Model Order Reduction of Linear Networks With Massive Ports via Frequency-Dependent Port Packing

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ABSTRACT

Model order reduction has been a driving force for reducing analysis complexity of VLSI systems containing large linear networks. However, most existing reduction techniques are only applicable to networks with a small number of ports, failing to fulfill an even stronger need of reducing massively interconnected subsystems such as power grids and wide buses. In this paper, a port packing scheme is presented wherein the correlation between circuit ports is explored in a *frequency-dependent* manner. In the proposed *McPack* (**M**ultiport **C**ircuit **Pack**ing) algorithm, port packing is combined with a practical realization of the recently developed tangential interpolation scheme for model reduction. *McPack* performs *feasible* moment matching for networks with many ports in the sense of tangential interpolation. With guaranteed passivity, extensibility to multi-point expansion as well as comparable complexity, *McPack* systematically introduces frequency-domain port packing into the existing projection-based model order reduction framework. For several large networks with high port count, the presented algorithm is shown to be significantly more accurate than the standard block-moment matching algorithm as well as other recently developed alternative.

Categories and Subject Descriptors: B.7.2 [Hardware]: INTEGRATED CIRCUITS - *Simulation*

General Terms: Algorithms, Performance, Verification.

Keywords: Model order reduction, multi-port networks.

1. INTRODUCTION

Model order reduction is an established technique for handling large linear networks in circuit simulation. A network with a large number of circuit elements may be reduced to a much more compact model with a sufficient accuracy such that the simulation CPU time can be significantly reduced [1, 2, 3, 4]. However, most existing algorithms are only applicable to networks with a small number of ports and become ineffective for networks with massive interfaces such as power grids and wide buses. In practice, these massive networks are common and may constitute the ultimate bottleneck for chip-level analysis. The need for model order reduction of these networks is even more pressing.

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The inefficiency of Krylov subspace methods for networks with many ports originates from the direct matching of block transfer function moments so that these techniques are keyed to the I/O ports [2, 3, 4]. The size of the dense reduced order model grows with the number of ports, leading to a quadratic increase in the number of dense matrix entries. Truncated balance realization (TBR) is in general less sensitive to the I/O count. However, it is computationally expensive and requires special treatment when applied to networks with many terminals [5]. An extended Krylov subspace method was introduced in [6] where the inputs to a system are expressed using moment expansions and are absorbed into the model order reduction process. The requirement of explicit moment expansions of input signals a *priori* casts the approach more into a simulation method.

As a matrix approximation technique, singular value decomposition (SVD) has been applied under many VLSI modeling contexts to handle massive coupling. In [7], SVD is employed to sparsify the integral operator matrix in 3D parasitic extraction while in [8] it is used to more efficiently extract resistive substrate models. More recently, SVD is adopted in [9] for model reduction of dynamical systems with a large number of ports. The introduced *SVDMOR* algorithm (extended to a hierarchical version in [10]) extracts correlation between different circuit ports seen from the matrix transfer function using SVD. The dominant singular vectors are then taken to approximate the input and output matrices of the network so that a system with a smaller number of ports is produced which can be reduced using any existing model reduction algorithm. A related approach is presented in [11].

In this paper, a general model order reduction algorithm *McPack* (**M**ultiport **C**ircuit **Pack**ing) targeting at networks with massive ports is proposed. Although under a spirit similar to that of *SVDMOR*, the presented technique facilities a general port-packing moment-matching based model reduction framework via following distinguishing features:

1)Although correlation between different network ports does exist for many practical circuits (e.g. power grids), in dynamical systems such correlation is often strongly frequency dependent. For instance, in an RLC circuit, correlation strongly depends on electrical connections between ports, which are in turn strongly frequency-dependent. Unlike in [9, 10], where correlation is extracted statically (e.g. using DC transfer functions), in *McPack*, such dependency is captured by employing moment expansion of correlation, leading to frequency-dependent port packing. 2)To perform moment-matching based model reduction while considering port correlation, *McPack* interleaves frequency-dependent port packing with a subsequent model reduction process. For the latter, the theoretical framework of *tangential interpolation* proposed in [12, 13] from the numerical analysis community is adopted. It is important to note that the success of tangential interpolation is critically dependent on the selection of one key component, namely, the interpolation matrix polynomial, which is *application specific*. For large practical VLSI circuit application, in *McPack* the interpolation matrix polynomial is chosen by incorporating frequency-dependent port packing. *McPack* preserves multi-port transfer function moments of the original system in the sense of tangential interpolation while maintaining the favorable algorithm complexity of standard Krylov subspace methods. 3)Under the projection-based framework, it is possible to explore port correlation at multiple frequency expansion point, in a way identical to multiple point moment matching. Furthermore, model passivity can be guaranteed.

2. BACKGROUND

A multi-input multi-output (MIMO) system can be described using a generalized state-space model

$$C\frac{d}{dt} + Gx = Bu \tag{1}$$

$$y = L^T x,$$

where $G, C \in \mathbf{R}^{n\times n}$ describe the resistive and energy storage elements in the circuit, $u \in \mathbf{R}^m$ is the input vector, $x \in \mathbf{R}^n$ is the vector of unknown voltages and currents, and $B, L \in \mathbf{R}^{n\times m}$ are the input and output matrices, respectively. The input output transfer function of the MIMO system is

$$H(s) = L^T(G + sC)^{-1}B.$$

The *SVDMOR* algorithm proposed in [9, 10] is briefly reviewed as follows. For many practical circuits, the input/output correspondence at various ports is highly correlated, which can be extracted by applying SVD to the matrix transfer function of the circuit. In this case, the input/output matrices can be approximated using low-rank matrices based on a few dominant singular vectors. Applying the standard SVD to the system transfer function at DC gives

$$H_{DC} = L^T G^{-1} B = U\Sigma V^T, \tag{2}$$

where $\Sigma = diag(\sigma_1, \ldots, \sigma_m)$, and U and V are the left and right singular vectors. If there exists a strong correlation between the responses at different I/O ports, the transfer function matrix can be well approximated based on $r(r < m)$ dominant left and right singular vectors V_r and U_r. In [9, 10], these singular vectors are also used to find rank-r approximations for B and L

$$B \approx b_r V_r^T,\ L \approx l_r U_r^T, \tag{3}$$

where $b_r \in \mathbf{R}^{n\times r}$ and $l_r \in \mathbf{R}^{n\times r}$. Now the original circuit transfer function is approximated as

$$H(s) \approx U_r l_r^T (G + sC)^{-1} b_r V_r^T \tag{4}$$

Since $H_r(s) = l_r^T(G + sC)^{-1}b_r$ represents a MIMO network with r $(r < M)$ ports, it can be more easily reduced by any existing method.

As can be seen, *SVDMOR* operates not only on the existence of correlation between various I/O ports, which is the case for many practical circuits, but also on the premise that B and C matrices can be well approximated by the dominant singular vectors of the matrix transfer function. Notice that the SVD of B (or C) are usually different from that of the matrix transfer function. Since the input/output matrices are compressed simply using a matrix approximation technique, the overall algorithm does not maintain the moment matching property even if a Krylov subspace based moment matching method is used in the second step. For the same reason, there does not exist a natural mechanism to capture the variation of port correlation along the frequency axis and facilitate multipoint moment expansion commonly employed in a standard Krylov method.

3. TANGENTIAL INTERPOLATION

Before we address the issues outlined in the previous section, in this section the notion of tangential interpolation developed in [12, 13] is first introduced. Then we formally show that tangential interpolation can be achieved via subspace projection that is computationally efficient for large practical VLSI circuit applications.

3.1 Theoretical notion

Instead of applying a multi-port block moment matching method to reduce the system in (1), let us consider left-multiplying the circuit transfer function with a matrix $P(s)^T$ where $P(s) \in \mathbf{C}^{m\times r},\ r < m$, is a matrix polynomial in s:

$$P(s) = \sum_{k=0}^{g-1} P_k(s - z_P)^k,$$

where z_p is a point on the complex plane. To specify a new moment matching concept suitable under this case, we have the following definitions.

Definition 1. Let $P(s) \in \mathbf{C}^{m\times r}$ be a matrix polynomial of degree $g - 1$ and not equal to zero at z_P, it is said that $\widehat{H}(s)$ left interpolates $H(s)$ at $\{P(s), z_P\}$ if

$$P^T(s)\big(H(s) - \widehat{H}(s)\big) = O(s - z_P)^g. \tag{5}$$

Similarly, with some other matrix polynomial

$$Q(s) = \sum_{k=0}^{g-1} Q_k(s - z_Q)^k \in \mathbf{C}^{m\times r}, r < m,$$

we may want to right multiply the system transfer function such that we have

Definition 2. Let $Q(s) \in \mathbf{C}^{m\times r}$ be a matrix polynomial of degree $g - 1$ and not equal to zero at z_Q, it is said that $\widehat{H}(s)$ right interpolates $H(s)$ at $\{Q(s), z_Q\}$ if

$$\big(H(s) - \widehat{H}(s)\big)Q(s) = O(s - z_Q)^g. \tag{6}$$

Notice that the above definitions are somewhat different from those in [12, 13]. Equations (5-6) are referred to as the *left and right tangential interpolation* conditions. $P(s)$ and $Q(s)$ are referred to interpolation matrix polynomials. It is also possible to define *two side tangential interpolation* condition. We skip this possibility due to the limitation of the scope of this paper.

Under tangential interpolation, the model reduction problem is to seek an approximate $\widehat{H}(s)$ such that one or both interpolation conditions are satisfied. For instance, if the right interpolation condition is considered, essentially, one seeks $\widehat{H}(s)$ such that moments of $H_Q(s) = H(s)Q(s)$ are matched up to $(g-1)$th order, with a given $Q(s)$. If $Q(s)$ has r columns, where $r < m$, so does $H_Q(s)$. Hence, the degrees of freedom in the newly defined model order reduction problem is less than that is in the standard block-moment matching, underscoring the possibility of achieving a more compact reduced order model. We establish the means by which interpolation conditions can be satisfied.

LEMMA 1. *Let* $H(s) = L^T(G + sC)^{-1}B$ *and* $\widehat{H}(s) = \widehat{L}^T(\widehat{G} + s\widehat{C})^{-1}\widehat{B}$ *be two* $m \times m$ *matrix transfer functions,*

$Q(s)$ is a matrix polynomial of degree $g-1$:

$$Q(s) = \sum_{k=0}^{g-1} Q_k(s-z_Q)^k \in \mathbf{C}^{m\times r},$$

z_Q is a point on the complex plane, $\widehat{H}(s)$ right interpolates $H(s)$ at $\{Q(s), z_Q\}$ if and only if

$$L^T K_g(A_{z_Q}, R_{z_Q})\mathcal{Q} = \widehat{L}^T \widehat{K}_g(\widehat{A}_{z_Q}, \widehat{R}_{z_Q})\widehat{\mathcal{Q}}, \qquad (7)$$

where $A_{z_Q} = -(G+z_QC)^{-1}C$, $R_{z_Q} = (G+z_QC)^{-1}B$,

$$K_g(A_{z_Q}, R_{z_Q}) = [R_{z_Q}\ A_{z_Q}R_{z_Q}\ A_{z_Q}^2R_{z_Q}\cdots A_{z_Q}^{g-1}R_{z_Q}],$$

$$\mathcal{Q} = \begin{bmatrix} Q_0 & Q_1 & \dots & \dots & Q_{g-1} \\ & Q_0 & Q_1 & \dots & Q_{g-2} \\ & & Q_0 & \ddots & \vdots \\ & & & \ddots & Q_1 \\ & & & & Q_0 \end{bmatrix}, \qquad (8)$$

and $\widehat{A}_{z_Q}$, $\widehat{R}_{z_Q}$, $\widehat{K}_g(\widehat{A}_{z_Q}, \widehat{R}_{z_Q})$ and $\widehat{\mathcal{Q}}$ are defined accordingly for the second matrix transfer function.

Notice that $L^TK_g(A_{z_Q}, R_{z_Q})$ and $\widehat{L}^T\widehat{K}_g(\widehat{A}_{z_Q}, \widehat{R}_{z_Q})$ consist of moments of two transfer functions expanded at $s = z_Q$, respectively. *Lemma 1* can be readily proved by expanding (6) at $s = z_Q$ and collect the first g block vectors ($g\times r$ columns) in a matrix form. A similar result can be reached for the left interpolation in (5).

3.2 Subspace projection

In [13], it is shown that a reduced order model that interpolates (1) can be obtained by projection. The projection matrices are solutions of some Sylvester equations. It is also suggested that the interpolation conditions are satisfied if the projection matrices contain some useful subspace. We state the result for right interpolation condition in the following theorem and then provide a sketch of a formal proof for the subspace projection based reduction.

THEOREM 1. *For the linear matrix transfer function defined in Lemma 1, let us assume that there exist two full-rank projecting matrices $Z, V \in \mathbf{C}^{n\times p}$ such that*

$$Colsp\big(K_g(A_{z_Q}, R_{z_Q}\mathcal{Q})\big) \subseteq V. \qquad (9)$$

Construct $\widehat{G}$, $\widehat{C} \in \mathbf{C}^{p\times p}$, $\widehat{B}, \widehat{L} \in \mathbf{C}^{p\times m}$ as

$$\widehat{G} = Z^TGV,\ \widehat{C} = Z^TCV,\ \widehat{B} = Z^TB,\ \widehat{L} = V^TL.$$

Then, $\widehat{H}(s) = \widehat{L}^T(\widehat{G}+s\widehat{C})^{-1}\widehat{B}$, right interpolates $H(s) = L^T(G+sC)^{-1}B$ at $\{Q(s), z_Q\}$.

PROOF. (sketch) Rewrite the left hand side of (7) as

$$L^TK_g(A_{z_Q}, R_{z_Q})\mathcal{Q} = L^T[X_0\ X_1 \dots X_{g-1}],$$

where $X_i \in \mathbf{C}^{n\times r}$. The right hand side of equation can be partitioned accordingly. The following recursion exists

$$X_i = A_{z_Q}X_{i-1} + R_{z_Q}Q_i,\ for\ i > 0.$$

Denote G_+z_QC as G_{z_Q}. We first show that $X_i = V\widehat{X}_i,\ for\ i = 0, \dots, g-1$ is true by induction. The above can be verified for $i = 0$ easily. Now assuming that it is true for any $i,\ 0 \le i < g-1$, we have

$$-G_{z_Q}X_i = CV\widehat{X}_{i-1} + BQ_i.$$

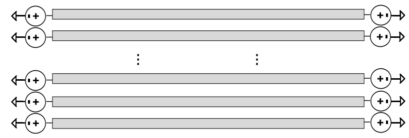


Figure 1: An n-bit bus structure.

Since $X_i \subseteq V$, it can be expressed as $X_i = V\widetilde{X}_i, \widetilde{X}_i \in \mathbf{C}^{p\times r}$. This leads to

$$-G_{z_Q}V\widetilde{X}_i = CV\widehat{X}_{i-1} + BQ_i.$$

Multiplying both sides of the above equation with Z^T yields

$$-Z^TG_{z_Q}V\widetilde{X}_i = Z^TCV\widehat{X}_{i-1} + Z^TBQ_i,$$

which is the same as

$$-\widehat{G}_{z_Q}\widetilde{X}_i = \widehat{C}\widehat{X}_{i-1} + \widehat{B}Q_i.$$

It can be readily seen that $\widetilde{X}_i = \widehat{X}_i$, i.e., $X_i = V\widehat{X}_i$ and $L^TX_i = L^TV\widehat{X}_i$. *Theorem 1* follows immediately from *Lemma 1.* $\square$

4. FREQUENCY-DEPENDENT PORT PACKING

We have shown that model order reduction of a MIMO system can be carried out using tangential interpolation at $\{Q(s), z_Q\}$. Now we show how to seek a proper matrix polynomial $Q(s)$. Let us consider an n-bit bus structure modeled as a coupled RLC $2n$-port shown in Fig. 1. To extract useful port correlation information for model order reduction, one may compute the SVD of the DC matrix transfer function as described in Section 2. Since there does not exist any DC path from a node to the ground and between different lines, it is not difficult to show that $H_{DC} = U\Sigma V^T$, with $\Sigma = diag(\sigma_1, \sigma_2, \dots, \sigma_n, 0, \dots, 0)$, i.e., H_{DC} is exactly equal to a low-rank presentation obtained by retaining the first n left and right singular vectors in the decomposition. However, as we move to a higher frequency, the capacitive and inductive coupling effects are becoming more visible. This will inevitably change the way various ports are correlated. Hence, only using correlation information obtained from DC will introduce large errors for high frequency band in the reduced order model.

The above observation motivates us to extract the *frequency dependent* port correlation information and exploit such information directly in the model order reduction. This choice is made possible under the framework of tangential interpolation we outlined in the previous section. The proposed frequency-dependent port packing is now described as follows.

To fully exploit the correlation between different ports, one may want to use the right interpolation based multiport model order reduction (it is also possible to use left interpolation or both) by computing a few dominant left singular vectors of $H(s)$, where $H(s) \approx U_r(s)\Sigma_r(s)V_r(s)^T$, and $U_r(s), V_r(s) \in \mathbf{C}^{m\times r}$. The correlation between columns and rows of $H(s)$ are captured in U_r and V_r. Notice that $U_r(s)$ and $V_r(s)$ are functions of s. In fact for all the passive circuits of interest, $U_r = V_r$ since $H(s)$ is symmetric due to *reciprocity*.

Getting the exact $U_r(s)$ at various frequency points via a direct SVD is expensive. Instead we seek a good approximation for it. To this end, we expand the transfer function at a given expansion point z_Q

$$H(s) = \sum_{k=0}^{\infty} L^T A_{z_Q}^k R_{z_Q} (s - z_Q)^k. \qquad (10)$$

The above expansion can be approximated by computing a SVD based rank-r approximation for each moment. Now collect the first g block moments and form

$$\mathcal{V} = [V_0 \; V_1 \; \dots V_{g-1}], \; V_k = L^T A_{z_Q}^k R_{z_Q}. \qquad (11)$$

To find the best r block vectors to approximate $\mathcal{V}$ in a 2-norm sense, we compute the SVD of $\mathcal{V}$

$$\mathcal{V} \approx \mathcal{U}_r \; \Sigma_r \mathcal{V}_r^T. \qquad (12)$$

We use the resulting right singular vectors $\mathcal{V}$ to approximate each moment as

$$V_k = L^T A_{z_Q}^k R_{z_Q} \approx U_{k,r} \mathcal{V}_r^T, \qquad (13)$$

which leads to

$$U_{k,r} = L^T A_{z_Q}^k R_{z_Q} \mathcal{V}_r. \qquad (14)$$

Substituting (13) and (14) into (10) yields

$$H(s) \approx \sum_{k=0}^{g-1} U_{k,r} \mathcal{V}_r^T (s - z_Q)^k = \Big(\sum_{k=0}^{g-1} U_{k,r} (s - z_Q)^k\Big) \mathcal{V}_r^T. \qquad (15)$$

In other words, we have computed an approximate low-rank SVD for $H(s)$ in which $U_r(s) = \sum_{k=0}^{g-1} U_{k,r}(s - z_Q)^k$ and $V_r(s) = \mathcal{V}_r$. Since $U_r(s)$ represents the dominant vector directions among the rows of $H(s)$ ($H(s)$ is symmetric), it is quite natural to impose $H(s)U_r(s) \approx \widehat{H}(s)U_r(s)$ in model order reduction. In the light of *Lemma 1*, we choose $Q(s) = \sum_{k=0}^{g-1} U_{k,r}(s - z_Q)^k$, and accordingly, $Q_k = U_{k,r}$ for the following model order reduction process.

It is possible to determine a proper projection matrix Z in *Theorem1* such that the reduced order model also satisfies the left interpolation condition. In the following section, in fact we choose $Z = V$ such that the reduced order model is guaranteed to be passive.

5. MCPACK ALGORITHM

5.1 Practical implementation

It should be noted that capturing the frequency-dependent port correlation and computing the project matrices for interpolation based model reduction require explicit computation of transfer function moments. For numerical stability reasons, this is suitable only for low order moments. This is typically not a true limitation for the reduction of networks with a large port count since the block moment matching order is usually low for these systems. Nevertheless, we construct a more general interpolation based model reduction by modifying *Definition* 2 (or 1) to eliminate this potential problem.

Definition 3. Let $Q(s) \in \mathbf{C}^{m \times r}$ be a matrix polynomial of degree $g-1$ and not equal to zero at z_Q, it is said that $\widehat{H}(s)$ right interpolates $H(s)$ at $\{Q(s), z_Q\}$ up to $(k-1)$th order with respect to $H(s)$, where $g \le k$, if

$$\big(H(s) - \widehat{H}(s)\big)Q(s) = O(s - z_Q)^k. \qquad (16)$$

According to this new definition, the overall moment matching order for $H(s)Q(s)$ can be greater than the degree of the matrix polynomial $Q(s)$. In this case, both *Lemma 1* and *Theorem 1* hold if we modify (8) as

$$\mathbb{Q} = \begin{bmatrix} Q_0 & \dots & Q_{g-1} & & & & \\ & \ddots & \vdots & Q_{g-1} & & & \\ & & Q_0 & \vdots & \ddots & & \\ & & & Q_0 & \vdots & Q_{g-1} & \\ & & & & \ddots & \vdots & \\ & & & & & Q_0 & \end{bmatrix}, \qquad (17)$$

where $\mathbb{Q} \in \mathbf{C}^{mk \times rk}$.

To use *Theorem 1* for the right interpolation based reduction, we first partition the required subspace based on (17) as

$$K_k(A_{z_Q}, R_{z_Q})\mathbb{Q} = [X_0 \; X_1 \dots X_{k-1}]. \qquad (18)$$

Interestingly, the above equation leads to the following recursion for $X_i \in \mathbf{C}^{n \times r}$

$$\begin{aligned} X_i &= A_{z_Q} X_{i-1} + R_{z_Q} Q_i, \qquad i = 1, \dots, g-1 \quad (19) \\ X_i &= A_{z_Q} X_{i-1}, \qquad i = g, \dots, k-1 \quad (20) \end{aligned}$$

This means that the first g blocks in the projection matrix should be computed explicitly while the following $k-g$ blocks fall into a Krylov subspace of A_{z_Q}. Hence, once X_{g-1} is available, the remaining vectors can be computed robustly using an Arnoldi process with orthogonalization.

We outline the flow of *McPack* algorithm as follows.

ALGORITHM 1. *McPack Algorithm (single expansion point)*

- ***Input:*** $C, G \in \mathbf{R}^{n \times n}, B = L \in \mathbf{R}^{n \times m}, \; z_Q \in \mathbf{C}, k, g, r \in \mathbf{N}$.
- ***Output:*** $\widehat{C}, \widehat{G} \in \mathbf{C}^{p \times p}, \widehat{B} = \widehat{L} \in \mathbf{C}^{p \times m}$, *with* $p = k \times r$.

1. *Expand* $H(s)$ *at* $s = z_Q$ *(eqn10);*
2. *Compute the SVD of the first g block moments (eqn12);*
3. *Approximate each block moment using a rank-r approximation (eqn13-14);*
4. *Find* $Q(s)$ *and* $Q_j, \; j = 1, \dots, g$ *(eqn15);*
5. *Compute the first* rg *vectors* $\mathbb{X}_1 = [X_0, \dots, X_{g-1}]$ *of the projection subspace* V *(eqn19);*
6. *Compute an orthonormal basis* $\mathbb{X}_2$ *of the Krylov subspace* $K_{k-g}(A_{z_Q}, X_{g-1}) = [X_{g-1}, \dots, A_{z_Q}^{k-g-1} X_{g-1}]$ *using an Arnoldi process;*
7. *Find an orthonormal basis* V *of the combined subspaces* $[\mathbb{X}1 \; \mathbb{X}2]$ *and form the reduced model:*

$$\widehat{G} = V^T G V, \; \widehat{C} = V^T C V, \; \widehat{B} = \widehat{L} = V^T B.$$

Based on the above discussions, it is not difficult to see that the reduced order model computed by *Algorithm 1* satisfies the general single point right interpolation condition defined in (16). The dominant complexity of the algorithm is the same as a standard moment matching method, i.e., due to a linear matrix factorization. Although SVD is employed in the above procedure, it is typically not the dominant cost since SVD is only applied to the transfer function matrix relating ports.

5.2 Passivity

It is also worth noting that if the system matrices of the full model are formulated as in the *PRIMA* algorithm [4], the interpolation based model computed by the above procedure is guaranteed to be passive. This is because we have purposely chosen to use the congruence transform to obtain the reduced order model. Also notice that when expanding the transfer function at a complex point, a real projection matrix can be found simply by combining the real and imaginary parts of V in the above algorithm such that a real reduced order model can be formed. Although a PRIMA-like formulation is used to achieve passive reduced order modeling under tangential interpolation, our *McPack* algorithm can work with any projection-based algorithm including recent *SPRIM* algorithm [14]. we expect the adoption of the structure-preserving formulation in [14] will further increase the accuracy of *McPack* as it is being applied to networks with a large number of I/O ports.

5.3 Multipoint moment expansions

We have shown a single point model order reduction procedure in *Algorithm* 1 by combining frequency-dependent port packing and tangential interpolation. Such procedure can be straightforwardly extended to the multipoint moment expansion case, analogous to its counterpart in a standard Krylov subspace method. Say, we have l expansion points, $z_{Q1}, \ldots, z_{Ql}$. For each expansion point, z_{Qi}, we follow the procedure described in *Algorithm* 1 to first extract the matrix transfer function correlation at $s = Z_{Qi}$ by forming matrix $\mathbb{Q}$ as in (17), then compute a set of projection vectors V_{Qi} by performing interpolation based moment matching at the same expansion point. The final projection matrix can be obtained by finding an orthonormal basis for the space spanned by all the projected vectors combined: $V = Colspan[V_{Q1}, \ldots, V_{Ql}]$. It is straightforward to show that the resulting reduced order model satisfies the interpolation condition for each of l expansion points.

It is also possible to extract port correlation by performing SVD at several expansion points in *SVDMOR*. However, in *SVDMOR* the combined correlation information collected at different expansion points is only used to find low-rank matrix approximations for input/output matrices. Then a complete separate reduction procedure follows. Differently, in *McPack* port packing and the following moment matching based model reduction are intrinsically interleaved. That is, the port correlation found at a particular frequency expansion point will be only used to drive the transfer function moment matching expanded at the same expansion point. Hence, consistency between port correlation and the following moment matching step is maintained.

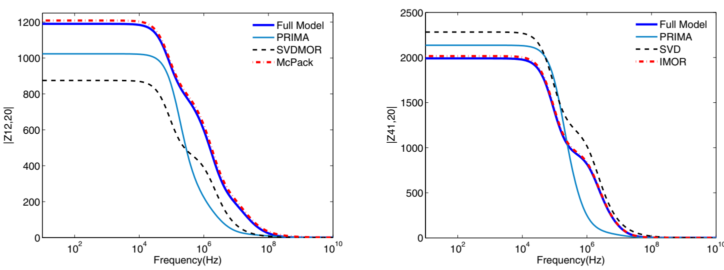
6. EXPERIMENTAL RESULTS

We demonstrate the efficacy of *McPack* algorithm using three challenging model order reduction examples and compare it against with *PRIMA* [4] and *SVDMOR* [9]. The comparisons are based on MATLAB prototyped implementation running on a 3GHz Pentium-4 machine.

6.1 Two coupled RC lines

We first consider a circuit containing two capacitively coupled RC lines with 1,000 nodes, 2,593 elements and 98 current sources as inputs. We compare three algorithms by computing a 98-port reduced Z-parameter model using each algorithm. In a *PRIMA* procedure, the zero-th order block moment (98 vectors) is computed first and then SVD is employed to find the most dominant 95 vectors to form the projection matrix, leading to a model of size 95. The two models computed using *SVDMOR* and *McPack* are both of size 6 and obtained by expanding the transfer function at DC. For the *McPack* model, $Q(s)$ is a matrix polynomial of degree 2 (see *Algorithm 1*).

Two trans-impedances relating two different ports due to various models are shown in Fig. 2(a) and Fig. 2(b). The large 95-state SVD-packed *PRIMA* model generates noticeable errors for both cases. Comparing the two much smaller 6-state models produced by *SVDMOR* and *McPack*, the former incurs large errors toward to DC while the latter follows very well with the full model over a wide frequency range. It takes 0.45s, 0.42s and 0.54s to compute the *PRIMA*, *SVDMOR* and *McPack* models, respectively. This circuit example indicates that standard model order reduction algorithms are not tuned to systems of a large number of ports and hence cannot produce compact reduced order models while there does exist strong correlations between ports.



(a) $|Z_{12,20}|$ (b) $|Z_{41,20}|$

Figure 2: The 98-input two coupled RC lines.

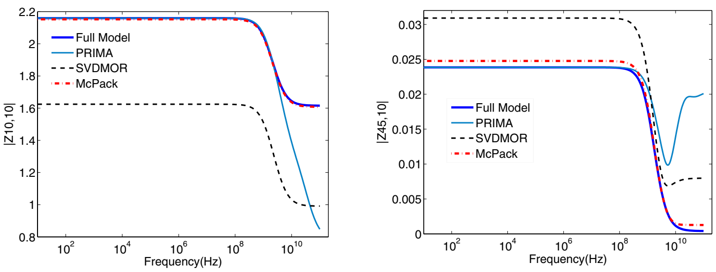
6.2 An RC mesh

Next, we consider a 1600-node RC mesh with 169 current inputs distributed within the mesh resembling a portion of a power grid. This circuit has a total of 3,378 circuit elements. Each of the three algorithms is employed to compute a 210-state model. We use a *PRIMA* procedure to match the zero-th order block moments (169 vectors) of the full model. Then we compute the first order block moments and use SVD to find out the most dominant 41 vectors. Combining these 210 projection vectors leads to a model with 210 state variables. The model due to *SVDMOR* is based on rank-70 approximations of input and output matrices which are obtained by performing SVD of the DC transfer function matrix. The *McPack* model employs a rank-70 matrix approximation of the frequency-dependent transfer function port correlation. Here, $Q(s)$ is a matrix polynomial of degree 2. The run-times for computing the *PRIMA*, *SVDMOR* and *McPack* modes are 2.75s, 2.09s and 3.73s, respectively.

Two trans-impedances are chosen to demonstrate the modeling results, which are shown in Fig. 3(a) and Fig. 3(b). In the first plot, The model produced by *McPack* is indistinguishable from the full model. As can be seen, the *PRIMA* model is very accurate for the low frequency range while it fails to follow the full model at high frequency. The SVDMOR model produces noticeable errors. For the second comparison, the *PRIMA* model again is accurate at low frequency but deviates from the full model at high frequency while the SVDMOR model is not accurate toward to DC. The *McPack* model produces some small error at low frequency but becomes very accurate toward to high frequency due to the frequency-dependent port packing.

6.3 A densely coupled 64-bit bus

The last circuit example is a 64-bit data bus. Each line has a cross-section dimension of $0.5um \times 0.5um$, and is $1000um$ long. The line pitch is $1um$. Parasitic extraction is applied



(a) $|Z_{10,10}|$ (b) $|Z_{45,10}|$

Figure 3: The 169-input RC mesh.

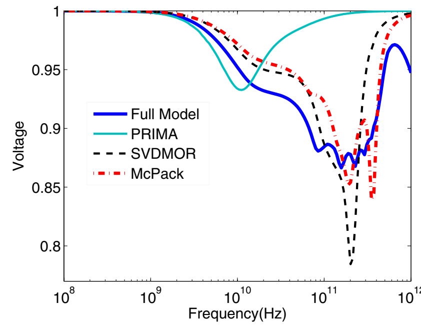


Figure 4: 128-port bus: near end of the 20th bit.

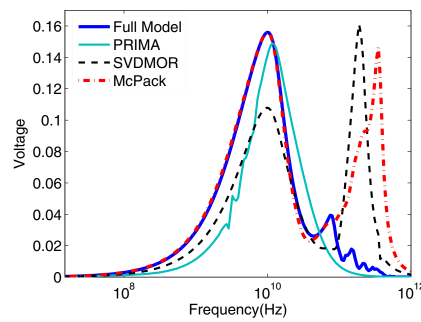


Figure 5: 128-port bus: far end of the 21st bit.

to extract a densely coupled 128-port RLC model. The extracted full model has 1,344 nodes, 1,280 resistors and inductors, 5,310 mutual inductors and 16,840 coupling capacitors. We use a *PRIMA* procedure to compute a 228-state model. The projection matrix consists of 128 zero-th order moment vectors expanded at DC and 200 zero-th order moment vectors expanded at 20GHz (along the imaginary axis) that are packed to 100 vectors using SVD. A 204-state SVDMOR model is obtained by extracting the port correlation using two SVDs of the matrix transfer function, one at DC and the other at 20GHz along the imaginary axis. 40 singular vectors at DC and 28 singular vectors at 20GHz are combined to approximate the input/output matrices. The 200-state McPack model performs a multipoint interpolation based model order reduction expanded at DC and 20GHz (along the imaginary axis). For both expansion points, 40 singular vectors are used to characterize the port correlation. The degree of $Q(s)$ is 2 at DC and 1 at 20GHz.

Once the three reduced, 128-port Y-parameter models are computed, they are compared against with the full model in a simulation where an ideal voltage source is applied to the near end of the 20-th bit via a resistor. All the other near ends are grounded and each far end is connected to ground through a capacitor. The voltage responses at the near end of the 20-th bit line and the far end of the 21-st bit line are shown in Fig. 4 and 5, respectively. Compared with previous examples, this large 128-port RLC model is more challenging to model compactly. However, as can be seen from, the reduced order model computed by *McPack* is the most accurate one for both cases, confirming the effectiveness the proposed algorithm. It takes 5.7s, 7.4s and 10.6s to compute the *PRIMA*, *SVDMOR* and *McPack* models.

7. CONCLUSIONS

Traditional model order reduction techniques are incapable of reducing linear networks with a large number of I/O ports. We propose to combine frequency-dependent port packing and tangential interpolation for more efficient model order reduction of these challenging networks. The proposed *McPack* algorithm systematically identifies the correlation between I/O ports seen from the system matrix transfer function and incorporates the port packing information directly as part of model order reduction. It is shown that *McPack* maintains the moment matching property under tangential interpolation and achieves excellent accuracy improvement over other techniques.

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