Advanced Image Processing: Assignment 3 (Due Mar 10, 2017)

Note: Please provide detailed comments for code that may be written to solve the following problems. The assignment will be evaluated not just based on the final results but also how you obtained them. Late submissions will be penalized.

Problem 1: JPEG Implementation
Implement a toy version of JPEG through the following steps:

1. Transform: Compute an 8x8 discrete cosine transform (DCT) for every non-overlapping block in the input grey scale image. Subtract 127 from all grey scale values **before** computing the DCT.

2. Quantization: Use the following quantization matrix to quantize each DCT coefficient in a given 8x8 block.

$$Q = \begin{bmatrix} d & d & a & a & b & b & b & b \\ d & d & a & a & b & b & b & b \\ a & a & a & a & b & b & b & b \\ a & a & a & a & b & b & b & b \\ b & b & b & b & c & c & c & c \\ b & b & b & b & c & c & c & c \\ b & b & b & b & c & c & c & c \\ b & b & b & b & c & c & c & c \end{bmatrix}$$

   with $d = 10, a = 20, b = 40, c = 100$. Note that the quantized index of the DCT coefficient $x(i, j)$ is given by

$$y(i, j) = \left\lfloor \frac{x(i, j)}{Q(i, j)} + 0.5 \right\rfloor$$

   and the reconstruction is given by $\hat{x}(i, j) = y(i, j)Q(i, j)$.

3. Lossless source coding: Use the following table to encode the quantized index corresponding to each DCT coefficient.

Quantized DCT index	Code
0	0
-1,1	10x
-3,-2,2,3	110xx
-7,-6,-5,-4,4,5,6,7	1110xxx
...	...

The output bitstream (or file) is given by the concatenation of the sequence of bits produced for each 8x8 block.

Using the JPEG implementation described above:

1. Compute the size of the output file generated for the cameraman.tif image provided to you. Also compute the mean squared error between the original image and reconstructed image. The reconstructed image is obtained by taking the inverse DCT for each block of quantized reconstructions of DCT coefficients. Calculate the compression ratio (defined as the ratio of the input image in bits and size of the output file in bits).

2. Compute the file size and mean squared error for a JPEG implementation where each DCT coefficient is merely rounded to the nearest integer without any quantization. Compute the compression ratio.

3. Find (a, b, c) in the quantization matrix where $a, b, c \in \{10, 20, \ldots, 100\}$ that minimizes the mean squared error of the reconstructed image such that the size of the output file is less than or equal to the size of the output file obtained in (1). Compare the reconstructed image with the image reconstructed in (1) both visually as well as using mean squared error.

4. What is the source distribution for which the lossless source code given in the table above achieves the optimal codeword length?

Problem 2: Bit allocation for uniform scalar quantization of independent sources: Consider a pair of uniformly distributed continuous independent sources X_1 and X_2, both with mean zero and variances 5 and 10 respectively. Let the rate of the uniform scalar quantizers used for X_1 and X_2 be R_1 and R_2 respectively where $R_i = \log_2 K_i$ for $i \in \{1, 2\}$ and K_i denotes the number of quantization points. Given a budget sum rate constraint of $R_1 + R_2 = 3$, compute the rate pair (R_1, R_2) that minimizes the sum of squared error in the respective reconstructions $\mathbb{E}\left[(X_1 - \hat{X}_1)^2\right] + \mathbb{E}\left[(X_2 - \hat{X}_2)^2\right]$. Comment on the choice of (R_1, R_2) obtained with respect to the optimal choices of a, b, c obtained in Problem 1-3.

Prepare a report containing the answers to both the problems and submit along with relevant code.