Decomposition - Class 2

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Recap

- We decompose to better understand aggregate measures changes/differences
- Decomposition methods improve on existing standardisation methods

Kitagawa decomposition

Decomposing a crude rate into

- Difference in rates
- Difference in population composition
  $\rightarrow$ can be along any categorical dimension
  $\rightarrow$ age, marital status, gender, education

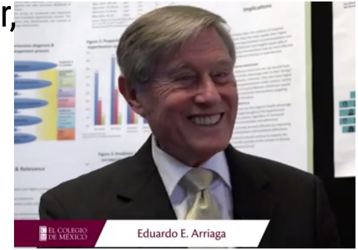
Quiz time!

What about other measures?

- Kitagawa: analytical decomposition of a crude rate
- One can do that with more aggregate measures
- For example, life expectancy

Eduardo E. Arriaga

- Spanish-born, Argentinian and US demographer, worked in US and Argentinian universities
- Mostly worked on mortality, also fertility, population change and urbanisation
- Focus on Latin America
- *Measuring and explaining the change in life expectancies*, 1984
  $\rightarrow$ similar approaches developed by other researchers at the same time



Arriaga decomposition

- Decompose difference in life expectancy by age
- For every age we have

  $\rightarrow$ direct effect: years gained in specific age group

  $\rightarrow$ indirect effect: years gained in following age groups

Arriaga decomposition

Total change is the sum of age-specific changes:

$$\Delta LE = \sum_x ({}_n\Delta_x)$$

Arriaga decomposition

$$
{}_n\Delta_x = \underbrace{\frac{l_x^1}{l_0^1}\left(\frac{{}_nL_x^2}{l_x^2} - \frac{{}_nL_x^1}{l_x^1}\right)}_{\text{Direct effect}} + \underbrace{\frac{T_{x+n}^2}{l_0^1}\left(\frac{l_x^1}{l_x^2} - \frac{l_{x+n}^1}{l_{x+n}^2}\right)}_{\text{Indirect and interaction effects}}
$$

Change in mortality rates between ages x and $x+n$ affects

- Number of years lived between ages x and $x+n$ $\rightarrow$ direct effect
- Number of years lived after age $x+n$ by new survivors $\rightarrow$ indirect and interaction effect

Arriaga decomposition

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Change in mortality rates between ages x and $x+n$ affects

- Number of years lived between ages x and $x+n$ $\rightarrow$ direct effect
- Number of years lived after age $x+n$ by new survivors $\rightarrow$ indirect and interaction effect

For open-ended age-group there is only a direct effect:

$$
{}_\infty\Delta_x = \frac{l_x^1}{l_0^1}\left(\frac{T_x^2}{l_x^2} - \frac{T_x^1}{l_x^1}\right)
$$

An example

PNAS RESEARCH ARTICLE | DEMOGRAPHY

OPEN ACCESS

Significant impacts of the COVID-19 pandemic on race/ethnic differences in US mortality

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Edited by Douglas Massey, Princeton University, Princeton, NJ; received April 2, 2022; accepted June 20, 2022

An example

Let’s try ourselves

Arriaga decomposition by causes of death

We can extend this method to account for contributions from different causes of death *i*:

$$
\begin{aligned}
{}_n\Delta_x^i &= {}_n\Delta_x \frac{{}_n m_{x,i}^2 - {}_n m_{x,i}^1}{{}_n m_x^2 - {}_n m_x^1} \\
&= {}_n\Delta_x \frac{{}_n R_{x,i}^2({}_n m_{,i}^2) - {}_n R_{x,i}^1({}_n m_{x,i}^1)}{{}_n m_x^2 - {}_n m_x^1}
\end{aligned}
$$

Where

- ${}_n R_{x,i}^t$ is the proportion of deaths from cause i in ages x to $x+n$ and population t
- ${}_n\Delta_x$ is the contribution of all-cause mortality differences in the same age group
- ${}_n m_x^t$ is the mortality rate in the same age group of population t

Arriaga decomposition by causes of death

All-cause contribution, weighted by the change in cause-specific mortality as a proportion of change in all-cause mortality

An example

Back to R!

Group work

Could the Arriaga decomposition be useful for your research? What if you could modify it?

References

E.E. Arriaga (1984). "Measuring and explaining the change in life expectancies", *Demography*, 21(1): 83–96.