "how should the trader respond to UMMs?" that price dynamics without fundamental parameters cannot.

As a consequence of the findings from section 2.2, the price forecast distribution $F^P_{t\tau}(\cdot)$ is modeled as a function of $p^C_{(t-1)\tau}$, $\Delta p^C_{(t-1)\tau} = p^C_{(t-1)\tau} - p^C_{(t-2)\tau}$, $t^R_\tau = \bar{t}^{GC}_\tau - t$ and the forecasted residual demand $\Delta\widetilde{d}_\tau$. It is assumed that the forecasted residual demand of delivery product $\tau$ has the martingale property, and that it is a function of the wind forecast $F^W_{t\tau}(\cdot)$, the solar forecast $F^S_{t\tau}(\cdot)$, the load forecast $F^L_{t\tau}(\cdot)$ and a forecast of the total production impact of UMMs, $F^{\mathrm{UMM}}_{t\tau}(\cdot)$. That is, $\Delta\widetilde{d}_\tau = \Delta\widetilde{d}_\tau(F^W_{t\tau}(\cdot), F^S_{t\tau}(\cdot), F^L_{t\tau}(\cdot), F^{\mathrm{UMM}}_{t\tau}(\cdot))$. If these forecasts are unavailable or too expensive, $\Delta\widetilde{d}_\tau$ will have to be based on Intraday orderbook data instead. The optimal forecasting of the fundamental input parameters to the forecasted residual demand will not be considered in this report.

A Kernel Density Estimator $\Phi(p^C_{t\tau}, p^C_{(t-1)\tau}, \Delta p^C_{(t-1)\tau}, t^R_\tau, \Delta\widetilde{d}_\tau)$ is used to estimate the probability distribution of the succeeding price, $F^P_{t\tau}(\cdot)$. For $t = \underline{t}$, $p^C_{(\underline{t}-1)\tau} = P^{DA}_\tau$, and $\Delta p^C_{(\underline{t}-1)\tau}$ is set to zero. As ID transaction prices correlate strongly with DA prices (as suggested by figure 2.6), "*A better way to model Intraday prices (...) is by modeling their spreads (...) to spot prices*" according to Farinelli and Tibiletti (2017). Therefore, in the price forecasting section, the price random variable $P_{t\tau}$ refers to the ID premium over the DA price.

$$\begin{aligned} F^P_{t\tau}(p^C_{t\tau}) = \Pr\{P_{t\tau} = p^C_{t\tau} \quad | \quad & p^C_{(t-1)\tau}, \Delta p^C_{(t-1)\tau}, t^R_\tau, \Delta\widetilde{d}_\tau\} \\ = \Phi(p^C_{t\tau}, & p^C_{(t-1)\tau}, \Delta p^C_{(t-1)\tau}, t^R_\tau, \Delta\widetilde{d}_\tau) \end{aligned} \tag{5.1}$$

The Kernel density estimation uses a hypercube as a window function as described in (Parzen, 1962) with sides $h$, and places weights with an exponential decay rate of $\alpha$ on historical observations of prices $\omega$ as a function of their maturity $M_\omega$ on the entire set $\Omega$ of historical observations. The maturity may represents the time since a historical observation happened, and may for instance be measured in months. The input vector $\vec{u} = (p^C_\tau, p^C_{(t-1)\tau}, \Delta p^C_{(t-1)\tau}, t^R_\tau, \Delta\widetilde{d}_\tau)$ is four-dimensional, so the probability distribution has support in all parts of $\mathcal{R}^5$ (the input parameters plus the stochastic variable) that are sufficiently close to at least one historical observation. $\phi$ is here a function that defines "sufficiently close"; it is 1 for inputs smaller than $\frac{1}{2}$, and 0 otherwise. Thus, if the $l_1$-norm (Wikipedia, 2017j) of $u - u_n$ is smaller than the width of the sides of the hypercube, $\phi$ is 1.

$$\Phi(\vec{u}) \quad = \quad \frac{\sum_{\omega \in \Omega} e^{-\alpha M_\omega} \phi(\frac{\|\vec{u} - \vec{u_\omega}\|_1}{h})}{h^4 \sum_{\omega \in \Omega} e^{-\alpha M_\omega}} \tag{5.2}$$

Note that if the data becomes too sparse with four input parameters, a *factor analysis* (Wikipedia, 2017f) can be used to determine the subset of the input factors that has the best predictive power without overfitting. With fewer input parameters, it may be possible to increase the granularity of the estimate by reducing the side lengths of the hypercube without overfitting. Ultimately, the optimal tradeoff between overfitting and underfitting will depend on the distribution of the data and should be determined empirically. A more thorough discussion of the overfitting/underfitting tradeoff can be found in 6.1, together with a discussion of parametric versus nonparametric methods.

To summarize, a price transition function is approximated by a kernel density estimator with four inputs, in order to simulate the price dynamics in a forward pass in a dynamic programming algorithm. The inputs include both fundamental data and orderbook data. Note that this procedure becomes computationally heavy and requires a large amount of expensive data as input. If it does not deliver significantly superior results to a GBM, then that should be used instead as it is the academic standard in this field.

## 5.2 Modeling the Cost Estimation Problem

As formerly mentioned, the Cost Estimation Problem is not the focus of this report. It will therefore be assumed that the individual producer has access to an algorithm that calculates the lowest production cost, given the state of the production facilities, including (if relevant) a forecast of intermittent production, forecasted fuel prices, plant outage status and the Value of Storage. As the goal is to provide an estimate of the variable costs of production for the Strategy Forecasting problem, the scope of this section is to briefly outline how it is computed. Thus, the Cost Estimation Problem is modeled in equation (5.3)-(5.6). For more detailed descriptions of how to compute the Value of Storage and other inputs to the model below, the reader is referred to the papers in section 3.2.

Table 5.2: Section-specific symbols (Cost Estimation Problem)

| **Symbol** | **Interpretation** |
|---|---|
| $\mathcal{G}$ | Set of the trader's production generators. |
| $g$ | Production generator ($g \in \mathcal{G}$). |
| $q_{\tau g}$ | Production at time $\bar{t}^D_\tau$ for generator $g$. |
| $\hat{q}_{\tau g}$ | The production decision that minimizes expected production costs over a day. |
| $y(\cdot)$ | Linear regression function used to compute imbalance prices. |
| $C^P_{\tau g}$ | The cost of production of generator $g$ in timeslot $\bar{t}^D_\tau$. |
| $\mathbf{E}_t(P^{\text{WA}}_{t\tau})$ | The expected weighted average transaction price of delivery product $\tau$ at time $t$. |
| $\bar{Q}_{\tau g}$ | Max production at time $\bar{t}^D_\tau$ for generator $g$. |
| $\underline{Q}_{\tau g}$ | Min production at time $\bar{t}^D_\tau$ for generator $g$. |
| $V^{C_{total}}_\tau$ | Total volume committed in all markets for delivery product $\tau$. |

$$\min_{q_{\tau g}} \sum_{\tau \in \mathcal{T}} \sum_{g \in \mathcal{G}} C^P_{\tau g}(q_{(\tau-1)g}, q_{\tau g}) \tag{5.3}$$

$$\sum_{g \in \mathcal{G}} q_{\tau g} = V^{C_{total}}_\tau \quad \tau \in \mathcal{T} \tag{5.4}$$

$$q_{\tau g} \geqslant \underline{Q}_{\tau g}, \quad \tau \in \mathcal{T}, g \in \mathcal{G} \tag{5.5}$$

$$q_{\tau g} \leqslant \bar{Q}_{\tau g}, \quad \tau \in \mathcal{T}, g \in \mathcal{G} \tag{5.6}$$

In this model, $q_{\tau g}$ marks the production of a generator $g$ for one delivery product $\tau$, which is bounded below by $\underline{Q}_{\tau g}$ and above by $\bar{Q}_{\tau g}$. $C^P_{\tau g}(q_{(\tau-1)g}, q_{\tau g})$ is the cost of production for one generator in one hour, which due to intertemporal costs is a function of both the current and former production decision. $V^{C_{total}}_\tau$ is the total commitment from all markets. Thus, the algorithm computes the cost of the cheapest unit commitment schedule for any production plan. In the edge case, when $\tau = 0$, $q_{\tau-1}$ equals the known production of the last hour of the previous day.

As the Strategy Formulation Problem considers production on an aggregated level rather than a per-generator level, $C^P_\tau(q_{(\tau-1)}, q_\tau)$ is returned to the Strategy Formulation Problem. As the future production decisions will be unknown due to non-anticipativity, $\hat{q}_{\tau g}$ denotes the production decision for generator $g$ for delivery product $\tau$ that minimizes the expected cost over a set of likely future dispatch plans. The unit variable cost aggregated over generators is computed as in equation (5.8). The production decisions for $\tau$ and $\tau - 1$ are defined in equation (5.7).

$$q_\tau - \sum_{g \in \mathcal{G}} \hat{q}_{\tau g} = 0 \quad \tau \in \mathcal{T} \tag{5.7}$$

$$C^P_\tau(q_{(\tau-1)}, q_\tau) = \sum_{g \in \mathcal{G}} C^P_{\tau g}(\hat{q}_{(\tau-1)g}, \hat{q}_{\tau g}), \quad \tau \in \mathcal{T} \tag{5.8}$$

In section 2.5.2 it was stated that the VoS is a function of the remaining storage, which is affected by the production decision in the Strategy Formulation Problem. Ideally, the VoS should be recomputed to reflect the new storage level if the assumed dispatch plan is violated significantly, but this is infeasible for high-frequency decision making due to the complexity of the problem. One suggestion for suitable heuristic could be to implement empirical approximations stating that if production overshoots the expectation with x% of the remaining storage, a z% premium will be placed on the water value, or something similar. For the rest of this report, it is assumed that suitable heuristics have been applied and that the VoS accurately reflects the actual alternative cost of production.

Recall from section 2.2 that the expected imbalance prices at time $t$, $\mathbf{E}_t(P^{BM+}_\tau)$ and $\mathbf{E}_t(P^{BM-}_\tau)$, are very hard to forecast accurately. Here, a simple model using linear regression with expected weighted average Intraday transaction prices for delivery product $\tau$ at time $t$, $\mathbf{E}_t(P^{\mathrm{WA}}_{t\tau})$ as inputs is proposed. As the expected drift of the transaction prices must be computed in the Price Forecasting Problem, it is trivial to find $\mathbf{E}_t(P^{\mathrm{WA}}_{t\tau})$ price at any time $t$ in the trading window of delivery product $\tau$. Introducing $y(\cdot)$ as the linear regression function, this relationship can be expressed as in equation (5.9):

$$\mathbf{E}_t(P^{BM+}_\tau) = \mathbf{E}_t(P^{BM-}_\tau) = y(\mathbf{E}_t(P^{\mathrm{WA}}_{t\tau})), \quad \tau \in \mathcal{T} \tag{5.9}$$

As $\mathbf{E}_t(P^{BM+}_\tau)$ and $\mathbf{E}_t(P^{BM-}_\tau)$ are parameters in the Strategy Formulation Problem, they are upper-case also in this section due to consistency reasons, despite being variables in the Cost Estimation Problem.

## 5.3 Modeling the Strategy Formulation Problem

The model presented in section 5.3.1 is denoted *the basic Multistage Stochastic Integer Program* (MSIP). The non-linearities and some relevant extensions of this model are presented in 5.3.2 and 5.3.3 respectively. The linearized version of the model is denoted *the basic Multistage Stochastic Integer Linear Program* (MSILP).

An overview of the costs of Intraday trading handled by the model of section 5.3.1 is provided in table 5.3. As can be inferred from this table, the majority of the relevant costs are included in the basic MSIP. The imbalance cost, production cost, transaction cost and subsidy cost are all represented explicitly in the objective function. The volume costs are represented implicitly. That is, even though they are not present in the objective function, they will play an important role in the execution of the model due to the chosen representation of the problem. Specifically, the reason why the volume cost is included is that the demand faced by a certain trader on the sell side will be reduced as bids are matched. The potential inclusion of Order-to-Trade-Ratio fine and market saturation costs is discussed as potential model extensions in section 5.3.3.

Table 5.3: The trading costs taken into account in the proposed model

| **Type** | **Magnitude** | **Included in model** |
|---|---|---|
| Imbalance cost | High | Yes |
| Production cost | High/Low | Yes |
| Volume cost | High/Low | (Yes) |
| Market saturation cost | High/Low | (No) |
| Order-to-Trade-Ratio fine | Low | (No) |
| Transaction cost | Low | Yes |
| Subsidy cost | Low/- | Yes |

### 5.3.1 Multistage Stochastic Integer Model

This section contains a formulation of the ITP of a specific trader as a multistage stochastic operational model. The model aims at being accurate for any type of trader on the EPEX Intraday market. The syntax used in this section is defined in the Nomenclature. The producer's decision variables at time $t_b$ for delivery product $\tau$ is if the trader should place a bid for a certain delivery product, $b_{t_b\tau}$, and eventually the price, $b^P_{t_b\tau}$, and volume, $b^V_{t_b\tau}$, features of this particular bid. That is, only one bid per delivery product can be placed in each timeslot. In the basic MSIP, the price and volume features are assumed continuous. In each timestep $t$, the trader should also decide whether to kill each of her already placed bids. In modeling terms, the action of killing the bid placed in timeslot $t_b$ for delivery product $\tau$ in timeslot $t$ corresponds to setting $\delta^K_{t_b t\tau} = 1$. Having limited computational power, a relatively higher bid frequency has been weighted heavier than the increased bidding flexibility of being able to place multiple bids for the same delivery product simultaneously. The bid placement decisions are repeated from the Intraday gate opening ($t = \underline{t}$) until gate closure at the end of timeslot ($t = \bar{t}^{GC}_{|\mathcal{T}|}$). In timeslots $t = \bar{t}^D_\tau$, a production decision $q_\tau$ must be made and the balancing market volumes will be set corresponding to the difference between the market commitment and the physical production. Here, we assume no delay between commitment and production, that no Over-the-Counter-trade can happen and that the production within a delivery product is constant. Also, the trader is assumed to be a Balance Responsible Party herself, so there are no risk pooling benefits to be reaped. An example of a potential set of realized decisions can be seen in figure 5.1. Here, the trader places four bids in the Intraday market for delivery product $\tau$. At the time of delivery of the delivery product, $t = \bar{t}^D_\tau$, the trader must determine her production volume $q_\tau$.

The objective function of this model, equation (5.10), states that the producer is trying to maximize her profits from Intraday trading, i.e. its revenues minus costs. Here, the revenue is simply the sum of the products of transaction price $p^C_{t_b t\tau}$ and transaction volume $v^C_{t_b t\tau}$ for all bids placed by the trader. The costs correspond to those of table 5.3. Transaction cost, variable cost of production and costs from trading in the balancing markets are all included in the objective function.

$$
\begin{aligned}
\max z = \sum_{\tau\in\mathcal{T}} \sum_{t_b\in T_\tau} \sum_{\tilde{t}=t_b}^{\bar{t}^{GC}_\tau} (p^C_{t_b\tilde{t}\tau} v^C_{t_b\tilde{t}\tau} - C^C v^C_{t_b\tilde{t}\tau}) - \sum_{\tau\in\mathcal{T}} C^P_\tau(q_\tau) q_\tau \\
- \sum_{\tau\in\mathcal{T}} (P^{BM+}_\tau v^{BM+}_\tau - P^{BM-}_\tau v^{BM-}_\tau)
\end{aligned}
\tag{5.10}
$$

Going forward, two classes of constraints of the Intraday Trading Problem, namely physical constraints and financial constraints, are presented in section 5.3.1.1 and 5.3.1.2. These are required to handle the two respective sides of the problem. Also, some optional, behavioral constraints are described in section

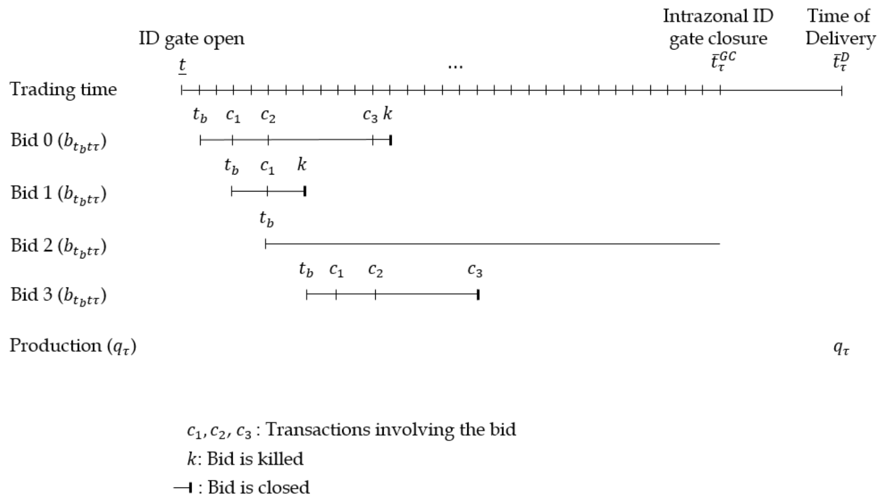



Figure 5.1: Timeline for the trading of one particular delivery product

5.3.1.3, and the variable domains are covered in section 5.3.1.4. Section 5.3.2 highlight non-linearities in the model.

#### 5.3.1.1 Physical constraints

The physical constraints of this report's model ensure that the power production and power storage comply to the relevant physical rules. Equation (5.11) and (5.12) restricts the energy storage of a power producer: equation (5.11) is a flow constraint guaranteeing that the stored power at the beginning of the delivery slot $\tau + 1$, nominated $\bar{t}^D_{\tau+1}$, follows from the stored value from the beginning of delivery slot $\tau$, nominated $\bar{t}^D_\tau$, and the production and inflow volume during delivery slot $\tau$. Remark that the edge case $\tau = |\mathcal{T}|$ is not really an edge case as $s_{|\mathcal{T}|} + 1$ of day $i$ is equal to $s_0$ of the consecutive day. In (5.12), the stored energy $s_{\tau+1}$ at the end of delivery slot $\tau + 1$, specifically at time $\bar{t}^D_{\tau+1}$, is limited by an upper bound $\bar{S}_{\tau+1}$. This cap is set at the end of the delivery slot to make sure that the model doesn't recommend to overflow the storage capacity during the last delivery slot of the planning horizon.Equations (5.13) and (5.14) define the bounds of the production variables.

$$\text{s.t.} \quad s_{\tau+1} - s_\tau - q_\tau \quad = \quad -f_\tau, \quad \tau \in \mathcal{T} \tag{5.11}$$

$$s_{\tau+1} \quad \leqslant \quad \bar{S}_{\tau+1}, \quad \tau \in \mathcal{T} \tag{5.12}$$

$$q_\tau \quad \geqslant \quad \underline{Q}_\tau, \quad \tau \in \mathcal{T} \tag{5.13}$$

$$q_{\tau} \quad \leqslant \quad \bar{Q}_{\tau}, \quad \tau \in \mathcal{T} \tag{5.14}$$

#### 5.3.1.2 Financial constraints

The technicalities of Intraday power trading are handled by constraint (5.15) - (5.26). These constraints aims at replicating the real-world market microstructure. As they are more complex than the physical constraints, they are presented more thoroughly. Also, the authors recommend that readers understand the logic of figure 5.2 before examining the financial constraints. Some of the most relevant support variables and their development during the lifetime of a bid are explained here. The underlying bid of the support variables in this example, $b_{t_b\tau}$ is created in timeslot $t = t_b$ and killed at the time of event $k$. In between these two timeslots, the bid is partially matched three times, at time $t = t_1$, $t = t_2$ and $t = t_3$ with clearing volumes $v^C_{t_bt_1\tau}, v^C_{t_bt_2\tau}$ and $v^C_{t_bt_3\tau}$ for the three trades respectively. Recall that $v^C_{t_bt_1\tau}$ reads as "volume cleared for bid $t_b$ in time $t_1$ for delivery product $\tau$", and that this index syntax is consistent throughout the entire model.

From this figure one can infer that $\delta^B_{t_bt\tau}$ is set to 1 as soon as the corresponding bid is placed in $t = t_b$. Please note that $\delta^B_{t_bt\tau}$ is introduced here for convenience, despite only being used in the behavior-shaping constraints of section 5.3.1.3. Once the bid is cleared for the first time, $\delta^C_{t_bt\tau}$ is set to 1. When the bid is killed, $\delta^K_{t_bt\tau}$ is set to 1. After a bid is killed, no more trades can happen. If the bid had not been killed, it could have cleared several more times until $\bar{t}^{GC}_{\tau}$.

One can also see how the residual volume $\widetilde{b}^V_{t_bt\tau}$ of the bid develops as the bid is placed, partially matched and killed. Specifically, the reduction in remaining bid volume from one timeslot to another, $\widetilde{b}^V_{t_b(t+1)\tau} - \widetilde{b}^V_{t_bt\tau}$ is equal to the traded volume of the bid in that timeslot, $v^C_{t_bt\tau}$. As this bid doesn't find a match immediately, the residual volume is set equal to the initial bid volume $b^V_{t_b\tau}$. Otherwise, the initial residual volume would have equalled the initial bid volume less the initial clearing volume. The residual bid volume does not decrease after the bid is killed, as no more transactions can occur. If the entire residual bid volume had cleared at some point, the residual bid volume would have fallen to zero and later clearings would have been impossible.

Note that one could imagine that $\delta^C_{t_bt\tau}$ should have been a continuous variable with a fractional value indicating what share of the corresponding bid volume was traded in period $t$. However, in order to comply to the Order-to-Trade-Ratio (OTR) restriction modeled in equation (5.29), $\delta^C_{t_bt\tau}$ is binary in this model. That is, $\delta^C_{t_bt\tau}$ is set to 1 even if bid $b_{t_b\tau}$ is only partially matched in timeslot $t$.

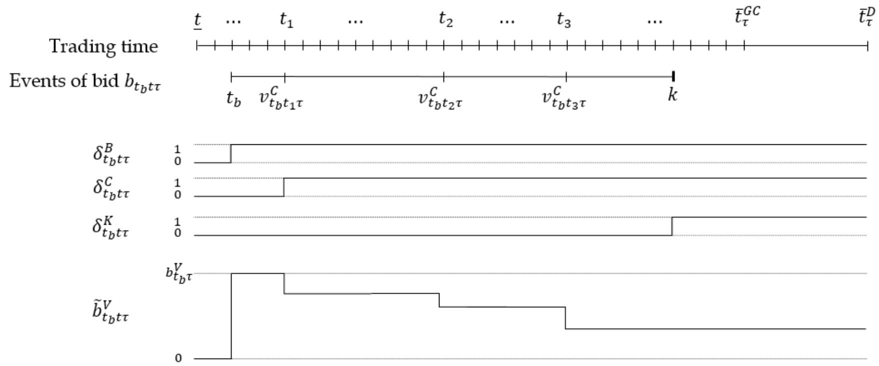



Figure 5.2: The support variables of the mathematical model

Equation (5.15) ensures that the volume traded in the balancing markets equals the difference between the generated volume and the sum of the volumes sold in Day-Ahead and Intraday markets.

$$\sum_{t_b \in T_\tau} \sum_{\widetilde{t}=t_b}^{\bar{t}_\tau^{GC}} v_{t_b \widetilde{t} \tau}^C + v_\tau^{BM-} - v_\tau^{BM+} - q_\tau \quad = \quad -V_\tau^{DA}, \quad \tau \in \mathcal{T} \tag{5.15}$$

Equation (5.16) caps the maximum volume per bid. This is done primarily to reduce the decision space.

$$b_{t_b \tau}^V \quad \leqslant \quad \bar{V}, \quad t_b \in T_\tau, \tau \in \mathcal{T} \tag{5.16}$$

(5.17) initiates important binary variables indicating whether a given bid had been (at least partially) cleared at time $t$. The tightest $\mathcal{M}_{t_b \tau}^1$ is equal to $b_{t_b \tau}^V$, the initial bid volume. However, setting the $\mathcal{M}_{t_b \tau}^1$ equal to a former decision would make the model quadratic. Therefore, $\mathcal{M}_{t_b \tau}^1 = \mathcal{M}^1$ is equal for all bids and set to the largest value that $b_{t_b \tau}^V$ can take, which is $\bar{V}$. This is a significantly less tight formulation, but the preserved linearity still makes it more computationally efficient.

$$\mathcal{M}_{t_b \tau}^1 \delta_{t_b t \tau}^C - \sum_{\widetilde{t}=t_b}^{t} v_{t_b \widetilde{t} \tau}^C, \quad \geqslant \quad 0, \quad t_b \leqslant t, t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T} \tag{5.17}$$

(5.18) prevents bids from being traded once they are killed, and this time, the tightest $\mathcal{M}_{t_b \tau}^2$ is equal to $\widetilde{b}_{t_b t \tau}^V$, the residual bid volume. Again, $\widetilde{b}_{t_b t \tau}^V$ is a function of a former decision. For the same reason as for $\mathcal{M}^1$, $\mathcal{M}_{t_b \tau}^2 = \mathcal{M}^2 = \sup\{\widetilde{b}_{t_b t \tau}^V\} = \bar{V}$.

$$\mathcal{M}_{t_b \tau}^2 (\delta_{t_b t \tau}^K - 1) + \sum_{\widetilde{t}=t}^{\bar{t}_\tau^{GC}} v_{t_b \widetilde{t} \tau}^C \quad \leqslant \quad 0, \quad t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T} \tag{5.18}$$

Constraints (5.19) and (5.20) make the clearing and killing of a bid irreversible, respectively. That is, once a bid is cleared and/or killed, it will stay cleared and/or killed throughout the trading window of the corresponding delivery product respectively.

$$\delta^C_{t_b(t+1)\tau} - \delta^C_{t_b t\tau} \geqslant 0, \quad t_b \in T_\tau, t \in T_\tau \setminus \{\bar{t}^{GC}_\tau\}, \tau \in \mathcal{T} \tag{5.19}$$

$$\delta^K_{t_b(t+1)\tau} - \delta^K_{t_b t\tau} \geqslant 0, \quad t_b \in T_\tau, t \in T_\tau \setminus \{\bar{t}^{GC}_\tau\}, \tau \in \mathcal{T} \tag{5.20}$$

Equation (5.21) states that the cleared volume of a specific bid in a time step equals either the residual volume of the bid or the total volume of the accessible residual demand for the bid, depending on which is lower. Note that the residual demand is defined at the end of a time step, which is why $\widetilde{b}^V_{t_b(t-1)\tau}$ is used. Equation (5.22) has the same purpose, except that it is only defined for the time step when the bid is placed, $t_b$. In this time step, $\widetilde{b}^V_{t_b(t-1)\tau}$ is zero, so $b^V_{t_b\tau}$ must be used instead. Both equation (5.21) and (5.22) can be modeled using logical binary variables. The constraints can not be modeled using continuous linear constraints, as the maximal possible clearing volume will not necessarily be chosen. A more in-depth explanation as well as the explicit formulation with binary variables can be found in section 5.3.2.

$$v^C_{t_b t\tau} - \min\{\widetilde{b}^V_{t_b(t-1)\tau}, \sum_{i \in D_{t_b t\tau}} d^V_{t_b t\tau i}\} \quad = \quad 0, \quad t_b < t, t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T} \tag{5.21}$$

$$v^C_{t_b t_b\tau} - \min\{b^V_{t_b\tau}, \sum_{i \in D_{t_b t\tau}} d^V_{t_b t_b\tau i}\} \quad = \quad 0, \quad t_b \in T_\tau, \tau \in \mathcal{T} \tag{5.22}$$

Equation (5.23) sets the remaining volume of a bid at time $t$ equal to the difference between the initial bid volume and the traded volume of that certain bid. This relationship could also be represented recursively, but it is stated explicitly here for readability reasons:

$$\sum_{\widetilde{t}=t_b}^{t} v^C_{t_b\widetilde{t}\tau} + \widetilde{b}^V_{t_b t\tau} - b^V_{t_b\tau} \quad = \quad 0, \quad t_b \leqslant t, t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T} \tag{5.23}$$

Equation (5.24) states that the cleared volume of a demand bid at time $t$ must be lower than the residual volume of that bid at the same time. Recall that the accessible residual demand bids $D_{t_b t\tau}$ are updated externally from timestep to timestep. Equation (5.25) links the cleared volume of a sell bid and the cleared volume for a set of accessible buy bids. This is elaborated further in section 5.3.2.

$$v^D_{t_b t\tau i} \leqslant d^V_{t_b t\tau i} \quad t_b \leqslant t, t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T}, i \in D_{t_b t\tau} \tag{5.24}$$

$$v^C_{t_b t\tau} - \sum_{i \in D_{t_b t\tau}} v^D_{t_b t\tau i} = 0, \quad t_b \leqslant t, t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T} \tag{5.25}$$

Finally, (5.26) sets the sell bid transaction price for all immediate transactions equal to the weighted average transaction price of the transactions in the time step when the bid is places. If the total available residual demand is larger than the volume of the sell bid, this constraint does not enforce that the buy bids with the best prices should be selected. However, it will always be more profitable to do so, so it will happen by default. (5.27) sets the clearing price for $t > t_b$ equal to the sell bid price.

$$p^C_{t_b t_b \tau} v^C_{t_b t_b \tau} - \sum_{i \in D_{t_b t \tau}} d^P_{t_b t_b \tau i} v^C_{t_b t_b \tau i} = 0 \quad t_b \in T_\tau, \tau \in \mathcal{T} \tag{5.26}$$

$$p^C_{t_b t \tau} - b^P_{t_b \tau} = 0 \quad t_b < t, t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T} \tag{5.27}$$

#### 5.3.1.3 Behavior-shaping constraints

The following constraints are not strictly necessary to comply to the short-term requirements of the market microstructure of EPEX Intraday trading. However, they will push the solutions towards more desirable behavior through restricting the volume traded in the balancing markets and reducing the number of overly optimistic bids. Such *soft constraints* are normally placed in the objective function together with a corresponding penalty of violating the constraint. In this case, it might be more appropriate to model these constraints as hard constraints since the contrary would incur risk of being banned from the market. This would probably reduce the model's attractiveness among risk averse traders, even if it is a more accurate representation of the actual situation. Also, moving the constraint (5.29) to the objective function would potentially encourage traders to placing fewer bids than necessary.

Specifically, (5.28) restricts the volume traded in the balancing markets by an upper bound $R^{BM}$ as done by Skajaa et al. (2015), and (5.29) limits the number of placed bids per cleared bid. Equation (5.29) states that the number of placed bids cannot be larger than the OTR limit. To handle the case where no bids are cleared, an indicator variable $\delta^C_\tau$ is introduced. This indicator is equal to 1 if no bid for the corresponding delivery product is cleared, allowing $R^{OTR}$ uncleared bids if no bids are cleared. Equation (5.29) is in fact not a valid inequality of the real-world problem, as it does not allow for solutions exceeding the OTR limit for any delivery product. Recall that the OTR fine is not given before the number of delivery products exceeding the OTR limit is greater than four per month. However, the authors believe that this is not a very attractive subspace of the decision space for solution algorithms to explore. This being said, a valid representation of the OTR restriction is discussed in section 5.3.3.

$$\sum_{\tau \in \mathcal{T}} (v^{BM+}_\tau + v^{BM-}_\tau) \quad \leqslant \quad R^{BM} \tag{5.28}$$

$$\sum_{t_b \in T_\tau} \delta^B_{t_b t \tau} - R^{OTR}(\delta^C_\tau + \sum_{t_b \in T_\tau} \delta^C_{t_b t \tau}) \quad \leqslant \quad 0, \quad t = \bar{t}^{GC}_\tau, \tau \in \mathcal{T} \tag{5.29}$$

Equation (5.30), (5.31) and (5.32) ensure correct initialization of the indicators necessary to fulfill constraint (5.29). In (5.30), the bid existence indicator $\delta^B_{t_b t\tau}$ is forced to 1 if the bid volume $b^V_{t_b\tau} > 0$. In (5.31), once a bid has been created, the bid existence indicator is forced to stay equal to 1 throughout the remaining trading window of the delivery product. The tightest bound for $\mathcal{M}^3$ is $\bar{V}$, the maximum bid size. In (5.32), $\delta^C_\tau$ is set equal to 1 of none of the trader's bids for delivery product $\tau$ has been matched.

$$\mathcal{M}^3 \delta^B_{t_b t\tau} - b^V_{t_b\tau} \quad \geqslant \quad 0, \quad t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T} \tag{5.30}$$

$$\delta^B_{t_b(t+1)\tau} - \delta^B_{t_b t\tau} \quad \geqslant \quad 0, \quad t_b \in T_\tau, t \in T_\tau \setminus \{\bar{t}^{GC}_\tau\}, \tau \in \mathcal{T} \tag{5.31}$$

$$\delta^C_\tau + \sum_{t_b \in T_\tau} \delta^C_{t_b t\tau} \quad \geqslant \quad 1, \quad t = \bar{t}^{GC}_\tau, \tau \in \mathcal{T} \tag{5.32}$$

#### 5.3.1.4 Variable domains

In the following equations, the variable domains are described.

$$q_\tau \quad free, \quad \tau \in \mathcal{T} \tag{5.33}$$

$$v^{BM-}_\tau, v^{BM+}_\tau, s_\tau \quad \geqslant 0, \quad \tau \in \mathcal{T} \tag{5.34}$$

$$b^V_{t_b\tau} \quad \geqslant 0, \quad t_b \in T_\tau, \tau \in \mathcal{T} \tag{5.35}$$

$$\widetilde{b}^V_{t_b t\tau}, v^C_{t_b t\tau} \quad \geqslant 0, \quad t_b \leqslant t, t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T} \tag{5.36}$$

$$\widetilde{b}^V_{t_b t\tau}, v^C_{t_b t\tau} \quad = 0, \quad t_b > t, t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T} \tag{5.37}$$

$$b^P_{t_b\tau}, p^C_{t_b\tau} \quad free, \quad t_b \in T_\tau, \tau \in \mathcal{T} \tag{5.38}$$

$$v^D_{t_b t\tau i} \geqslant 0, \quad t_b \leqslant t, t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T}, i \in D_{t_b t\tau} \tag{5.39}$$

$$\delta^C_{t_b t\tau}, \delta^B_{t_b t\tau}, \delta^K_{t_b t\tau} \quad \in \quad \{0,1\}, \quad t_b \leqslant t, t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T} \tag{5.40}$$

$$\delta^C_{t_b t\tau}, \delta^B_{t_b t\tau}, \delta^K_{t_b t\tau} \quad = \quad 0, \quad t_b > t, t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T} \tag{5.41}$$

### 5.3.2 Non-linearities

The model outlined in section 5.3.1 has three non-linearities: in the goal function, the revenue term $p^C_{t_b t\tau} v^C_{t_b t\tau}$ is the product of two support variables, corresponding to the two decision variables $b^P_{t_b\tau}$ and $b^V_{t_b\tau}$ respectively; the production cost term $C^P_\tau(q_\tau)\cdot q_\tau$ is the product of the production quantity and the marginal production costs, which are a function of the production quantity; and finally, constraints (5.21) and (5.22) chooses the minimum of two alternatives. In this section, all three non-linearities are linearized.

#### 5.3.2.1 The revenue non-linearity

In this section, the linearization of the revenue term of the goal function is explained. This linearization is easier understood in a dynamic model. As such a model is proposed in chapter 6, a dynamic formulation of the basic MSIP is assumed in this section. The decision then consists of determining the optimal price and volume of one single bid. The concepts presented also generalize to a non-dynamic MSIP formulation, but a more complicated formulation is then required to ensure global optimality and feasibility. In the end of the section it is discussed how to generalize the proposed method to the non-dynamic MSIP.

Table 5.4: Section-specific symbols (The revenue non-linearity)

| **Symbol** | **Interpretation** |
|---|---|
| $\mathcal{V}$ | The set of bid volume categories. |
| $\nu_{t_b\tau}$ | Bid volume category ($\nu \in \mathcal{V}$). |
| $\hat{\nu}$ | The chosen bid volume category ($\hat{\nu} \in \mathcal{V}$). |
| $b^P_{t_b\tau}$ | Bid price decision variable in subproblem $\nu$. |
| $\hat{b}_{t_b\tau\nu}$ | The optimal bid if volume category $\nu$ was chosen as bid volume. |
| $\hat{b}^P_{t_b\tau\nu}$ | The optimal price if volume category $\nu$ was chosen as bid volume. |
| $\hat{b}^V_{t_b\tau\nu}$ | The optimal volume if volume category $\nu$ was chosen as bid volume. |
| $\delta_{t_b\tau\nu}$ | 1 if $\nu$ was the best volume category. |
| $V_\nu$ | The bid volume parameter in subproblem $\nu$. |

The revenue non-linearity is also discussed in Engmark and Sandven (2017), who solve it by discretizing the bid price into categories of low, medium and high. In this report, the bid volumes are categorized instead. The proposed bid volume categories are zero (0 MWh), tiny bids (1 MWh), very small bids (2 MWh) small bids (5 MWh), medium-sized bids (10 MWh), large bids (15 MWh), very large bids (20 MWh) and huge bids (25 MWh). This is handled using a nested problem structure as illustrated in figure 5.3: creating one subproblem per bid volume category and one master problem coordinating between the subproblems.

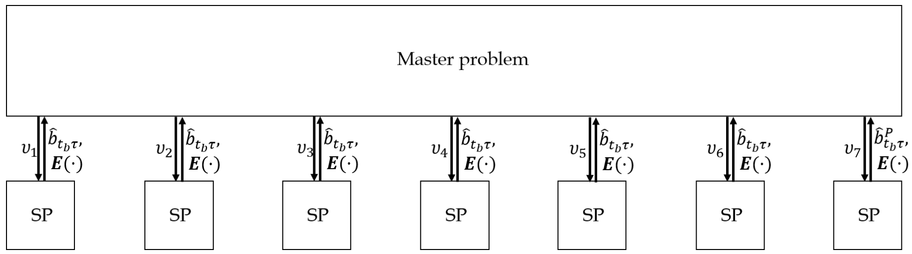



Figure 5.3: Optimal prices and volumes are solved in separate problems, creating a nested problem structure

The subproblems take an input volume $V_\nu$ and compute the optimal bid price given this bid volume. The only decision in each subproblem is to determine the price which gives the bid the highest expected value. In a dynamic framework, the expected value can easily be found using the value function or value lookup table. Thus, solving each of the subproblems is equivalent to solving one step of the dynamic formulation based on the basic MSIP with an equality constraint on the bid volume. That is, with $V_\nu$ as the input bid volume category parameter, the constraint $b^V_{t_b\tau} = V_\nu, t_b \in T, \tau \in \mathcal{T}$ is added to the dynamic version of the basic MSIP in each volume category. The optimal bid prices for each volume category are found under the assumption that no other bid will be placed by the trader in the given time step. In the master problem, the results of each of the subproblems are compared, and the best bid feature combination is chosen as the bid to place. First, the master problem is described mathematically. Let $\mathcal{V}$ be the set of bid volume categories with indices $\nu \in \mathcal{V}$ and let $\hat{b}_{t_b\tau\nu}$ be the optimal bid returned from the subproblem. Recall that the bid is represented as a two-tuple, $\hat{b}^P_{t_b\tau\nu}$, $\hat{b}^V_{t_b\tau\nu}$. Then, the master problem chooses the bid category that maximizes the expected value, and the attributes of the placed bid are equal to those of the bid from the winning subproblem. These relationships are modeled in equation (5.42)-(5.44).

$$\hat{\nu}_{t_b\tau} = \text{argmax}_{\nu\in\mathcal{V}}\{\mathbf{E}(\hat{b}_{t_b\tau\nu})\}, \quad t_b \in T_\tau, \tau \in \mathcal{T} \tag{5.42}$$

$$b^V_{t_b\tau} = V_{\hat{\nu}_{t_b\tau}}, \quad t_b \in T_\tau, \tau \in \mathcal{T} \tag{5.43}$$

$$b^P_{t_b\tau} = \hat{b}^P_{t_b\tau\hat{\nu}}, \quad t_b \in T_\tau, \tau \in \mathcal{T} \tag{5.44}$$

If the zero volume bid is optimal, in practice, no bid is placed. Thus, seven ($|\mathcal{V}| - 1$) integer linear optimization problems (subproblems) are solved per time step per delivery product, and the best of the eight alternatives (including the zero bid) is chosen in a master problem. This process is illustrated in figure 5.3.

In practice, equation (5.42) is implemented using the following constraints in the volume optimization master problem:

$$\sum_{\nu \in \mathcal{V}} \delta_{t_b \tau \nu} = 1, \quad t_b \in T_\tau, \tau \in \mathcal{T} \tag{5.45}$$

$$b^V_{t_b \tau} - \sum_{\nu \in \mathcal{V}} V_\nu \delta_{t_b \tau \nu} = 0, \quad t_b \in T_\tau, \tau \in \mathcal{T} \tag{5.46}$$

$$b^P_{t_b \tau} - \sum_{\nu \in \mathcal{V}} \delta_{t_b \tau \nu} \hat{b}^P_{t_b \tau \nu} = 0, \quad t_b \in T_\tau, \tau \in \mathcal{T} \tag{5.47}$$

$$\delta_{t_b \tau \nu} \in \{0, 1\}, \quad t_b \in T_\tau, \tau \in \mathcal{T}, \nu \in \mathcal{V} \tag{5.48}$$

Note that to ease notation, $\hat{\nu} = \hat{\nu}_{t_b \tau}$. Equation (5.45) states that only one of the bud categories can be chosen, whereas (5.46) forces the bid volume, $b^V_{t_b \tau}$, equal to the chosen bid volume. The goal function then chooses the $b^V_{t_b \tau}$ that gives the highest expected value. These are the only constraints in the master problem, as all other constraints have already been satisfied in the subproblems for each volume category.

The rationale for choosing to discretize volume instead of price is twofold; firstly, volume is additive while price is not - if two bids with volumes of 2 MWh and 15 MWh clears fully, 17 MWh has cleared, whereas if bids with prices of 35€/MWh and 50€/MWh clears, this does not equal that a bid with price of 85€/MWh cleared. Besides, radical alterations of the price disrupts the probability that a bid will clear. The same can not be said for variations in volumes. Thus, consecutive decisions with volumetric categories can approximate a continuous decision space in a way that consecutive decisions with price categories cannot. Secondly, it already seems like the market opted for a solution with discretized bid levels. As observed in figure 2.7, traders today mostly place a combination of small bids and multiples of fives. This is also the reason why the specific volume categories were chosen; they span the entire range of bid volumes that are most common in the market.

An additional problem-specific bonus of categorizing by volume is that in the price-optimization problems for each volume $b^V_{t_b \tau \nu}$, the big-Ms in the original model can use a tighter formulation since it no longer is a function of a previous decision. That is, $\mathcal{M}^1_{t_b \tau}, \mathcal{M}^2_{t_b \tau}$ and $\mathcal{M}^3$ can be set to $b^V_{t_b \tau}$. Note that $\widetilde{b}^V_{t_b t \tau}$ cannot replace the original bound as they are still variable in the subproblems. Also, please note that equation (5.16) is superfluous using this linearization.

The reason why this linearization is harder to grasp in the non-dynamic MSIP, is that all bid placements must be determined simultaneously in the beginning of the day. Thus, determining the expected value of one bid is non-trivial since it depends on all other bids placed the same day. Thus, the expected value of one bid in the non-dynamic MSIP equals the value of the goal function for the entire day on the condition that the bid is placed, minus the value of the goal function for the entire day on the condition that the bid is not placed. In the dynamic algorithm, the value table contains information about the value of future actions, so this is not an issue.

#### 5.3.2.2 The cost function non-linearity

Table 5.5: Section-specific symbols (Cost function non-linearity)

| Symbol | Interpretation |
|---|---|
| $\Gamma$ | The set of production locations. |
| $\mathcal{N}$ | The set of natural numbers. |
| $\gamma$ | A given production location $\gamma \in \Gamma$. |
| $\delta^q_{\tau\gamma}$ | 1 if the production at location $\gamma$ is maximal for $\tau$, 0 otherwise. |
| $q_{\tau\gamma}$ | The volume produced for delivery product $\tau$ at location $\gamma$. |
| $C^P_{\tau\gamma}$ | The cost of producing power for delivery product $\tau$ at a given location $\gamma$. |
| $\bar{Q}_{\tau\gamma}$ | Upper bound on the production volume for delivery product $\tau$ at location $\gamma$. |

The cost function non-linearity is solved by approximating the cost function with linear cost curves, as illustrated by figure 5.4. Conceptually, this is similar to viewing the production volume $q_\tau$ as a resource with non-homogenous prices depending on which location it is sourced from, with limited supply from the cheapest locations. Using this metaphor, $\Gamma$ represents a set of production locations and $\gamma$ is a location with production price $c^P_{\tau\gamma}$ and production capacity $\bar{Q}_{\tau\gamma}$. The actual production at $\gamma$ for delivery product $\tau$ is $q_{\tau\gamma}$. Note that the production isn't necessarily actually sourced from different actual locations on a map, it's simply a conceptual shift for communicating the linearization of the term $C^P_\tau(q_\tau) \cdot q_\tau$.

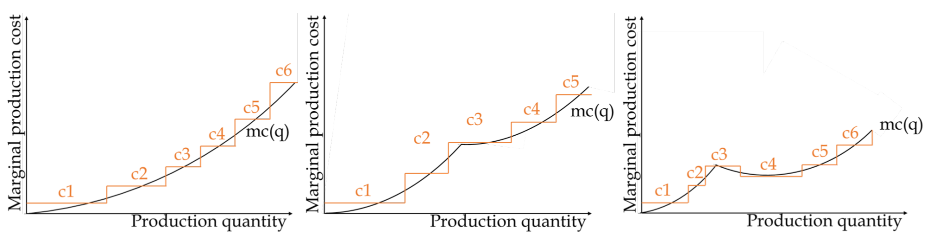



Figure 5.4: Linearization of convex monotone, non-convex monotone and non-convex non-monotone cost functions, respectively.

The goal function automatically chooses the cheapest production location. Thus, c1 in the leftmost figure represents the production cost at the cheapest production location, $C^P_{\tau 1}$. The location costs and production capacities should be set so that the linearization approximates the non-linear cost function as well as possible while keeping the runtime acceptible. A higher number of production locations will be able to approximate the cost function better, but is also likely to increase the running time of potential solution methods.

As observed in figure 5.4, the goal function will be incentivized to choose the "correct" costs as long as the cost function is monotonely increasing. In the rightmost chart, c4 is lower than c3 and will therefore be preferred, breaking the intention when the linearization was constructed. If $c_{\tau\gamma} \geqslant c_{\tau(\gamma+1)}$ for a specific location $\gamma$, it can be handled by logical constraints. Equation (5.49) allows $\delta^q_{\tau\gamma} = 1$ if and only if $\bar{Q}_{\tau\gamma} - q_{\tau\gamma} = 0$, that is, if and only if the production of location $\gamma$ is maximized. Then, equation (5.50) allows $q_{\tau(\gamma+1)}$ to be nonzero if and only if $\delta_{\tau\gamma} = 1$. Here, $\mathcal{M}^q_{\tau\gamma} = \bar{Q}_{\tau\gamma}$ and $\mathcal{M}^q_{\tau(\gamma+1)} = \bar{Q}_{\tau(\gamma+1)}$.

$$\mathcal{M}^q_{\tau\gamma}(1 - \delta^q_{\tau\gamma}) - \bar{Q}_{\tau\gamma} + q_{\tau\gamma} \leqslant 0, \quad \tau \in \mathcal{T} \tag{5.49}$$

$$q_{\tau(\gamma+1)} - \mathcal{M}^q_{\tau(\gamma+1)}\delta^q_{\tau\gamma} \leqslant 0, \quad \tau \in \mathcal{T} \tag{5.50}$$

$$\delta^q_{\tau\gamma} \in \{0, 1\}, \quad \tau \in \mathcal{T} \tag{5.51}$$

The actual production for delivery product $\tau$ equals the sum of the productions at all locations, and the variable production cost equals the weighted average cost from all locations:

$$q_\tau - \sum_{\gamma\in\Gamma} q_{\tau\gamma} = 0, \quad \tau \in \mathcal{T} \tag{5.52}$$

$$C^P_\tau(q_\tau) \cdot q_\tau - \sum_{\gamma\in\Gamma} c^P_{\tau\gamma}(q_{\tau\gamma}) \cdot q_{\tau\gamma} = 0, \quad \tau \in \mathcal{T} \tag{5.53}$$

$$0 \leqslant q_{\tau\gamma} \leqslant \bar{Q}_{\tau\gamma}, \quad \tau \in \mathcal{T}, \gamma \in \Gamma \tag{5.54}$$

In figure 5.4 and the following equations, non-negative production has been assumed. However, the concept generalizes to units with potential for negative production too (for instance pumped hydro). If there are several cases where $c_{\tau\gamma} \geqslant c_{\tau(\gamma+m)}$ for any $m \in \mathcal{N}$, many such constraints must be implemented. Also, figure 5.4 and the following equations demonstrate the case where the cost takes a one-dimensional input, namely the current production level. If significant intertemporal costs dictate that the cost function should take the former production level as input too, the concept generalizes to two-dimensional inputs too. In this case, the cost function is a surface in a three-dimensional space, that can be approximated by rectangular planar segments.

#### 5.3.2.3 The bid clearing non-linearities

Constraints (5.21) and (5.22) must be handled by logical variables indicating whether the clearing of a bid in time step $t$ is volume-constrained or price constrained.

Table 5.6: Section-specific symbols (Bid clearing non-linearities)

| Symbol | Interpretation |
|---|---|
| $\delta^{PC}_{t_b t\tau}$ | Logical variable indicating that the clearing of the bid $b_{t_b\tau}$ is price constrained in $t$. |
| $\hat{v}^{C}_{t_b t\tau}$ | The expectation in $t$ of the clearing volume for the bid $b_{t_b\tau}$ that will yield the most profit. |
| $\delta^{VC}_{t_b t\tau}$ | Logical variable indicating that the clearing of the bid $b_{t_b\tau}$ is volume constrained in $t$. |

If the residual bid volume $\widetilde{b}^{V}_{t_b t\tau}$ is smaller than the total accessible demand $\sum_{i\in D_{t_b t\tau}} d^{V}_{t_b t\tau i}$, the bid clearing is volume-constrained. Otherwise, it is price-constrained. Before formulating the constraints, it is explained why it is insufficient to say that $v^{C}_{t_b t\tau} - \widetilde{b}^{V}_{t_b t\tau} \leqslant 0$ and $v^{C}_{t_b t\tau} - \sum_{i\in D_{t_b t\tau}} d^{V}_{t_b t\tau i} \leqslant 0$ must hold.

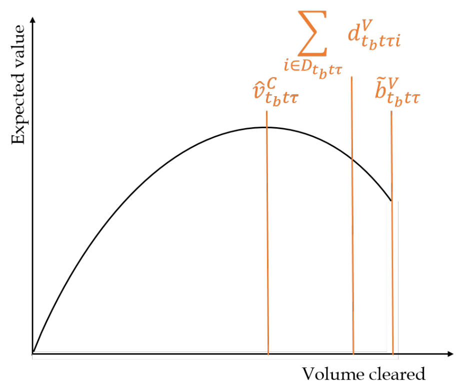



Figure 5.5: The expected value of bid clearing as a function of clearing volume for one hypothetical bid.

As can be observed from figure 5.5, there may exist bids for which the optimal clearing volume $\hat{v}^{C}_{t_b t\tau}$ is smaller than both $\widetilde{b}^{V}_{t_b(t-1)\tau}$ and $\sum_{i\in D_{t_b t\tau}} d^{V}_{t_b t\tau i}$. Recall that the former is the residual volume of a certain bid in a certain timeslot and that the latter is the sum of the buy bid volumes that can be used to clear the bid in the given timeslot. This may be the case even if the expected value of bid clearing is positive for all clearing volumes, which is a sufficient but not necessary criterium to determine that the bid has not been killed. The reason why this problem may arise is that bid clearing is semi-continuous whereas bid killing is binary; therefore, if a formerly placed bid at a later point is deemed to have a larger-than-optimal residual volume, the residual bid volume cannot be reduced at will. One example of when the payoff curve in figure 5.5 may be accurate is if $t = \bar{t}^{GC}_{\tau}$ and the expected production of a photovoltaic plant equals previous commitments plus $\hat{v}^{C}_{t_b t\tau}$; in this case, clearing $\hat{v}^{C}_{t_b t\tau}$ is equal to minimizing the expected imbalance cost. If it is the case that $\hat{v}^{C}_{t_b t\tau}$ is smaller than both $\widetilde{b}^{V}_{t_b(t-1)\tau}$ and $\sum_{i\in D_{t_b t\tau}} d^{V}_{t_b t\tau i}$, none of the constraints are binding, even though at least one of them should be binding according to the market rules. Therefore, binary variables will have to be used to make the bid either volume constrained or price constrained.

Constraints (5.24) and (5.25) already encertain that the clearing volume is smaller than or equal to the accessible residual demand. The clearing volume should also be smaller than or equal to the residual bid volume. This is handled in equations (5.55) and (5.56).

$$v^C_{t_b t\tau} - \widetilde{b}_{t_b(t-1)\tau} \leqslant 0, \quad t_b < t, t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T} \tag{5.55}$$

$$v^C_{t_b t_b \tau} - b_{t_b \tau} \leqslant 0, \quad t_b \in T_\tau, \tau \in \mathcal{T} \tag{5.56}$$

In addition, the cleared volume should be either price-constrained or volume-constrained. In equation (5.57), this is denoted by $\delta^{PC}_{t_b t\tau}$ and $\delta^{VC}_{t_b t\tau}$ respectively.

$$\delta^{PC}_{t_b t\tau} + \delta^{VC}_{t_b t\tau} = 1 \quad t_b \leqslant t, t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T} \tag{5.57}$$

$\delta^{PC}_{t_b t\tau}$ and $\delta^{VC}_{t_b t\tau}$ should only be allowed to take value 1 if the respective constraints actually are binding. This is handled by equations (5.58), (5.59) and (5.60), all of which are on the same form. In equation (5.58), $\mathcal{M}^{PC}_{t_b t\tau} = \sum_{i \in D_{t_b t\tau}} d^V_{t_b t\tau i}$, and in equations (5.59) and (5.60) $\mathcal{M}^{VC}_{t_b t\tau} = \widetilde{b}^V_{t_b(t-1)\tau}, \quad t_b < t$ and $\mathcal{M}^{VC}_{t_b t_b \tau} = b^V_{t_b \tau}$ respectively. Equation (5.61) defines the values that the logical variables can take.

$$\mathcal{M}^{PC}_{t_b t\tau}(1 - \delta^{PC}_{t_b t\tau}) - \sum_{i \in D_{t_b t\tau}} d_{t_b t\tau i} + v^C_{t_b t_b \tau} \geqslant 0, \quad t_b \leqslant t, t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T} \tag{5.58}$$

$$\mathcal{M}^{VC}_{t_b t\tau}(1 - \delta^{VC}_{t_b t\tau}) - \widetilde{b}^V_{t_b(t-1)\tau} + v^C_{t_b t_b \tau} \geqslant 0, \quad t_b < t, t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T} \tag{5.59}$$

$$\mathcal{M}^{VC}_{t_b t_b \tau}(1 - \delta^{VC}_{t_b t\tau}) - b^V_{t_b \tau} + v^C_{t_b t_b \tau} \geqslant 0, \quad t_b \in T_\tau, \tau \in \mathcal{T} \tag{5.60}$$

$$\delta^{PC}_{t_b t\tau}, \delta^{VC}_{t_b t\tau} \in \{0, 1\} \quad t_b \leqslant t, t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T} \tag{5.61}$$

**To summarize**, there were three non-linearities in the original model. In this section, all of them have been reformulated to integer-linear formulations. When incorporating these changes in the basic MSIP, a new problem formulation is referred as the basic *Multistage Stochastic Integer Linear Program* (MSILP).

### 5.3.3 Model extensions

While the basic MSILP may be sufficient to trade in the ID market, there are several extensions that may make the model more realistic or flexible without adding too much complexity. In this section, some moderate extensions are proposed. Wherever new symbols are introduced, a table describing each new symbol is also included.

#### 5.3.3.1 Chance-constrained OTR with target/shortfall

Table 5.7: Section-specific symbols (Chance-constrained OTR)

| Symbol | Interpretation |
|---|---|
| $N^{OTR}$ | Allowed frequency of exceedance per month. |
| $\mathcal{D}$ | The set of days of the current month. |

The OTR restriction can be represented as a chance constraint, stating that the probability of placing fewer bids than $R^{OTR}$ should exceed the probability of this constraint to hold set by the regulating authorities. As the allowed *frequency of exceedance* of the OTR limit $R^{OTR}$ is denoted $N^{OTR}$, and the number of delivery products per month is equal to the number of delivery products per day ($|\mathcal{T}|$) multiplied by the number of days of the current month ($|\mathcal{D}|$), the probability of complying with the constraint should be $1 - \frac{N^{OTR}}{|\mathcal{T}||\mathcal{D}|}$. The chance constraint is represented mathematically in equation (5.62):

$$\Pr\{\sum_{t_b \in T_\tau} \delta^B_{t_b t \tau} - R^{OTR} \sum_{t_b \in T_\tau} (\delta^C_\tau + \delta^C_{t_b t \tau} \quad \leqslant \quad 0\} \geqslant 1 - \frac{N^{OTR}}{|\mathcal{T}||\mathcal{D}|}, t = \bar{t}^{GC}_\tau, \tau \in \mathcal{T} \tag{5.62}$$

Recall from section 2.4 that for a given delivery product, the acceptable OTR before an alert is triggered is currently $R^{OTR} = 50$. The first 4 alerts are free every month, but some OTR alerts may be acceptable every month. Recall that the reason for putting a hard constraint on the number of OTRs is to avoid legal repercussions, not to avoid the OTR fine altogether. Therefore, $N^{OTR} = 10$ would probably serve that purpose well without restricting the model unreasonably.

In that case, equation (5.62) states that if 100 bids are placed for a delivery product, the probability that at least two bids clear should be at least 98.6%. This may look non-convex, as it is an either/or statement ("either these two bids should clear or those two bids should clear" et cetera) and either/or statements are non-convex. However, the order in which bids are cleared is known; the clearing is sorted by the bid price.

Therefore, the original statement is equivalent to saying that the probability of the second cheapest bid clearing should be 98.6%. Recall from section 4.1 that this is the exceedance probability of the price with the bid price as the critical value. While this is non-trivial to learn for a non-parametric probability distribution for the price, the ADP algorithm proposed in section 6.3.2 will have to learn it anyways in order to estimate the expected value of a bid. In the mean time, in this constraint it may be assumed that the price is a Gaussian process, for which the exceedance probability is known Wikipedia (2017e). This is a conservative assumption, since the autoregressive properties that create correlation between consecutive time steps is disregarded. Thus, the price change over time is assumed to be the sum of uncorrelated variables, which has less variance than the sum of positively correlated variables ceteris paribus.

To incorporate the costs if more than four alerts are sent within a month, a target-shortfall model as described in King and Wallace (2012) could be used. The reward for overperforming compared to the target would be zero, and the penalty for breaching the target would equal the cost of 100€ per alert.

#### 5.3.3.2 Subsidy cost

As described in section 2.4, renewable producers receive a market premium on the energy they sell in the ID and DA markets. This premium is on a per-unit-sold basis, so it will simply be added to the transaction costs with an opposite sign (positive contribution to the profits). This is referred to as *subsidy costs* in section 2.5.2.

#### 5.3.3.3 Over-the-counter trade

Table 5.8: Section-specific symbols (OTC trade)

| **Symbol** | **Interpretation** |
|---|---|
| $t_\tau^{OTC}$ | Timeslot where OTC trade is allowed |
| $p_\tau^{OTC}$ | Predetermined OTC price of $\tau$. |
| $v_\tau^{VOTC+}$ | Volume of power bought in OTC of $\tau$. |
| $v_\tau^{VOTC-}$ | Volume of power sold in OTC of $\tau$. |
| $b_\tau^{VOTC+}$ | Volume of trader's buy-bid in OTC of $\tau$. |
| $b_\tau^{VOTC-}$ | Volume of trader's sell-bid in OTC of $\tau$. |
| $d_\tau^{VOTC+}$ | Stochastic volume of opposite party's buy-bid in OTC of $\tau$. |
| $d_\tau^{VOTC-}$ | Stochastic volume of opposite party's sell-bid in OTC of $\tau$. |

As mentioned in section 2.1, OTC trade is bilateral trade outside of the spot power exchanges. One typical use case for OTC trade is that if a trader was unable to close their position in the ID market, the OTC market may be used as a last resort to avoid imbalance costs. As the window between ID gate closure $\bar{t}_\tau^{GC}$ and time of delivery $\bar{t}_\tau^{D}$ is very short, a price norm may be negotiated in bilateral agreements in advance, or promoted by the spot exchange (Rademaekers et al., 2008). Here, a prior bilateral agreement is assumed with price $p_\tau^{OTC}$. It is assumed that each actor may unilaterally determine their maximum trading volume in the OTC for delivery product $\tau$, and that the clearing transaction volume will equal the smallest of the bid volumes.

If this is the case, the expression $-p_\tau^{OTC} \cdot \sum_{\tau \in \mathcal{T}} (v_\tau^{OTC+} - v_\tau^{OTC-})$ may be added at the end of the goal function. Here, $v_\tau^{OTC+}, v_\tau^{OTC-}$ denotes the volumes of positive and negative volume OTC transactions. Also, let $t_\tau^{OTC}$ be the timestep

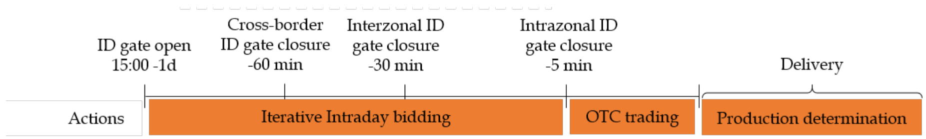



Figure 5.6: Market structure including OTC trade

when OTC trade is allowed and $b_\tau^{VOTC+}$ or $b_\tau^{VOTC-}$ be the OTC bid volume (decision variable). Similar to the ID market, the OTC transaction volume is bounded by the opposite party's OTC bid volume, which is stochastic, as the counterparty in the bilateral contract may or may not place a matching bid. This is expressed in (5.63)-(5.67). Also, equation (5.15) must be updated to include the net OTC volume, as illustrated in equation (5.68).

$$b_\tau^{VOTC+} - v_\tau^{OTC+} \geqslant 0, \quad \tau \in \mathcal{T} \tag{5.63}$$

$$b_\tau^{VOTC-} - v_\tau^{OTC-} \geqslant 0, \quad \tau \in \mathcal{T} \tag{5.64}$$

$$d_\tau^{VOTC-} - v_\tau^{OTC+} \geqslant 0, \quad \tau \in \mathcal{T} \tag{5.65}$$

$$d_\tau^{VOTC+} - v_\tau^{OTC-} \geqslant 0, \quad \tau \in \mathcal{T} \tag{5.66}$$

$$b_\tau^{VOTC-},\ d_\tau^{VOTC-},\ v_\tau^{OTC-}, b_\tau^{VOTC+},\ d_\tau^{VOTC+},\ v_\tau^{OTC+} \geqslant 0, \quad \tau \in \mathcal{T} \tag{5.67}$$

$$\sum_{t_b \in T_\tau} \sum_{t=t_b}^{\bar{t}_\tau^{GC}} v_{t_b t \tau}^{C} + V_\tau^{DA} + v_\tau^{OTC-} - v_\tau^{OTC+} + v_\tau^{BM-} - v_\tau^{BM+} - q_\tau = 0, \quad \tau \in \mathcal{T} \tag{5.68}$$

Note that unlike in the Intraday market, OTC bid volume is a continuous variable. Thus, the issues in section 5.3.2.3 don't arise in this case, which simplifies the equations. Also, while other use cases for OTC trade also exists, they have been disregarded here as they have less impact on the optimal Intraday trading strategy.

#### 5.3.3.4 Production curtailment

Table 5.9: Section-specific symbols (Production curtailment)

| **Symbol** | **Interpretation** |
|---|---|
| $o_\tau$ | Overflow (MWh). Storage voluntarily discarded by the producer. |

Producers with co-located storage often have the opportunity to curtail their production without having to store the energy. For instance, overflow tunnels may be installed in hydropower dams. Production curtailment can be incorporated in the model by introducing overflow variables for each delivery product $o_\tau$ in equation (5.11) as illustrated in equation (5.69).

$$s_{\tau+1} - s_\tau - o_\tau - q_\tau \quad = \quad -f_\tau, \quad \tau \in \mathcal{T} \tag{5.69}$$

The overflow, naturally, is non-negative;

$$o_\tau \quad \geqslant \quad 0, \quad \tau \in \mathcal{T} \tag{5.70}$$

#### 5.3.3.5 Cap on the monetary value of bids

Table 5.10: Section-specific symbols (Cap on the monetary value of bids)

| Symbol | Interpretation |
|---|---|
| $C^{max}$ | Maximum monetary value of the bids in a day. |

As described in Jansen (2016), clearing members (see section 2.3) may cap the total monetary value of the bids placed in the ID market throughout a day. If this monetary cap is denoted $C^{\max}$, this can be modeled using constraint (5.71):

$$\sum_{t_b \in T_\tau} \sum_{\tau \in \mathcal{T}} b^P_{t_b \tau} b^V_{t_b \tau} \quad \leqslant \quad C^{\max} \tag{5.71}$$

#### 5.3.3.6 Bid execution restrictions

Recall from table 2.1 that bids may be placed with the execution restrictions IOC, FOK, AON or regular. At face value this might look like added complexity, but in section 6.3.4.1 it is demonstrated that for any bid price/volume combination there exists one optimal and easily identifiable bid restriction. This is therefore trivial to implement; for all bids, choose the optimal execution restriction.

#### 5.3.3.7 Market impact

The EPEX Intraday is still not a perfectly efficient market, so large traders may be able to impact the price trajectory for a delivery product. While short-term price impact is handled by the assumption that the set of revealed residual demand bids is finite and with heterogeneous prices, the long-term price impact from placing large bids (or many consecutive bids) has been neglected in the model so far. If the bids from one large trader has the potential to shift the market equilibrium, and the trader doesn't take it into account, the shift is always going to be in a disfavorable direction for the trader.

The impact this has on the goal function is hard to evaluate, as the counterfactual price trajectory if the bid is not placed cannot be observed. An intuitive approach would be to measure if prices fall in the wake of large bids being placed in the market, but this may be inaccurate too; if large bids are only placed when traders expect the price to drop, there might be a selection bias where large bids are only placed before price drops that would have happened anyways. Aid et al. (2015) and Tan and Tankov (2016) both handle the case with market impact, but make assumptions about the form of the impact without justifying them rigorously. The insights from the former paper are proposed as priors for good trading policies in section 6.3.2. While this to some degree will reward policies that have low market impact, more research is needed in this area to determine how large bids (or many consecutive bids) affect the price.

#### 5.3.3.8 Ramping constraints

Table 5.11: Section-specific symbols (Ramping Constraints)

| Symbol | Interpretation |
|---|---|
| $R_{\tau}^{ramp}$ | Highest possible ramping-up speed. |

As seen in the papers on the Unit Commitment Problem in section 4.2 ((Pang and Chen, 1976), (Ostrowski et al., 2012), (Trivedi et al., 2012) and (Bertsimas et al., 2013)), thermal power plants are typically modeled with ramping constraints. While this report doesn't focus on the per-plant modeling of production, this might impose ramping constraints on a per-trader level. If that is the case, the following constraint (which is a simplified version of the constraint proposed by Ostrowski et al. (2012)) should be added to the physical constrains:

$$q_{\tau} - q_{\tau-1} \quad \leqslant \quad R_{\tau}^{ramp}, \quad \tau \in \mathcal{T} \tag{5.72}$$

It may also be necessary to constrain the speed of ramping down, if the physics of the production facility dictates it. As is further discussed in section 6.3.2.5, these constraints create much stronger interconnections between different delivery products, and therefore makes the problem considerably harder to solve.

#### 5.3.3.9 Partial bid killing

Table 5.12: Section-specific symbols (Partial bid killing)

| Symbol | Interpretation |
|---|---|
| $v_{t_b \tilde{t} \tau}^{K}$ | Volume killed in time step $t$ of bid $b_{t_b \tau}$. |

In the EPEX Technical documentation EPEX (2017i), it is referred to a feature for modifying bids after they have been placed. While the feature is not described in detail, the documentation clearly shows that it is possible to adjust the size of placed bids downwards. This can be implemented removing $\delta^K_{t_b t\tau}$ and replacing it with a $v^K_{t_b t\tau}$. The killed volume variable will then be added to equation (5.23), preventing the residual volume to go below zero:

$$\sum_{\widetilde{t}=t_b}^{t} (v^K_{t_b\widetilde{t}\tau} + v^C_{t_b\widetilde{t}\tau}) + \widetilde{b}^V_{t_b t\tau} - b^V_{t_b\tau} \quad = \quad 0, \quad t_b \leqslant t, t_b \in T_\tau, t \in T_\tau, \tau \in \mathcal{T} \tag{5.73}$$

To avoid having to change other constraints, any (partial) killing of a bid will have to be determined after the clearing in that time step happened. Otherwise, the sum of clearing and killing in a time step may have sent the residual volume below zero. Formerly, all decisions including the bid killing happened before the clearing.

#### 5.3.3.10 Imbalance premium

Table 5.13: Section-specific symbols (Imbalance premium)

| Symbol | Interpretation |
|---|---|
| $C^{BM+}$ | Multiplier on the cost of trading in the up-regulating market. |
| $C^{BM-}$ | Multiplier on the cost of trading in the down-regulating market. |

In addition to capping the maximum volume traded in the balancing markets, it may be desirable to add a premium to the imbalance price to capture the disutility of the risk associated with BM trades in the model. This premium can be implemented as a multiplier, $C^{BM+}, C^{BM-}$ on the imbalance price. In the objective function, the balancing market term is replaced by: $-\sum_{\tau\in\mathcal{T}}(C^{BM+}p^{BM+}_\tau v^{BM}_\tau \quad - \quad C^{BM-}p^{BM-}_\tau v^{BM}_\tau)$.

## 5.4 Applying the model to the needs of a specific type of trader

The model in section 5.3.1 includes all kinds of traders in the EPEX Intraday market; producers of hydropower, thermal power, intermittent power and combinations of these, demand aggregators, power retailers, and purely financial traders. In this section, the simplifications that each class of trader may do are considered more closely.

**Hydropower producers** is a group composed of three different kinds of power plants: flow-of-the-river hydropower, hydropower dams with storage, and pumped hydro. The first category has neither storage capacity nor flexibility of production; $s_\tau = 0$, $\tau \in \mathcal{T}$, so equation (5.11) reduces to say that production is equal to inflow of energy - similarly to an intermittent producer. Equation (5.12) is also superfluous, and equation (5.13) becomes a non-negativity constraint on the production variable. Equation (5.14) is superfluous as long as the support of the stochastic inflow variable $f_\tau$ is limited to $[0, \bar{Q}_\tau]$. hydropower dams provide the storage described by equation (5.11) and (5.12), but production is still limited below to 0 absent hydro pumping technology. For pumped hydro, negative production is possible. The inefficiencies in the pumping process are modeled as positive costs related to negative production, extending the marginal cost curve into the negative domain. It is worth noting that the marginal cost of production for hydropower is near 0, save for the alternative cost of later usage in the case of hydropower dams. This is what is is referred to as Value of Storage (see section 2.5.2). As explained in section 2.4 and 5.3.3, hydropower counts as a renewable energy source according to the EEG 2014 and thus receives a market premium for each MWh sold. This is added to the transaction cost with an opposite sign, and referred to as a subsidy cost.

**Thermal power producers** typically don't have storage capacity nor access to negative production, due to the irreversibility of the energy conversion processes in such plants. Equations (5.11) and (5.12) can therefore be discarded, and (5.13) is transformed into a non-negativity constraint on the production variable. The intertemporal costs are particularly high for thermal producers. Also, ramping constraints may force a thermal producer to make the production decision $q_\tau$ before $\bar{t}_\tau^{GC}$, but this does not make the problem significantly harder (Aid et al., 2015). Some thermal producers count as renewables according to the EEG 2014, and thus also receive the subsidy costs.

**Intermittent producers** without co-located storage can be modeled similarly to flow-of-the-river hydropower producers, albeit with a markedly different probability distribution for the inflow of power. Notably, solar- and wind production often has higher short-term variance, and solar power has a strong daily cyclic component. Intermittents with co-located storage can be modeled similarly to pumped hydro producers, as the battery park can also draw power from the grid. Current battery technology makes the storage limitations on such producers much tighter than for pumped hydro plants, however. As elaborated in section 3.2, the Value of Storage in batteries is conceptually similar to the water value of stored hydro and denotes the alternative cost per unit of current production. Intermittents also count as renewables according to the EEG 2014, and thus also receive the subsidy costs.

**Multi-asset producers**, assuming that they have solved the unit commitment problem for different production volumes and scenarios throughout the day, have a marginal cost curve composed of several kinds of production units. This marginal cost curve will contain all the costs that are relevant for at least one of their power plants. Similarly, all of the constraints that are relevant for single-asset producers with similar production units as at least one of the production units of the multi-asset producer, are relevant for multi-asset producers too. However, the production and storage capacities of the multi-asset producer equals the sum of the capacities of the different power plants, giving the multi-asset producer more flexibility than single-asset producers with similar plants would have had. This flexibility is one of the reasons why some producers form BRPs together, as they may be able to partly cancel out their individual imbalances with over-the-counter trade post-gate closure.

**Demand aggregators** typically have some flexible loads, and thus there is a difference between their lower and upper bound in equations (5.13) and (5.14) respectively. Unless the given consumers have access to local production, both of these bounds will be negative. The flexible load may for instance be boilers that need to keep the water within certain temperature bounds, in which case it can be modeled similarly to a heat reservoir, using the constraints of equations (5.11) and (5.12). The marginal cost of production will consist of the alternative cost known as the value of storage, in addition to the cost of reduced comfort for the consumers, if that is quantified in the agreement with the demand aggregator.

**Power retailers**, unlike demand aggregators, rarely have neither flexibility of consumption nor storage capacity. In this sense, they are similar to intermittent producers without co-located storage, except that the expected production is negative. As covered in section 2.1, the demand has several seasonalities with different cycle durations.

**Purely financial traders** also have no flexibility of consumption nor storage capacity, but unlike intermittent producers and power retailers, their expected production is typically deterministic, and equal to 0. As all inflexible producers they have marginal costs of production equal to 0, but are exposed to balancing market risks.

To summarize, the financial constraints are equal for all traders, but the physical constraints as well as the marginal costs of production are different depending on the accessibility of storage and the production flexibility. Also, inflexible producers are more exposed to balancing market risk. *Renewable producers* as defined in the EEG 2014 receive a market premium that is added to the transaction costs. Table 5.14 shows which factors determine the modeling choices of each trader.
This paragraph contains an explanation of table 5.14. Here, "Production flexibility" denotes that $\underline{Q}_\tau$ and $\bar{Q}_\tau$ may take different values, and values different from $f_\tau$. "$s_\tau$?" denotes whether energy may be stored in some form of a co-located storage device or reservoir. The "Energy inflow" column describes the nature of the primary energy resource. Note that it is zero for purely financial traders, and controllable only for thermal producers. Moreover, it is trivial to observe that either storage capacity or a controllable energy inflow is needed to make the production flexible. "Production reversibility" denotes whether $\underline{Q}_\tau$ and $\bar{Q}_\tau$ may have opposite signs. Finally, the "Marginal costs" column describes the key drivers of marginal costs for the different producers. *VoS* is short for Value of Storage, which is the alternative cost of spending stored energy now

Table 5.14: Determining factors for how to model each type of trader

| **Trader type** | **Production flexibility** | $s_\tau$**?** | **Energy inflow** | **Production reversibility** | **Marginal costs** |
|---|---|---|---|---|---|
| Hydro (Flow-of-the-river) | No | No | Stochastic | No | 0, but high BM risk |
| Intermittent w/o storage | No | No | Stochastic | No | 0, but high BM risk |
| Power retailer | No | No | Stochastic | No | 0, but high BM risk |
| Financial trader | No | No | Deterministic | No | 0, but high BM risk |
| Thermal power | Yes | No | Controllable | No | Fuel, intertemporals |
| Hydro (with dam) | Yes | Yes | Stochastic | No | VoS, (intertemportals) |
| Demand aggregators | Yes | Yes | Stochastic | Rarely | VoS, "comfort cost" |
| Intermittent w/ storage | Yes | Yes | Stochastic | Yes | VoS |
| Hydro (Pumped) | Yes | Yes | Stochastic | Yes | VoS, (intertemportals) |

rather than later.

# 6 Discussion of solution methods

This chapter contains discussions about how the mathematical models presented in chapter 5 affect the selection of solution methods. In particular, in section 6.1 some of the most relevant issues of the different types of price forecasting models are presented. Section 6.2 briefly discusses the Cost Estimation Problem, before section 6.3 contains a description of appropriate solution methods for the Strategy Formulation Problem. The proposed solution framework is evaluated in light of the requirements of a Automated Intraday Trading algorithm proposed in section 2.6.1 in section 6.4. Note that neither of the presented solution methods have been implemented in conjunction with this report and the discussions are therefore performed in a cursory manner.

## 6.1 Solving The Price Forecasting Problem

This section starts out discussing some tradeoffs related to the choice of parametric or non-parametric forecasting. Based on this discussion and the model in 5.1, solution methods that address the key challenges related to the proposed non-parametric model are proposed.

### 6.1.1 Parametric versus non-parametric forecasts

As explained in sections 3.3 and 5.1 the fundamental non-parametric model is chosen because it looks neglected, and the information gain from attempting to model prices more rigorously may result in improved methods. It is pointed out that the currently popular assumption of a Gaussian process as the price dynamic is unable to capture both negative prices (in the case of a GBM) and skewness and kurtosis (in the case of a SBM) simultaneously.

In addition to this, parametric methods fail to capture shifts in the shape of the probability distribution if the price dynamics fundamentally change. Garnier and Madlener (2015) states that "*the real benefit of the model lies in its flexibility. The Intraday market is fairly illiquid, subject to frequent changes (...) Fixed strategies (...) cannot provide sufficient flexibility.*". This argument works in favor of a non-parametric model, as no assumptions about the shape of the distribution are made. It also supports the exponential smoothing employed in section 5.1, as recent data will be more relevant.

Non-parametric models also have their drawbacks. In section 5.1, it is pointed out that they can grow computationally expensive, as well as costly if the required training data is expensive. While avoiding assumptions about the shape of the probability distribution reduces the modeling risk, it drastically increases the degrees of freedom and thus is exposed to overfitting. To avoid overfitting (Wikipedia, 2017n) in the regions where data is sparse, the window of the kernel density estimation must be so large that it underfits in the dense regions of the data set (Dyrdal, 2017), or the data set must be very large.

Another drawback with non-parametric models is that less research exists on each distribution, as the distribution will be uniquely defined by the data set. One example of this is seen in section 5.3.3.1, where the exceedance probability for the normal distribution is used since it is possible to derive theoretically. The lack of fundamental theory can to some degree be compensated by even more empirical estimation.

### 6.1.2 Overfitting versus underfitting

As the window size is kept constant for the kernel density estimator, whereas the data density is not, there may be orders of magnitude of difference between the amount of information within windows in different regions of the support of the probability distribution. Thus, the algorithm will inevitably overfit in some regions and underfit in others. One way to compensate for this is to make the window size variable, and rather keep the number of data points within the window constant. This is called a k-Nearest Neighbors (kNN) method for estimating the posterior probability distribution (Dyrdal, 2017).

Both kernel density estimators and kNN-methods face issues with long solution times and large memory requirements. For both methods, several speedups exist (Dyrdal, 2017). The kNN method may be harder to combine with exponential smoothing of the weights of the data points, since it uses the number of data points as the definition of the size of the window. In the end, empirical analysis will have to determine which method is better suited to forecast future prices within the practical constraints.

### 6.1.3 Transaction price forecasting versus buy bid forecasting

As explained in section 4.1, transaction price forecasting has several advantages over buy bid forecasting; primarily that it contains information about the supply side too, and thus is more decision relevant. The weakness with transaction price forecasting is that it contains no information about the order depth or the slope of the residual demand curve. It is thus better suited the more liquid the market is, and vice versa. Until the Intraday market becomes more liquid, information about the order depth of the residual demand as a function of price is necessary, however. Thus, the transaction price forecast should be coupled with a method to estimate the order depth. A simple model could be based on the average slope or shape of the residual demand curve. More advanced models could estimate the conditional probability distribution, based on the current state.

## 6.2 Solving The Cost Estimation Problem

As explained in section 3.2, this problem is likely too hard to solve to optimality, so the algorithm will have to be heuristics-based. Also, it must be supported by a forecast of the power produced by intermittent producers, an imbalance price forecast, a Value of Storage estimate, a generator availability forecast and forecasted fuel prices. The Cost Estimation Problem is assumed solved by the producer and will not be explained further here.

## 6.3 Solving The Strategy Formulation Problem

### 6.3.1 Comparison of candidate solution methods

Going forward, two inexact methods for solving the Strategy Formulation Problem are discussed, namely Approximate Dynamic Programming and Stochastic Dual Dynamic Programming. The focus of this section is on what makes the solution methods appropriate for the Strategy Formulation Problem or not, and not particularly on how the algorithms operate. Discussions of what adaptations of the MSIP presented in section 5.3.1 that are necessary for the algorithms to converge is also provided.

#### 6.3.1.1 Approximate Dynamic Programming

Approximate Dynamic Programming (ADP) is a method where the value function of a Markov Decision Process (MDP) is approximated iteratively using Monte Carlo simulation. The method is thoroughly described in (Powell, 2014). To be able to apply the ADP method to the SFP, first of all it is necessary to reformulate the basic Multistage Stochastic Integer Program to its MDP equivalent. For a decision process to be a MDP, the *Markov property* must hold. That is, conditional on the current state, the upcoming state must be independent of all states prior to the current state (Ghahramani, 2001). Powell (2014) defines the state variable of a MDP as "*(...) the minimally dimensioned function of history that is necessary and sufficient to compute the decision function, the transition function, and the contribution function.*". Thus, a requirement of the reformulation of the MSIP of section 5.3.1 to a MDP is that enough state history is encapsulated into each stage so that no previous state is required to calculate the transition probabilities at any timestep. Also, the model reformulation should be dynamic in trading time, whereas the weakly coupled delivery products will be handled in a resource allocation master problem.

In ADP, the value function is represented as a look-up table. With an exponentially growing lookup table size as a function of the state variables, this does not scale very well itself. There are two reasons for this. One is that computers executing the ADP algorithm may run out of memory, and another is that only a relatively small subset of the entries of the lookup table is likely to be updated throughout the execution of the algorithm. One potential solution of the former is to approximate the lookup-table using an implicit representation, as in artificial neural networks. The latter could be resolved through updating related states in each iteration. That is, if the value of having placed an array of bids so far in timeslot $t$ is updated, then states corresponding to having placed similar bids so far in the same timeslot should also be updated.

#### 6.3.1.2 Stochastic Dual Dynamic Programming

Stochastic Dual Dynamic Programming (SDDP) is a method applied to multistage stochastic linear problems (Shapiro, 2011). In the same publication, the following assumptions of the SDDP are mentioned:

- Noises are time-independent, with finite support.
- Decision and state constraint sets are compact convex subset of finite dimensional space.
- There is a strict relatively complete recourse assumption.

When comparing these assumptions to the features of the Intraday Trading Problem, it is possible to argue that SDDP might not be the optimal solution method for solving the problem. As suggested by figure 2.6, the volatility of the Intraday transaction prices tend to increase as time of delivery is approaching, thus the noise is not time-independent. The fact that the bid withdrawal-variables are binary in the mathematical formulation of section 5.3.1 violates the second assumption. Finally, the behavior-shaping constraints violates the relative complete recourse assumption. This assumption can be described using the ITP as: *no matter what bid you place now, all combinations of bids placed in the future that are feasible now must also be feasible later.* This is clearly not true for the balancing market trading volume (equation 5.28) and the Order-to-Trade-Ratio (equation 5.29) restrictions.

SDDP can therefore not be applied directly to the SFP. To be able to take advantage of the SDDP approach, either some parts of the ITP must be relaxed, or some extensions of the SDDP must be used. Here, we will consider a combination of the two of them. Without some major upcoming change of EPEX regulation, the assumption regarding time-independent noise is hard to fulfill. However, Leclere (2015) states that this assumption is not necessary in order to have theoretical convergence if a tree-view is taken. In the case of multistage optimization problems, nested decomposition, as suggested in Birge (1985), takes this view and is an interesting path to follow. This being said, to the authors' best knowledge, the methods suggested in this paper are only valid for linear programs. If the decision variables of the problem have to be continuous, this means that the killing-variables must allow for values within the range $[0, 1]$. In practice, allowing continuous killing-variables corresponds to allowing bid volume changes, as discussed in section 5.3.3. Zou et al. (2016) presents another interesting approach, namely the *Stochastic Dual Dynamic integer Programming* (SDDiP). This method assumes only binary state variables, which is not the case in ITP. However, they also present a technique for approximating continuous and discrete state variables as binary state variables, so this challenge can potentially be omitted. The relative complete recourse assumption does only hold if the behavior-shaping constraints of the ITP are relaxed. One relevant relaxation method is Lagrangian relaxation as described by Lemarechal (2001), among others. Lagrangian relaxation corresponds to moving the relaxed constraints to the objective function and adding a cost of violating each of the relaxed constraints. In the case of ITP, this would introduce an incentive to reduce the number of placed bids and also to reduce the volume traded in the balancing market, even though this may not be optimal actions.

#### 6.3.1.3 Concluding remarks on the selection of solution method

Now, having discussed the two methods briefly, one can conclude that our formulation of the ITP is more likely to fit ADP than SDDP. The relaxations needed for the SDDP solution method to function do not maintain the desired structure of the problem. Although ADP is a good approximation for solving MDP, the algorithm may be insufficiently fast at approximating the value function of the ITP. Hassler (2017) faced the same challenge when applying ADP to a similar formulation of the short-term trading of renewable energy with co-located energy storage. Therefore, measures should be taken to make the algorithm as fast as possible. This is explored further in the following sections.

### 6.3.2 Applying Approximate Dynamic Programming

There are several ways the ADP framework can be applied to the ITP. In this section, two of them are described, the "Inter-$\tau$ forward pass" model and the "Construction heuristic, local search" model. The main difference between the two proposed models concerns the way inter-delivery product relationships are handled. As the authors have not implemented these particular solution methods, there is no guarantee that they overperforms others. When applying the ADP framework to ITP, it is therefore suggested to maintain an explorative mindset. This being said, most of the activities of the ADP framework coincide. This includes the representation of states, the set of actions, the transition functions and the reward function. In order to describe how the ADP framework could be applied to the ITP, these four elementary building blocks of any dynamic problem are explained first.

In the dynamic model corresponding to the MSIP of section 5.3.1, the state should encapsulate information about (for each delivery product) the total volume traded sold so far, the number of opened and closed bids, the revealed residual demand, the remaining time until gate closure, the most recent forecasts mentioned in section 5.1. Including all this information within each state is unlikely to be a good idea, as the state space grows exponentially with the number of state variables, referred to as the *curse of dimensionality* in relevant literature ((Lincoln and Rantzer, 2006), (Donoho et al., 2000)). Thus, some way to *merge similar states* must be considered in the state representation. The set of actions in a given state must equal those actions that do not violate the physical, financial or behavioral-shaping constraints of the MSIP. The policy is a mapping from state to action. The transition functions must describe possible next-step stages given a current stage, an action and some realization of the stochastic parameters of the MSIP. The reward equals the revenue from each of the matched bid of the trader less the costs associated with it. Thus, the reward clearly depends on the stage, the action and the realization of the stochastic parameters in the dynamic version of the MSIP.

Before revealing the details on the two approaches mentioned earlier, a brief, qualitative overview of the steps of the general ADP framework, described in (Powell, 2014), in the context of the ITP is presented. Note that the motivation for including the following paragraphs in this report is not to describe the method in detail. We refer to the (Powell, 2014) for further details.

#### 6.3.2.1 Initiate value table

Before entering the ADP loop, the value table must be initialized with some a priori approximation of the value function. Here, some heuristic rules should be applied, stating that for instance, the value of being in a state with $t_1$ remaining time until gate closure of the delivery product is better than to be in a state with $t_2$ remaining time if $t_1 > t_2$, ceteris paribus. Section 6.3.3.1 outlines examples of how to set accurate priors in the value table.

#### 6.3.2.2 Sample market data

In this phase, the market data is sampled. The sampling is the first part of the simulation process of this algorithm. The most relevant market data includes the orderbook data and some updated forecasts on future market quantities. The second part of the simulation process is to process the market data. That is, the newly arrived bids must be compared to the existing market data to create new transactions and thus also updating the trader's bids and their revealed residual demand.

#### 6.3.2.3 Compute optimal bids to be placed in all timeslots

This corresponds to what is referred to as *forward-passing* or *stepping forward* in the ADP literature. In this phase, optimal bids in given states are computed to create a path through the state space from $t = 0$ to $t = \bar{t}_{\tau}^{D}$ based on the values from the lookup table and the sampled market data. In this context, the optimal action (optimal bid) is determined using the Bellman equation. In short, the Bellman equation identifies the action that maximizes immediate profit plus discounted expected value of being in the state after the action is performed. Section 6.3.3.2 further outlines how to choose optimal actions.

#### 6.3.2.4 Recompute the entries of the value table

After having computed an optimal path through the state space, the new information is incorporated into the value table. That is, given the bid sequence suggested in the previous step, the expected value of being in the states before each of the bids were placed are updated. Thus, this must be done iteratively going from the newest stage until the first stage, which is also the reason for denoting the process *backward-pass* in the literature. Section 6.3.3.3 elaborates further on this concept.

The steps described in section 6.3.2.2-6.3.2.4 are repeated until some stopping criteria is met. At this point, when training is finished, the output of the algorithm will be a table indicating the expected value of placing a certain bid in a certain state.

#### 6.3.2.5 Coordinating decisions for consecutive delivery products

As already mentioned, the two proposed methods handles coordination between delivery products differently. That is, the behavior-shaping constraints of section 5.3.1 and eventual ramping constraints, covered as a potential model extension in section 5.3.3, must be handled in order to create globally feasible bidding strategies. The way that the inter-delivery product coordination is done in the two proposed models is illustrated in figure 6.1 and 6.2. Here, the rectangular boxes represent processes and diamonds represent decisions in the proposed framework. The steps of the ADP solution method described above are easily recognized as the diamond and processes inside the double-lined grey box of the flowcharts.

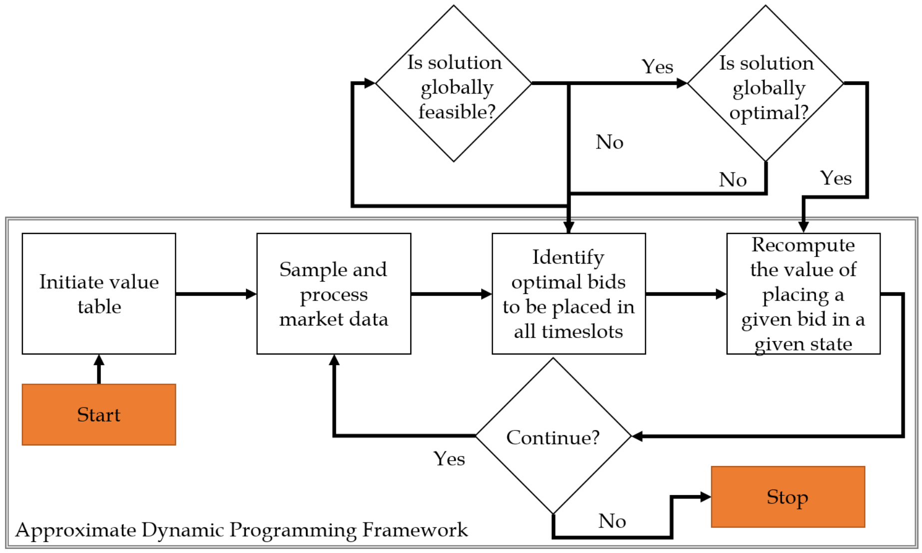



Figure 6.1: Flowchart of the "Inter-$\tau$ forward pass model" method

In the "Inter-$\tau$ forward pass model" method, the coordination between the delivery products are done in the forward pass step of the ADP framework. That is, the paths created through the state space for each delivery product are coordinated so that they are both globally feasible and globally optimal going forward.

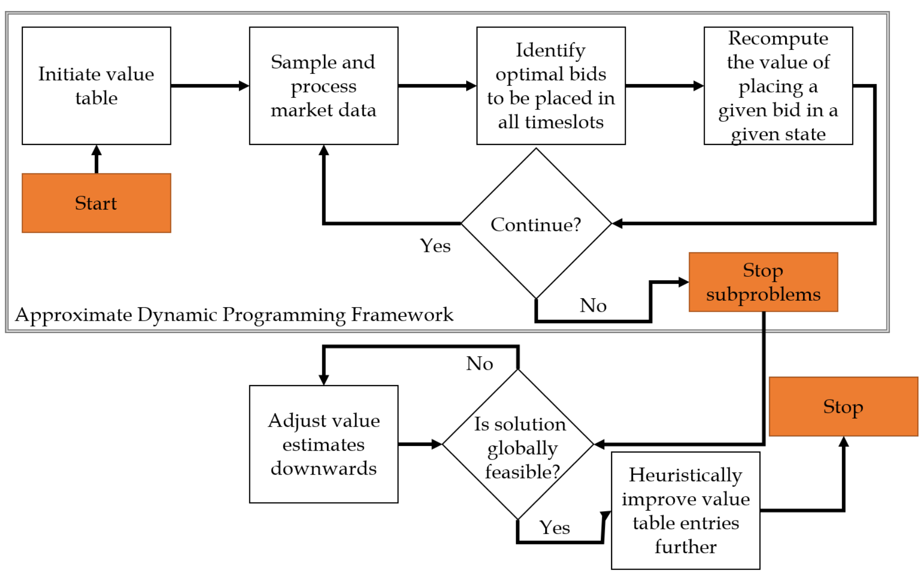



Figure 6.2: Flowchart of the "Construction heuristic, local search" method

In the "Construction heuristic, local search" method (figure 6.2), the coordination between the delivery products are done after the value tables of the different delivery products are created. In the construction phase, the problem is completely decoupled across the set of delivery products, and the ADP framework is used to build the value lookup table for each delivery product separately. Thus, otimal strategies are found for each decoupled subproblem, but these are not coordinated globally. The value tables created in this phase are therefore likely to suggest actions that are not globally feasible, thus a neighborhood search must be done in order to guarantee feasible trading strategies. Potentially, once feasible strategies are guaranteed, more neighborhood search could be applied to improve the already found feasible strategy.

### 6.3.3 Speeding up the learning process

There are several complex elements of the problem and the proposed solution methods. First of all, solving the problem require efficient handling of stochasticities in the Intraday bids and the imbalance prices. The state- and decision spaces of the dynamic representation of the ITP are huge, so some thoughtful techniques must be applied in order to solve the problem efficiently as possible. As the model of the Strategy Formulation Problem in this report is more extensive than those suggested in former literature, the temporal complexity of finding sufficiently good solutions to these problems are likely lower bounds on the temporal performance of our algorithms. The exact performance of the given model will vary with different traders, and can not be accurately estimated without a more rigorous case study. Having mentioned the performance challenges of the solution method described above, some potential enhancements of the initial solution method are introduced below. This section covers speedup in the process of estimating the entries in the value table, whereas section 6.3.4 focuses on the speedups that are relevant when actions need to be chosen in real-time.

#### 6.3.3.1 Setting accurate priors in the value table (simple heuristics)

The heuristics mentioned in section 6.3.2.1 can for instance be developed based on the existing literature on optimal Intraday trading. Aid et al. (2015) state that if prices are martingales, so is the trading rate. Thus, if changes in prices are unpredictable, so should changes in trading rate be. Note that this is not a *sufficient* criteria for optimality even in the paper of Aid et al. (2015), but it is a *necessary* feature of an optimal solution.

In contrast to Aid et al. (2015), Garnier and Madlener (2015) include stochastic clearing of bids, or *counterparty risk* as they call it. In this case, the trading rate is biased towards early trades if prices are volatile. As Garnier and Madlener (2015) deals with balancing forecasting errors for inflexible producers, early trading is preferred if prices are supermartingales. If the production forecasts are uncertain, trading happens later on average. The insights from Aid et al. (2015) and Garnier and Madlener (2015) can be combined to a prior for the Order Execution Problem. The prior might for instance assign higher value to policies where bid volumes and recent prices correlate positively, where the variance in trading throughout the day can be explained largely by variations in price, and where trading happens earlier if prices are more volatile.

For producers with storage capacity, Hassler (2017) offer simple heuristics for how to distribute the production capactity throughout the day, based on the current storage level and market prices. It is shown that some of these heuristics are near-competitive with an ADP algorithm for several types of batteries throughout large parts of the year. Thus, they may serve as priors for the Dynamic Resource Allocation Problem. This prior might state that trading volumes should be lower if the current storage level is lower and vice versa, ceteris paribus. Dispatch should also be larger in hours with higher prices.

#### 6.3.3.2 Choosing the right actions (exploration vs. exploitation)

In section 6.3.2.3 it is proposed that actions with optimal estimated values should be chosen. The problem with choosing only the actions that are currently expected to be optimal, when the estimated values in the lookup table contain uncertainty, is that the algorithm may end up always choosing solutions that were identified as fairly good early on. While it is expected that far better solutions exist, no single solution is expected to be better than the one identified as fairly good. To avoid this, unexplored options must sometimes be evaluated, even if they are worse on expectation. The question of how often good, known solutions should be exploited and how often unknown solutions should be explored is known as the *Exploration versus Exploitation tradeoff* (Wikipedia, 2017d). Note that this tradeoff is a version of the precision versus accuracy tradeoff, as pure exploitation is biased towards the solutions explored initially but evaluates them very accurately, whereas pure exploration yields unbiased solutions with volatile performance.

While partial randomization is commonly proposed as a solution to this tradeoff (Kaelbling et al., 1996), methods from Bayesian Global Optimization such as the *Knowledge Gradient* described by Frazier and Wang (2016) may offer the opportunity to discover solutions along the entire Pareto frontier without wasting too much computational power on inefficient solutions. Similar methods have recently been applied successfully by Bellemare et al. (2017). In figure 6.3, the lower rightmost orange dot represents pure exploration, and the upper leftmost orange dot represents pure exploitation. The Knowledge gradient evaluates solutions along the orange line representing the Pareto frontier. Partially randomized solutions may end up anywhere in the outcome space.

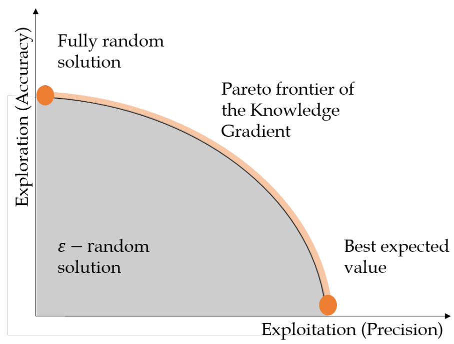



Figure 6.3: The exploration vs. exploitation tradeoff

#### 6.3.3.3 Aggregation across states (propagating learning)

Section 6.3.2.4 states that the estimated values of taking an action in a given state should be reevaluated after a simulation is performed and a payoff is observed. However, many states and actions resemble each other to a degree where it is likely that the value of an action in one state is a good estimate of the same action in a similar state, or a similar action in the same state. In that case, not only should the value table be updated for the given action in the given state, but also in similar actions and states throughout the value table.

Examples of similar states in the ITP includes states that are identical except for slightly different time steps - for instance, placing a bid for an evening delivery product in the morning may be optimal regardless of whether it is placed at 09:32 or 09:41. Thus, learning may be propagated along the time axis. If market prices are very similar (say, within a range of ±2€/MWh), the same may apply. Also space variables that apply to the specific trader, such as the amount of stored energy, may be an axis to aggregate along.

It is also possible to update the values of similar decisions, if the state is held constant. That is, if placing a given bid is optimal in a given state, then placing a bid with a slightly smaller or larger volume may also be quite good. The same isn't necessarily true of price, as small changes in the price may severely affect the probability of clearing. While this list is not exhaustive, some examples of how aggregation could be used have been mentioned.

#### 6.3.3.4 Exploiting goal function structure(Using monotonicity)

If the goal function has a nice structure, it is often possible to exploit it to develop faster solution algorithms. For instance, the $t$-dynamic papers are able to explicitly solve the HJB PDE and arrive at a closed-form expression for the optimal policy. Farinelli and Tibiletti (2017) uses the convexity of the given goal function to apply Interior Points Methods, which converge fast (Boyd and Vandenberghe, 2004). Jiang and Powell (2015b) demonstrate that monotonicity in the goal function can be exploited to better aggregate across states and decisions. In particular, bounds can be put on the performance of unexplored solutions.

For this particular problem, especially the latter approach looks promising, since it doesn't require monotonicity in the entire state space or decision space to work. If subspaces are monotone, some of the value function updates may still be propagated to other states or decisions. For instance, ceteris paribus larger volumes should be traded at higher clearing prices rather than lower. Clearing prices can be expected to be higher when recent transaction prices have been higher, so if recent prices have been higher, higher trading volumes should be expected. Thus, bounds-in-expectation may be placed on the actions even in states that have not been visited.

#### 6.3.3.5 Exploiting the state structure (FMDPs)

*Factored Markov Decision Processes* (FMDPs) attempt to resolve the issue of exponentially growing state spaces in large, structured MDPs. Guestrin et al. (2003) describes how this is handled using Dynamic Bayesian Networks under the assumption that only a few variables influence the transition of each variable. In the case of ITP, this assumption obviously holds given the state description above: the volume traded of a given delivery product (as well as other decision variables) is almost independent of all the state variables that apply to different delivery products. Thus, instead of thinking about the ITP as one dynamic problem with weakly coupled state variables, in the FMDP paradigm, the problem is decomposed into 24 weakly coupled subproblems (one per delivery product). Each subproblem is an MDP, and the decisions in the subproblems are coordinated by the FMDP formulation. It is expected that the majority of the states of each of the subproblems are present in more than one subproblem, thus some learning across states should be possible. The same article contains a description of how this approach provided results in problems with $10^{40}$ states, so this could also apply to other problems with huge state spaces like the ITP.

#### 6.3.3.6 Using effective samples (Variance reduction)

After having learned different candidate bid policies, these must be evaluated and compared in order to tell which policy is better. This is rarely a trivial task, as the performance of a policy on some data may not represent the general performance of a policy. There are several generic tricks on how to improve performance evaluation. One is to evaluate each policy using a set of diverse samples representing the width of the sample space. In the field of statistics, this is referred to as sampling *Antithetic variates* (Goldsman, 2017). In the case of the Intraday Trading Problem, this corresponds to evaluate candidate bid policies using samples from all seasons and where the Intraday price trajectories follows a variety of different patterns. Another way to improve policy evaluation is to evaluate each of the policies using the same subset of available data, referred to as *Common Random Numbers* (Goldsman, 2017). That is, each bid policy should be evaluated on the same set of days.

### 6.3.4 Speeding up the execution process

As formerly mentioned, section 6.3.3 covered speedups in the process of learning the entries in the value table. In this paragraph, potential speed-ups to the real-time trading are proposed. Techniques that speed up both processes are covered in this section.

#### 6.3.4.1 Reducing the decision space (Restrictions on bids)

Some actions are obviously sub-optimal upon simple inspection, and can be discarded prior to any simulations. For instance, for bids with a volume $V$, there's no need to evaluate prices below the price of the demand bid that makes the total available demand volume equal to $V$. On the other side of the price spectrum, it can be expected that prices too far above the last weighted average transaction price will never occur. This could be implemented by a constraint saying that no bid without at least x % probability of clearing should be placed.

For bids above the highest price of the currently revealed residual demand, execution constraints such as FoK and IoC make no sense. The AoN restriction only applies to block bids, and is thus outside of the scope of this report. For bids that are fully below the revealed residual demand curve, IoC constraints can always be applied. If the bid clears, the execution constraint is of no regard so such a rule simply breaks symmetry; in the rare case where the residual demand is (partially) cleared immediately before the bid is placed, the IoC bid will clear the rest of the demand and otherwise not appear in the orderbook, and thus it has no unintentional adverse long-term price impact. For bids that are partly below the revealed residual demand curve, the bid may be partitioned into one IoC bid and one regular bid. Thus, optimal execution restrictions exist and are trivial to determine, as hinted at in section 5.3.3.6.

Bids with zero residual volume can immediately be killed, as no further transactions can occur. This is a symmetry-breaking constraint, as the solution algorithm without the constraint would spend computational resources contemplating whether an empty bid should be killed or not.

It is expected that even more options can be removed with closer inspection.

#### 6.3.4.2 Reducing the state space (Factor analysis, state variable discretization)

One obvious way to try to handle enormous state spaces is to try to reduce the number of state variables. That is, mapping the states to lower dimensions to disregard redundant information. Within the field of statistics, this is often referred to as factor analysis, as also mentioned in section 5.1. Another way to reduce the state spaces is to reduce the number of state variable realizations by discretizing (semi-)continuous variables, as Gönsch and Hassler (2016) successfully do.

#### 6.3.4.3 Simplifying the model (discarding constraints)

If absolutely necessary, it may be possible to speed up the model even further by relaxing or disregarding some constraints. In particular, the Order-to-Trade Ratio constraints may be disregarded if producers are still willing to purchase the product. In order to comply with the rules, at least one in every fifty bids would have to be sufficiently below the revealed residual demand curve to be 100% certain that the bid clears. This could be solved by solving the problem without the OTR constraint, and then simply placing bid at a very low price whenever the OTR is approaching the threshold. Such a solution would be significantly less robust to errors, however. The behavior-shaping constraint on the use of Balancing Markets should not be discarded.

#### 6.3.4.4 Separating the mapping from the trading (Simple lookup)

If the algorithm has evaluated a sufficient amount of actions in all likely states and identified at least one action that is likely to be quite good, it would not be necessary to simulate which action it should take in real-time. It would suffice to do a simple lookup in the value table (or the approximation that is used for the value table; for instance, a neural network) in order to identify an acceptable solution. Thus, the execution of trades and the exploration of the value space would be separate processes.

This process is akin to approach 4 in section 2.6.2, where the value table approximation serves as an estimate of what the ADP algorithm would have proposed. As the ADP algorithm has more optimal and less noisy behavior than a human trader, this is both an improvement and a simplification of approach 4. This is the only technique that would speed up only the execution process without also speeding up the learning process.

## 6.4 Assessment of the proposed solution framework

In section 2.6.1, a list of requirements for autonomous algorithms was proposed. Here, the proposed solution method will be discussed in light of each of these requirements.

The superhuman performance of similar algorithms in similar game structures (DeepMind, 2017) makes it look probable that the proposed approach could beat humans on average. The algorithm is also adaptable, for instance because the price forecasting algorithms weights the newest observations the most. Due to issues of overfitting in the price forecast, the algorithm would likely improve on expectation with access to more data. Through the decomposition of the ITP into the Price Forecasting Problem, the Cost Estimation Problem and the Strategy Formulation Problem the algorithm has been modularized. A sensitivity analysis could also provide a rationale for the decisions made by the algorithm.

However, no lower bound for the performance of the algorithm is provided. Unlike several previous papers it is not immediately obvious that the algorithm not scales to a Nash equilibrium; however, neither is it proven that it does. It may be entirely possible to "hack" the algorithm by placing bids in a pattern that confuses it.

Customization is provided for individual cost structures, but not for individual risk preferences.

As no case study has been performed, the computational resources and time required have not been assessed. Neither has the performance in different market situations throughout the year. As all decision variables in the EPEX trading software are determined by the algorithm, it has the potential to trade autonomously.

With the proposed model, the rules of the EPEX Intraday market are followed.

To summarize, the algorithm satisfies more of the requirements to an autonomous algorithm than former papers. However, there are still several issues to consider before such an algorithm can be allowed to trade on it's own.

# 7 Conclusion and future research

In this report, the potential for automated trading in the EPEX Intraday market has been discussed. A list of requirements for autonomous trading algorithms has been presented.

The Intraday Trading Problem has been defined and decomposed into the Price Forecasting Problem, the Cost Estimation Problem and the Strategy Formulation Problem. As the state of the existing literature on the Strategy Formulation Problem was found the most critical with regards to automation of the ITP, the SFP has been modeled in the most detail. Specifically, the model allows all types of traders and strategies, the double dynamics of the SFP have been modeled for the first time and the market microstructure - including both physical, financial and juridical technicalities - has been accounted for. A non-parametric price forecast with fundamental explanatory variables is proposed. Finally, we have discussed two heuristic methods for solving the Intraday Trading Problem, Stochastic Dual Dynamic Programming and Approximate Dynamic Programming, of which the latter is assumed to be more appropriate due to its flexibility and moderate set of underlying assumptions.

However, this is only a first step towards solving the Automated Intraday Trading Problem. The most critical next step is to perform a case study. This is necessary both in order to determine the computational requirements and the financial performance of the proposed algorithm, and to tune adjust the implementation of the algorithm so that it balances the proposed tradeoffs well (including underfitting versus overfitting, exploration versus exploitation et cetera).

Later, the modeling may be improved to include heterogeneous risk preferences and incorporate defense strategies such as randomization of actions in the case of adversaries attempting to "hack" the algorithm. Theoretical research is needed to improve the modeling of market power, and to determine if the proposed solution scales to a Nash Equilibrium in the case of broad adopotion. If possible, it is also desirable to determine a lower bound on the performance of the algorithm. However, all of these issues are secondary to performing a case study to receive empirical feedback on the proposed model.

# 8 Bibliography

H. Abgottspon, M. Bucher, and G. Andersson. Stochastic dynamic programming for unified short and medium-term planning of hydro power considering market products. In *12th IEEE International Conference on Probabilistic Methods Applied to Power Systems (PMAPS)*, 2014.

S. K. Aggarwal, L. M. Saini, and A. Kumar. Electricity price forecasting in deregulated markets: A review and evaluation. *International Journal of Electrical Power and Energy Systems*, 2008.

R. Aid, P. Gruet, and H. Pham. An optimal trading problem in intraday electricity markets. *Mathematics and Financial Economics*, 2015.

R. Almgren and N. Chriss. Optimal execution of portfolio transactions. *Journal of Risk*, 3:5–40, 2001.

K. J. Arrow, T. Harris, and J. Marschak. Optimal inventory policy. *Econometrica Journal of Econometric Society*, 1951.

X. Ayón, M. Á. Moreno, and J. Usaola. Aggregators' optimal bidding strategy in sequential day-ahead and intraday electricity spot markets. *Energies*, 10 (4):450, 2017.

M. G. Bellemare, W. Dabney, and R. Munos. A distributional perspective on reinforcement learning. *arXiv preprint arXiv:1707.06887*, 2017.

M. Belsnes, O. Wolfgang, T. Follestad, and E. Aasgård. Applying successive linear programming for stochastic short-term hydropower optimization. *Management Science Volume 33, Issue 12*, 2016.

D. Bertsimas, E. Litvinov, X. A. Sun, J. Zhao, and T. Zheng. Adaptive robust optimization for the security constrained unit commitment problem. In *IEEE Transactions on Power Systems Volume 28, Issue 1*, 2013.

J. R. Birge. Decomposition and partitioning methods for multistage stochastic linear programs. *Operations research*, 33(5):989–1007, 1985.

BMWI. Act on the development of renewable energy sources (renewable energy sources act - res act 2014). `http://www.bmwi.de/Redaktion/EN/Downloads/renewable-energy-sources-act-eeg-2014.pdf?__blob=publicationFile&v=1`, 2014. [Last Accessed: 2017-12-11].

S. Boyd and L. Vandenberghe. *Convex Optimization*. Cambridge University Press, 2004.

S. Braun. Hydropower storage optimization considering spot and intraday auction market. *Energy Procedia*, 2016.

S. Braun and R. Hoffmann. Intraday optimization of pumped hydro power plants in the german electricity market. *Energy Procedia*, 2016.

DeepMind. Alphago zero: Learning from scratch. `https://deepmind.com/blog/alphago-zero-learning-scratch/`, 2017. [Last Accessed: 2017-12-08].

H. Ding, Z. Hu, and Y. Song. Rolling optimization of wind farm and energy storage system in electricity markets. *IEEE Transactions on Power Systems Volume 30, Issue 5*, 2017.

D. L. Donoho et al. High-dimensional data analysis: The curses and blessings of dimensionality. *AMS Math Challenges Lecture*, 1:32, 2000.

G. Doorman, R. van de Veen, and A. Abassy. Balancing market design. *Sintef, August*, 2011.

I. Dyrdal. MØnstergjenkjenning. `http://kybernetikk.unik.no/unik4590-ttk4205/Kopier/Unik4590Notater.pdf`, 2017. [Last Accessed: 2017-12-15].

E. Edoli, S. Fiorenzani, and T. Vargiolu. *Optimization Methods for Gas and Power Markets*. Palgrave Macmillan UK, 2016.

P. G. Efthymoglou. Optimal use and the value of water resources in electricity generation. *INFORMS*, 1987.

E. Engmark and H. Sandven. Stochastic multistage bidding optimisation for a nordic hydro power producer in the post-spot markets. Master's thesis, Norwegian University of Science and Technology, 2017.

ENTSO-e. Imbalance. `https://transparency.entsoe.eu/balancing/r2/imbalance/show`, 2017a. [Last Accessed: 2017-12-08].

ENTSO-e. Total load - day-ahead/actual. `https://transparency.entsoe.eu/load-domain/r2/totalLoadR2/show`, 2017b. [Last Accessed: 2017-12-07].

EPEX. Power trading results on epex spot and eex power derivatives in 2009. `https://www.epexspot.com/document/7233/20100113_EPEX_Volumes_Year2009.pdf`, 2009. [Last Accessed: 2017-12-14].

EPEX. Epex spot intraday markets reach all-time high in 2016. `https://www.epexspot.com/document/36851/2017-01-11_EPEX%20SPOT_2016_Annual%20Press%20Release.pdf`, 2016. [Last Accessed: 2017-12-14].

EPEX. Intraday lead time overview. `http://www.epexspot.com/en/product-info/intradaycontinuous/intraday_lead_time`, 2017a. [Last Accessed: 2017-12-07].

EPEX. Epexspot intraday continuous. `https://www.epexspot.com/en/market-data/intradaycontinuous/intraday-table/2017-10-28/DE`, 2017b. [Last Accessed: 2017-12-07].

EPEX. Market rules: Code of conduct. https://www.epexspot.com/en/extras/download-center, 2017c. [Last Accessed: 2017-12-11].

EPEX. Market data day-ahead auction. http://www.epexspot.com/en/market-data/dayaheadauction/auction-table/2016-12-21/DE/24, 2017d. [Last Accessed: 2017-12-07].

EPEX. Market data day-ahead auction. http://www.epexspot.com/en/market-data/dayaheadauction/chart/auction-chart/2017-12-07/DE, 2017e. [Last Accessed: 2017-12-07].

EPEX. Market data intraday continuous. http://www.epexspot.com/en/market-data/intradaycontinuous/chart/intraday-chart/2017-12-07/DE, 2017f. [Last Accessed: 2017-12-07].

EPEX. Intraday market with delivery on the german tso zone. https://www.epexspot.com/en/product-info/intradaycontinuous/germany, 2017g. [Last Accessed: 2017-12-07].

EPEX. Market rules: Operational rules. https://www.epexspot.com/en/extras/download-center, 2017h. [Last Accessed: 2017-12-11].

EPEX. M7 intraday trading system – comtrader 2.7. http://www.epexspot.com/en/extras/download-center/technical_documentation, 2017i. [Last Accessed: 2017-12-13].

EPEX. Become a member: Who are our members? https://www.epexspot.com/en/membership/who_are_our_members, 2017j. [Last Accessed: 2017-12-08].

EPEX. Become a member: Faq. https://www.epexspot.com/en/membership/faq, 2017k. [Last Accessed: 2017-12-08].

Eurostat. Share of energy from renewable sources in gross final consumption of energy, 2004-2015 %. http://ec.europa.eu/eurostat/statistics-explained/index.php/File:Share_of_energy_from_renewable_sources_in_gross_final_consumption_of_energy,_2004-2015_%25._T1_Newpng.png, 2017. [Last Accessed: 2017-12-14].

A. Faghih, M. Roozbehani, and M. A. Dahleh. Optimal utilization of storage and the induced price elasticity of demand in the presence of ramp constraints. *50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC)*, 2011.

S. Farinelli and L. Tibiletti. Hydroassets portfolio management for intraday electricity trading from a discrete time stochastic optimization perspective. *Energy Systems*, 2017.

P. I. Frazier and J. Wang. Bayesian optimization for materials design. In *Information Science for Materials Discovery and Design*, pages 45–75. Springer, 2016.

E. Garnier and R. Madlener. Day-ahead versus intraday valuation of demand-side flexibility for photovoltaic and wind power systems. 2014.

E. Garnier and R. Madlener. Balancing forecast errors in continuous-trade intraday markets. *Energy Systems Volume 6, Issue 3*, 2015.

Z. J. C. Ghahramani. An introduction to hidden markov models and bayesian networks. *IJPRAI*, 15:9–42, 2001.

D. Goldsman. Comparing systems. `http://www2.isye.gatech.edu/~sman/courses/6644/Module10-ComparingSystems_171115.pdf`, 2017.

C. Guestrin, D. Koller, R. E. Parr, and S. Venkataraman. Efficient solution algorithms for factored mdps. *J. Artif. Intell. Res.*, 19:399–468, 2003.

J. Gönsch and M. Hassler. Sell or store? an adp approach to marketing renewable energy. *OR Spectrum Volume 38, Issue 3*, 2016.

M. Hassler. Heuristic decision rules for short-term trading of renewable energy with co-located energy storage. *Computers & Operations Research Volume 83*, 2017.

M. Jansen. Economics of control reserve provision by fluctuating renewable energy sources. In *European Energy Market (EEM), 2016 13th International Conference on the*, pages 1–5. IEEE, 2016.

T. Javanainen, T. KORKEAKOULU, D. TIIVISTELMAA, T. FYSIIKAN, and J. M. OSASTO. Analysis of short-term hydro power production in the nordic electricity market. 2005.

D. R. Jiang and W. B. Powell. Optimal hour-ahead bidding in the real-time electricity market with battery storage using approximate dynamic programming. *INFORMS Journal on Computing*, 2015a.

D. R. Jiang and W. B. Powell. An approximate dynamic programming algorithm for monotone value functions. *Operations Research*, 63(6):1489–1511, 2015b.

L. P. Kaelbling, M. L. Littman, and A. W. Moore. Reinforcement learning: A survey. *J. Artif. Intell. Res.*, 4:237–285, 1996.

R. Kiesel and F. Paraschiv. Econometric analysis of 15-minute intraday electricity prices. *Energy Economics*, 64, 03 2017.

M. S. Kim. Modeling special-day effects for forecasting intraday electricity demand. *European Journal of Operational Research*, 2013.

A. J. King and S. W. Wallace. *Modeling with Stochastic Programming*. Springer, 2012.

G. Klaboe and O. B. Fosso. Optimal bidding in sequential physical markets—a literature review and framework discussion. In *PowerTech (POWERTECH), 2013 IEEE Grenoble*, pages 1–6. IEEE, 2013.

E. Lazarczyk. Market-specific news and its impact on forward premia on electricity markets. *Energy Economics*, 2016.

V. Leclere. An introduction to stochastic dual dynamic programming (sddp). http://gdrro.lip6.fr/sites/default/files/JourneeCOSdec2015-Leclere.pdf, 2015.

C. Lemarechal. Lagrangian relaxation. In *Computational Combinatorial Optimization*, 2001.

B. Lincoln and A. Rantzer. Relaxing dynamic programming. *IEEE Trans. Automat. Contr.*, 51:1249–1260, 2006.

N. Löhndorf, D. Wozabal, and S. Minner. Optimizing trading decisions for hydro storage systems using approximate dual dynamic programming. *Operations Research Volume 61, Issue 4*, 2013.

A. Malic. *Determinants of price volatility of German Intraday power market.* PhD thesis, University of Ljubljana, 2017.

D. Negoescu, P. Frazier, and W. Powell. Optimal learning policies for the newsvendor problem with censored demand and unobservable lost sales. 12 2017.

J. Nowotarski, E. Raviv, S. Trück, and R. Weron. An empirical comparison of alternative schemes for combining electricity spot price forecasts. *Energy Economics*, 2014.

J. Ostrowski, M. F. Anjos, and A. Vannelli. Tight mixed integer linear programming formulations for the unit commitment problem. In *IEEE Transactions on Power Systems Volume 27, Issue 1*, 2012.

A. Panagiotelis and M. Smith. Bayesian density forecasting of intraday electricity prices using multivariate skew t distributions. *International Journal of Forecasting*, 2008.

C. Pang and H. Chen. Optimal short-term thermal unit commitment. *IEEE Transactions on Power Apparatus and Systems Volume 95, Issue 4*, 1976.

C. Pape, S. Hagemann, and C. Weber. Are fundamentals enough? explaining price variations in the german day-ahead and intraday power market. *Energy Economics*, 2016.

E. Parzen. On estimation of a probability density function and mode. *The annals of mathematical statistics*, 33(3):1065–1076, 1962.

Powel. Powel intraday trading: Link your portfolio directly to the market. https://www.powel.com/globalassets/product-information/smart-energy/fs-powel-intraday_trading_english_sept-2017.pdf, 2017a. [Last Accessed: 2017-12-08].

Powel. Powel intraday trading released. `https://www.powel.com/news/powel-intraday-trading-released/`, 2017b. [Last Accessed: 2017-12-14].

W. B. Powell. Clearing the jungle of stochastic optimization. In *Bridging Data and Decisions*, pages 109–137. Informs, 2014.

W. B. Powell, A. George, B. Bouzaiene-Ayari, and H. P. Simao. Approximate dynamic programming for high dimensional resource allocation problems. In *Neural Networks, 2005. IJCNN'05. Proceedings. 2005 IEEE International Joint Conference on*, volume 5, pages 2989–2994. IEEE, 2005.

A. Purkus, E. Gawel, M. Deissenroth, K. Nienhaus, and S. Wassermann. Market integration of renewable energies through direct marketing-lessons learned from the german market premium scheme. *Energy, Sustainability and Society*, 5(1):12, 2015.

K. Rademaekers, A. Slingenberg, and S. Morsy. Review and analysis of eu wholesale energy markets. Technical report, ECORYS Nederland BV, 2008.

M. Rao, M. Chowdhury, Y. Zhao, T. Javidi, and A. Goldsmith. Value of storage for wind power producers in forward power markets. *Proceedings of the American Control Conference*, 2015:5686–5691, 07 2015.

Regelleistung. Method for determinig the rebap. `https://www.regelleistung.net/ext/static/rebap?lang=en`, 2017. [Last Accessed: 2017-12-08].

M. Robinius, F. t. Stein, A. Schwane, and D. Stolten. A top-down spatially resolved electrical load model. *Energies*, 10(3):361, 2017.

E. B. Rudlang, C. F. Tjeransen, S.-E. Fleten, and G. Klaboe. Value of price dependent bidding for thermal power producers. In *11th International Conference on the European Energy Market (EEM)*, 2014.

O. Ruhnau, P. Hennig, and R. Madlener. Economic implications of enhanced forecast accuracy: The case of photovoltaic feed-in forecasts. 2015.

A. Shapiro. Analysis of stochastic dual dynamic programming method. *European Journal of Operational Research*, 209:63–72, 2011.

A. Skajaa, K. Edlund, and J. M. Morales. Intraday trading of wind energy. *IEEE Transactions on Power Systems Volume 30, Issue 6*, 2015.

H. I. Skjelbred, J. Kong, T. J. Larsen, and F. Kristiansen. Operational use of marginal cost curves for hydropower plants as decision support in real-time balancing markets. *14th International Conference on the European Energy Market (EEM)*, 2017.

M. Tahanan, W. van Ackooij, and A. Frangioni. Large-scale unit commitment under uncertainty. *4OR Volume 13, Issue 2*, 2015.

Z. Tan and P. Tankov. Optimal trading policies for wind energy producer. *HAL*, 2016.

Y. Tang, J. Ling, T. Ma, N. Chen, X. Liu, and B. Gao. A game theoretical approach based bidding strategy optimization for power producers in power markets with renewable electricity. *Energies*, 2017.

A. Trivedi, D. Sharma, and D. Srinivasan. Multi-objectivization of short-term unit commitment under uncertainty using evolutionary algorithm. *IEEE Congress on Evolutionary Computation (CEC)*, 2012.

A. Ugedo, E. Lobato, A. Franco, L. Rouco, M. IEEE, J. C. J. Femindez-Caro, J. de-Benito, and J. D. la Hoz. Stochastic model of residual demand curves with decision trees. *Power Engineering Society General Meeting, IEEE*, 2003.

R. Weron. *Modeling and forecasting of electricity loads and prices.* Wiley, 2006.

R. Weron. Electricity price forecasting: A review of the state-of-the-art with a look into the future. *International Journal of Forecasting*, 2014.

Wikipedia. Geometric brownian motion. `https://en.wikipedia.org/wiki/Geometric_Brownian_motion`, 2017a. [Last Accessed: 2017-12-15].

Wikipedia. Accuracy and precision. `https://en.wikipedia.org/wiki/Accuracy_and_precision`, 2017b. [Last Accessed: 2017-12-15].

Wikipedia. Bias of an estimator. `https://en.wikipedia.org/wiki/Bias_of_an_estimator`, 2017c. [Last Accessed: 2017-12-15].

Wikipedia. Reinforcement learning. `https://en.wikipedia.org/wiki/Reinforcement_learning`, 2017d. [Last Accessed: 2017-12-16].

Wikipedia. Frequency of exceedance. `https://en.wikipedia.org/wiki/Frequency_of_exceedance`, 2017e. [Last Accessed: 2017-12-15].

Wikipedia. Factor analysis. `https://en.wikipedia.org/wiki/Factor_analysis`, 2017f. [Last Accessed: 2017-12-15].

Wikipedia. Gaussian process. `https://en.wikipedia.org/wiki/Gaussian_process`, 2017g. [Last Accessed: 2017-12-15].

Wikipedia. Hamilton–jacobi–bellman equation. `https://en.wikipedia.org/wiki/Hamilton-Jacobi-Bellman_equation`, 2017h. [Last Accessed: 2017-12-18].

Wikipedia. Kurtosis. `https://en.wikipedia.org/wiki/Kurtosis`, 2017i. [Last Accessed: 2017-12-15].

Wikipedia. Norm (mathematics). `https://en.wikipedia.org/wiki/Norm_(mathematics)`, 2017j. [Last Accessed: 2017-12-15].

Wikipedia. Martingale (probability theory). https://en.wikipedia.org/wiki/Martingale_(probability_theory), 2017k. [Last Accessed: 2017-12-15].

Wikipedia. Nash equilibrium. https://en.wikipedia.org/wiki/Nash_equilibrium, 2017l. [Last Accessed: 2017-12-18].

Wikipedia. Ornstein–uhlenbeck process. https://en.wikipedia.org/wiki/Ornstein%E2%80%93Uhlenbeck_process, 2017m. [Last Accessed: 2017-12-15].

Wikipedia. Overfitting. https://en.wikipedia.org/wiki/Overfitting, 2017n. [Last Accessed: 2017-12-15].

Wikipedia. Pareto efficiency. https://en.wikipedia.org/wiki/Pareto_efficiency, 2017o. [Last Accessed: 2017-12-15].

Wikipedia. Path dependence. https://en.wikipedia.org/wiki/Path_dependence, 2017p. [Last Accessed: 2017-12-18].

Wikipedia. Skewness. https://en.wikipedia.org/wiki/Skewness, 2017q. [Last Accessed: 2017-12-15].

Wikipedia. Weiner process. https://en.wikipedia.org/wiki/Wiener_process, 2017r. [Last Accessed: 2017-12-15].

Wärtsilä. Delivering flexibility in the german electricity markets: are current arrangements fit for purpose? Technical report, Wärtsilä, 2014.

Y. Zhou, C. Wang, J. Wu, J. Wang, M. Cheng, and G. Li. Optimal scheduling of aggregated thermostatically controlled loads with renewable generation in the intraday electricity market. *Applied Energy*, 2016.

J. Zou, S. Ahmed, and X. A. Sun. Stochastic dual dynamic integer programming. *Submitted for publication*, 2016.