Closed-Form Expressions of Distributed RLC Interconnects for Analysis of On-Chip Inductance Effects

Yuichi Tanji
Dept. of RISE, Kagawa University
Hayashi-cho 2217–20
Takamatsu, 761-0396, Japan
tanji@eng.kagawa-u.ac.jp

Hideki Asai
Dept. of Systems Eng., Shizuoka University
Johoku3–5–1
Hamamatsu, 432-8561, Japan
hideasai@sys.eng.shizuoka.ac.jp

ABSTRACT

The closed-form expressions of distributed RLC interconnects are proposed for analysis of on-chip inductance effects in order to insert optimally the repeaters. The transfer function of a circuit with driver-interconnect-load structure is approximated by the 5th order rational functions. The step responses computed by using the proposed expressions give the good agreement with the SPICE simulations.

Categories and Subject Descriptors: B.7 [Integrated Circuits]: Design Aids

General Terms: Design, Theory.

Keywords: Inductance Effects, RLC Distributed Interconnects

1. INTRODUCTION

Increasing clock speed and low resistance metal on high performance integrated circuits make on-chip inductance effects prominent. Since the reactive component is comparable to resistive one due to the lower resistance, the inductance gives rise to over and undershoot on propagation signals, which results in fault switching and large power dissipation. The inductance with complex 3D structures of integrated circuits is extracted as equivalent circuits of electromagnetic fields[1]. The resulting equivalent circuits are very large scale and the timing simulations for evaluating the inductance effects waste huge CPU times. Hence, the model reduction algorithms of the equivalent circuits have received much attentions [6]-[11]. On the other hand, since global wires of the integrated circuits are far from the substrate, they are modeled by RLC distributed interconnects driven by the load resistance and output parasitic capacitance and driving the load capacitance [4]. The global wires are the most susceptible to large variations in the current return path. Therefore, suitable repeaters obtained as driven/driving passive elements must be inserted in order to diminish the on-chip inductance effects. In advance of the repeater size decision, the transfer function of the interconnect with linear termination has to be modeled accurately and efficiently. Some models [4], [5] have been presented using Taylor expansion, although they are not necessarily sufficient.

In this paper, the closed-form expressions of RLC distributed interconnects are proposed, where the transfer function of the networks are approximated by the 5th order rational functions. These expressions are based on the analytical functions for hyperbolic sine and cosine functions and the approximate one for e^s which are composed of powers of complex variable s. Therefore, the proposed expressions are compact and accurate, and these expressions would be useful for repeater size decision of global wires. In numerical examples, using the proposed closed-form expressions, the step-responses of the RLC distributed interconnects are computed, comparing with the SPICE simulations. It will be confirmed that the proposed closed-form expressions give good approximation results.

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2. PRELIMINARY

On-chip interconnects, especially global wires, are modeled by an RLC distributed interconnect driven by a repeater of resistance R_S and output parasitic capacitance C_P, and driving a repeater with load capacitance C_L as shown in Fig. 1 [4]. The optimum repeaters size must be determined in order to ease the interconnect effects such as delay and reflections. In advance of the decision, the transfer function has to be modeled accurately and the compactness is also required.

The chain matrix formulation of the circuit shown in Fig. 1 is given by

$$\begin{pmatrix} V_i \\ I_i \end{pmatrix} = \begin{pmatrix} 1 & R_S \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ sC_P & 1 \end{pmatrix} \times \begin{pmatrix} \cosh\gamma d & Z_0 \sinh\gamma d \\ \frac{1}{Z_0}\sinh\gamma d & \cosh\gamma d \end{pmatrix} \begin{pmatrix} 1 & 0 \\ sC_L & 1 \end{pmatrix} \begin{pmatrix} V_o \\ I_o \end{pmatrix} \quad (1)$$

where $Z_0 = \sqrt{(r+sl)/(sc)}$, $\gamma = \sqrt{(r+sl)(sc)}$, d is the length of the transmission line, and r, l, and c are resistance, inductance, capacitance per unit length of the line, respectively. The transfer function of the circuit is written by

$$\frac{V_i}{V_o} = \frac{N(s)}{D(s)} \quad (2)$$

$$= \{(1+sR_SC_P)(\cosh\gamma d + sC_L Z_0 \sinh\gamma d) + R_S((1/Z_0)\sinh\gamma d + sC_L\cosh\gamma d)\}^{-1}. \quad (3)$$

Recently, the compact models of the transfer function (3) have been proposed, based on the Taylor expansion of hyperbolic functions [4], [5]. Taylor expansion of circuit components is known as moment generation in the AWE methods [6] that are model reduction algorithms of large scale linear lumped circuits and distributed networks. Compact model is not obtained by only the moment generation, the moment matching technique is imperative. This means the previous works [4], [5] do not necessarily obtain an accurate model. The AWE method and other model reduction methods are classified into the method of numerical analysis. Therefore, we can not obtain the closed-form expression of the transfer function, using these model reduction algorithms.

In this paper, we propose the closed-form expressions of the

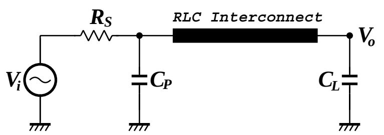


Figure 1: An RLC distributed interconnects driven by some passive elements

Table 1: Coefficients of denominator polynomials of transfer functions obtained from infinite product expansions.

degree	coefficient
0	1
1	$\frac{1}{2}R_TC_T + R_SC_P + R_TC_L + R_SC_T + C_LR_S$
2	$\frac{1}{2}L_TC_T + \frac{1}{2}R_SC_PC_TR_T + C_LR_TR_SC_P + C_LL_T$ $+\frac{1}{6}C_LR_T^2C_T + \frac{1}{6}R_SC_T^2R_T + \frac{1}{2}C_LR_SR_TC_T$
3	$\frac{1}{2}R_SC_PC_TL_T + C_LL_TR_SC_P + \frac{1}{6}C_LR_TL_TC_T$ $+\frac{1}{6}C_LR_T^2R_SC_PC_T + \frac{1}{6}C_LL_TR_TC_T + \frac{1}{6}R_SC_T^2L_T$ $+\frac{1}{2}C_LR_SL_TC_T$
4	$\frac{1}{6}C_LR_TR_SC_PL_TC_T + \frac{1}{6}C_LL_T^2C_T + \frac{1}{6}C_LL_TR_SC_PR_TC_T$
5	$\frac{1}{6}C_LL_T^2R_SC_PC_T$

transfer function using the following scalar functions [14], [15].

$$\sinh z = z\prod_{n=1}^{\infty}\left(1+\frac{z^2}{n^2\pi^2}\right) \quad (4)$$

$$\cosh z = \prod_{n=1}^{\infty}\left[1+\frac{z^2}{\{(2n-1)\pi/2\}^2}\right] \quad (5)$$

$$\frac{\cosh z}{\sinh z} = \frac{1}{z} + 2z\sum_{n=1}^{\infty}\frac{1}{z^2+(n\pi)^2} \quad (6)$$

$$\frac{1}{\sinh z} = \frac{1}{z} + 2z\sum_{n=1}^{\infty}\frac{(-1)^n}{z^2+(n\pi)^2} \quad (7)$$

$$e^z = \frac{\sum_{j=0}^{M}\frac{(M+N-j)!M!}{(M+N)!j!(M-j)!}z^j}{\sum_{j=0}^{N}\frac{(M+N-j)!N!}{(M+N)!j!(N-j)!}(-z)^j} \quad (8)$$

The first two formulae (4) and (5) are infinite product expansions of hyperbolic functions, the second two formulae (6) and (7) are partial fraction expansions, and (8) is Páde approximation.

The three compact models of the transfer function (3) are provided with good accuracy in Sect. IV.

3. FORMULATION OF GENERAL LOSSY TRANSMISSION LINES

Before providing the closed-form expressions of RLC distributed interconnects, the formulations of general lossy transmission lines are obtained without loss of generality, where the transmission lines are assumed to be frequency dependent and arbitrary number of conductors. Since we focus on global wires on VLSI circuits in this paper, single conductor transmission line is enough to be considered. However, the formulations being presented here would be very useful for crosstalk modeling considering the coupling inductance effects [3].

Table 2: Coefficients of denominator polynomials of transfer functions obtained from partial fraction expansions.

degree	coefficient
0	π^2
1	$R_TC_L\pi^2 + 3R_TC_T + R_SC_P\pi^2 + 8R_SC_T + C_LR_S\pi^2$
2	$3L_TC_T + R_T^2C_TC_L + L_TC_L\pi^2 + R_SC_PR_TC_L\pi^2$ $+3R_SC_PR_TC_T + 3R_TC_TC_LR_S$
3	$2R_TL_TC_TC_L + 3R_SC_PL_TC_T + R_SC_PR_T^2C_TC_L$ $+R_SC_PL_TC_L\pi^2 + 3L_TC_TC_LR_S$
4	$2R_SC_PR_TL_TC_TC_L + L_T^2C_TC_L$
5	$R_SC_PL_T^2C_TC_L$

3.1 Infinite Product Expansion

The Telegrapher's equations of n-conductor lossy transmission lines are described by

$$\frac{d\boldsymbol{V}(x)}{dx} = -\boldsymbol{Z}_s\boldsymbol{I}(x), \quad \frac{d\boldsymbol{I}(x)}{dx} = -\boldsymbol{Y}_p\boldsymbol{V}(x) \quad (9)$$

where

$$\boldsymbol{Z}_s = \boldsymbol{R}(s) + s\boldsymbol{L}(s), \quad \boldsymbol{Y}_p = \boldsymbol{G}(s) + s\boldsymbol{C}(s).$$

In (9), $\boldsymbol{V}(x)$ and $\boldsymbol{I}(x)$ are voltage and current vectors at x, and $\boldsymbol{R}(s)$, $\boldsymbol{L}(s)$, $\boldsymbol{C}(s)$, and $\boldsymbol{G}(s)$ are respectively resistance, inductance, capacitance, and conductance matrices per unit length which are assumed to be frequency dependent and symmetric matrices.

The multiport networks of the transmission lines are described in admittance, impedance, and chain matrices [12], associated with the eigenvalue problem:

$$\left(\boldsymbol{Z}_s\boldsymbol{Y}_p - \gamma_i^2\boldsymbol{I}\right)\boldsymbol{p}_i = \boldsymbol{0}, \quad (i=1,\ldots,m). \quad (10)$$

Assuming the transform matrix and diagonal one with the eigenvalues as

$$\boldsymbol{P} = (\boldsymbol{p}_1,\ldots,\boldsymbol{p}_m) \quad (11)$$

$$\boldsymbol{\Gamma}^2 = \mathrm{diag}\left(\gamma_1^2,\ldots,\gamma_m^2\right), \quad (12)$$

we write the multiport networks using the chain matrix as

$$\begin{pmatrix}\boldsymbol{V}(0)\\ \boldsymbol{I}(0)\end{pmatrix} = \boldsymbol{F}(s)\begin{pmatrix}\boldsymbol{V}(d)\\ \boldsymbol{I}(d)\end{pmatrix} \quad (13)$$

where

$$\boldsymbol{F}(s) = \begin{pmatrix}\boldsymbol{P}\boldsymbol{E}_1\boldsymbol{P}^{-1} & \boldsymbol{P}\boldsymbol{E}_2\boldsymbol{P}^{-1}\boldsymbol{Z}_s\\ \boldsymbol{Z}_s^{-1}\boldsymbol{P}\boldsymbol{E}_3\boldsymbol{P}^{-1} & \boldsymbol{Z}_s^{-1}\boldsymbol{P}\boldsymbol{E}_1\boldsymbol{P}^{-1}\boldsymbol{Z}_s\end{pmatrix} \quad (14)$$

$$\boldsymbol{E}_1 = \mathrm{diag}\left(\cosh\gamma_1 d,\ldots,\cosh\gamma_m d\right)$$

$$\boldsymbol{E}_2 = \mathrm{diag}\left(\frac{\sinh\gamma_1 d}{\gamma_1},\ldots,\frac{\sinh\gamma_m d}{\gamma_m}\right)$$

$$\boldsymbol{E}_3 = \mathrm{diag}\left(\gamma_1\sinh\gamma_1 d,\ldots,\gamma_m\sinh\gamma_m d\right), \quad (15)$$

and d is the length of transmission lines.

The closed-form expression of the chain matrix $\boldsymbol{F}(s)$ in a rational matrix form of s is obtained by using the infinite product expansions (4) and (5). Let us consider first the submatrix $\boldsymbol{P}\boldsymbol{E}_1\boldsymbol{P}^{-1}$. Using (5), we write it as

$$\begin{aligned}\boldsymbol{P}\boldsymbol{E}_1\boldsymbol{P}^{-1} &= \boldsymbol{P}\mathrm{diag}\left(\prod_{n=1}^{\infty}\left[1+\frac{\{\gamma_1 l\}^2}{\{(2n-1)\pi/2\}^2}\right],\right.\\ &\left.\cdots,\prod_{n=1}^{\infty}\left[1+\frac{\{\gamma_m l\}^2}{\{(2n-1)\pi/2\}^2}\right]\right)\boldsymbol{P}^{-1}\\ &= \boldsymbol{P}\boldsymbol{D}_1\boldsymbol{D}_2\cdots\boldsymbol{D}_\infty\boldsymbol{P}^{-1}\end{aligned} \quad (16)$$

where

$$\boldsymbol{D}_i = \mathrm{diag}\left(1+\frac{\{\gamma_1 d\}^2}{\{(2n-1)\pi/2\}^2},\cdots,1+\frac{\{\gamma_m d\}^2}{\{(2n-1)\pi/2\}^2}\right).$$

Since $\boldsymbol{P}\boldsymbol{\Gamma}^2\boldsymbol{P}^{-1} = \boldsymbol{Z}_s\boldsymbol{Y}_p$, (16) is rewritten by

$$\begin{aligned}\boldsymbol{P}\boldsymbol{E}_1\boldsymbol{P}^{-1} &= \boldsymbol{P}\boldsymbol{D}_1\boldsymbol{P}^{-1}\boldsymbol{P}\boldsymbol{D}_2\boldsymbol{P}^{-1}\cdots\boldsymbol{P}\boldsymbol{D}_\infty\boldsymbol{P}^{-1}\\ &= \left(\boldsymbol{I}+\frac{d^2}{\{\pi/2\}^2}\boldsymbol{Z}_s\boldsymbol{Y}_p\right)\\ &\quad\times\left(\boldsymbol{I}+\frac{d^2}{\{3\pi/2\}^2}\boldsymbol{Z}_s\boldsymbol{Y}_p\right)\cdots\boldsymbol{I}.\end{aligned} \quad (17)$$

Similarly, using (4) and (5), other submatrices in (13) are expressed by

$$\begin{aligned} \boldsymbol{P E}_2 \boldsymbol{P}^{-1} \boldsymbol{Z}_s &= d\left(\boldsymbol{I}+\frac{d^2}{\pi^2}\boldsymbol{Z}_s\boldsymbol{Y}_p\right) \\ &\times\left(\boldsymbol{I}+\frac{d^2}{\{2\pi\}^2}\boldsymbol{Z}_s\boldsymbol{Y}_p\right)\cdots\boldsymbol{I}\boldsymbol{Z}_s. \end{aligned} \quad (18)$$

$$\begin{aligned} \boldsymbol{Z}_s^{-1}\boldsymbol{P E}_3 \boldsymbol{P}^{-1} &= d\boldsymbol{Y}_p\left(\boldsymbol{I}+\frac{d^2}{\pi^2}\boldsymbol{Z}_s\boldsymbol{Y}_p\right) \\ &\times\left(\boldsymbol{I}+\frac{d^2}{\{2\pi\}^2}\boldsymbol{Z}_s\boldsymbol{Y}_p\right)\cdots\boldsymbol{I}. \end{aligned} \quad (19)$$

$$\begin{aligned} \boldsymbol{Z}_s^{-1}\boldsymbol{P E}_1 \boldsymbol{P}^{-1}\boldsymbol{Z}_s &= \boldsymbol{Z}_s^{-1}\left(\boldsymbol{I}+\frac{d^2}{\{\pi/2\}^2}\boldsymbol{Z}_s\boldsymbol{Y}_p\right) \\ &\times\left(\boldsymbol{I}+\frac{d^2}{\{3\pi/2\}^2}\boldsymbol{Z}_s\boldsymbol{Y}_p\right)\cdots\boldsymbol{Z}_s. \end{aligned} \quad (20)$$

All submatrices of (13) are represented by matrix polynomials. Therefore, the chain matrix (13) is written by a rational matrix of s, if the series impedance $\boldsymbol{Z}_s$ and the parallel admittance $\boldsymbol{Y}_p$ matrices per unit length are described by rational matrices of s.

3.2 Partial Fraction Expansion

The closed-form expression of the admittance matrix $\boldsymbol{Y}(s)$ has been obtained in [13]. However, it is not shown that the derivation is based on the partial faction expansions of (6) and (7). So, the derivation is again given based on their expansions.

The admittance matrix of the transmission lines is written by

$$\boldsymbol{Y}(s) = \begin{pmatrix} \boldsymbol{Z}_s^{-1}\boldsymbol{P E}_4\boldsymbol{P}^{-1} & -\boldsymbol{Z}_s^{-1}\boldsymbol{P E}_5\boldsymbol{P}^{-1} \\ -\boldsymbol{Z}_s^{-1}\boldsymbol{P E}_5\boldsymbol{P}^{-1} & \boldsymbol{Z}_s^{-1}\boldsymbol{P E}_4\boldsymbol{P}^{-1} \end{pmatrix} \quad (21)$$

where

$$\boldsymbol{E}_4 = \operatorname{diag}\left(\gamma_1\frac{\cosh\gamma_1 l}{\sinh\gamma_1 l},\ldots,\gamma_m\frac{\cosh\gamma_m l}{\sinh\gamma_m l}\right) \quad (22)$$

$$\boldsymbol{E}_5 = \operatorname{diag}\left(\gamma_1\frac{1}{\sinh\gamma_1 l},\ldots,\gamma_m\frac{1}{\sinh\gamma_m l}\right) \quad (23)$$

From (7), the submatrix $\boldsymbol{Z}_s^{-1}\boldsymbol{P E}_4\boldsymbol{P}^{-1}$ is expressed by

$$\boldsymbol{Z}_s^{-1}\boldsymbol{P E}_4\boldsymbol{P}^{-1} = \frac{1}{d}\boldsymbol{Z}_s^{-1}+\frac{2}{d}\sum_{n=1}^{\infty}\left\{\boldsymbol{Z}_s^{-1}+\left(\frac{n\pi}{d}\right)\boldsymbol{Y}_p^{-1}\right\}^{-1} \quad (24)$$

Representing the submatrix $-\boldsymbol{Z}_s^{-1}\boldsymbol{P E}_5\boldsymbol{P}^{-1}$ in the similar form of (24), we describe the admittance matrix $\boldsymbol{Y}(s)$ as

$$\begin{aligned} \boldsymbol{Y}(s) &= \frac{1}{d}\boldsymbol{Z}_s^{-1}\begin{pmatrix} \boldsymbol{I} & -\boldsymbol{I} \\ -\boldsymbol{I} & \boldsymbol{I}\end{pmatrix}+\frac{2}{d}\sum_{n=1}^{\infty}\{\boldsymbol{Z}_s \\ &+\left(\frac{n\pi}{d}\right)^2\boldsymbol{Y}_p^{-1}\Big\}^{-1}\begin{pmatrix} \boldsymbol{I} & \boldsymbol{I}(-1)^{n+1} \\ \boldsymbol{I}(-1)^{n+1} & \boldsymbol{I}\end{pmatrix}. \end{aligned} \quad (25)$$

3.3 Páde Approximation

The third expression is one proposed in [16]. The reciprocal expression of (13) is expressed by

$$\begin{pmatrix} \boldsymbol{V}(d) \\ \boldsymbol{I}(d)\end{pmatrix} = e^{\boldsymbol{Z}}\begin{pmatrix} \boldsymbol{V}(0) \\ \boldsymbol{I}(0)\end{pmatrix} \quad (26)$$

where

$$\boldsymbol{Z} = \begin{pmatrix} \boldsymbol{0} & -\boldsymbol{Z}_s \\ -\boldsymbol{Y}_p & \boldsymbol{0}\end{pmatrix}, \quad (27)$$

and $e^{\boldsymbol{Z}}$ is the matrix exponential.

Using Páde approximation (8) for $M = N$, we can write (26) into

$$\begin{pmatrix} \boldsymbol{V}(d) \\ \boldsymbol{I}(d)\end{pmatrix} = \boldsymbol{H}^{-1}\boldsymbol{Q}\begin{pmatrix} \boldsymbol{V}(0) \\ \boldsymbol{I}(0)\end{pmatrix} \quad (28)$$

where

$$\boldsymbol{Q} = \sum_{j=0}^{N}\tfrac{(2N-j)!N!}{(2N)!j!(N-j)!}\boldsymbol{Z}^j, \quad \boldsymbol{H} = \sum_{j=0}^{N}\tfrac{(2N-j)!N!}{(2N)!j!(N-j)!}(-\boldsymbol{Z})^j \quad (29)$$

The input-output relations are expressed by the two matrix polynomials $\boldsymbol{Q}$ and $\boldsymbol{H}$ in (28). As same with the previous two expressions, the matrix $\boldsymbol{H}^{-1}\boldsymbol{Q}$ is a rational matrix of s, if $\boldsymbol{Z}_s$ and $\boldsymbol{Y}_p$ are rational matrices.

4. TRANSFER FUNCTION

4.1 Infinite Product Expansion

From (13) and (16)-(20), the element of the ABCD matrix of single-conductor transmission line is rewritten in the infinite series as

$$\cosh\gamma d = 1+\frac{4d^2}{\pi^2}\left(\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\right)Z_sY_p+\cdots \quad (30)$$

$$Z_0\sinh\gamma d = \left(1+\frac{d^2}{\pi^2}\left(\sum_{n=1}^{\infty}\frac{1}{n^2}\right)Z_sY_p+\cdots\right)Z_s d \quad (31)$$

$$\frac{1}{Z_0}\sinh\gamma d = dY_p\left(1+\frac{d^2}{\pi^2}\left(\sum_{n=1}^{\infty}\frac{1}{n^2}\right)Z_sY_p+\cdots\right). \quad (32)$$

where Z_s and Y_p are series impedance and parallel admittance per unit length.

To obtain the low order approximation of the transfer function (3), the infinite series of Z_sY_p is truncated until the second term. Using the relation

$$\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} = \frac{1}{8}, \quad \sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{1}{6} \quad (33)$$

we approximate the transfer function with [0/5] degree rational function. The coefficients of denominator polynomial are listed in Tab. 1 and the numerator polynomial is given by $N(s) = 1$, where $dZ_s = R_T + sL_T$ and $dY_p = sC_T$ are total series impedance and parallel admittance of the transmission line. R_T, L_T, and C_T imply resistance, inductance, and capacitance.

4.2 Partial Fraction Expansion

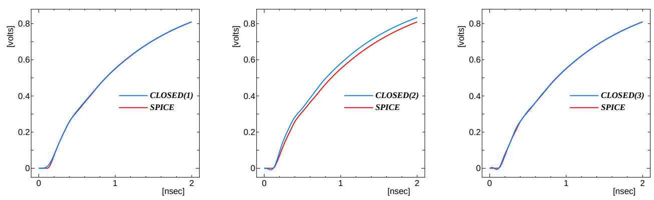
The closed-form expression (25) using the partial fraction expansions of (6) and (7) are truncated until the third term ($n = 2$). The expression is converted into the chain matrix in order to obtain the transfer function (3). As a result, we can obtain [2/5] degree rational function. The coefficients of denominator polynomial are listed in Tab. 2 and the numerator polynomial is given by $N(s) = -L_TC_Ts^2 - R_TC_Ts + \pi^2$.

4.3 Páde Approximation

Putting the coefficients of (8) into $q_i = \sum_{j=0}^{N}\frac{(2N-j)!N!}{(2N)!j!(N-j)!}$, the transfer function of the distributed RLC interconnect is approximated using (28). We can obtain [4/5] degree rational function. The coefficients of denominator polynomial are listed in Tab. 3 and the numerator polynomial is given by $N(s) = (q_2C_TL_T)^2s^4 + 2q_2^tC_T^2R_TL_Ts^3+\left\{(q_2C_TR_T)^2+2q_0q_2C_TL_T-q_1^2C_TL_T\right\}s^2+(2q_0q_2C_TR_T-q_1^2C_TR_T)s+q_0^2$.

Table 3: Coefficients of denominator polynomials of transfer functions obtained from Páde approximation.

degree	coefficient
0	q_0^2
1	$2q_0q_2C_TR_T + q_1^2C_TR_T + 2q_0q_1C_LR_T + q_0^2R_SC_P$ $+q_0^2R_SC_L + 2q_0q_1R_SC_T$
2	$(q_2C_TR_T)^2 + 2q_0q_2C_TL_T + q_1^2C_TL_T + 2q_0q_1C_LC_T$ $+2q_1q_2C_LC_TR_T^2 + 2q_0q_2C_TR_TR_SC_P + q_1^2R_SC_PC_TR_T$ $+2q_0q_1R_SC_PC_LR_T + 2q_0q_2R_SC_LC_TR_T$ $+q_1^2R_SC_LC_TR_T + 2q_1q_2R_SC_T^2R_T$
3	$+2q_2^2C_T^2R_TL_T + 4q_1q_2C_LC_TR_TL_T + (q_2C_TR_T)^2R_SC_P$ $+2q_0q_2R_SC_PC_TL_T + q_1^2R_SC_PC_TL_T + 2q_0q_1R_SC_PC_LL_T$ $+2q_1q_2R_SC_PC_LC_TR_T^2 + (q_2C_TR_T)^2R_SC_L$ $+2q_0q_2R_SC_LC_TL_T + q_1^2R_SC_LC_TL_T + 2q_1q_2R_SC_T^2L_T$
4	$(q_2C_TL_T)^2 + 2q_1q_2C_LC_TL_T^2 + 2q_2^2R_SC_PC_T^2R_TL_T$ $+4q_1q_2R_SC_PC_LC_TR_TL_T + 2q_2^2R_SC_LC_T^2R_TL_T$
5	$R_SC_P(q_2C_TL_T)^2 + 2q_1q_2R_SC_PC_LC_TL_T^2$ $+R_SC_L(q_2C_TL_T)^2$



(a) (b) (c)

Figure 2: Step responses of the circuit shown in Fig. 1, where $R_S = 457$ [Ω], $C_P = 750$ [fF], and $C_L = 750$ [fF]. Results are obtained from the closed-form expressions based on (a)infinite product expansion (b)partial fraction expansion (c)Páde approximation.

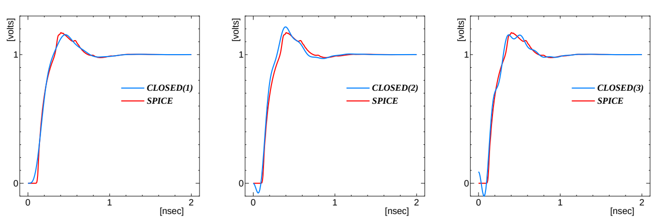
5. RESULTS AND DISCUSSION

The closed-form expressions of the transfer function obtained in the previous section are estimated by a sample which is the 180-nm technology node (ITRS 1999 [17]) with $R = 36281$ [Ω/m], $C = 269$ [pF/m], $L = 4$ [μ/m], and 3.3 [mm]. We computed the step responses of the circuit shown in Fig. 1. The step responses in the frequency-domain are described by

$$\frac{1}{s} + \sum_{n=1}^{5} \frac{k_n}{s - p_n} \tag{34}$$

where the pole p_n is calculated by foot finding algorithm and the residue k_n is obtained by

$$k_n = \left. \frac{N(s)}{d/ds(sD(s))} \right|_{s=p_n}. \tag{35}$$



(a) (b) (c)

Figure 3: Step responses of the circuit shown in Fig. 1, where $R_S = 0$ [Ω], $C_P = 0$ [fF], and $C_L = 750$ [fF]. Results are obtained from the closed-form expressions based on (a)infinite product expansion (b)partial fraction expansion (c)Páde approximation.

Figures 2 and 3 show the step responses of far-end of the transmission line obtained by using the closed-form expressions in the previous section. For comparisons, the same circuits were analyzed by Berkley SPICE3, where the transmission line was discretized by 100 T-sections. All expressions capture the profiles of the step responses as shown in Fig. 2 and 3. Especially, the result obtained from the expression based on infinite product expansion seems to be better than other expressions, and the expression based on partial fraction expansion is somewhat inaccurate in estimating the delay time.

6. CONCLUSIONS

The closed-form expressions of RLC distributed interconnects have been presented for analysis of the on-chip inductance effects. The step responses computed by using the closed-form expressions give the good agreement with the SPICE simulations. Since these derivations are based on modeling of general lossy coupled transmission lines, the closed-form expressions would be easily extended to crosstalk modeling to evaluate the on-chip inductance/capacitance coupling effects.

7. REFERENCES

[1] M. Kammon, M. J. Tsuk, and J. K. White, "FASTHENRY: A multipole-accelerated 3-D inductance extraction program," *IEEE Trans. Microwave Theory Techn.*, vol. 42, no. 1, pp. 1750-1758, Sept. 1994.

[2] E. E. Davidson and B. D. McCredie, and W. V. Vilkelis. "Long lossy lines (L^3) and their impact upon large chip performance," *IEEE Trans. Components, Packaging, and Manufacturing Technology, Part B. Advanced Packaging.*, vol. 20, pp. 361-375, Nov. 1998.

[3] T. Sakurai, "Closed-form expressions for interconnection delay, coupling, and crosstalk in VLSI's," *IEEE Trans. Electron. Devices*, vol. 40, no. 1, pp. 118-124, Jan. 1993.

[4] K. Banerjee and A. Mehrotra, "Analysis of on-chip inductance effects for distributed RLC interconnects," *IEEE Trans. Computer-Aided Design*, vol. 21, no. 8, pp. 904-915, Aug. 2002.

[5] Y. I. Ismail and E. G. Friedman, "Effects of inductance on the propagation delay and repeater insertion in VLSI circuits," *IEEE Trans. Computer-Aided Design*, vol. 8, no. 2, pp. 195-206, Apr. 2000.

[6] L. T. Pillage and R. A. Rohrer, "Asymptotic waveform evaluation for timing analysis," *IEEE Trans. Computer-Aided Design*, vol. 9, no. 4, pp. 352-366, Apr. 1990.

[7] E. Chiprout and M. S. Nakhla, "Analysis of interconnect networks using complex frequency hopping (CFH)," *IEEE Trans. Computer-Aided Design*, vol. 14, no. 2, pp. 186-220, Feb. 1995.

[8] P. Feldmann and R. W. Freund, "Efficient linear analysis by Padé approximation via the Lanczos process," *IEEE Trans. Computer-Aided Design*, vol. 14, no. 5, pp. 639-649, May 1995.

[9] L. M. Silveira, M. Kamon, I. Elfadel, and J. White, "A coordinate-transformed Arnoldi algorithm for generating guaranteed stable reduced-order models of RLC circuits," *in Proc. ICCAD'96*, pp. 288-294, 1996.

[10] A. Odabasioglu, M. Celik, and L. T. Pilleggi, "PRIMA: passive reduced-order interconnect macromodeling algorithm," *IEEE Trans. Computer-Aided Design*, vol. 17, no. 8, pp. 645-654, Aug. 1998.

[11] Y. I. Ismail and E. G. Friedman, "DTT: direct truncation of the transfer function– alternative to moment matching for tree structured interconnect," *IEEE Trans. Computer-Aided Design*, vol. 21, no. 2, pp. 131-144, Feb. 2002.

[12] D. H. Xie and M. Nakhla, "Delay and crosstalk simulation of high-speed VLSI interconnects with nonlinear terminations," *IEEE Trans. Computer-Aided Design*, vol. 12, no. 11, pp. 1798-1811, Nov. 1993.

[13] Y. Tanji, A. Ushida, and M. S. Nakhla, "Passive closed-form expression of RLCG transmission lines," *in Proc. ISCAS'02*, vol. 3, pp. 795-798, May 2002.

[14] S. Moriguchi, K. Udagawa, S. Ichimatsu, *Suugaku Koushiki II*, Kadokawa, Tokyo, 1957 (in Japanese).

[15] J. Vlach and K. Singhal, *Computer Methods for Circuits Analysis and Design*, New York: Van Nostrand Reinholdt, 1983.

[16] A. Dounavis, R. Achar, and M. Nakhla, "Efficient passive circuit models for distributed networks with frequency-dependent parameters," *IEEE Trans. Microwave Theory Tech.*, vol. 23, pp. 382-392, Aug 2000.

[17] *Int. Technol. Roadmap for Semiconductors (ITRS)*, 1999.