1) $m\ddot{\mathbf{r}} = \sum_i \mathbf{F}_i$

$$\Rightarrow m \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \sum_i \begin{bmatrix} \mathbf{F}_x \\ \mathbf{F}_y \\ F_z \end{bmatrix}$$

$$m\ddot{x} = \sum_i F_{x,i}$$

Einzel partikel:

$$\Rightarrow \begin{bmatrix} \dot{x} \\ \dot{y} \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ B_0 \end{bmatrix} = \begin{bmatrix} B_0 \dot{y} \\ -B_0 \dot{x} \\ 0 \end{bmatrix}$$

$$m\ddot{x} = F_x = q E_x + (q \mathbf{v} \times \mathbf{B})_x$$

$$= -q \frac{\partial V}{\partial x} + q \left. \begin{bmatrix} \dot{x} \\ \dot{y} \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ B_0 \end{bmatrix} \right|_x$$

$$= -q \left(-\frac{V_0}{d^2} x \right) + q B_0 \dot{y}$$

$$\Rightarrow \ddot{x} - \frac{qB_0}{m}\dot{y} - \frac{qV_0}{md^2}x = 0$$

$$= \ddot{x} - \omega_0 \dot{y} - \tfrac{1}{2}\omega_z^2 x = 0$$

$$m\ddot{y} = F_y = qE_y + (qV \times B)_y$$

$$= -q\frac{\partial V}{\partial y} - qB_0\dot{x}$$

$$= \frac{qV_0}{d^2}y - qB_0\dot{x}$$

$$\Rightarrow \ddot{y} + \frac{qB_0}{m}\dot{x} - \frac{qV_0}{md^2}x = 0$$

$$m\ddot{z} = qE_z = -q\frac{\partial V}{\partial z} = \frac{-4qV_0}{2d^2}z$$

$$\Rightarrow \ddot{z} + \omega_z^2 = 0$$

$$\Rightarrow z = A\cos(\omega_z z) + B\sin(\omega_z z)$$

$\blacksquare$

2) $\ddot{x} - \omega_0 \dot{y} - \frac{1}{2}\omega_z^2 x +$

$i(\ddot{y} + \omega_0 \dot{x} - \frac{1}{2}\omega_z^2 y) = 0$

$\Rightarrow \ddot{x} + i\ddot{x} + i\omega_0 \dot{x} - \omega_0 \dot{y} -$

$\frac{1}{2}\omega_z^2 (x + iy) = 0$

$\Rightarrow \ddot{\rho} + i\omega_0(\dot{x} + i\dot{y}) - \frac{1}{2}\omega_z^2 \rho = 0$

$\Rightarrow \ddot{\rho} + i\omega_0 \dot{\rho} - \frac{1}{2}\omega_z^2 \rho = 0 \quad \blacksquare$

3) hrever $\omega_\pm \in \mathbb{R}$:

$\Rightarrow \omega_0^2 - 2\omega_z^2 > 0$

$\Rightarrow \omega_0^2 > 2\omega_z^2$

$\left(\frac{qB_0}{m}\right)^2 > \left(\frac{2qV_0}{md^2}\right)$

$\Rightarrow \frac{q}{m}B_0^2 > \frac{4V_0}{d^2} \quad \blacksquare$

4) $f(t) = A_+ (\cos \omega_+ t + i \sin \omega_+ t)$

$+ A_- (\cos \omega_- t + i \sin \omega_- t)$

$x(t) = Re(f(t))$

$= A_+ \cos \omega_+ t + A_- \cos \omega_- t$

$y(t) = Im(f(t))$

$= A_+ \sin \omega_+ t + A_- \sin \omega_- t$

vi får altså max når disse er i samme fase:

$R_+ = A_+ + A_-$

og min når disse er i mot fase:

$R_- = |A_+ + A_-|$

5) $x(0) = Re(f(0)) = A_+ + A_- = x_0$

$$y(0) = \operatorname{Im}(f(0)) = 0$$

$$\dot{f} = \frac{d}{dt}\left[A_+ e^{-i\omega_+ t} + A_- e^{-i\omega t}\right]$$

$$= -A_+ i\omega_+ e^{-i\omega t} - A_- i\omega e^{-i\omega t}$$

$$\dot{y} = \operatorname{Im}(\dot{f}) = -A_+ \omega_+ \cos \omega_+ t$$

$$- A_- \omega_- \cos \omega_- t$$

$$\dot{y}(0) = V_0$$

$$\Rightarrow A_+ \omega_+ - A_- \omega_- = V_0$$

$$\dot{x} = \operatorname{Re}(\dot{f}) = A_+ \omega_+ \sin \omega_+ t$$

$$+ A_- \omega_- \sin \omega_- t$$

$$= 0$$

$X(0) = X_0$

$\Rightarrow A_+ + A_- = X_0$ (i)

$Y(0) = V_0$

$\Rightarrow -A_+ \omega_+ - A_- \omega_- = V_0$ (ii)

(i) gir $A_- = X_0 - A_+$ (iii)

Sette (iii) inn i (ii)

$\Rightarrow -A_+ \omega_+ - X_0 \omega_- + A_+ \omega_- = V_0$

$\Rightarrow A_+ (\omega_- - \omega_+) = V_0 + X_0 \omega_-$

$\Rightarrow A_+ = \dfrac{V_0 + X_0 \omega_-}{\omega_- - \omega_+}$ (iv)

Sette (iv) inn i (iii)

$\Rightarrow A_- = X_0 - \dfrac{V_0 + X_0 \omega_-}{\omega_- - \omega_+}$

$$= \frac{x_0(\omega_- - \omega_+) - v_0 - x_0\omega_-}{\omega_- - \omega_+}$$

$$= \frac{-x_0\omega_+ - v_0}{\omega_- - \omega_+}$$

$$= -\frac{v_0 + x_0\omega_+}{\omega_- - \omega_+}$$

$$z(0) = z_0$$

$$\Rightarrow A\cos\omega_z 0 + B\sin\omega_z 0$$

$$= z_0$$

$$\Rightarrow A = z_0$$

$$\dot{z} = \omega_z(-z_0\sin\omega_z t + B\cos\omega_z t)$$

$$\dot{z}(0) = \omega_z(-z_0\sin\omega_z \cdot 0 + B\cos\omega_z 0)$$

$$= \omega_z B = 0$$

$$\Rightarrow B = 0$$

$$\Rightarrow z(t) = \underline{z_0 \cos(\omega_z t)}$$