Homework 4, due March 22.

This assignment is to explore different numerical algorithms of numerical differentiation and integration and estimate their rates of convergence. You need to write Matlab programs for the required calculations.

1. Compute the derivative of $f(x) = \sin x$ at $x = \frac{\pi}{3}$ using i) forward difference formula, ii) backward difference formula, iii) central difference formula. Use the step size $h = 2^{-n}, n = 0, 1, 2, ...15$. We know that $f'(\pi/3) = \frac{1}{2}$. Perform all computations and output the computed derivatives and errors in double precision. Estimate the rate of convergence for each formula by examining the errors.

2. Compute the derivative of $f(x) = \sin x$ at $x = \frac{\pi}{3}$ using the forward different formula and Richardson extrapolation algorithm. First use the step size $h = 2^{-n}, n = 0, 1, 2, ...15$ in forward difference formula to generate the values of the first column. Estimate the rate of convergence from the first column, then perform the first level of Richard extrapolation, and store the values in the second column. Estimate the rate of convergence from the second column, then perform the second level of Richard extrapolation, and store the results in the third column.

3. Compute $\int_0^{\pi/2} \sin(x)dx$ using composite i) rectangle, ii) mid-point, iii) trapezoid, and iv) Simpson's rules. Use the number of subintervals $m = 2^n, n = 0, ..., 10$. We know that $\int_0^{\pi/2} \sin(x)dx = -\cos(\pi/2) + \cos(0) = 1$. Perform all computations and output computed results and errors in double precision. Estimate the rate of convergence for each rule by examining the errors.

4. Compute $\int_0^{\pi/2} \sin(x)dx$ using Richard extrapolation using the composite rectangle rule. Perform similar calculations as question 2.