Book Name: Selina Concise

EXERCISE 13 (A)

Solution 1:

(i) Let the co-ordinates of the point P be (x, y)

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1\times5 + 2\times1}{1+2} = \frac{7}{3}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{1\times9 + 2\times3}{1+2} = \frac{15}{3} = 5$$

Thus, the co-ordinates of point P are $\left(\frac{7}{3}, 5\right)$

(ii) Let the co-ordinates of the point P be (x, y).

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{3\times3 + 2\times(-4)}{3+2} = \frac{1}{5}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{3\times(-5) + 2\times6}{3+2} = \frac{-3}{5}$$

Thus, the co-ordinates of point P are $\left(\frac{1}{5}, \frac{-3}{5}\right)$

Solution 2:

Let the line joining points A (2, −3) and B (5, 6) be divided by point P (x, 0) in the ratio k: 1.

$$y = \frac{Ky_2 + y_1}{k+1}$$

$$0 = \frac{k\times6 + 1\times(-3)}{k+1}$$

$$0 = 6k - 3$$

$$k = \frac{1}{2}$$

Thus, the required ratio is 1: 2.

Solution 3:

Let the line joining points A (2, -4) and B (-3, 6) be divided by point P (0, y) in the ratio k: 1.

$$x = \frac{Kx_2 + x_1}{k+1}$$

$$0 = \frac{k \times (-3) + 1 \times 2}{k+1}$$
$$0 = -3k + 2$$
$$k = \frac{2}{3}$$

Thus, the required ratio is 2: 3.

Solution 4:

K 1

A(-1, 4) P(1, a) B(4, −1)

Let the point P (1, a) divides the line segment AB in the ratio k: 1.
Using section formula, we have:

$$1 = \frac{4K-1}{K+1}$$
$$\Rightarrow K + 1 = 4K - 1$$
$$\Rightarrow 2 = 3K$$
$$\Rightarrow K = \frac{2}{3} \qquad \text{........}(1)$$
$$\Rightarrow a = \frac{-k+4}{k+1}$$

Hence, the required is 2:3 and the value of a is 2.

Solution 5:

Let the point P (a, 6) divides the line segment joining A (-4, 3) and B (2, 8) in the ratio k: 1.
Using section formula, we have:

$$6 = \frac{8K+3}{K+1}$$
$$\Rightarrow 6K + 6 = 8K + 3$$
$$\Rightarrow \ 3 = 2k$$
$$\Rightarrow \ k = \frac{3}{2} \qquad \text{.......}(1)$$
$$\Rightarrow a = \frac{2k-4}{k+1}$$

$$\Rightarrow a=\frac{2\times\frac{3}{2}-4}{\frac{3}{2}+1}\quad(\text{from}(1))$$

$$\Rightarrow a=-\frac{2}{5}$$

Hence, the required ratio is 3:2 and the value of a is $-\frac{2}{5}$

Solution 6:

Let the point P (x, 0) on x-axis divides the line segment joining A (4, 3) and B (2, -6) in the ratio k: 1.

Using section formula, we have:

$$0=\frac{-6k+3}{k+1}$$

$$0=-6k+3$$

$$k=\frac{1}{2}$$

Thus, the required ratio is 1: 2.

Also, we have:

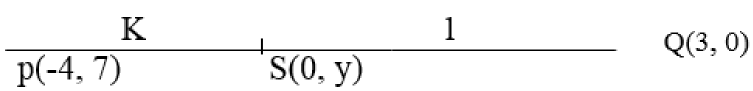
$$x=\frac{2k+4}{k+1}$$

$$=\frac{2\times\frac{1}{2}+4}{\frac{1}{2}+1}$$

$$=\frac{10}{3}$$

Thus, the required co-ordinates of the point of intersection are $\left(\frac{10}{3},0\right)$

Solution 7:



$$0=\frac{3k-4}{k+1}$$

$$3k=4$$

$$k=\frac{4}{3} \qquad \ldots\ldots(1)$$

$$y=\frac{0+7}{k+1}$$

$$y=\frac{7}{\frac{4}{3}+1}\quad(\text{from}(1))$$

$$y=3$$

Hence, the required is 4:3 and the required point is S(0, 3)

Solution 8:

(5, -10) (0, 0)

Point A divides PO in the ratio 1: 4.

Co-ordinates of point A are:

$$\left(\frac{1\times0+4\times5}{1+4},\frac{1\times0+4\times(-10)}{1+4}\right)=\left(\frac{20}{5},\frac{-40}{5}\right)=(4,-8)$$

Point B divides PO in the ratio 2: 3.

Co-ordinates of point B are:

$$\left(\frac{2\times0+3\times5}{2+3},\frac{2\times0+3\times(-10)}{2+3}\right)=\left(\frac{15}{5},\frac{-30}{5}\right)=(3,-6)$$

Point C divides PO in the ratio 3: 2.

Co-ordinates of point C are:

$$\left(\frac{3\times0+2\times5}{3+2},\frac{3\times0+2\times(-10)}{3+2}\right)=\left(\frac{10}{5},\frac{-20}{5}\right)=(2,-4)$$

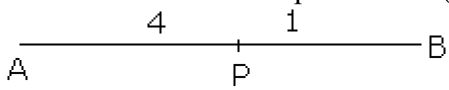
Point D divides PO in the ratio 4: 1.

Co-ordinates of point D are:

$$\left(\frac{4\times0+1\times5}{4+1},\frac{4\times0+1\times(-10)}{4+1}\right)=\left(\frac{5}{5},\frac{-10}{5}\right)=(1,-2)$$

Solution 9:

Let the co-ordinates of point P are (x, y).



(-3, -10) (x, y) (-2, 6)

Given: PB : AB = 1 : 5

∴ PB : PA = 1 : 4

Coordinates of P are

$$(x, y) = \left(\frac{4\times(-2)+1\times(-3)}{5}, \frac{4\times6+1\times(-10)}{5}\right) = \left(\frac{-11}{5}, \frac{14}{5}\right)$$

Solution 10:

5AP = 2BP

$$\frac{AB}{BP} = \frac{2}{5}$$

The co-ordinates of the point P are

$$\left(\frac{2\times(-2)+5\times4}{2+5}, \frac{2\times6+5\times3}{2+5}\right)$$

$$\left(\frac{16}{7}, \frac{27}{7}\right)$$

Solution 11:

The co-ordinates of every point on the line x = 2 will be of the type (2, y).

Using section formula, we have:

$$x = \frac{m_1\times5+m_2\times(-3)}{m_1+m_2}$$

$$2 = \frac{5m_1-3m_2}{m_1+m_2}$$

$$2m_1+2m_2 = 5m_1-3m_2$$

$$5m_2 = 3m_1$$

$$\frac{m_1}{m_2} = \frac{5}{3}$$

Thus, the required ratio is 5: 3.

$$y = \frac{m_1 \times 7 + m_2 \times (-1)}{m_1 + m_2}$$

$$y = \frac{5 \times 7 + 3 \times (-1)}{5 + 3}$$

$$y = \frac{35 - 3}{8}$$

$$y = \frac{32}{8} = 4$$

Thus, the required co-ordinates of the point of intersection are (2, 4).

Solution 12:

The co-ordinates of every point on the line y = 2 will be of the type (x, 2).

Using section formula, we have:

$$y = \frac{m_1 \times (-3) + m_2 \times 5}{m_1 + m_2}$$

$$2 = \frac{-3m_1 + 5m_2}{m_1 + m_2}$$

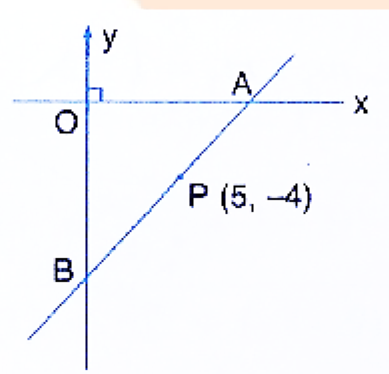
$$2m_1 + 2m_2 = -3m_1 + 5m_2$$

$$5m_1 = 3m_2$$

$$\frac{m_1}{m_2} = \frac{3}{5}$$

Thus, the required ratio is 3 : 5.

Solution 13:



Point A lies on x-axis. So, let the co-ordinates of A be (x, 0).

Point B lies on y-axis. So, let the co-ordinates of B be (0, y).
P divides AB in the ratio 2: 5.
We have:

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$5 = \frac{2 \times 0 + 5 \times x}{2+5}$$

$$5 = \frac{5x}{7}$$

$$x = 7$$

Thus, the co-ordinates of point A are (7, 0).

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$-4 = \frac{2 \times y + 5 \times 0}{2+5}$$

$$-4 = \frac{2y}{7}$$

$$-2 = \frac{y}{7}$$

$$y = -14$$

Thus, the co-ordinates of point B are (0, -14).

Solution 14:

Let P and Q be the point of trisection of the line segment joining the points A (-3, 0) and B (6, 6).
So, AP = PQ = QB
We have AP: PB = 1 : 2
Co-ordinates of the point P are

$$\left(\frac{1 \times 6 + 2 \times (-3)}{1+2}, \frac{1 \times 6 + 2 \times 0}{1+2}\right)$$

$$= \left(\frac{6-6}{3}, \frac{6}{3}\right)$$

$$= (0, 2)$$

We have AQ : QB = 2 : 1
Co-ordinates of the point Q are

$$\left(\frac{2\times6+1\times(-3)}{2+1},\frac{2\times6+1\times0}{2+1}\right)$$
$$=\left(\frac{9}{3},\frac{12}{3}\right)$$
$$=(3,4)$$

Solution 15:

Let P and Q be the point of trisection of the line segment joining the points A (-5, 8) and B (10, −4).

So, AP = PQ = QB

We have AP:PB = 1 : 2

Co-ordinates of the point P are

$$\left(\frac{1\times10+2\times(-5)}{1+2},\frac{1\times(-4)+2\times8}{1+2}\right)$$
$$=\left(\frac{10-10}{3},\frac{12}{3}\right)$$
$$=(0,4)$$

So, point P lies on the y-axis

We have AQ : QB = 2: 1

Co-ordinates of the point Q are

$$\left(\frac{2\times10+1\times(-5)}{2+1},\frac{2\times(-4)+1\times8}{2+1}\right)$$
$$=\left(\frac{20-5}{3},\frac{-8+8}{3}\right)$$
$$=(5,0)$$

So, point Q lies on the x-axis.

Hence, the line segment joining the given points A and B is trisected by the co-ordinate axes.

Solution 16:

Let A and B be the point of trisection of the line segment joining the points P (2, 1) and Q (5, −8).

So, PA = AB = BQ

We have PA : AQ = 1 : 2

Co-ordinates of the point A are

$$\left(\frac{1\times5+2\times2}{1+2},\frac{1\times(-8)+2\times1}{1+2}\right)$$

$$=\left(\frac{9}{3},\frac{-6}{3}\right)$$

$$=(3,-2)$$

Hence, A (3, −2) is a point of trisection of PQ.

We have PB : BQ = 2 : 1

Co-ordinates of the point B are

$$\left(\frac{2\times5+1\times2}{2+1},\frac{2\times(-8)+1\times1}{2+1}\right)$$

$$=\left(\frac{10+2}{3},\frac{-16+1}{3}\right)$$

$$=(4,-5)$$

Solution 17:

(i) A (-4,3) and B (8, -6)

$$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{(8+4)^2+(-6-3)^2}$$

$$=\sqrt{144+81}$$

$$=\sqrt{225}$$

= 15 units

(ii) Let P be the point, which divides AB on the x-axis in the ratio k : 1.

Therefore, y-co-ordinate of P = 0.

$$\Rightarrow\frac{-6k+3}{k+1}=0$$

$\Rightarrow -6k+3=0$

$\Rightarrow k=\frac{1}{2}$

∴ Required ratio is 1 : 2.

Solution 18:

Since, point L lies on y-axis, its abscissa is 0.
Let the co-ordinates of point L be (0, y). Let L divides MN in the ratio k: 1.
Using section formula, we have:

$$x = \frac{k \times (-3) + 1 \times 5}{k+1}$$

$$0 = \frac{-3k+5}{k+1}$$

$$-3k + 5 = 0$$

$$k = \frac{5}{3}$$

Thus, the required ratio is 5 : 3.

$$\text{Now,}\;\; y = \frac{k \times 2 + 1 \times 7}{k+1}$$

$$= \frac{\frac{5}{3} \times 2 + 7}{\frac{5}{3} + 1}$$

$$= \frac{10+21}{5+3}$$

$$= \frac{31}{8}$$

Solution 19:

(i) Co-ordinates of P are

$$\left(\frac{1 \times (-1) + 2 \times 2}{1+2}, \frac{1 \times 2 + 2 \times 5}{1+2} \right)$$

$$= \left(\frac{3}{3}, \frac{12}{3} \right)$$

$$= (1, 4)$$

Co-ordinates of Q are

$$\left(\frac{1 \times 5 + 2 \times 2}{1+2}, \frac{1 \times 8 + 2 \times 5}{1+2} \right)$$

$$= \left(\frac{9}{3}, \frac{18}{3} \right)$$

$$= (3, 6)$$

(ii) Using distance formula, we have:

$$BC = \sqrt{(5+1)^2 + (8-2)^2} = \sqrt{36+36} = 6\sqrt{2}$$

$$PQ = \sqrt{(3-1)^2 + (6-4)^2} = \sqrt{4+4} = 2\sqrt{2}$$

Hence, $PQ = \frac{1}{3}BC$.

Solution 20:

BP: PC = 2: 3

Co-ordinates of P are

$$\left(\frac{2\times(-2)+3\times3}{2+3}, \frac{2\times4+3\times(-1)}{2+3}\right)$$

$$= \left(\frac{-4+9}{5}, \frac{8-3}{5}\right)$$

$$= (1, 1)$$

Using distance formula, we have:

$AP = \sqrt{(1+3)^2 + (1-4)^2} = \sqrt{16+9} = \sqrt{25} = 5$ units.

Solution 21:

Since, point K lies on x-axis, its ordinate is 0.

Let the point K (x, 0) divides AB in the ratio k: 1.

We have,

$$y = \frac{k\times(-5)+1\times3}{k+1}$$

$$0 = \frac{-5k+3}{k+1}$$

$$k = \frac{3}{5}$$

Thus, K divides AB in the ratio 3: 5.

Also, we have:

$$x = \frac{k\times6+1\times2}{k+1}$$

$$x = \frac{\frac{3}{5}\times6+2}{\frac{3}{5}+1}$$

$$x = \frac{18+10}{3+5}$$

$$x = \frac{28}{8} = \frac{7}{2} = 3\frac{1}{2}$$

Thus, the co-ordinates of the point K are $\left(3\frac{1}{2}, 0\right)$.

Solution 22:

Since, point K lies on y-axis, its abscissa is 0.
Let the point K (0, y) divides AB in the ratio k : 1
We have,

$$x = \frac{k\times(-6)+1\times 4}{k+1}$$

$$0 = \frac{-6k+4}{k+1}$$

$$k = \frac{4}{6} = \frac{2}{3}$$

Thus, K divides AB in the ratio 2: 3.
Also, we have:

$$y = \frac{k\times(-2)+1\times 7}{k+1}$$

$$y = \frac{-2k+7}{k+1}$$

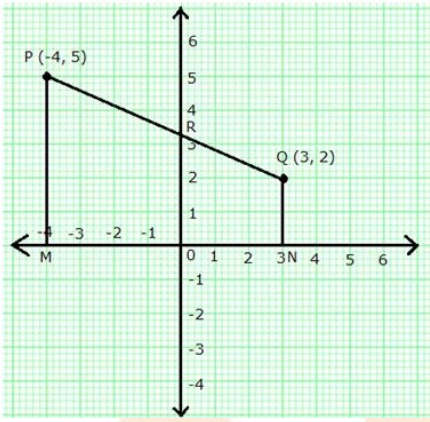
$$y = \frac{-2\times\frac{2}{3}+7}{\frac{2}{3}+1}$$

$$y = \frac{-4+21}{2+3}$$

$$y = \frac{17}{5}$$

Thus, the co-ordinates of the point K are $\left(0, \frac{17}{5}\right)$

Solution 23:



(i) Let point R (0, y) divides PQ in the ratio k: 1.

We have:

$$x = \frac{k \times 3 + 1 \times (-4)}{k+1}$$

$$0 = \frac{3k-4}{k+1}$$

$$0 = 3k - 4$$

$$k = \frac{4}{3}$$

Thus, PR: RQ = 4: 3

(ii) Also, we have:

$$y = \frac{k \times 2 + 1 \times 5}{k+1}$$

$$y = \frac{2k+5}{k+1}$$

$$y = \frac{2 \times \frac{4}{3} + 5}{\frac{4}{3} + 1}$$

$$y = \frac{8+15}{4+3}$$

$$y = \frac{23}{7}$$

Thus, the co-ordinates of point R are $\left(0, \frac{23}{7}\right)$.

(iii) Area of quadrilateral PMNQ

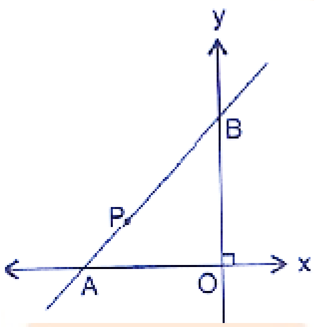
$= \frac{1}{2} \times (PM + QN) \times MN$

$= \frac{1}{2} \times (5 + 2) \times 7$

$= \frac{1}{2} \times 7 \times 7$

$= 24.5$ sq units

Solution 24:



Given, A lies on x-axis and B lies on y-axis.
Let the co-ordinates of A and B be (x, 0) and (0, y) respectively.
Given, P is the point (-4, 2) and AP: PB = 1: 2.
Using section formula, we have:

$$-4 = \frac{1 \times 0 + 2 \times x}{1 + 2}$$

$$-4 = \frac{2x}{3}$$

$$x = \frac{-4 \times 3}{2} = -6$$

Also,

$$2 = \frac{1 \times y + 2 \times 0}{1 + 2}$$

$$2 = \frac{y}{3}$$

$$y = 6$$

Thus, the co-ordinates of points A and B are (-6, 0) and (0, 6) respectively.

Solution 25:

(i) Let the required ratio be $m_1 : m_2$

Consider A(-4, 6) = (x_1, y_1); B(8, −3) = (x_2, y_2) and let
P(x, y) be the point of intersection of the line segment
And the y-axis
By section formula, we have,

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}, y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$\Rightarrow x = \frac{8m_1 - 4m_2}{m_1 + m_2}, y = \frac{-3m_1 + 6m_2}{m_1 + m_2}$$

The equation of the y-axis is x = 0

$$\Rightarrow x = \frac{8m_1 - 4m_2}{m_1 + m_2} = 0$$

$$\Rightarrow 8m_1 - 4m_2 = 0$$

$$\Rightarrow 8m_1 = 4m_2$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{4}{8}$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{1}{2}$$

(ii) from the previous subpart, we have,

$$\frac{m_1}{m_2} = \frac{1}{2}$$

$\Rightarrow m_1 = k$ and $m_2 = 2k$, where k

Is any constant.

Also, we have,

$$x = \frac{8m_1 - 4m_2}{m_1 + m_2}, y = \frac{-3m_1 + 6m_2}{m_1 + m_2}$$

$$\Rightarrow x = \frac{8 \times k - 4 \times 2k}{k + 2k}, y = \frac{-3 \times k + 6 \times 2k}{k + 2k}$$

$$\Rightarrow x = \frac{8k - 8k}{3k}, y = \frac{-3k + 12k}{3k}$$

$$\Rightarrow x = \frac{0}{3k}, y = \frac{9k}{3k}$$

$\Rightarrow x = 0, y = 3$

Thus, the point of intersection is p (0, 3)

(iii) The length of AB = distance between two points A and B.

The distance between two given points

$A(x_1 , y_1)$ and $B(x_2 , y_2)$ is given by,

Distance AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(8+4)^2 + (-3-6)^2}$$
$$= \sqrt{(12)^2 + (9)^2}$$
$$= \sqrt{144+81}$$
$$= \sqrt{225}$$
$$= 15 \text{ units}$$

EXERCISE. 13 (B)

Solution 1:

(i) A (-6, 7) and B (3, 5)

Mid-point of AB $= \left(\frac{-6+3}{2}, \frac{7+5}{2}\right) = \left(\frac{-3}{2}, 6\right)$

(ii) A (5, -3) and B (-1, 7)

Mid-point of AB $= \left(\frac{5-1}{2}, \frac{-3+7}{2}\right) = (2,2)$

Solution 2:

Mid-point of AB = (2, 3)

$$\therefore \left(\frac{3+x}{2}, \frac{5+y}{2}\right) = (2,3)$$
$$\Rightarrow \frac{3+x}{2} = 2 \text{ and } \frac{5+y}{2} = 3$$
$$\Rightarrow 3+x = 4 \text{ and } 5+y = 6$$
$$\Rightarrow x = 1 \text{ and } y = 1$$

Solution 3:

Given, L is the mid-point of AB and M is the mid-point of AC.

Co-ordinates of L are

$$\left(\frac{5-1}{2}, \frac{3+1}{2}\right) = (2,2)$$

Co-ordinates of M are

$$\left(\frac{5+7}{2},\frac{3-3}{2}\right)=(6,0)$$

Using distance formula, we have:

$$BC=\sqrt{(7+1)^2+(-3-1)^2}=\sqrt{64+16}=\sqrt{80}=4\sqrt{5}$$

$$LM=\sqrt{(6-2)^2+(0-2)^2}=\sqrt{16+4}=\sqrt{20}=2\sqrt{5}$$

Hence, LM $=\frac{1}{2}$BC

Solution 4:

(i) Let the co-ordinates of A be (x, y).

$$\therefore\ (1,7)=\left(\frac{x-5}{2},\frac{y+10}{2}\right)$$

$$\Rightarrow 1=\frac{x-5}{2}\quad\text{and}\quad 7=\frac{y+10}{2}$$

$$\Rightarrow 2=x-5\quad\text{and}\quad 14=y+10$$

$$\Rightarrow x=7\quad\text{and}\quad y=4$$

Hence, the co-ordinates of A are (7, 4).

(ii) Let the co-ordinates of B be (x, y).

$$\therefore\ (-1,3)=\left(\frac{3+x}{2},\frac{-1+y}{2}\right)$$

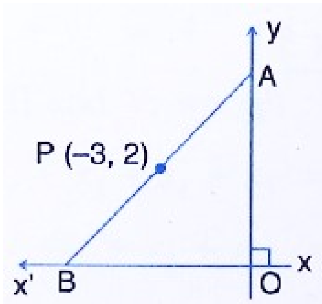
$$\Rightarrow -1=\frac{3+x}{2}\quad\text{and}\quad 3=\frac{-1+y}{2}$$

$$\Rightarrow -2=3+x\quad\text{and}\quad 6=-1+y$$

$$\Rightarrow x=-5\quad\text{and}\quad y=7$$

Hence, the co-ordinates of B are (-5, 7).

Solution 5:



Point A lies on y-axis, so let its co-ordinates be (0, y).
Point B lies on x-axis, so let its co-ordinates be (x, 0).
P (-3, 2) is the mid-point of line segment AB.

$$\therefore\ (-3,2)=\left(\frac{0+x}{2},\frac{y+0}{2}\right)$$

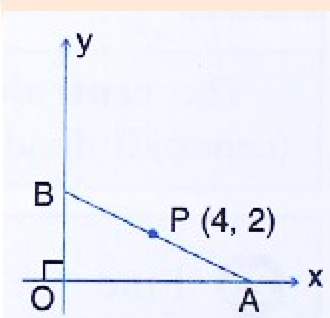
$$\Rightarrow (-3,2)=\left(\frac{x}{2},\frac{y}{2}\right)$$

$$\Rightarrow -3=\frac{x}{2} \quad \text{and} \quad 2=\frac{y}{2}$$

$$\Rightarrow -6=x \quad \text{and} \quad 4=y$$

Thus, the co-ordinates of points A and B are (0, 4) and (−6, 0) respectively.

Solution 6:



Point A lies on x-axis, so let its co-ordinates be (x, 0).
Point B lies on y-axis, so let its co-ordinates be (0, y).
P (4, 2) is mid-point of line segment AB.

$$\therefore\ (4,2)=\left(\frac{x+0}{2},\frac{0+y}{2}\right)$$

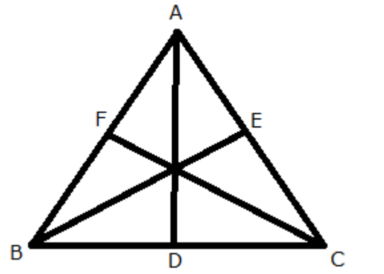
$$\Rightarrow 4=\frac{x}{2} \quad \text{and} \quad 2=\frac{y}{2}$$

$$\Rightarrow 8=x \quad \text{and} \quad 4=y$$

Hence, the co-ordinates of points A and B are (8, 0) and (0, 4) respectively.

Solution 7:

Let A (-5, 2), B (3, -6) and C (7, 4) be the vertices of the given triangle.
Let AD be the median through A, BE be the median through B and CF be the median through C.



We know that median of a triangle bisects the opposite side.

Co-ordinates of point F are

$$\left(\frac{-5+3}{2},\frac{2-6}{2}\right)=\left(\frac{-2}{2},\frac{-4}{2}\right)=(-1,-2)$$

Co-ordinates of point D are

$$\left(\frac{3+7}{2},\frac{-6+4}{2}\right)=\left(\frac{10}{2},\frac{-2}{2}\right)=(5,-1)$$

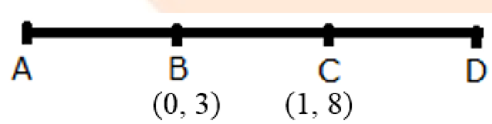
Co-ordinates of point E are

$$\left(\frac{-5+7}{2},\frac{2+4}{2}\right)=\left(\frac{2}{2},\frac{6}{2}\right)=(1,3)$$

The median of the triangle through the vertex B(3, -6) is BE
Using distance formula,

$$BE=\sqrt{(1-3)^2+(3+6)^2}=\sqrt{4+81}=\sqrt{85}=9.22$$

Solution 8:



Given, AB = BC = CD
So, B is the mid-point of AC. Let the co-ordinates of point A be (x, y).

$$\therefore (0,3)=\left(\frac{x+1}{2},\frac{y+8}{2}\right)$$

$$\Rightarrow 0=\frac{x+1}{2} \quad \text{and} \quad 3=\frac{y+8}{2}$$

$\Rightarrow 0 = x + 1 \quad \text{and} \quad 6 = y + 8$

$\Rightarrow -1 = x \quad \text{and} \quad -2 = y$

Thus, the co-ordinates of point A are (-1, -2).

Also, C is the mid-point of BD. Let the co-ordinates of point D be (p, q).

$\therefore (1,8) = \left(\dfrac{0+p}{2}, \dfrac{3+q}{2}\right)$

$\Rightarrow 1 = \dfrac{0+p}{2} \quad \text{and} \quad 8 = \dfrac{3+q}{2}$

$\Rightarrow 2 = 0 + p \quad \text{and} \quad 16 = 3 + q$

$\Rightarrow -2 = p \quad \text{and} \quad 13 = y$

Thus, the co-ordinates of point D are (2, 13).

Solution 9:

We know that the centre is the mid-point of diameter.

Let the required co-ordinates of the other end of mid-point be (x, y).

$\therefore (2,-1) = \left(\dfrac{-2+x}{2}, \dfrac{5+y}{2}\right)$

$\Rightarrow 2 = \dfrac{-2+x}{2} \quad \text{and} \quad -1 = \dfrac{5+y}{2}$

$\Rightarrow 4 = -2 + x \quad \text{and} \quad -2 = 5 + y$

$\Rightarrow 6 = x \quad \text{and} \quad -7 = y$

Thus, the required co-ordinates are (6, −7).

Solution 10:

Co-ordinates of the mid-point of AC are

$\left(\dfrac{2-4}{2}, \dfrac{5+3}{2}\right) = \left(\dfrac{-2}{2}, \dfrac{8}{2}\right) = (-1,4)$

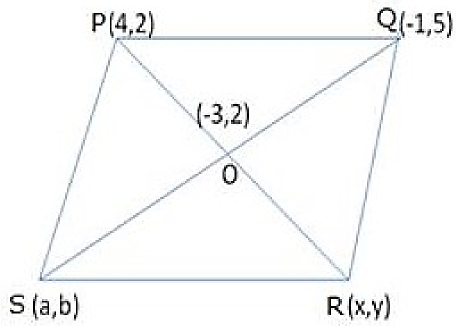
Co-ordinates of the mid-point of BD are

$\left(\dfrac{1-3}{2}, \dfrac{0+8}{2}\right) = \left(\dfrac{-2}{2}, \dfrac{8}{2}\right) = (-1,4)$

Since, mid-point of AC = mid-point of BD

Hence, ABCD is a parallelogram.

Solution 11:



Let the coordinates of R and S be (x, y) and (a, b) respectively.
Mid-point of PR is O.

$$\therefore\ O(-3,2) = O\left(\frac{4+x}{2}, \frac{2+y}{2}\right)$$

$$-3 = \frac{4+x}{2}, 2 = \frac{2+y}{2}$$

$-6 = 4 + x$, $4 = 2 + y$

$x = -10$, $y = 2$

Hence, R = (−10,2)

Similarly, the mid-point of SQ is O.

$$\therefore\ O(-3,2) = O\left(\frac{a-1}{2}, \frac{b+5}{2}\right)$$

$$-3 = \frac{a-1}{2}, 2 = \frac{b+5}{2}$$

$$-6 = a - 1, 4 = b + 5$$

$$a = -5, b = -1$$

Hence, $S = (-5, -1)$

Thus, the coordinates of the point R and S are (-10, 2) and (-5, -1).

Solution 12:

Let the co-ordinates of vertex C be (x, y).
ABCD is a parallelogram.
∴ Mid-point of AC = Mid-point of BD

$$\left(\frac{-1+x}{2}, \frac{0+y}{2}\right) = \left(\frac{1+3}{2}, \frac{3+5}{2}\right)$$

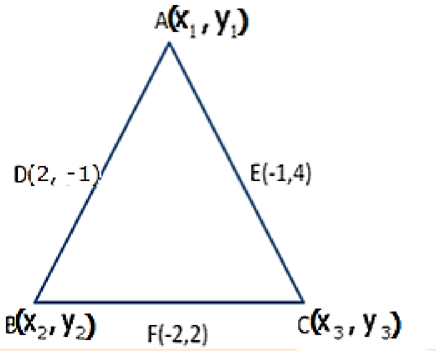
$$\left(\frac{-1+x}{2}, \frac{y}{2}\right) = (2, 4)$$

$$\frac{-1+x}{2}=2 \quad \text{and} \quad \frac{y}{2}=4$$

$$x=5 \quad \text{and} \quad y=8$$

Thus, the co-ordinates of vertex C is (5, 8).

Solution 13:



Let A(x_1, y_1), B(x_2,y_2) and C (x_3,y_3) be the co-ordinates of the vertices of Δ ABC.
Midpoint of AB, i.e. D

$$D(2,-1)=D\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$2=\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}=\ldots\ldots\ldots\ldots\ldots(1)$$

$$x_1+x_2=4--(1)y_1+y_2=-2\ldots\ldots..(2)$$

Similarly

$$x_1+x_3=-2\ldots\ldots\ldots\ldots(3)$$

$$y_1+y_3=8\ldots\ldots\ldots\ldots\ldots(4)$$

$$x_1+x_3=-4\ldots\ldots\ldots\ldots..(5)$$

$$y_2+y_3=4\ldots\ldots\ldots\ldots\ldots..(6)$$

Adding (1), (3) and (5), we get,

$$2(x_1+x_2+x_3)=-2$$

$$x_1+x_2+x_3=-1$$

$$4+x_3=-1[\text{from}(1)]$$

$$x_3=-5$$

From (3)

$x_1 - 5 = -2$

$x_1 = 3$

From (5)

$x_2 - 5 = -4$

$x_2 = 1$

Adding (2), (4) and (6), we get,

$2(y_1 + y_2 + y_3) = 10$

$y_1 + y_2 + y_3 = 5$

$-2 + y_3 = 5[\text{from}(2)]$

$y_3 = 7$

From (4)

$y_1 + 7 = 8$

$y_1 = 1$

From (6)

$y_2 + 7 = 4$

$y_2 = -3$

Thus, the co-ordinates of the vertices of Δ ABC are (3, 1), (1, −3) and (−5, 7).

Solution 14:

Given, AB = BC, i.e., B is the mid-point of AC.

$\therefore (y, 7) = \left(\frac{-5+1}{2}, \frac{x-3}{2}\right)$

$(y, 7) = \left(-2, \frac{x-3}{2}\right)$

$\Rightarrow y = -2 \text{ and } 7 = \frac{x-3}{2}$

$\Rightarrow y = -2 \text{ and } x = 17$

Solution 15:

Given, PR = 2QR

Now, Q lies between P and R, so, PR = PQ + QR

∴ PQ + QR = 2QR

$\Rightarrow$PQ = QR

$\Rightarrow$ Q is the mid-point of PR.

$$\therefore (-2,b)=\left(\frac{a+0}{2},\frac{-4+2}{2}\right)$$

$$(-2,b)=\left(\frac{a}{2},-1\right)$$

$$\Rightarrow a=-4 \text{ and } b=-1$$

Solution 16:

Co-ordinates of the centroid of triangle ABC are

$$\left(\frac{7+0-1}{3},\frac{-2+1+4}{3}\right)$$

$$=\left(\frac{6}{3},\frac{3}{3}\right)$$

$$=(2,1)$$

Solution 17:

Let G be the centroid of DPQR whose coordinates are (2, −5) and let (x, y) be the coordinates of vertex P.

Coordinates of G are,

$$G(2,-5)=G\left(\frac{x-6+11}{3},\frac{y+5+8}{3}\right)$$

$$2=\frac{x+5}{3},\quad -5=\frac{y+13}{3}$$

6 = x + 5, −15 = y + 13

x = 1, y = −28

Coordinates of vertex P are (1, −28)

Solution 18:

Given, centroid of triangle ABC is the origin.

$$\therefore\ (0,0)=\left(\frac{5-4+y}{3},\frac{x+3-2}{3}\right)$$

$$(0,0)=\left(\frac{1+y}{3},\frac{x+1}{3}\right)$$

$$0=\frac{1+y}{3} \quad \text{and} \quad 0=\frac{x+1}{3}$$

$$y=-1 \quad \text{and} \quad x=-1$$

EXERCISE 13 (C)

Solution 1:

Given, BP: PC = 3: 2

Using section formula, the co-ordinates of point P are

$$\left(\frac{3\times5+2\times0}{3+2},\frac{3\times10+2\times5}{3+2}\right)$$

$$=\left(\frac{15}{5},\frac{40}{5}\right)$$

$$=(3,8)$$

Using distance formula, we have:

$$AP=\sqrt{(3-4)^2+(8+4)^2}=\sqrt{1+144}=\sqrt{145}=12.04$$

Solution 2:

Given, $3PB=AB$

$$\Rightarrow.\ \frac{AB}{PB}=\frac{3}{1}$$

$$\Rightarrow\frac{AB-PB}{PB}=\frac{3-1}{1}$$

$$\Rightarrow\frac{AP}{PB}=\frac{2}{1}$$

Using section formula,

Coordinates of P are

$$P(x,y)=P\left(\frac{2\times10+1\times20}{2+1},\frac{2x(-20)+1\times0}{2+1}\right)$$

$$=p\left(\frac{40}{3},-\frac{40}{3}\right)$$

Given, AB = 6AQ

$$\Rightarrow \frac{AQ}{AB}=\frac{1}{6}$$

$$\Rightarrow \frac{AQ}{AB-AQ}=\frac{1}{6-1}$$

$$\Rightarrow \frac{AQ}{QB}=\frac{1}{5}$$

Using section formula,

Coordinates of Q are

$$Q(x,y)=Q\left(\frac{1\times 10+5\times 20}{1+5},\frac{1\times(-20)+5\times 0}{1+5}\right)$$

$$=Q\left(\frac{110}{6},-\frac{20}{6}\right)$$

$$=Q\left(\frac{55}{3},-\frac{10}{3}\right)$$

Solution 3:

Given that, point P lies on AB such that AP: PB = 3: 5.

The co-ordinates of point P are

$$\left(\frac{3\times 0+5x(-8)}{3+5},\frac{3\times 16+5\times 0}{3+5}\right)$$

$$=\left(\frac{-40}{8},\frac{48}{8}\right)$$

$$=(-5,6)$$

Also, given that, point Q lies on AB such that AQ: QC = 3: 5.

The co-ordinates of point Q are

$$\left(\frac{3\times 0+5x(-8)}{3+5},\frac{3\times 0+5\times 0}{3+5}\right)$$

$$=\left(\frac{-40}{8},\frac{0}{8}\right)$$

$$=(-5,0)$$

Using distance formula,

$$PQ=\sqrt{(-5+5)^2+(0-6)^2}=\sqrt{0+36}=6$$

$$BC=\sqrt{(0-0)^2+(0-16)^2}=\sqrt{0+(16)^2}=16$$

$$\text{Now,}\frac{3}{8}BC=\frac{3}{8}\times16=6=PQ$$

Hence, proved

Solution 4:

Let P and Q be the points of trisection of the line segment joining A (6, -9) and B (0, 0).

P divides AB in the ratio 1: 2. Therefore, the co-ordinates of point P are

$$\left(\frac{1\times0+2\times6}{1+2},\frac{1\times0+2x(-9)}{1+2}\right)$$

$$=\left(\frac{12}{3},\frac{-18}{3}\right)$$

$$=(4,-6)$$

Q divides AB in the ratio 2: 1. Therefore, the co-ordinates of point Q are

$$\left(\frac{2\times0+1\times6}{2+1},\frac{2\times0+1\times(-9)}{2+1}\right)$$

$$=\left(\frac{6}{3},\frac{-9}{3}\right)$$

$$=(2,-3)$$

Thus, the required points are (4, −6) and (2, −3).

Solution 5:

Since, the line segment AB intersects the y-axis at point P, let the co-ordinates of point P be (0, y).

P divides AB in the ratio 1: 3.

$$\therefore\ (0,y)=\left(\frac{1\times a+3x(-1)}{1+3},\frac{1\times5+3\times\frac{5}{3}}{1+3}\right)$$

$$(0,y)=\left(\frac{a-3}{4},\frac{10}{4}\right)$$

$$0=\frac{a-3}{4} \quad \text{and} \quad y=\frac{10}{4}$$

$$a=3 \quad \text{and} \quad y=\frac{5}{2}=2\frac{1}{2}$$

Thus, the value of a is 3 and the co-ordinates of point P are $\left(0,2\frac{1}{2}\right)$

Solution 6:

Let the line segment AB intersects the x-axis by point P (x, 0) in the ratio k: 1.

$$\therefore \ (x,0)=\left(\frac{k\times 4+1\times 0}{k+1},\frac{k\times(-1)+1\times 3}{k+1}\right)$$

$$(x,0)=\left(\frac{4k}{k+1},\frac{-k+3}{k+1}\right)$$

$$\Rightarrow 0=\frac{-k+3}{k+1}$$

$$\Rightarrow k=3$$

Thus, the required ratio in which P divides AB is 3: 1.

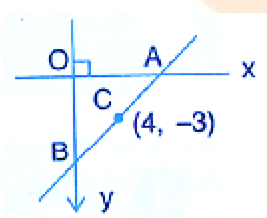
Also, we have:

$$x=\frac{4k}{k+1}$$

$$\Rightarrow x=\frac{4\times 3}{3+1}=\frac{12}{4}=3$$

Thus, the co-ordinates of point P are (3, 0).

Solution 7:



Since, point A lies on x-axis, let the co-ordinates of point A be (x, 0).

Since, point B lies on y-axis, let the co-ordinates of point B be (0, y).

Given, mid-point of AB is C (4, −3).

$$\therefore\ (4,-3)=\left(\frac{x+0}{2},\frac{0+y}{2}\right)$$

$$\Rightarrow (4-3)=\left(\frac{x}{2},\frac{y}{2}\right)$$

$$\Rightarrow 4=\frac{x}{2} \quad \text{and} \quad -3=\frac{y}{2}$$

$$\Rightarrow x=8 \quad \text{and} \quad y=-6$$

Thus, the co-ordinates of point A are (8, 0) and the co-ordinates of point B are (0, −6).

Solution 8:

(i) Radius AC = $\sqrt{(3+2)^2+(-7-5)^2}$

$$=\sqrt{5^2+(-12)^2}$$

$$=\sqrt{25+144}$$

$$=\sqrt{169}$$

$$=13\ \text{units}$$

(ii) Let the co-ordinates of B be (x , y)

Using mid – point formula, we have

$$-2=\frac{3+x}{2} \quad \text{and} \quad 5=\frac{-7+y}{2}$$

$$\Rightarrow -4=3+x \quad \text{and} \quad 10=-7+y$$

$$\Rightarrow x=-7 \quad \text{and} \quad y=17$$

Thus, the coordinates of B are $(-7,17)$

Solution 9:

Co- ordinates of the centroid of triangle ABC are

$$\left(\frac{-1+1+5}{3},\frac{3-1+1}{3}\right)$$

$$=\left(\frac{5}{3},1\right)$$

Solution 10:

It is given that the mid-point of the line-segment joining (4a, 2b - 3) and (-4, 3b) is (2, -2a).

$$\therefore\ (2,-2a)=\left(\frac{4a-4}{2},\frac{2b-3+3b}{2}\right)$$

$$\Rightarrow 2=\left(\frac{4a-4}{2}\right)$$

$$\Rightarrow 4a-4=4$$

$$\Rightarrow 4a=8$$

$$\Rightarrow a=2$$

Also,

$$-2a=\frac{2b-3+3b}{2}$$

$$\Rightarrow -2\times 2=\frac{5b-3}{2}$$

$$\Rightarrow 5b-3=-8$$

$$\Rightarrow 5b=-5$$

$$\Rightarrow b=-1$$

Solution 11:

Mid-point of (2a, 4) and (−2, 2b) is (1, 2a + 1), therefore using mid-point formula, we have:

$$x=\frac{x_1+x_2}{2}\ y=\frac{y_1+y_2}{2}$$

$$1=\frac{2a-2}{2}\ 2a+1=\frac{4+2b}{2}$$

$$1=a-1$$

$$\therefore\ a=2\ 2a+1\ =2+b$$

Putting, a = 2 in 2a + 1 = 2 + b, we get,

5 − 2 = b $\Longrightarrow$ b = 3

Therefore, a = 2, b = 3.

Solution 12:

(i) Co-ordinates of point P are

$$\left(\frac{1\times 17+2\times(-4)}{1+2},\frac{1\times 10+2\times 1}{1+2}\right)$$

$=\left(\frac{17-8}{3},\frac{10+2}{3}\right)$

$=\left(\frac{9}{3},\frac{12}{3}\right)$

$=(3,4)$

(ii) OP = $\sqrt{(0-3)^2+(0-4)^2}=\sqrt{9+16}=\sqrt{25}$ = 5 units

(iii) Let AB be divided by the point P(0, y) lying on y-axis in the ratio k : 1

$\therefore (0,y)=\left(\frac{k\times 17+1\times(-4)}{k+1},\frac{k\times 10+1\times 1}{k+1}\right)$

$\Rightarrow (0,y)=\left(\frac{17k-4}{k+1},\frac{10k+1}{k+1}\right)$

$\Rightarrow 0=\frac{17k-4}{k+1}$

$\Rightarrow 17k-4=0$

$\Rightarrow k=\frac{4}{17}$

Thus, the ratio in which the y-axis divide the line AB is 4: 17.

Solution 13:

We have:

$AB=\sqrt{(-1+5)^2+(-2-4)^2}=\sqrt{16+36}=\sqrt{52}$

$BC=\sqrt{(-1+5)^2+(-2-2)^2}=\sqrt{36+16}=\sqrt{52}$

$AC=\sqrt{(5+5)^2+(2-4)^2}=\sqrt{100+4}=\sqrt{104}$

$AB^2+BC^2=52+52=104$

$AC^2=104$

$\because$ AB = BC and $AB^2+BC^2=AC^2$

$\therefore$ ABC is an isosceles right-angled triangle.

Let the coordinates of D be (x, y).

If ABCD is a square, then,

Mid-point of AC = Mid-point of BD

$\left(\frac{-5+5}{2},\frac{4+2}{2}\right)=\left(\frac{x-1}{2},\frac{y-2}{2}\right)$

$0=\frac{x-1}{2},3=\frac{y-2}{2}$

x = 1, y = 8

Thus, the co-ordinates of point D are (1, 8).

Solution 14:

Given, M is the mid-point of the line segment joining the points A (−3, 7) and B (9, −1).
The co-ordinates of point M are

$$\left(\frac{-3+9}{2},\frac{7-1}{2}\right)$$
$$=\left(\frac{6}{2},\frac{6}{2}\right)$$
$$=(3,3)$$

Also, given that, R (2, 2) divides the line segment joining M and the origin in the ratio p : q.

$$\therefore\ (2,2)=\left(\frac{p\times0+q\times3}{p+q},\frac{p\times0+q\times3}{p+q}\right)$$
$$\Rightarrow\frac{p\times0+q\times3}{p+q}=2$$
$$\Rightarrow\frac{3q}{p+q}=2$$
$$\Rightarrow 3q=2p+2q$$
$$\Rightarrow 3q-2q=2p$$
$$\Rightarrow q=2p$$
$$\Rightarrow\frac{p}{q}=\frac{1}{2}$$

Thus the ratio p : q is 1 : 2.