1 NUMBER SYSTEMS

EXERCISE 1.1

Q.1. *Is zero a rational number? Can you write it in the form* $\frac{p}{q}$*, where p and q are integers and* $q \neq 0$*?*

Sol. Yes, zero is a rational number. It can be written as $\frac{0}{1}, \frac{0}{2}$, etc., in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. **Ans.**

Q.2. *Find six rational numbers between* 3 *and* 4.

Sol. To find six rational numbers between 3 and 4 denominator should be made equal to 6 + 1 = 7.

Therefore, $3 = \frac{3 \times 7}{7} = \frac{21}{7}$ $\quad 4 = \frac{4 \times 7}{7} = \frac{28}{7}$

Six rational numbers between 3 and 4 can be found by varying the numerator between 21 and 28.

Or, the numbers are $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$. **Ans.**

Q.3. *Find five rational numbers between* $\frac{3}{5}$ *and* $\frac{4}{5}$.

Sol. To find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$, we may add the given numbers and divide by 2, and repeat the process.

$$\frac{\frac{3}{5}+\frac{4}{5}}{2} = \frac{7}{5 \times 2} = \frac{7}{10} = x_1$$

$$\frac{7}{10}+\frac{4}{5} = \frac{7+8}{10} = \frac{15}{10}$$

$$\text{Next rational number} = \frac{15}{10 \times 2} = \frac{15}{20} = \frac{3}{4} = x_2$$

$$\frac{3}{4}+\frac{4}{5} = \frac{15+16}{20} = \frac{31}{20}$$

$$\text{Next rational number} = \frac{31}{20 \times 2} = \frac{31}{40} = x_3$$

$$\frac{31}{40}+\frac{4}{5} = \frac{31+32}{40} = \frac{63}{40}$$

$$\text{Next rational number} = \frac{63}{40 \times 2} = \frac{63}{80} = x_4$$

$$\frac{63}{80}+\frac{4}{5} = \frac{63+64}{80} = \frac{127}{80}$$

$$\text{Next rational number} = \frac{127}{80 \times 2} = \frac{127}{160} = x_5$$

$x_1 = \frac{7}{10}, x_2 = \frac{3}{4}, x_3 = \frac{31}{40}, x_4 = \frac{63}{80}, x_5 = \frac{127}{160}$. **Ans.**

(**Note :** Many answers are possible. There are of course infinitely many rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.)

Q.4. *State whether the following statements are true or false. Give reasons for your answers.*

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Sol. (i) True, since the collection of whole numbers contains all the natural numbers and in addition zero.

(ii) False. Negative integers are not whole numbers.

(iii) False. Numbers such as $\frac{2}{3}, \frac{3}{4}, \frac{-3}{5}$, etc., are rational numbers but not whole numbers.

EXERCISE 1.2

Q.1. *State whether the following statements are true or false. Justify your answers.*

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Sol. (i) True. All irrational and rational numbers together make up the collection of real numbers R.

(ii) False, e.g. between $\sqrt{2}$ and $\sqrt{3}$ there are infinitely many numbers and these can not be represented in the form $\sqrt{m}$, where m is a natural number.

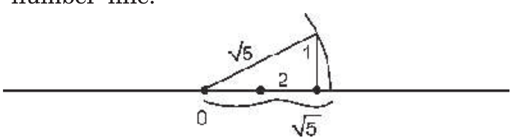
(iii) False. All rational numbers are also real numbers.

Q.2. *Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.*

Sol. Square roots of positive integers are not irrational. For example, $\sqrt{4} = 2$, which is a rational number.

Q.3. *Show how $\sqrt{5}$ can be represented on the number line.*

Sol. To represent $\sqrt{5}$ on the number line we take a length of two units from 0 on the number line in positive direction and one unit perpendicular to it. The hypotenuse of the triangle thus formed is of length $\sqrt{5}$. Then with the help of a divider a length equal to the hypotenuse of $\sqrt{5}$ units can be cut on the number line.



EXERCISE 1.3

Q.1. *Write the following in decimal form and say what kind of decimal expansion each has :*

(i) $\frac{36}{100}$ *(ii)* $\frac{1}{11}$ *(iii)* $4\frac{1}{8}$ *(iv)* $\frac{3}{13}$ *(v)* $\frac{2}{11}$ *(vi)* $\frac{329}{400}$

Sol. (i) 0.36, terminating. (ii) $0.\overline{09}$, recurring non-terminating.

(iii) 4.125, terminating. (iv) $0.\overline{230769}$, recurring non-terminating.

(v) $0.\overline{18}$, non-terminating recurring. (vi) 0.8225, terminating.

Q.2. *You know that* $\frac{1}{7} = 0.\overline{142857}$. *Can you predict what the decimal expansions of* $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ *are, without actually doing the long division? If so, how?*

Sol. $\frac{2}{7} = 2\times\frac{1}{7} = 0.\overline{285714}$, $\frac{3}{7} = 3\times\frac{1}{7} = 0.\overline{428571}$ $\frac{4}{7} = 4\times\frac{1}{7} = 0.\overline{571428}$,

$\frac{5}{7} = 5\times\frac{1}{7} = 0.\overline{714285}$ $\frac{6}{7} = 6\times\frac{1}{7} = 0.\overline{857142}$

Q.3. *Express the following in the form of* $\frac{p}{q}$*, where p and q are integers and* $q \neq 0$.

(i) $0.\overline{6}$ *(ii)* $0.4\overline{7}$ *(iii)* $0.\overline{001}$

Sol. (i) $x = 0.\overline{6} = 0.666\ ...$ to be written in $\frac{p}{q}$ form.

One digit 6 is repeating.
We multiply it with 10 on both sides.

$$10x = 6.\overline{6}$$
$$\Rightarrow \quad 10x = 6 + x$$
$$\Rightarrow \quad 10x - x = 6$$
$$\Rightarrow \quad 9x = 6$$
$$\Rightarrow \quad x = \frac{6}{9} = \frac{2}{3} \quad \textbf{Ans.}$$

(ii) $x = 0.4\overline{7} = 0.4777\ ...$
One digit is repeating.
We multiply by 10 on both sides.

$$\therefore \quad 10x = 4.\overline{7}$$
$$= 4.3 + .4\overline{7}$$
$$= 4.3 + x$$
$$\Rightarrow \quad 9x = 4.3$$
$$\Rightarrow \quad x = \frac{4.3}{9} = \frac{43}{90} \quad \textbf{Ans.}$$

(iii) $x = 0.\overline{001}$.
Here three digits repeats; we multiply with 1000.

$$\therefore \quad 1000x = 1.\overline{001}$$
$$1000x = 1 + x$$

$$\Rightarrow \quad 1000x - x = 1$$
$$\Rightarrow \quad 999x = 1$$
$$\Rightarrow \quad x = \frac{1}{999} \quad \textbf{Ans.}$$

Q.4. *Express* 0.99999 ... *in the form* $\frac{p}{q}$. *Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.*

Sol. $x = 0.99999 \ldots = 0.\overline{9}$

One digit repeat; we multiply by 10.

$$10x = 9.\overline{9}$$
$$\Rightarrow 10x = 9 + x$$
$$\Rightarrow 9x = 9$$
$$x = 1 \quad \textbf{Ans.}$$

The answer makes sense as $0.\overline{9}$ is infinitely close to 1, i.e., we can make the difference between 1 and 0.99 ...... as small as we wish by taking enough 9's.

Q.5. *What can the maximum number of digits be in the repeating block of digits in the decimal expansion of* $\frac{1}{17}$? *Perform the division to check your answer.*

Sol. The maximum number of digits in the repeating block is 16 (< 17).

Division gives $\frac{1}{17} = 0.\overline{0588235294117647}$

The repeating block has 16 digits. **Ans.**

Q.6. *Look at several examples of rational numbers in the form* $\frac{p}{q}$ *(q ≠ 0), where p and q are integers with no common factors other than* 1 *and having terminating decimal representations (expansions). Can you guess what property q must satisfy?*

Sol. $\frac{2}{5} = 0.4, \ \frac{3}{2} = 1.5, \ \frac{7}{8} = 0.875, \ \frac{7}{10} = 0.7$.

All the denominators are either 2 (or its power), 5 (or its power) or a combination of both. **Ans.**

Q.7. *Write three numbers whose decimal expansion are non-terminating non-recurring.*

Sol. 7.314114111411114..........

0.101002000300004...........

$\pi = 3.1416$........ **Ans.**

Q.8. *Find three different irrational numbers between the rational numbers* $\frac{5}{7}$ *and* $\frac{9}{11}$.

Sol. $\frac{5}{7} = 0.\overline{714285} \qquad \frac{9}{11} = 0.\overline{81}$

There are an infinite number of irrational numbers between these two numbers. We may choose any three of them, e.g.

0.7234596..........
0.7425735..........
0.78123957........ **Ans.**

Q.9. *Classify the following numbers as rational or irrational :*

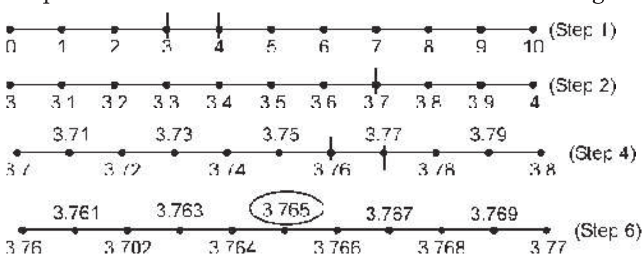
(i) $\sqrt{23}$ *(ii)* $\sqrt{225}$ *(iii)* 0.3796
(iv) 7.478478 ...... *(v)* 1.101001000100001....

Sol. Rational — $\sqrt{225}$ = 15, and 0.3796
Irrational — $\sqrt{23}$, 7.478478....., 1.101001000100001.... **Ans.**

EXERCISE 1.4

Q.1. *Visualise* 3.765 *on the number line, using successive magnification.*

Sol. Step one — The given number lies between 3 and 4.
Step two — Magnify the interval between 3 and 4 and divide it into 10 equal parts.
Step three — The given number lies between 3.7 and 3.8.
Step four — Divide the interval between 3.7 and 3.8 into ten equal parts and magnify it.
Step five — The given number lies between 3.76 and 3.77.
Step six — Magnify the interval between 3.76 and 3.77 and divide it into ten equal parts.
Step seven — 3.675 is the fifth division in this magnification.



Q.2. *Visualise* $4.\overline{26}$ *on the number line, up to* 4 *decimal places.*

Sol. Step one — On the number line the given number $4.\overline{26}$ lies between 4 and 5. (For four decimal places number is 4.2626.)

Step two — Magnify the interval between 4 and 5 and divide it into 10 equal parts.

Step three — The given number 4.2626 lies between 4.2 and 4.3.

Step four — Magnify the interval between 4.2 and 4.3 and divide it into ten equal parts.

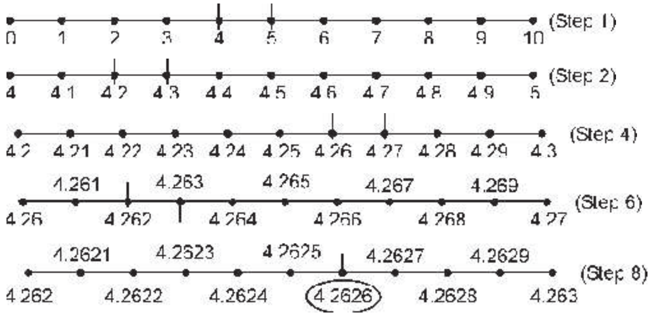
Step five — The given number falls between 4.26 and 4.27.

Step six — Magnify the interval between 4.26 and 4.27 and divide it into ten equal parts.

Step seven — The given number lies between 4.262 and 4.263.

Step eight — Magnify the interval between 4.262 and 4.263 and divide it into ten equal parts.

Step nine — The given number is the sixth division of the given interval.



EXERCISE 1.5

Q.1. *Classify the following numbers as rational or irrational :*

(i) $2-\sqrt{5}$ *(ii)* $(3+\sqrt{23})-\sqrt{23}$ *(iii)* $\dfrac{2\sqrt{7}}{7\sqrt{7}}$ *(iv)* $\dfrac{1}{\sqrt{2}}$ *(v)* 2π

Ans. (i) $2-\sqrt{5}$, (iv) $\dfrac{1}{\sqrt{2}}$ and, (v) 2π are irrational

(ii) $(3+\sqrt{23})-\sqrt{23}$ = 3, and (iii) $\dfrac{2\sqrt{7}}{7\sqrt{7}} = \dfrac{2}{7}$ are rational **Ans.**

Q.2. *Simplify each of the following expressions :*

(i) $(3+\sqrt{3})(2+\sqrt{2})$ *(ii)* $(3+\sqrt{3})(3-\sqrt{3})$

(iii) $(\sqrt{5}+\sqrt{2})^2$ *(iv)* $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Ans. (i) $(3+\sqrt{3})(2+\sqrt{2}) = 3\times2+3\times\sqrt{2}+\sqrt{3}\times2+\sqrt{3}\times\sqrt{2}$

$= 6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$ **Ans.**

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

Using identity $(a + b)(a - b) = a^2 - b^2$, it equals, $3^2 - 3 = 9 - 3 = 6$. **Ans.**

(iii) $(\sqrt{5}+\sqrt{2})^2$

Using identity $(a + b)^2 = a^2 + b^2 + 2ab$, we have

$(\sqrt{5}+\sqrt{2})^2 = 5 + 2 + 2\sqrt{2}\sqrt{5} = 7+2\sqrt{10}$ **Ans.**

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

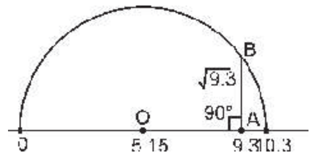
Using identity $(a + b)(a - b) = a^2 - b^2$ we have

$(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (5 - 2) = 3$ **Ans.**

Q.3. *Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is,* $\pi = \dfrac{c}{d}$. *This seems to contradict the fact that π is irrational. How will you resolve this contradiction?*

Ans. With a scale or tape we get only an approximate rational number as the result of our measurement. That is why π can be approximately represented as a quotient of two rational numbers. As a matter of mathematical truth it is irrational.

Q.4. *Represent* $\sqrt{9.3}$ *on the number line.*



Sol. To represent $\sqrt{9.3}$, draw a segment of 9.3 units on the number line. Let A represent 9.3 Extend it by 1 cm. Show point $\frac{10.3}{2}$ = 5.15 by on the number line. With 'O' as centre and radius 5.15 units, draw a semicircle. Draw AB perpendicular to OA to cut the hemisphere at B. The length AB is $\sqrt{9.3}$ units.

Q.5. *Rationalise the denominators of the following :*

(i) $\frac{1}{\sqrt{7}}$ *(ii)* $\frac{1}{\sqrt{7}-\sqrt{6}}$ *(iii)* $\frac{1}{\sqrt{5}+\sqrt{2}}$ *(iv)* $\frac{1}{\sqrt{7}-2}$

Sol. (i) $\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$ **Ans.**

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$ **Ans.**

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{5-2} = \frac{\sqrt{5}-\sqrt{2}}{3}$ **Ans.**

(iv) $\frac{1}{\sqrt{7}-2} = \frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2} = \frac{\sqrt{7}+2}{7-4} = \frac{\sqrt{7}+2}{3}$ **Ans.**

EXERCISE 1.6

Q.1. *Find :* *(i)* $64^{\frac{1}{2}}$ *(ii)* $32^{\frac{1}{5}}$ *(iii)* $125^{\frac{1}{3}}$

Sol. (i) $64^{\frac{1}{2}} = (8^2)^{\frac{1}{2}} = 8$ **Ans.** (ii) $32^{\frac{1}{5}} = (2\times2\times2\times2\times2)^{\frac{1}{5}} = (2^5)\ (2^5)^{\frac{1}{5}} = 2$ **Ans.**

(iii) $125^{\frac{1}{3}} = (5\times5\times5)^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = 5$ **Ans.**

Q.2. *Find :* *(i)* $9^{\frac{3}{2}}$ *(ii)* $32^{\frac{2}{5}}$ *(iii)* $16^{\frac{3}{4}}$ *(iv)* $125^{\frac{-1}{3}}$

Sol. (i) $9^{\frac{3}{2}} = \left(9^{\frac{1}{2}}\right)^3 = (3)^3 = 27$ **Ans.** (ii) $32^{\frac{2}{5}} = \left(32^{\frac{1}{5}}\right)^2 = (2)^2 = 4$ **Ans.**

(iii) $16^{\frac{3}{4}} = \left(16^{\frac{1}{4}}\right)^3 = (2)^3 = 8$ **Ans.** (iv) $125^{\frac{-1}{3}} = \frac{1}{(125)^{\frac{1}{3}}} = \frac{1}{5}$ **Ans.**

Q.3. *Simplify :* *(i)* $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$ *(ii)* $\left(\frac{1}{3^3}\right)^7$ *(iii)* $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ *(iv)* $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$

Sol. (i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}} = 2^{\left(\frac{2}{3}+\frac{1}{5}\right)} = 2^{\frac{13}{15}}$ **Ans.** (ii) $\left(\frac{1}{3^3}\right)^7 = (3^{-3})^7 = (3)^{-21}$ **Ans.**

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}} \cdot 11^{-\frac{1}{4}} = 11^{\frac{1}{2}-\frac{1}{4}} = 11^{\frac{1}{4}}$ **Ans.** (iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = (56)^{\frac{1}{2}}$ **Ans.**