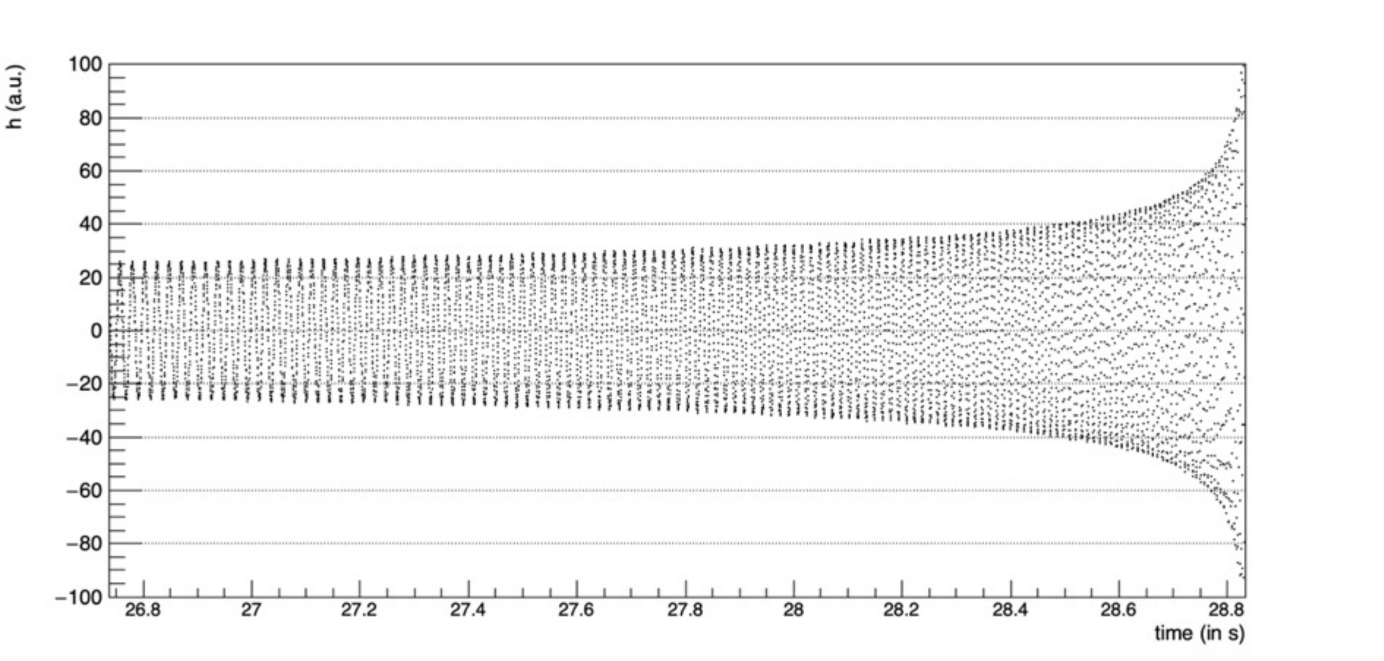
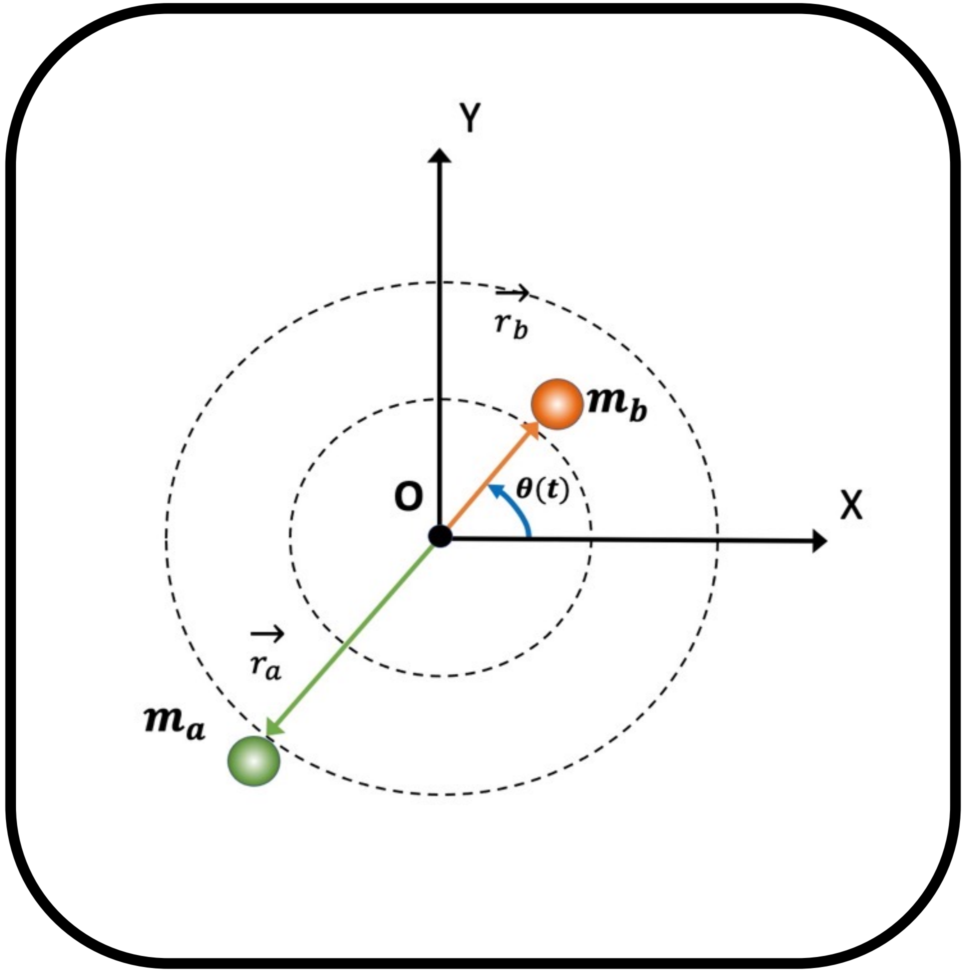
Chirp detection

An introduction



1. Signal shape
2. Detection

1. Signal shape: why a chirp?



➔ The system considered is made of **2 massive objects (*A and B*) in rotation**

➔ **System radiates energy via gravitational waves.** When objects are very dense (*Neutron star, black holes*) and close to each other, **the amount of energy radiated becomes significant.**

➔ **The system is isolated, so it's total energy E_{tot} decreases.**

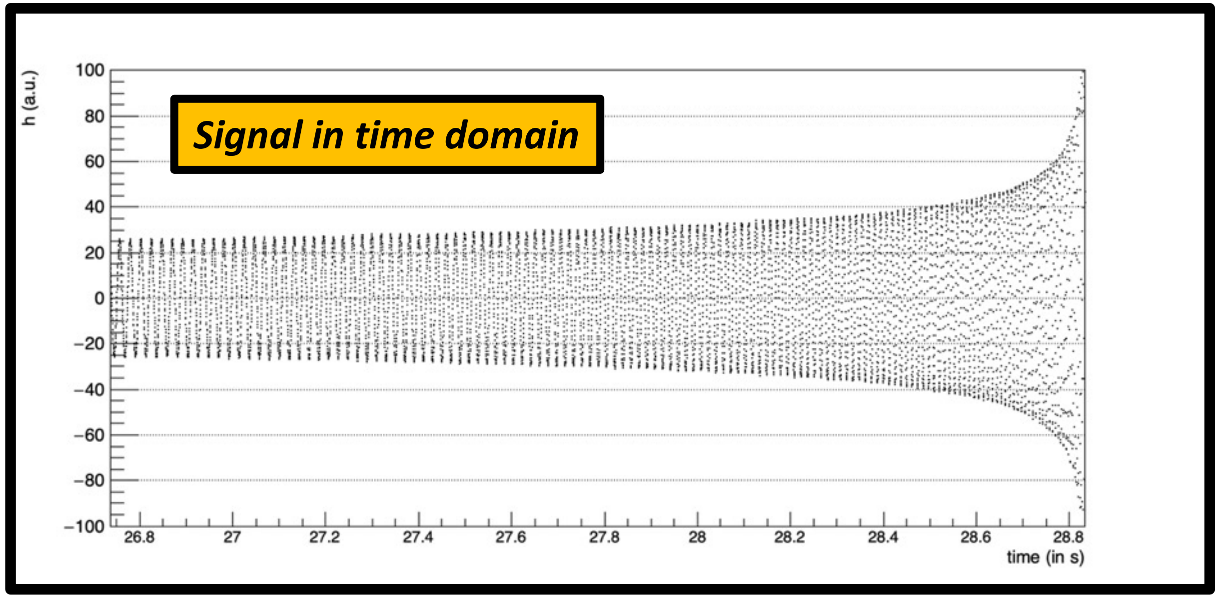
$$\boxed{\frac{dE_{tot}}{dt} = G\frac{m_a m_b \dot{r}}{2r^2}}$$

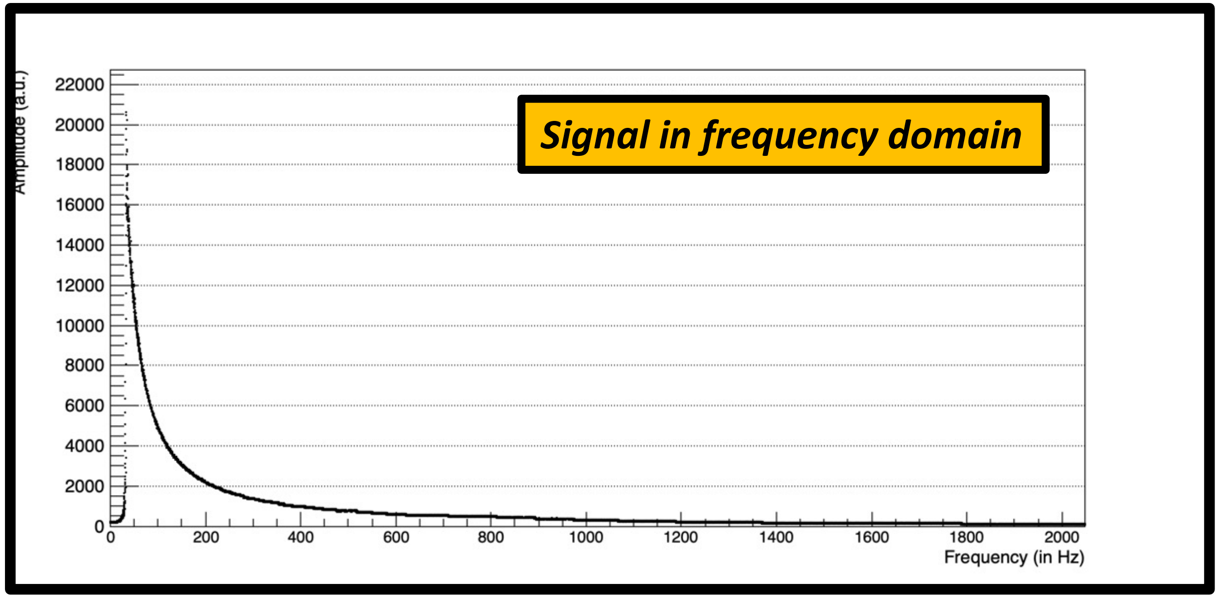
➔ The two objects will **get closer**, rotation **speed will increase** (*Kepler's 3rd law*), and the amount of **radiated energy will increase.**

➔ The shape of the gravitational wave signal will therefore **have increasing amplitude and frequency until r~0 (***in reality before that, at least you know that below $r=r_s$ something should happen***)**

1. Signal shape: the chirp

➔ The signal we are looking for is the following

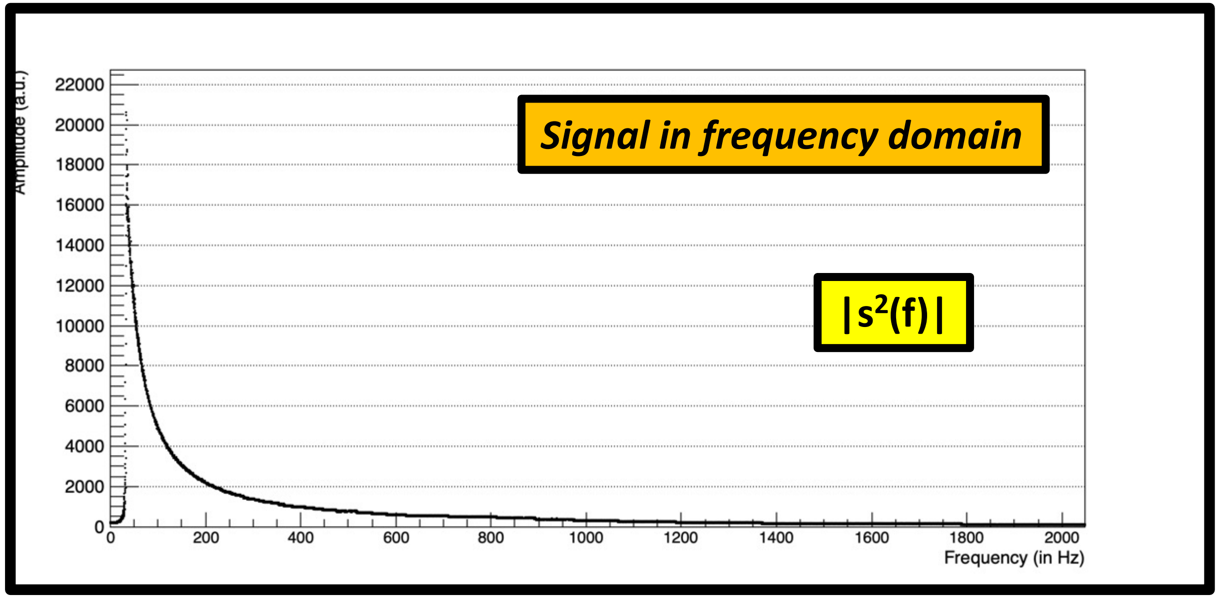


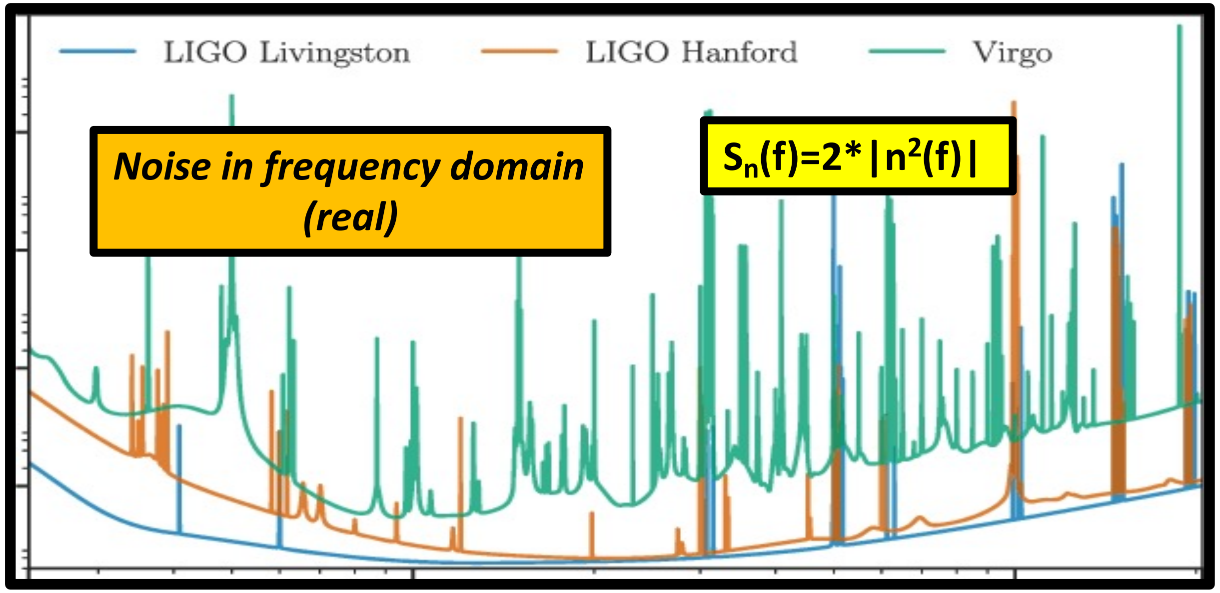


➔ The time during which we can detect it depends on the sensitivity range of the detector, **basically between 40 and 1000 Hz.**

➔ Corresponding time range depends on the signal origin, in particular on the masses of the 2 objects. It can be between 30s (*low masses*)and less than 1s (*high masses*).

2. Detection: the problem





➔ **Problem:** we measure a strain **h(t)**, and we need to detect when we have a signal (**h(t)=s(t)+n(t)**) and to stay quiet when there is nothing (**h(t)=n(t)**).

➔ **Question:** how to do that?

➔ Look in the frequency domain in order to reduce the number of operations

➔ Idea is to transform the signal in such a way that **signal to noise ratio (S/N) gets maximized.**

$$x(t) = \int_{-\infty}^{+\infty} \frac{\tilde{h}(f)\tilde{s}^*(f)}{S_n} e^{2i\pi f t} df$$

Matched filter

➔ **Matched filter is doing exactly that.** When noise is larger, filter suppress the contribution, and when it gets small it enhances it.

2. Detection: the algorithm

Template signal

$$x(t) = \int_{-\infty}^{+\infty} \frac{\tilde{h}(f)\tilde{s}^*(f)}{S_n} e^{2i\pi f t} df$$

➔ Fourier transform of the measured strain will be continuously compared to a big collection of precomputed signals: **the templates.**

➔ Templates are stored in a bank, which covers a given phase space (*depending on what you're looking for*).

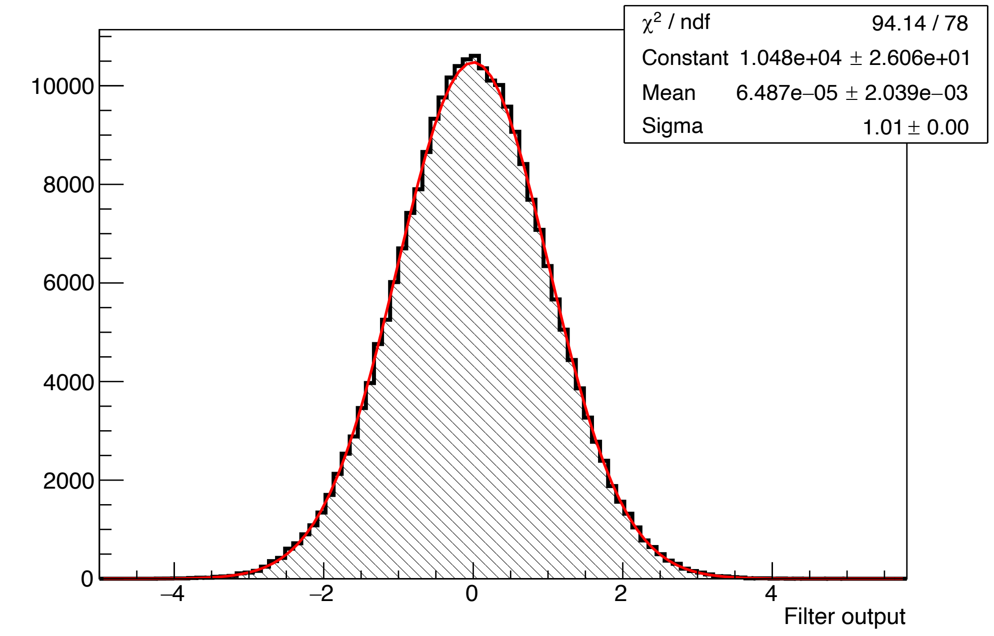
➔ The noise also needs to be measured precisely. It is used to compute the matched filter, but also to normalize the template amplitude. If **h(t)=n(t)** one has indeed:

$$\langle x^2(t) \rangle = \int_{-\infty}^{+\infty} \frac{|\tilde{s}^2(f)|}{2S_n} df = \frac{\sigma_s^2}{2}$$

➔ Normalizing each template with this factor, the output of the filter is exactly the S/N ratio **ρ(t)**

$$\rho_s(t) = \frac{\sqrt{2}}{\sigma_s} x(t)$$

Filter output if signal not matched

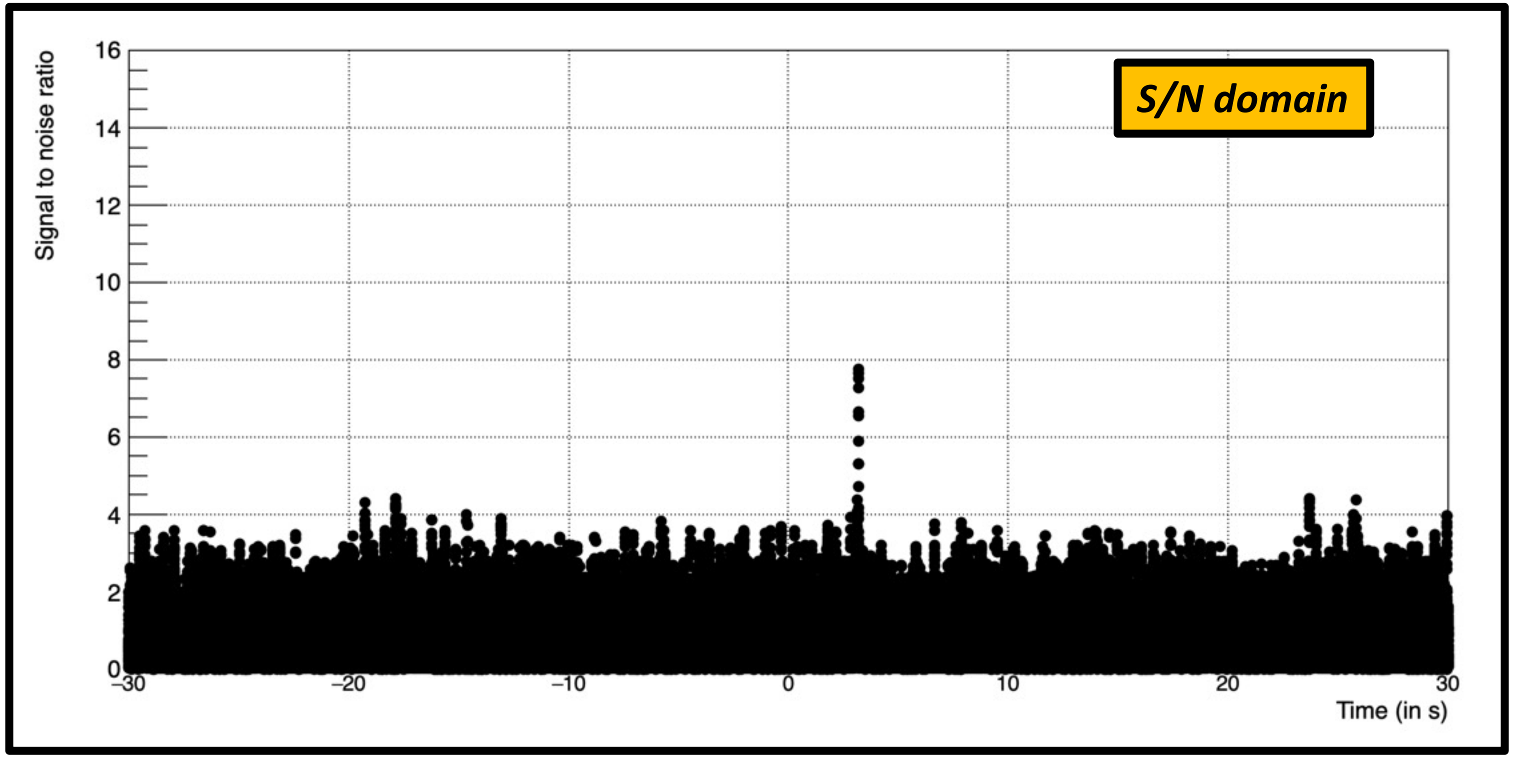


➔ In the absence of signal, output is gaussian with σ=1.

2. Detection: the algorithm

➔ Whenever $|\rho(t)|$ gets larger than a **given threshold**, we can conclude that there is a match with a certain confidence level

➔ This threshold is a tradeoff between the efficiency we want to obtain, and the maximum proportion of fake signals we can afford (*false alarms*)



➔ For a given signal, there could be **more than one template passing the threshold**

➔ A detailed analysis of the signal is then necessary to precisely identify its properties. **This is the parameter estimation.**

Documentation

- Document detailing the calculations: http://sviret.web.cern.ch/sviret/docs/Chirps.pdf
- Webpage with code and exercises: http://sviret.web.cern.ch/sviret/Welcome.php?n=Virgo.Ana