

# 18

# Electrostatics

## 18.1 Electric Charge

The electrical nature of matter is inherent in atomic structure. An atom consists of a small, relatively massive nucleus that contains particles called protons and neutrons. A proton has a mass  $1.673 \times 10^{-27}$  kg, while a neutron has a slightly greater mass  $1.675 \times 10^{-27}$  kg. Surrounding the nucleus is a diffuse cloud of orbiting particles called electrons. An electron has a mass of  $9.11 \times 10^{-31}$  kg.

Like mass **electric charge** is an intrinsic property of protons and electrons, and only two types of charge have been discovered positive and negative. A proton has a positive charge, and an electron has a negative charge. A neutron has no net electric charge.

The magnitude of the charge on the proton exactly equals the magnitude of the charge on the electron. The proton carries a charge  $+e$  and the electron carries a charge  $-e$ . The SI unit of charge is Coulomb (C) and  $e$  has the value,  $e = 1.6 \times 10^{-19}$  C

Regarding charge following points are worth noting:

1. Like charges repel each other and unlike charges attract each other.
2. Charge is a scalar and can be of two types positive or negative.
3. Charge is quantized. The quantum of charge is  $e$ . The charge on any body will be some integral multiple of  $e$ , i.e.,

$$q = \pm ne$$

where,  $n = 1, 2, 3, \dots$

Charge on any body can never be  $\left(\frac{1}{3}e\right)$ ,  $1.5e$ , etc.

### Note

1. Apart from charge, energy, angular momentum and mass are also quantized. The quantum of energy is  $h\nu$  and that of angular momentum is  $\frac{h}{2\pi}$ . Quantum of mass is yet not known.

### Chapter Snapshot

- Electric Charge
- Conductors and Insulators
- Charging of a Body
- Coulomb's Law
- Electric Field
- Electric Potential Energy
- Electric Potential
- Relation between Electric Field and Potential
- Equipotential Surfaces
- Electric Dipole
- Gauss's Law
- Properties of a Conductor
- Electric Field and Potential due to Charged Spherical Shell of Solid Conducting Sphere
- Electric Field and Potential due to a Solid Sphere of Charge
- Capacitance

2. The protons and neutrons are combination of other entities called **quarks**, which have charges  $\frac{1}{3}e$  and  $\pm \frac{2}{3}e$ . However, isolated quarks have not been observed, so, quantum of charge is still  $e$ .

4. During any process, the net electric charge of an isolated system remains constant or we can say that charge is conserved. Pair production and pair annihilation are two examples of conservation of charge.
5. A charge particle at rest produces electric field. A charge particle in an unaccelerated motion produces both electric and magnetic fields but does not radiate energy. But an accelerated charged particle not only produces an electric and magnetic fields but also radiates energy in the form of electromagnetic waves.

6.  $1 \text{ Coulomb} = 3 \times 10^9 \text{ esu} = \frac{1}{10} \text{ emu of charge.}$

**Example 18.1** How many electrons are there in one coulomb of negative charge?

**Solution** The negative charge is due to the presence of excess electrons, since they carry negative charge. Because an electron has a charge whose magnitude is  $e = 1.6 \times 10^{-19}$  C, the number of electrons is equal to the charge  $q$  divided by the charge  $e$  on each electron. Therefore, the number  $n$  of electrons is

$$n = \frac{q}{e} = \frac{1.0}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$$

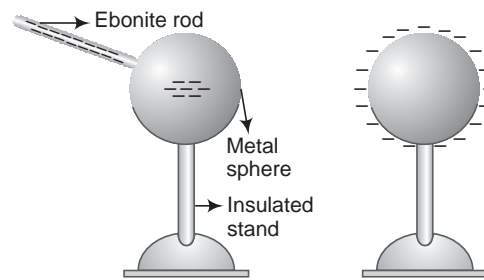


Fig. 18.1

electrons from the rod are transferred to the sphere. Once the electrons are on the metal sphere, where they can move readily, they repel one another and spread out over the sphere's surface. The insulated stand prevents them from flowing to the earth. When the rod is removed the sphere is left with a negative charge distributed over its surface. In a similar manner the sphere will be left with a positive charge after being rubbed with a positively charged rod. In this case, electrons from the sphere would be transferred to the rod. The process of giving one object a net electric charge by placing it in contact with another object that is already charged is known as **charging by contact**.

### (iii) Charging by Induction

It is also possible to charge a conductor in a way that does not involve contact.

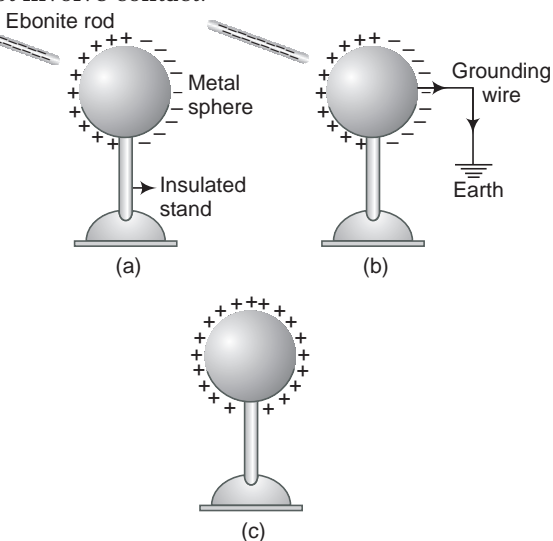


Fig. 18.2

In Fig. (a) a negatively charged rod brought close to (but does not touch) a metal sphere. In the sphere, the free electrons close to the rod move to the other side (by repulsion). As a result, the part of the sphere nearer to the rod becomes positively charged and the part farthest from the rod negatively charged. This phenomenon is called induction. Now if the rod is removed, the free electrons return to their original places and the charged regions disappear. Under most conditions the earth is a

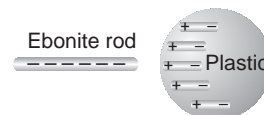


Fig. 18.3

## 18.2 Conductors and Insulators

For the purpose of electrostatic theory all substances can be divided into two main groups — conductors and insulators. In conductors electric charges are free to move from one place to another, whereas in insulators they are tightly bound to their respective atoms. In an uncharged body there are equal number of positive and negative charges.

The examples of conductors of electricity are the metals, human body and the earth and that of insulators are glass, hard rubber and plastics. In metals, the free charges are free electrons known as conduction electrons.

**Semiconductors** are a third class of materials, and their electrical properties are somewhere between those of insulators and conductors. Silicon and germanium are well known examples of semiconductors.

## 18.3 Charging of a Body

Mainly there are following three methods of charging a body:

### (i) Charging by Rubbing

The simplest way to experience electric charges is to rub certain bodies against each other. When a glass rod is rubbed with a silk cloth the glass rod acquires some positive charge and the silk cloth acquires negative charge by the same amount. The explanation of appearance of electric charge on rubbing is simple. All material bodies contain large number of electrons and equal number of protons in their normal state. When rubbed against each other, some electrons from one body pass onto the other body. The body that donates the electrons becomes positively charged while that which receives the electrons becomes negatively charged. For example when glass rod is rubbed with silk cloth, glass rod becomes positively charged because it donates the electrons while the silk cloth becomes negatively charged because it receives electrons. Electricity so obtained by rubbing two objects is also known as **frictional electricity**. The other places where the frictional electricity can be observed are when amber is rubbed with wool or a comb is passed through a dry hair. Clouds also become charged by friction.

### (ii) Charging by Contact

When a negatively charged ebonite rod is rubbed on a metal object, such as a sphere, some of the excess

good electric conductor. So when a metal wire is attached between the sphere and the ground as in Fig. (b) some of the free electrons leave the sphere and distribute themselves on the much larger earth. If the grounding wire is then removed, followed by the ebonite rod, the sphere is left with a net positive charge. The process of giving one object a net electric charge without touching the object to a second charged object is called **charging by induction**. The process could also be used to give the sphere a net negative charge, if a positively charged rod were used. Then, electrons would be drawn up from the ground through the grounding wire and onto the sphere.

If the sphere were made from an insulating material like plastic, instead of metal, the method of producing a net charge by induction would not work, because very little charge would flow through the insulating material and down the grounding wire. However, the electric force of the charged rod would have some effect as shown in figure. The electric force would cause the positive and negative charges in the molecules of the insulating material to separate slightly, with the negative charges being pushed away from the negative rod. The surface of the plastic sphere does acquire a slight induced positive charge, although no net charge is created.

**Example 18.2** If we comb our hair on a dry day and bring the comb near small pieces of paper, the comb attracts the pieces, why?

**Solution** This is an example of frictional electricity and induction. When we comb our hair it gets positively charged by rubbing. When the comb is brought near the pieces of paper some of the electrons accumulate at the edge of the paper piece which is closer to the comb. At the farther end of the piece there is deficiency of electrons and hence, positive charge appears there. Such a redistribution of charge in a material, due to presence of a nearby charged body is called induction. The comb exerts larger attraction on the negative charges of the paper piece as compared to the repulsion on the positive charge. This is because the negative charges are closer to the comb. Hence, there is a net attraction between the comb and the paper piece.

**Example 18.3** Does the attraction between the comb and the piece of papers last for longer period of time?

**Solution** No, because the comb loses its net charge after some time. The excess charge of the comb transfers to earth through our body after some time.

**Example 18.4** Can two similarly charged bodies attract each other?

**Solution** Yes, when the charge on one body ( $q_1$ ) is much greater than that on the other ( $q_2$ ) and they are close enough to each other so that force of attraction between  $q_1$  and induced charge on the other exceeds the force of repulsion between  $q_1$  and  $q_2$ . However two similar point charges can never attract each other because no induction will take place here.

**Example 18.5** Does in charging the mass of a body change?

**Solution** Yes, as charging a body means addition or removal of electrons and electron has a mass.

**Example 18.6** Why a third hole in a socket provided for grounding?

**Solution** All electric appliances may end with some charge due to faulty connections. In such a situation charge will be accumulated on the appliance. When the user touches the appliance he may get a shock. By providing the third hole for grounding all accumulated charge is discharged to the ground and the appliance is safe.

## 18.4 Coulomb's Law

The law that describes how charges interact with one another was discovered by Charles Augustin de Coulomb in 1785. With a sensitive torsion balance, Coulomb measured the electric force between charged spheres. In Coulomb's experiment the charged spheres were much smaller than the distance between them so that the charges could be treated as point charges. The results of the experiments of Coulomb and others are summarized in **Coulomb's law**.

The electric force  $F_e$  exerted by one point charge on another acts along the line between the charges. It varies inversely as the square of the distance separating the charges and is proportional to the product of charges. The force is repulsive if the charges have the same sign and attractive if the charges have opposite signs.

The magnitude of the electric force exerted by a charge  $q_1$  on another charge  $q_2$  a distance  $r$  away is thus, given by

$$F_e = \frac{k |q_1 q_2|}{r^2} \quad \dots (i)$$

The value of the proportionality constant  $k$  in Coulomb's law depends on the system of units used. In SI units the constant  $k$  is,

$$k = 8.987551787 \times 10^9 \frac{\text{N-m}^2}{\text{C}^2} \\ \approx 8.988 \times 10^9 \frac{\text{N-m}^2}{\text{C}^2}$$

The value of  $k$  is known to such a large number of significant digits because this value is closely related to the speed of light in vacuum. This speed is defined to be exactly  $c = 2.99792458 \times 10^8 \text{ m/s}$ . The numerical value of  $k$  is defined in terms of  $c$  to be precisely,

$$k = \left( 10^{-7} \frac{\text{N-s}^2}{\text{C}^2} \right) c^2$$

This constant  $k$  is often written as  $\frac{1}{4\pi\epsilon_0}$ , where  $\epsilon_0$

(epsilon-nought) is another constant. This appears to complicate matters, but it actually simplifies many formulae that we will encounter in later chapters. Thus, Eq. (i) can be written as,

$$F_e = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2} \quad \dots (ii)$$

Here, 
$$\frac{1}{4\pi\epsilon_0} = \left(10^{-7} \frac{\text{N} \cdot \text{s}^2}{\text{C}^2}\right) c^2$$

Substituting value of  $c = 2.99792458 \times 10^8 \text{ m/s}$ , we get

$$\frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

In examples and problems we will often use the approximate value,

$$\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

Here, the quantity  $\epsilon_0$  is called the **permittivity of free space**. It has the value,

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$$

Regarding Coulomb's law following points are worth noting:

- (1) Coulomb's law stated above describes the interaction of two point charges. When two charges exert forces simultaneously on a third charge, the total force acting on that charge is the vector sum of the forces that the two charges would exert individually. This important property, called the **principle of superposition of forces**, holds for any number of charges. Thus,

$$\mathbf{F}_{\text{net}} = \mathbf{F}_1 + \mathbf{F}_2 + \dots + \mathbf{F}_n$$

- (2) The electric force is an action reaction pair, i.e., the two charges exert equal and opposite forces on each other.

- (3) The electric force is conservative in nature.

- (4) Coulomb's law as we have stated above can be used for point charges in **vacuum**. If some dielectric (insulator) is present in the space between the charges, the net force acting on each charge is altered because charges are induced in the molecules of the intervening medium. We will describe this effect later. Here at this moment it is enough to say that the force decreases  $K$  times if the medium extends till infinity. Here  $K$  is a dimensionless constant which depends on the medium and called dielectric constant of the medium. Thus,

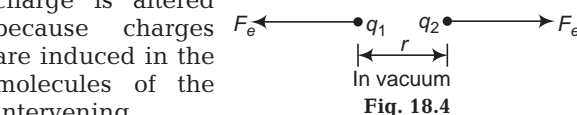


Fig. 18.4

$$F_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad (\text{in vacuum})$$

$$\frac{F_e}{K} = \frac{F_e}{K} = \frac{1}{4\pi\epsilon_0 K} \cdot \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon} \cdot \frac{q_1 q_2}{r^2} \quad (\text{in medium})$$

Here,  $\epsilon = \epsilon_0 K$  is called permittivity of the medium.

## Objective Galaxy

1. In few problems of electrostatics Lami's theorem is very useful.

According to this theorem, 'if three concurrent forces  $\mathbf{F}_1$ ,  $\mathbf{F}_2$  and  $\mathbf{F}_3$  as shown in figure are in equilibrium or if  $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$ , then

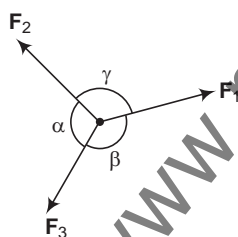


Fig. 18.5

$$\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}$$

2. Suppose the position vectors of two charges  $q_1$  and  $q_2$  are  $\mathbf{r}_1$  and  $\mathbf{r}_2$ , then, electric force on charge  $q_1$  due to charge  $q_2$  is,

$$\mathbf{F}_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} (\mathbf{r}_1 - \mathbf{r}_2)$$

Similarly, electric force on  $q_2$  due to charge  $q_1$  is

$$\mathbf{F}_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|\mathbf{r}_2 - \mathbf{r}_1|^3} (\mathbf{r}_2 - \mathbf{r}_1)$$

Here  $q_1$  and  $q_2$  are to be substituted with sign.

$$\mathbf{r}_1 = x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}$$

and

$$\mathbf{r}_2 = x_2 \hat{i} + y_2 \hat{j} + z_2 \hat{k}$$

where  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  are the co-ordinates of charges  $q_1$  and  $q_2$ .

**Example 18.7** What is the smallest electric force between two charges placed at a distance of 1.0 m.

**Solution**

$$F_e = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \quad \dots (i)$$

For  $F_e$  to be minimum  $q_1 q_2$  should be minimum. We know that

$$(q_1)_{\min} = (q_2)_{\min} \\ = e = 1.6 \times 10^{-19} \text{ C}$$

Substituting in Eq. (i), we have

$$(F_e)_{\min} = \frac{(9.0 \times 10^9) (1.6 \times 10^{-19}) (1.6 \times 10^{-19})}{(1.0)^2} \\ = 2.304 \times 10^{-28} \text{ N}$$

**Example 18.8** Three charges  $q_1 = 1 \mu\text{C}$ ,  $q_2 = -2 \mu\text{C}$  and  $q_3 = 3 \mu\text{C}$  are placed on the vertices of an equilateral triangle of side 1.0 m. Find the net electric force acting on charge  $q_1$ .

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**How to Proceed** Charge  $q_2$  will attract charge  $q_1$  (along the line joining them) and charge  $q_3$  will repel charge  $q_1$ . Therefore, two forces will act on  $q_1$ , one due to  $q_2$  and another due to  $q_3$ . Since, the force is a vector quantity both of these forces (say  $\mathbf{F}_1$  and  $\mathbf{F}_2$ ) will be added by vector method. Following are two methods of their addition.

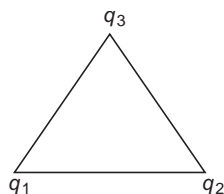


Fig. 18.6

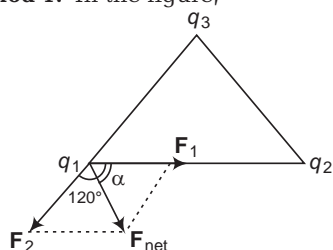


Fig. 18.7

$$|\mathbf{F}_1| = F_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

= magnitude of force between  $q_1$  and  $q_2$

$$= \frac{(9.0 \times 10^9) (1.0 \times 10^{-6}) (2.0 \times 10^{-6})}{(1.0)^2}$$

$$= 1.8 \times 10^{-2} \text{ N}$$

Similarly,

$$|\mathbf{F}_2| = F_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_3}{r^2}$$

= magnitude of force between  $q_1$  and  $q_3$

$$= \frac{(9.0 \times 10^9) (1.0 \times 10^{-6}) (3.0 \times 10^{-6})}{(1.0)^2}$$

$$= 2.7 \times 10^{-2} \text{ N}$$

Now,  $|\mathbf{F}_{\text{net}}| = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos 120^\circ}$

$$= \left( \sqrt{(1.8)^2 + (2.7)^2 + 2(1.8)(2.7)\left(-\frac{1}{2}\right)} \right) \times 10^{-2} \text{ N}$$

$$= 2.38 \times 10^{-2} \text{ N}$$

and  $\tan \alpha = \frac{F_2 \sin 120^\circ}{F_1 + F_2 \cos 120^\circ}$

$$= \frac{(2.7 \times 10^{-2}) (0.87)}{(1.8 \times 10^{-2}) + (2.7 \times 10^{-2}) \left(-\frac{1}{2}\right)}$$

or  $\alpha = 79.2^\circ$

Thus, the net force on charge  $q_1$  is  $2.38 \times 10^{-2} \text{ N}$  at an angle  $\alpha = 79.2^\circ$  with a line joining  $q_1$  and  $q_2$  as shown in the figure.

**Method 2.** In this method let us assume a co-ordinate axes with  $q_1$  at origin as shown in figure. The co-ordinates of  $q_1$ ,  $q_2$  and  $q_3$  in this co-ordinate system are  $(0, 0, 0)$ ,  $(1 \text{ m}, 0, 0)$  and  $(0.5 \text{ m}, 0.87 \text{ m}, 0)$  respectively. Now,

$\mathbf{F}_1$  = force on  $q_1$  due to charge  $q_2$

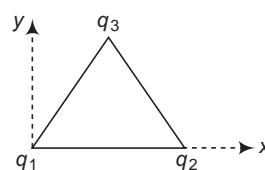


Fig. 18.8

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} (\mathbf{r}_1 - \mathbf{r}_2)$$

$$= \frac{(9.0 \times 10^9) (1.0 \times 10^{-6}) (-2.0 \times 10^{-6})}{(1.0)^3}$$

$$\times [(0-1)\hat{i} + (0-0)\hat{j} + (0-0)\hat{k}]$$

$$= (1.8 \times 10^{-2}) \hat{i} \text{ N}$$

and

$\mathbf{F}_2$  = force on  $q_1$  due to charge  $q_3$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_3}{|\mathbf{r}_1 - \mathbf{r}_3|^3} (\mathbf{r}_1 - \mathbf{r}_3)$$

$$= \frac{(9.0 \times 10^9) (1.0 \times 10^{-6}) (3.0 \times 10^{-6})}{(1.0)^3}$$

$$\times [(0-0.5)\hat{i} + (0-0.87)\hat{j} + (0-0)\hat{k}]$$

$$= (-1.35\hat{i} - 2.349\hat{j}) \times 10^{-2} \text{ N}$$

Therefore, net force on  $q_1$  is,

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (0.45\hat{i} - 2.349\hat{j}) \times 10^{-2} \text{ N}$$

**Note** Once you write a vector in terms of  $\hat{i}$ ,  $\hat{j}$  and  $\hat{k}$ , there is no need of writing the magnitude and direction of vector separately.

**Example 18.9** Two identical balls each having a density  $\rho$  are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium each string makes an angle  $\theta$  with vertical. Now, both the balls are immersed in a liquid. As a result the angle  $\theta$  does not change. The density of the liquid is  $\sigma$ . Find the dielectric constant of the liquid.

**Solution** Each ball is in equilibrium under the following three forces:

- (i) tension
- (ii) electric force and
- (iii) weight

So, Lami's theorem can be applied.

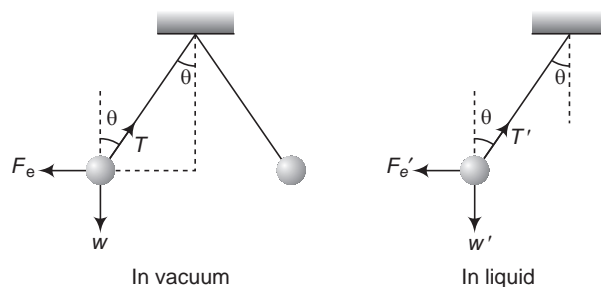


Fig. 18.9



In the liquid,  $F'_e = \frac{F_e}{K}$

where,  $K$  = dielectric constant of liquid

and  $w' = w - \text{upthrust}$

Applying Lami's theorem in vacuum

$$\frac{w}{\sin(90^\circ + \theta)} = \frac{F_e}{\sin(180^\circ - \theta)}$$

$$\text{or } \frac{w}{\cos \theta} = \frac{F_e}{\sin \theta} \quad \dots(i)$$

Similarly in liquid,

$$\frac{w'}{\cos \theta} = \frac{F'_e}{\sin \theta} \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{w}{w'} = \frac{F_e}{F'_e}$$

$$\text{or } K = \frac{w}{w - \text{upthrust}} \quad \left( \text{as } \frac{F_e}{F'_e} = K \right)$$

$$= \frac{V\rho g}{V\rho g - V\sigma g} \quad (V = \text{volume of ball})$$

$$\text{or } K = \frac{\rho}{\rho - \sigma}$$

**Note** In the liquid  $F_e$  and  $w$  have been changed. Therefore,  $T$  will also change.

## 18.5 Electric Field

A charged particle cannot directly interact with another particle kept at a distance. A charge produces something called an electric field in the space around it and this electric field exerts a force on any other charge (except the source charge itself) placed in it.

Thus, the region surrounding a charge or distribution of charge in which its electrical effects can be observed is called the **electric field** of the charge or distribution of charge. Electric field at a point can be defined in terms of either a vector function  $\mathbf{E}$  called **electric field strength** or a scalar function  $V$  called **electric potential**. The electric field can also be visualised graphically in terms of **lines of force**.

### Electric Field Strength ( $\mathbf{E}$ )

Like its gravitational counterpart, the electric field strength (often called electric field) at a point in an electric field is defined as the electrostatic force  $\mathbf{F}_e$  per unit positive charge. Thus, if the electrostatic force experienced by a small test charge  $q_0$  is  $\mathbf{F}_e$ , then field strength at that point is defined as,

$$\mathbf{E} = \lim_{q_0 \rightarrow 0} \frac{\mathbf{F}_e}{q_0}$$

The electric field is a vector quantity and its direction is the same as the direction of the force  $\mathbf{F}_e$  on a positive test charge. The SI unit of electric field is N/C. Here, it should be noted that the test charge  $q_0$  does not disturb other charges which produces  $\mathbf{E}$ . With the concept of electric field, our description of electric interactions has two parts. First, a given charge distribution acts as a source of electric field. Second, the electric field exerts a force on any charge that is present in this field.

### An Electric Field Leads to a Force

Suppose there is an electric field strength  $\mathbf{E}$  at some point in an electric field, then the electrostatic force acting on a charge  $+q$  is  $q\mathbf{E}$  in the direction of  $\mathbf{E}$ , while on the charge  $-q$  it is  $q\mathbf{E}$  in the opposite direction of  $\mathbf{E}$ .

**Example 18.10** An electric field of  $10^5$  N/C points due west at a certain spot. What are the magnitude and direction of the force that acts on a charge of  $+2 \mu\text{C}$  and  $-5 \mu\text{C}$  at this spot?

**Solution** Force on  $+2 \mu\text{C} = q\mathbf{E}$   
 $= (2 \times 10^{-6}) (10^5)$   
 $= 0.2 \text{ N (due west)}$   
 Force on  $-5 \mu\text{C} = (5 \times 10^{-6}) (10^5)$   
 $= 0.5 \text{ N (due east)}$

### Electric Field Due to a Point Charge

The electric field produced by a point charge  $q$  can be obtained in general terms from Coulomb's law. First note that the magnitude of the force exerted by the charge  $q$  on a test charge  $q_0$  is,

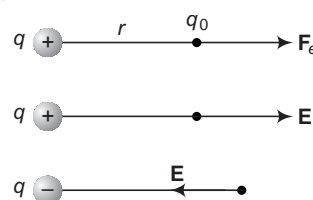


Fig. 18.10

$$F_e = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq_0}{r^2}$$

then divide this value by  $q_0$  to obtain the magnitude of the field.

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

If  $q$  is positive  $\mathbf{E}$  is directed away from  $q$ . On the other hand if  $q$  is negative, then  $\mathbf{E}$  is directed towards  $q$ .

The electric field at a point is a vector quantity. Suppose  $\mathbf{E}_1$  is the field at a point due to a charge  $q_1$  and  $\mathbf{E}_2$  is the field at the same point due to a charge  $q_2$ . The resultant field when both the charges are present is

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

If the given charge distribution is continuous, we can use the technique of integration to find the resultant electric field at a point.

**Example 18.11** Two positive point charges  $q_1 = 16 \mu\text{C}$  and  $q_2 = 4 \mu\text{C}$ , are separated in vacuum by a distance of 3.0 m. Find the point on the line between the charges where the net electric field is zero.

**Solution** Between the charges the two field contributions have opposite directions, and the net electric field is zero at a point (say  $P$ ), where the magnitudes of  $\mathbf{E}_1$  and  $\mathbf{E}_2$  are equal. However, since,  $q_2 < q_1$ , point  $P$  must be closer to  $q_2$ , in order that the field of the smaller charge can balance the field of the larger charge.

$$\text{At } P, \quad E_1 = E_2$$

$$\text{or} \quad \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1^2} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2^2}$$

$$\therefore \quad \frac{r_1}{r_2} = \sqrt{\frac{q_1}{q_2}}$$

$$= \sqrt{\frac{16}{4}} = 2 \quad \dots(i)$$

$$\text{Also,} \quad r_1 + r_2 = 3.0 \text{ m} \quad \dots(ii)$$

Solving these equations, we get

$$r_1 = 2 \text{ m}$$

$$\text{and} \quad r_2 = 1 \text{ m}$$

Thus, the point  $P$  is at a distance of 2 m from  $q_1$  and 1 m from  $q_2$ .

### Electric Field of a Ring of Charge

Electric field at distance  $x$  from the centre of uniformly charged ring of total charge  $q$  on its axis is given by,

$$E_x = \left( \frac{1}{4\pi\epsilon_0} \right) \frac{qx}{(x^2 + R^2)^{3/2}}$$

Direction of this electric field is along the axis and away from the ring in case of positively charged ring and towards the ring in case of negatively charged ring.

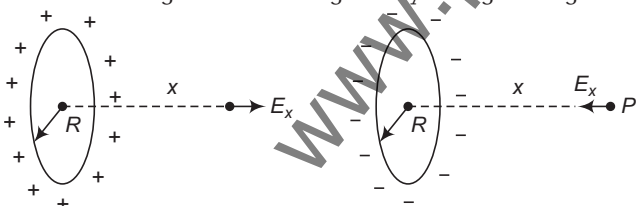


Fig. 18.12

From the above expression, we can see that

(i)  $E_x = 0$  at  $x = 0$ , i.e., field is zero at the centre of the ring. We should expect this, charges on opposite sides of the ring would push in opposite directions on a test charge at the centre, and the forces would add to zero.

(ii)  $E_x = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{x^2}$  for  $x \gg R$ , i.e., when the point  $P$  is much farther from the ring, its field is the same as

that of a point charge. To an observer far from the ring, the ring would appear like a point, and the electric field reflects this.

(iii)  $E_x$  will be maximum where  $\frac{dE_x}{dx} = 0$ . Differentiating

$E_x$  w.r.t.  $x$  and putting it equal to zero, we get  $x = \frac{R}{\sqrt{2}}$  and  $E_{\text{max}}$  comes out to be,  $\frac{2}{3\sqrt{3}} \left( \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2} \right)$ .

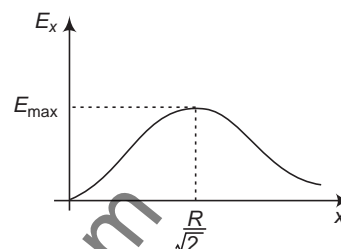


Fig. 18.13

### Electric Field of an Infinitely Long Line Charge

Electric field at distance  $r$  from an infinitely long line charge is given by

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

Here  $\lambda$  is charge per unit length. Direction of this electric field is away from the line charge in case of positively charged line charge and towards the line charge in case of negatively charged line charge.

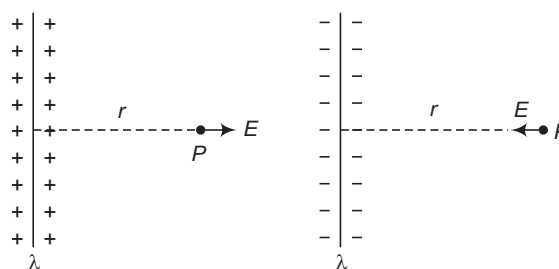


Fig. 18.14

$$\text{or} \quad E \propto \frac{1}{r}$$

Thus,  $E$ - $r$  graph is as shown in Fig. 18.15.

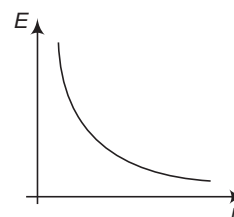


Fig. 18.15

The direction of  $\mathbf{E}$  is radially outward from the line.

## Objective Galaxy

1. Suppose a charge  $q$  is placed at a point whose position vector is  $\mathbf{r}_q$  and we want to find the electric field at a point  $P$  whose position vector is  $\mathbf{r}_p$ . Then in vector form the electric field is given by,

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{|\mathbf{r}_p - \mathbf{r}_q|^3} (\mathbf{r}_p - \mathbf{r}_q)$$

Here,

$$\mathbf{r}_p = x_p \hat{i} + y_p \hat{j} + z_p \hat{k}$$

and

$$\mathbf{r}_q = x_q \hat{i} + y_q \hat{j} + z_q \hat{k}$$

**Example 18.12** A charge  $q = 1 \mu\text{C}$  is placed at point  $(1 \text{ m}, 2 \text{ m}, 4 \text{ m})$ . Find the electric field at point  $P (0 \text{ m}, -4 \text{ m}, 3 \text{ m})$ .

**Solution** Here,  $\mathbf{r}_q = \hat{i} + 2\hat{j} + 4\hat{k}$

and  $\mathbf{r}_p = -4\hat{j} + 3\hat{k}$

$$\therefore \mathbf{r}_p - \mathbf{r}_q = -\hat{i} - 6\hat{j} - \hat{k}$$

$$\text{or } |\mathbf{r}_p - \mathbf{r}_q| = \sqrt{(-1)^2 + (-6)^2 + (-1)^2} \\ = \sqrt{38} \text{ m}$$

$$\text{Now, } \mathbf{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{|\mathbf{r}_p - \mathbf{r}_q|^3} (\mathbf{r}_p - \mathbf{r}_q)$$

Substituting the values, we have

$$\mathbf{E} = \frac{(9.0 \times 10^9) (1.0 \times 10^{-6})}{(38)^{3/2}} (-\hat{i} - 6\hat{j} - \hat{k}) \\ = (-38.42 \hat{i} - 230.52 \hat{j} - 38.42 \hat{k}) \frac{\text{N}}{\text{C}}$$

### Electric Field Lines

As we have seen, electric charges create an electric field in the space surrounding them. It is useful to have a kind of map that gives the direction and indicates the strength of the field at various places. **Field lines**, a concept introduced by Michael Faraday, provide us with an easy way to visualize the electric field.

"An **electric field line** is an imaginary line or curve drawn through a region of space so that its tangent at any point is in the direction of the electric field vector at that point. The relative closeness of the lines at some place give an idea about the intensity of electric field at that point."

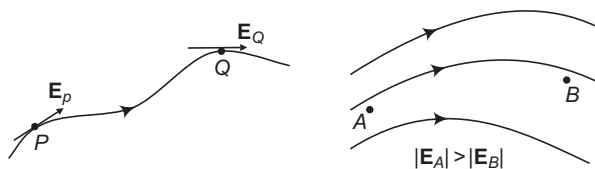


Fig. 18.16

The electric field lines have the following properties:

1. The tangent to a line at any point gives the direction of  $\mathbf{E}$  at that point. This is also the path on which a positive test charge will tend to move if free to do so.

2. Electric field lines always begin on a positive charge and end on a negative charge and do not start or stop in midspace.
3. The number of lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge. This means, for example that if 100 lines are drawn leaving a  $+4 \mu\text{C}$  charge then 75 lines would have to end on a  $-3 \mu\text{C}$  charge.

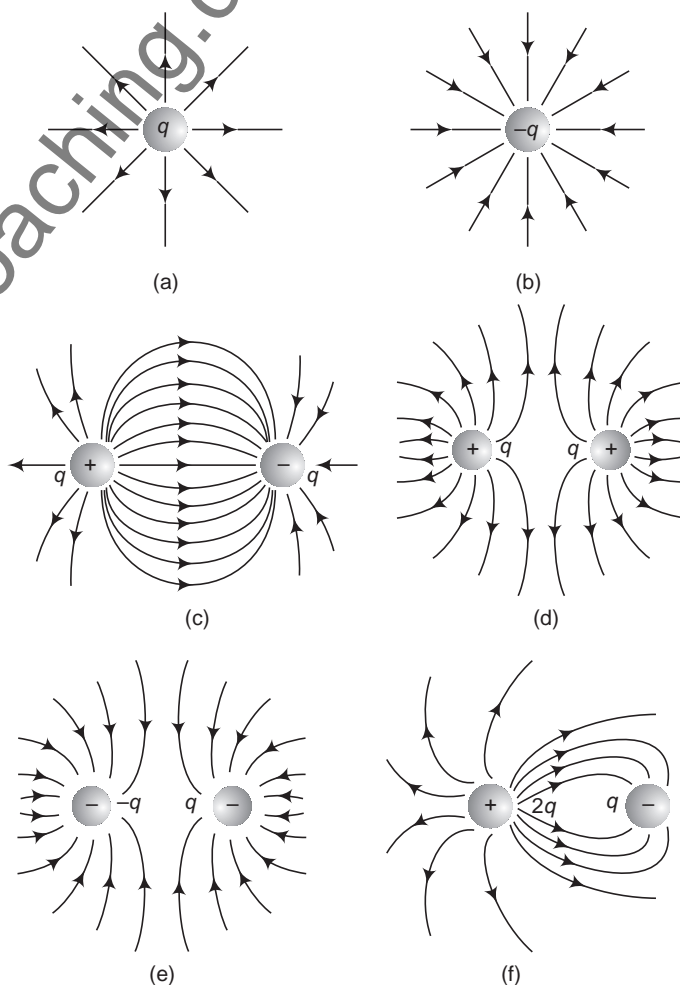


Fig. 18.17

4. Two lines can never intersect. If it happens then two tangents can be drawn at their point of intersection, i.e., intensity at that point will have two directions which is absurd.



5. In a uniform field, the field lines are straight parallel and uniformly spaced.
6. The electric field lines can never form closed loops as a line can never start and end on the same charge.
7. Electric field lines also give us an indication of the equipotential surface (surface which has the same potential)
8. Electric field lines always flow from higher potential to lower potential.
9. In a region where there is no electric field, lines are absent. This is why inside a conductor (where electric field is zero) there, cannot be any electric field line.
10. Electric lines of force ends or starts normally from the surface of a conductor.

**Note** Here work done by electrostatic force is positive. Hence, the potential energy is decreasing.

## Electric Potential Energy of Two Charges

The idea of electric potential energy is not restricted to the special case of a uniform electric field as in example 18.13. Let us now calculate the work done on a test charge  $q_0$  moving in a non-uniform electric field caused by a single, stationary point charge  $q$ .

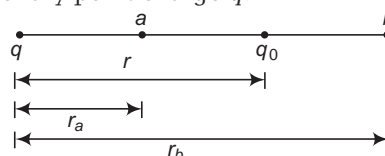


Fig. 18.19

The Coulomb's force on  $q_0$  at a distance  $r$  from a fixed charge  $q$  is,

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq_0}{r^2}$$

If the two charges have same signs, the force is repulsive and if the two charges have opposite signs, the force is attractive. The force is not constant during the displacement, so we have to integrate to calculate the work  $W_{a \rightarrow b}$  done on  $q_0$  by this force as  $q_0$  moves from  $a$  to  $b$ .

$$\begin{aligned} W_{a \rightarrow b} &= \int_{r_a}^{r_b} F dr = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0} \cdot \frac{qq_0}{r^2} dr \\ &= \frac{qq_0}{4\pi\epsilon_0} \left( \frac{1}{r_a} - \frac{1}{r_b} \right) \end{aligned}$$

Being a conservative force this work is path independent. From the definition of potential energy,

$$\begin{aligned} U_b - U_a &= -W_{a \rightarrow b} \\ &= -\frac{qq_0}{4\pi\epsilon_0} \left( \frac{1}{r_b} - \frac{1}{r_a} \right) \end{aligned}$$

We choose the potential energy of the two charge system to be zero when they have infinite separation. This means  $U_\infty = 0$ . The potential energy when the separation is  $r$  is  $U_r$

$$\therefore U_r - U_\infty = \frac{qq_0}{4\pi\epsilon_0} \left( \frac{1}{r} - \frac{1}{\infty} \right)$$

$$\text{or } U_r = \frac{qq_0}{4\pi\epsilon_0} \frac{1}{r}$$

This is the expression for electric potential energy of two point charges kept at a separation  $r$ . In this expression both the charges  $q$  and  $q_0$  are to be substituted with sign.

## Electric Potential Energy of a System of Charges

To find electrical potential energy of a system of charges, we make pairs. If there are total  $n$  charges then total number of pairs are  $\frac{n(n-1)}{2}$ . Neither of the pair should be repeated, nor should be left.

## 18.6 Electric Potential Energy

If the force  $\mathbf{F}$  is conservative, the work done by  $\mathbf{F}$  can always be expressed in terms of a **potential energy**  $U$ . When the particle moves from a point where the potential energy is  $U_a$  to a point where it is  $U_b$ , the change in potential energy is,  $\Delta U = U_b - U_a$ . This is related by the work  $W_{a \rightarrow b}$  as

$$W_{a \rightarrow b} = U_a - U_b = -(U_b - U_a) = -\Delta U \quad \dots (i)$$

Here  $W_{a \rightarrow b}$  is the work done in displacing the particle from  $a$  to  $b$  by the **conservative force** (here electrostatic) not by us. Moreover we can see from Eq. (i) that if  $W_{a \rightarrow b}$  is positive,  $\Delta U$  is negative and the potential energy decreases. So, **whenever the work done by a conservative force is positive, the potential energy of the system decreases and vice versa**. That's what happens when a particle is thrown upwards, the work done by gravity is negative, and the potential energy increases.

**Example 18.13** A uniform electric field  $E_0$  is directed along positive  $y$ -direction. Find the change in electric potential energy of a positive test charge  $q_0$  when it is displaced in this field from  $y_i = a$  to  $y_f = 2a$  along the  $y$ -axis.

**Solution** Electrostatic force on the test charge,  
 $F_e = q_0 E_0$  (along positive  $y$ -direction)

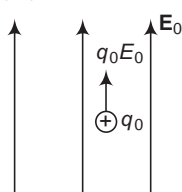
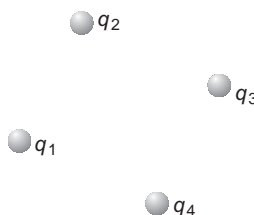


Fig. 18.18

$$\begin{aligned} \therefore W_{i \rightarrow f} &= -\Delta U \\ \text{or } \Delta U &= -W_{i \rightarrow f} = -[q_0 E_0 (2a - a)] \\ &= -q_0 E_0 a \end{aligned}$$

For example electric potential energy of four point charges  $q_1, q_2, q_3$  and  $q_4$  would be given by,



$$U = \frac{1}{4\pi\epsilon_0} \left[ \frac{q_4 q_3}{r_{43}} + \frac{q_4 q_2}{r_{42}} + \frac{q_4 q_1}{r_{41}} + \frac{q_3 q_2}{r_{32}} + \frac{q_3 q_1}{r_{31}} + \frac{q_2 q_1}{r_{21}} \right] \quad \dots(ii)$$

Here, all the charges are to be substituted with sign.

**Example 18.14** Four charges  $q_1 = 1 \mu\text{C}$ ,  $q_2 = 2 \mu\text{C}$ ,  $q_3 = -3 \mu\text{C}$  and  $q_4 = 4 \mu\text{C}$  are kept on the vertices of a square of side 1 m. Find the electric potential energy of this system of charges.

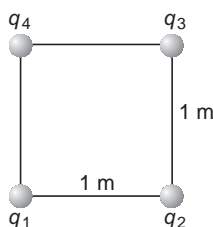


Fig. 18.20

**Solution** In this problem,

$$r_{41} = r_{43} = r_{32} = r_{21} = 1 \text{ m}$$

and  $r_{42} = r_{31} = \sqrt{(1)^2 + (1)^2} = \sqrt{2} \text{ m}$

Substituting the proper values with sign in Eq. (ii), we get

$$U = (9.0 \times 10^9)(10^{-6})(10^{-6})$$

$$\left[ \frac{(4)(-3)}{1} + \frac{(4)(2)}{\sqrt{2}} + \frac{(4)(1)}{1} + \frac{(-3)(2)}{1} + \frac{(-3)(1)}{\sqrt{2}} + \frac{(2)(1)}{1} \right]$$

$$= (9.0 \times 10^{-3}) \left[ -12 + \frac{5}{\sqrt{2}} \right] = -7.62 \times 10^{-2} \text{ J}$$

**Note** Here negative sign of  $U$  implies that positive work has been done by electrostatic forces in assembling these charges at respective distances from infinity.

**Example 18.15** Two point charges are located on the x-axis,  $q_1 = -1 \mu\text{C}$  at  $x = 0$  and  $q_2 = +1 \mu\text{C}$  at  $x = 1 \text{ m}$ .

(a) Find the work that must be done by an external force to bring a third point charge  $q_3 = +1 \mu\text{C}$  from infinity to  $x = 2 \text{ m}$ .

(b) Find the total potential energy of the system of three charges.

**Solution** (a) The work that must be done on  $q_3$  by an external force is equal to the difference of potential energy  $U$  when the charge is at  $x = 2 \text{ m}$  and the potential energy when it is at infinity.

$$\therefore W = U_f - U_i = \frac{1}{4\pi\epsilon_0} \left[ \frac{q_3 q_2}{(r_{32})_f} + \frac{q_3 q_1}{(r_{31})_f} + \frac{q_2 q_1}{(r_{21})_f} \right]$$

$$- \frac{1}{4\pi\epsilon_0} \left[ \frac{q_3 q_2}{(r_{32})_i} + \frac{q_3 q_1}{(r_{31})_i} + \frac{q_2 q_1}{(r_{21})_i} \right]$$

Here,  $(r_{21})_i = (r_{21})_f$  and  $(r_{32})_i = (r_{31})_i = \infty$

$$\therefore W = \frac{1}{4\pi\epsilon_0} \left[ \frac{q_3 q_2}{(r_{32})_f} + \frac{q_3 q_1}{(r_{31})_f} \right]$$

Substituting the values, we have

$$W = (9.0 \times 10^9) (10^{-12}) \left[ \frac{(1)(1)}{(1.0)} + \frac{(1)(-1)}{(2.0)} \right]$$

$$= 4.5 \times 10^{-3} \text{ J}$$

(b) The total potential energy of the three charges is given by,

$$U = \frac{1}{4\pi\epsilon_0} \left( \frac{q_3 q_2}{r_{32}} + \frac{q_3 q_1}{r_{31}} + \frac{q_2 q_1}{r_{21}} \right)$$

$$= (9.0 \times 10^9) \left[ \frac{(1)(1)}{(1.0)} + \frac{(1)(-1)}{(2.0)} + \frac{(1)(-1)}{(1.0)} \right] (10^{-12})$$

$$= -4.5 \times 10^{-3} \text{ J}$$

**Example 18.16** Two point charges  $q_1 = q_2 = 2 \mu\text{C}$  are fixed at  $x_1 = +3 \text{ m}$  and  $x_2 = -3 \text{ m}$  as shown in figure. A third particle of mass  $1 \text{ g}$  and charge  $q_3 = -4 \mu\text{C}$  are released from rest at  $y = 4.0 \text{ m}$ . Find the speed of the particle as it reaches the origin.

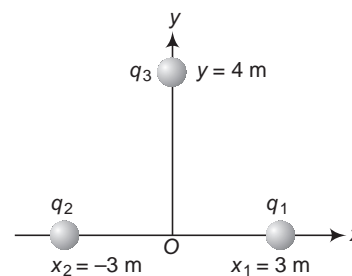


Fig. 18.21

**How to Proceed** Here the charge  $q_3$  is attracted towards  $q_1$  and  $q_2$  both. So, the net force on  $q_3$  is towards origin.

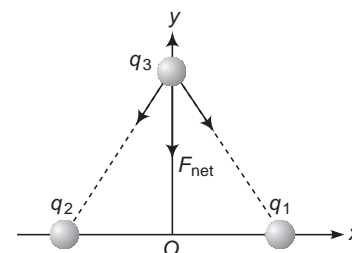


Fig. 18.22

By this force charge is accelerated towards origin, but this acceleration is not constant. So, to obtain the speed of particle at origin by kinematics, we will have to find first the acceleration at same intermediate position and then will have to integrate it with proper limits. On the other hand it is easy to use energy conservation principle as the only forces are conservative.

**Solution** Let  $v$  be the speed of particle at origin. From conservation of mechanical energy,

$$U_i + K_i = U_f + K_f$$

$$\text{or } \frac{1}{4\pi\epsilon_0} \left[ \frac{q_3 q_2}{(r_{32})_i} + \frac{q_3 q_1}{(r_{31})_i} + \frac{q_2 q_1}{(r_{21})_i} \right] + 0$$

$$= \frac{1}{4\pi\epsilon_0} \left[ \frac{q_3 q_2}{(r_{32})_f} + \frac{q_3 q_1}{(r_{31})_f} + \frac{q_2 q_1}{(r_{21})_f} \right] + \frac{1}{2} mv^2$$

Here,  $(r_{21})_i = (r_{21})_f$

Substituting the proper values, we have

$$(9.0 \times 10^9) \left[ \frac{(-4)(2)}{(5.0)} + \frac{(-4)(2)}{(5.0)} \right] \times 10^{-12}$$

$$= (9.0 \times 10^9) \left[ \frac{(-4)(2)}{(3.0)} + \frac{(-4)(2)}{(3.0)} \right] \times 10^{-12} + \frac{1}{2} \times 10^{-3} \times v^2$$

$$\therefore (9 \times 10^{-3}) \left( -\frac{16}{5} \right) = (9 \times 10^{-3}) \left( -\frac{16}{3} \right) + \frac{1}{2} \times 10^{-3} \times v^2$$

$$(9 \times 10^{-3}) (16) \left( \frac{2}{15} \right) = \frac{1}{2} \times 10^{-3} \times v^2$$

$$\therefore v = 6.2 \text{ m/s}$$

## 18.7 Electric Potential

As we have discussed in Article 18.5 that an electric field at any point can be defined in two different ways:

- by the field strength  $\mathbf{E}$ , and
- by the electric potential  $V$  at the point under consideration.

Both  $\mathbf{E}$  and  $V$  are functions of position and there is a fixed relationship between these two. Of these, the field strength  $\mathbf{E}$  is a vector quantity while the electric potential  $V$  is a scalar quantity. In this article we will discuss about the electric potential and in the next the relationship between  $\mathbf{E}$  and  $V$ . **Potential** is the potential energy per unit charge. Electric potential at any point in an electric field is defined as the potential energy per unit charge, same as the field strength is defined as the force per unit charge. Thus,

$$V = \frac{U}{q_0}$$

or

$$U = q_0 V$$

The SI unit of potential is **volt (V)** which is equal to joule per coulomb. So,

$$1 \text{ V} = 1 \text{ volt} = 1 \text{ J/C} = 1 \text{ joule / coulomb}$$

The work done by the electrostatic force in displacing a test charge  $q_0$  from  $a$  to  $b$  in an electric field is defined as the negative of change in potential energy between them, or

$$\Delta U = -W_{a-b}$$

$$\therefore U_b - U_a = -W_{a-b}$$

We divide this equation by  $q_0$ ,

$$\frac{U_b}{q_0} - \frac{U_a}{q_0} = -\frac{W_{a-b}}{q_0}$$

$$\text{or } V_a - V_b = \frac{W_{a-b}}{q_0} \quad \text{as } V = \frac{U}{q_0}$$

Thus, the work done per unit charge by the **electric force** when a charged body moves from  $a$  to  $b$  is equal to the potential at  $a$  minus the potential at  $b$ . We sometimes abbreviate this difference as  $V_{ab} = V_a - V_b$ .

Another way to interpret the potential difference  $V_{ab}$  is that the potential at  $a$  minus potential at  $b$ , equals the work that must be done to move a unit positive charge slowly from  $b$  to  $a$  against the electric force.

$$V_a - V_b = \frac{(W_{b-a})_{\text{external force}}}{q_0}$$

### Absolute Potential at Some Points

Suppose we take the point  $b$  at infinity and as a reference point assign the value  $V_b = 0$ , the above equations can be written as

$$V_a - V_b = \frac{(W_{a-b})_{\text{electric force}}}{q_0} = \frac{(W_{b-a})_{\text{external force}}}{q_0}$$

$$\text{or } V_a = \frac{(W_{a-\infty})_{\text{electric force}}}{q_0} = \frac{(W_{\infty-a})_{\text{external force}}}{q_0}$$

Thus, the absolute electric potential at point  $a$  in an electric field can be defined as the work done in displacing a unit positive charge from infinity to  $a$  by the external force or the work done per unit positive charge in displacing it from  $a$  to infinity.

## Objective Galaxy

Following three formulae are very useful in the problems related to work done in electric field.

$$(W_{a-b})_{\text{electric force}} = q_0 (V_a - V_b)$$

$$(W_{a-b})_{\text{external force}} = q_0 (V_b - V_a) = -(W_{a-b})_{\text{electric force}}$$

$$(W_{\infty-a})_{\text{external force}} = q_0 V_a$$

Here,  $q_0$ ,  $V_a$  and  $V_b$  are to be substituted with sign.

**Example 18.17** The electric potential at point  $A$  is 20 V and at  $B$  is -40 V. Find the work done by an external force and electrostatic force in moving an electron slowly from  $B$  to  $A$ .

**Solution** Here, the test charge is an electron, i.e.,

$$q_0 = -1.6 \times 10^{-19} \text{ C}$$

$$V_A = 20 \text{ V}$$

and

$$V_B = -40 \text{ V}$$

**Work done by external force**

$$(W_{B-A})_{\text{external force}} = q_0 (V_A - V_B)$$

$$= (-1.6 \times 10^{-19}) [(20) - (-40)]$$

$$= -9.6 \times 10^{-18} \text{ J}$$

### Work done by electric force

$$(W_{B-A})_{\text{electric force}} = -(W_{B-A})_{\text{external force}}$$

$$= -(-9.6 \times 10^{-18} \text{ J})$$

$$= 9.6 \times 10^{-18} \text{ J}$$

**Note** Here we can see that the electron (a negative charge) moves from  $B$  (lower potential) to  $A$  (higher potential) and the work done by electric force is positive. Therefore, we may conclude that whenever a negative charge moves from a lower potential to higher potential work done by the electric force is positive or when a positive charge moves from lower potential to higher potential the work done by the electric force is negative.

**Example 18.18** Find the work done by some external force in moving a charge  $q = 2 \mu\text{C}$  from infinity to a point where electric potential is  $10^4 \text{ V}$ .

**Solution** Using the relation,

$$(W_{\infty-a})_{\text{external force}} = qV_a$$

$$\text{We have, } (W_{\infty-a})_{\text{external force}} = (2 \times 10^{-6}) (10^4)$$

$$= 2 \times 10^{-2} \text{ J}$$

### Electric Potential Due to a Point Charge $q$

From the definition of potential,

$$V = \frac{U}{q_0} = \frac{\frac{1}{4\pi\epsilon_0} \cdot \frac{q q_0}{r}}{q_0}$$

or

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$$

Here,  $r$  is the distance from the point charge  $q$  to the point at which the potential is evaluated.

If  $q$  is positive, the potential that it produces is positive at all points; if  $q$  is negative, it produces a potential that is negative everywhere. In either case,  $V$  is equal to zero at  $r = \infty$ .

### Electric Potential Due to a System of Charges

Just as the electric field due to a collection of point charges is the vector sum of the fields produced by each charge, the electric potential due to a collection of point charges is the scalar sum of the potentials due to each charge.

$$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

## Objective Galaxy

In the equation  $V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$  if the whole charge is at equal distance  $r_0$  from the point where  $V$  is to be evaluated, then we can write,

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_{\text{net}}}{r_0}$$

where  $q_{\text{net}}$  is the algebraic sum of all the charges of which the system is made.

Here are few examples:

**Examples 1.** Four charges are placed on the vertices of a square as shown in figure. The electric potential at centre of the square is zero as all the charges are at same distance from the centre and

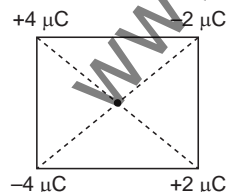


Fig. 18.23

$$q_{\text{net}} = 4 \mu\text{C} - 2 \mu\text{C} + 2 \mu\text{C} - 4 \mu\text{C}$$

$$= 0.$$

**Example 2.** A charge  $q$  is uniformly distributed over the circumference of a ring in Fig. (a) and is non-uniformly distributed in Fig. (b).

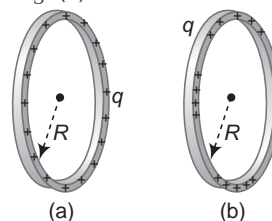


Fig. 18.24

The electric potential at the centre of the ring in both the cases is

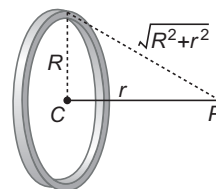


Fig. 18.25

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R} \quad (\text{where } R = \text{radius of ring})$$

and at a distance  $r$  from the centre of ring on its axis would be,

**Example 18.19** Three point charges  $q_1 = 1 \mu\text{C}$ ,  $q_2 = -2 \mu\text{C}$  and  $q_3 = 3 \mu\text{C}$  are placed at  $(1 \text{ m}, 0, 0)$ ,  $(0, 2 \text{ m}, 0)$  and  $(0, 0, 3 \text{ m})$  respectively. Find the electric potential at origin.

**Solution** The net electric potential at origin is,

$$V = \frac{1}{4\pi\epsilon_0} \left[ \frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} \right]$$

Substituting the values, we have

$$V = (9.0 \times 10^9) \left( \frac{1}{1.0} - \frac{2}{2.0} + \frac{3}{3.0} \right) \times 10^{-6} \\ = 9.0 \times 10^3 \text{ V}$$

**Example 18.20** A charge  $q = 10 \mu\text{C}$  is distributed uniformly over the circumference of a ring of radius  $3 \text{ m}$  placed on  $x$ - $y$  plane with its centre at origin. Find the electric potential at a point  $P(0, 0, 4 \text{ m})$ .

**Solution** The electric potential at point  $P$  would be,

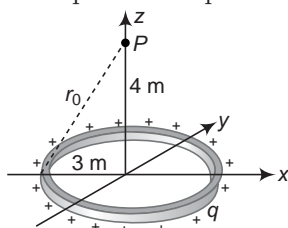


Fig. 18.26

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r_0}$$

Here  $r_0$  = distance of point  $P$  from the circumference of ring

$$= \sqrt{(3)^2 + (4)^2} = 5 \text{ m}$$

and  $q = 10 \mu\text{C} = 10^{-5} \text{ C}$

Substituting the values, we have

$$V = \frac{(9.0 \times 10^9) (10^{-5})}{(5.0)} = 1.8 \times 10^4 \text{ V}$$

### Variation of Electric Potential on the Axis of a Charged Ring

We have discussed in Medical Galaxy that the electric potential at the centre of a charged ring (whether charged uniformly or non-uniformly) is  $\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R}$  and at a distance  $r$  from

the centre on the axis of the ring

is  $\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{\sqrt{R^2 + r^2}}$ . From these expressions we can see that

electric potential is maximum at the centre and decreases as we move away from the centre on the axis. Thus potential varies with distance  $r$  as shown in figure.

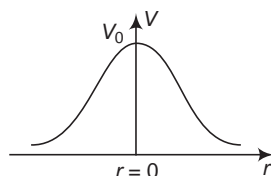


Fig. 18.27

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{\sqrt{R^2 + r^2}}$$

In the figure,  $V_0 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R}$

**Example 18.21** Find out the points on the line joining two charges  $+q$  and  $-3q$  (kept at a distance of  $1.0 \text{ m}$ ), where electric potential is zero.

**Solution** Let  $P$  be the point on the axis either to the left or to the right of charge  $+q$  at a distance  $r$  where potential is zero. Hence,

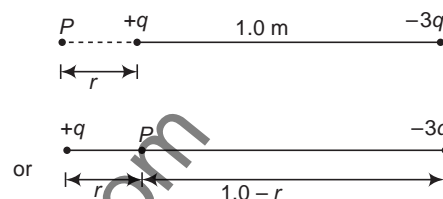


Fig. 18.28

$$V_P = \frac{q}{4\pi\epsilon_0 r} - \frac{3q}{4\pi\epsilon_0 (1+r)} = 0$$

Solving this, we get  $r = 0.5 \text{ m}$

$$\text{Further, } V_P = \frac{q}{4\pi\epsilon_0 r} - \frac{3q}{4\pi\epsilon_0 (1-r)} = 0,$$

which gives  $r = 0.25 \text{ m}$ .

Thus, the potential will be zero at point  $P$  on the axis which is either  $0.5 \text{ m}$  to the left or  $0.25 \text{ m}$  to the right of charge  $+q$ .

## 18.8 Relation between Electric Field and Potential

Let us first consider the case when electric potential  $V$  is known and we want to calculate  $\mathbf{E}$ . The relation is as under,

### In Case of Cartesian Coordinates

$$\mathbf{E} = E_x \hat{\mathbf{i}} + E_y \hat{\mathbf{j}} + E_z \hat{\mathbf{k}}$$

Here,  $E_x = -\frac{\partial V}{\partial x} = -(\text{partial derivative of } V \text{ wrt } x)$

$$E_y = -\frac{\partial V}{\partial y} = -(\text{partial derivative of } V \text{ wrt } y)$$

$$E_z = -\frac{\partial V}{\partial z} = -(\text{partial derivative of } V \text{ wrt } z)$$

$$\therefore \mathbf{E} = -\left[ \frac{\partial V}{\partial x} \hat{\mathbf{i}} + \frac{\partial V}{\partial y} \hat{\mathbf{j}} + \frac{\partial V}{\partial z} \hat{\mathbf{k}} \right]$$

This is also sometimes written as,

$$\mathbf{E} = -\text{gradient } V \\ = -\text{grad } V$$



$$= -\nabla V$$

**Example 18.22** The electric potential in a region is represented as,

$$V = 2x + 3y - z$$

obtain expression for electric field strength.

**Solution**

$$\mathbf{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

$$\text{Here, } \frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(2x + 3y - z) = 2$$

$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}(2x + 3y - z) = 3$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}(2x + 3y - z) = -1$$

$$\therefore \mathbf{E} = -2\hat{i} - 3\hat{j} + \hat{k}$$

We have determined how to calculate electric field  $\mathbf{E}$  from the electrostatic potential  $V$ . Let us now consider how to calculate potential difference or absolute potential if electric field  $\mathbf{E}$  is known. For this use the relation,

$$dV = -\mathbf{E} \cdot d\mathbf{r}$$

$$\text{or } \int_A^B dV = - \int_A^B \mathbf{E} \cdot d\mathbf{r}$$

$$\text{or } V_B - V_A = - \int_A^B \mathbf{E} \cdot d\mathbf{r}$$

$$\text{Here, } d\mathbf{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

### When $\mathbf{E}$ is Uniform

Let us take this case with the help of an example.

**Example 18.23** Find  $V_{ab}$  in an electric field

$$\mathbf{E} = (2\hat{i} + 3\hat{j} + 4\hat{k}) \frac{N}{C}$$

$$\text{where } \mathbf{r}_a = (\hat{i} - 2\hat{j} + \hat{k}) \text{ m}$$

$$\text{and } \mathbf{r}_b = (2\hat{i} + \hat{j} - 2\hat{k}) \text{ m.}$$

**Solution** Here the given field is uniform (constant). So using,

$$dV = -\mathbf{E} \cdot d\mathbf{r}$$

$$\begin{aligned} \text{or } V_{ab} &= V_a - V_b = - \int_b^a \mathbf{E} \cdot d\mathbf{r} \\ &= - \int_{(2, 1, -2)}^{(1, -2, 1)} (2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k}) \\ &= - \int_{(2, 1, -2)}^{(1, -2, 1)} (2dx + 3dy + 4dz) \\ &= -[2x + 3y + 4z]_{(2, 1, -2)}^{(1, -2, 1)} = -1 \text{ volt} \end{aligned}$$

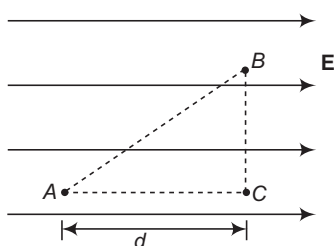


Fig. 18.29

**Note** In uniform electric field we can also apply,  
 $V = Ed$

Here  $V$  is the potential difference between any two points,  $E$  the magnitude of electric field and  $d$  is the projection of the distance between two points along the electric field.

For example, in the figure if we are interested in the potential difference between points  $A$  and  $B$  we will have to keep two points in mind,

(i)  $V_A > V_B$  as electric lines always flow from higher potential to lower potential.

(ii)  $d \neq AB$  but  $d = AC$

Hence, in the above figure,  $V_A - V_B = Ed$ .

**Example 18.24** In uniform electric field  $\mathbf{E} = 10 \text{ N/C}$  as shown in figure, find :

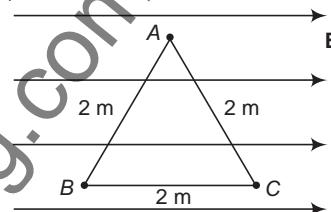


Fig. 18.30

(a)  $V_A - V_B$

(b)  $V_B - V_C$

**Solution** (a)  $V_B > V_A$ , so,  $V_A - V_B$  will be negative.

Further  $d_{AB} = 2 \cos 60^\circ = 1 \text{ m}$

$$\begin{aligned} \therefore V_A - V_B &= -Ed_{AB} \\ &= (-10)(1) = -10 \text{ V} \end{aligned}$$

(b)  $V_B > V_C$ , so,  $V_B - V_C$  will be positive

Further,  $d_{BC} = 2.0 \text{ m}$

$$\therefore V_B - V_C = (10)(2) = 20 \text{ V}$$

**Example 18.25** A uniform electric field of  $100 \text{ V/m}$  is directed at  $30^\circ$  with the positive  $x$ -axis as shown in figure. Find the potential difference  $V_{BA}$  if  $OA = 2 \text{ m}$  and  $OB = 4 \text{ m}$ .

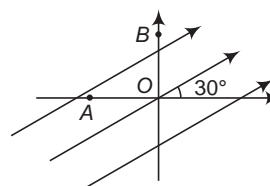


Fig. 18.31

**Solution** This problem can be solved by both the methods as discussed above.

**Method 1.** Electric field in vector form can be written as,

$$\mathbf{E} = (100 \cos 30^\circ \hat{i} + 100 \sin 30^\circ \hat{j}) \text{ V/m}$$

$$= (50\sqrt{3} \hat{i} + 50\hat{j}) \text{ V/m}$$

$$A \equiv (-2 \text{ m}, 0, 0)$$

$$\text{and } B \equiv (0, 4 \text{ m}, 0)$$

$$\therefore V_{BA} = V_B - V_A$$

$$\begin{aligned}
 &= - \int_A^B \mathbf{E} \cdot d\mathbf{r} \\
 &= - \int_{(-2\text{ m}, 0, 0)}^{(0, 4\text{ m}, 0)} (50\sqrt{3}\hat{i} + 50\hat{j}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k}) \\
 &= - [50\sqrt{3}x + 50y]_{(-2\text{ m}, 0, 0)}^{(0, 4\text{ m}, 0)} \\
 &= -100(2 + \sqrt{3})\text{ V}
 \end{aligned}$$

**Method 2.** We can also use,  $V = Ed$

With the view that  $V_A > V_B$  or  $V_B - V_A$  will be negative.

Here,  $d_{AB} = OA \cos 30^\circ + OB \sin 30^\circ$

$$= 2 \times \frac{\sqrt{3}}{2} + 4 \times \frac{1}{2} = (\sqrt{3} + 2)$$

$$\therefore V_B - V_A = -Ed_{AB} = -100(2 + \sqrt{3})$$

## 18.9 Equipotential Surfaces

The equipotential surfaces in an electric field have the same basic idea as topographic maps used by civil engineers or mountain climbers. On a topographic map, contour lines are drawn passing through the points having the same elevation. The potential energy of a mass  $m$  does not change along a contour line as the elevation is same everywhere.

By analogy to contour lines on a topographic map, an **equipotential surface** is a three dimensional surface on which the electric potential  $V$  is the same at every point on it. An equipotential surface has the following characteristics.

1. Potential difference between any two points in an equipotential surface is zero.
2. If a test charge  $q_0$  is moved from one point to the other on such a surface, the electric potential energy  $q_0 V$  remains constant.
3. No work is done by the electric force when the test charge is moved along this surface.
4. Two equipotential surfaces can never intersect each other because otherwise the point of intersection will have two potentials which is of course not acceptable.
5. As the work done by electric force is zero when a test charge is moved along the equipotential

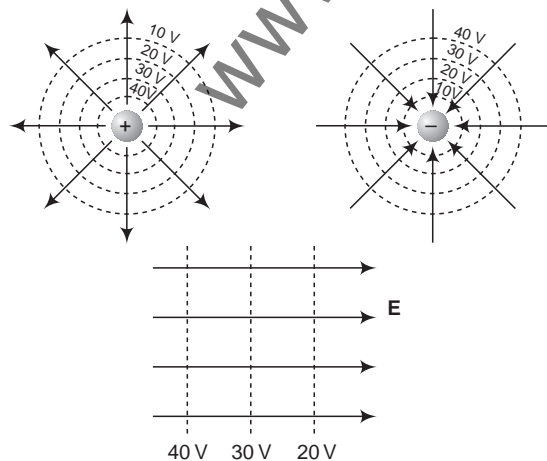


Fig. 18.32

surface, it follows that  $\mathbf{E}$  must be perpendicular to the surface at every point so that the electric force  $q_0\mathbf{E}$  will always be perpendicular to the displacement of a charge moving on the surface. Thus, **field lines and equipotential surfaces are always mutually perpendicular.** Some equipotential surfaces are shown in Fig. 18.32.

The equipotential surfaces are a family of concentric spheres for a point charge or a sphere of charge and are a family of concentric cylinders for a line of charge or cylinder of charge. For a special case of a uniform field, where the field lines are straight, parallel and equally spaced the equipotentials are parallel planes perpendicular to the field lines.

**Note** While drawing the equipotential surfaces we should keep in mind the two main points.

1. These are perpendicular to field lines at all places.
2. Field lines always flow from higher potential to lower potential.

**Example 18.26** Equipotential spheres are drawn round a point charge. As we move away from the charge, will the spacing between two spheres having a constant potential difference decrease, increase or remain constant.

**Solution**

$$V_1 > V_2$$

$$V_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r_1}$$

$$V_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r_2}$$

and

$$\text{Now, } V_1 - V_2 = \frac{q}{4\pi\epsilon_0} \left( \frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$= \frac{q}{4\pi\epsilon_0} \left( \frac{r_2 - r_1}{r_1 r_2} \right)$$

$$\therefore (r_2 - r_1) = \frac{(4\pi\epsilon_0)(V_1 - V_2)}{q} (r_1 r_2)$$

For a constant potential difference ( $V_1 - V_2$ ),

$$r_2 - r_1 \propto r_1 r_2$$

i.e., the spacing between two spheres ( $r_2 - r_1$ ) increases as we move away from the charge, because the product  $r_1 r_2$  will increase.

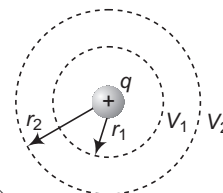


Fig. 18.33

## 18.10 Electric Dipole

A pair of equal and opposite point charges  $\pm q$ , that are separated by a fixed distance is known as **electric dipole**. Electric dipole occurs in nature in a variety of situations. The hydrogen fluoride (HF) molecule is typical. When a hydrogen atom combines with a fluorine atom, the single electron of the former is strongly attracted to the latter and spends most of its time near the fluorine atom. As a result, the molecule consists of a strongly negative fluorine ion



Fig. 18.34

some (small) distance away from a strongly positive ion, though the molecule is electrically neutral overall.

Every electric dipole is characterized by its **electric dipole moment** which is a vector  $\mathbf{p}$  directed from the negative to the positive charge.

The magnitude of dipole moment is,

$$p = (2a)q$$

Here,  $2a$  is the distance between the two charges.

## Electric Potential and Field Due to an Electric Dipole

Consider an electric dipole lying along positive y-direction with its centre at origin.

$$\mathbf{p} = 2aq \hat{\mathbf{j}}$$

The electric potential due to this dipole at point A ( $x, y, z$ ) as shown is simply the sum of the potentials due to the two charges. Thus,

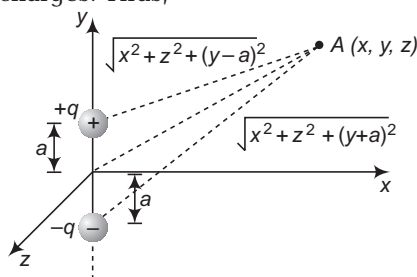


Fig. 18.35

$$V = \frac{1}{4\pi\epsilon_0} \left[ \frac{q}{\sqrt{x^2 + (y-a)^2 + z^2}} - \frac{q}{\sqrt{x^2 + (y+a)^2 + z^2}} \right]$$

By differentiating this function, we obtain the electric field of the dipole.

$$\begin{aligned} E_x &= -\frac{\partial V}{\partial x} \\ &= \frac{q}{4\pi\epsilon_0} \left\{ \frac{x}{[x^2 + (y-a)^2 + z^2]^{3/2}} - \frac{x}{[x^2 + (y+a)^2 + z^2]^{3/2}} \right\} \\ E_y &= -\frac{\partial V}{\partial y} \\ &= \frac{q}{4\pi\epsilon_0} \left\{ \frac{y-a}{[x^2 + (y-a)^2 + z^2]^{3/2}} - \frac{y+a}{[x^2 + (y+a)^2 + z^2]^{3/2}} \right\} \\ E_z &= -\frac{\partial V}{\partial z} \\ &= \frac{q}{4\pi\epsilon_0} \left\{ \frac{z}{[x^2 + (y-a)^2 + z^2]^{3/2}} - \frac{z}{[x^2 + (y+a)^2 + z^2]^{3/2}} \right\} \end{aligned}$$

## Special Cases

### (i) On the axis of the dipole (i.e., along y-axis)

$$x=0, z=0$$

$$\therefore V = \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{y-a} - \frac{1}{y+a} \right] = \frac{2aq}{4\pi\epsilon_0 (y^2 - a^2)}$$

$$\text{or } V = \frac{p}{4\pi\epsilon_0 (y^2 - a^2)} \quad (\text{as } 2aq = p)$$

i.e., at a distance  $r$  from the centre of the dipole ( $y = r$ )

$$V = \frac{p}{4\pi\epsilon_0 (r^2 - a^2)}$$

$$\text{or } V_{\text{axis}} \approx \frac{p}{4\pi\epsilon_0 r^2} \quad (\text{for } r \gg a)$$

$V$  is positive when the point under consideration is towards positive charge and negative if it is towards negative charge.

Moreover the components of electric field are as under,

$$E_x = 0, E_z = 0 \quad (\text{as } x=0, z=0)$$

$$\text{and } E_y = \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{(y-a)^2} - \frac{1}{(y+a)^2} \right]$$

$$= \frac{4ayq}{4\pi\epsilon_0 (y^2 - a^2)^2}$$

$$\text{or } E_y = \frac{1}{4\pi\epsilon_0} \frac{2py}{(y^2 - a^2)^2}$$

Note that  $E_y$  is along positive y-direction or parallel to  $\mathbf{p}$ .

Further, at a distance  $r$  from the centre of the dipole ( $y = r$ ).

$$E_y = \frac{1}{4\pi\epsilon_0} \frac{2pr}{(r^2 - a^2)^2}$$

$$\text{or } E_{\text{axis}} \approx \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3} \quad (\text{for } r \gg a)$$

### (ii) On the perpendicular bisector of dipole

Say along x-axis (it may be along z-axis also).

$$y=0, z=0$$

$$\therefore V = \frac{1}{4\pi\epsilon_0} \left[ \frac{q}{\sqrt{x^2 + a^2}} - \frac{q}{\sqrt{x^2 + a^2}} \right] = 0$$

$$\text{or } V_{\perp \text{ bisector}} = 0$$

Moreover the components of electric field are as under,

$$E_x = 0, E_z = 0$$

$$\text{and } E_y = \frac{q}{4\pi\epsilon_0} \left\{ \frac{-a}{(x^2 + a^2)^{3/2}} - \frac{a}{(x^2 + a^2)^{3/2}} \right\}$$

$$= \frac{-2aq}{4\pi\epsilon_0 (x^2 + a^2)^{3/2}}$$

$$\text{or } E_y = -\frac{1}{4\pi\epsilon_0} \frac{p}{(x^2 + a^2)^{3/2}}$$

Here negative sign implies that the electric field is along negative y-direction or antiparallel to  $\mathbf{p}$ .

Further, at a distance  $r$  from the centre of dipole ( $x = r$ ), the magnitude of electric field is,

$$E = \frac{1}{4\pi\epsilon_0} \frac{p}{(r^2 + a^2)^{3/2}}$$

$$\text{or } E_{\perp \text{ bisector}} \approx \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3} \quad (\text{for } r \gg a)$$

## Electric Dipole in Uniform Electric Field

As we have said earlier also a uniform electric field means, at every point the direction and magnitude of electric field is constant. A uniform electric field is shown by parallel equidistant lines. The field due to a point charge or due to an electric dipole is non-uniform in nature. Uniform electric field is found between the plates of a parallel plate capacitor. Now let us discuss the behaviour of a dipole in uniform electric field.

## Force on Dipole

Suppose an electric dipole of dipole moment  $|\mathbf{p}| = 2aq$  is placed in a uniform electric field  $\mathbf{E}$  at an angle  $\theta$ . Here  $\theta$  is the angle between  $\mathbf{p}$  and  $\mathbf{E}$ . A force  $\mathbf{F}_1 = q\mathbf{E}$  will act on positive charge and  $\mathbf{F}_2 = -q\mathbf{E}$  on negative charge. Since,  $\mathbf{F}_1$  and  $\mathbf{F}_2$  are equal in magnitude but opposite in direction. Hence,

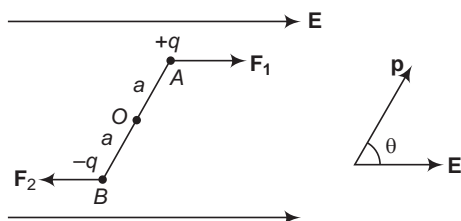


Fig. 18.36

$$\mathbf{F}_1 + \mathbf{F}_2 = 0$$

$$\text{or } \mathbf{F}_{\text{net}} = 0$$

Thus, net force on a dipole in uniform electric field is zero. While in a nonuniform electric field it may or may not be zero.

## Torque on Dipole

The torque of  $\mathbf{F}_1$  about O,

$$\begin{aligned} \tau_1 &= \mathbf{OA} \times \mathbf{F}_1 \\ &= q(\mathbf{OA} \times \mathbf{E}) \end{aligned}$$

and torque of  $\mathbf{F}_2$  about O is,

$$\begin{aligned} \tau_2 &= \mathbf{OB} \times \mathbf{F}_2 \\ &= -q(\mathbf{OB} \times \mathbf{E}) \\ &= q(\mathbf{BO} \times \mathbf{E}) \end{aligned}$$

The net torque acting on the dipole is,

$$\begin{aligned} \tau &= \tau_1 + \tau_2 \\ &= q(\mathbf{OA} \times \mathbf{E}) + q(\mathbf{BO} \times \mathbf{E}) \\ &= q(\mathbf{OA} + \mathbf{BO}) \times \mathbf{E} \\ &= q(\mathbf{BA} \times \mathbf{E}) \end{aligned}$$

$$\text{or } \tau = \mathbf{p} \times \mathbf{E}$$

Thus, the magnitude of torque is  $\tau = pE \sin \theta$ . The direction of torque is perpendicular to the plane of paper inwards. Further this torque is zero at  $\theta = 0^\circ$  or  $\theta = 180^\circ$ , i.e., when the dipole is parallel or antiparallel to  $\mathbf{E}$  and maximum at  $\theta = 90^\circ$ .

## Potential Energy of Dipole

When an electric dipole is placed in an electric field  $\mathbf{E}$ , a torque  $\tau = \mathbf{p} \times \mathbf{E}$  acts on it. If we rotate the dipole through a small angle  $d\theta$ , the work done by the torque is,

$$dW = \tau d\theta$$

$$dW = -pE \sin \theta d\theta$$

The work is negative as the rotation  $d\theta$  is opposite to the torque. The change in electric potential energy of the dipole is therefore

$$\begin{aligned} dU &= -dW \\ &= pE \sin \theta d\theta \end{aligned}$$

Now at angle  $\theta = 90^\circ$ , the electric potential energy of the dipole may be assumed to be zero as net work done by the electric forces in bringing the dipole from infinity to this position will be zero. Integrating,

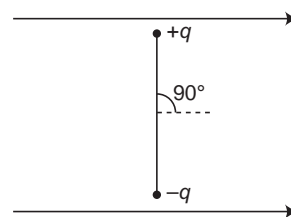


Fig. 18.37

$$dU = pE \sin \theta d\theta$$

From  $90^\circ$  to  $\theta$ , we have

$$\int_{90^\circ}^{\theta} dU = \int_{90^\circ}^{\theta} pE \sin \theta d\theta$$

$$\text{or } U(\theta) - U(90^\circ) = pE [-\cos \theta]_{90^\circ}^{\theta}$$

$$\begin{aligned} \therefore U(\theta) &= -pE \cos \theta \\ &= -\mathbf{p} \cdot \mathbf{E} \end{aligned}$$

If the dipole is rotated from an angle  $\theta_1$  to  $\theta_2$ , then work done by external forces  $= U(\theta_2) - U(\theta_1)$

$$\text{or } W_{\text{external forces}} = -pE \cos \theta_2 - (-pE \cos \theta_1)$$

$$\text{or } W_{\text{external forces}} = pE (\cos \theta_1 - \cos \theta_2)$$

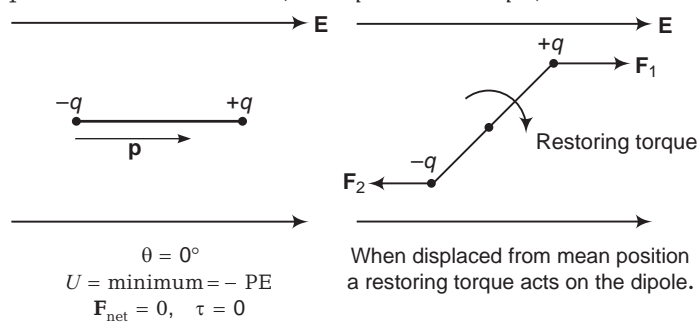
and work done by electric forces,

$$\begin{aligned} W_{\text{electric force}} &= -W_{\text{external force}} \\ &= pE (\cos \theta_2 - \cos \theta_1) \end{aligned}$$

## Equilibrium of Dipole

When an electric dipole is placed in a uniform electric field net force on it is zero for any position of the dipole in the electric field. But torque acting on it is zero only at  $\theta = 0^\circ$  and  $180^\circ$ . Thus, we can say that at these two positions of the dipole, net force or torque on it is zero or the dipole is

in equilibrium. Of this  $\theta = 0^\circ$  is the **stable equilibrium position** of the dipole because potential energy in this position is minimum ( $U = -pE \cos 0^\circ = -pE$ ) and when



other hand at  $\theta = 180^\circ$ , the potential energy of the dipole is maximum ( $U = -pE \cos 180^\circ = +pE$ ) and when it is displaced from this position, the torque has a tendency to

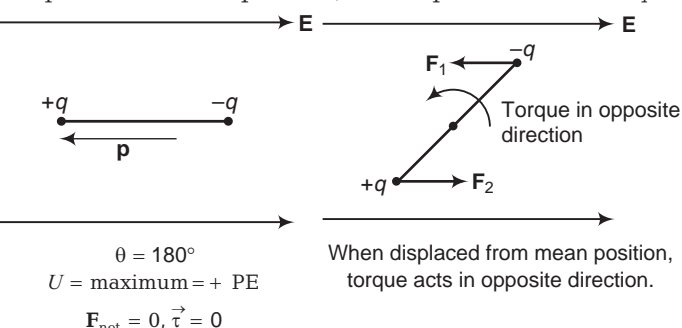


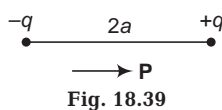
Fig. 18.38

displaced from this position a torque starts acting on it which is restoring in nature and which has a tendency to bring the dipole back in its equilibrium position. On the

rotate it in other direction. This torque is not restoring in nature. So this equilibrium is known as **unstable equilibrium position**.

## Objective Galaxy

1. As there are too many formulae in electric dipole, we have summarised them as under:



$$|p| = (2a)q$$

2. On the axis of dipole :

$$(i) \quad V = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^2 - a^2}$$

or  $V_{\text{axis}} \approx \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^2}$  if  $r \gg a$

$$(ii) \quad E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2pr}{(r^2 - a^2)^2}$$

or  $E_{\text{axis}} \approx \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3}$  for  $r \gg a$

3. On the perpendicular bisector of dipole :

$$(i) \quad V = 0$$

$$(ii) \quad E = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{(r^2 + a^2)^{3/2}}$$

(opposite to  $p$ )

$$\approx \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3}$$

(for  $r \gg a$ )

4. Dipole in uniform electric field

$$(i) \quad F_{\text{net}} = 0$$

$$(ii) \quad \tau = p \times E \quad \text{and} \quad |\tau| = pE \sin \theta$$

$$(iii) \quad U(\theta) = -P \cdot E = -pE \cos \theta \quad \text{with} \quad U(90^\circ) = 0$$

$$(iv) \quad (W_{\theta_1 \rightarrow \theta_2})_{\text{external force}} = pE (\cos \theta_2 - \cos \theta_1)$$

$$(v) \quad (W_{\theta_1 \rightarrow \theta_2})_{\text{electric force}} = pE (\cos \theta_2 - \cos \theta_1)$$

$$= -(W_{\theta_1 \rightarrow \theta_2})_{\text{external force}}$$

$$(vi) \quad \text{At } \theta = 0^\circ, F_{\text{net}} = 0, \tau_{\text{net}} = 0, U = \text{minimum}$$

(stable equilibrium position)

$$(vii) \quad \text{At } \theta = 180^\circ, F_{\text{net}} = 0, \tau_{\text{net}} = 0, U = \text{maximum}$$

(unstable equilibrium position)

**Example 18.27** An electric dipole of dipole moment  $p$  is placed in a uniform electric field  $E$  in stable equilibrium position. Its moment of inertia about the centroidal axis is  $I$ . If it is displaced slightly from its mean position find the period of small oscillations.

**Solution** When displaced at an angle  $\theta$ , from its mean position the magnitude of restoring torque is,

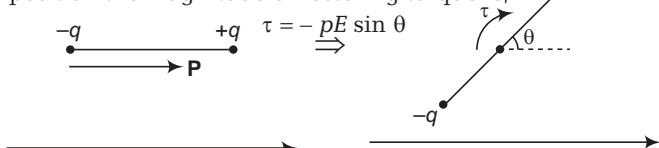


Fig. 18.40

For small angular displacement  $\sin \theta \approx \theta$

$$\tau = -pE \theta$$

The angular acceleration is,

$$\alpha = \frac{\tau}{I} = -\left(\frac{pE}{I}\right) \theta = -\omega^2 \theta$$

where

$$\omega^2 = \frac{pE}{I}$$

$\therefore$

$$T = 2\pi \sqrt{\frac{I}{pE}}$$



## 18.11 Gauss's Law

Gauss's law is a tool of simplifying electric field calculations where there is symmetrical distribution of charge. Gauss's law is a relation between the field at all the points on the surface and the total charge enclosed within the surface.

### Electric Flux

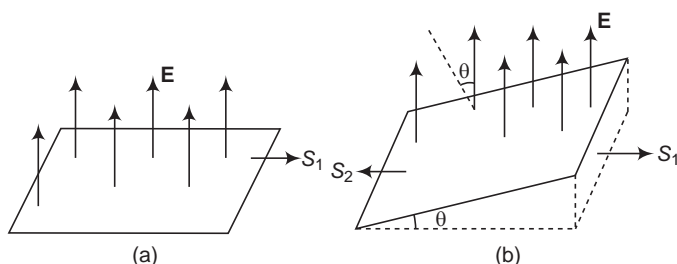


Fig. 18.41

Electric flux ( $\phi_e$ ) is a measure of the number of field lines crossing a surface.

Fig. (a) shows a planar surface of area  $S_1$  that is perpendicular to a uniform electric field  $E$ . The number of field lines passing through per unit area ( $N/S_1$ ) will be proportional to the electric field. Or,

$$\frac{N}{S_1} \propto E$$

$$\therefore N \propto ES_1 \quad \dots (i)$$

The quantity  $ES_1$  is the **electric flux** through  $S_1$ . It is a scalar quantity and has the SI unit  $\text{N}\cdot\text{m}^2/\text{C}$ . Eq. (i) can also be written as,

$$N \propto \phi_e$$

Now let us consider a planar surface that is not perpendicular to the field. How would the electric flux then be represented? Fig. (b) shows a surface  $S_2$  that is inclined at an angle  $\theta$  to the electric field. Its projection perpendicular to  $E$  is  $S_1$ . The areas are related by,

$$S_2 \cos \theta = S_1$$

Because the same number of field lines cross both  $S_1$  and  $S_2$ , the fluxes through both surfaces must be the same. The flux through  $S_2$  is therefore,

$$\phi_e = ES_1 = ES_2 \cos \theta$$

Designating  $\hat{n}_2$  as a unit vector normal to  $S_2$ , we obtain

$$\phi_e = E \cdot \hat{n}_2 S_2$$

This result can be easily generalized to the case of an arbitrary electric field  $E$  varying over an arbitrary surface  $S$ .

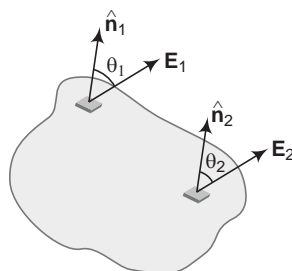


Fig. 18.42

We first divide  $S$  into infinitesimal elements with areas  $dS_1, dS_2, \dots$ , etc. and unit normals  $\hat{n}_1, \hat{n}_2, \dots$ , etc. Since, the elements are infinitesimal, they may be assumed to be planar and  $E$  may be taken as constant over any element. The flux  $d\phi_e$  through an area  $dS$  is given by,

$$d\phi_e = E dS \cos \theta = E \cdot \hat{n} dS$$

The net flux is the sum of the infinitesimal flux elements over the entire surface.

$$\therefore \phi_e = \int_S E \cdot \hat{n} dS \quad (\text{open surface})$$

To distinguish between the flux through an open surface and the flux through a closed surface, we represent flux for the latter case by,

$$\phi_e = \oint_S E \cdot \hat{n} dS \quad (\text{closed surface})$$

### Direction of $\hat{n}$

Since,  $\hat{n}$  is a unit vector normal to a surface, it has two possible directions at every point on that surface. On a closed surface  $\hat{n}$  is usually chosen to be the outward normal at every point. For an open surface we can use either direction, as long as we are consistent over the entire surface. It is sometimes helpful to visualize electric flux as the flow of the field lines through a surface. When field lines leave (or flow out of) a closed surface,  $\phi_e$  is positive, when they enter (or flow into) the surface,  $\phi_e$  is negative. But remember that field lines are just a visual aid, these are just imaginary lines.

Here are two special cases for calculating the electric flux passing through a surface  $S$  of finite size (whether closed or open)

**Case 1.**  $\phi_e = ES$

If at every point on the surface, the magnitude of electric field is constant and perpendicular (to the surface).

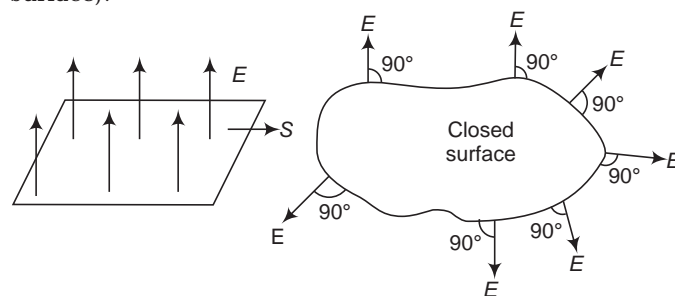


Fig. 18.43

**Case 2.**  $\phi_e = 0$

If at all points on the surface the electric field is tangential to the surface.

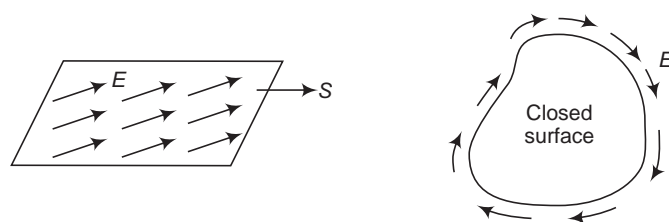


Fig. 18.44

## Gauss's Law

This law gives a relation between the net electric flux through a closed surface and the charge enclosed by the surface.

According to this law, "the net electric flux through any closed surface is equal to the net charge inside the surface divided by  $\epsilon_0$ ." In symbols it can be written as,

$$\phi_E = \oint_S \mathbf{E} \cdot \hat{\mathbf{n}} dS = \frac{q_{in}}{\epsilon_0} \quad \dots(i)$$

where  $q_{in}$  represents the net charge inside the surface and  $\mathbf{E}$  represents the electric field at any point on the surface.

In principle Gauss's law is valid for the electric field of any system of charges or continuous distribution of charge. In practice however, the technique is useful for calculating the electric field only in situations where the degree of symmetry is high. Gauss's law can be used to evaluate the electric field for charge distributions that have spherical, cylindrical or plane symmetry.

Gauss's law in simplified form can be written as under,

$$ES = \frac{q_{in}}{\epsilon_0} \quad \dots(ii)$$

but this form of Gauss's law is applicable only under following two conditions :

- the electric field at every point on the surface is either perpendicular or tangential.
- magnitude of electric field at every point where it is perpendicular to the surface has a constant value (say  $E$ ).

Here  $S$  is the area where electric field is perpendicular to the surface.

## Applications of Gauss's Law

As Gauss's law does not provide expression for electric field but provides only for its flux through a closed surface. To calculate  $E$  we choose an imaginary closed surface (called Gaussian surface) in which Eq. (i) or (ii) can be applied easily. In most of the cases we will use equation number (ii). Let us discuss few simple cases.

### (i) Electric field due to a point charge

The electric field due to a point charge is every where radial. We wish to find the electric field at a distance  $r$  from the charge  $q$ . We select Gaussian surface, a sphere at distance  $r$  from the charge. At every point of this sphere the

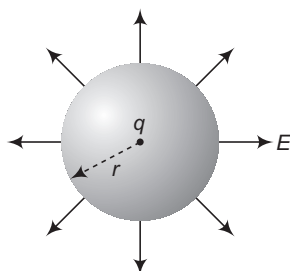


Fig. 18.45

electric field has the same magnitude  $E$  and it is perpendicular to the surface itself. Hence, we can apply the simplified form of Gauss law,

$$ES = \frac{q_{in}}{\epsilon_0}$$

Here,  $S$  = area of sphere =  $4\pi r^2$  and

$q_{in}$  = net charge enclosing the Gaussian surface =  $q$

$$\therefore E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$\therefore E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

It is nothing but Coulomb's law.

### (ii) Electric field due to a linear charge distribution

Consider a long line charge with a linear charge density (charge per unit length)  $\lambda$ . We have to calculate the electric field at a point, a distance  $r$  from the line charge. We construct a Gaussian surface, a cylinder of any arbitrary length  $l$  of radius  $r$  and its axis coinciding with the axis of the line charge. This cylinder have three surfaces. One is curved surface and the two plane parallel surfaces. Field lines at plane parallel surfaces are tangential (so flux passing through these surfaces is zero). The magnitude of electric field is having the same magnitude (say  $E$ ) at curved surface and simultaneously the electric field is perpendicular at every point of this surface.

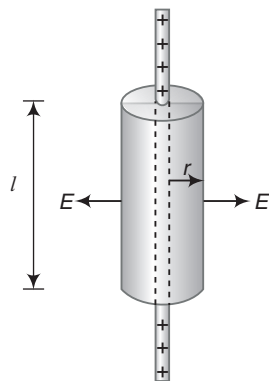
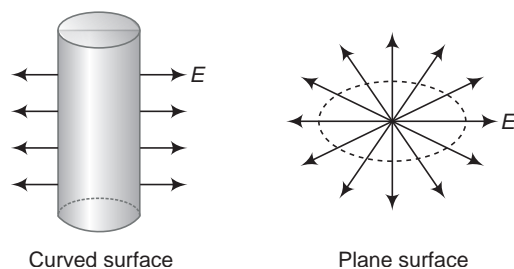


Fig. 18.46

Hence, we can apply the Gauss law as,

$$ES = \frac{q_{in}}{\epsilon_0}$$



Curved surface

Plane surface

Fig. 18.47

Here,  $S$  = area of curved surface =  $(2\pi rl)$

and  $q_{in}$  = net charge enclosing this cylinder =  $\lambda l$

$$\therefore E(2\pi rl) = \frac{\lambda l}{\epsilon_0}$$

$$\therefore E = \frac{\lambda}{2\pi\epsilon_0 r}$$