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AE 142-02

Professor Hunter

June 5, 2023

AE 142 Project 1 Final Report: Exploring Patched Conic Interplanetary Earth-Jupiter Trajectories

Background and Inspiration:

The main source of inspiration for this project is the ongoing Europa Clipper mission. The Europa Clipper mission, a currently ongoing NASA flagship mission aimed to launch in October 2024 with the SpaceX Falcon Heavy rocket out of Kennedy Space Center Launch Complex 39C and follow a Mars-Earth Gravity Assist (MEGA) trajectory, is one of the most exciting interplanetary missions of this decade. As one of the most promising places for habitable environments beyond Earth, its objectives are to increase the understanding of Europa's surface and potential habitability, as well as to gain valuable insight for landing site selections for future Europa missions. The spacecraft will do this by performing dozens of close flybys of Jupiter's moon Europa, gathering detailed measurements to investigate whether the moon could have conditions suitable for life. Europa Clipper is not a life detection mission – its main science goal is to determine whether there are places below Europa's surface that could support life. It is also the largest spacecraft NASA has ever developed for a planetary mission, mainly a product of its massive solar arrays and radar antennas.

In terms of the planned interplanetary trajectory, the original plan for this mission was to inject the spacecraft into Europa's orbit. However, due to the harsh radiation effects from Jupiter's magnetosphere in Europa's orbit, limiting scientific return and spacecraft operational time. It is worth noting that had this plan went through, the planned operational time would only be about 30 days long. The ultimate solution was to inject the spacecraft into an elliptical orbit around Jupiter and make 44-45 close flybys of Europa instead, some as low as 25 km, effectively making use of the best of both worlds by soaring over a different location during each flyby to scan nearly the entire moon while diminishing prolonged durations that would damage the spacecraft. The actual interplanetary trajectory involves a Mars gravity assist in February 2025, followed by an Earth gravity assist in December 2026 before arriving to Jupiter's system in April 2030. Therefore, the total mission time consists of a 5.5 year interplanetary trajectory to the Jovian system, starting with a 21 day launch window from October 10th to the 30th and followed by a ~4 year science period. Some actual parameters that orbital planners for this mission considered when taking into account radiational effects included Europa's orbital resonance, since the radiation environment varies with the moon's position relative to Jupiter's magnetic field, the spacecraft's shielding thickness and material, and, of course, classic trajectory parameters such as altitude and inclination.

What's interesting to note is that the selected trajectory was not actually the shortest duration possible that was considered by mission planners. In fact, there was one option involving the use of a NASA Space Launch System (SLS) rocket that would have entailed a direct trajectory to Jupiter taking less than three years. However, the SpaceX Falcon Heavy rocket was ultimately chosen due to the much cheaper launch costs and the lack of guaranteed availability for the SLS rockets. Additionally, solid rocket boosters are a key component of the SLS rockets, while the SpaceX Falcon Heavy rocket does not use such boosters. As such, looking to reduce the risk of vibrational damage on the sensitive and expensive scientific payload, the Falcon Heavy was ultimately the prime contender for the Europa mission.

For this project, the original objective was essentially a trajectory optimization problem involving a direct interplanetary trajectory for the Europa Clipper spacecraft in minimum, using the current launch date of October 10, 2024 and SpaceX Falcon Heavy rocket. To do so, a patched conic trajectory was originally planned consisting of a gravity assist maneuver using Earth to slingshot to Jupiter, reducing fuel and significantly boost the speed of the spacecraft, and then use a Hohmann transfer from Jupiter to Europa for more precise/efficient maneuvers. However, once I started to actually solve and simulate the problem, this proved out to be an extremely difficult project objective with far too many unknowns, especially when coupled with the time constraints of this project. In fact, upon further research into similar problems and trajectory analysis research papers for mission development, I soon realized that these types of problems are not only often the focus of entire masters theses, but also are the exact reason astrodynamics engineers are paid six figures to do their jobs.

The original trajectory idea was also fundamentally inaccurate; the gravity assist maneuver, also known as a planetary flyby or slingshot maneuver, is essentially an application of conservation of energy which takes advantage of a planet's gravitational field to increase or decrease the velocity and alter a spacecraft's trajectory. Since it always results in a change in direction, my original idea of establishing an orbit around Earth that is gradually more and more elliptical, pointing towards Jupiter, and then slingshotting directly towards Jupiter is not probable. While carefully planned flybys around a planet or moon, such as in the case of the MEGA trajectory in the actual Europa Clipper mission, are incredibly beneficial in conserving fuel and enabling exploration of distant destinations in our solar system and beyond through the resulting change in velocity that happens when under the influence of the planetary body, this requires more detailed analysis, visualization, and optimization of existing planetary ephemeris data to understand which gravity assist options are available during a particular mission period. Additionally, a gravity assist maneuver cannot be used in lieu of any other maneuvers or propagations, they can simply be used to provide that extra boost of speed in an existing trajectory.

Additionally, my original trajectory idea of going from a parking orbit around Jupiter to a final orbit around Europa using a two-body Hohmann transfer is also fundamentally inaccurate. A Hohmann transfer maneuver is often a reasonable assumption in interplanetary travel due to the majority of the travel being spent in the Sun's sphere of influence, however, in this case, due to the Jupiter-Europa system being well within Jupiter's sphere of influence, this in combination with the large gravitational effects of Jupiter within its system resembles the Earth-Moon trajectory situation, in which a Planar Circular/Elliptical Restricted Three Body Problem (PCR3BP or PER3BP) would be a more realistic model for the Jupiter-Europa transit. Ultimately, I decided to further simplify the problem for the purposes of this project, focusing on just the transit from Earth to Jupiter, and use a patched conic Hohmann elliptical transfer orbit for the interplanetary trajectory. While it is an idealized case scenario that requires the alignment of the planets to align perfectly, the assumptions made are reasonable enough for preliminary calculations, and the skills built upon in this project can be adapted into more complicated orbital mechanics applications.

Problem Statement:

The objective of this project is to design a direct interplanetary trajectory for the Europa Clipper spacecraft from Earth to Jupiter using the current launch date of October 10, 2024, and rocket provider, the SpaceX Falcon Heavy. The problem starts at the circular orbit around Earth, already on its ecliptic plane. To do so, we will use a patched conic trajectory consisting of a circular orbit around Earth at an altitude of 200 km, followed by a heliocentric Hohmann transfer ellipse upon departure from Earth's sphere of influence, and ends at at circular orbit around Jupiter at an altitude of 1271889 km (twice the orbital altitude of Europa around Jupiter). We will also explore the use of GMAT and Matlab to attempt simulating the results of such a trajectory, and compare hand calculated results to that we have simulated.

Assumptions:

1) All orbits and trajectories lie within the ecliptic plane
2) Idealized, circular orbits
3) Nested Two-Body Problem
    a) Earth-Spacecraft
    b) Spacecraft-Sun
    c) Jupiter-Spacecraft
4) Constant accelerations and gravitational pulls
5) No atmospheric drag on spacecraft
6) Spacecraft is treated as point mass
    a) Actual total mass at start of trajectory: 2968 kg, but mass can be assumed to be negligible
7) Impulsive burns are assumed to be instantaneous
    a) Impulsive burns can be assumed to be instantaneous if its a small correction burn (ie. the duration of each thrust is much smaller compared to the trajectory's timescale)
    b) In real life, must use finite burns/propulsive burns to accurately model interplanetary trajectories
8) Simplifying gravity assist design: target planet is assumed to have no angular velocity during encounter
    a) Reasonable for preliminary design
    b) Assuming planet's orientation is constant so it has no angular velocity with respect to the Sun frame

Variables and Constants:

1) Gravitational Constant: $G = 6.67 \times 10^{-20}\ km^3/kg*s^2$
2) Mass, Radius, altitudes, and semi-major axes of the Sun, Earth, Jupiter
    a) $M_{Earth} = 5.972 \times 10^{24}$ kg
    b) $R_{Earth} = 6378$ km
    c) $a_{Earth} = 149.6 \times 10^6$ km
    d) alt of circular orbit around Earth = 200 km
    e) $M_{Jupiter} = 1.898 \times 10^{27}$ kg
    f) $R_{Jupiter} = 69911$ km
    g) $a_{Jupiter} = 778.6 \times 10^6$ km
    h) alt of circular orbit around Jupiter = 1271889 km
    i) $M_{Sun} = 1.989 \times 10^{30}$ kg
    j) $R_{Sun} = 695700$ km
3) Ephemerides of Earth and Jupiter (position of celestial bodies with respect to time)
    a) Position of celestial bodies with respect to time
    b) Ephemeris data for Earth about heliocentric orbit (JPL Horizons)
        i)

ome / Tools / Horizons System

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*******************************************************************************
 Revised: April 12, 2021                 Earth                              399

 GEOPHYSICAL PROPERTIES (revised May 9, 2022):
  Vol. Mean Radius (km)    = 6371.01+-0.02   Mass x10^24 (kg)= 5.97219+-0.0006
  Equ. radius, km          = 6378.137        Mass layers:
  Polar axis, km           = 6356.752          Atmos         = 5.1   x 10^18 kg
  Flattening               = 1/298.257223563   oceans        = 1.4   x 10^21 kg
  Density, g/cm^3          = 5.51              crust         = 2.6   x 10^22 kg
  J2 (IERS 2010)           = 0.00108262545     mantle        = 4.043 x 10^24 kg
  g_p, m/s^2  (polar)      = 9.8321863685      outer core    = 1.835 x 10^24 kg
  g_e, m/s^2  (equatorial) = 9.7803267715      inner core    = 9.675 x 10^22 kg
  g_o, m/s^2               = 9.82022         Fluid core rad  = 3480 km
  GM, km^3/s^2             = 398600.435436   Inner core rad  = 1215 km
  GM 1-sigma, km^3/s^2     =      0.0014     Escape velocity = 11.186 km/s
  Rot. Rate (rad/s)        = 0.00007292115   Surface area:
  Mean sidereal day, hr    = 23.9344695944     land          = 1.48 x 10^8 km
  Mean solar day 2000.0, s = 86400.002         sea           = 3.62 x 10^8 km
  Mean solar day 1820.0, s = 86400.0         Love no., k2    = 0.299
  Moment of inertia        = 0.3308          Atm. pressure   = 1.0 bar
  Mean surface temp (Ts), K= 287.6           Volume, km^3    = 1.08321 x 10^12
  Mean effect. temp (Te), K= 255             Magnetic moment = 0.61 gauss Rp^3
  Geometric albedo         = 0.367           Vis. mag. V(1,0)= -3.86
  Solar Constant (W/m^2)   = 1367.6 (mean), 1414 (perihelion), 1322 (aphelion)
 HELIOCENTRIC ORBIT CHARACTERISTICS:
  Obliquity to orbit, deg  = 23.4392911  Sidereal orb period  = 1.0000174 y
  Orbital speed, km/s      = 29.79       Sidereal orb period  = 365.25636 d
  Mean daily motion, deg/d = 0.9856474   Hill's sphere radius = 234.9
*******************************************************************************
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Home / Tools / Horizons System

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Ephemeris / WWW_USER Thu Jun  1 15:06:28 2023 Pasadena, USA      / Horizons
*******************************************************************************
Target body name: Earth (399)                     {source: DE441}
Center body name: Sun (10)                        {source: DE441}
Center-site name: BODY CENTER
*******************************************************************************
Start time      : A.D. 2024-Oct-10 00:00:00.0000 TDB
Stop  time      : A.D. 2029-Oct-10 00:00:00.0000 TDB
Step-size       : 1440 minutes
*******************************************************************************
Center geodetic : 0.0, 0.0, 0.0                   {E-lon(deg),Lat(deg),Alt(km)}
Center cylindric: 0.0, 0.0, 0.0                   {E-lon(deg),Dxy(km),Dz(km)}
Center radii    : 696000.0, 696000.0, 696000.0 km {Equator_a, b, pole_c}
Keplerian GM    : 1.3271283864171489E+11 km^3/s^2
Output units    : KM-S, deg, Julian Day Number (Tp)
Calendar mode   : Mixed Julian/Gregorian
Output type     : GEOMETRIC osculating elements
Output format   : 10
Reference frame : Ecliptic of J2000.0
*******************************************************************************
JDTDB
   EC    QR   IN
   OM    W    Tp
   N     MA   TA
   A     AD   PR
*******************************************************************************
$$SOE
2460593.500000000 = A.D. 2024-Oct-10 00:00:00.0000 TDB
 EC= 1.709540723578720E-02 QR= 1.470564042964451E+08 IN= 3.347399350545583E-03
 OM= 1.780274471404182E+02 W = 2.847932613204633E+02 Tp=  2460678.768266428728
 N = 1.140565508619520E-05 MA= 2.759725062703836E+02 TA= 2.740201869510648E+02
 A = 1.496141185818247E+08 AD= 1.521718328672043E+08 PR= 3.156329007666775E+07
2460594.500000000 = A.D. 2024-Oct-11 00:00:00.0000 TDB
 EC= 1.708568664225113E-02 QR= 1.470299494397347E+08 IN= 3.822851020687870E-03
 OM= 1.805059558412661E+02 W = 2.829499581844326E+02 Tp=  2460679.387771653477
```

c) Ephemeris data for Jupiter about heliocentric orbit (JPL Horizons)

i)

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*******************************************************************************
 Revised: April 12, 2021               Jupiter                              599

 PHYSICAL DATA:
  Mass x 10^22 (g)      = 189818722 +- 8817 Density (g/cm^3)  = 1.3262 +- .0003
  Equat. radius (1 bar) = 71492+-4 km       Polar radius (km)     = 66854+-10
  Vol. Mean Radius (km) = 69911+-6          Flattening            = 0.06487
  Geometric Albedo      = 0.52              Rocky core mass (Mc/M)= 0.0261
  Sid. rot. period (III)= 9h 55m 29.71 s    Sid. rot. rate (rad/s)= 0.00017585
  Mean solar day, hrs   = ~9.9259
  GM (km^3/s^2)         = 126686531.900     GM 1-sigma (km^3/s^2) =  +- 1.2732
  Equ. grav, ge (m/s^2) = 24.79             Pol. grav, gp (m/s^2) =  28.34
  Vis. magnitude V(1,0) = -9.40
  Vis. mag. (opposition)= -2.70             Obliquity to orbit    =  3.13 deg
  Sidereal orbit period = 11.861982204 y    Sidereal orbit period = 4332.589 d
  Mean daily motion     = 0.0831294 deg/d   Mean orbit speed, km/s= 13.0697
  Atmos. temp. (1 bar)  = 165+-5 K          Escape speed, km/s    =  59.5
  A_roche(ice)/Rp       =  2.76             Hill's sphere rad. Rp = 740
                               Perihelion   Aphelion    Mean
  Solar Constant (W/m^2)       56           46          51
  Maximum Planetary IR (W/m^2) 13.7         13.4        13.6
  Minimum Planetary IR (W/m^2) 13.7         13.4        13.6
*******************************************************************************
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me / Tools / Horizons System

```
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Ephemeris / WWW_USER Thu Jun  1 16:09:13 2023 Pasadena, USA      / Horizons
*******************************************************************************
Target body name: Jupiter (599)                   {source: jup365_merged}
Center body name: Sun (10)                        {source: jup365_merged}
Center-site name: BODY CENTER
*******************************************************************************
Start time      : A.D. 2024-Oct-10 00:00:00.0000 TDB
Stop  time      : A.D. 2029-Oct-10 00:00:00.0000 TDB
Step-size       : 1440 minutes
*******************************************************************************
Center geodetic : 0.0, 0.0, 0.0                   {E-lon(deg),Lat(deg),Alt(km)}
Center cylindric: 0.0, 0.0, 0.0                   {E-lon(deg),Dxy(km),Dz(km)}
Center radii    : 696000.0, 696000.0, 696000.0 km {Equator_a, b, pole_c}
Keplerian GM    : 1.3283912657317976E+11 km^3/s^2
Output units    : KM-S, deg, Julian Day Number (Tp)
Calendar mode   : Mixed Julian/Gregorian
Output type     : GEOMETRIC osculating elements
Output format   : 10
Reference frame : Ecliptic of J2000.0
*******************************************************************************
JDTDB
   EC     QR   IN
   OM     W    Tp
   N      MA   TA
   A      AD   PR
*******************************************************************************
$$SOE
2460593.500000000 = A.D. 2024-Oct-10 00:00:00.0000 TDB
 EC= 4.840177729278788E-02 QR= 7.406318102371855E+08 IN= 1.303745031674669E+00
 OM= 1.005137799490223E+02 W = 2.734805213084002E+02 Tp=  2459965.240108585451
 N = 9.617507414493423E-07 MA= 5.220542157615656E+01 TA= 5.675198020903498E+01
 A = 7.783030616956744E+08 AD= 8.159743131541634E+08 PR= 3.743173615416049E+08
2460594.500000000 = A.D. 2024-Oct-11 00:00:00.0000 TDB
 EC= 4.832144898039221E-02 QR= 7.405880244093616E+08 IN= 1.303822535666742E+00
 OM= 1.005118446324126E+02 W = 2.733475453372224E+02 Tp=  2459963.751103654504
 N = 9.619578265379686E-07 MA= 5.242353155308079E+01 TA= 5.697475506050949E+01
```

4) Sphere of Influence (SOI) of Earth, Jupiter, and Europa
    a) $SOI_{Earth} = 924540$ km
    b) $SOI_{Jupiter} = 4.8196 \times 10^7$ km

General Calculation Process:

To calculate such a patched conic trajectory for the Europa Clipper mission, we can think of it as a methodology in three steps. The first step is to gather all of the necessary information required for the patched conic trajectory, including orbital data, mass and radii of the planetary bodies, and Keplerian angles, which can be obtained from web searches and online databases such as the NASA JPL Horizon. Since we are traveling to an outer planet, this will be a front door approach, as opposed to a back door approach with the case of interplanetary travel to inner planets.

Once we have all of our data and we draw out a diagram of the problem, we can actually solve our patched conic trajectory. We will be using a simple three-segment mission using an interplanetary Hohmann transfer from Earth to Jupiter, which include three phases - departure, transit, and rendezvous/arrival. In the departure phase, the spacecraft departs on a hyperbolic departure trajectory from Earth after being in a circular orbit around Earth, already situated on the ecliptic plane for simplification purposes, and in this initial phase of escaping Earth's orbit, the two bodies of interest are the Europa Clipper and Earth. Once the spacecraft leaves Earth's SOI, it travels on a heliocentric ellipse, hence the two bodies of interest are the spacecraft and the Sun. Finally, once the spacecraft enters

Jupiter's SOI, it will be on a hyperbolic arrival trajectory with Jupiter, and the two bodies of interest are the spacecraft and Jupiter. Since we want to transfer into a circular orbit around Jupiter, a Δv burn is necessary to prevent the spacecraft from performing a flyby of Jupiter instead.

After defining our perifocal basis vectors ($\mathbf{s}_x$, $\mathbf{s}_y$, $\mathbf{s}_z$) in the Sun-centered Newtonian reference frame, S, particle Q as the spacecraft, P_E as Earth and P_J as Jupiter, we can solve the transit phase first to obtain our hyperbolic excess speed v_∞ needed to design the departure and rendezvous hyperbolas. The v_∞ for the departure hyperbola is essentially the difference between the velocity of the spacecraft, Q, with respect to the Sun at the perihelion of the transfer ellipse, and the heliocentric orbit speed of Earth along the $\mathbf{s}_y$ direction. Likewise, the v_∞ for the arrival hyperbola is the difference between the velocity of the spacecraft, Q, with respect to the Sun at the apohelion of the transfer ellipse, and the heliocentric orbit speed of Jupiter along the $\mathbf{s}_y$ direction.

Next, we can calculate the departure phases. Using calculations, we can determine the Δv required to transfer from the hyperbolic departure trajectory following the parking orbit around Earth to the heliocentric ellipse, with the hand-off point being the SOI boundary. Defining r_p as the periapsis radius of the departure hyperbola (aka the radius of the initial parking orbit around Earth), we can use variations of the orbit equation and a few more equations expanded upon in the hand-calculations to solve for angular momentum, h, eccentricity, e, velocity at periapsis of the departure hyperbola, v_p, aiming radius, Δ, angle between the departure hyperbola line of apsides and its asymptote (parallel to the velocity vector of Earth), β, and the Δv necessary to transfer the spacecraft from the initial circular orbit about Earth to the departure hyperbola.

To calculate the rendezvous or arrival phase, the process is similar to the departure phase. The Europa Clipper will be approaching Jupiter in the $+\mathbf{s}_y$ direction since at the instant of rendezvous, both the spacecraft and Jupiter are moving in the $-\mathbf{s}_y$ direction, however, because Jupiter moves faster on the large circular orbit and overtakes the spacecraft, the relative motion of the spacecraft with respect to Jupiter will be in the $+\mathbf{s}_y$ direction. Since we want to place the spacecraft in final orbit about Jupiter, we need to determine the Δv required to transfer from the hyperbolic departure trajectory following the parking orbit around Earth to the heliocentric ellipse, with the hand-off point being the SOI boundary. Again, we can use the periapsis radius of the departure (or arrival) hyperbola, variations of the orbit equation, and the same equations used in the departure phase to solve for angular momentum, h, eccentricity, e, velocity at periapsis of the departure hyperbola, v_p, aiming radius, Δ, angle between the departure hyperbola line of apsides and its asymptote (parallel to the velocity vector of Earth), β, and the Δv necessary to transfer the spacecraft from the arrival hyperbola to the final circular orbit about Jupiter.

Calculations

The following images show my process as I calculated the relevant values that are required to solve this interplanetary patched conic trajectory.

Project 1 Calcs

↳ Knowns:

Departure Date/Time:

October 10, 2024

12:00:00 UT

$R_E = 6578\,km$ (alt: 200 km)

$R_J = 1411711\,km$ (alt: 1271889 km)

↳ Orbital Radius of Europa around Jupiter: 670900 km

• 2(670900) = 1411711 km

↳ SOI's

$$r_{SOI} = r_p \left(\frac{M_{planet}}{M_{sun}}\right)^{2/5}$$

$$r_{SOI,Earth} = a_E \left(\frac{M_E}{M_S}\right)^{2/5}$$

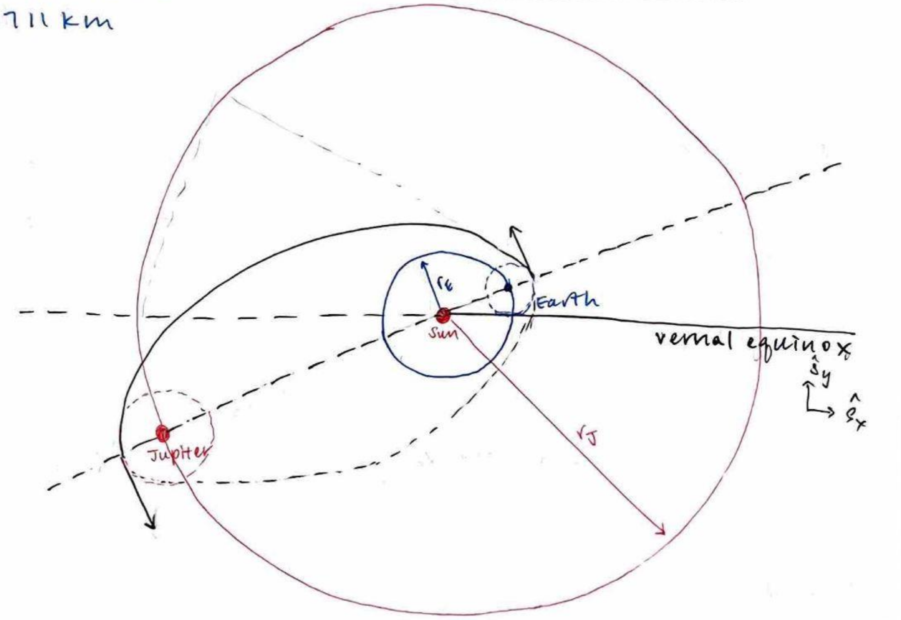
$$= (149.6\times10^6)\left(\frac{5.972\times10^{24}}{1.989\times10^{30}}\right)^{2/5}$$

$$r_{SOI,E} = 924540\ km$$

$$r_{SOI,Jupiter} = a_J \left(\frac{M_J}{M_S}\right)^{2/5}$$

$$= (778.299\times10^6)\left(\frac{1.898\times10^{27}}{1.989\times10^{30}}\right)^{2/5}$$

$$r_{SOI,J} \approx 48196000\ km$$



Direct Interplanetary Patched Conic Trajectory: Earth → Jupiter

↳ Using Hohmann Transfer Heliocentric Ellipse

↳ Three Phases: 1) Departure 2) Transit 3) Rendezvous / Arrival

- Starting w/ Phase 2: Transit *front door approach

*gives us hyperbolic excess speed, v_∞, needed to design the departure & rendezvous hyperbolas

Earth → Jupiter ← outer planet trajectory (dark side) → away from sun

Define: · Perifocal reference frame $(\hat{s}_x, \hat{s}_y, \hat{s}_z)$ in sun centered reference frame S (Newtonian frame)

· Particle Q → Europa clipper

· P_E = Earth, P_J = Jupiter

$${}^{S}\vec{v}^{Earth} = \sqrt{\frac{GM_S}{r_E}} = \sqrt{\frac{1.327\times10^{11}}{149.6\times10^{6}}}\,\hat{a}_x$$

$${}^{S}\vec{v}^{Earth} = 29.7831\ km/s$$

$${}^{S}\vec{v}^{Jup.} = -\sqrt{\frac{GM_S}{r_J}} = -\sqrt{\frac{1.327\times10^{11}}{778.6\times10^{6}}}\,\hat{a}_x$$

$${}^{S}\vec{v}^{Jupiter} = -13.055\ km/s$$

$v_{Pt} < v_{Jupiter}$: $\Delta v_2 = v_J - v_E > 0$

$${}^{Earth}\vec{v}^{Q} = \Delta v_1\,\hat{s}_y$$

$$= \sqrt{\frac{GM_S}{r_E}}\left\{\sqrt{\frac{2r_J}{r_E+r_J}} - 1\right\}\hat{s}_y$$

$$= \sqrt{\frac{1.327\times10^{11}}{149.6\times10^{6}}}\left\{\sqrt{\frac{2(778.6\times10^{6})}{(149.6\times10^{6}+778.6\times10^{6})}} - 1\right\}\hat{s}_y$$

$${}^{Earth}\vec{v}^{Q} = 8.80574\ km/s\ \hat{s}_y \rightarrow = \vec{v}_{\infty_{d_1}}$$ ($\vec{v}_\infty$ of departure hyperbola)

Phase 1: Burn from 200 km LEO to departure hyperbola:

*additional departure parameters on next page

$$\Delta v = v_p - v_{circ}$$

$$= \sqrt{v_\infty^2 + \frac{2GM_E}{R_E}} - \sqrt{\frac{GM_E}{R_E}}$$

$$\Delta v = \sqrt{8.80574^2 + \frac{2(6.67\times10^{-20})(5.972\times10^{24})}{6378+200}} - \sqrt{\frac{(6.67\times10^{-20})(5.972\times10^{24})}{6378+200}}$$

$$\Delta v = 6.31265\ km/s$$

Phase 3:

Arrival @ Jupiter from heliocentric ellipse

*additional arrival parameters on last page

$${}^{Jup}\vec{v}^{Q} = \Delta v_2\,\hat{s}_y$$

$$= \sqrt{\frac{GM_S}{r_J}}\left\{1 - \sqrt{\frac{2r_E}{r_E+r_J}}\right\}\hat{s}_y$$

$$= \sqrt{\frac{1.327\times10^{11}}{778.6\times10^{6}}}\left\{1 - \sqrt{\frac{2(149.6\times10^{6})}{(149.6\times10^{6}+778.6\times10^{6})}}\right\}\hat{s}_y$$

$${}^{Jup}\vec{v}^{Q} = 5.64299\ km/s\ \hat{s}_y$$ $\vec{v}_\infty$ of arrival hyperbola ↳ $\vec{v}_{\infty_{d_2}}$

Burn from arrival hyperbola to LJO (Low Jupiter Orbit)

1271889 km circular orbit:

$$\Delta V = V_p - V_{circ}$$

$$= \sqrt{V_\infty^2 + \frac{2GM_J}{R_J}} - \sqrt{\frac{GM_J}{R_J}}$$

$$= \sqrt{(6.31265)^2 + \frac{2(6.67\times10^{-20}\cdot 1.898\times10^{27})}{1271889}} - \sqrt{\frac{(6.67\times10^{-20})(1.898\times10^{27})}{1271889}}$$

$$\Delta V = 5.48029 \text{ km/s}$$

Total ΔV: $\Delta V = 6.31265 \text{ km/s} + 5.48029 \text{ km/s}$

$$\boxed{\Delta V = 11.7929 \text{ km/s}}$$

* only includes actual propulsive burns from
  - ↳ 200 km LEO to departure hyperbola
  - ↳ arrival hyperbola to 1271889 km LJO

Additional Parameters

Earth Departure

$r_p = 6578$ km (periapsis radius of departure hyperbola; radius of initial circular orbit around Earth)

$$r_p = \frac{h^2}{GM_E}\left(\frac{1}{1+e}\right), \quad v_{\infty,1} = \frac{GM_E}{h}\sqrt{e^2-1}, \quad h = \frac{GM_E\sqrt{e^2-1}}{v_{\infty,1}}$$

↳ $$e = 1 + \frac{r_p v_{\infty,1}^2}{GM_E} = 1 + \frac{6578(8.80574)^2}{(6.67\times10^{-20})(5.972\times10^{24})} \rightarrow \boxed{e = 2.2805}$$

$$h = \sqrt{\{r_p GM_{Earth}(1+e)\}} = \sqrt{\{6578(6.67\times10^{-20})(5.972\times10^{24})(1+2.2805)\}}$$

$$\boxed{h = 92712.8 \text{ km}^2/\text{s}}$$

$$\Delta = \frac{h^2}{GM}\frac{1}{\sqrt{e^2-1}} = \frac{(92712.8)^2}{(6.67\times10^{-20})(5.972\times10^{24})}\frac{1}{\sqrt{2.2805^2-1}} \rightarrow \boxed{\Delta = 10528.7 \text{ km}}$$

$$v_p = \frac{h}{r_p} = \frac{92712.8}{6578} \rightarrow \boxed{v_p = 14.0944 \text{ km/s}}$$

* velocity @ periapsis of departure hyperbola

$$\beta = \cos^{-1}\left(\frac{1}{e}\right) = \cos^{-1}\left(\frac{1}{2.2805}\right) \rightarrow \boxed{\beta = 63.9918°}$$

Jupiter Arrival

$$e = 1 + \frac{r_p v_{\infty_2}^2}{GM_J} = 1 + \frac{(1411711)(3.85988)^2}{(6.67\times10^{-20})(1.898\times10^{27})} \rightarrow \boxed{e = 1.16614}$$

$$h = \sqrt{r_p GM_J (1+e)} = \sqrt{(1411711)(6.67\times10^{-20})(1.898\times10^{27})(1+1.16614)}$$

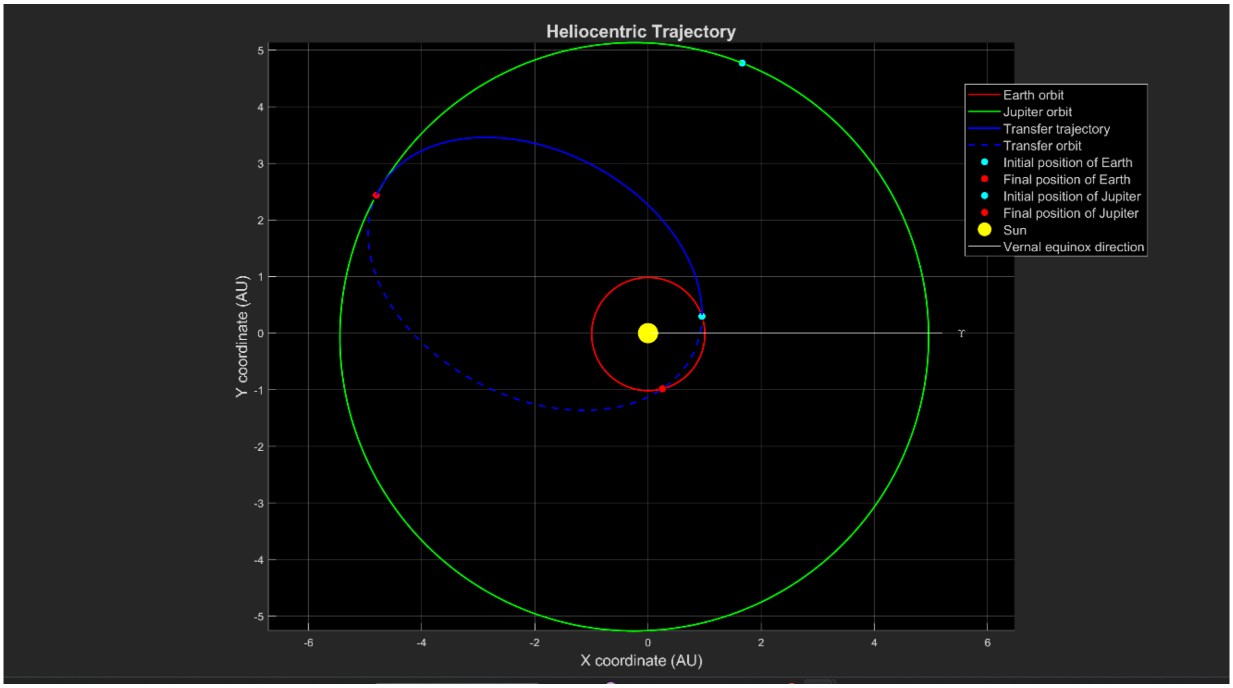
$$\boxed{h = 19675563.6\ km^2/s}$$

$$\Delta = \frac{h^2}{GM_J}\frac{1}{\sqrt{e^2-1}} = \frac{19675563.6^2}{(6.67\times10^{-20})(1.898\times10^{27})}\ \frac{1}{\sqrt{1.16614^2-1}} \rightarrow \boxed{\Delta = 5097438.05 km}$$

$$\boxed{v_{p_2} = \frac{h}{r_p} = 13.9374\ km/s}$$ * velocity @ periapsis of arrival hyperbola

$$\beta = \cos^{-1}\left(\frac{1}{e}\right) = \cos^{-1}\left(\frac{1}{1.16614}\right) \rightarrow \boxed{\beta = 30.9596°}$$

Simulations



Matlab simulation of interplanetary trajectory from Earth to Jupiter

- Uses Kepler's Equations and Lambert's Function to calculate heliocentric orbit trajectories using inputted departure and arrival planets, dates and times in UTC, and orbit altitudes about the planets' parking and arrival orbits.
- Result shown for Earth-Jupiter trajectory with a launch date of Oct 10, 2024 and an arrival time of July 7, 2027

```
  < Interplanetary mission design program >

  < Inputs >

Departure parameters

  Choose the planet of deprarture

  1 - Mercury
  2 - Venus
  3 - Earth
  4 - Mars
  5 - Jupiter
  6 - Saturn
  7 - Uranus
  8 - Neptune
  9 - Pluto
? 3

Input the calendar date for departure
(1 <= day <= 31/ 1 <= month <= 12/ year = four digits)
(Example: 1/12/2005)
? 10/10/2024

Input the universal time for departure
(0 <= hours <= 24: 0 <= minutes <= 60: 0 <= seconds <= 60)
(Example: 12:00:00)
? 12:00:00

Input the altitude of the circular departure parking orbit
? 200

Departure parameters

  Choose the planet of arrival

  1 - Mercury
  2 - Venus
  3 - Earth
  4 - Mars
  5 - Jupiter
  6 - Saturn
  7 - Uranus
  8 - Neptune
  9 - Pluto
? 5
```

```
Input the calendar date for arrival
(1 <= day <= 31/ 1 <= month <= 12/ year = four digits)
(Example: 1/12/2005)
? 7/7/2027

Input the universal time for arrival
(0 <= hours <= 24: 0 <= minutes <= 60: 0 <= seconds <= 60)
(Example: 12:00:00)
? 2:45:30

Input the altitude of the circular capture orbit
? 1271889

deg =

    0.0175


deg =

    0.0175


deg =

    0.0175


deg =

    0.0175


deg =

    0.0175


deg =

    0.0175
```

Matlab simulation: command line prompts and results (part 1).

```
 < Results >

Departure planet           Earth
Departure calendar date    10/10/2024
Departure universal time   12:00:00

Departure julian date      2460594.000000

Arrival planet             Jupiter
Arrival calendar date      7/7/2027
Arrival universal time     2:45:30

Arrival julian date        2461593.614931

Transfer time              999.614931  days


Heliocentric ecliptic orbital elements of the departure planet
---------------------------------------------------------------
       sma (AU)              eccentricity          inclination (deg)        argper (deg)
 +1.00012819740614e+01  +1.67007956616016e-02  -3.18036017948133e-03  +1.15544742844855e+02

      raan (deg)          true anomaly (deg)        longper(deg)            period (days)
 +3.47484911492509e+02  +2.74304956574033e+02  +1.03029654337364e+02  +3.65257008008152e+02


Heliocentric ecliptic orbital elements of the transfer orbit
prior to reaching the sphere of influence of the arrival planet
---------------------------------------------------------------
       sma (AU)              eccentricity          inclination (deg)        argper (deg)
 +3.13185046373035e+01  +7.20311430635355e-01  +1.48112086673230e+00  +3.14902083769929e+02

      raan (deg)          true anomaly (deg)       period (min)
 +1.73958321699015e+01  +4.50366744723666e+01  +2.02402827877955e+03

Heliocentric ecliptic orbital elements of the transfer orbit
prior to reaching the sphere of influence of the arrival planet
---------------------------------------------------------------
       sma (AU)              eccentricity          inclination (deg)        argper (deg)
 +3.13185046373034e+01  +7.20311430635354e-01  +1.48112086673230e+00  +3.14902083769929e+02

      raan (deg)          true anomaly (deg)       period (min)
 +1.73958321699015e+01  +1.80785766628617e+02  +2.02402827877955e+03
```

```
Heliocentric ecliptic orbital elements of the arrival planet
---------------------------------------------------------------
       sma (AU)              eccentricity          inclination (deg)        argper (deg)
 +5.20419667832272e+01  +4.83572250485132e-02  +1.30498285229324e+00  +2.74168870891350e+02

      raan (deg)          true anomaly (deg)        longper(deg)            period (days)
 +1.00649167511864e+02  +1.38282395919873e+02  +1.48180384032141e+01  +4.33556399745302e+03


Departure velocity vector and magnitude

x-component of departure velocity                                 0.899106  km/s
y-component of departure velocity                                38.626096  km/s
z-component of departure velocity                                 0.946094  km/s
departure velocity magnitude                                      0.946094  km/s

Arrival velocity vector and magnitude

x-component of arrival velocity                                  -2.859749  km/s
y-component of arrival velocity                                  -6.159795  km/s
z-component of arrival velocity                                  -0.129878  km/s
departure velocity magnitude                                      6.792504  km/s

Hyperbolic excess velocity vector and magnitude at departure

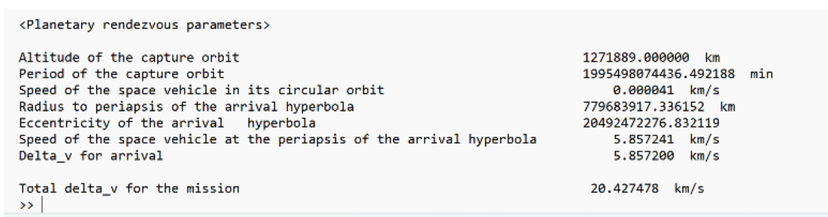
x-component of hyperbolic excess velocity at departure          10.259426  km/s
y-component of hyperbolic excess velocity at departure          10.302399  km/s
z-component of hyperbolic excess velocity at departure           0.947516  km/s
hyperbolic excess velocity at departure magnitude               14.570279  km/s

Hyperbolic excess velocity vector and magnitude at arrival

x-component of hyperbolic excess velocity at arrival             3.214158  km/s
y-component of hyperbolic excess velocity at arrival             4.886603  km/s
z-component of hyperbolic excess velocity at arrival            -0.312361  km/s
hyperbolic excess velocity at arrival magnitude                  5.857241  km/s

<Planetary departure parameters>

Altitude of the parking orbit                                          200.000000  km
Period of the parking orbit                                          27097715436102.296875  min
Speed of the space vehicle in its circular orbit                         0.000001  km/s
Radius to periapsis of the departure hyperbola                       149598087.455766  km
Eccentricity of the departure hyperbola                              635172623831189.250000
Speed of the space vehicle at the periapsis of the departure hyperbola     14.570279  km/s
Delta_v for departure                                                   14.570278  km/s
```

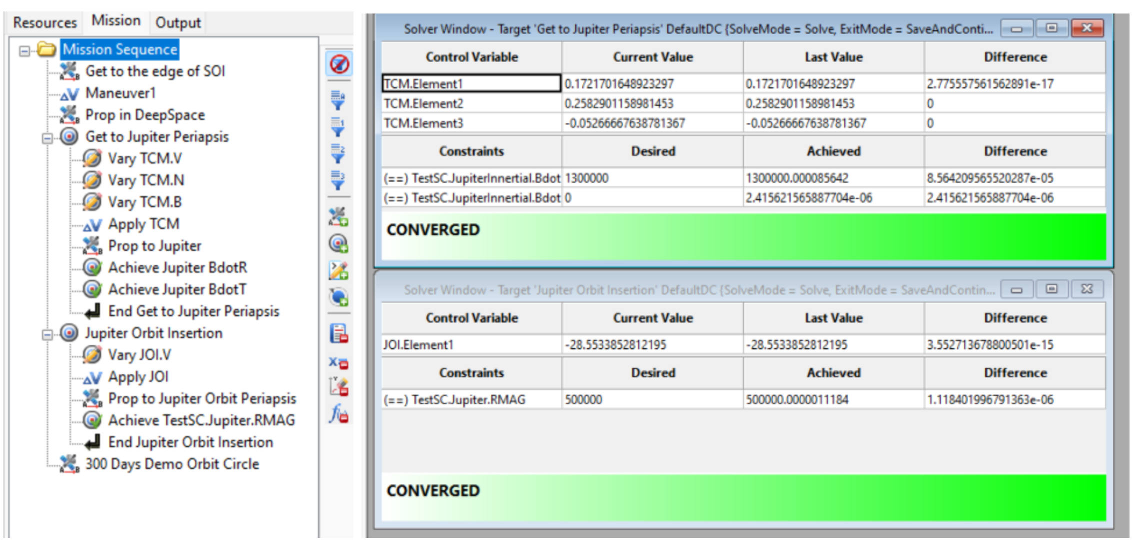


```
<Planetary rendezvous parameters>

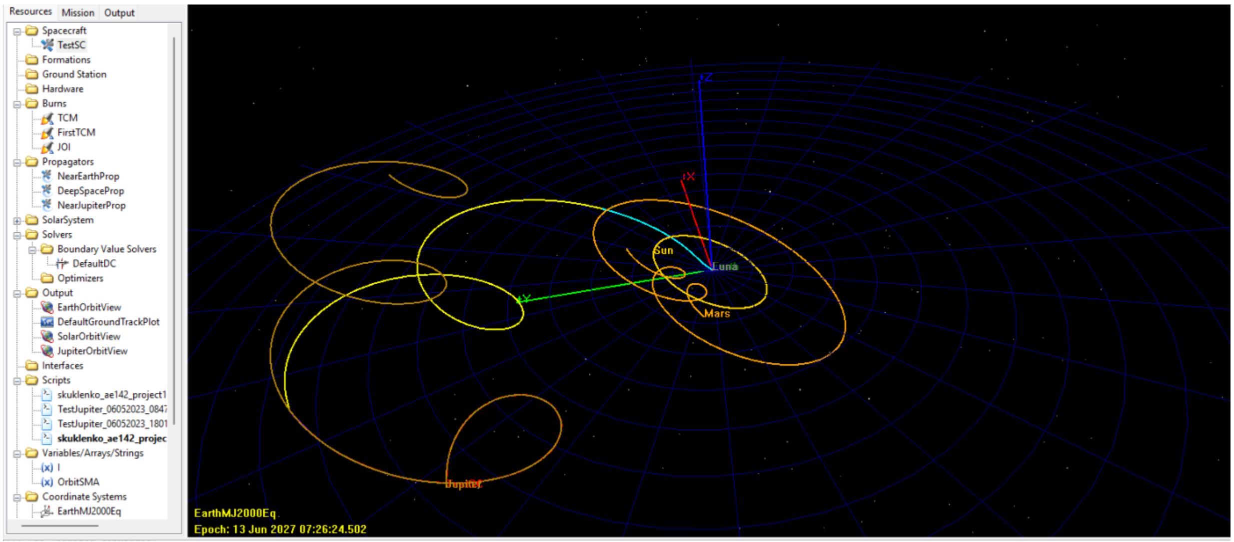
Altitude of the capture orbit                                       1271889.000000  km
Period of the capture orbit                                         1995498074436.492188  min
Speed of the space vehicle in its circular orbit                        0.000041  km/s
Radius to periapsis of the arrival hyperbola                        779683917.336152  km
Eccentricity of the arrival   hyperbola                             20492472276.832119
Speed of the space vehicle at the periapsis of the arrival hyperbola    5.857241  km/s
Delta_v for arrival                                                     5.857200  km/s

Total delta_v for the mission                                        20.427478  km/s
>>
```

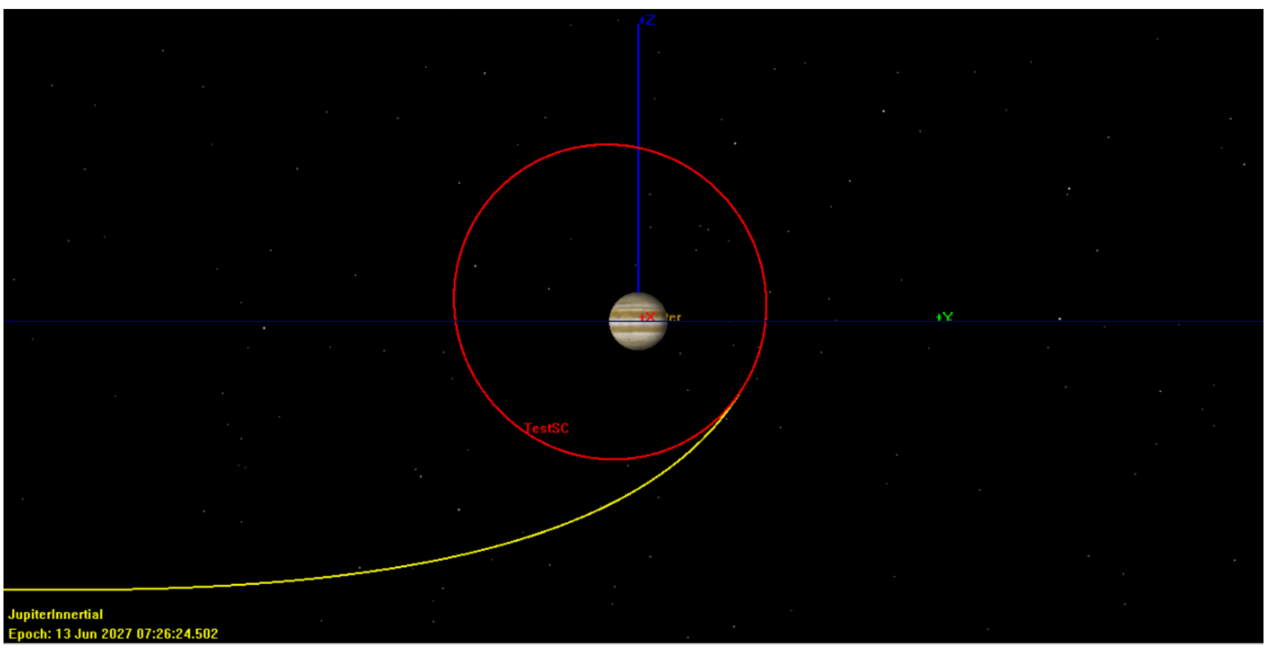
Matlab simulation: command line prompts and results (part 2).



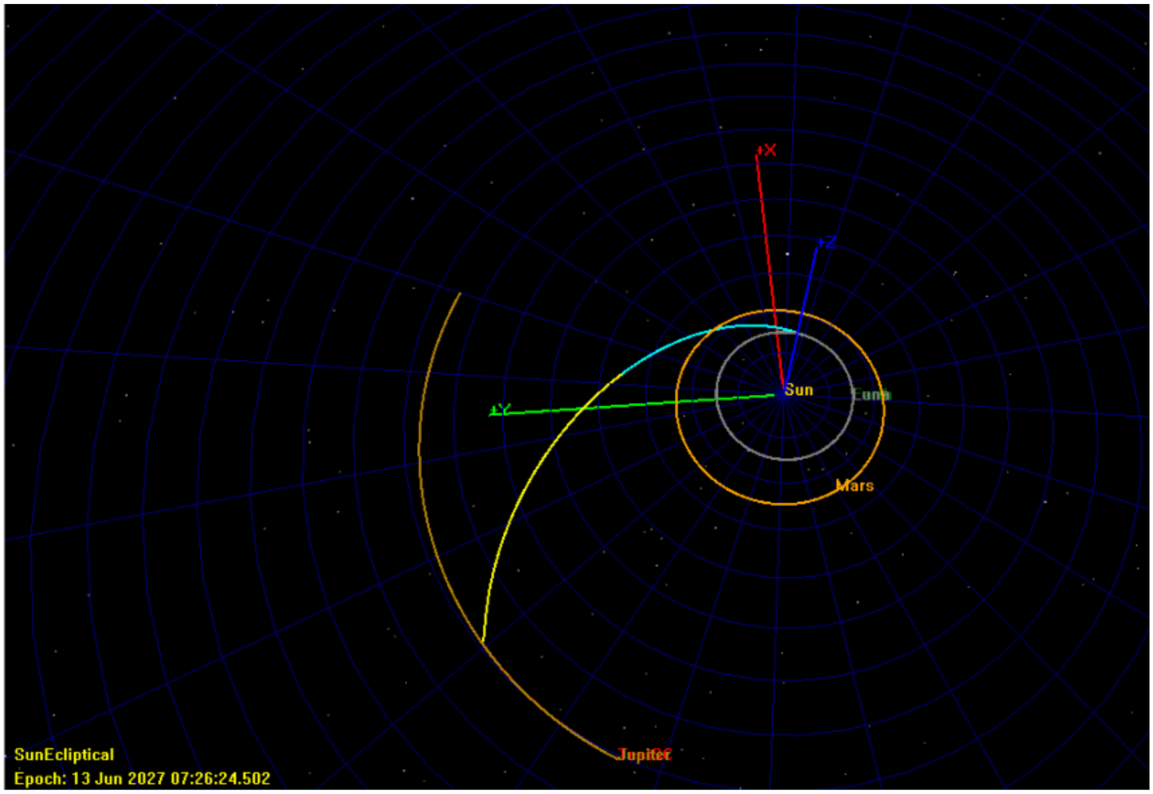
GMAT simulation: mission sequence tree and convergence windows signifying successful runthrough.



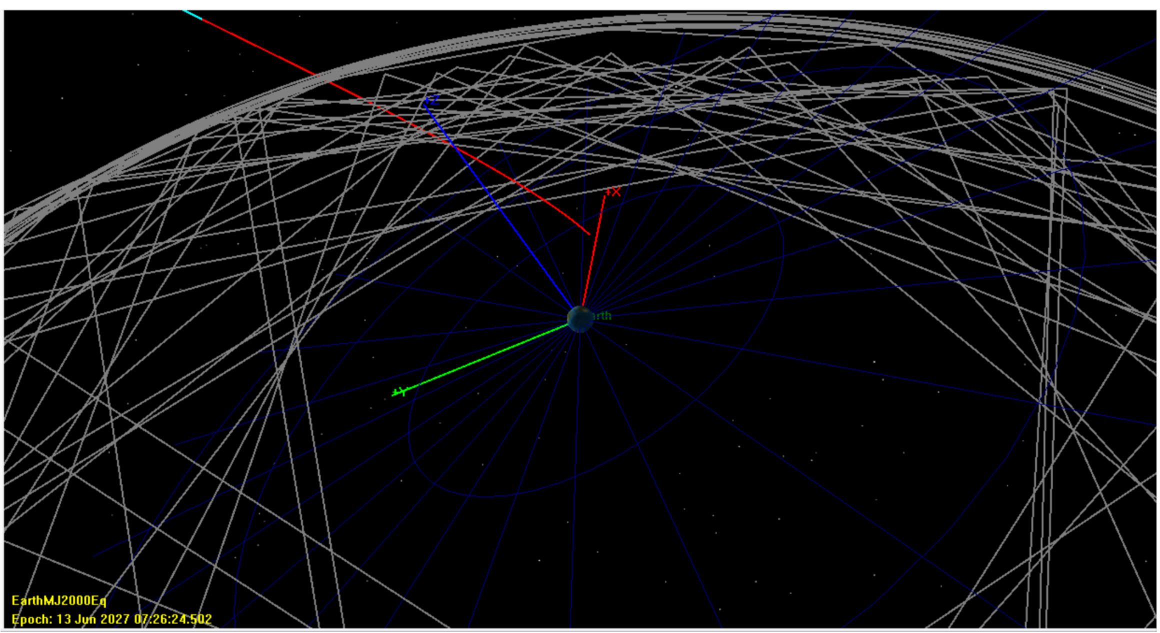
GMAT simulation: resource tree showcasing spacecraft properties, 3 burns (trajectory correction maneuver, final trajectory maneuver, and Jupiter orbit insertion), view outputs, variables, scripts, and more. Earth centered orbit view showcasing the entire mission trajectory from a distance.



GMAT simulation: Jupiter inertial reference frame orbit view showcasing Jupiter orbit insertion.



GMAT simulation: Sun centered elliptical frame orbit view showcasing entire mission trajectory.



GMAT simulation: Earth centered orbit view zoomed in. Crazy white lines are the moon's trajectory around the Earth due to large step sizes.

Results

When I first started solving the problem, one of my biggest hiccups was that I had a difficult time visualizing the problem at hand, especially trying to incorporate accurate planetary ephemeris data obtained from NASA JPL Horizons. While I learned how to look up and read the data results for each of my planetary bodies of interest, I had a very difficult time incorporating these values into accurately modeling the expected interplanetary orbit as expected on the anticipated launch date of October 10, 2024. This was before I was set on changing the focus of the interplanetary trajectory to just the Hohmann transfer ellipse from Earth and Jupiter.

I then decided to try to find online resources to help me visualize the problem, which led me to find very useful Matlab source code online, which takes inputs for the departure and arrival planets of interest (for any planet in the solar system), departure and arrival dates and times in UTC, and orbit altitudes about the planets' parking and arrival orbits, and outputs all necessary orbit parameters including Keplerian elements, velocities, eccentricities, and delta-v burns. When I inputted my required parameters for my problem, and assumed an arrival time of July 7th, 2027, it provided me a more helpful insight into how to set up the problem, and also helped me realize that a Hohmann transfer ellipse was probably the best way to model this idealized case scenario. However, the Matlab code centers on the use of the Lambert function, which is extremely beneficial in determining the trajectory of a spacecraft between two known positions using iterative numerical methods, but due to my practical unfamiliarity with the algorithm, I tried revising the code to incorporate the Hohmann transfer methodology instead. When this was not successful, as well as other attempts to include Europa into the code and calculate the arrival time and location while inputting orbital parameters such as delta-v burns, I decided to transition to GMAT.

GMAT has plenty of tutorials on modeling Hohmann transfer orbits from Earth to Mars, but learning how to adapt the mission sequence to include a patched conic trajectory with a circular Earth orbit, a heliocentric Earth-Jupiter Hohmann transfer ellipse, and a circular Jupiter final orbit was significantly more challenging. However, I was able to get a pretty successful attempt at modeling the interplanetary trajectory by manipulating the spacecraft data, burn sequence, gravity files, and mission sequence that converged! Although in the GMAT, this script does not converge with an initial launch epoch time of October 10, 2024, it was able to converge with many other dates, including August 26, 2024, which gave us a final arrival time of June 13, 2027. Considering that in the actual Europa Clipper mission, the direct trajectory option had a travel duration time of under 3 years, the GMAT program's result is reasonable in modeling the travel duration time for the Earth-Jupiter direct interplanetary trajectory. The GMAT function features three main orbit views (Earth-centered, Jupiter-centered, and Sun-centered), three burns, and three different propagators to model the gravitational spheres of influences for each of the three planetary bodies. Additionally, the mission sequence models the patched conic trajectory methodology, calculating trajectory parameters in Earth's orbit, upon leaving Earth's sphere of influence, deep-space travel, and the final orbit insertion into Jupiter. While I was learning how to build the trajectory in GMAT, I was able to actually gain a better understanding of the problem at hand and solving it on paper became much more straightforward as well*.

While neither the simulations nor the hand calculations are 100% accurate, they do illustrate the different ways we can model and determine interplanetary orbital trajectories very well, especially the fundamental concept of patched conic trajectory design.

<u>Conclusion</u>

To summarize, this report details the problem of the design of a direct interplanetary trajectory for the Europa Clipper spacecraft using a patched conic trajectory consisting of a circular parking orbit about Earth, followed by a heliocentric Hohmann transfer from Earth to Jupiter, concluded with a final orbit insertion about Jupiter. While the actual Europa Clipper mission will make use of a Mars-Earth Gravity Assist (MEGA) trajectory, this simplified trajectory optimization problem demonstrates the key considerations and methods for interplanetary trajectory planning. By taking into account variables such as gravitational pulls, Keplerian angles, and ephemerides, we can calculate the necessary launch velocity and trajectory adjustments to reach Jupiter. With the use of simplified models and assumptions, we can gain a deeper understanding of the interplanetary planning process and contribute to the ongoing field of space exploration. Through the unfortunate process of underestimating the complexity of a mission objective and working to simplify the problem down, I was able to learn about all the various parameters orbital trajectory designers actually keep in mind when optimizing trajectories for space missions, particularly interplanetary ones. In the case of the Europa Clipper mission, while it would be quicker to put the spacecraft into a direct gravity assist from Earth to Jupiter, numerous factors from the launch vehicle to the total monetary cost to the powerful magnetosphere's effect on the sensitive payload when in Jupiter's vicinities played a factor in determining the final trajectory of the spacecraft.

*** NOTE:**

- **When opening GMAT on a new computer, there is an issue in the script on line 105 that links the .cof gravity file for Jupiter's sphere of influence through my local folders on my computer. I'm not sure how to fix this, but if you go into "NearJupiterProp" and reselect the same gravity file (in tutorials the Earth gravity model JGM3 was recommended for Jupiter), the issue should be fixed and the program should run as expected.**