Problem 7 (1)

October 22, 2019

0.1 Problem 7

0.2 7.1: Weight Update Equations for Five Gradient Descent Optimizers:

0.2.1 a) AdaGrad

Adagrad uses a different learning rate η_i for every parameter θ_i, which it updates during each step t_i Let g_t denote the gradient at time step t. $g_{t,i}$ is then the partial derivative of the objective function w.r.t. to the parameter θ_i at time step t.

Then, the weight update equation: $ g_{t,i} = \Delta

Instead of accumulating all past squared gradients, Adadelta restricts the window of accumulated past gradients to some fixed size w. Starting from AdaGrad,

$$\theta_{t+1,i} = \theta_{t,i} + \frac{\eta}{\sqrt{G_{t,i} + \epsilon}} g_t$$

we replace the sum of squares of accumulated weights $G_{t,i}$ with $E[G_{t,i}$, sum of squares of weights only accumulated through the window, which is nothing but the Root Mean Squared error of the weights within the window. Additionally, to match the dimensions of the learning rate, the LR is also defined as an infinitely decaying RMS parameter. This results in the following weight update equation:

$$\theta_{t+1} = \theta_t - \frac{RMS[\Delta\theta]_{t-1}]}{RMS[g_t} \cdot g_t$$

0.2.2 c) RMSProp:

Developed to sovle the vanishing gradients problem in AdaDelta- retain η, and apply an exponentially decaying average of square gradients.

$$\theta_{t+1,i} = \theta_{t,i} + \frac{\eta}{\sqrt{E[g^2]_t + \epsilon}} g_t$$

HyperParameter: η. the learning rate and ϵ, any small value to smoothen out (avoid division by 0)

0.2.3 d) Adam

In addition to maintaining the average of squared decaying gradients, it also maintains the average of the decaying gradients themselves.

$$m_t = \beta_1 m_{t-1} + (1 - \beta_1) g_t$$

$$v_t = \beta_2 v_{t-1} + (1 - \beta_2) g_t^2$$

Thus, the update rule:

$$\theta_t = \theta_{t-1} - \frac{\eta}{\sqrt{v_t + \epsilon}} \hat{m}_t$$

where $\hat{m}$ and $\hat{v}$ are the gradients of m and v respectively

HyperParameters: β_1, the factor for momentum calculation, and β_2, the factor for sum of squared weights.

0.2.4 e) RMSProp + Nesterov: Nadam

Addition of a Momentum term to RMSProp.

$$\theta_t = \theta_{t-1} - \frac{\eta}{\sqrt{\hat{v} + \epsilon}} (\beta \hat{m}_t + \frac{1 - \beta g_t}{1 - \beta})$$

where $\hat{m}$ is the momentum (average of past exponentially decaying gradients) at time t, $\hat{v}$ the average of squares of past exponentially decaying gradients, and β the Adam Rule hyperparameter.

HyperParameters: β_1, the factor for momentum calculation, and β_2, the factor for sum of squared weights.

Disclaimer: Using a 300 unit dense layer since a model with 784x1000x1000 does not fit into memory (tried until 4 NVDIA P100 GPUs, 102GB RAM with 16 vCPUs)

[1]:
```
from keras.datasets import mnist
from keras.utils import np_utils

import time

from keras.optimizers import RMSprop,Adagrad,Adadelta,Adam,Nadam
from keras.models import Sequential
from keras.layers import Dense,Dropout,Flatten
import keras.layers as layers
from keras import regularizers
```

```
Using TensorFlow backend.
/usr/local/lib/python3.5/dist-
packages/tensorflow/python/framework/dtypes.py:516: FutureWarning: Passing
(type, 1) or '1type' as a synonym of type is deprecated; in a future version of
numpy, it will be understood as (type, (1,)) / '(1,)type'.
  _np_qint8 = np.dtype([("qint8", np.int8, 1)])
/usr/local/lib/python3.5/dist-
packages/tensorflow/python/framework/dtypes.py:517: FutureWarning: Passing
(type, 1) or '1type' as a synonym of type is deprecated; in a future version of
numpy, it will be understood as (type, (1,)) / '(1,)type'.
  _np_quint8 = np.dtype([("quint8", np.uint8, 1)])
/usr/local/lib/python3.5/dist-
packages/tensorflow/python/framework/dtypes.py:518: FutureWarning: Passing
(type, 1) or '1type' as a synonym of type is deprecated; in a future version of
numpy, it will be understood as (type, (1,)) / '(1,)type'.
```

```
  _np_qint16 = np.dtype([("qint16", np.int16, 1)])
/usr/local/lib/python3.5/dist-
packages/tensorflow/python/framework/dtypes.py:519: FutureWarning: Passing
(type, 1) or '1type' as a synonym of type is deprecated; in a future version of
numpy, it will be understood as (type, (1,)) / '(1,)type'.
  _np_quint16 = np.dtype([("quint16", np.uint16, 1)])
/usr/local/lib/python3.5/dist-
packages/tensorflow/python/framework/dtypes.py:520: FutureWarning: Passing
(type, 1) or '1type' as a synonym of type is deprecated; in a future version of
numpy, it will be understood as (type, (1,)) / '(1,)type'.
  _np_qint32 = np.dtype([("qint32", np.int32, 1)])
/usr/local/lib/python3.5/dist-
packages/tensorflow/python/framework/dtypes.py:525: FutureWarning: Passing
(type, 1) or '1type' as a synonym of type is deprecated; in a future version of
numpy, it will be understood as (type, (1,)) / '(1,)type'.
  np_resource = np.dtype([("resource", np.ubyte, 1)])
/usr/local/lib/python3.5/dist-
packages/tensorboard/compat/tensorflow_stub/dtypes.py:541: FutureWarning:
Passing (type, 1) or '1type' as a synonym of type is deprecated; in a future
version of numpy, it will be understood as (type, (1,)) / '(1,)type'.
  _np_qint8 = np.dtype([("qint8", np.int8, 1)])
/usr/local/lib/python3.5/dist-
packages/tensorboard/compat/tensorflow_stub/dtypes.py:542: FutureWarning:
Passing (type, 1) or '1type' as a synonym of type is deprecated; in a future
version of numpy, it will be understood as (type, (1,)) / '(1,)type'.
  _np_quint8 = np.dtype([("quint8", np.uint8, 1)])
/usr/local/lib/python3.5/dist-
packages/tensorboard/compat/tensorflow_stub/dtypes.py:543: FutureWarning:
Passing (type, 1) or '1type' as a synonym of type is deprecated; in a future
version of numpy, it will be understood as (type, (1,)) / '(1,)type'.
  _np_qint16 = np.dtype([("qint16", np.int16, 1)])
/usr/local/lib/python3.5/dist-
packages/tensorboard/compat/tensorflow_stub/dtypes.py:544: FutureWarning:
Passing (type, 1) or '1type' as a synonym of type is deprecated; in a future
version of numpy, it will be understood as (type, (1,)) / '(1,)type'.
  _np_quint16 = np.dtype([("quint16", np.uint16, 1)])
/usr/local/lib/python3.5/dist-
packages/tensorboard/compat/tensorflow_stub/dtypes.py:545: FutureWarning:
Passing (type, 1) or '1type' as a synonym of type is deprecated; in a future
version of numpy, it will be understood as (type, (1,)) / '(1,)type'.
  _np_qint32 = np.dtype([("qint32", np.int32, 1)])
/usr/local/lib/python3.5/dist-
packages/tensorboard/compat/tensorflow_stub/dtypes.py:550: FutureWarning:
Passing (type, 1) or '1type' as a synonym of type is deprecated; in a future
version of numpy, it will be understood as (type, (1,)) / '(1,)type'.
  np_resource = np.dtype([("resource", np.ubyte, 1)])
```

[2]:
```
(x_train, y_train), (x_test, y_test) = mnist.load_data()

# Set numeric type to float32 from uint8
x_train = x_train.astype("float32")
x_test = x_test.astype("float32")

# Transform lables to one-hot encoding
y_train = np_utils.to_categorical(y_train, 10)
y_test = np_utils.to_categorical(y_test, 10)

# Reshape the dataset into 4D array
x_train = x_train.reshape(x_train.shape[0], 28,28,1)
x_test = x_test.reshape(x_test.shape[0], 28,28,1)

#Normalize input
x_test /= 255
x_train /= 255
```

[3]:
```
model = Sequential()
model.add(Dense(300, input_shape=(28,28,1), activation='relu',␣
 ↪kernel_regularizer=regularizers.l2(0.1)))
model.add(Flatten())
model.add(Dense(300,activation='relu', kernel_regularizer=regularizers.l2(0.1)))
model.add(Dense(10,activation='softmax'))
```

[29]:
```
opts = ['Adagrad','RMSprop','Nadam','Adadelta','Adam']

histories = []
times = []
metrics = []

for opt in opts:
    model.compile(loss='categorical_crossentropy', optimizer=opt,␣
 ↪metrics=['accuracy'])
    print(" Using Optimizer: "+opt)
    start = time.time()
    history = model.fit(x_train, y_train,
        batch_size=128,
        epochs=200,
        verbose=0,
        )
    t = time.time()-start

    histories.append(history)
    times.append(t)
    metrics.append(model.evaluate(x_test,y_test))
```

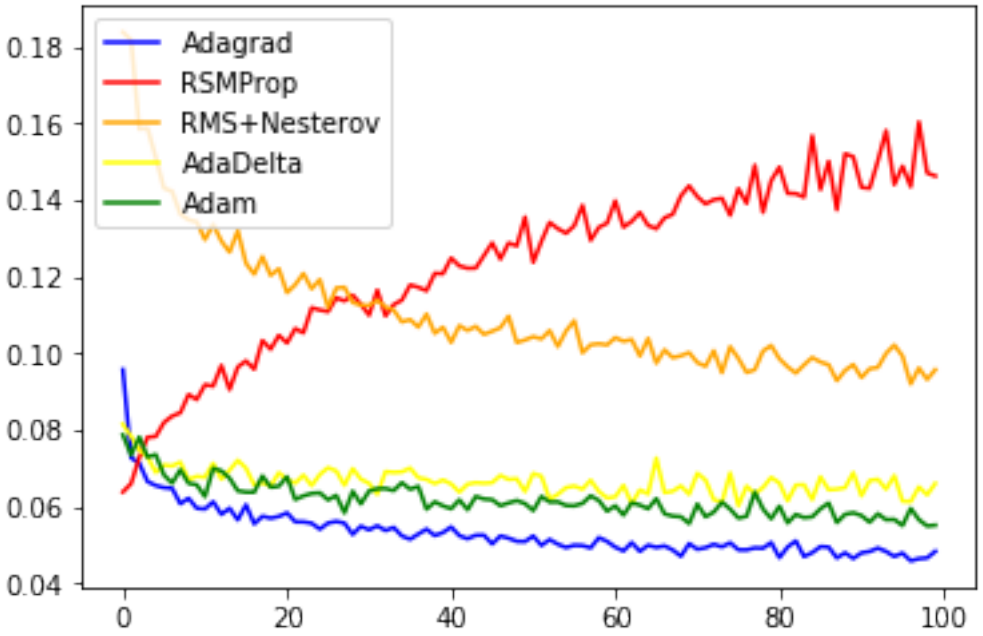
```
Using Optimizer: Adagrad
```

```
10000/10000 [==============================] - 1s 66us/step
 Using Optimizer: RMSprop
10000/10000 [==============================] - 1s 67us/step
 Using Optimizer: Nadam
10000/10000 [==============================] - 1s 68us/step
 Using Optimizer: Adadelta
10000/10000 [==============================] - 1s 68us/step
 Using Optimizer: Adam
10000/10000 [==============================] - 1s 70us/step
```

[30]:
```
print(times)
print(histories)
print(metrics)
```

```
[153.77004885673523, 158.54089736938477, 196.96231698989868, 176.2131621837616,
175.37222576141357]
[<keras.callbacks.callbacks.History object at 0x7f01840ff5f8>,
<keras.callbacks.callbacks.History object at 0x7f015c5facc0>,
<keras.callbacks.callbacks.History object at 0x7f015c3089b0>,
<keras.callbacks.callbacks.History object at 0x7f0154eb6588>,
<keras.callbacks.callbacks.History object at 0x7f01549c3e80>]
[[0.05396905895551717, 0.9846000075340271], [0.17004848983989665,
0.9818999767303467], [0.08229756782286436, 0.9830999970436096],
[0.09658989797149968, 0.9858999848365784], [0.07053784349224264,
0.9854000210762024]]
```

[34]:
```
import matplotlib.pyplot as plt

x = [i for i in range(100)]
plt.plot(x, histories[0].history['loss'], color='blue', label='Adagrad')
plt.plot(x, histories[1].history['loss'], color='red', label='RSMProp')
plt.plot(x, histories[2].history['loss'], color='orange', label='RMS+Nesterov')
plt.plot(x, histories[3].history['loss'], color='yellow', label='AdaDelta')
plt.plot(x, histories[4].history['loss'], color='green', label='Adam')
plt.legend()
plt.show()
```



0.2.5 A comparison of Test Loss, Accuracy and Training Time for the 5 different optimizers:

Time: AdaGrad trains the fastest , where as RMS+Nesterov trains the slowest (Computationally eexpensive- given a fixed set of epochs). This does not indicate time to convergence. Test Loss: AdaGrad has the least test loss (0.053), whereas RMSProp has the highest (0.17). Test Accuracy: AdaDelta has the highest test Accuracy (98.54), and RMSProp the least (98.18)

Optimizer	Test Loss	Test Accuracy	Training Time
AdaGrad	0.05396905895551717	0.9846000075340271	153.77004885673523
RMSProp	0.17004848983989665	0.9818999767303467	158.54089736938477
RMS+Nesterov	0.08229756782286436	0.9830999970436096	196.96231698989868
Nadam	0.09658989797149968	0.9858999848365784	176.2131621837616
AdaDelta	0.07053784349224264	0.9854000210762024	175.37222576141357

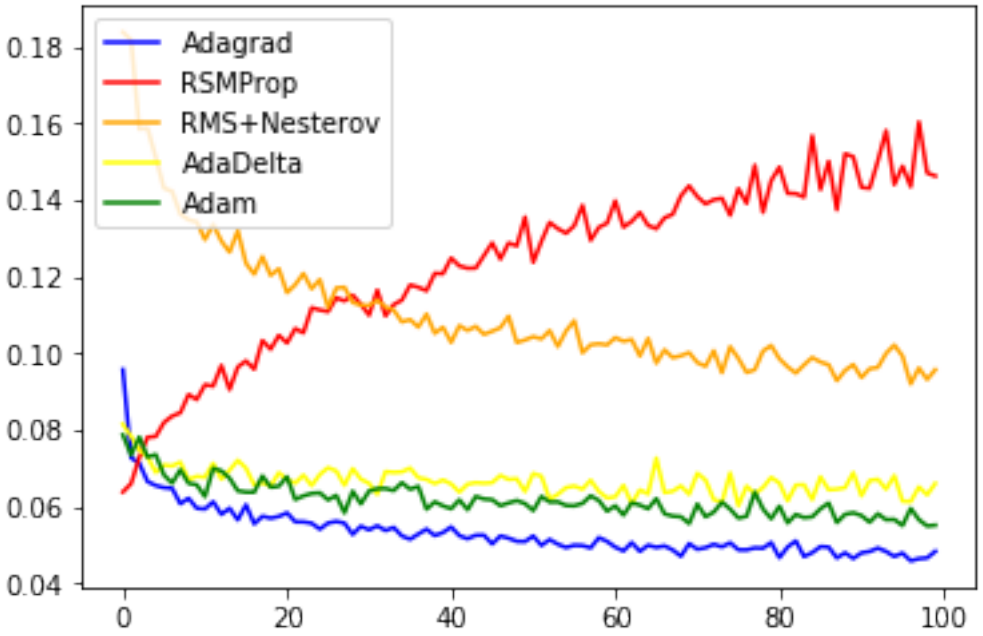
0.2.6 With dropout:

[25]:
```
model = Sequential()
model.add(Dropout(0.2,input_shape=(28,28,1)))
model.add(Flatten())
model.add(Dense(300,activation='relu'))
model.add(Dropout(0.5))
model.add(Dense(300,activation='relu'))
model.add(Dropout(0.5))
model.add(Dense(10,activation='softmax'))
```

[31]:
```
opts = ['Adagrad','RMSprop','Nadam','Adadelta','Adam']

histories_dropout = []
times_dropout = []
metrics_dropout = []

for opt in opts:
    model.compile(loss='categorical_crossentropy', optimizer=opt,␣
 ↪metrics=['accuracy'])
    print("Using Optimizer: ",opt)
    start = time.time()
    history = model.fit(x_train, y_train,
        batch_size=128,
        epochs=200,
        verbose=0,
        )
    t = time.time()-start

    histories_dropout.append(history)
    times_dropout.append(t)
    metrics_dropout.append(model.evaluate(x_test,y_test))
```

```
Using Optimizer:  Adagrad
10000/10000 [==============================] - 1s 70us/step
Using Optimizer:  RMSprop
10000/10000 [==============================] - 1s 71us/step
Using Optimizer:  Nadam
10000/10000 [==============================] - 1s 72us/step
Using Optimizer:  Adadelta
10000/10000 [==============================] - 1s 75us/step
Using Optimizer:  Adam
10000/10000 [==============================] - 1s 77us/step
```

[36]:
```
x = [i for i in range(100)]
plt.plot(x, histories[0].history['loss'], color='blue', label='Adagrad')
plt.plot(x, histories[1].history['loss'], color='red', label='RSMProp')
plt.plot(x, histories[2].history['loss'], color='orange', label='RMS+Nesterov')
plt.plot(x, histories[3].history['loss'], color='yellow', label='AdaDelta')
plt.plot(x, histories[4].history['loss'], color='green', label='Adam')
plt.legend()
plt.show()
```



[37]:
```
print(times_dropout)
print(histories_dropout)
print(metrics_dropout)
```

```
[155.4027111530304, 161.8654146194458, 198.07765197753906, 178.04522228240967,
177.54143357276917]
[<keras.callbacks.callbacks.History object at 0x7f01545e9e10>,
<keras.callbacks.callbacks.History object at 0x7f01542c1630>,
<keras.callbacks.callbacks.History object at 0x7f0154041fd0>,
<keras.callbacks.callbacks.History object at 0x7efd582a7e48>,
<keras.callbacks.callbacks.History object at 0x7efd37ecaac8>]
[[0.06117173416424048, 0.9864000082015991], [0.24486077860649916,
0.984000027179718], [0.09027380716234537, 0.9817000031471252],
[0.12017995868224567, 0.9853000044822693], [0.08639935254225972,
0.9833999872207642]]
```

	Optimizer	Test Loss	Test Accuracy	Training Time
AdaGrad	0.06117173416424048	0.9864000082015991	155.4027111530304
RMSProp	0.24486077860649916	0.984000027179718	161.8654146194458
RMS+Nesterov	0.09027380716234537	0.9817000031471252	198.07765197753906
Nadam	0.12017995868224567	0.9853000044822693	178.04522228240967
AdaDelta	0.08639935254225972	0.9833999872207642	177.54143357276917