Analytical Decision Making

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Homework 2

Corrected

Instructions:

1. The homework is a graded **individual** assignment.

2. Due date: **June 2nst, 2018 by 9am.**
   a. Late assignments won't be graded.
   b. No extension.

3. There are 2 parts

4. Please submit a Jupyter notebook with your codes. In addition, prepare a written document summarizing your results. Submit a PDF version of the rerpot.

5. In your optimization models, please make sure to be as clear as possible about your notations. For the Python codes, add comments throughout the code (e.g., "#This kind of comment") to help me follow what you are doing. No need to comment every line.

6. Please submit a copy of your assignment on Canvas AND email me a copy as a backup: orubel@ucdavis.edu

7. **Attention should be given to the presentation of the results.**

The homework has two parts and looks at portfolio optimization

Part A: Portfolio Optimization based on the Full Data Set

The objective of the homework is to consider different approaches with respect to portfolio optimization. On Canvas, you have a data set called "datahomework2.xls" where you have trading information about 6 stocks. Based on the evolution of these stocks, you are asked to recommend different portfolio allocations.

1. Preparing the data

A. Based on the data set, compute the rate of returns for each stock, i.e.,

$$r_{it} = \frac{I_{i,t} - I_{i,t-1}}{I_{i,t-1}}$$

where $I_{i,t}$ is the raw data for stock i at time t.

B. Based on the data set, compute the average returns that will be used for the optimization, i.e.,

This is the mistake

$$\boldsymbol{\mu_i = \left(\sum_{t=1}^{T}(1 + r_{it})\right)^{\frac{1}{T}} - 1}$$

Below is the correct formula

$$\boldsymbol{\mu_i = \left(\prod_{t=1}^{T}(1 + r_{it})\right)^{\frac{1}{T}} - 1}$$

Hint: the term $\left(\prod_{t=1}^{T}(1 + r_{it})\right)^{\frac{1}{T}}$ is a geometric mean

C. Based on the data set, compute the variance-covariance matrix of the stock returns, i.e.,

$$CoVar(r_i, r_j) = \frac{1}{T}\sum_{t=1}^{T}(r_{it} - \overline{r_{it}})(r_{jt} - \overline{r_{jt}})$$

D. Provide the vector μ and the variance-covariance matrix Σ for the 6 assets considered.

2. Portfolio Optimization: Minimizing Risk

What is the allocation $x = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ that solve the following optimization problem

$$\text{Minimize } \frac{1}{2} x^T \Sigma x$$

Subject to

$$\mu^T x \geq R$$
$$\sum_{i=1}^{6} x_i = 1$$
$$x \geq 0$$

where R is the *annual* return that the investor wants to achieve, with $R = 0.07$.

3. Portfolio Optimization: Maximizing Returns (or Utility)

Another approach to portfolio optimization is to optimize the expected returns that the portfolio would give while penalizing for volatility (variance). Specifically, what is the allocation $x = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ that solve the following optimization problem

$$\text{Maximize } \mu^T x - \gamma \sqrt{x^T \Sigma x}$$

Subject to

$$\sum_{i=1}^{6} x_i = 1$$
$$x \geq 0$$

where γ is the risk aversion coefficient of the investor. Report the optimal allocations for $\gamma = 0.1$, $\gamma = 0.15$ and $\gamma = 0.2$, as well as the value of the objective function.

4. Simulations

Based on the optimal allocations x^* obtained in the two questions above, μ and Σ, please simulate the value of your portfolio after 300 trading days assuming that you have invested $100,000.

- Report the mean and the variance of the portfolio in a table
- Provide visualizations of your results.

5. Recommendations

a. Explain the different allocations, i.e., why they are different.

b. What should an investor with risk aversion $\gamma = 0.1$ do? What about investors with $\gamma = 0.15$ and $\gamma = 0.2$?

Part B: Portfolio Optimization based on the last 400 trading days.

Redo Part A taking into account the last 400 trading days only.

1. Preparing the data

E. Based on the data set, compute the rate of returns for each stock, i.e.,

$$r_{it} = \frac{I_{i,t} - I_{i,t-1}}{I_{i,t-1}}$$

where $I_{i,t}$ is the raw data for stock i at time t.

F. Based on the data set, compute the average returns that will be used for the optimization, i.e.,

This is the mistake

$$\boldsymbol{\mu_i = \left(\sum_{t=1}^{T}(1 + r_{it})\right)^{\frac{1}{T}} - 1}$$

Below is the correct formula

$$\boldsymbol{\mu_i = \left(\prod_{t=1}^{T}(1 + r_{it})\right)^{\frac{1}{T}} - 1}$$

Hint: the term $\boldsymbol{\left(\prod_{t=1}^{T}(1 + r_{it})\right)^{\frac{1}{T}}}$ is a geometric mean

G. Based on the data set, compute the variance-covariance matrix of the stock returns, i.e.,

$$CoVar(r_i, r_j) = \frac{1}{T}\sum_{t=1}^{T}(r_{it} - \overline{r_{it}})(r_{jt} - \overline{r_{jt}})$$

H. Provide the vector μ and the variance-covariance matrix Σ for the 6 assets considered.

2. Portfolio Optimization: Minimizing Risk

What is the allocation $x = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ that solve the following optimization problem

$$\text{Minimize } \frac{1}{2} x^T \Sigma x$$

Subject to

$$\mu^T x \geq R$$
$$\sum_{i=1}^{6} x_i = 1$$
$$x \geq 0$$

where R is the *annual* return that the investor wants to achieve, with $R = 0.07$.

3. Portfolio Optimization: Maximizing Returns (or Utility)

Another approach to portfolio optimization is to optimize the expected returns that the portfolio would give while penalizing for volatility (variance). Specifically, what is the allocation $x = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ that solve the following optimization problem

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4. Simulations

Based on the optimal allocations x^* obtained in the two questions above, μ and Σ, please simulate the value of your portfolio after 300 trading days assuming that you have invested $100,000.

- Report the mean and the variance of the portfolio in a table
- Provide visualizations of your results.

5. Explanations

a. Explain the different allocations, i.e., why they are different.
b. What should an investor with risk aversion $\gamma = 0.1$ do? What about investors with $\gamma = 0.15$ and $\gamma = 0.2$?
c. **Why are the allocations different?**