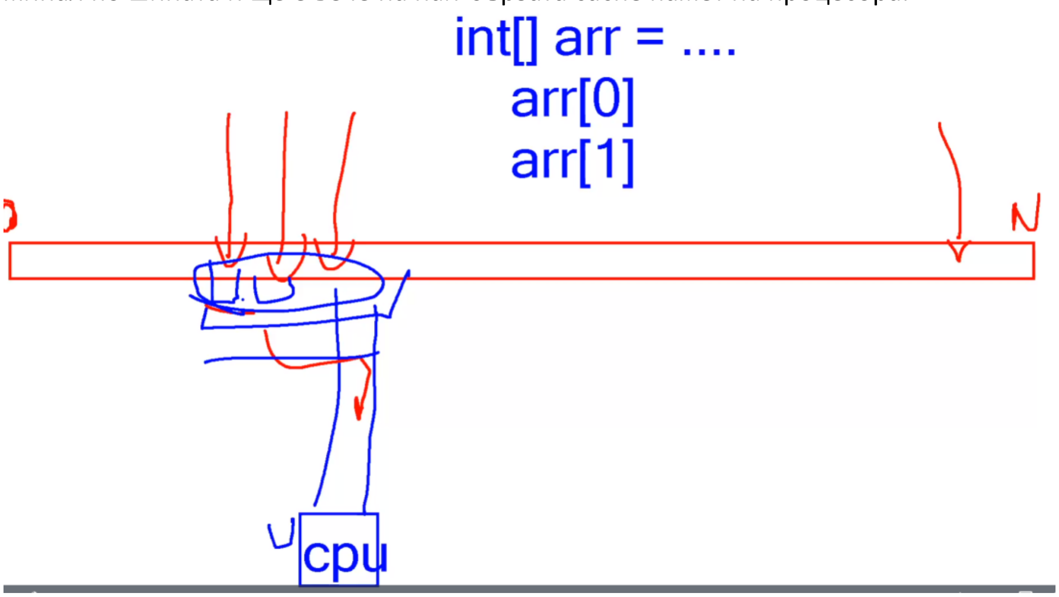
1. Data Structures and Complexity

1.1. Memory Storage

The term "memory", meaning "primary storage" or "**main memory**", is often associated with adressable **semiconductor(транзистор) memory**.

Много често като искаме единица данни от Рам паметта към процесора, шината заделя исканата единица данни плюс още много поредни следващи единици данни (до размера на шината). Много е вероятно, ако искаме да достъпим първия елемент на масив, то да искаме да достъпим и втория елемент. А втория елемент също ще е минал по шината и ще е вече на най-бързата cache памет на процесора.



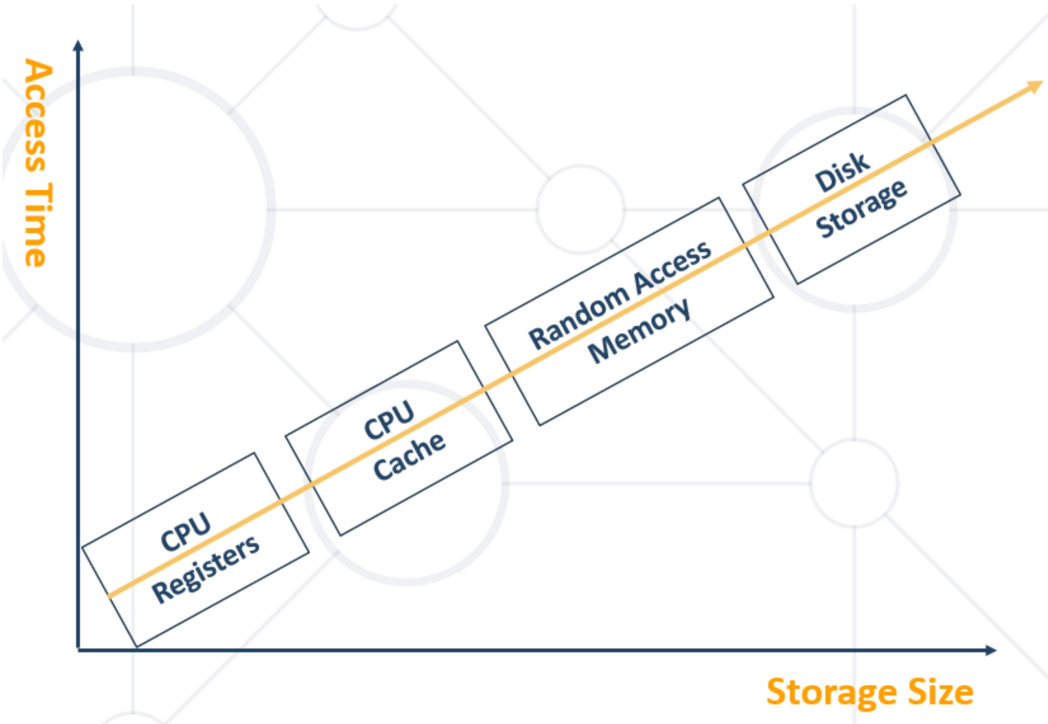
In computer science, memory usually is:

- a continuous, numbered – aka addressed – sequence of bytes
- storage for variables and functions created in programs
- random-access – equally fast accessing **5**th and **500**th byte
- addresses numbered in hexadecimal, prefixed with **0x**.

boolen -> 1 byte въпреки че може да е 1 бит (true/false или 1/0) – **защото най-малката адресируема част на паметта е 1 byte**
byte -> 1 byte
short -> 2 bytes = 16 bits
int -> 4 byte = 32 bits
long ->

Memory Hierarchy

Each memory level is **faster** and **smaller** than the **next memory level**. At the end we can say we have **nearly infinite memory** storage that **is also infinitely slow**.



1.2. Data Structures – Overview

What is Data?

- **"Data"** from Latin – datum, which originally meant "**something given**." Dates back to the 1600s.
- Data is **raw, unorganized** facts that need to be processed. Data can be something simple and seemingly **random** and **useless** until it is **organized.**
- Example:

The history of temperature readings all over the world for the past 100 years is data.

What is information?

- **"Information"** has Old French and Middle English origins. It has always referred to "**the act of informing**, " usually in regard to education, instruction, or other knowledge communication.
- When data is **processed, organized, structured or presented** in a **given context** so as to **make it useful**, it is called **information.**
- Example:

The history of temperature readings all over the world for the past 100, when organized and analyzed we find that global temperature is rising. – That is information.

Data in Computing

- Set of **symbols** gathered and translated for **some purpose.**
- Simplified – bits of information stored in memory. If those bits remain **unused,** they don't do anything.
- Example:

Binary Data	Translation
100 0001	65
100 0001	A

- It is easy to notice, that the way we **read** the data **retrieves the information** of the bits in different ways. However those bits have only **0** or **1** as values.
- Example:

Type	Binary Data	Translation
Integer	0000 0100 0001	65
Character	0000 0100 0001	'A'
Double	0000 0100 0001	65.0
Instruction Code	0000 0100 0001	Store 65
Color	0000 0100 0001	

Data Structures

- Data structure – an **object** which takes responsibility for data **organization**, **storage**, **management** in **effective** manner.
- Storing items **requires memory consumption**:

Data Structure	Size
int	= 4 bytes
float	= 4 bytes
long	= 8 bytes
int[]	≈ (Array length) * 4 bytes
List<Double>	≈ (List size) * 8 bytes
Map<Integer, int[]>	≈ (Map size) * Entry bytes

Abstract Data Structures (ADS)

- An **Abstract Data Structure (ADS = ADT type)** – the way the real objects will be modulated as **mathematical** objects, alongside the **set of operations** to be executed upon them, **without** the implementation itself.

```java
public interface List<E> {
    boolean add(E e);
    int size();
    boolean remove(Object o);
    boolean isEmplty();
}
```

Data Structures Implementation

- An **implementation** – definitive way of ADS to be presented inside the computer memory, alongside the implementation of the operations

```java
public class ArrayList<E> implements List<E> {
    public boolean add(E e) {
        this.elements[this.index++] = e;
        this.size++;
        return true;
    }
}
```

1.3. Algorithmic Complexity

Algorithm Analysis

- Why should we analyze algorithms?
  - Predict the **resources** the algorithm will need
    - Computational time (**CPU** consumption)
    - Memory space (**RAM** consumption)
    - Communication **bandwidth** consumption
    - **Hard disk** operations

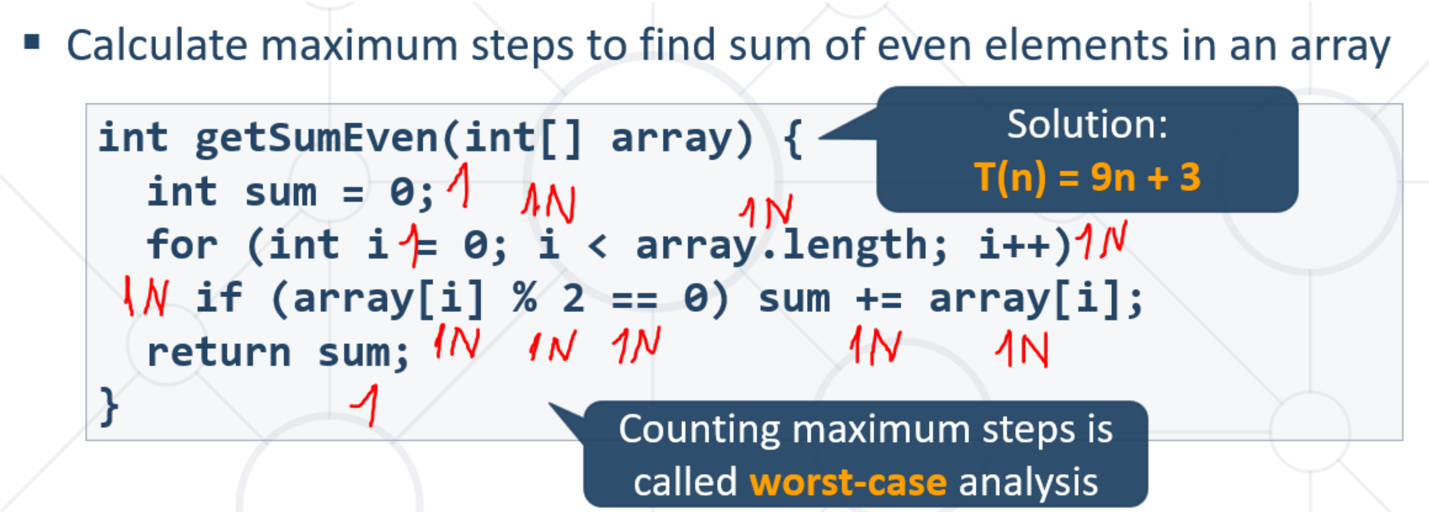
- There are three main properties we want to analyze:
  - **Simplicity** – this is really a matter of intuition and of course it is subjective quality
  - **Accuracy** – this seems easy to determine, however it may be very difficult to determine is the algorithm correct?
  - **Performance** – the consumption of CPU, Memory and other resources (we really care the most for the first two)

- The expected **running time** of an algorithm is:
  - The total number of **primitive operations** executed (machine independent steps)
  - Also known as **algorithm complexity**
  - Compare algorithms **ignoring details** such as **language** or **hardware**

Consumption of CPU

Step Count

Assume that a **single step** is a single CPU instruction.
Гледаме колко стъпки има алгортъма ни, а не колко памет е заета (което е различно)



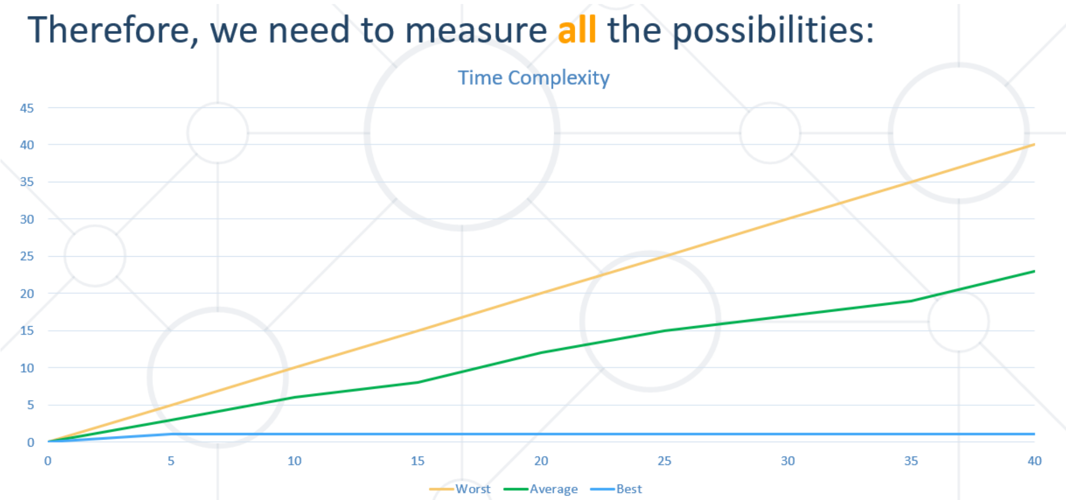
Инструкция на процесора можем да кажем, че е код завършващ с точка и запетая накрая.
Реално повече инструкции на процесора има на 1 ред код -аритметични/логически и т.н.

Simplifying Step Count

- Some parts of the equation **grow much faster** than others
  - $T(n) = \mathbf{3(n^2)} + 3n + 3$
  - We can **ignore** some part of this equation
  - Higher terms **dominate** lower terms – $\mathbf{n > 2}$, $\mathbf{n^2 > n}$, $\mathbf{n^3 > n^2}$
  - Multiplicative constants can be **omitted** – $\mathbf{12n \rightarrow n}$, $\mathbf{2n^2 \rightarrow n^2}$
  - The solution for $T(n) = \mathbf{3(n^2)} + 3n + 3$ becomes $\approx \mathbf{n^2}$

Time Complexity

- Worst-case
  - An **upper** bound on the running time
- Average-case
  - **Average** running time
- Best-case
  - The **lower** bound on the running time (the optimal case)



- From the previous chart we can deduce:
  - For smaller size of the input (**n**) we **don't care much for the runtime**. So we measure the time as **n** approaches **infinity**
  - If an algorithm **has to scale**, it **should compute** the result within a **finite and practical time**
  - We're concerned about the **order of an algorithm's complexity**, not the actual time in terms of **milliseconds**

Asymptotic notations:

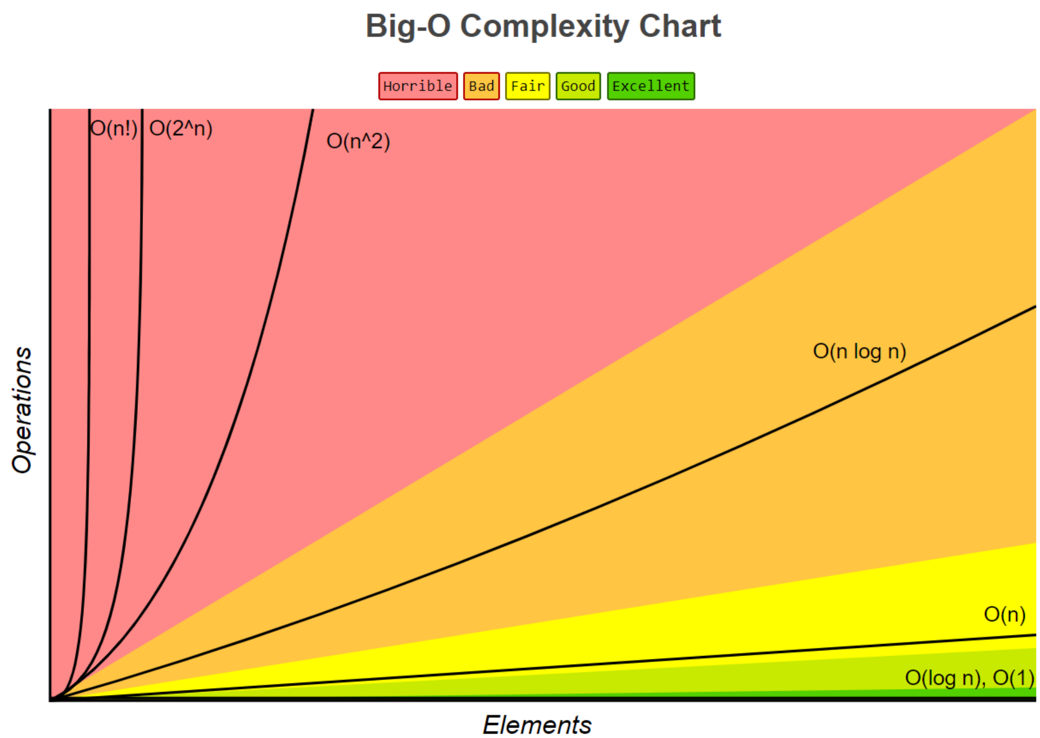
- **Asymptotic notations** are descriptions that allow us to examine an algorithm's running time by expressing its **performance** as the input size, **n**, of an algorithm or a function **f increases**. There are **three** common asymptotic notations:
  - Big **O – O(f(n)) – worst case - in our course**
  - Big **Theta – Θ(f(n)) – амортизиран amortized constant time**
  - Big **Omega – Ω(f(n))**

Algorithmic complexity – rough estimation of the number of steps performed by given computation, depending on the size of the input

- Measured with asymptotic notation
  - **O(f(n))** – **upper bound (worst case)**
  - **Θ(f(n))** – average case
  - **Ω(f(n))** – lower bound (best case)
    - Where **f(n)** is a function of the size of the input data

In this course we will analyze only the Big O – **O(f(n)) upper bound (worst case).**

Ако два алгоритъма имат Big O което е еднакво, то чак тогава може да се наложи да гледаме Theta или Omega.



- O(1) – Constant time – time does not depend on **N**
- O(log(N)) – Logarithmic time – grows with rate as **log(N)** *log2 64 = 6 (2^6 = 64)*
- O(N) – Linear time grows at the same rate as **N**
- O(N^2),O(N^3) – Quadratic, Cubic grows as square or cube of **N**
- O(2^N) – Exponential grows as **N** becomes the exponent worst algorithmic complexity

Typical Complexities

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Complexity	Notation	Description
constant	O(1)	n = 1 000 → 1-2 operations
logarithmic	O(log n)	n = 1 000 → 10 operations
linear	O(n)	n = 1 000 → 1 000 operations
linearithmic	O(n*log n)	n = 1 000 → 10 000 operations
quadratic	O(n2)	n = 1 000 → 1 000 000 operations
cubic	O(n3)	n = 1 000 → 1 000 000 000 operations
exponential	O(n^n)	n = 10 → 10 000 000 000 operations

Time Complexity and Program Speed

Complexity	10	20	50	100	1 000	10 000	100 000
O(1)	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s
O(log n)	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s
O(n)	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s
O(n*log n)	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s
O(n^2)	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	2 s	3-4 min
O(n^3)	< 1 s	< 1 s	< 1 s	< 1 s	20 s	5 hours	231 days
O(2^n)	< 1 s	< 1 s	260 days	hangs	hangs	hangs	hangs
O(n!)	< 1 s	hangs	hangs	hangs	hangs	hangs	hangs
O(n^n)	3-4 min	hangs	hangs	hangs	hangs	hangs	hangs

Performance of RAM

- **Memory consumption** should also be considered, for example:
  - Storing elements in a matrix of size N by N
    - Filling the matrix – Running time **$O(n^2)$**
    - Get element by index – Running time **O(1)**
    - Memory requirement **$O(n^2)$**
- However in this course we **won't be optimizing** memory consumption we will only point it out

1.4. Array Data Structure

- Ordered
- Very **lightweight**
- Has a **fixed size**
- Usually **built into the language**
- Many collections are implemented by using arrays, e.g.
  - **ArrayList<E>** in Java
  - **ArrayDeque<E>** in Java

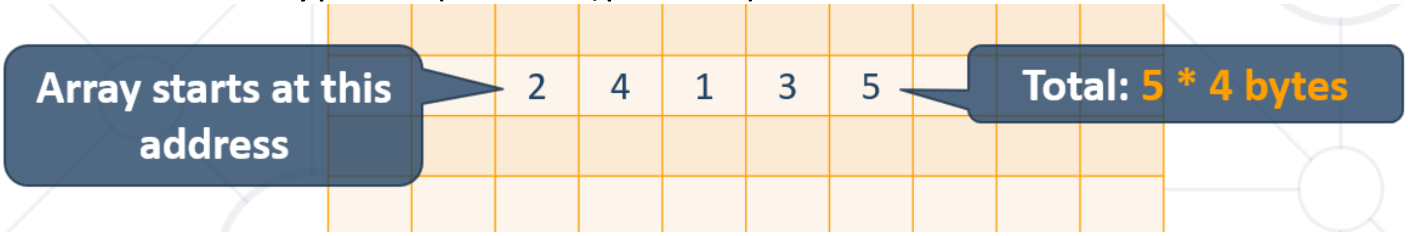
Why Arrays Are Fast?

- Arrays use a **single block of memory = size of the array * size of the data type**

```
int[] array = { 2, 4, 1, 3, 5 };
```

int size is 4 bytes

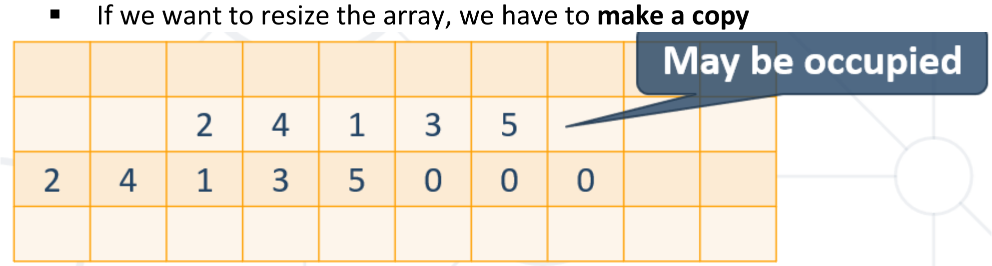
- Uses total of **array pointer + (N * element/pointer size)**



- **Array Address + (Element Index * Size) = Element Address**
- Array Element Lookup – **O(1)**

Arrays – Changing Array Size

- Arrays have a **fixed size**
- Memory after the array **may be occupied**
- If we want to resize the array, we have to **make a copy**



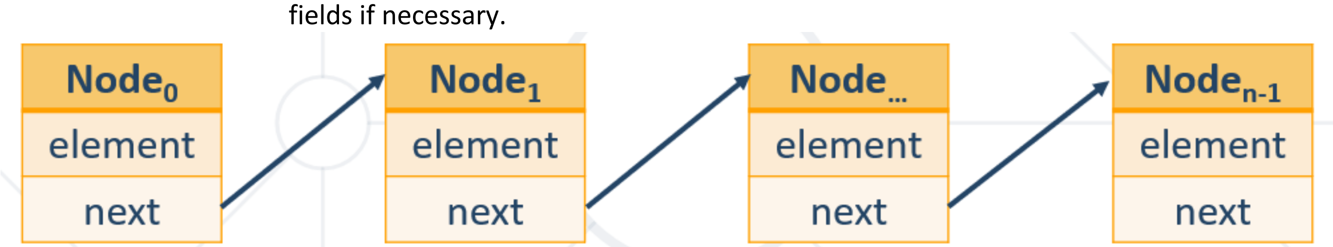
- Array Copy – **O(n)**
- Array Search element – **O(n)**

1.5. Data Structure Implementation - Elements Representation Approaches

How Do We Store the Elements?

- **Choose** the way to **store** the elements:
  - By **using an array**: - **статична реализация**
    - Stores the elements as a **sequence** inside the computer memory

  - By **using a Node<E>** class: - **динамична реализация**
    - Contains the **element** inside the Node. **Must** have **pointer to the next Node.** Can have **more** fields if necessary.




```java
public class ArrayStorage {
    private final int INITIAL_CAPACITY = 4;

    private int[] elements;
    private int index;
```

```java
    public ArrayStorage() {
        this.elements = new int[INITIAL_CAPACITY];
        this.index = 0;
    }

    public boolean add(int element) {
        add(element, ++index);
        return true;
    }

    private void add(int element, int index) {
        if (index == this.elements.length) {
            // TODO: Add grow method call here
        private void grow() { - What is the complexity? – O(n)
        // Create new array with larger size
        // Copy the elements from the old to the new array
        // Do additional operations if needed
   }

        }
        this.elements[index] = element;
    }

    // TODO: Implement additional operations like: remove(int element), contains(int element) and
more
}


public class NodeStorage {
    private Node node;

    class Node {
        private int element;
        private Node next;

        Node(int element) {
            this.element = element;
        }
    }

    public NodeStorage() {
        this.node = new Node(0);
    }

    public boolean add(int element) {
        this.node.next = new Node(element);
        return true;
    }

    // TODO: How do we iterate over the items? How do we remove? How do we iterate and access
data?
}
```

2. Linear Data Structures

Static and Dynamic Implementation

2.1. Dynamic Arrays – static implementation

- ArrayList is the **implementation** of ADS (Abstract Data structure) **List**
  - Built **atop an array**, which is able to dynamically **grow** and **shrink** as you **add/remove** elements
- Stores the **elements** inside an array

```
public class ArrayList<E> implements List<E> {
private Object[] elements;
}
```

Supported operations and complexity:

- **size()**, **isEmpty()**, **get()**, **set()** – **O(1)**
- **add()** – the operation runs in **amortized constant** time – повечето пъти времето е константно(рядко се преузмерява)
- adding **n** elements requires **O(n)** time
- all of the other operations like: **add(int index, E element)**, **contains()**, **indexOf(), remove(int index) etc.**, run in **linear time O(n)** (roughly speaking)

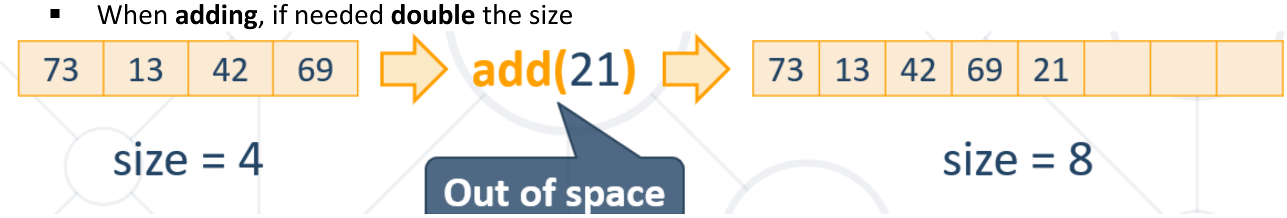
ArrayList – Add O(n)

- Implemented **using an array**
- Adding **new item** requires **new array**

- This approach will copy **all the elements** for each add operation – **O(n)**

ArrayList – Add O(1)

- Implemented **using an array**
- When **adding**, if needed **double** the size



- This approach will copy at **log(n)** → n = 10^9, only ~33 copies – **O(1) amortized**

Constructor and fields:

```
public class ArrayList<E> implements List<E> {
    private static final int DEFAULT_CAPACITY = 4;
    private Object[] elements;
    private int size;

    public ArrayList() {
    this.elements = new Object[DEFAULT_CAPACITY];
        }
}
```

Add - Adds an element after the last element:

```java
public boolean add(E element) {
    if(this.size == this.elements.length) {
        this.elements = grow();
    }

    this.elements[this.size++] = element;

    return true;
}
```

Get - Returns an element at index:

```java
public E get(int index) {
    checkIndex(index);
    return this.getElement(index);
}

private E getElement(int index) {
    return (E) this.elements[index];
}

private void checkIndex(int index) {
    if (index < 0 || index >= size) {
        throw new IllegalArgumentException();
    }
}
```

Set - Sets an element at index:

```java
public E set(int index, E element) {
    checkIndex(index);
    E oldElement = this.getElement(index);
    this.elements[index] = element;
    return oldElement;
}
```

Remove - Removes and returns an element at index:

```java
public E remove(int index) {
    this.checkIndex(index);
    E element = this.getElement(index);
    this.elements[index] = null;
    this.size--;
    shift(index);
    ensureCapacity();

    return element;
}
```

Grow and Shrink

```java
private Object[] grow() {
    return Arrays.copyOf(this.elements, this.elements.length * 2);
}
```

```
//increase the size of the list
if (size >= elements.length) {
    Object[] newElements = new Object[size * 2];
    for (int i = 0; i <elements.length; i++) {
        newElements[i] = elements[i];
    }

    elements = newElements;
}


private Object[] shrink() {
    return Arrays.copyOf(this.elements, this.elements.length / 2);
}
```

2.2. Nodes - Building Block

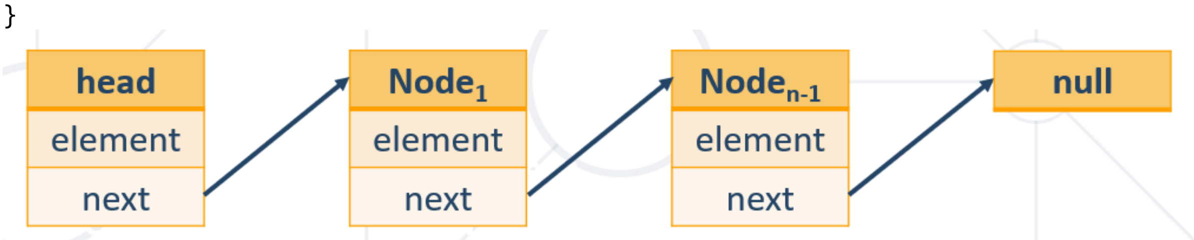
Node Class

- The **Node** class is the **build block** for many data structures
- Inside Node object we store **an element and pointer to the next node at least**
- However, we **can store anything else**

```
private static class Node<E> {
    private E element;          // Must have
    private Node<E> next;       // Must have
    private Node<E> previous;   // Additional
}
```

- Many data structures use **node chaining**

```
public class LinkedList<E> implements Deque<E> {
    private Node<E> head;
}
```



SinglyLinkedList – dynamic implementation

- Linear data structure where each **element** is a **separate object – Node**
- The elements are **not** stored at **contiguous** memory
- The entry point is commonly the **head** of the list

However we define what is the entry point

```
public class DoublyLinkedList<E> implements LinkedList<E> {
    private Node<E> head;
    private int size;
```

- Supported operations and complexity:
    - **addFirst**(), **removeFirst**(), **getFirst**(), **size**() – **O(1)**
    - How about operations on the **last element**?
        - **addLast**(), **removeLast**(), **getLast**() – again depends if we keep the reference to the last node or no can be constant – **O(1)** or linear – **O(n)**
    - operations that **index** into the list will run in **linear time O(n)** (roughly speaking)

DoublyLinkedList – dynamic implementation

However we define what is the entry point

```java
public class DoublyLinkedList<E> implements LinkedList<E> {
    private Node<E> head;
    private Node<E> tail;
    private int size;


public Node<E> getHead() {
    return head;
}

public static class Node<E> {
    private E element;          // Must have
    private Node<E> next;       // Must have
    private Node<E> previous;   // Additional

    public Node(E element, Node<E> next, Node<E> previous) {
        this.element = element;
        this.next = next;
        this.previous = previous;
    }

    public E getElement() {
        return element;
    }

    public Node<E> getNext() {
        return next;
    }

    public Node<E> getPrevious() {
        return previous;
    }

    public Node setNext(Node<E> next) {
        this.next = next;
        return this;
    }

    public Node setPrevious(Node<E> previous) {
        this.previous = previous;
        return this;
    }
}
```

хитро и работи 😊

```java
@Override
public void addLast(E element) {
    Node<E> newNnode = new Node<>(element, null, tail);
    if (head == null) {
        head = newNnode;
    }

    if (tail != null) {
        tail.setNext(newNnode);
```

```java
    }

    tail = newNnode;

    size++;
}
```

хитро и работи 😊

```java
public void remove(Node<E> node) {
    if (head == node) {
        head = node.getNext();
    }

    if (tail == node) {
        tail = node.getPrevious();
    }

    if (node.getPrevious() != null) {
        node.getPrevious().setNext(node.getNext());
    }

    if (node.getNext() != null) {
        node.getNext().setPrevious(node.getPrevious());
    }

    size--;
}
```

Built-in Class LinkedList in JAVA – doubly-linked queue with static implementation for quick performance

```java
public class LinkedList<E>
    extends AbstractSequentialList<E>
    implements List<E>, Deque<E>, Cloneable, java.io.Serializable

public interface List<E> extends Collection<E> {

public interface Collection<E> extends Iterable<E> {


LinkedList<String> builtInLinkedList = new LinkedList<>();

DoublyLinkedList<String> people = new DoublyLinkedList<>();
builtInLinkedList.addLast("joro");
builtInLinkedList.addLast("pesho");
builtInLinkedList.addLast("misho");
builtInLinkedList.addLast("grisho");
```

Обхождане с iter (foreach)

```java
for (String lrr : builtInLinkedList) {
    System.out.println(lrr);
}
```

Обхождане с iterator

```java
Iterator<String> iterator = builtInLinkedList.iterator();
while (iterator.hasNext()){
    String person = iterator.next(); //връща стринг
```

```java
    System.out.println(person);
}
```

Премахване на елемент – работи и с ArrayList<String> тъй като iterator работи с всяка колекция, която имплементира Iterable интерфейса

```java
while (iterator.hasNext()) {
    String person = iterator.next(); //връща стринг
    if (person.equals("pesho")) {
        iterator.remove();
    } else {
        System.out.println(person);
    }
}
```

2.3. Сравнение на статична и динамична имплементация

При статичната имплементация, при вмъкване или изтриване на елемент, пренареждаме всички останали, но пък имаме директен достъп до елементите.

При динамичната имплементация, при вмъкване или изтриване на елемент става веднага, но пък имаме обхождане през next / previous докато стигнем до даден елемент.

What is the time complexity to search (not to select a concrete) an element in an array (statis implemenation)? - **O(n)**

	add last	insert	remove	get(index)
Dynamic Array static implementation	O(1) *	O(n)	O(n)	O(1)
Linked List dynamic implementation	O(1)	O(1)	O(1)	O(n)

Time complexity of a hashset when contains operation - O(1) which is very clever implementation of a hashset.
https://stackoverflow.com/questions/6574916/hashset-look-up-complexity

2.4. Stacks - dynamic implementation

- Stack is the **implementation** of ADS **LIFO** **L**ast **I**n **F**irst **O**ut
  - Build by using **Node** class or atop an **array**
- Stack example using Node

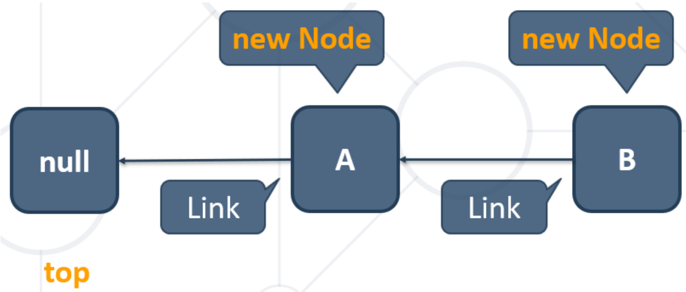
```java
public class Stack<E> implements AbstractStack<E> {
    private Node<E> top;
    private int size;
}
```

- Supported operations and complexity:
  - **size**(), **isEmplty**(), **push**(), **pop**(), **peek**() – **O(1)**
  - all of the other operations run in linear time (roughly speaking):
    - **forEach**()
    - **contains**()

etc…

Stack – Push

- Chain the nodes by using the **top** field:

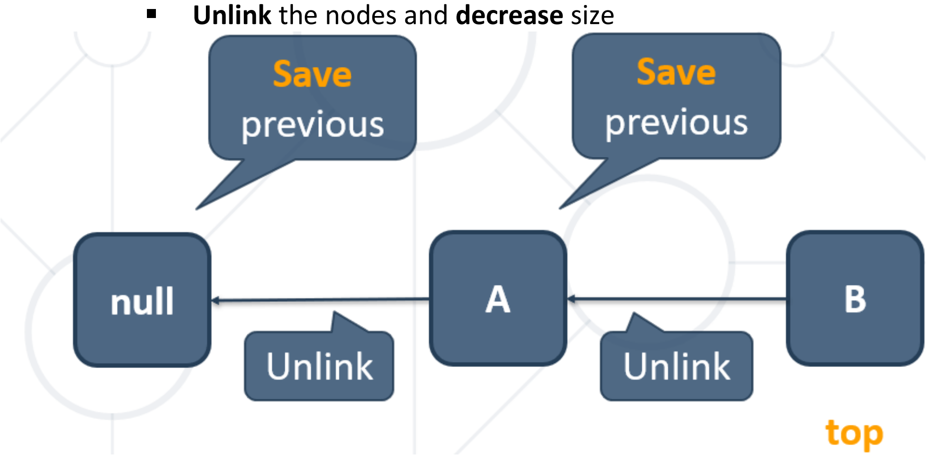


- Add element at the top
  - **Link** the nodes and **increment** size

```
public void push (E element){
    Node<E> newNode = new Node<>(element);
    newNode.previous = top;
    top = newNode;
    this.size++;
}
```

Stack – Pop

- Remove the **top** Node and return the element
  - **Unlink** the nodes and **decrease** size



- Remove and return element at the top:

```
public E pop () {
    ensureNonEmpty();
    E element = this.top.element;
    Node<E> temp = this.top.previous;
    this.top.previous = null;
    this.top = temp;
    this.size--;
    return element;
}
```

2.5. Queues – dynamic implementation

- Queue is the **implementation** of ADS **FIFO**   **F**irst **I**n **F**irst **O**ut
  - Build by using **Node** class or atop an **array**
- Queue example using Node

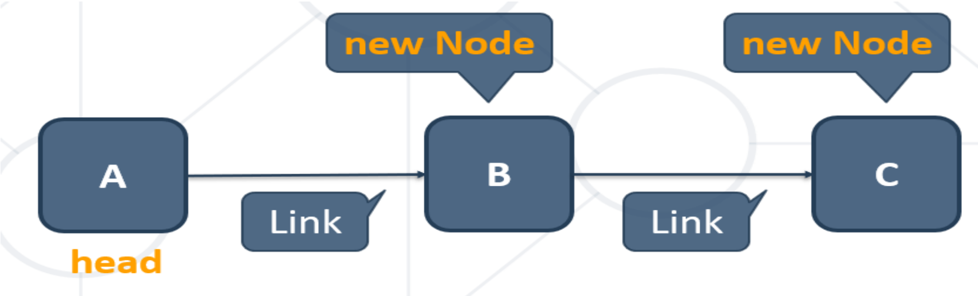
```
public class Queue<E> implements AbstractQueue<E> {
    private Node<E> head;
```

```
    private int size;
}
```

- Supported operations and complexity:
  - **size**(), **isEmplty**(), **poll**(), **peek**() – **O(1)**
  - **offer**():
    - if we keep the reference to the that node – **O(1)**
    - If we have to chase pointers to that node – **O(n)**
  - all of the other operations run in linear time (roughly speaking): **forEach**(), **contains**(), etc…

Queue – Offer

- Head **==** null → head **= new Node**
- Size **>** 0 → chain the nodes by adding **new Node** after the last one the so called **tail**:



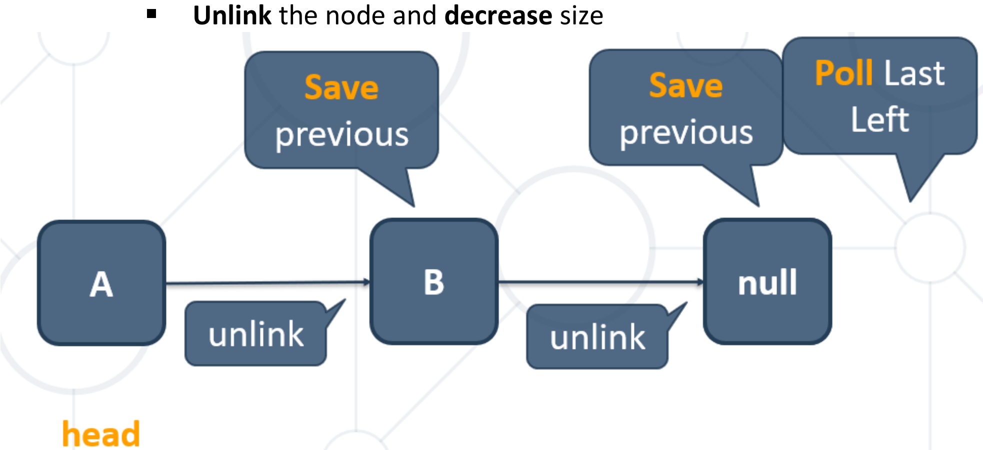
Queue – Offer

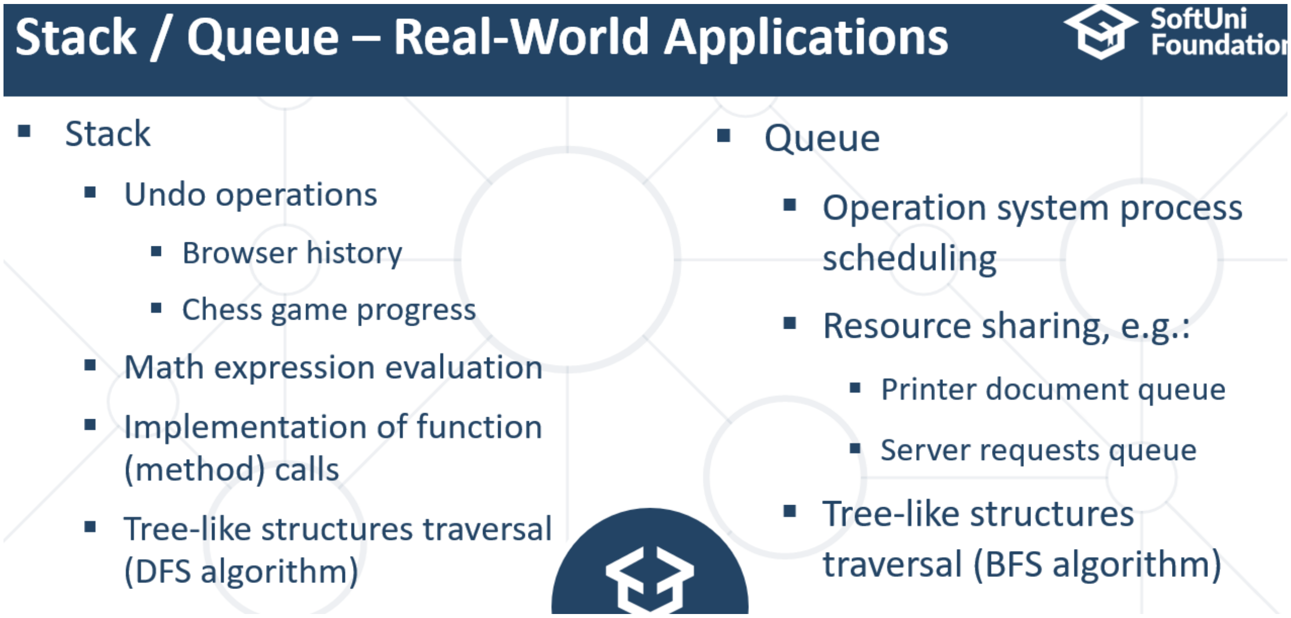
- Add element at the end – **Link** the nodes and **increase** size

```
public void offer (E element){
    Node<E> newNode = new Node<>(element);
    if (this.head == null) {
        this.head = newNode;
    } else {
        Node<E> current = this.head;
        while (current.next != null) {
            current = current.next;
        }
        current.next = newNode;
    }
    this.size++;
}
```

Queue – Poll

- Remove the **head** Node and return the element
  - **Unlink** the node and **decrease** size





3. Trees Representation and Traversal (BFS, DFS)

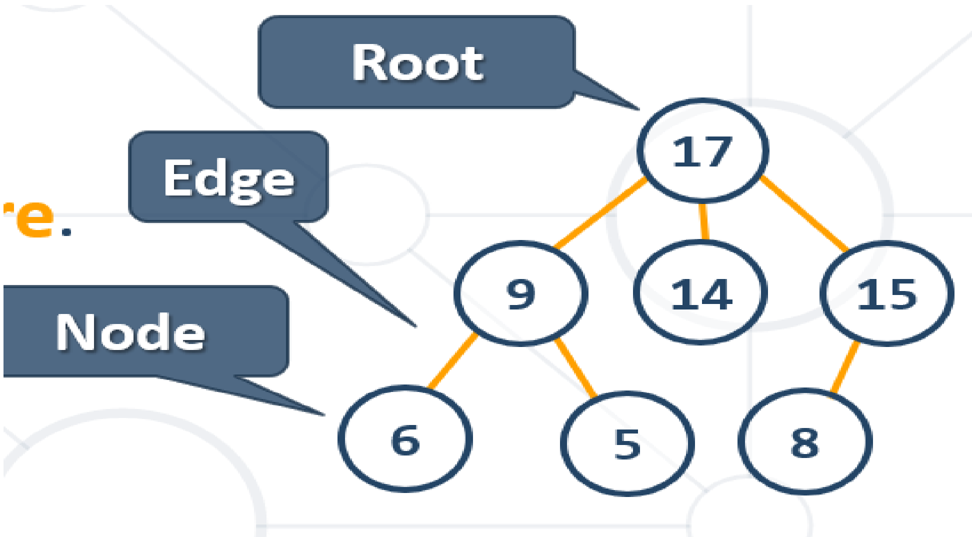
3.1. Trees and Related Terminology

Tree Definition

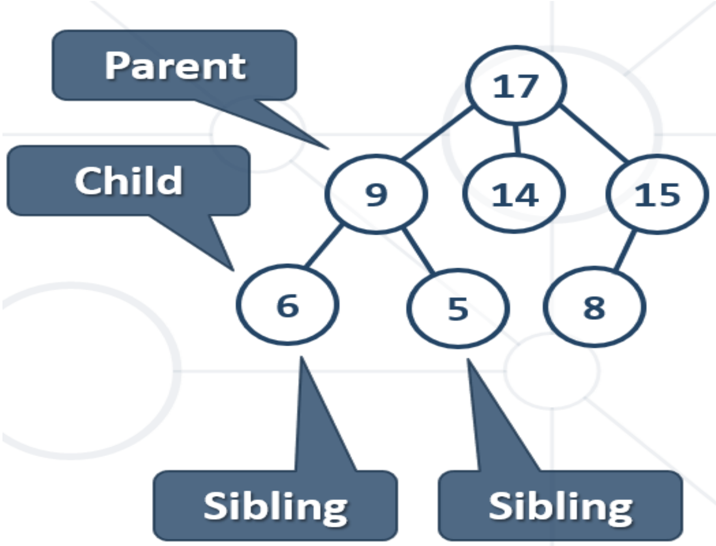
- Tree is a widely used **abstract data type** (ADT) that simulates a hierarchical **tree structure**, with a root value and subtrees of children with a **parent node**, represented as a set of linked **nodes**.
- **Recursive definition** – a tree consists of a value and a forest (the subtrees of its children)
- One **reference** can point to **any given node** (a node has at **most** a **single** parent), and **no node** in the **tree point to the root**. Every node (other than the root) **must** have exactly **one parent**, and the **root must** have **no parents**.

Tree Data Structure – Terminology

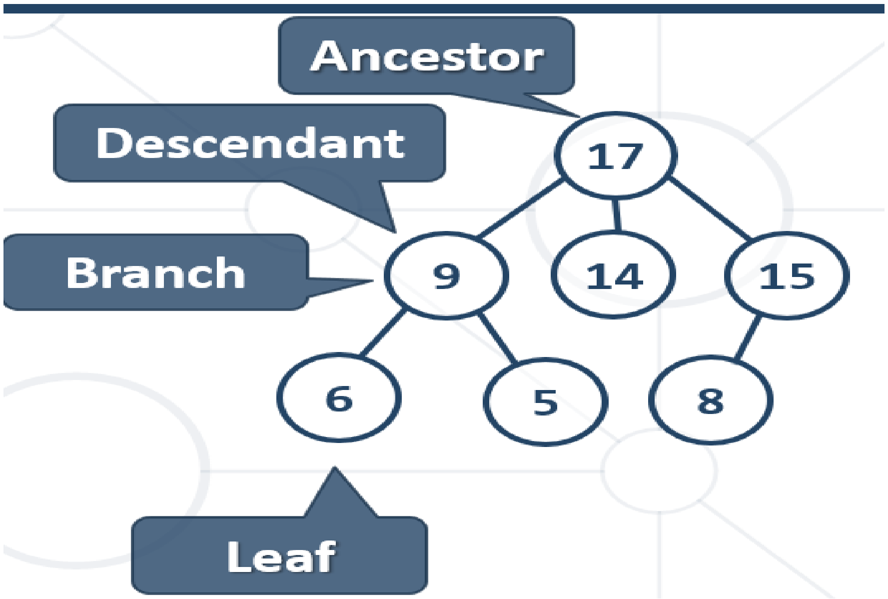
- **Node (Връх)** – a structure which may contain a **value** or condition, or represent a separate **data structure**.
- **Edge (Ребра)** – the **connection between** one **node** and **another**.
- **Root (Корен)** – the **top** node in a **tree**, the **prime ancestor**.



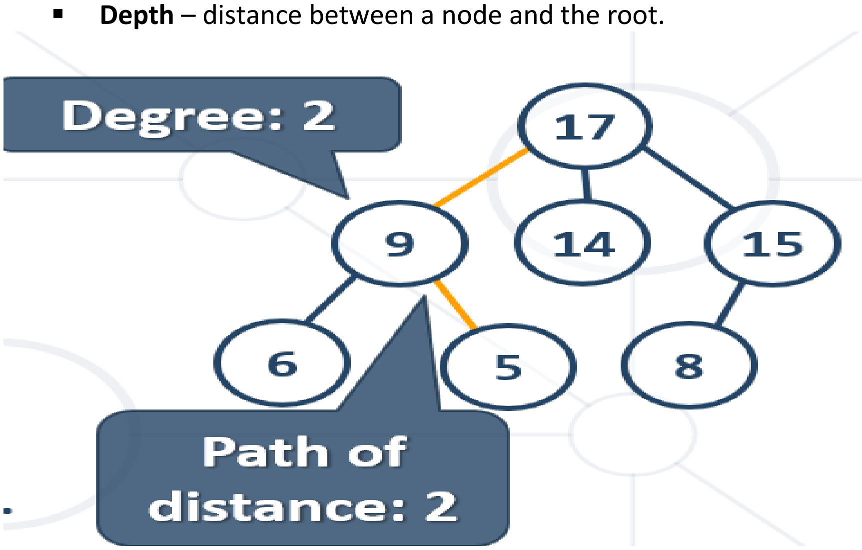
- **Parent (Родител)** – the **converse** notion of a **child**, an **immediate ancestor (прародител)**.
- **Child (Дете)** – node **directly** connected to **another** node when moving **away** from the **root**, an immediate descendant.
- **Siblings (Близнаци, братя, сестри, които имат общ родител)** – a **group** of **nodes** with the **same parent**.



- **Ancestor(Предшественик)** – node reachable by repeated proceeding **from child to parent**.
- **Descendant (Потомък)** – node reachable by repeated proceeding **from parent to child**.
- **Leaf (Листо)** – node with **no children**.
- **Branch (Клон)** – node with **at least one child**.

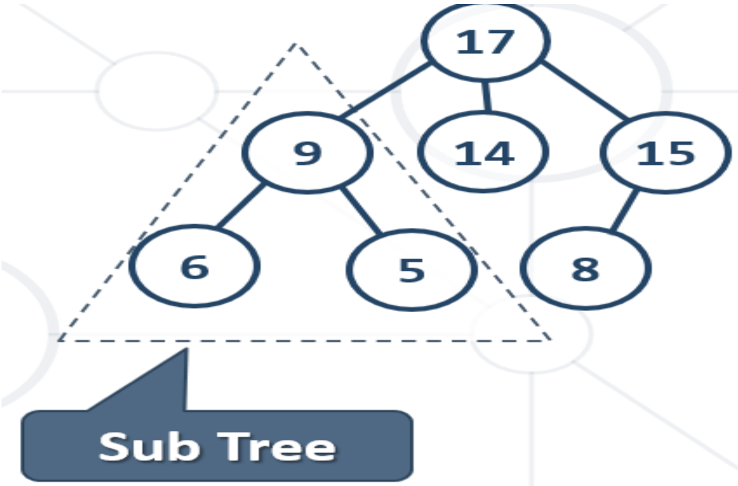


- **Degree (Степен)** – number of children for node zero for a leaf.
- **Path** – sequence of nodes and edges connecting a node with a descendant.
- **Distance** – number of edges along the shortest path between two nodes.
- **Depth** – distance between a node and the root.



- **Level** – depth + 1.
- **Height** – The number of edges on the longest path between a node and a descendant leaf. = **Height** – the maximum level in the tree.
- **Width** – number of nodes in a level.
- **Breadth** – number of leaves.

- **Forest** – set of disjoint trees.
  - {17}, {9, 6, 5}, {14}, {15, 8}
- **Sub Tree** – tree T is a tree consisting of a node in T and all of its descendants in T.



ВАЖНО:

Дървовидната структура е вид граф когато от root-а (корена) мога да стигна до всяко едно място (връх или листо) само по един път/ по един начин.

Докато при graph (граф) – неща, които са свързани помежду си без никакви изисквания

3.2. Implementing Trees - Recursive Tree Data Structure

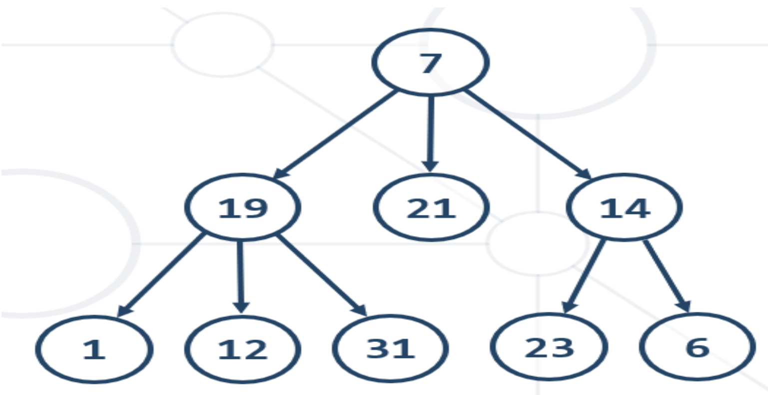
- The recursive definition for **tree** data structure:
  - A single node **is a tree**
  - Nodes have **zero or multiple children** that are **also trees**

```
public class Tree<E> implements AbstractTree<E> {
    private E key;
    private Tree<E> parent;
    private List<Tree<E>> children;

    public Tree(E key, Tree<E>... children) {
        this.key = key;
        this.children = new ArrayList<>();
        for (Tree<E> child : children) {
            this.children.add(child);
            child.parent = this;
        }
    }
}
```

Или използваме името Node вместо Tree

```
public class Node<E> implements NodeTree<E> {
    private E key;
    private Node<E> parent;
    private List<Node<E>> children;

    public Tree(E key, Node<E>... children) {
        this.key = key;
        this.children = new ArrayList<>();
        for (Node<E> child : children) {
            this.children.add(child);
            child.parent = this;
        }
    }
}
```

```java
public class Main {
    public static void main(String[] args) {
        Tree<Integer> tree =
                new Tree<>(7,
                        new Tree<>(19,
                                new Tree<>(1),
                                new Tree<>(12),
                                new Tree<>(31)),
                        new Tree<>(21),
                        new Tree<>(14,
                                new Tree<>(23),
                                new Tree<Integer>(6))
                );
    }
}
```



3.3. Traversing Tree-Like Structures – Обхождане на дървета

Traversing a tree means to visit each of its nodes exactly once
The **order of visiting nodes** may vary on the traversal algorithm

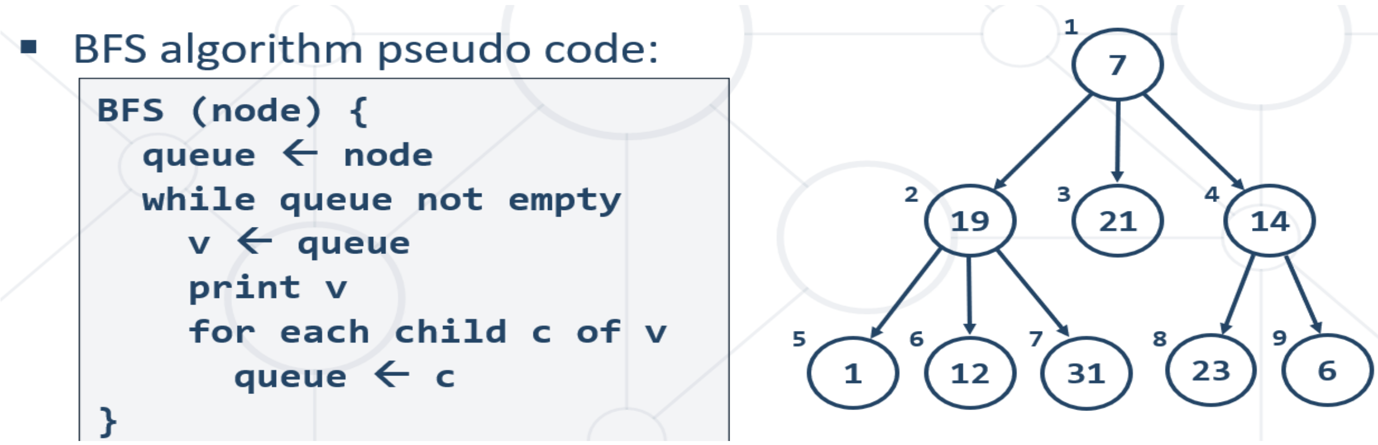
- **Breadth-First Search** (BFS)
  - Nearest nodes visited first
  - Implemented by a queue and a while loop (recursion also possble)
- **Depth-First Search** (DFS)
  - Visit node's successors first
  - Usually implemented by recursion (or implemented by a stack and a while loop)

Breadth-First Search (BFS)

- **Breadth-First Search** (BFS) first visits the neighbor nodes, then the neighbors of neighbors, etc.

Имплементация с цикъл и опашка

7 19 21 14 1 12 31 23 6




```java
public List<E> orderBfs() {
    List<E> result = new ArrayList<>();
    Deque<Tree<E>> queue = new ArrayDeque<>();
    queue.offer(this);

    while (queue.size() > 0) {
        Tree<E> current = queue.poll();
        result.add(current.key)

        for (Tree<E> child : current.children)
            queue.offer(child);
    }
    return result;
}
```

Имплементация с рекурсия

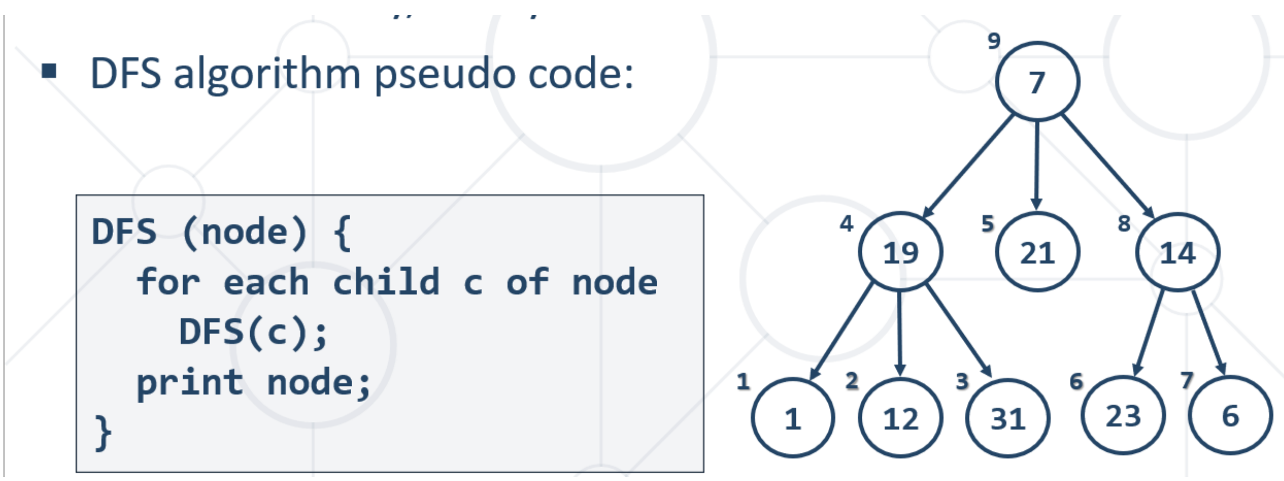
Не се получава лесно

Depth-First Search (DFS)

- **Depth-First Search (DFS)** first visits all descendants of given node recursively, finally visits the node itself
- DFS algorithm pseudo code:

Имплементация с рекурсия

1 12 31 19 21 23 6 14 7



```java
@Override
public List<E> orderDfs() {
    List<E> order = new ArrayList<>();
    this.dfs(this, order);
    return order;
}

private void dfs(Tree<E> tree, List<E> order) {
    for (Tree<E> child : tree.children) {
        this.dfs(child, order);
    }
    order.add(tree.key);
}
```

Имплементация с цикъл и стек - (в случая тръгва от последния в дълбочина)

7 14 6 23 21 19 31 12 1

```java
@Override
public List<E> orderDfs() {
    List<E> result = new ArrayList<>();
    Deque<Tree<E>> stack = new ArrayDeque<>();
    stack.push(this);

    while (stack.size() > 0) {
        Tree<E> current = stack.pop();
        result.add(current.key);

        for (Tree<E> child : current.children)
            stack.push(child);
    }

    return result;
}
```

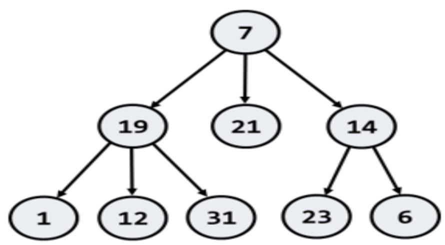
3.4. Инициализиране на дървото

Вариант 1)

```java
public class Tree<E> implements AbstractTree<E> {
    private E key;
    private Tree<E> parent;
    private List<Tree<E>> children;

    public Tree(E key, Tree<E>... children) {
        this.key = key;
        this.children = new ArrayList<>();
        for (Tree<E> child : children) {
            this.children.add(child);
            child.parent = this;
        }
    }
}

public class Main {
    public static void main(String[] args) {
        Tree<Integer> tree =
                new Tree<>(7,
                        new Tree<>(19,
                                new Tree<>(1),
                                new Tree<>(12),
```

```java
                                new Tree<>(31)),
                        new Tree<>(21),
                        new Tree<>(14,
                                new Tree<>(23),
                                new Tree<Integer>(6))
                );
    }
}
```

Вариант 2) – чрез TreeFactory наш клас когато четем поредни двойки родител – дете

Input	Output	Tree
9	7	
7 19	19	
7 21	1	
7 14	12	
19 1	31	
19 12	21	
19 31	14	
14 23	23	
14 6	6	

```java
public class TreeFactory {
    private Map<Integer, Tree<Integer>> nodesByKeys;

    public TreeFactory() {
        this.nodesByKeys = new LinkedHashMap<>();
    }

    public Tree<Integer> createTreeFromStrings(String[] input) {
        for (String line : input) {
            int[] nodeValues = Arrays.stream(line.split("\\s+"))
                    .mapToInt(Integer::parseInt)
                    .toArray();
            addEdge(nodeValues[0], nodeValues[1]);
        }
        return getRoot();

    }

    private Tree<Integer> getRoot() {
        for (Tree<Integer> node : this.nodesByKeys.values()) {
            if (node.getParent() == null) {
                return node;
            }
        }
        return null;
    }

    public Tree<Integer> createNodeByKey(int key) {
        this.nodesByKeys.putIfAbsent(key, new Tree<Integer>(key));
        return this.nodesByKeys.get(key);
    }

    public void addEdge(int parent, int child) {
        Tree<Integer> parentTree = this.createNodeByKey(parent);
        Tree<Integer> childTree = this.createNodeByKey(child);
```

```java
        parentTree.addChild(childTree);
        childTree.setParent(parentTree);
    }
}

public class Tree<E> implements AbstractTree<E> {
    private E value;
    private Tree<E> parent;
    private List<Tree<E>> children;

    public Tree(E value, Tree<E>... children) {
        this.value = value;
        this.children = this.initChildren(children);
    }

    private List<Tree<E>> initChildren(Tree<E>[] children) {
        List<Tree<E>> treeChildren = new ArrayList<>();

        for (Tree<E> child : children) {
            child.setParent(this);
            treeChildren.add(child);
        }
        return treeChildren;
    }

    @Override
    public void setParent(Tree<E> parent) {
        this.parent = parent;
    }

    @Override
    public void addChild(Tree<E> child) {
        this.children.add(child);
    }

    @Override
    public Tree<E> getParent() {
        return this.parent;
    }

    @Override
    public E getKey() {
        return this.value;
    }

    public List<Tree<E>> getChildren() {
        return this.children;
    }
}

B main-a:
String[] input = {
        "19 1",
        "19 12",
        "19 31",
        "14 23",
        "14 6",
        "7 19",
        "7 21",
        "7 14"
};
```

```
TreeFactory treeFactory = new TreeFactory();
Tree<Integer> tree = treeFactory.createTreeFromStrings(input);
```

3.5. Други команди по дървото

Добавяне на дете

Вземаме единият от алгоритмите за обхождане (в случая BFS), и в червен цвят е моята добавка

```
@Override
public void addChild(E parentKey, Tree<E> child) {
    Deque<Tree<E>> queue = new ArrayDeque<>();
    if (this.key == parentKey) {
        child.parent = this;
        this.children.add(child);
        return;
    }

    queue.offer(this);

    while (queue.size() > 0) {
        Tree<E> current = queue.poll();

        for (Tree<E> ch : current.children) {
            if (ch.key == parentKey) {
                child.parent = ch;
                ch.children.add(child);
                return;
            }
            queue.offer(ch);
        }
    }
}

public static void main(String[] args) {
    Tree<Integer> tree =
            new Tree<>(7,
                    new Tree<>(19,
                            new Tree<>(1),
                            new Tree<>(12),
                            new Tree<>(31)),
                    new Tree<>(21),
                    new Tree<>(14,
                            new Tree<>(23),
                            new Tree<Integer>(6))
            );

    tree.addChild(19, new Tree<Integer>(45));
}
```

Връщане на поддърво

```
//returns a subtree with root nodeKey
public Tree<E> getNode(E nodeKey) {
    if (this.key.equals(nodeKey)) {
        return this;
```

```java
    }

    Deque<Tree<E>> queue = new ArrayDeque<>();
    queue.offer(this);

    while (queue.size() > 0) {
        Tree<E> current = queue.poll();

        for (Tree<E> ch : current.children) {
            if (ch.key.equals(nodeKey)) {
                return ch;
            }
            queue.offer(ch);
        }
    }

    return null;
}
```

Изтриване на поддърво

```java
@Override
public void removeNode(E nodeKey) {
    if (this.key == nodeKey && this.parent == null) {
        this.children = new ArrayList<>();
        this.key = null;
        return;
    }

    Tree<E> nodeToDelete = getNode(nodeKey);
    if (nodeToDelete == null) {
        return;
    }

    if (nodeToDelete.children.isEmpty()) {
        List<Tree<E>> parentChildrenList = nodeToDelete.parent.children;
        parentChildrenList.remove(nodeToDelete);
        nodeToDelete.parent = null;
        return;
    } else {
        List<Tree<E>> parentChildrenList = nodeToDelete.parent.children;
        parentChildrenList.remove(nodeToDelete);
        nodeToDelete.parent = null;
        nodeToDelete.children = new ArrayList<>();
        return;
    }

}
```

Размени върхове

Направил съм го донякъде само

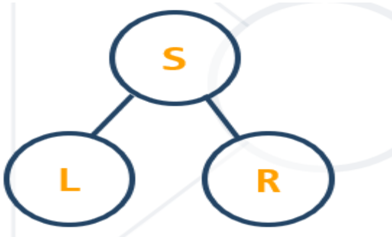
```java
@Override
public void swap(E firstKey, E secondKey) {
    if (firstKey.equals(secondKey)) {
        return;
    }

    Tree<E> firstSubtree = getNode(firstKey);
```

```java
    Tree<E> secondSubtree = getNode(secondKey);

    if (firstSubtree.parent == null) {
        if (secondSubtree.children.isEmpty()) { //ако е листо
            inTheParentListDeleteThatLeaf(secondSubtree);
            secondSubtree.children.add(firstSubtree);
            firstSubtree.parent = secondSubtree;
        } else { //ако е среден връх

        }

        return;
    }

    if (secondSubtree.parent == null) {
        if (firstSubtree.children.isEmpty()) { //ако е листо
            inTheParentListDeleteThatLeaf(firstSubtree);
            firstSubtree.children.add(secondSubtree);
            secondSubtree.parent = firstSubtree;
        }

        return;
    }

    switchNodes(firstSubtree, secondSubtree);
}

private void switchNodes(Tree<E> subtree1, Tree<E> subtree2) {
    Tree<E> parent1 = subtree1.parent;
    Tree<E> parent2 = subtree2.parent;
    int indexSubtree1 = parent1.children.indexOf(subtree1);
    int indexSubtree2 = parent2.children.indexOf(subtree2);

    parent1.children.set(indexSubtree1, subtree2);
    parent2.children.set(indexSubtree2, subtree1);
    subtree1.parent = parent2;
    subtree2.parent = parent1;
}


private void inTheParentListDeleteThatLeaf(Tree<E> nodeToDelete) {
    nodeToDelete.parent.children.remove(nodeToDelete); //ако е листо
    nodeToDelete.parent = null; //ако е листо
}
```

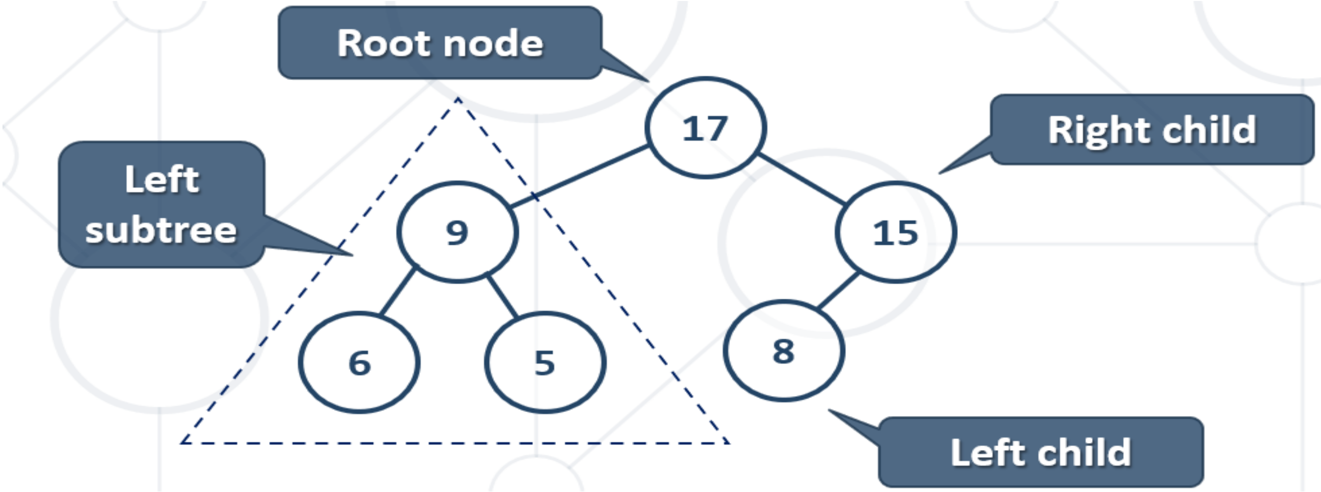
4. Binary Trees, Heaps and BST

4.1. Binary Tree – двоично дърво – неподредено / небалансирано !!!

- Each node has **at most two** children
    - Children are called **left** and **right**
    - The **parent** is also called **source**

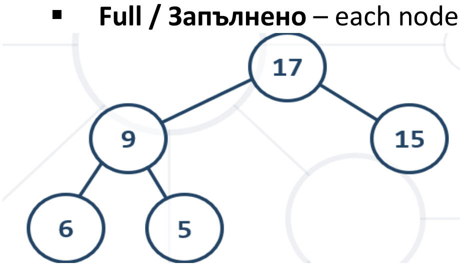


- **Binary trees**: the most widespread form - Each node **has at most 2 children** (**left** and **right**)

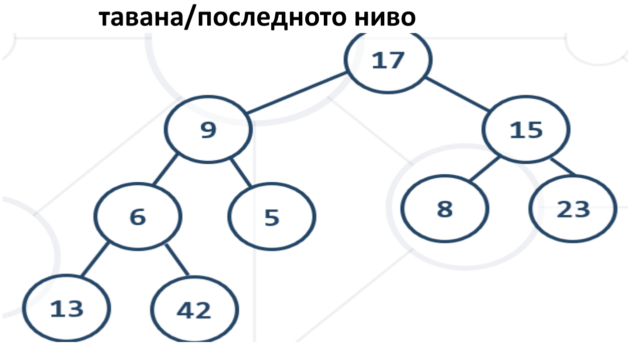


Types of Binary Trees

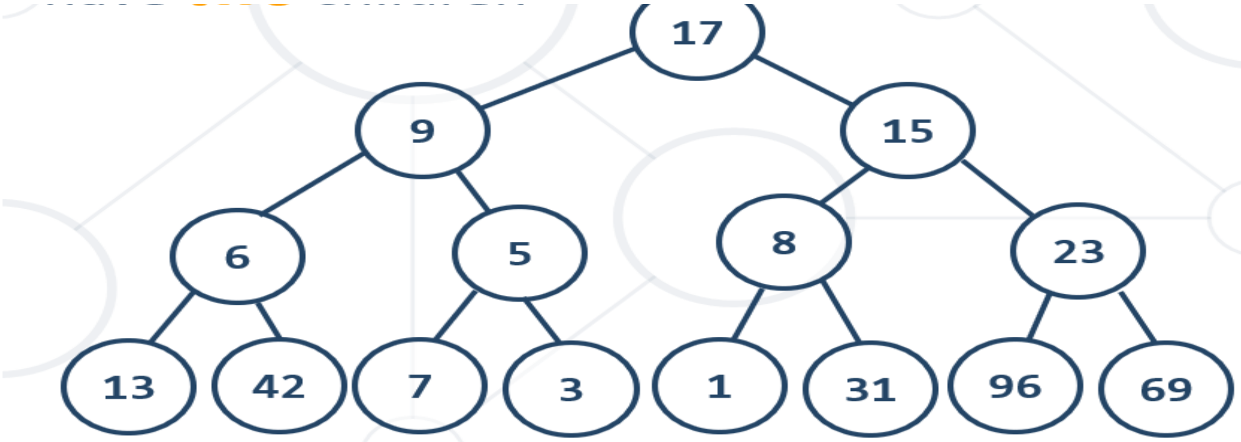
- **Full / Запълнено** – each node has **0** or **2** children - **всичко освен листата има деца**



- **Complete /** – nodes are filled **top** to **bottom** and **left** to **right – на всяко едно ниво е еднакво, без тавана/последното ниво**



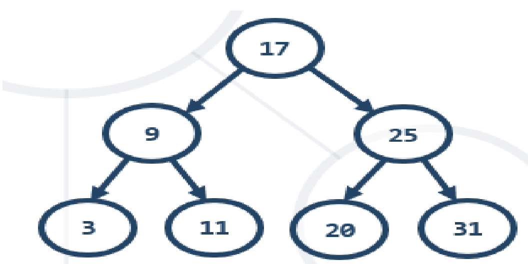
- **Perfect** – combines **complete** and **full**
  - leafs are at the **same level**, other nodes have **two** children



Инициализиране на двуично дърво

Вариант 1 – ръчно инициализиране / hardcore-нати стойности

```
public class BinaryTree<E> implements AbstractBinaryTree<E> {
    private E key;
    private BinaryTree<E> left;
    private BinaryTree<E> right;

    public BinaryTree(E key, BinaryTree<E> left, BinaryTree<E> right) {
        this.key = key;
        this.left = left;
        this.right = right;
    }
}

main
BinaryTree tree = new BinaryTree<>(17,
        new BinaryTree<>(9, new BinaryTree<>(3, null, null),
                new BinaryTree<>(11, null, null)),
        new BinaryTree<>(25, new BinaryTree<>(20, null, null),
                new BinaryTree<>(31, null, null))
);
```



Вариант 2 – все пак ги подреждаме стойностите – отляво по-малки, отдясно по-големи

```
public class MessagingSystem implements DataTransferSystem {

    static class Node {
        Message message;
        Node left;
        Node right;

        public Node(Message message) {
            this.message = message;
        }
```

```
        int getWeight() {
            return this.message.getWeight();
        }
    }

    Node root;
    int size;


@Override
public void add(Message message) {
    if (root == null) {
        root = new Node(message);
    } else {
        add(root, message);
    }

    size++;
}

private void add(Node node, Message message) {
    if (node.getWeight() == message.getWeight()) {
        throw new IllegalArgumentException();
    }

    if (message.getWeight() < node.getWeight()) {
        if (node.left == null) {
            node.left = new Node(message);
        } else {
            add(node.left, message);
        }
    } else {
        if (node.right == null) {
            node.right = new Node(message);
        } else {
            add(node.right, message);
        }
    }
}
```

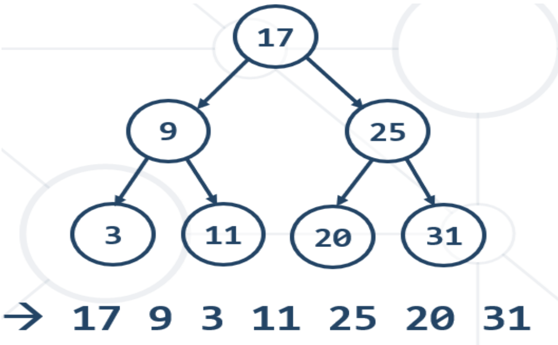
Traversing binary tree

Pre-Order – по нормален начин – a kind of DFS

Root -> Left -> Right

First we **add** the **visiting** node then we **continue** with the **left** and **right** child

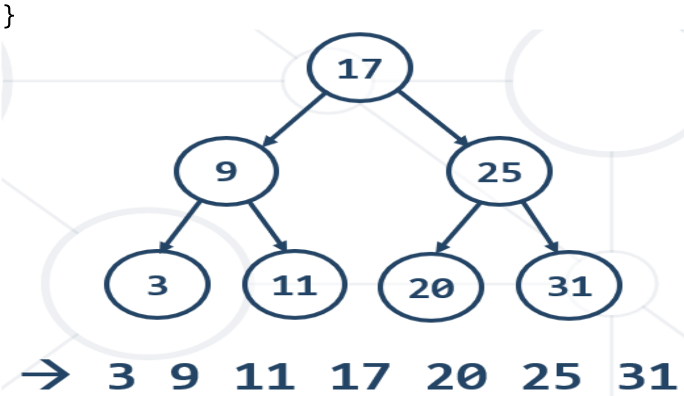
```
preOrder (node) {
    if (node != null) {
        print node.key
        preOrder(node.left)
        preOrder(node.right)
    }
}
```



In-Order - a kind of DFS

Left → Root → Right

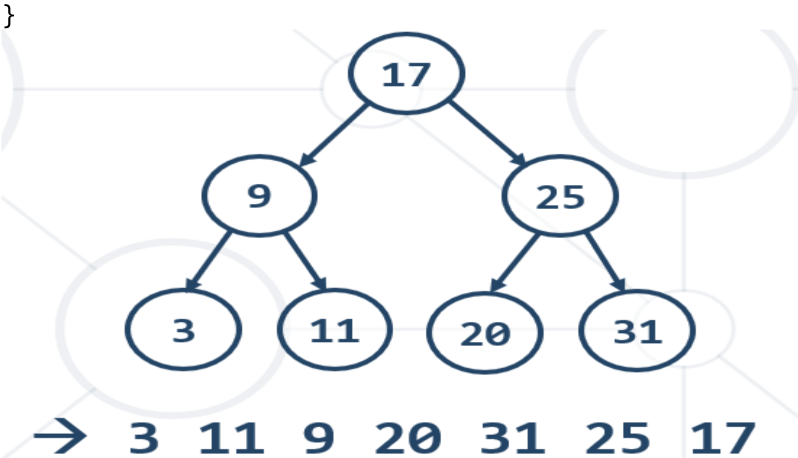
```
inOrder (node) {
    if (node != null) {
        inOrder(node.left)
        print node.key
        inOrder(node.right)
    }
}
```



Post-Order - a kind of DFS

Left → Right → Root

```
postOrder (node) {
    if (node != null) {
        postOrder(node.left)
        postOrder(node.right)
        print node.key
    }
}
```



How to copy a tree

Pre-Order – по нормален начин

```
public BinaryTree<E> copyTree(){
    return copy(this); //това е корена
}

private BinaryTree<E> copy(BinaryTree<E> root) {
    if (root == null) {
        return null;
    }

    BinaryTree<E> copiedTree = new BinaryTree<E>(root.getKey(), null, null);
    copiedTree.left = copy((BinaryTree<E>) root.getLeft());
    copiedTree.right = copy((BinaryTree<E>) root.getRight());

    return copiedTree;
}

main
BinaryTree tree = new BinaryTree<>(17,
        new BinaryTree<>(9, new BinaryTree<>(3, null, null),
                new BinaryTree<>(11, null, null)),
        new BinaryTree<>(25, new BinaryTree<>(20, null, null),
                new BinaryTree<>(31, null, null))
);

BinaryTree<Integer> secondCopiedTree = tree.copyTree();
```

How to foreach a binary tree

```
The case of in-order
@Override
public void forEachInOrder(Consumer<E> consumer) {
    if (this.getLeft() != null) {
        this.getLeft().forEachInOrder(consumer);
    }
    consumer.accept(this.getKey());

    if (this.getRight() != null) {
        this.getRight().forEachInOrder(consumer);
    }
}

StringBuilder builder = new StringBuilder();
tree.forEachInOrder(key -> builder.append(key).append(", "));
String actual = builder.toString();
```

How to delete a node in a (binary) tree

Post-Order – изтриваме левия, десния, и се връщаме на бащата

Ако искаме да върнем всички изтрити елементи, то не можем просто да изтрием връзката, а трябва да изтрием всички под/sub-nodes. И тогава използваме Post-Order

Lowest Common Ancestor algorithm

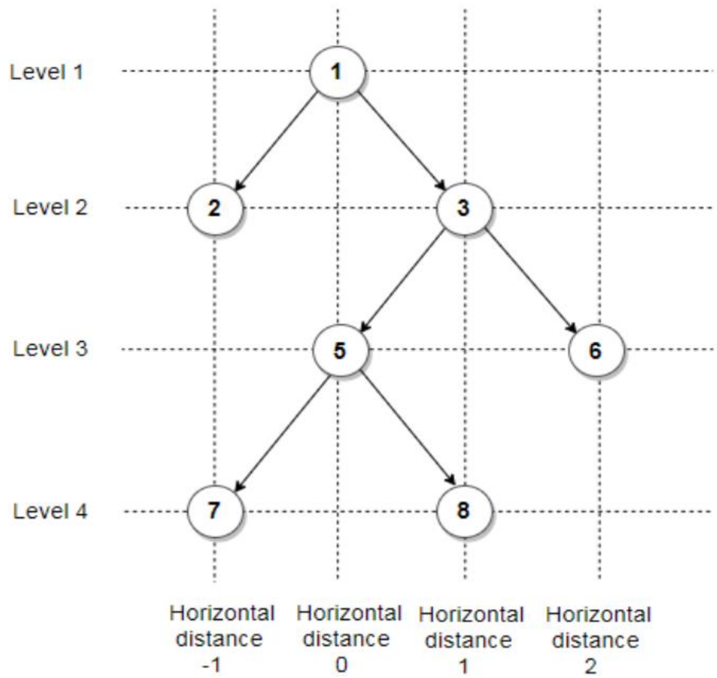
In other words you can **ignore** the **value** you **should only care for the distance**.

```
public class BinaryTree {
    private int value;
    private BinaryTree left;
    private BinaryTree right;
    private BinaryTree parent;


    public BinaryTree(int key, BinaryTree left, BinaryTree right) {
        this.value = key;
        this.left = left;
        this.right = right;
        this.setParent(null);
        if (this.left != null) {
            this.left.setParent(this);
        }
        if (this.right != null) {
            this.right.setParent(this);
        }
    }

private BinaryTree findNode(BinaryTree current, int value) {
    if (current == null) {
        return null;
    }

    if (current.getValue() == value) {
        return current;
    } else {
        BinaryTree foundNode = this.findNode(current.getLeft(), value);

        if (foundNode == null) {
            foundNode = this.findNode(current.getRight(), value);
        }

        return foundNode;
    }
}

private List<BinaryTree> findAncestors(int value) {
    List<BinaryTree> result = new ArrayList<>();
    BinaryTree foundNode = this.findNode(this, value);

    while (foundNode.getParent() != null) {
        foundNode = foundNode.getParent();
        result.add(foundNode);
    }

    return result;
}

public Integer findLowestCommonAncestor(int first, int second) {
```

```java
    List<BinaryTree> firstAncestors = this.findAncestors(first);
    List<BinaryTree> secondAncestors = this.findAncestors(second);

    for (int i = 0; i < firstAncestors.size(); i++) {
        if (secondAncestors.contains(firstAncestors.get(i))) {
            return firstAncestors.get(i).getValue();
        }
    }

    return null;
}
```

Top view elements algorithm

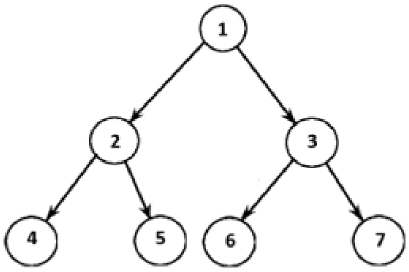
The following figure shows the horizontal distance and level of each node in the above binary tree:



Horizontal distance = offset

The final values in the map will be:

```
horizontal distance -> (node's value, node's level)
-1 -> (2, 2)
0 -> (1, 1)
1 -> (3, 2)
2 -> (6, 3)
```



Given the above tree the result should be: **1, 2, 4, 3, 7,** where order of **output does not matter.**

```
public class Pair<K, V> {
    private K key;
    private V value;

    public Pair(K key, V value) {
        this.key = key;
        this.value = value;
    }

    public K getKey() {
        return key;
    }

    public void setKey(K key) {
        this.key = key;
    }

    public V getValue() {
        return value;
    }

    public void setValue(V value) {
        this.value = value;
    }
}


public class BinaryTree {
    private int value;
    private BinaryTree left;
    private BinaryTree right;
    private BinaryTree parent;

        public BinaryTree(int key, BinaryTree left, BinaryTree right) { …….}

public List<Integer> topView() {
//първият елемент е offset – заема стойности от -3 до +3
//вторият параметър е стойността на върха
//третият елемент е нивото, на което се намира върха
    Map<Integer, Pair<Integer, Integer>> offsetToValueLevel = new TreeMap<>();

    traverseTree(this, 0, 1, offsetToValueLevel);

    List<Integer> collect = offsetToValueLevel.values()
            .stream()
            .map(e -> e.getKey())
            .collect(Collectors.toList());
```

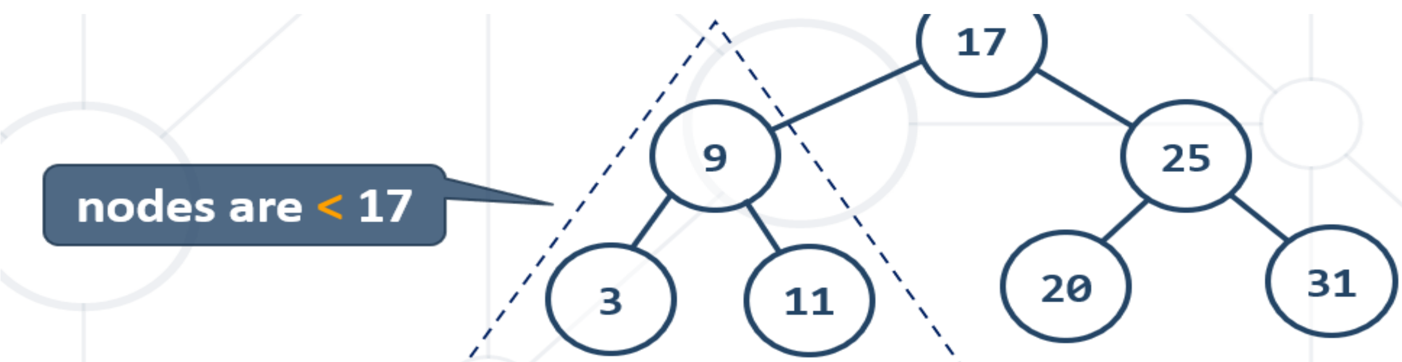
```java
        return collect;
    }

    private void traverseTree(BinaryTree tree, int offset, int level, Map<Integer,
            Pair<Integer, Integer>> offsetToValueLevel) {
        if (tree == null) {
            return;
        }

        Pair<Integer, Integer> currentValueLevel = offsetToValueLevel.get(offset);
        if (currentValueLevel == null || level < currentValueLevel.getValue()) {
            offsetToValueLevel.put(offset, new Pair<>(tree.value, level));
        }

        traverseTree(tree.left, offset - 1, level + 1, offsetToValueLevel);
        traverseTree(tree.right, offset + 1, level + 1, offsetToValueLevel);
    }
```

4.4. Binary Search Trees – подредено!!!

Двоичните дървета за търсене предразполагат много добре за подреждане на елементи.
Приема се, че няма повтарящи се елементи!!!

Всички двуични дървета имат операции с логаритмична сложност
find -> O(log(n))
insert -> O(log(n))
delete -> O(log(n))

- **Binary search trees** are **ordered**
    - For each node ***x***
        - Elements in left subtree of ***x*** are **<** ***x***
        - Elements in right subtree of ***x*** are **>** ***x***



BST – Insert - инициализация на BinarySearchTree

- if node is **null** → insert x
- else if x **<** node.value → **go left**
- else if x **>** node.value → **go right**
- else → node **exists**

Вариант с една стойност на Node

```java
public interface AbstractBinarySearchTree<E extends Comparable<E>> {
```

```java
public static class Node<E> {
        public E value;
        //        public E ;
        public Node<E> leftChild;
        public Node<E> rightChild;

        public Node(E value) {
            this.value = value;
        }
}



public class BinarySearchTree<E extends Comparable<E>> implements AbstractBinarySearchTree<E> {
    private Node<E> root;
    private Node<E> leftChild;
    private Node<E> rightChild;
```

```java
public static class Node<E> {
    private E value;
    private Node<E> leftChild;
    private Node<E> rightChild;

    public Node() {
    }

    public Node(E value) {
        this.value = value;
    }

    public Node(E value, Node<E> leftChild, Node<E> rightChild) {
        this.value = value;
        this.leftChild = leftChild;
        this.rightChild = rightChild;
    }

    public void setLeftChild(Node<E> leftChild) {
        this.leftChild = leftChild;
    }

    public void setRightChild(Node<E> rightChild) {
        this.rightChild = rightChild;
    }

    public Node<E> getLeft() {
        return this.leftChild;
    }

    public Node<E> getRight() {
        return this.rightChild;
    }

    public E getValue() {
        return this.value;
    }
}


      public BinarySearchTree(E value) {
      this.root = new Node<E>(value);
      }

        public BinarySearchTree() {
      }



}


    @Override
    public void insert(E key) {
        Node<E> node = new Node<>(key, null, null);

        if (this.getRoot() == null) {
            this.root = node;
        } else {
            // TODO: Find the place to insert
            insertRecursive(key, this.root);
```

```java
        }
    }

      private void insertRecursive(E key, Node<E> node) {
          int compareResult = key.compareTo(node.value);

          if (compareResult == 0) {
              return;
          }

          if (compareResult < 0) {
              if (node.leftChild == null) {
                  node.leftChild = new Node<>(key, null, null);
              } else {
                  insertRecursive(key, node.leftChild);
              }

          } else {
              if (node.rightChild == null) {
                  node.rightChild = new Node<>(key, null, null);
              } else {
                  insertRecursive(key, node.rightChild);
              }
          }
      }
}

main
BinarySearchTree<Integer> bst = new BinarySearchTree<>();
bst.insert(12);
bst.insert(21);
bst.insert(5);
bst.insert(1);
bst.insert(8);
bst.insert(18);
bst.insert(23);
```

Вариант с две стойности на Node = TreeMap

```java
public interface AbstractBinarySearchTree<E extends Comparable<E>> {

    public static class Node<E> {
        public E key;
        public E value;
        public Node<E> leftChild;
        public Node<E> rightChild;

        public Node(E key, E value) {
            this.key = key;
            this.value = value;
        }

        public Node(E key, E value, Node<E> leftChild, Node<E> rightChild) {
            this.key = key;
            this.value = value;
            this.leftChild = leftChild;
            this.rightChild = rightChild;
        }
}
```

```java
public class BinarySearchTree<E extends Comparable<E>> implements AbstractBinarySearchTree<E> {
    private Node<E> root;

    @Override
    public void insert(E key, E value) {
        Node<E> node = new Node<>(key, value, null, null);

        if (this.getRoot() == null) {
            this.root = node;
        } else {
            // TODO: Find the place to insert
            insertRecursive(key, value, this.root);
        }
    }

      private void insertRecursive(E key, E value, Node<E> node) {
          int compareResult = key.compareTo(node.key); //сравняваме по key

          if (compareResult == 0) {
              return;
          }

          if (compareResult < 0) {
              if (node.leftChild == null) {
                  node.leftChild = new Node<>(key, value, null, null);
              } else {
                  insertRecursive(key, value, node.leftChild);
              }

          } else {
              if (node.rightChild == null) {
                  node.rightChild = new Node<>(key, value, null, null);
              } else {
                  insertRecursive(key, value, node.rightChild);
              }
          }
      }
}
```

*In-order (**Left** → **Root** → **Right**) на двоично дърво за търсене връща елементите в сортиран вид!!!*

```java
main
BinarySearchTree<Integer> bst = new BinarySearchTree<>();
bst.insert(12);
bst.insert(21);
bst.insert(5);
bst.insert(1);
bst.insert(8);
bst.insert(18);
bst.insert(23);
```

bst.print(); връща

```
"C:\Program Fil
    1
  5
    8
12
    18
  21
    23
```

```java
public void print(){
        printRecursive(this.root, 0);
    }

    private void printRecursive(Node<E> node, int level){
        if (node == null) {
            return;
        }

        StringBuilder padding = new StringBuilder();
        for (int i = 0; i < level; i++) {
            padding.append("  ");
        }

        printRecursive(node.leftChild, level + 1);
        System.out.println(padding.append(node.getValue()));
        printRecursive(node.rightChild, level + 1);
    }
}
```

How to foreach a binary search tree(BST)

```java
The case of in-order
public void eachInOrder(Consumer<E> consumer) {
    this.internalEachInOrder(this.root, consumer);
}

private void internalEachInOrder(Node<E> node, Consumer<E> consumer) {
    if (node == null) {
        return;
    }

    this.internalEachInOrder(node.getLeft(), consumer);
    consumer.accept(node.getValue());
    this.internalEachInOrder(node.getRight(), consumer);
}
```

BST - Search

- Search for **x** in BST
    - if node is not null
        - if x **<** node.value → **go left**
        - else if x **>** node.value → **go right**
        - else if x **==** node.value → **return**

Вариант 1 – смесен вариант

```java
@Override
public AbstractBinarySearchTree<E> search(E searchedElement) {
```

```
    AbstractBinarySearchTree<E> result = new BinarySearchTree<>();
    Node<E> current = this.root;
    while (current != null){
        if(searchedElement.compareTo(current.value) < 0){
            current = current.leftChild;
        } else if(searchedElement.compareTo(current.value) > 0){
            current = current.rightChild;
        } else { // ако елемента съвпада
            return new BinarySearchTree<E>(current);
        }
    }
    return result;
}

//генерирай дърво с метода insert с определен корен
private BinarySearchTree(Node<E> root) {
    this.copy(root);
}

//генерирай дърво с метода insert с определен корен – ИЗПОЛЗВА Pre-Order
private void copy(Node<E> node) {
    if(node!=null) {
        this.insert(node.value);
        this.copy(node.leftChild);
        this.copy(node.rightChild);
    }
}
```

2ри вариант само с рекурсия за търсене на елемент

```
private Node<E> internalSearchReturnNodeRecursive(Node<E> node, E element) {
    if (node == null) {
        return null;
    }
    if (node.getValue().compareTo(element) < 0) {
        return this.internalSearchReturnNodeRecursive(node.getRight(), element);
    } else if (node.getValue().compareTo(element) > 0) {
        return this.internalSearchReturnNodeRecursive(node.getLeft(), element);
    }

    return node;
}

public BinarySearchTree<E> search(E searchedElement) {
Node<E> current = internalSearchReturnNodeRecursive(this.root, element);

return current == null ? null :
new BinarySearchTree<E>(current.getValue());
}
```

BST – Contains

Итеративен вариант, съществува и рекурсивен вариант

```
@Override
public boolean contains(E element) {
    Node<E> current = this.root;
    while (current != null){
        if (element.compareTo(current.value) < 0){
```

```java
            current = current.leftChild;
        } else if (element.compareTo(current.value) > 0){
            current = current.rightChild;
        } else {
            break;
        }
    }
    return current != null;
}
```

- Binary search trees can be **balanced**
  - Balanced trees have for each node
    - Nearly equal number of nodes in its subtrees

Balanced trees have **height of ~ log(n)**

BST – deleteMin

```java
public void deleteMin() {
    isEmptyTree();

    if (this.root.getLeft() == null) {
        this.root = this.root.getRight();
    } else {
        Node<E> current = this.root;

        while (current.getLeft().getLeft() != null){
            current = current.getLeft();
        }

        current.setLeftChild(current.getLeft().getRight());
    }
}
```

BST – deleteMax

```java
public void deleteMax() {
    isEmptyTree();

    if (this.root.getRight() == null) {
        this.root = this.root.getLeft();
    } else {
        Node<E> current = this.root;

        while (current.getRight().getRight() != null){
            current = current.getRight();
        }

        current.setRightChild(current.getRight().getLeft());
    }
}
```

BST – count elements

```java
public int count() {
    return this.internalCount(this.root);
}
```

```java
private int internalCount(Node<E> node) {
    if (node == null) {
        return 0;
    } else {
        return 1 + (node.getLeft() == null ? 0 : this.internalCount(node.getLeft()))
                + (node.getRight() == null ? 0 : this.internalCount(node.getRight()));
    }
}
```

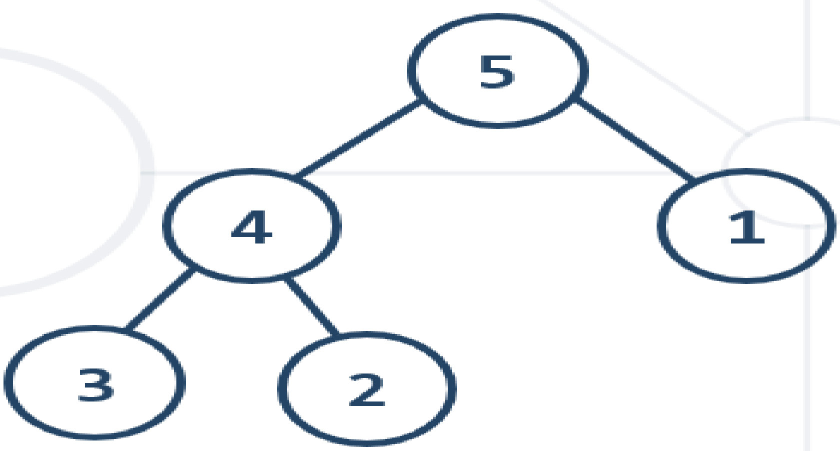
4.2. Heap – пирамида/куп

What is Heap?

- **Heap**
  - Tree-based data structure
  - Stored in an array
- Heaps hold the **heap property** for each node:
  - **Min Heap**
    - parent ≤ children
  - **Max Heap**
    - parent ≥ children

Няма изискване отляво да са по-малки, а отдясно да са по-големи

- **Binary heap**
  - Represents a Binary Tree
- **Shape property** - Binary heap is a **complete binary tree**:
    - Every level, except the last, is **completely filled**
    - Last is filled **from left to right**



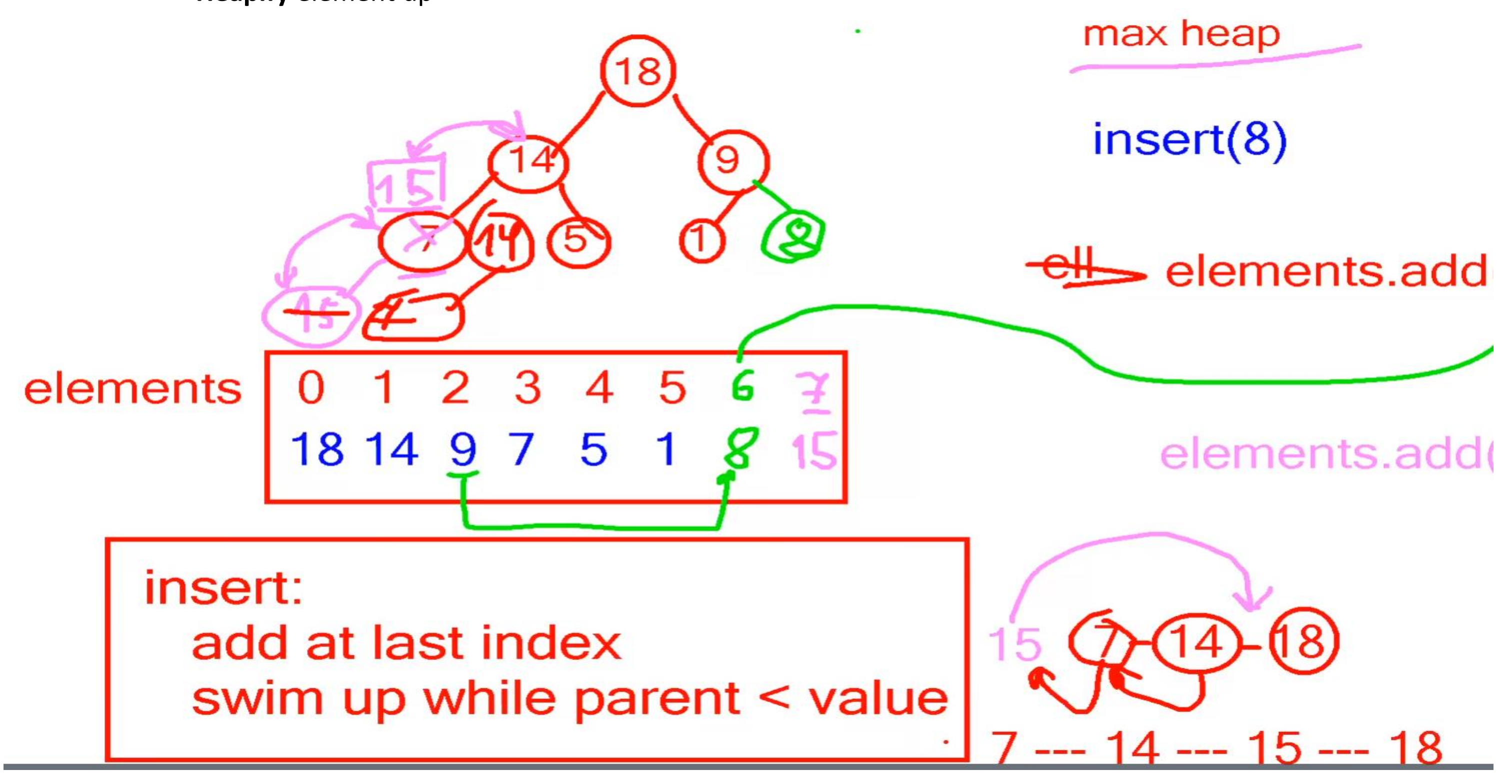
Binary Heap – Array Implementation

- Binary heap can be efficiently stored in an array
- **Parent(i)** = (i - 1) / 2
- **Left(i)** = 2 * i + 1; **Right(i)** = 2 * i + 2

Heap Insertion = Initialization на MaxHeap

Insert element -> add last and SWIM

- To preserve **heap properties**:
  - **Insert** at the end
  - **Heapify** element up




```java
public class MaxHeap<E extends Comparable<E>> implements Heap<E> {
    private List<E> elements;

    public MaxHeap() {
        this.elements = new ArrayList<>();
    }

@Override
public void add(E element) {
    this.elements.add(element);
    this.heapifyUp(this.size() - 1); //heapifyUp = SWIM = изплува нагоре ако има нужда
}

private void heapifyUp(int index) {
    while (hasParent(index) && less(parent(index), elements.get(index))) {
        int parentAt = getParentAt(index);
        Collections.swap(this.elements, parentAt, index);
        index = parentAt;
```

```
    }
}

private int getParentAt(int index) {
    return (index - 1) / 2;
}

private boolean less(E parent, E child) {
    int result = parent.compareTo(child); //ако parent е по-малък от детето, което изплува
    if (result < 0) {
        return true;
    }

    return false;
}

private E parent(int index) {
    return this.elements.get((index - 1) / 2);
}

private boolean hasParent(int index) {
    if (index == 0) {
        return false;
    }

    return true;
}
```

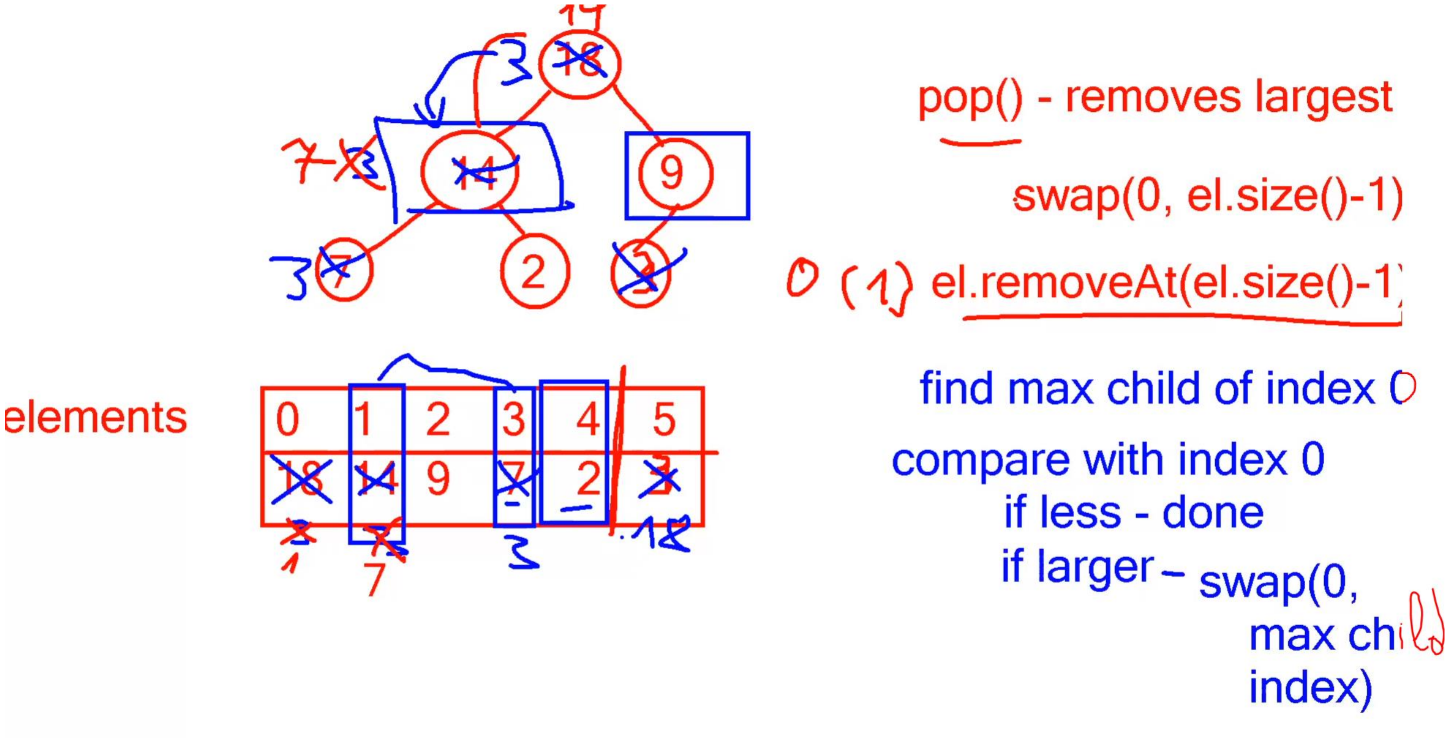
Heap peek – get the 1st element in the array – MaxHeap and MinHeap

```
@Override
public E peek() {
    if (this.size() == 0) {
        throw new IllegalStateException("Heap is empty upon peek attempt");
    }
    return this.elements.get(0);
}
```

Heap pop/poll – removes largest element - MaxHeap

Кофти се имплементира с код – да видя имплентацията на PriorityQueue

Pop max element -> swap first and last; resize size-1 (намаляваме масива с 1 – без последния елемент); SINK first



Heap decrease element – the MinHeap case

```
@Override
public void decrease(E element) {
    int decreasedElementIndex = this.elements.indexOf(element);
    E heapElement = this.elements.get(decreasedElementIndex);
    heapElement.decrease(); //it decreases by 1

    this.heapifyUp(decreasedElementIndex);
}
```

Или

```
@Override
public void decrease(E element) {
    this.elements.remove(element);
    element.decrease(); //it decreases by 1

    this.add(element);
}
```

4.3. Priority Queue – най-често се реализира като Heap!!!

При приоритетната опашка, най-големият елемент/приоритетен отива най-първи на върха

Dequeue Most Significant Element

- ADS representing queue or stack like DS
    - Each element is **served** in **priority**
    - High priority is served **before** low priority
    - Elements with **equal** priority
    - Served in **order** of **input** or **undefined**

- Retains a **specific order** to the elements
- **Higher priority** elements are **pushed to the beginning** of the queue

- **Lower priority** elements are **pushed to the end** of the queue

- **Priority queue** abstract data type (ADT) supports:
    - **Insert(element)**
    - **Pull() → max/min element**
    - **Peek() → max/min element**
- Where **element** has a priority

Priority

- In Java usually the priority is passed as comparator
    - E.g. **Comparable<E>**

PriorityQueue – Array Implementation

Insertation/initialization – like the MaxHeap

Insert element -> add last and SWIM

```
public class PriorityQueue<E extends Comparable<E>> implements AbstractQueue<E> {
    private List<E> elements;

    public PriorityQueue() {
        this.elements = new ArrayList<>();
    }

etc.
```

PriorityQueue peek – like the Heap

```
@Override
public E peek() {
    ensureNonEmpty();
    return this.elements.get(0);
}

private void ensureNonEmpty() {
    if (this.size() == 0) {
        throw new IllegalStateException("Heap is empty upon peek/poll attempt");
    }
}
```

PriorityQueue poll – removes largest element - MaxHeap

Pop/poll max element ->

swap first and last;

resize size-1 (намаляваме масива с 1 – без последния елемент);

SINK first

Save the element on the top of the heap (index 0), **swap** the **first** and **last elements**, **exclude** the **last element** and **demote** the one **at the top until it has correct position**

```
@Override
public E poll() {
    ensureNonEmpty();
```

```
    E removedElement = this.elements.get(0);
    Collections.swap(this.elements, 0, this.size() - 1);
    this.elements.remove(this.elements.size() - 1);
    this.heapifyDown(0);

    return removedElement;
}


/*function will demote the element at a given index until it has no children
 or it is greater than its both children. The first check will be our loop condition*/
private void heapifyDown(int index) {
    while (index < this.elements.size() / 2) {
        int childLeftOrRightIndex = 2 * index + 1;
        int childRightIndex = 2 * index + 2;

        //ако левия клон е по-малък от десния, до десния по-голям изплува нагоре
        if (childRightIndex < this.elements.size() &&
                less(this.elements.get(childLeftOrRightIndex),
this.elements.get(childRightIndex))) {
            childLeftOrRightIndex += 1;
        }

        //ако детето стане първия елемент с нулев индекс
        if (less(this.elements.get(childLeftOrRightIndex), this.elements.get(index))) {
            break;
        }

        Collections.swap(this.elements, index, childLeftOrRightIndex);
        index = childLeftOrRightIndex;
    }
}
```

4.5. O notation table comparison

	insert	remove	search	max
BST (well-balanced)	log(n)	log(n)	log(n)	log(n)
sorted array	n	n	log(n)	O(1)
heap	log(n)	log(n)	-----	O(1)