Advanced Image Processing
Assignment #5

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Problem 1

Spearman Rank Order Correlation Coefficients (SROCC):

Metric 1 (MSE): 0.782274
Metric 2 (SSIM): -0.903680
Metric 3 (CS): -0.90135
Metric 4 (MSE$_{mod}$): 0.90135

Observations:

1. We know from the definition of DMOS measure that smaller the value of DMOS score better is the quality of the image. The positive SROCC value for MSE tells us that the MSE and DMOS are positive correlated which is intuitively also true because, lower MSE signifies that the distorted image is close to the original image.

2. The closer the value of SSIM is to 1 better is the quality of the image. SSIM therefore, should be negatively correlated with the DMOS value which is indeed supported by the above results.

3. CS is negatively correlated with DMOS which tells us that the larger value of CS measure signifies that the quality of image is better

4. MSE$_{mod}$ is positively correlated with DMOS which tells us that the smaller value of MSE$_{mod}$ signifies better quality of the image.

Problem 2

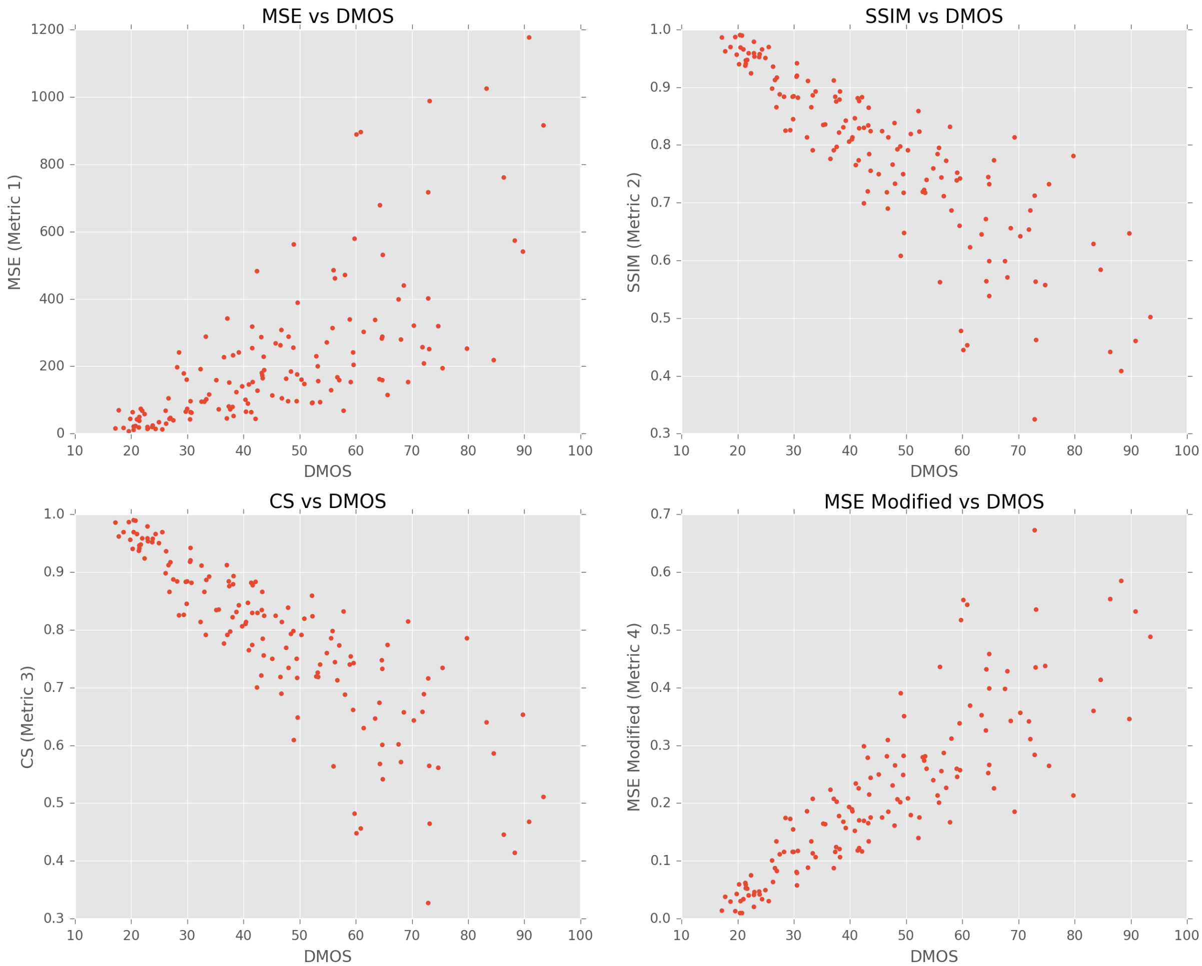

Figure 1: Comparing DMOS with all metrics

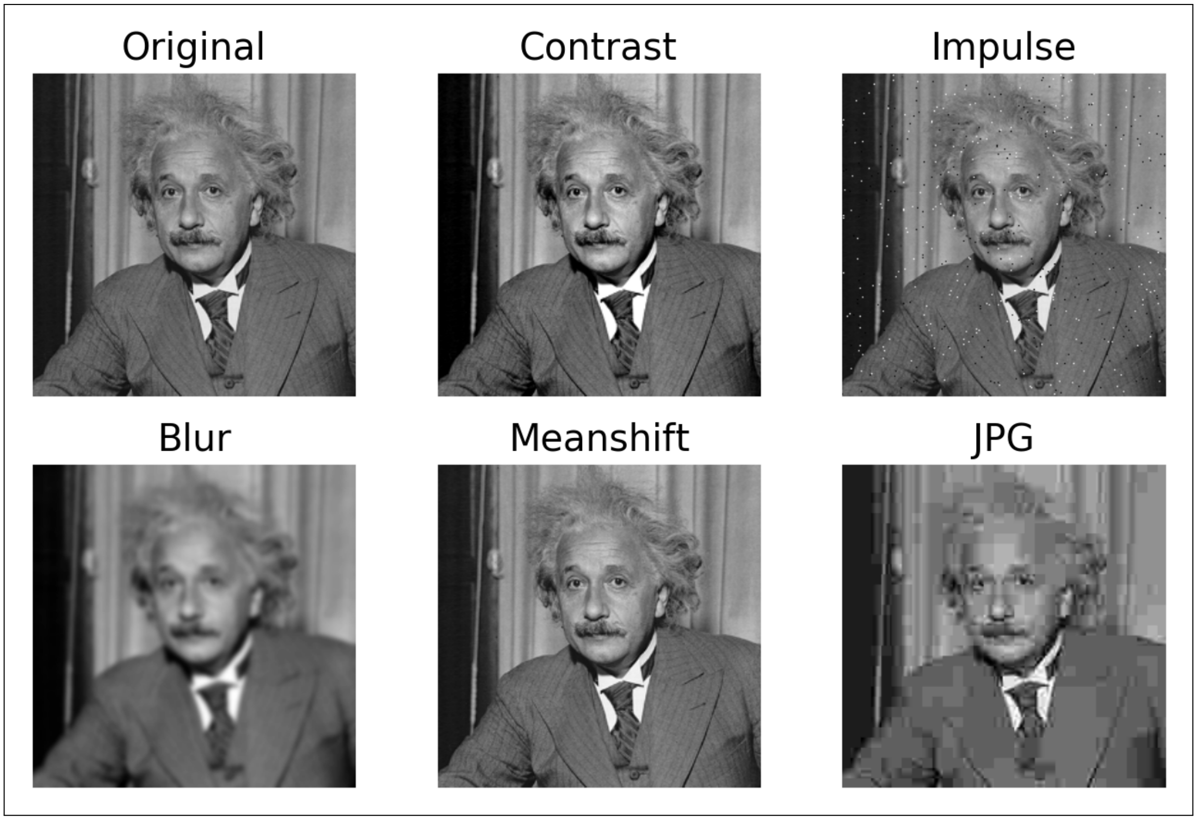

Figure 2: Comparing metrics for different distortions

Observations:

1. From the results given in the table we can see that MSE gives the same score for all the images no matter how good or bad the image is. So it is the worst metric among all the gives metrics.

2. We can see from the result table that smaller the value of MSE_{mod} more closer is the image to the original image. And the opposite holds for the other two metrics SSIM and CS. More closer is their value to 1, more closer is the image to the original.

	MSE	**SSIM**	**CS**	**MSE$_{mod}$**
Contrast	144.2188	0.9012	0.9762	0.0237
Blur	143.9085	0.7022	0.7031	0.2968
Impulse	143.9390	0.8393	0.8394	0.1605
JPG	141.9529	0.6700	0.6716	0.3283
Meanshift	143.9944	0.9873	0.9999	5.4904e-07

Table 1: Score of different metrics for images in Figure 2

3. From the results given in the table, we can see that the sum of the score given by CS and MSE_{mod} is always equal to 1. Therefore, both the metrics are equivalent.

4. SSIM and CS gives almost the similar results for most of the distortion but their results significantly differ in case of contrast changed image. This is because of the fact that CS is actually SSIM without luminance component.

Problem 3

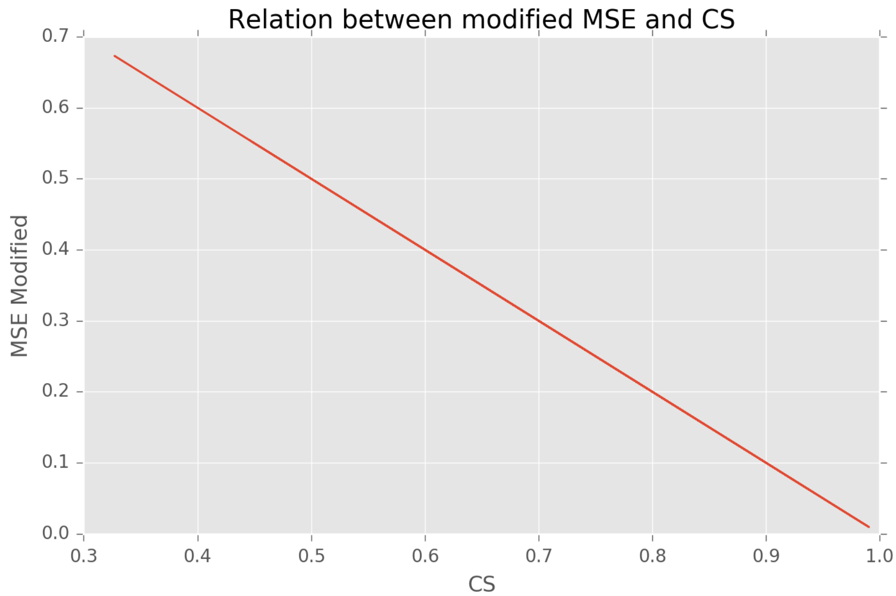

Figure 3: Finding relation between CS and MSE$_{mod}$

Observations:

1. The graph shows that there exist a linear relation between MSE$_{mod}$ and CS metric. The following derivation proves that the actual relation is:

$$MSE_{mod} = 1 - CS$$

Problem 3

$$MSE_{mod}(i,j) = \frac{\sum_{l=-M}^{M} \sum_{k=-M}^{M} w(k,l)\left(x(i+k,j+l) - \mu_x(i,j) - y(i+k,j+l) + \mu_y(i,j)\right)^2}{\sigma_x^2(i,j) + \sigma_y^2(i,j) + C_2}$$

Numerator: $\sum_{l=-M}^{M} \sum_{k=-M}^{M} w(k,l)\left[\left(x(i+k,j+l) - \mu_x(i,j)\right) - \left(y(i+k,j+l) - \mu_y(i,j)\right)\right]^2$

$$= \sum_{l=-M}^{M} \sum_{k=-M}^{M} \left[w(k,l)\left[x(i+k,j+l) - \mu_x(i,j)\right]^2 + w(k,l)\left[y(i+k,j+l) - \mu_y(i,j)\right]^2\right]$$

$$- 2 \sum_{l=-M}^{M} \sum_{k=-M}^{M} w(k,l)\left(x(i+k,j+l) - \mu_x(i,j)\right)\left(y(i+k,j+l) - \mu_y(i,j)\right)$$

$$= \sigma_x^2(i,j) + \sigma_y^2(i,j) - 2\sigma_{xy}(i,j)$$

$$\boxed{MSE_{mod}(i,j) = \frac{\sigma_x^2(i,j) + \sigma_y^2(i,j) - 2\sigma_{xy}(i,j)}{\sigma_x^2(i,j) + \sigma_y^2(i,j) + C_2}}$$

$$CS(i,j) = \left(\frac{\cancel{2\sigma_x(i,j)\sigma_y(i,j) + C_2}}{\sigma_x^2(i,j) + \sigma_y^2(i,j) + C_2}\right)\left(\frac{2\sigma_{xy}(i,j) + C_2}{\cancel{2\sigma_x(i,j)\sigma_y(i,j) + C_2}}\right)$$

[Replaced $C_3 = C_2/2$]

$$\boxed{CS(i,j) = \frac{2\sigma_{xy}(i,j) + C_2}{\sigma_x^2(i,j) + \sigma_y^2(i,j) + C_2}}$$

$$MSE_{mod}(i,j) = \frac{\sigma_x^2(i,j) + \sigma_y^2(i,j) + C_2 - (2\sigma_{xy}(i,j) + C_2)}{\sigma_x^2(i,j) + \sigma_y^2(i,j) + C_2}$$

$$= 1 - \frac{2\sigma_{xy}(i,j) + C_2}{\sigma_x^2(i,j) + \sigma_y^2(i,j) + C_2}$$

$$\boxed{MSE_{mod}(i,j) = 1 - CS(i,j)}$$

(P_3) SROCC $= \dfrac{\sum_{i=1}^{N} (p_i-\bar{p})(q_i-\bar{q})}{\sqrt{\sum_{i=1}^{N} (p_i-\bar{p})^2} \cdot \sqrt{\sum_{i=1}^{N} (q_i-\bar{q})^2}}$

We know from last derivation that

$MSE_{mod} = 1 - CS$

So the rank given by MSE_{mod} will be the reverse of what will be given by CS.

Let $\cdot\, p_i \Rightarrow$ rank of i^{th} image acc to MSE_{mod}

$r_i \Rightarrow$ rank of i^{th} image acc to CS

$r_i = (N+1) - p_i$. where N is the total number of images

$$\boxed{\bar{r} = (N+1) - \bar{p}}$$

SROCC for MSE_{mod} $= \dfrac{\sum_{i=1}^{N} (p_i-\bar{p})(q_i-\bar{q})}{\sqrt{\sum_{i=1}^{N} (p_i-\bar{p})^2} \cdot \sqrt{\sum_{i=1}^{N} (q_i-\bar{q})^2}}$

SROCC for CS $= \dfrac{\sum_{i=1}^{N} (r_i-\bar{r})(q_i-\bar{q})}{\sqrt{\sum_{i=1}^{N} (r_i-\bar{r})^2} \cdot \sqrt{\sum_{i=1}^{N} (q_i-\bar{q})^2}}$

$$= \frac{\sum_{i=1}^{N} \left((N+1-p_i) - (N+1-\bar{p}) \right)(q_i-\bar{q})}{\sqrt{\sum_{i=1}^{N} (N+1-p_i-(N+1-\bar{p}))^2} \cdot \sqrt{\sum_{i=1}^{N} (q_i-\bar{q})^2}}$$

$$= \frac{\sum_{i=1}^{N} (-p_i+\bar{p})(q_i-\bar{q})}{\sqrt{\sum_{i=1}^{N} (-p_i+\bar{p})^2} \cdot \sqrt{\sum_{i=1}^{N} (q_i-\bar{q})^2}}$$

$$= - \frac{\sum_{i=1}^{N} (p_i-\bar{p})(q_i-\bar{q})}{\sqrt{\sum_{i=1}^{N} (p_i-\bar{p})^2} \cdot \sqrt{\sum_{i=1}^{N} (q_i-\bar{q})^2}} \neq -\text{SROCC for } MSE_{mod}$$