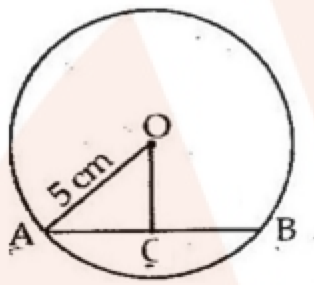
Book Name: Selina Concise

EXERCISE 17 (A)

Solution 1:

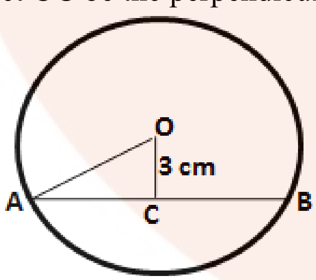
Let AB be the chord and O be the centre of the circle.
Let OC be the perpendicular drawn from O to AB.



We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.
∴ AC = CB = 3 cm
In ΔOCA,
$OA^2 = OC^2 + AC^2$ (By Pythagoras theorem)
$\Rightarrow OC^2 = (5)^2 - (3)^2 = 16$
$\Rightarrow OC = 4$ cm

Solution 2:

Let AB be the chord and O be the centre of the circle.
Let OC be the perpendicular drawn from O to AB.



We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.
∴ AB = 8 cm

$$\Rightarrow AC = CB = \frac{AB}{2}$$

$$\Rightarrow AC = CB = \frac{8}{2}$$

$\Rightarrow$ AC = CB = 4 cm

In ΔOCA,

$OA^2 = OC^2 + AC^2$ (By Pythagoras theorem)

$\Rightarrow OA^2 = (4)^2 + (3)^2 = 25$

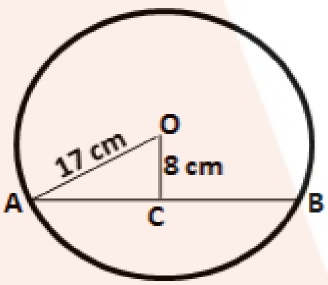
$\Rightarrow OA = 5$ cm

Hence, radius of the circle is 5 cm.

Solution 3:

Let AB be the chord and O be the centre of the circle.

Let OC be the perpendicular drawn from O to AB.



We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.

$\therefore$ AC = CB

In ΔOCA,

$OA^2 = OC^2 + AC^2$ (By Pythagoras theorem)

$\Rightarrow AC^2 = (17)^2 - (8)^2 = 225$

$\Rightarrow AC = 51$ cm

$\therefore$ AB = 2 AC = 2 × 15 = 30 cm

Solution 4:

Let AB be the chord of length 24 cm and O be the centre of the circle.

Let OC be the perpendicular drawn from O to AB.

We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.

$\therefore$ AC = CB = 12 cm

In ΔOCA,

$OA^2 = OC^2 + AC^2$ (By Pythagoras theorem)

$= (5)^2 + (12)^2 = 169$

$\Rightarrow OA = 13$ cm

∴ radius of the circle = 13 cm

Let $A'B'$ be new chord at a distance of 12 cm from the centre.

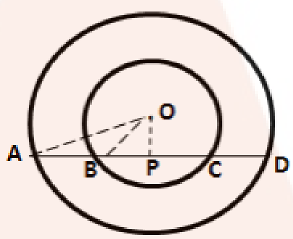
$\therefore (OA')^2 = (OC')^2 + (A'C')^2$

$\Rightarrow (A'C')^2 = (13)^2 - (12)^2 = 25$

$\therefore A'C' = 5cm$

Hence, length of the new chord = 2 × 5 = 10 cm

Solution 5:



For the inner circle, BC is a chord and $OP \perp BC$.

We know that the perpendicular to a chord, from the centre of a circle, bisects the chord.

∴ BP = PC

By Pythagoras theorem,

$OA^2 = OP^2 + BP^2$

$\Rightarrow BP^2 = (20)^2 - (16)^2 = 144$

$\therefore BP = 12cm$

For the outer circle, AD is the chord and $OP \perp AD$.

We know that the perpendicular to a chord, from the centre of a circle, bisects the chord.

∴ AP = PD

By Pythagoras Theorem,

$OA^2 = OP^2 + AP^2$

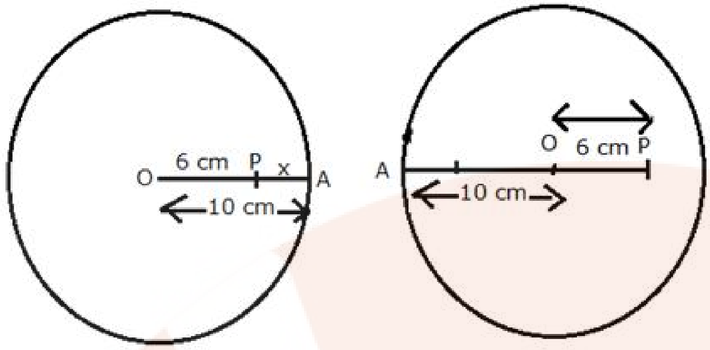
=> $AP^2 = (34)^2 - (16)^2 = 900$

=> AP = 30 cm

AB = AP − BP = 30 − 12 = 18 cm

Solution 6:

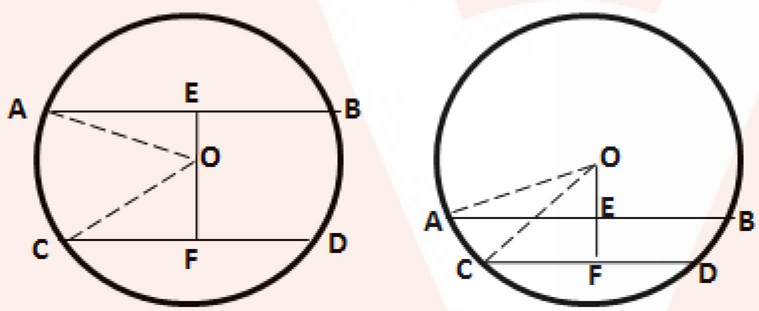
The least value of x will be when A is on OP produced, i.e. O, P and A are collinear.



$\therefore$ AP = OA - OP = 10 - 6 = 4 cm.
The maximum value of x will be when A is on PO produced, i.e. A, O and P are collinear.
$\therefore$ AP = OA + OP = 10 + 6 = 16 cm.

Solution 7:

Let O be the centre of the circle and AB and CD be the two parallel chords of length 30 cm and 16 cm respectively.
Drop OE and OF perpendicular on AB and CD from the centre O.



$OP \perp AB$ and $OF \perp CD$
$\therefore$ OE bisects AB and OF bisects CD
(Perpendicular drawn from the centre of a circle to a chord bisects it)

$$\Rightarrow AE = \frac{30}{2} = 15\text{cm}; \quad CF = \frac{16}{2} = 8\text{cm}$$

In right ΔOAE,

$$OA^2 = OE^2 + AE^2$$

$$\Rightarrow OE^2 = OA^2 - AE^2 = (17)^2 - (15)^2 = 64$$

$$\therefore OE = 8\text{cm}$$

In right ΔOCF,

$OC^2 = OF^2 + CF^2$

$\Rightarrow OF^2 = OC^2 - CF^2 = (17)^2 - (8)^2 = 225$

$\therefore OF = 15 \text{ cm}$

(i) The chords are on the opposite sides of the centre:

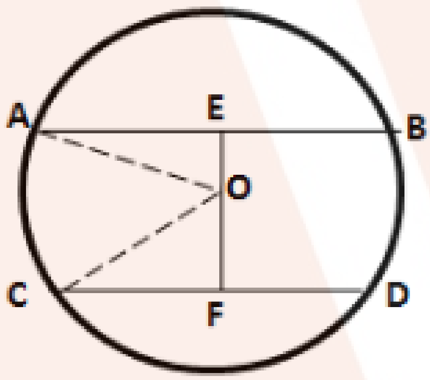
$\therefore EF = EO + OF = (8+15) = 23 \text{ cm}$

(ii) The chords are on the same side of the centre:

$\therefore EF = OF - OE = (15-8) = 7 \text{ cm}$

Solution 8:

Since the distance between the chords is greater than the radius of the circle (15 cm), so the chords will be on the opposite sides of the centre.



Let O be the centre of the circle and AB and CD be the two parallel chords such that AB = 24 cm.
Let length of CD be 2x cm.
Drop OE and OF perpendicular on AB and CD from the centre O.

$OE \perp AB$ and $OF \perp CD$

$\therefore$ OE bisects AB and OF bisects CD

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\Rightarrow AE = \frac{24}{2} = 12\text{cm};\ \ CF = \frac{2x}{2} = x \text{ cm}$

In right ΔOAE,

$OA^2 = OE^2 + AE^2$

$\Rightarrow OE^2 = OA^2 - AE^2 = (15)^2 - (12)^2 = 81$

$\therefore OE = 9 \text{ cm}$

$\therefore OF = EF - OE = (21-9) = 12 \text{ cm}$

In right ΔOCF,

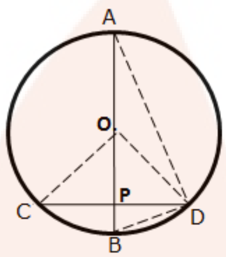
$OC^2 = OF^2 + CF^2$

$\Rightarrow x^2 = OC^2 - OF^2 = (15)^2 - (12)^2 = 81$

$\therefore x = 9$ cm

Hence, length of chord $CD = 2x = 2 \times 9 = 18$ cm

Solution 9:



$OP \perp CD$

$\therefore$ OP bisects CD

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\Rightarrow CP = \frac{CD}{2}$

In right ΔOPC,

$OC^2 = OP^2 + CP^2$

$\Rightarrow CP^2 = OC^2 - OP^2 = (15)^2 - (9)^2 = 144$

$\therefore CP = 12$ cm

$\therefore CD = 12 \times 2 = 24$ cm

(ii) Join BD

$\therefore BP = OB - OP = 15 - 9 = 6$ cm

In right ΔBPD,

$BD^2 = BP^2 + PD^2$

$= (6)^2 + (12)^2 = 180$

In ΔADB, $\angle ADB = 90°$

(Angle in a semicircle is a right angle)

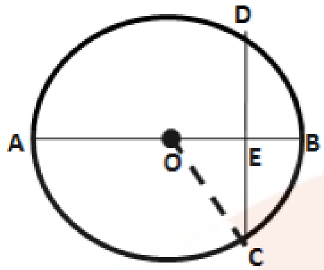
$\therefore AB^2 = AD^2 + BD^2$

$\Rightarrow AD^2 = AB^2 - BD^2 = (30)^2 - 180 = 720$

$\therefore AD = \sqrt{720} = 26.83$ cm

(iii) Also, $BC = BD = \sqrt{180} = 13.42$ cm

Solution 10:



Let the radius of the circle be r cm.

$\therefore OE = OB - EB = r - 4$

Join OC

In right ΔOEC,

$OC^2 = OE^2 + CE^2$

$\Rightarrow r^2 = (r-4)^2 + (8)^2$

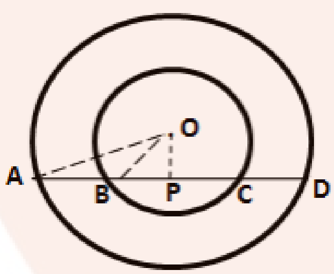
$\Rightarrow r^2 = r^2 - 8r + 16 + 64$

$\Rightarrow 8r = 80$

$\therefore r = 10$ cm

Hence, radius of the circle is 10 cm.

Solution 11:



Drop $OP \perp AD$

$\therefore$ OP bisects AD

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\Rightarrow AP = PD$ .... (i)

Now, BC is a chord for the inner circle and $OP \perp BC$

$\therefore$OP bisects BC

(Perpendicular drawn from the centre of a circle to a chord bisects it)

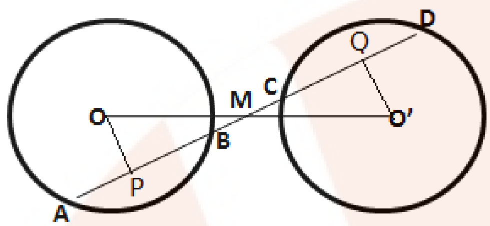
$\Rightarrow BP = PC$ .... (ii)

Subtracting (ii) from (i),

$AP - PB = PD - PC$

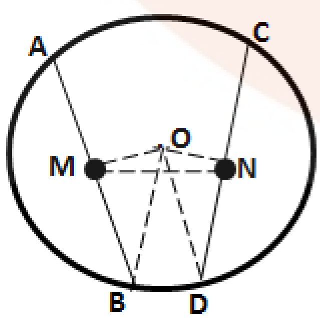
$\Rightarrow AB = CD$

Solution 12:



Given: A straight line Ad intersects two circles of equal radii at A, B, C and D.
The line joining the centres OO' intersect AD at M
And M is the midpoint of OO'.
To prove: AB = CD
Construction: From O, draw $OP \perp AB$ and from O', draw $O'Q \perp CD$.
Proof:
In ΔOMP and $\Delta O'MQ$,
$\angle OMP = \angle O'MQ$ (vertically opposite angles)
$\angle OPM = \angle O'QM$ (each $= 90°$)
$OM = O'M$ (Given)
By Angle – Angle – Side criterion of congruence,
$\therefore \Delta OMP \cong \Delta O'MQ$, (by AAS)
The corresponding parts of the congruent triangle are congruent
$\therefore OP = O'Q$ (c.p.ct)
We know that two chords of a circle or equal circles which are equidistant from the centre are equal.
$\therefore AB = CD$

Solution 13:



Drop $OM \perp AB$ and $ON \perp CD$
$\therefore$ OM bisects AB and ON bisects CD
(Perpendicular drawn from the centre of a circle to a chord bisects it)

$$\Rightarrow BM = \frac{1}{2}AB = \frac{1}{2}CD = DN \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

Applying Pythagoras theorem,

$$OM^2 = OB^2 - BM^2$$
$$= OD^2 - DN^2 \qquad \text{(by (1))}$$
$$= ON^2$$

$\therefore OM = ON$

$\Rightarrow \angle OMN = \angle ONM \qquad \ldots\ldots\ldots\ldots\ldots(2)$

(Angles opp to equal sides are equal)

(i) $\angle OMB = \angle OND$ (both 90°)

Subtracting (2) from above,

$\angle BMN = \angle DNM$

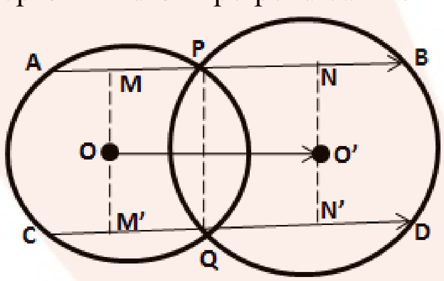
(ii) $\angle OMA = \angle ONC$ (both 90°)

Adding (2) to above,

$\angle AMN = \angle CNM$

Solution 14:

Drop OM and O'N perpendicular on AB and OM' and O'N' perpendicular on CD.



$$\therefore\ MP = \frac{1}{2}AP,\ PN = \frac{1}{2}BP,\ M'Q = \frac{1}{2}CQ,\ QN' = \frac{1}{2}QD$$

$$\text{Now, } OO' = MN = MP + PN = \frac{1}{2}(AP + BP) = \frac{1}{2}AB \text{---------(i)}$$

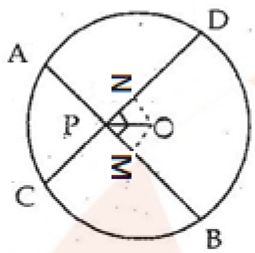
$$\text{And } OO' = M'N' = M'Q + QN' = \frac{1}{2}(CQ + QD) = \frac{1}{2}CD \text{-------(ii)}$$

By (i) and (ii)

AB = CD

Solution 15:

Drop OM and ON perpendicular on AB and CD.
Join OP, OB and OD.



∴ OM and ON bisect AB and CD respectively
(Perpendicular drawn from the centre of a circle to chord bisects it)

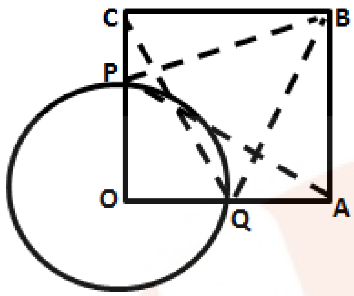
$\therefore MP = \frac{1}{2}AB = \frac{1}{2}CD = ND$ ....(i)

In rt Δ OMB, $OM^2 = OB^2 - MB^2$ ....(ii)

In rt Δ OND, $ON^2 = OD^2 - ND^2$ ....(iii)

From (i),(ii) and (iii)
OM = ON
In ΔOPM and ΔOPN,
$\angle OMP = \angle ONP$ (both 90°)
OP = OP (Common)
OM = ON (Proved above)
By Right Angle – Hypotenuse – Side criterion of congruence,
∴ $\Delta OPM \cong \Delta OPN$ (by RHS)
The corresponding parts of the congruent triangles are congruent.
∴ PM = PN (c.p.c.t)
Adding (i) to both sides,
MB + PM = ND + PN
⟹ BP = DP
Now, AB = CD
∴ AB – BP = CD – DP (∵ BP = DP)
⟹ AP = CP

Solution 16:



(i)

In ΔOPA and ΔOQC,

OP = OQ (radii of same circle)

∠AOP = ∠COQ (both 90°)

OA = OC (Sides of the square)

By Side – Angle – Side criterion of congruence,

∴ ΔOPA ≅ ΔOQC (by SAS)

(ii)

Now, OP = OQ (radii)

And OC = OA (sides of the square)

∴ OC – OP = OA – OQ

⇒ CP = AQ …………… (1)

In ΔBPC and ΔBQA,

BC = BA (Sides of the square)

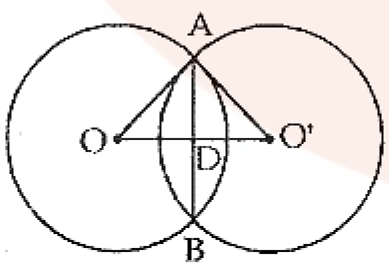
∠PCB = ∠QAB (both 90°)

PC = QA (by (1))

By Side – Angle – Side criterion of congruence,

∴ ΔBPC ≅ ΔBQA (by SAS)

Solution 17:



OA = 25 cm and AB = 30 cm

$\therefore\ AD = \frac{1}{2} \times AB = \left(\frac{1}{2} \times 30\right) cm = 15\ cm$

Now in right angled ΔADO,

$OA^2 = AD^2 + OD^2$

$\Rightarrow OD^2 = OA^2 - OD^2 = 25^2 - 15^2$

$= 625 - 225 = 400$

$\therefore\ OD = \sqrt{400} = 20\ cm$

Again, we have $O'A = 17$ cm

In right angle $\Delta ADO'$

$O'A^2 = AD^2 + O'D^2$

$\Rightarrow O'D^2 = O'A^2 - AD^2 = 17^2 - 15^2$

$= 289 - 225 = 64$

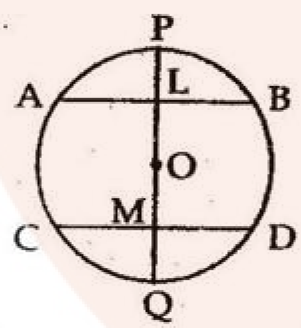
$\therefore O'D = 8\ cm$

$\therefore OO' = (OD + O'D)$

$= (20 + 8) = 28\ cm$

$\therefore$ the distance between their centres is 28 cm

Solution 18:



Given: AB and CD are the two chords of a circle with centre O.

L and M are the midpoints of AB and CD and O lies in the line joining ML

To prove: AB || CD

Proof: AB and CD are two chords of a circle with centre O.

Line LOM bisects them at L and M

Then, $OL \perp AB$

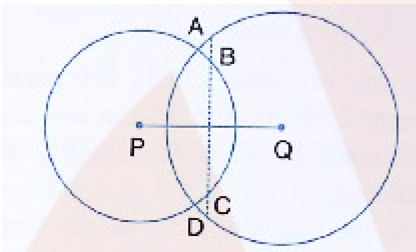
And, $OM \perp CD$

$\therefore\quad \angle ALM = \angle LMD = 90°$

But they are alternate angles

$\therefore$ AB || CD.

Solution 19:



In the circle with centre Q, $QO \perp AD$

$\therefore \quad OA = OD \quad \ldots\ldots\ldots\ldots\ldots (1)$

(Perpendicular drawn from the centre of a circle to a chord bisects it)

In the circle with centre P, $PO \perp BC$

$\therefore \ OB = OC \quad \ldots\ldots\ldots\ldots\ldots..(2)$

(Perpendicular drawn from the centre of a circle to a chord bisects it)

(i)

(1) – (2) Gives,

$AB = CD \quad \ldots\ldots\ldots\ldots\ldots(3)$

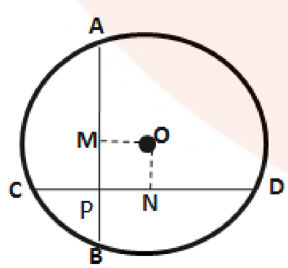
(ii)

Adding BC to both sides of equation (3)

$AB + BC = CD + BC$

$\Rightarrow AC = BD$

Solution 20:



Clearly, all the angles of OMPN are 90°

$OM \perp AB$ and $ON \perp CD$

$\therefore$ $BM = \frac{1}{2}AB = \frac{1}{2}CD = CN$ ………….. (i)

(Perpendicular drawn from the centre of a circle to a chord bisects it)
As the two equal chords AB and CD intersect at point P inside
The circle,

$\therefore$ $AP = DP$ and $CP = BP$ ……………. (ii)

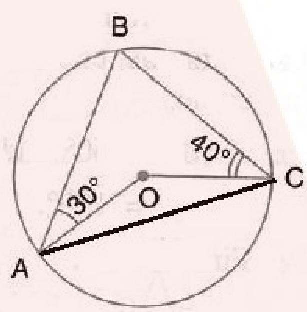
Now, $CN - CP = BM - BP$ (by (i) and (ii))

$\Rightarrow PN = MP$

$\therefore$ Quadrilateral OMPN is a square

EXERCISE. 17 (B)

Solution 1:



Join AC,

Let $\angle OAC = \angle OCA = x$ (say)

$\therefore \angle AOC = 180° - 2x$

Also, $\angle BAC = 30° + x$

$\angle BCA = 40° + x$

In ΔABC,

$\angle ABC = 180° - \angle BAC - \angle BCA$

$= 180° - (30° + x) - (40° + x) = 110° - 2x$

Now, $\angle AOC = 2\angle ABC$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

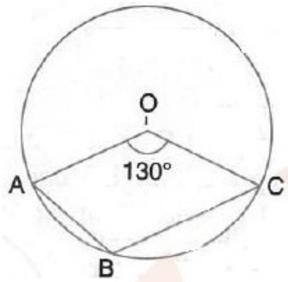
$\Rightarrow 180° - 2x = 2(110° - 2x)$

$\Rightarrow 2x = 40°$

$\therefore x = 20°$

$\therefore \angle AOC = 180° - 2 \times 20° = 140°$

Solution 2:



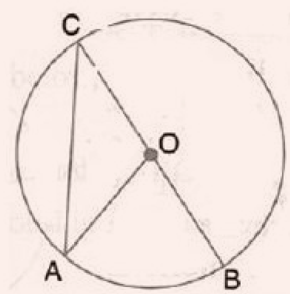
Here, Reflex $\angle AOC = 2\angle ABC$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\Rightarrow 360° - 130° = 2\angle ABC$

$$\Rightarrow \angle ABC = \frac{230°}{2} = 115°$$

Solution 3:



Here, $\angle AOB = 2\angle ACB$

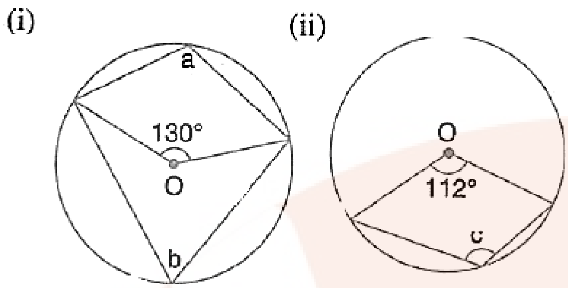
(Angle at the center is double the angle at the circumference by the same chord)

$$\Rightarrow \angle ACB = \frac{70°}{2} = 35°$$

Now, $OC = OA$ (radii of same circle)

$\Rightarrow \angle OCA = \angle OAC = 35°$

Solution 4:



(i) Here, $b = \frac{1}{2} \times 130°$

(Angle ate he centre is double the angle at the circumference subtended by the same chord)

$\Rightarrow b = 65°$

Now, $a + b = 180°$

(Opposite angles of a cyclic quadrilateral are supplementary)

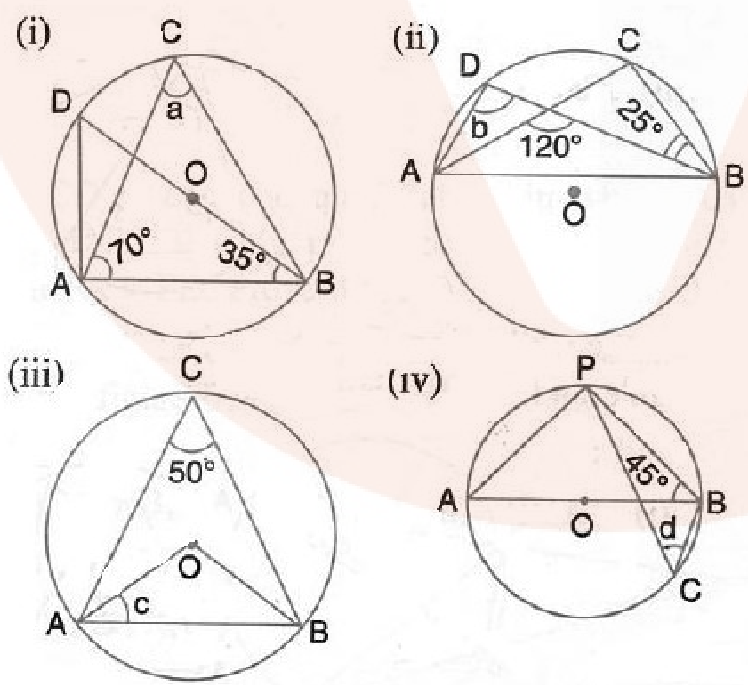
$\Rightarrow a = 180° - 65° = 115°$

(ii) Here, $c = \frac{1}{2}$ Reflex $(112°)$

(Angle at the cente is double the angle at the circumference subtended by the same chord)

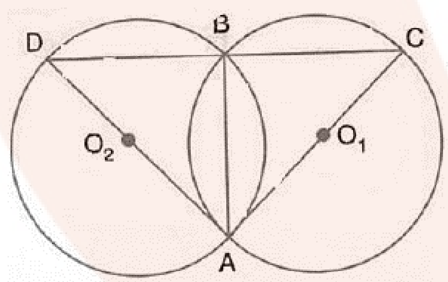
$\Rightarrow c = \frac{1}{2} \times (360° - 112°) = 124°$

Solution 5:



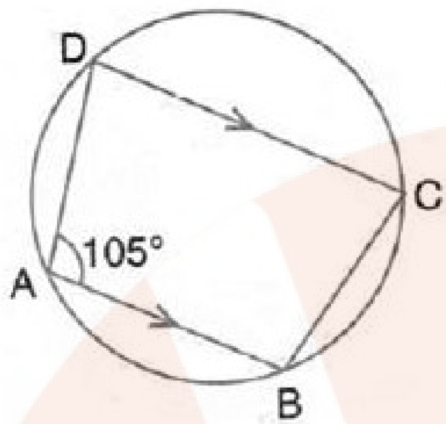
(i) Here, $\angle BAD = 90°$ (Angle in a semicircle)
$\therefore \angle BDA = 90° - 35° = 55°$
Again, $a = \angle ACB = \angle BDA = 55°$
(Angle subtended by the same chord on the circle are equal)
(ii) Here, $\angle DAC = \angle CBD = 25°$
(Angle subtended by the same chord on the circle are equal)
Again, $120° = b + 25°$
(In a triangle, measure of exterior angle is equal to the sum of pair of opposite interior angles)
$\Rightarrow b = 95°$
(iii) $\angle AOB = 2\angle AOB = 2 \times 50° = 100°$
(Angle at the centre is double the angle at the circumference subtended by the same chord)
Also, $OA = OB$
$\Rightarrow \angle OBA = \angle OAB = C$
$\therefore c = \dfrac{180° - 100°}{2} = 40°$
(iv) $\angle APB = 90°$ (Angle in a semicircle)
$\therefore \angle BAP = 90° - 45° = 45°$
Now, $d = \angle BCP = \angle BAP = 45°$
(Angle subtended by the same chord on the circle are equal)

Solution 6:



$\angle DBA = 90°$ and $\angle CBA = 90°$
(Angles in a semicircle is a right angle)
Adding both we get,
$\angle DBC = 180°$
$\therefore$ D, B and C form a straight line.

Solution 7:



(i) $\angle BCD + \angle BAD = 180°$

(Sum of opposite angles of a cyclic quadrilateral is $180°$)

$\Rightarrow \angle BCD = 180° - 105° = 75°$

(ii) Now, AB || CD

$\therefore \angle BAD + \angle ADC = 180°$

(Interior angles on the same side of parallel lines is $180°$)

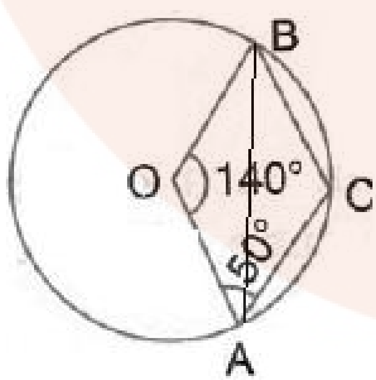
$\Rightarrow \angle ADC = 180° - 105° = 75°$

(iii) $\angle ADC + \angle ABC = 180°$

(Sum of opposite angles of a cyclic quadrilateral is $180°$)

$\Rightarrow \angle ABC = 180° - 75° = 105°$

Solution 8:



Here, $\angle ACB = \frac{1}{2}$ Reflex $(\angle AOB) = \frac{1}{2}(360° - 140°) = 110°$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

Now, OA = OB (Radii of same circle)

$\therefore \angle OBA = \angle OAB = \dfrac{180° - 140°}{2} = 20°$

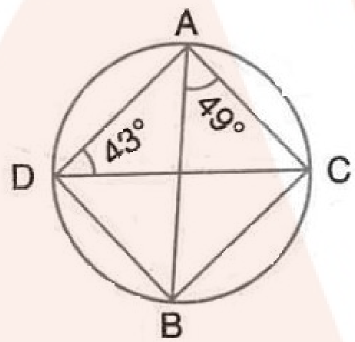
$\therefore \angle CAB = 50° - 20° = 30°$

$\Delta CAB,$

$\angle CBA - 180° - 110° - 30° = 40°$

$\therefore \angle OBC = \angle CBA + \angle OBA = 40° + 20° = 60°$

Solution 9:



Here,

$\angle CDB = \angle BAC = 49°$

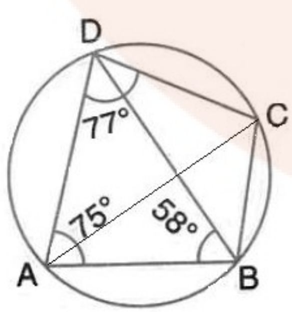
$\angle ABC = \angle ADC = 43°$

(Angle subtend by the same chord on the circle are equal)

By angle – sum property of a triangle,

$\angle ACB = 180° - 49° - 43° = 88°$

Solution 10:



(i) By angle – sum property of triangle ABD,

$\angle ADB = 180° - 75° - 58° = 47°$

$\therefore \angle BDC = \angle ADC - \angle ADB = 77^\circ - 47^\circ = 30^\circ$

(ii) $\angle BAD + \angle BCD = 180^\circ$

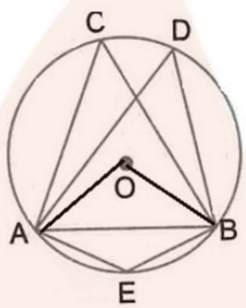
(Sum of opposite angles of a cyclic quadrilateral is 180°)

$\Rightarrow \angle BCD = 180^\circ - 75^\circ = 105^\circ$

(iii) $\angle BDA = \angle ADB = 47^\circ$

(Angle subtended by the same chord on the circle are equal)

Solution 11:



Since $\angle ACB$ and $\angle ADB$ are in the same segment,

$\angle ADB = \angle ACB = 60^\circ$

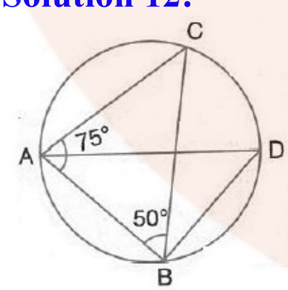
Join OA and OB

Here, $\angle AOB = 2\angle ACB = 2 \times 60^\circ = 120^\circ$

$$\angle AEB = \frac{1}{2}\text{Reflex}(\angle AOB) = \frac{1}{2}(360^\circ - 120^\circ) = 120^\circ$$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

Solution 12:



In $\Delta ABC, \angle CBA = 50^\circ, \angle CAB = 75^\circ$

$$\angle ACB = 180^\circ - (\angle CBA + \angle CAB)$$
$$= 180^\circ - (50^\circ + 75^\circ)$$
$$= 180^\circ - 125^\circ$$

$= 55°$

But $\angle ADB = \angle ACB = 55°$

(Angle subtended by the same chord on the circle are equal)

Now consider ΔABD,

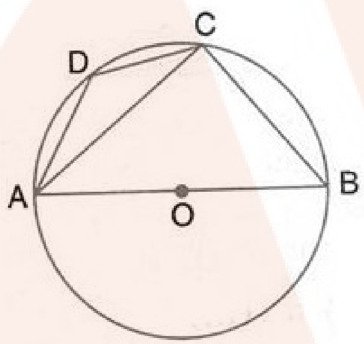
$\angle DAB + \angle ABD + \angle ADB = 180°$

$\Rightarrow \angle DAB + \angle ABD + 55° = 180°$

$\Rightarrow \angle DAB + \angle ABD = 180° - 55°$

$\Rightarrow \angle DAB + \angle ABD = 125°$

Solution 13:



Here $\angle ACB = 90°$

(Angle in a semicircle is right angle)

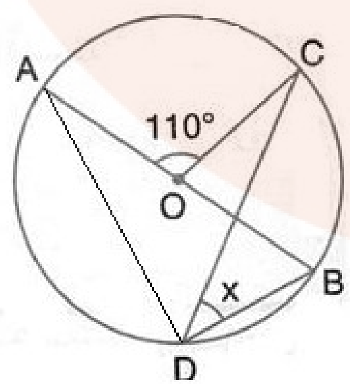
Also, $\angle ABC = 180° - \angle ADC = 180° - 130° = 50°$

(pair of opposite angles in a cy clic quadrilateral are supplementary)

By angle sum property of right triangle ACB,

$\angle BAC = 90° - \angle ABC = 90° - 50° = 40°$

Solution 14:



Join AD.

Here, $\angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2} \times 110° = 55°$

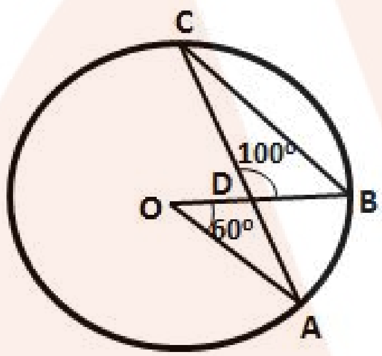
(Angle at the centre is double the angle at the circumference subtended by the same chord)

Also, $\angle ADB = 90°$

(Angle in a semicircle is a right angle)

$\therefore \angle BDC = 90° - \angle ADC = 90° - 55° = 35°$

Solution 15:



Here, $\angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 60° = 30°$

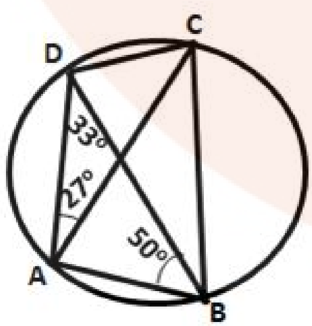
(Angle at the centre is double the angle at the circumference subtended by the same chord)

By angle sum property of ΔBDC,

$\therefore \angle DBC = 180° - 100° - 30° = 50°$

Hence, $\angle OBC = 50°$

Solution 16:



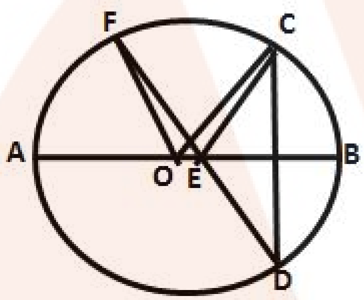
(i) $\angle DBC = \angle DAC = 27°$

(Angle subtended by the same chord on the circle are equal)

(ii) $\angle ACB = \angle ADB = 33°$

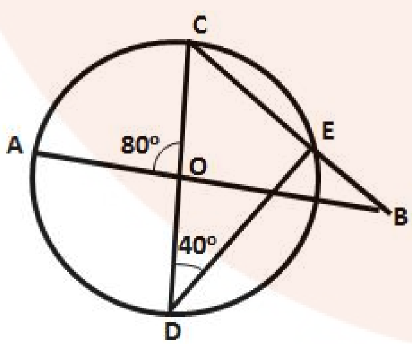
$\angle ACD = \angle ABD = 50°$
(Angle subtended by the same chord on the circle are equal)
$\therefore \angle DCB = \angle ACD + \angle ACB = 50° + 33° = 83°$
(iii) $\angle DAB + \angle DCB = 180°$
(Pair of opposite angles in a cyclic quadrilateral are supplementary)
$\Rightarrow 27° + \angle CAB + 83° = 180°$
$\Rightarrow \angle CAB = 180° - 110° = 70°$

Solution 17:



Here, $\angle COF = 2\angle CDF = 2 \times 32° = 64°$ ……………… (i)
(Angle at the centre is double the angle at the circumference subtended by the same chord)
In ΔECD,
$\angle CEF = \angle ECD + \angle EDC = 32° + 32° = 64°$ …………(ii)
(Exterior angle of a Δ is equal to the sum of pair of interior opposite angles)
From (i) and (ii), we get
$\angle COF = \angle CEF$

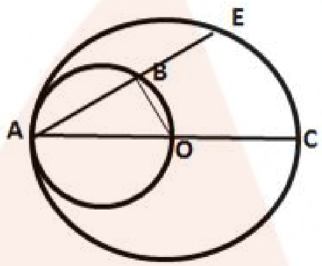
Solution 18:



(i) Here, $\angle CED = 90°$
(Angle in a semicircle is a right angle)
$\therefore \angle DCE = 90° - \angle CDE = 90° - 40° = 50°$
$\therefore \angle DCE = \angle OCB = 50°$

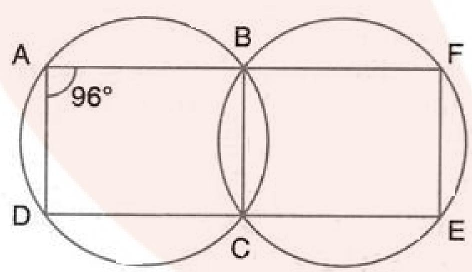
(ii) In ΔBOC,
$\angle AOC = \angle OCB + \angle OBC$
(Exterior angle of a Δ is equal to the sum of pair of interior opposite angles)
$\Rightarrow \angle OBC = 80° - 50° = 30° \quad [\angle AOC = 80°, \text{given}]$
Hence, $\angle ABC = 30°$

Solution 19:



Join OB,
Then $\angle OBA = 90°$
(Angle in a semicircle is a right angle)
i.e. $OB \perp AE$
We know the perpendicular drawn from the centre to a chord bisects the chord.
$\therefore AB = BE$

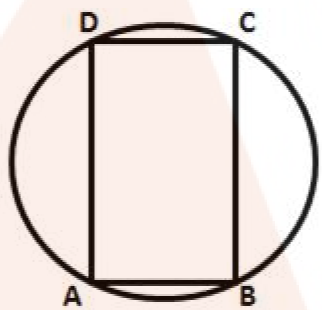
Solution 20:



(i) ABCD is a cyclic quadrilateral
$\therefore \angle BAD + \angle BCD = 180°$
(pair of opposite angles in a cyclic quadrilateral are supplementary)
$\Rightarrow \angle BCD = 180° - 96° = 84°$
$\therefore \angle BCE = 180° - 84° = 96°$
Similarly, BCEF is a cyclic quadrilateral

$\therefore \angle BCE + \angle BFE = 180°$
(pair of opposite angles in a cyclic quadrilateral are supplementary)
$\therefore \angle BFE = 180° - 96° = 84°$
(ii) Now, $\angle BAD + \angle BFE = 96° + 84° = 180°$
But these two are interior angles on the same side
of a pair of lines AD and FE
$\therefore$ AD || FE

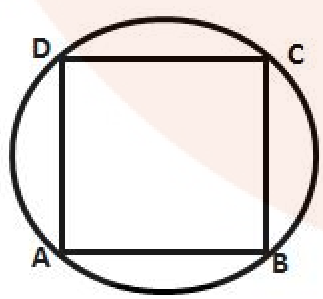
Solution 21:



(i) Let ABCD be a parallelogram, inscribe in a circle,
Now, $\angle BAD = \angle BCD$
(Opposite angles of a parallelogram are equal)
And $\angle BAD = \angle BCD = 180°$
(pair of opposite angles in a cyclic quadrilateral are supplementary)

$$\therefore \angle BAD = \angle BCD = \frac{180°}{2} = 90°$$

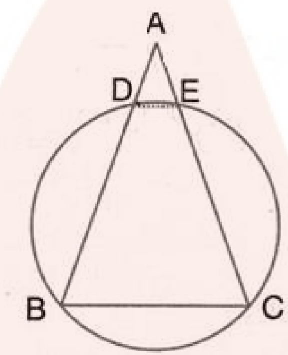
||y, the other two angles are $90°$ and opposite pair of sides
Are equal.
$\therefore$ ABCD is a rectangle.



(ii) Let ABCD be a rhombus, inscribed in a circle
Now, $\angle BAD = \angle BCD$
(Opposite angles of a parallelogram are equal)

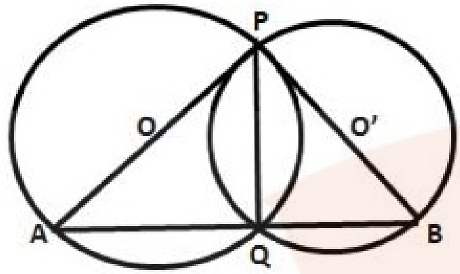
And $\angle BAD = \angle BCD = 180°$
(pair of opposite angles in a cyclic quadrilateral are supplementary)
$\therefore \angle BAD = \angle BCD = \dfrac{180°}{2} = 90°$
||y, the other two angles are $90°$ and all the sides
are equal.
$\therefore$ ABCD is a square.

Solution 22:



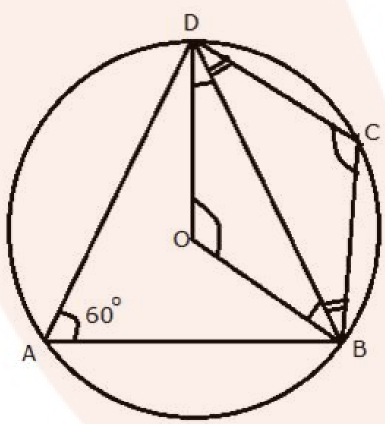
Here, AB = AC
$\Rightarrow \angle B = \angle C$
$\therefore$ DECB is a cyclic quadrilateral
(Ina triangle, angles opposite to equal sides are equal)
Also, $\angle B + \angle DEC = 180°$ ………. (1)
(pair of opposite angles in a cyclic quadrilateral are supplementary
$\Rightarrow \angle C + \angle DEC = 180°$ [from (1)]
But this is the sum of interior angles
On one side of a transversal.
$\therefore$ DE || BC
But $\angle ADE = \angle B$ and $\angle AED = \angle C$ [Corresponding angles]
Thus, $\angle ADE = \angle AED$
$\Rightarrow AD = AE$
$\Rightarrow AB - AD = AC - AE(\because \quad C)$
$\Rightarrow BD = CE$
Thus, we have, DE || BC and $BD = CE$
Hence, DECB is an isosceles trapezium

Solution 23:



Let O and O' be the centres of two intersecting circle, where
Points of intersection are P and Q and PA and PB are their diameter respectively.
Join PQ, AQ and QB.
$\therefore \angle AQP = 90°$ and $\angle BQP = 90°$
(Angle in a semicircle is a right angle)
Adding both these angles,
$\angle AQP + \angle BQP = 180° \quad \Rightarrow \angle AQB = 180°$
Hence, the points A, Q and B are collinear.

Solution 24:



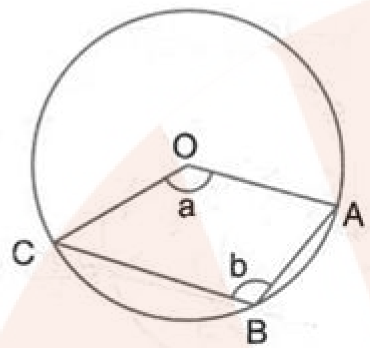
$\angle BOD = 2\angle BAD = 2 \times 60° = 120°$

And $\angle BCD = \frac{1}{2} \text{Re felx} (\angle BOD) = \frac{1}{2}(360° - 120°) = 120°$

(Angle at the centre is double the angle at the circumference subtended by the same chord
$\therefore \angle CBD + \angle CDB = 180° - 120° = 60°$
(By angle sum property of triangle CBD)
Again, $\angle OBD + \angle ODB = 180° - 120° = 60°$
(By angle sum property of triangle OBD)

$\therefore \angle OBD + \angle ODB = \angle CBD + \angle CDB$

Solution 25:



(i) $\angle ABC = \frac{1}{2}$ Reflex $(\angle COA)$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\Rightarrow b = \frac{1}{2}(360° - a)$

$\Rightarrow a + 2b = 180°$

(ii) Since OABC is a parallelogram, so opposite angles are equal

$\therefore a = b$

Using relationship in (i)

$3a = 180°$

$\therefore a = 60°$

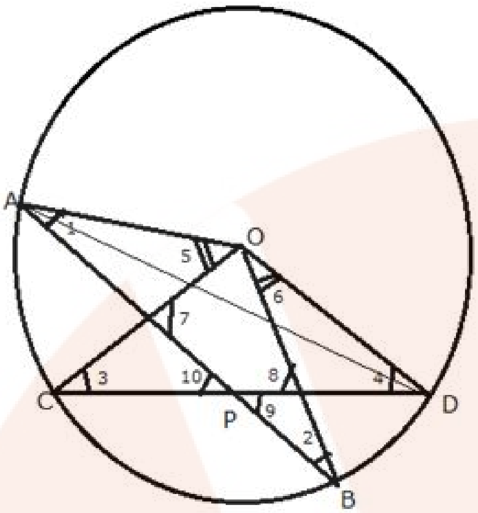
Also, $OC \parallel BA$

$\therefore \angle COA + \angle OAB = 180°$

$\Rightarrow 60° + \angle OAB = 180°$

$\Rightarrow \angle OAB = 120°$

Solution 26:



Given: two chords AB and CD intersect each other at P inside the circle. OA, OB, OC and OD are joined.

To prove: $\angle AOC + \angle BOD = 2\angle APC$

Construction: Join AD.

Proof: Arc AC subtends $\angle AOC$ at the centre and $\angle ADC$ at the remaining Part of the circle.

$\angle AOC = 2\angle ADC$ ....(1)

Similarly,

$\angle BOD = 2\angle BAD$ ....(2)

Adding (1) and (2),

$\angle AOC + \angle BOD = 2\angle ADC + 2\angle BAD$

$= 2(\angle ADC + \angle BAD)$ ....(3)

But ΔPAD,

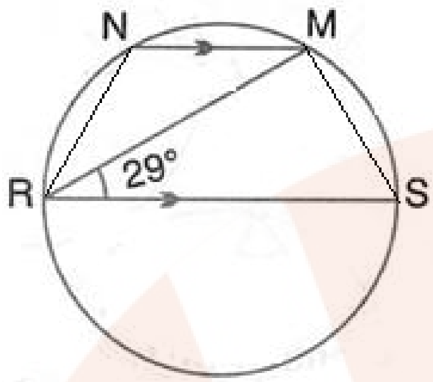
Ext. $\angle APC = \angle PAD + \angle ADC$

$= \angle BAD + \angle ADC$ ....(4)

From (3) and (4),

$\angle AOC + \angle BOD = 2\angle APC$

Solution 27:



(i) Join RN and MS.

$\therefore \angle RMS = 90°$

(Angle in a semicircle is a right angle)

$\therefore \angle RSM = 90° - 29° = 61°$

(By angle sum property of triangle RMS)

$\therefore \angle RNM = 180° \angle RSM = 180° - 61° = 119°$

(pair of opposite angles in a cyclic quadrilateral are supplementary)

(ii) Also, $RS \parallel NM$

$\therefore \angle NMR = \angle MRS = 29°$ (Alternate angles)

$\therefore \angle NMS = 90° + 29° = 119°$

Also, $\angle NRS + \angle MS = 180°$

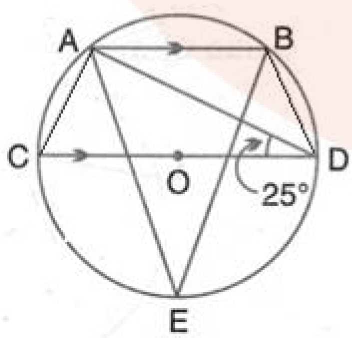
(pair of opposite angles in a cyclic quadrilateral are supplementary)

$\Rightarrow \angle NMR + 29° + 119° = 180°$

$\Rightarrow \angle NRM = 180° - 148°$

$\therefore \angle NRM = 32°$

Solution 28:



Join AC and BD

$\therefore \angle CAD = 90^\circ$ and $\angle CBD = 90^\circ$

(Angle in a semicircle is a right angle)

Also, $AB \parallel CD$

$\therefore \angle BAD = \angle ADC = 25^\circ$ (alternate angles)

$\angle BAC = \angle BAD + \angle CAD = 25^\circ + 90^\circ = 115^\circ$

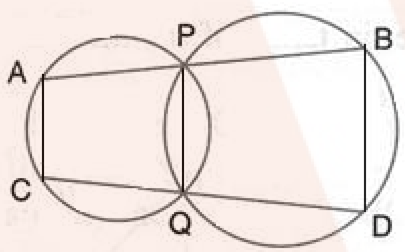
$\therefore \angle ADB = 180^\circ - 25^\circ - \angle BAC = 180^\circ - 25^\circ - 115^\circ = 40^\circ$

(pair of opposite angles in a cyclic quadrilateral are supplementary)

Also, $\angle AEB = \angle ADB = 40^\circ$

(Angle subtended by the same chord on the circle are equal)

Solution 29:



Join AC, PQ and BD

ACQP is a cyclic quadrilateral

$\therefore \angle CAP + \angle PQC = 180^\circ$ ………….(i)

(pair of opposite in a cyclic quadrilateral are supplementary)

PQDB is a cyclic quadrilateral

$\therefore \angle PQD + \angle DBP = 180^\circ$ ………….(ii)

(pair of opposite angles in a cyclic quadrilateral are supplementary)

Again, $\angle PQC + \angle PQD = 180^\circ$ …………. (iii)

(CQD is a straight line)

Using (i), (ii) and (iii)

$\therefore \angle CAP + \angle DBP = 180^\circ$

Or $\therefore \angle CAB + \angle DBA = 180^\circ$

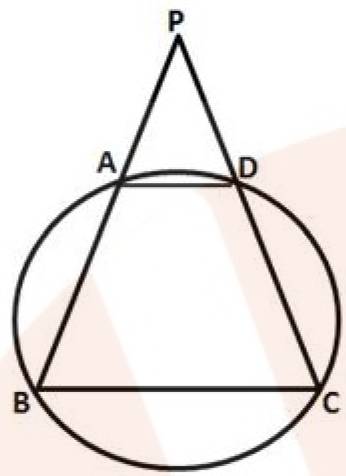
We know, if a transversal intersects two lines such

That a pair of interior angles on the same side of the

Transversal is supplementary, then the two lines are parallel

$\therefore AC \parallel BD$

Solution 30:



Let ABCD be the given cyclic quadrilateral

Also, PA = PD (Given)

$\therefore \angle PAD = \angle PDA$ ……..(1)

$\therefore \angle BAD == 180° - \angle PAD$

And $\angle CDA = 180° - \angle PDA = 180° - \angle PAD$ (From (1))

We know that the opposite angles of a cyclic quadrilateral are supplementary

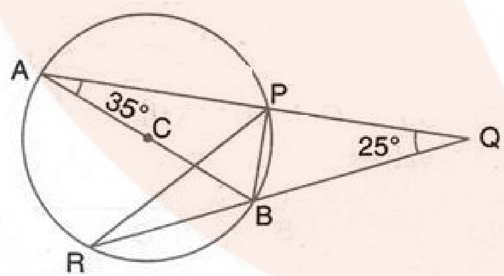
$\therefore \angle ABC = 180° - \angle CDA = 180° - (180° - \angle PAD) = \angle PAD$

And $\angle DCB = 180° - \angle BAD = 180° - (180° - \angle PAD) = \angle PAD$

$\therefore \angle ABC = \angle DCB = \angle PAD = \angle PAD$

That means $AD \parallel BC$

Solution 31:



(i) $\angle PRB = \angle PAB = 35°$

(Angles subtended by the same chord on the circle are equal)

(ii) $\angle BPA = 90°$

(angle in a semicircle is a right angle)

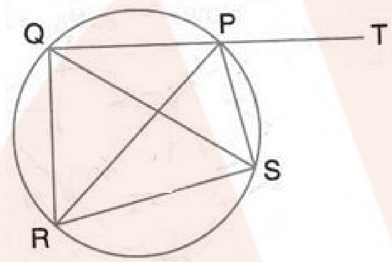
$\therefore \angle BPQ = 90°$

$\therefore \angle PBR = \angle BQP + \angle BPQ = 25° + 90° = 115°$

(Exterior angle of a Δ is equal to the sum of pair of interior opposite angles)
(iii) $\angle ABP = 90° - \angle BAP = 90° - 35° = 55°$
$\therefore \angle ABR = \angle PBR = \angle ABP = 115° - 55° = 60°$

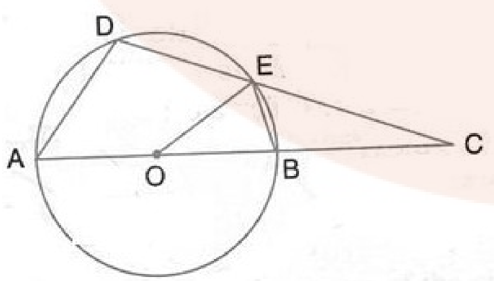
$\therefore \angle APR = \angle ABR = 60°$
(Angles subtended by the same chord on the circle are equal)
$\therefore \angle BPR = 90° - \angle APR = 90° - 60° = 30°$

Solution 32:



PQRS is a cyclic quadrilateral
$\therefore \angle QRS + \angle QPS = 180°$ …………(i)
(pair of opposite angles in a cyclic quadrilateral are supplementary)
Also, $\angle QPS + \angle SPT = 180°$ ……(ii)
(Straight line QPT)
From (i) and (ii)
$\angle QRS = \angle SPT$ ………. (iii)
Also, $\angle RQS = \angle RPS$ ……(iv)
(Angle subtended by the same chord on the circle are equal)
And $\angle RPS = \angle SPT$ (PS bisects $\angle RPT$) …… (v)
From (iii), (iv) and (v)
$\angle QRS = \angle RQS$
$\Rightarrow SQ = SR$

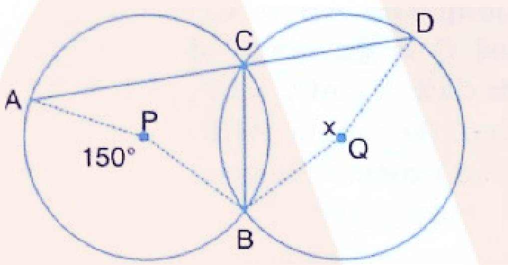
Solution 33:



$$\angle ADE = \frac{1}{2} \text{Reflex}(\angle AOE) = \frac{1}{2}(360° - 150°) = 105°$$

(Angle at the center is double the angle ate the circumference subtended by the same chord)
$\angle DAB + \angle BED = 180°$
(pair of opposite angles in a cyclic quadrilateral are supplementary)
$\Rightarrow \angle BED = 180° - 51° = 129°$
$\therefore \angle CEB = 180° - \angle BED$ (straight line)
$= 180° - 129° = 51°$
Also, by angle sum property of ΔADC,
$\angle OCE = 180° - 51° - 105° = 24°$

Solution 34:



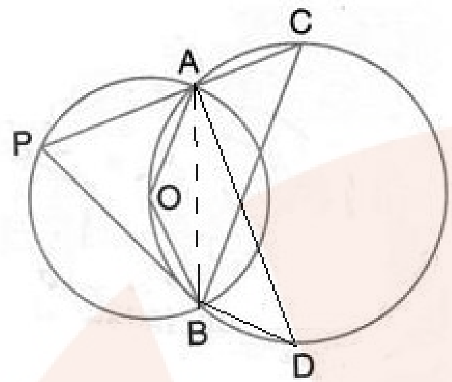
$\angle ACB = \frac{1}{2}\angle APB = \frac{1}{2}\times 150° = 75°$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\angle ACB + \angle BCD = 180°$

(Straight line)
$\Rightarrow \angle BCD = 180° - 75° = 105°$
Also, $\angle BCD = \frac{1}{2}$ reflex $\angle BQD = \frac{1}{2}(360° - x)$
(Angle at the center is double the angle at the circumference subtended by the same chord)
$\Rightarrow 105° = 180° - \frac{x}{2}$
$\therefore x = 2(180° - 105°) = 2\times 75° = 150°$

Solution 35:



(i) obtuse $\angle AOB = 2\angle APB = 2a°$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

(ii) OABC is a cyclic quadrilateral

$\therefore \angle AOB + \angle ACB = 180°$

(Pair of opposite angles in a cyclic quadrilateral are supplementary)

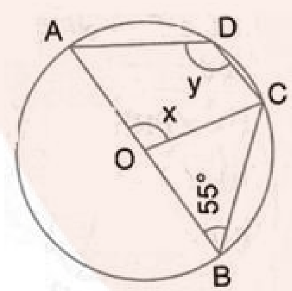
$\Rightarrow ACB = 180° - 2a°$

(iii) Join AB.

$\angle ADB = \angle ACB = 180° - 2a°$

(Angle subtended by the same arc on the circle are equal)

Solution 36:



$\angle AOC = 2\angle ABC = 2 \times 55°$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\therefore x = 110°$

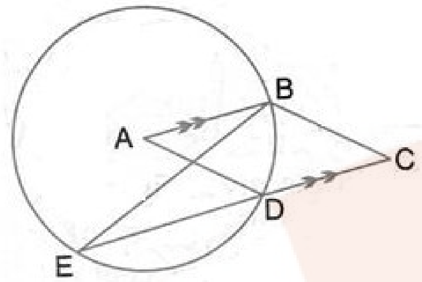
ABCD is cyclic quadrilateral

$\therefore \angle ADC + \angle ABC = 180°$

(pair of opposite angles in a cyclic quadrilateral are supplementary

$\Rightarrow y = 180° - 55° = 125°$

Solution 37:



$\angle BAD = 2\angle BED$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

And $\angle BED = \angle ABE$ (alternate angles)

$\therefore \angle BAD = 2\angle ABE$ ……… (i)

ABCD is a parallelogram

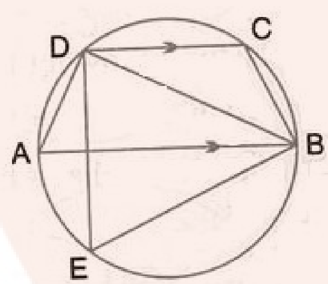
$\therefore \angle BAD = \angle BCD$ ………. (ii)

(opposite angles in a parallelogram are equal)

From (i) and (ii),

$\angle BCD = 2\angle ABE$

Solution 38:



(i) $\angle DAB = \angle BED = 65°$

(Angle subtended by the same chord on the circle are equal)

(ii) $\angle ADB = 90°$

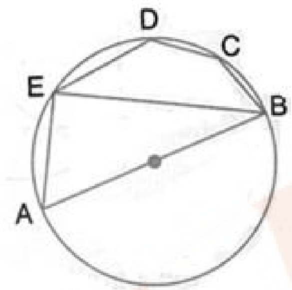
(Angle in a semicircle is a right angle)

$\therefore \angle ABD = 90° - \angle DAB = 90° - 65° = 25°$

$AB \parallel DC$

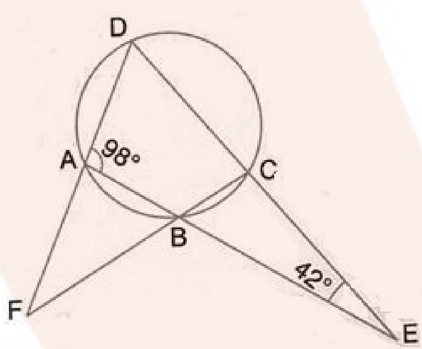
$\therefore \angle BDC = \angle ABD = 25°$ (Alternate angles)

Solution 39:



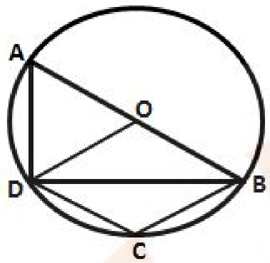
(i) $\angle AEB = 90°$
(Angle in a semicircle is a right angle)
Therefore $\angle EBA = 90° - \angle EAB = 90° - 63° = 27°$
(ii) $AB \parallel ED$
Therefore $\angle DEB = EBA = 27°$ (Alternate angles)
Therefore BCDE is a cyclic quadrilateral
Therefore $\angle DEB + \angle BCD = 180°$
[pair of opposite angles in a cyclic quadrilateral are supplementary]
Therefore $\angle BCD = 180° - 27° = 153°$

Solution 40:



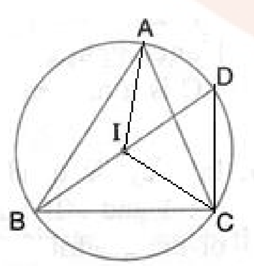
By angle sum property of ΔADE,
$\angle ADC = 180° - 98° - 42° = 40°$
Also, $\angle ADC + \angle ABC = 180°$
(pair of opposite angles in a cyclic quadrilateral are supplementary)
$\therefore \angle ABC = 180° - 40° = 140°$
Also, $\angle BAF = 180° - \angle BAD = 180° - 98° = 82°$
$\therefore \angle ABC = \angle AFB + \angle BAF$
(Exterior angle of a Δ is equal to the sum of pair of interior opposite angles)
$\Rightarrow \angle AFB = 140° - 82° = 58°$
Thus, $\angle AFB = 58°$ and $\angle ADC = 40°$

Solution 41:



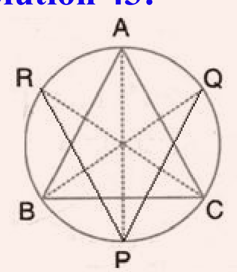
(i) ABCD is a cyclic quadrilateral
$\therefore \angle DCB + \angle DAB = 180°$
(pair of opposite angles in a cyclic quadrilateral are supplementary)
$\Rightarrow \angle DAB = 180° - 120° = 60°$
(ii) $\angle ADB = 90°$
(Angle in a semicircle is a right angle)
$\therefore \angle DBA = 90° - \angle DAB = 90° - 60° = 30°$
(iii) $OD = OB$
$\therefore \angle ODB = \angle OBD$
Or $\angle ABD = 30°$
Also, $AB \parallel ED$
$\therefore \angle DBC = \angle ODB = 30°$ (Alternate angles)
(iv) $\angle ABD + \angle DBC = 30° + 30° = 60°$
$\Rightarrow \angle ABC = 60°$
In cyclic quadrilateral ABCD,
$\angle ADC + \angle ABC = 180°$
(pair of opposite angles in a cyclic quadrilateral are supplementary)
$\Rightarrow \angle ADC = 180° - 60° = 120°$
In ΔAOD, OA = OD (radii of the same circle)
$\angle AOD = \angle DAO$ Or $\angle DAB = 60^\circ$ [proved in (i)]
$\Rightarrow \angle AOD = 60°$
$\angle ADO = \angle AOD = \angle DAO = 60°$
∴ΔAOD is an equilateral triangle.

Solution 42:



Join IA, IC and CD
(i) IB is the bisector of $\angle ABC$
$\Rightarrow \angle ABD = \frac{1}{2}\angle BC = \frac{1}{2}(180° - 65° - 55°) = 30°$
$\angle DCA = \angle ABD = 30°$
(Angle in the same segment)
(ii) $\angle DAC = \angle CBD = 30°$
(Angle in the same segment)
(iii) $\angle ACI = \frac{1}{2}\angle ACB = \frac{1}{2} \times 65° = 32.5°$
(CI is the angular bisector of $\angle ACB$)
$\therefore DCI = \angle DCA + \angle ACI = 30° + 32.5° = 62.5°$
(iv) $\angle IAC = \frac{1}{2}\angle BAC = \frac{1}{2} \times 55° = 27.5°$
(AI is the angular bisector of $\angle BAC$)
$\therefore \angle AIC = 180° - \angle IAC - \angle ICA = 180° - 27.5° - 32.5° = 120°$

Solution 43:



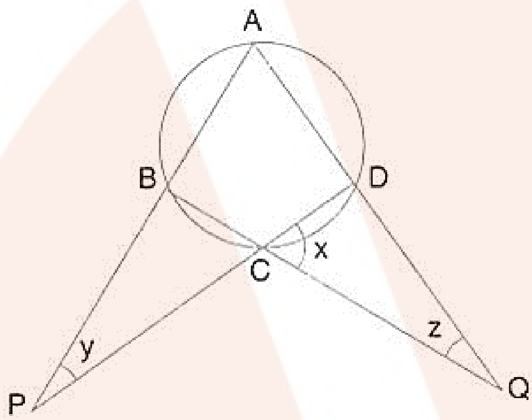
Join PQ and PR
(i) BQ is the bisector of $\angle ABC$
$\Rightarrow \angle ABQ = \frac{1}{2}\angle ABC$
Also, $\angle APQ = \angle ABQ$
(Angle in the same segment)
$\therefore \angle ABC = 2\angle APQ$
(ii) CR is the bisector of $\angle ACB$
$\Rightarrow ACR = \frac{1}{2}\angle ACB$
Also, $\angle ACR = \angle APR$
(Angle in the same segment)
$\therefore \angle ACB = 2\angle APR$
(iii) Adding (i) and (ii)
We get

$\angle ABC + \angle ACB\angle = 2(\angle APR + \angle APQ) = 2\angle QPR$

$\Rightarrow 180^{\circ} - \angle BAC = 2\angle QPR$

$\Rightarrow \angle QPR = 90^{\circ} - \frac{1}{2}\angle BAC$

Solution 44:



Let x = 3k, y = 4k and z = 5k

$\angle ADB = x + z = 8k$ and $\angle ABC = x + y = 7k$

(Exterior angle of a Δ is equal to the sum of pair of interior opposite angles)

Also, $\angle ABC + \angle ADC = 180^{\circ}$

(pair of opposite angles in a cyclic quadrilateral are supplementary)

$\Rightarrow 8k + 7k = 180^{\circ}$

$\Rightarrow 15k = 180^{\circ}$

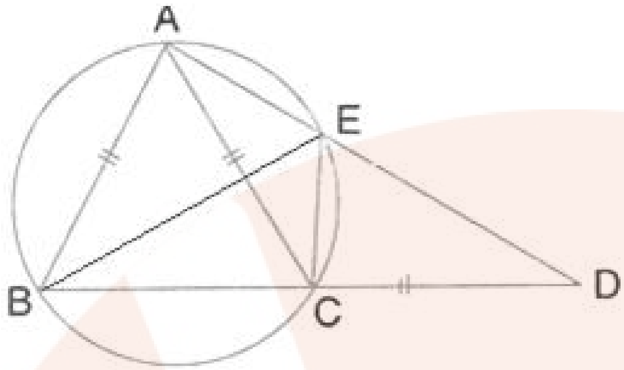
$\therefore k = \frac{180^{\circ}}{15} = 12^{\circ}$

$\therefore x = 3 \times 12^{\circ} = 36^{\circ}$

$y = 4 \times 12^{\circ} = 48^{\circ}$

$z = 5 \times 12^{\circ} = 60^{\circ}$

Solution 45:



(i) AC = CD

$\therefore \angle CAD = \angle CDA = 38°$

$\therefore \angle ACD = 180° - 2 \times 38° = 104°$

$\therefore \angle ACB = 180° - 104° = 76°$ (Straight line)

Also, AB = AC

$\therefore \angle ABC = \angle ACB = 76°$

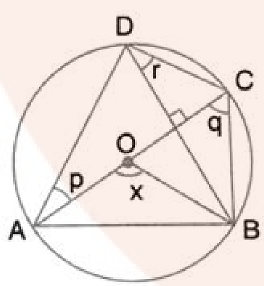
(ii) By angle sum property,

$\angle BAC = 180° - 2 \times 76° = 38°$

$\therefore \angle BEC = \angle BAC = 38°$

(Angles in the same chord)

Solution 46:



$\angle AOB = 2\angle ACB = 2ADB$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\Rightarrow x = 2q \text{ and } \angle ADB = \frac{x}{2} \therefore q = \frac{x}{2}$

Also, $\angle ADC = 90°$

(Angle in a semicircle)

$\Rightarrow r + \frac{x}{2} = 90°$

$\Rightarrow r = 90^\circ - \dfrac{x}{2}$

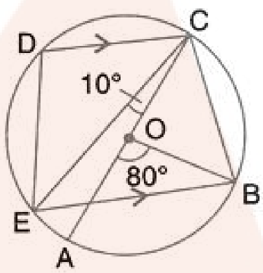
Again, $\angle DAC = \angle DBC$

(Angle in the same segment)

$\Rightarrow p = 90^\circ - q$

$\Rightarrow p = 90^\circ - \dfrac{x}{2}$

Solution 47:



(i) $\angle BOC = 180^\circ - 80^\circ = 100^\circ$ (Straight line)

And $\angle BOC = 2\angle BEC$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$\Rightarrow \angle BEC = \dfrac{100^\circ}{2} = 50^\circ$

(ii) $DC \parallel EB$

$\therefore DCE = \angle BEC = 50^\circ$ (Alternate angles)

$\because \qquad = 80^\circ$

$\Rightarrow \angle ACB = \dfrac{1}{2}\angle AOB = 40^\circ$

(Angle at the center is double the angle at the circumference subtended by the same chord)

We have,

$\angle BCD = \angle ACB + \angle ACE + \angle DCE = 40^\circ + 10^\circ + 50^\circ = 100^\circ$

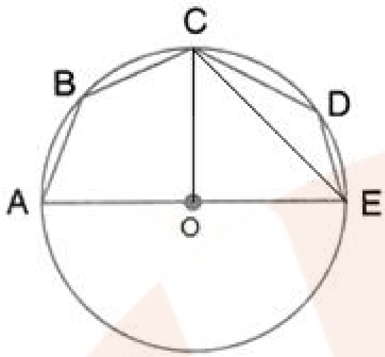
(iii) $\angle BED = 180^\circ - \angle BCD = 180^\circ - 100^\circ = 80^\circ$

(pair of opposite angles in a cyclic quadrilateral are supplementary)

$\Rightarrow \angle CED + 50^\circ = 80^\circ$

$\Rightarrow \angle CED = 30^\circ$

Solution 48:



Join centre O and C and EC.

$$\angle AOC = \frac{180^\circ}{2} = 90^\circ$$

And $\angle AOC = 2\angle AEC$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

$$\Rightarrow \angle AEC = \frac{90^\circ}{2} = 45^\circ$$

Now, ABCE is a cyclic quadrilateral

$\therefore \angle ABC + \angle AEC = 180^\circ$

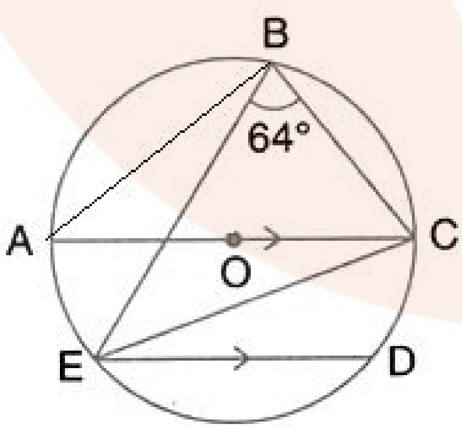
(pair of opposite angles in a cyclic quadrilateral are supplementary)

$\Rightarrow \angle ABC = 180^\circ - 45^\circ = 135^\circ$

Similarly, $\angle CDE = 135^\circ$

$\therefore \angle ABC + \angle CDE - 135^\circ + 135^\circ = 270^\circ$

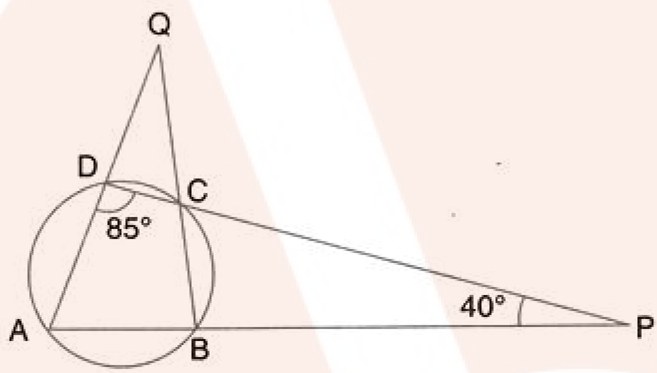
Solution 49:



Join AB,

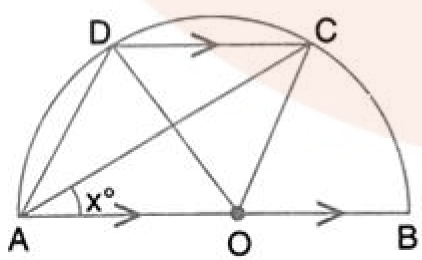
$\angle ABC = 90^\circ$
(Angle in a semi circle)
$\therefore \angle ABE = 90^\circ - 64^\circ = 26^\circ$
Now, $\angle ABE = \angle ACE = 26^\circ$
(Angle in the same segment)
Also, $AC \parallel ED$
$\therefore \angle DEC = \angle ACE = 26^\circ$ (Alternate angles)

Solution 50:



(i) By angle sum property of ΔADP,
$\angle BAD = 180^\circ - 85^\circ - 40^\circ = 55^\circ$
(ii) $\angle ABC = 180^\circ - \angle ADC = 180^\circ - 85^\circ = 95^\circ$
(pair of opposite angles in a cyclic quadrilateral are supplementary)
By angle sum property,
$\angle AQB = 180^\circ - 95^\circ - 55^\circ$
$\Rightarrow \angle DQB = 30^\circ$

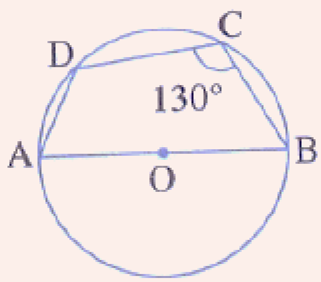
Solution 51:



(i) $\angle COB = 2\angle CAB = 2x$

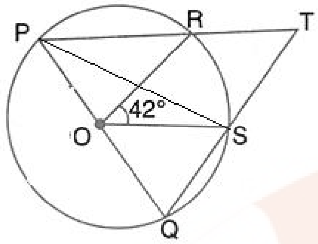
(Angle ate the centre is double the angler at the circumference subtended by the same order)
(ii) $\angle OCD = \angle COB = 2x$ (Alternate angles)
In ΔOCD, $OC = OD$
$\therefore \angle ODC = \angle OCD = 2x$
By angle sum property of ΔOCD,
$\angle DOC = 180° - 2x - 2x = 180° - 4x$
(iii) $\angle DAC = \frac{1}{2}\angle DOC = \frac{1}{2}(180° - 4x) = 90° - 2x$
(Angle at the centre is double the angle at the circumference subtended by the same chord)
(iv) $DC \parallel AO$
$\therefore \angle ACD = \angle OAC = x$ (Alternate angles)
By angle sum property,
$\angle ADC = 180° - \angle DAC - \angle ACD = 180° - (90° - 2x) - x = 90° + x$

Solution 52:



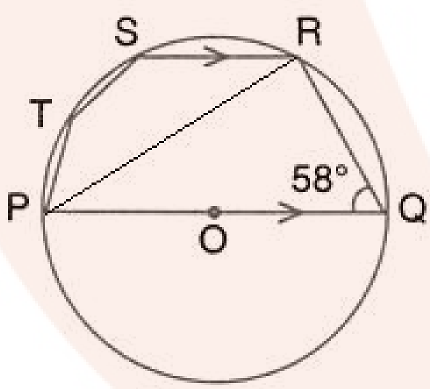
i. ABCD is a cyclic quadrilateral
$m\angle DAB = 180° - \angle DCB$
$= 180° - 130°$
$= 50°$
ii. In ΔADB,
$m\angle DAB + m\angle ADB + m\angle DBA = 180°$
$\Rightarrow 50° + 90° + m\angle DBA = 180°$
$\Rightarrow m\angle DBA = 40°$

Solution 53:



Join PS.
$\angle PSQ = 90°$
(Angle in a semicircle)
Also, $\angle SPR = \frac{1}{2}\angle ROS$
(Angle ate the centre is double the angle at the circumference subtended by the same chord)
$\Rightarrow SPT = \frac{1}{2} \times 42° = 21°$
$\therefore$ In right triangle PST,
$\angle PTS = 90° - \angle SPT$
$\Rightarrow \angle RTS = 90° - 21° = 69°$

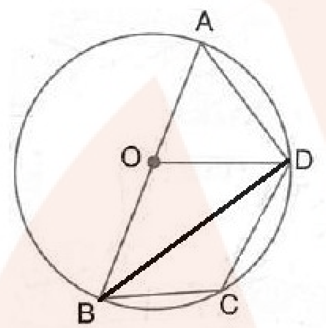
Solution 54:



Join PR.
(i) $\angle PRQ = 90°$
(Angle in a semicircle)
$\therefore$ In right triangle PQR,
$\angle RPQ = 90° - \angle PQR = 90° - 58° = 32°$
(ii) Also, $SR \parallel PQ$
$\therefore \angle PRS = \angle RPQ = 32°$ (Alternate angles)

In cyclic quadrilateral PRST,
$\angle STP = 180° - \angle PRS = 180° - 32° = 148°$
(pair of opposite angles in a cyclic quadrilateral are supplementary)

Solution 55:



Join BD.

(i) $\angle ABD = \frac{1}{2}\angle AOD = \frac{1}{2} \times 60° = 30°$

(Angle at the first is double the angle at the circumference subtended by the same chord)

(ii) $\angle BDA = 90°$

(Angle in a semicircle)

Also, ΔOAD is equilateral $(\because \angle OAD = 60°)$

$\therefore \angle ODB = 90° - \angle ODA = 90° - 60° = 30°$

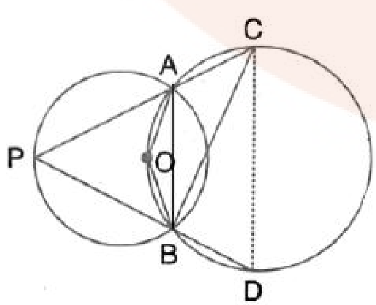
Also, $OD \parallel BC$

$\therefore \angle DBC = \angle ODB = 30°$ (Alternate angles)

(iii) $\angle ABC = \angle ABD + \angle DBC = 30° + 30° = 60°$

In cyclic quadrilateral ABCD,
$\angle ADC = 180° - \angle ABC = 180° - 60° = 120°$
(pair of opposite angles in a cyclic quadrilateral are supplementary)

Solution 56:



Join AB and AD

(i) $\angle AOB = 2\angle APB = 2\times 75° = 150°$

(Angle at the centre is double the angle at the circumference subtended by the same chord)

(ii) In cyclic quadrilateral AOBC,

$\angle ACB = 180° - \angle AOB = 180° - 150° = 30°$

(pair of opposite angles in a cyclic quadrilateral are supplementary)

(iii) In cyclic quadrilateral ABDC

$\angle ABD = 180° - \angle ACD = 180° - (40° + 30°) = 110°$

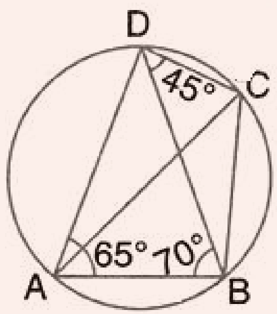
(pair of opposite angles in a cyclic quadrilateral are supplementary

(iv) In cyclic quadrilateral AOBD,

$\angle ADB = 180° - \angle AOB = 180° - 150° = 30°$

(pair of opposite angles in a cyclic quadrilateral are supplementary

Solution 57:



(i) In cyclic quadrilateral ABCD,

$\angle BCD = 180° - \angle BAD = 180° - 65° = 115°$

(pair of opposite angles in a cyclic quadrilateral are supplementary)

(ii) By angle sum property of ΔABD,

$\angle ADB = 180° - 65° - 70° = 45°$

Again, $\angle ACB = \angle ADB = 45°$

(Angle in the same segment)

$\therefore \angle ADC = \angle ADB + \angle BDC = 45° + 45° = 90°$

Hence, AC is a semicircle.

(since angle in a semicircle is a right angle)

Solution 58:

Let $\angle A$ and $\angle C$ be 3x and x respectively

In cyclic quadrilateral ABCD,

$\angle A+\angle C=180°$

(pairs of opposite angles in a cyclic quadrilateral are supplementary)

$\Rightarrow 3x+x=180°$

$\Rightarrow x=\dfrac{180°}{4}=45°$

$\therefore \angle A=135°$ and $\angle C=45°$

Let the measure of $\angle B$ and $\angle D$ be y and 5y respectively

In cyclic quadrilateral ABCD,

$\angle B+\angle D=180°$

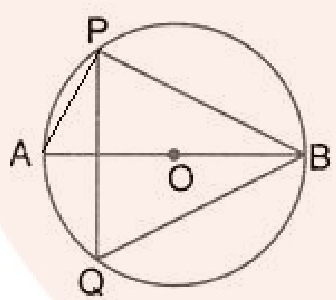
(pair of opposite angles in a cyclic quadrilateral are supplementary are supplementary)

$\Rightarrow y+5y=180°$

$\Rightarrow y=\dfrac{180^{\circ}}{6}=30°$

$\therefore \angle B=30^{\circ}$ and $\angle D=150^{\circ}$

Solution 59:



Join AP.

(i) $\angle APB=90°$

(Angle in a semicircle)

$\therefore \angle BAP=90°-\angle ABP=90°-42°=48°$

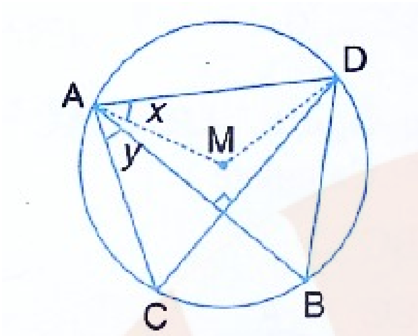
Now, $\angle PQB=\angle BAP=48°$

(Angle in the same segment)

(ii) By angle sum property of ΔBPQ,

$\angle QPB+\angle PBQ=180°-\angle PQB=180°-48°=132°$

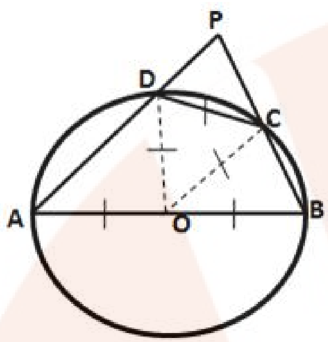
Solution 60:



In the figure, M is the centre of the circle.
Chords AB and CD are perpendicular to each other at L.
$\angle MAD = x$ and $\angle BAC = y$
(i) In ΔAMD,
$MA = MD$
$\therefore \angle MAD = \angle MDA = x$
But in ΔAMD,
$\angle MAD + \angle MDA + \angle AMD = 180°$
$\Rightarrow x + x + \angle AMD = 180°$
$\Rightarrow 2x + \angle AMD = 180°$
$\Rightarrow \angle AMD = 180° - 2x$
(ii) ∴Arc $AD \angle AMD$ at the centre and at the remaining
(Angle in the same segment)
(Angle at the centre is double the angle at the circumference subtended by the same chord)
$\Rightarrow \angle AMD = 2\angle ABD$
$\Rightarrow \angle ABD = \frac{1}{2}(180° - 2x)$
$\Rightarrow \angle ABD = 90° - x$
$AB \perp CD, \angle ALC = 90^\circ$
In ΔALC,
$\therefore \angle LAC + \angle LCA = 90°$
$\Rightarrow \angle BAC + \angle DAC = 90°$
$\Rightarrow y + \angle DAC = 90°$
$\therefore \angle DAC = 90° - y$
We have, $\angle DAC =$ [Angles in the same segment]
$\therefore \angle ABD = 90° - y$
(iii) we have, $\angle ABD = 90° - y$ and $\angle ABD = 90° - x$ [proved]
$\therefore 90° - x = 90° - y$

$\Rightarrow x = y$

Solution 61:



Join OD and OC.

$\Delta OCD,\ OD = OD = CD$

$\therefore \Delta OCD$ is an equilateral triangle

$\therefore \angle ODC = 60°$

Also, in cyclic quadrilateral ABCD

$\angle ADC + \angle ABC = 180°$

(pair of opposite angles in cyclic quadrilateral are supplementary)

$\Rightarrow \angle ODA + 60° + \angle ABP = 180°$

$\Rightarrow \angle OAD + \angle ABP = 90°\ (\because OA = OD)$

$\Rightarrow \angle PAB + \angle ABP = 120°$

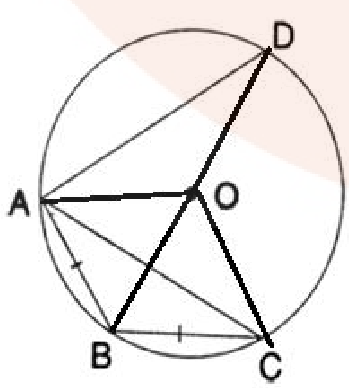
By angle sum property of ΔPAB,

$\therefore \angle APB = 180° - \angle PAB - \angle ABP = 180° - 120° = 60°$

EXERCISE. 17 (C)

Solution 1:

Join OA, OB, OC and OD.

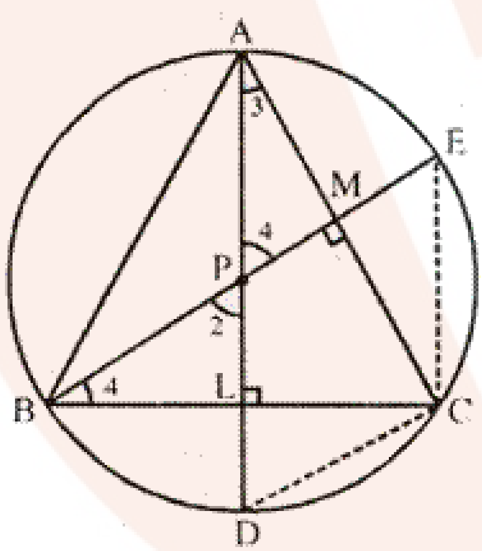


(i) Arc AB = Arc BC [$\because$ Equal chords subtends equal arcs]
(ii) $\angle AOB = \angle BOC$ [$\because$ Equal chords subtends equal arcs]
(iii) If arc AD > arc ABC, then chord AD > AC
(iv) $\angle AOB = 50°$
But $\angle AOB = \angle BOC$ [from (ii) above]
$\therefore \angle BOC = 50°$
Now arc BC subtends $\angle BOC$ at the center and $\angle BAC$ at
The remaining part of the circle.

$$\therefore \angle BAC = \frac{1}{2}\angle BOC = \frac{1}{2}\times 50° = 25°$$

Solution 2:

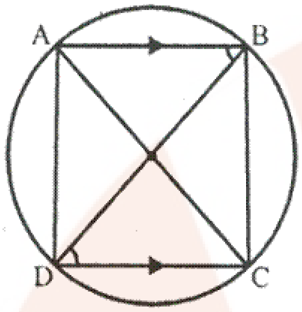
Given: In ΔABC, the perpendiculars from vertices A and B on their opposite sides meet (when produced) the circumcircle of the triangle at points D and E respectively.
To prove: Arc CD = Arc CE
Construction: Join CE and CD
Proof:



In ΔAPM and ΔBPL
$\angle AMP = \angle BLP$ [each = $90°$]
$\angle 1 = \angle 2$ [vertically opposite angles]
$\Delta APM \sim \Delta BPL$ [AA postulate]
$\therefore$ Third angle = Third angle
$\therefore \angle 3 = \angle 4$
$\therefore$ Arc which subtends equal angle at the

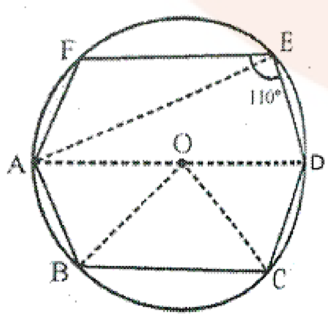
Circumference of the circle are also equal.
∴ Arc CD = Arc CE

Solution 3:



A cyclic trapezium ABCD in which AB ∥ DC and AC and BD are joined.
To prove-
(i) AD = BC
(ii) AC = BD
Proof:
∵ chord AD subtends and chord BC subtends $\angle BDC$
At the circumference of the circle.
But $\angle ABD = \angle BDC$ [proved]
Chord AD = Chord BC
$\Rightarrow$ AD = BC
Now in ΔADC and Δ BCD
DC = DC [Common]
$\angle CAD = \angle CBD$ [angles in the same segment]
And AD = BC [proved]
By Side – Angle – Side criterion of congruence, we have
$\therefore \Delta ADC \cong \Delta BCD$ [SAS axion]
The corresponding parts of the congruent triangle are congruent
$\therefore$ AC = BD [c.p.c.t]

Solution 4:



Join AE, OB and OC

(i) $\because$ AOD is the diameter,

$\therefore \angle AED = 90^\circ$ [Angle in a semi-circle]

But $\angle DEF = 110^\circ$ [given]

$\therefore \angle AEF = \angle DEF - \angle AED$

$= 110^\circ - 90^\circ = 20^\circ$

(ii) $\because$ Chord AB = Chord BC = Chord CD [given]

$\therefore \angle AOB = \angle BOC = \angle COD$ (Equal chords subtends equal angles at the centre)

But $\angle AOB + \angle BOC + \angle COD = 180^\circ$ [AOD is a straight line]

$\therefore \angle AOB = \angle BOC = \angle COD = 60^\circ$

In ΔOAB, $OA = OB$

$\therefore \angle OAB = \angle OBA$ [radii of the same circle]

But $\angle OAB + \angle OBA = 180^\circ - \angle AOB$

$= 180^\circ - 60^\circ$

$= 120^\circ$

$\therefore \angle OAB = \angle OBA = 60^\circ$

In cyclic quadrilateral ADEF,

$\angle DEF + \angle DAF = 180^\circ$

$\Rightarrow \angle DAF = 180^\circ - \angle DEF$

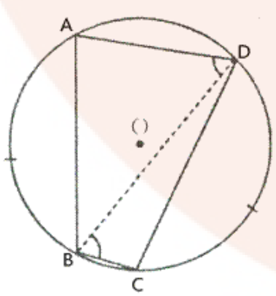
$= 180^\circ - 110^\circ$

$= 70^\circ$

Now, $\angle FAB = \angle DAF + \angle OAB$

$= 70^\circ + 60^\circ = 130^\circ$

Solution 5:



Given –

In the figure, O is the centre of a circle and arc AB = arc CD

To prove –

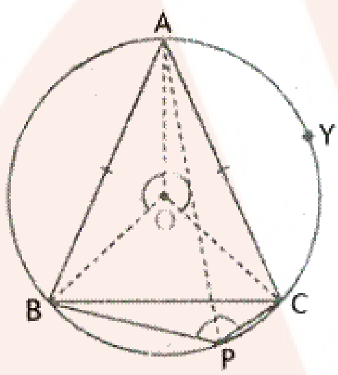
ABCD is an isosceles trapezium.

Construction – Join BD, AD and BC.

Proof – Since equal arcs subtends equal angles at the circumference of a circle.

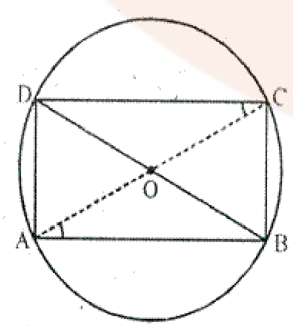
$\therefore \angle ADB = \angle DBC$ [$\because$ arc AB = arc CD]
But, these are alternate angles.
$\therefore$ AD || BC
$\therefore$ ABCD is a trapezium
$\because$ Arc AB = Arc CD [Given]
$\therefore$ Chord AB = Chord CD
$\therefore$ ABCD is an isosceles trapezium

Solution 6:



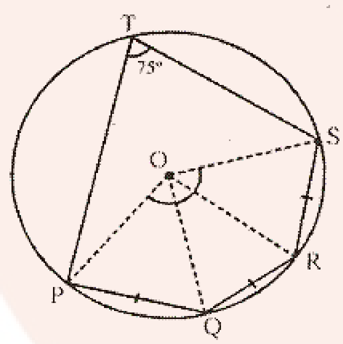
Given –
ΔABC is an isosceles triangle and O is the centre of its circumcircle
To prove –
AP bisects $\angle BPC$
Proof –
Chord AB subtends and chord AC subtends $\angle APC$ at the
Circumference of the circle.
But chord AB = Chord AC
$\therefore \angle APB = \angle APC$
$\therefore$ AP is the bisector of $\angle BPC$

Solution 7:



Given –
ABCD is a cyclic quadrilateral in which AB ∥ DC. AC and BD are its diagonals.
To prove –
(i) $AD = BC$
(ii) $AC = BD$
Proof –
(i) $AB \parallel DC \Rightarrow \angle DCA = \angle CAB$ [Alternate angles]
Now, chord AD subtends $\angle DCA$ and chord BC subtends $\angle CAB$
At the circumference of the circle.
$\therefore \angle DCA = \angle CAB$ [proved]
∴ Chord AD = Chord BC or AD = BC
(ii) Now in ΔABC and ΔADB,
$AB = AB$ [Common]
$\angle ACB = \angle ADB$ [Angles in the same segment]
$BC = AD$ [Proved]
By Side – Angle – Side criterion of congruence, we have
$\Delta ACB \cong \Delta ADB$ [SAS postulate]
The corresponding parts of the congruent triangles are congruent.
$\therefore AC = BD$ [c.p.c.t]

Solution 8:



Join OP, OQ and OS.
$\because$ R = RS,
$\angle POQ = \angle QOR = \angle ROS$ [Equal chords subtends equal angles at the centre]
Arc PQRS subtends $\angle POS$ at the center and $\angle PTS$ at the remaining
Parts of the circle.
$\therefore \angle POS = 2\angle PTS = 2 \times 75° = 150°$
$\Rightarrow \angle POQ + \angle QOR + \angle ROS = 150°$
$$\Rightarrow \angle POQ = \angle QOR = \angle ROS = \frac{150°}{3} = 50°$$
In ΔOPQ, $OP = OQ$ [radii of the same circle]
$\therefore \angle OPQ = \angle OQP$

But $\angle OPQ + \angle OQP + \angle POQ = 180°$

$\therefore \angle OPQ + \angle QP = 50° = 180°$

$\Rightarrow \angle OPQ + \angle OQP = 180° - 50°$

$\Rightarrow \angle OPQ + \angle OPQ = 130°$

$\Rightarrow 2\angle OPQ = 130°$

$\Rightarrow \angle OPQ = \angle OQP = \dfrac{130°}{2} = 65°$

Similarly, we can prove that

In $\Delta OQR, \angle OQR = \angle ORQ = 65°$

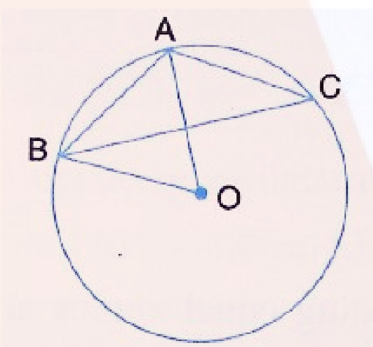
And in $\Delta ORS, \angle ORS = \angle OSR = 65°$

(i) Now $\angle POS = 150°$

(ii) $\angle QOR = 50°$ and

(iii) $\angle PQR = \angle PQO + \angle OQR = 65° + 65° = 130°$

Solution 9:



(i) Arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of the circle.

$\therefore \angle ACB = \dfrac{1}{2}\angle AOB$

Since AB is the side of a regular hexagon,

$\angle AOB = 60°$

(ii) $\angle AOB = 60° \Rightarrow \angle ACB = \dfrac{1}{2} \times 60° = 30°$

(iii) Since AC is the side of a regulare octagon,

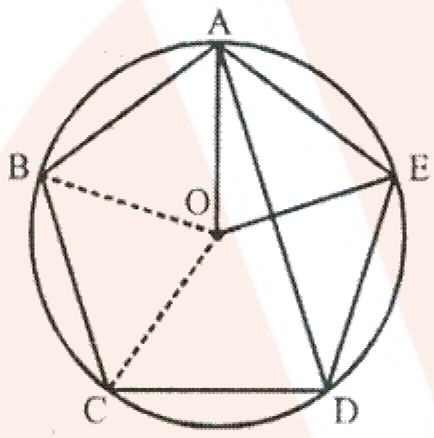
$\angle AOC = \dfrac{360}{8} = 45°$

Again, Arc AC subtends $\angle AOC$ at the center and $\angle ABC$ at the remaining part of the circle.

$$\Rightarrow \angle ABC = \frac{1}{2}\angle AOC$$

$$\Rightarrow \angle ABC = \frac{45^\circ}{2} = 22.5^\circ$$

Solution 10:



Arc AE subtends $\angle AOE$ at the centre and

$\angle ADE$ at the remaining part of the circle.

$$\therefore \angle ADE = \frac{1}{2}\angle AOE$$

$$= \frac{1}{2}\times 72^\circ$$

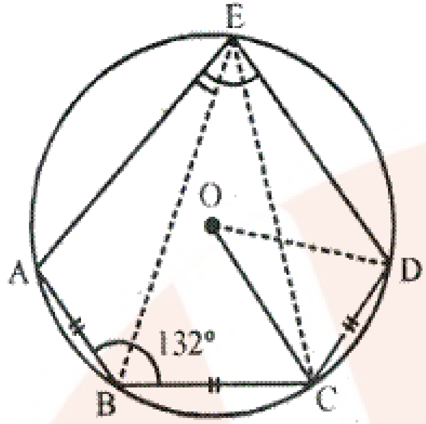
$$= 36^\circ$$ [central angle is a regular pentagon at O]

$\angle ADC = \angle ADB + \angle BDC$

$= 36^\circ + 36^\circ + 72^\circ$

$\therefore \angle ADE : \angle ADC = 36^\circ : 72^\circ = 1:2$

Solution 11:



In the figure, O is the centre of circle, with AB = BC = CD.

Also, $\angle ABC = 132°$

(i) In cyclic quadrilateral ABCE

$\angle ABC + \angle AEC = 180°$ [sum of opposite angles]

$\rightarrow 132° + \angle AEC = 180°$

$\rightarrow \angle AEC = 180° - 132°$

$\rightarrow \angle AEC = 48°$

Since, $AB = BC$, $\angle AEB = \angle BEC$ [equal chords subtends equal angles]

$$\therefore \angle AEB = \frac{1}{2}\angle AEC$$

$$= \frac{1}{2} \times 48°$$

$$= 24°$$

(ii) Similarly, $AB = BC = CD$

$\angle AEB = \angle BEC = \angle CED = 24°$

$\angle AED = \angle AEB + \angle BEC + \angle CED$

$= 24° + 24° + 24° = 72°$

(iii) Arc CD subtends $\angle COD$ at the centre and

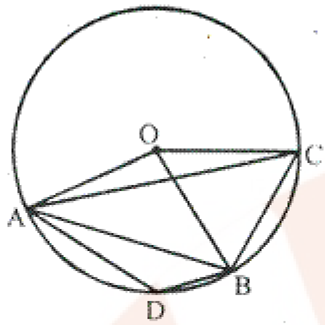
$\angle CED$ at the remaining part of the circle.

$\therefore COD = 2\angle CED$

$= 2 \times 24°$

$= 48°$

Solution 12:



(i) Join AD and DB

Arc B = 2 arc BC and $\angle AOB = 180°$

$$\therefore \angle BOC = \frac{1}{2}\angle AOB$$
$$= \frac{1}{2} \times 108°$$
$$= 54°$$

Now, Arc BC subtends $\angle BOC$ at the centre and $\angle CAB$ at the remaining part of the circle.

$$\therefore CAB = \frac{1}{2}\angle BOC$$
$$= \frac{1}{2} \times 54°$$
$$= 27°$$

(ii) Again, Arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of the circle.

$$\therefore ACB = \frac{1}{2}\angle AOB$$
$$= \frac{1}{2} \times 108°$$
$$= 54$$

In cyclic quadrilateral ADBC

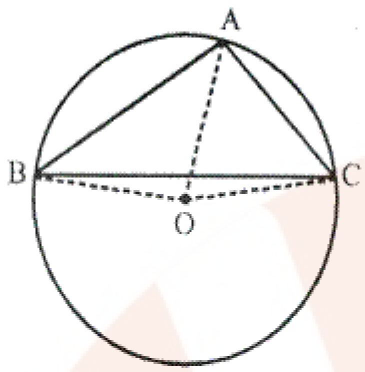
$\angle ADB + \angle ACB = 180°$ [sum of opposite angles]

$\Rightarrow \angle ADB + 54° = 180°$

$\Rightarrow \angle ADB = 180° - 54°$

$\Rightarrow \angle ADB = 126°$

Solution 13:



Join OA, OB and OC

Since AB is the side of a regular pentagon,

$\angle AOB = \dfrac{360°}{5} = 72°$

Again AC is the side of a regular hexagon,

$\angle AOC = \dfrac{360°}{6} = 60°$

But $\angle AOB + \angle AOC + \angle BOC = 360°$ [Angles at a point]

$\Rightarrow 72° + 60° + \angle BOC = 360°$

$\Rightarrow 132° + \angle BOC = 360°$

$\Rightarrow \angle BOC = 360° - 132°$

$\Rightarrow \angle BOC = 228°$

Now, Arc BC subtends $\angle BOC$ at the centre and $\angle BAC$ at the remaining part of the circle.

$\Rightarrow \angle BAC = \dfrac{1}{2}\angle BOC$

$\Rightarrow \angle BAC = \dfrac{1}{2} \times 228° = 114°$

Similarly, we can prove that

$\Rightarrow \angle ABC = \dfrac{1}{2}\angle AOC$

$\Rightarrow \angle ABC = \dfrac{1}{2} \times 60° = 30°$

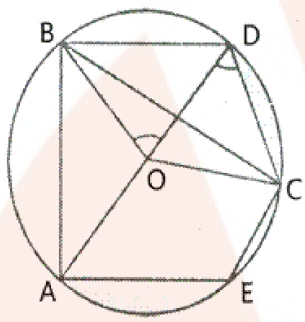
And

$\Rightarrow \angle ACB = \dfrac{1}{2}\angle AOB$

$\Rightarrow \angle ACB = \frac{1}{2} \times 72° = 36°$

Thus, angles of the triangle are, 114°, 30° and 36°

Solution 14:



Join BC, BO, CO and EO

Since BD is the side of a regular hexagon,

$\angle BOD = \frac{360}{6} = 60°$

Since DC is the side of a regular pentagon,

$\angle COD = \frac{360}{5} = 72°$

In ΔBOD. $\angle BOD = 60°$ and OB = OD

$\therefore \angle OBD = \angle ODB = 60°$

(i) In ΔOCD, $\angle COD = 72°$ and OC = OD

$\therefore \angle ODC = \frac{1}{2}(180° - 72°)$

$= \frac{1}{2} \times 108°$

$= 54°$

Or, $\angle ADC = 54°$

(ii) $\angle BDO = 60°$ or $\angle BDA = 60°$

(iii) Arc AC subtends $\angle AOC$ at the centre and

$\angle ABC$ at the remaining part of the circle.

$$\therefore ABC = \frac{1}{2}\angle AOC$$
$$= \frac{1}{2}[\angle AOD - \angle COD]$$
$$= \frac{1}{2}\times(180^\circ - 72^\circ)$$
$$= \frac{1}{2}\times 108^\circ$$
$$= 54^\circ$$

(iv) In cyclic quadrilateral AECD

$\angle AEC + \angle ADC = 180^\circ$ [sum of opposite angles]

$\Rightarrow \angle AEC + 54^\circ = 180^\circ$

$\Rightarrow \angle AEC = 180^\circ - 54^\circ$

$\Rightarrow \angle AEC = 126^\circ$

EXERCISE. 17 (D)

Solution 1:

$\angle ABD = \angle ACD = 30^\circ$ [Angle in the same segment]

Now in ΔADB,

$\angle BAD + \angle ADB + \angle ABD = 180^\circ$ [angles of a triangle]

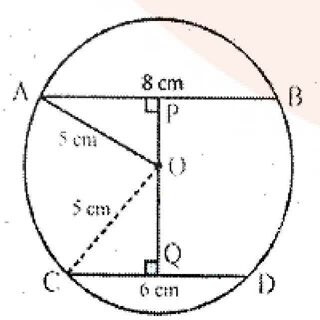
But, $\angle ADB = 90^\circ$ [Angle in a semi-circle]

$\therefore x + 90^\circ + 30^\circ = 180^\circ$

$\Rightarrow x = 180^\circ - 120^\circ$

$\Rightarrow x = 60^\circ$

Solution 2:



Radius of the circle whose centre is 0 = 5 cm

OP ⊥ AB and OQ ⊥ CD, AB = 8 cm and CD = 6 cm.

Join OA and OC, then OA = OC + 5 cm

Since OP ⊥ AB, P is the midpoint of AB

Similarly, Q is the midpoint of CD

In right ΔOAP,

$OA^2 = OP^2 + AP^2$ [Pythagoras Theorem]

$\Rightarrow (5)^2 = OP^2 + (4)^2$ $[\because$ $B = \frac{1}{2} \times 8 = 4$ cm]

$\Rightarrow 25 = OP^2 + 16$

$\Rightarrow OP^2 = 25 - 16$

$\Rightarrow OP^2 = 9$

$\Rightarrow OP = 3$ cm

Similarly, in right ΔOCQ,

$OC^2 = OQ^2 + CQ^2$ [Pythagoras theorem]

$\Rightarrow (5)^2 = OQ^2 + (3)^2$

$\Rightarrow 25 = OQ^2 + 9$

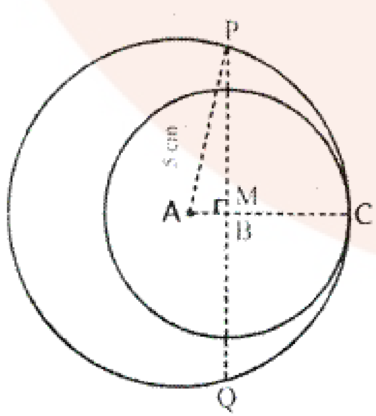
$\Rightarrow OQ^2 = 25 - 9$

$\Rightarrow OQ^2 = 16$

$\Rightarrow OQ = 4$cm

Hence, $PQ = OP + OQ = 3 + 4 = 7$ cm

Solution 3:



Join AP and produce AB to meet the bigger circle at C.

AB = AC – BC = 5 cm – 3 cm = 2 cm
But, M is the mid – point of AB.

$\therefore AM = \frac{2}{2} = 1\text{cm}$

Now in right ΔAPM,

$AP^2 = MP^2 + AM^2$ [Pythagoras Theorem]

$\Rightarrow (5)^2 = MP^2 + 1^2$

$\Rightarrow 25 = MP^2 1$

$\Rightarrow MP^2 = 25 - 1$

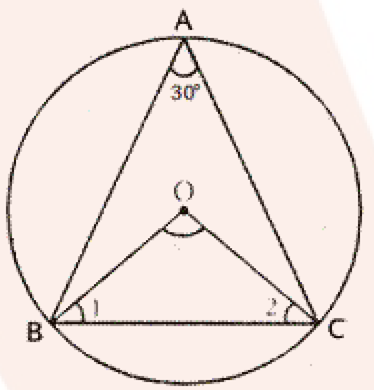
$\Rightarrow MP^2 = 24$

$\Rightarrow MP = \sqrt{24} = \sqrt{4\times 6} = 2\sqrt{6}$ cm

$\therefore PQ = 2MP = 2\times 2\sqrt{6} = 4\sqrt{6}$ cm

$\Rightarrow PQ = 4\times 2.45 = 9.8$ cm

Solution 4:



Given – In the figure ABC is a triangle in which $\angle A = 30°$
To prove – BC is the radius of circumcircle of ΔABC whose centre is O.
Construction – join OB and OC.
Proof:

$\angle BOC = 2\angle BAC = 2\times 30° = 60°$

Now in ΔOBC,

$OB = OC$ [Radii of the same circle]

$\angle OBC = \angle OCB$

But, in ΔBOC,

$\angle OBC + \angle OCB + \angle BOC = 180°$ [Angles of a triangle]

$\Rightarrow \angle OBC + \angle OBC + 60° = 180°$

$\Rightarrow 2\angle OBC + 60° = 180°$

$\Rightarrow 2\angle OBC = 180° - 60°$

$\Rightarrow 2\angle OBC = 120°$

$\Rightarrow \angle OBC = \frac{120°}{2} = 60°$

$\Rightarrow \angle OBC = \angle OCB = \angle BOC = 60°$

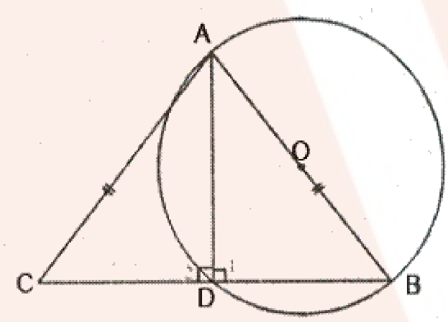
$\Rightarrow \Delta BOC$ is an equilateral triangle

$\Rightarrow BC = OB = OC$

But, OB and OC are the radii of the circum – circle

$\therefore$ BC is also the radius of the circum – circle

Solution 5:



Given – In ΔABC, AB = AC and a circle with AB as diameter is drawn

Which intersects the side BC and D.

To prove – D is the mid point of BC

Construction – Join AD.

Proof - $\angle 1 = 90°$ [Angle in a semi circle]

But $\angle 1 + \angle 2 = 180°$ [Linear pair]

$\therefore \angle 2 = 90°$

Now in right ΔABD and ΔACD,

Hyp. AB = Hyp. AC [Given]

Side AD = AD [Common]

$\therefore$ By the right Angle – Hypotenuse – side criterion of congruence, we have

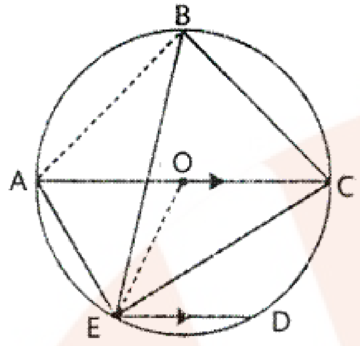
$\Delta ABD \cong \Delta ACD$ [RHS criterion of congruence]

The corresponding parts of the congruent triangle are congruent.

$\therefore$ BD = DC [c.p.c.t]

Hence D is the mid point of BC.

Solution 6:



Join OE.

Arc EC subtends $\angle EOC$ at the centre and $\angle EBC$ at the remaining Part of the circle.

$\angle EOC = 2\angle EBC = 2\times 65° = 130°$

Now in ΔOEC, $OE = OC$ [radii of the same circle]

$\therefore \angle OEC = \angle OCE$

But, in ΔEOC,

$\angle OEC + \angle OCE + \angle EOC = 180°$ [Angles of a triangle]

$\Rightarrow \angle OCE + \angle OCE + \angle EOC = 180°$

$\Rightarrow 2\angle OCE + 130° = 180°$

$\Rightarrow 2\angle OCE = 180° - 130°$

$\Rightarrow 2\angle OCE = 50°$

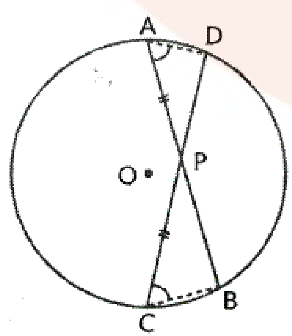
$\Rightarrow \angle OCE = \dfrac{50°}{2} = 25°$

$\therefore AC \parallel ED$ [Given]

$\therefore \angle DEC = \angle OCE$

$\Rightarrow \angle DEC = 25°$

Solution 7:

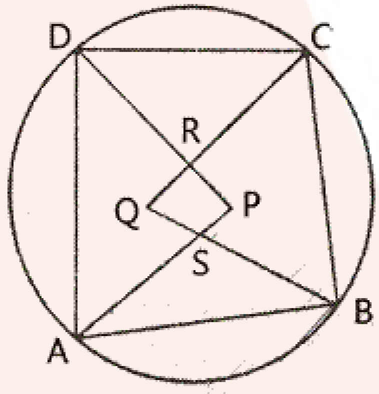


Given – two chords AB and CD intersect
Each other at P inside the circle
With centre O and AP = CP
To prove – AB = CD
Proof – Two chords AB and CD intersect each other inside the circle at P.
$\therefore AP \times PB = CP \times PD$

$\Rightarrow \frac{AP}{CP} = \frac{PD}{PB}$

But $AP = CP$ ....(1) [given]
$\therefore PD = PB$ or $PB = PD$ .... (2)
Adding (1) and (2)
$AP + PB = CP + PD$
$\Rightarrow AB = CD$

Solution 8:



Given – ABCD is a cyclic quadrilateral and PQRS is a
Quadrilateral formed by the angle
Bisectors of angle $\angle A, \angle B, \angle C$ and $\angle D$
To prove – PQRS is a cyclic quadrilateral.
Proof – In ΔAPD,
$\angle PAD + \angle ADP + \angle APD = 180°$ .... (1)
Similarly, IN ΔBQC,
$\angle QBC + \angle BCQ + \angle BQC = 180°$ ....(2)
Adding (1) and (2), we get
$\angle PAD + \angle ADP + \angle APD + \angle QBC + \angle BCQ + \angle BQC = 180° + 180°$
$\Rightarrow \angle PAD + \angle ADP + \angle QBC + \angle BCQ + \angle APD + \angle BQC = 360°$

But $\angle PAD + \angle ADP + \angle QBC + \angle BCQ = \frac{1}{2}[\angle A + \angle B + \angle C + \angle D]$

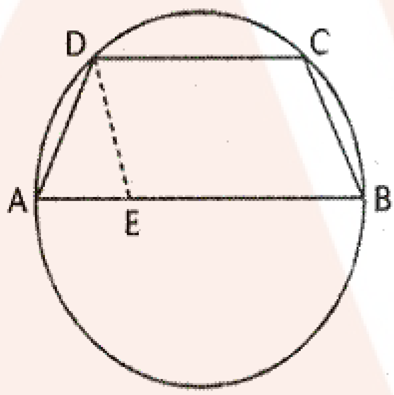
$$= \frac{1}{2} \times 360° = 180°$$

$\therefore \angle APD + \angle BQC = 360° - 180° = 180°$ [from (3)]

But these are the sum of opposite angles of quadrilateral PRQS.

∴ Quad. PRQS is a cyclic quadrilateral.

Solution 9:



Given – ABCD is a trapezium in which AB || CD and AD = BC

To prove – ABCD is cyclic

Construction – draw DE || BC

Proof –

DCBE is a parallelogram [by construction]

$\angle DEB = \angle DCB$ [Opposite angles of parallelogram]

Also, $\angle DEB = \angle EDA + \angle DAE$ [Exterior angle property]

In ΔADE, $\angle DAE = \angle DAE$ ……. (1) [since AD = BC = DE Or AD = DE]

Also, $\angle DEB + \angle EDA = 180°$ ………. (2)

From (1) and (2),

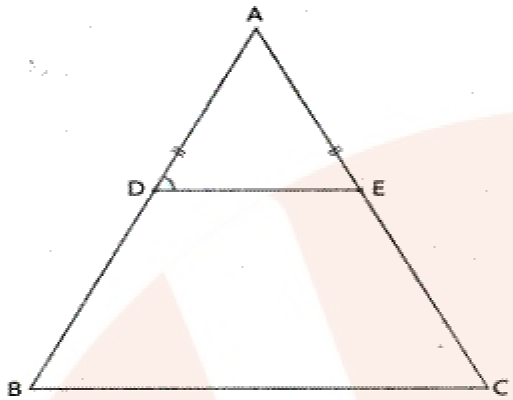
$\angle DEB + \angle DAE = 180°$

$\Rightarrow \angle DCB + \angle DAE = 180°$

$\Rightarrow \angle C + \angle A = 180°$

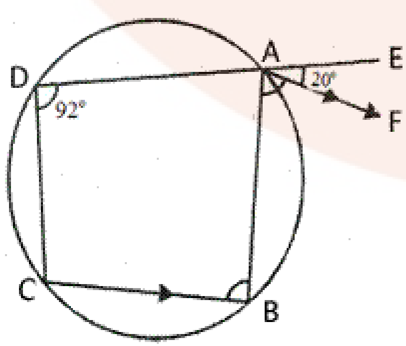
Hence ABCD is cyclic trapezium

Solution 10:



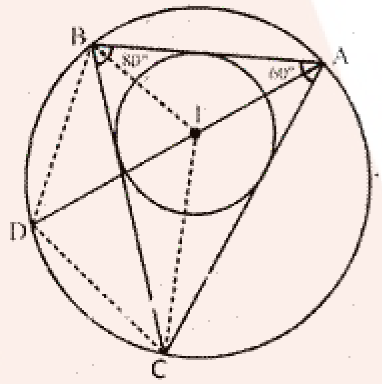
Given – In ΔABC, AB = AC and D and E are points on AB and AC
Such that AD = AE. DE is joined.
To prove B, C, E, D are concyclic.
Proof – In ΔABC, AB = AC
$\therefore \angle B = \angle C$ [Angles opposite to equal sides]
Similarly, In ΔADE, AD = AE [Given]
$\therefore \angle ADE = \angle AED$ [Angles opposite to equal sides]
In ΔABC,
$\because \dfrac{AP}{AB} = \dfrac{AE}{AC}$
$\therefore DE \parallel BC$
$\therefore \angle ADE = \angle B$ [corresponding angles]
But $\angle B = \angle C$ [proved]
$\therefore$ Ext $\angle ADE$ = its interior opposite $\angle C$
$\therefore$ BCED is a cyclic quadrilateral
Hence B, C, E and D are concyclic.

Solution 11:



In cyclic quad. ABCD,
$AF \parallel CB$ and DA is produced to E such that $\angle ADC = 92°$ and $\angle FAE = 20°$
Now we need to find the measure of $\angle BCD$
In cyclic quad. ABCD,
$\angle B + \angle = 180°$
$\Rightarrow \angle B + 92° = 180°$
$\Rightarrow \angle B = 180° - 92°$
$\Rightarrow \angle B = 88°$
Since $AF \parallel CB$, $\angle FAB = \angle B = 88°$
But, $\angle FAE = 20°$ [given]
Ext. $\angle BAE = \angle BAF + \angle FAE$
$= 88° + 22° = 108°$
But, Ext. $\angle BAE = \angle BCD$
$\therefore \angle BCD = 108°$

Solution 12:



Join DB and DC, IB and IC
$\angle BAC = 66°$, $\angle ABC = 80°$, I is the incentre of the ΔABC,

(i) since $\angle DBC$ and $\angle DAC$ are in the same segment,
$\angle DBC = \angle DAC$
But, $\angle DAC = \frac{1}{2} \angle BAC = \frac{1}{2} \times 66° = 33°$
$\therefore \angle DBC = 33°$
(ii) Since I is the incentre of ΔABC, IB bisects $\angle ABC$
$\therefore \angle IBC = \frac{1}{2} \angle ABC = \frac{1}{2} \times 80° = 40°$

(iii) $\therefore \angle BAC = 66^\circ$ and $\angle ABC = 80^\circ$

In $\Delta ABC, \angle ACB = 180^\circ - (\angle ABC + \angle BAC)$

$\Rightarrow \angle ACB = 180^\circ - (80^\circ + 66^\circ)$

$\Rightarrow \angle ACB = 180^\circ - (156^\circ)$

$\Rightarrow \angle ACB = 34^\circ$

Since IC bisects the $\angle C$

$\therefore \angle ICB = \frac{1}{2}\angle C = \frac{1}{2} \times 34^\circ = 17^\circ$

Now in ΔIBC

$\angle IBC + \angle ICB + \angle BIC = 180^\circ$

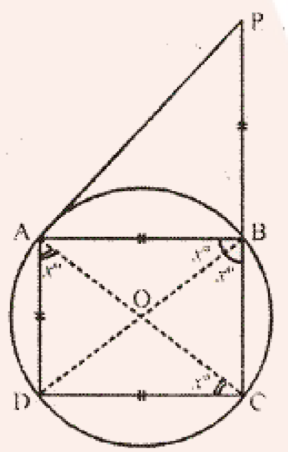
$\Rightarrow 40^\circ + 17^\circ + \angle BIC = 180^\circ$

$\Rightarrow 57^\circ + \angle BIC = 180^\circ$

$\Rightarrow \angle BIC = 180^\circ - 57^\circ$

$\Rightarrow \angle BIC = 123^\circ$

Solution 13:



Given – In the figure, AB = AD = DC = PB and $DBC = X$

Join AC and BD

To find : the measure of $\angle ABD$ and $\angle APB$

Proof : $\angle DAC = \angle DBC = X$ [angles in the same segment]

But $\angle DCA = \angle DAC = X$ [$\because$ AD = DC]

Also, we have, $\angle ABD = \angle DAC$ [angles in the same segment]

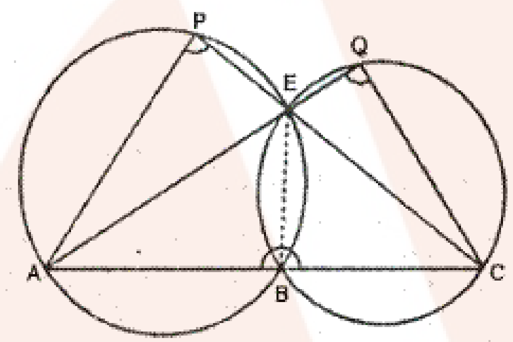
In ΔABP, ext $\angle ABC = \angle BAP + \angle APB$

But, $\angle BAP = \angle APB$ [$\because$ AB = BP]

$2 \times X = \angle APB + \angle APB = 2\angle APB$

$\therefore 2\angle APB = 2X$
$\Rightarrow \angle APB = X$
$\angle APB = \angle DBC = X$,
But these are corresponding angles
$\therefore AP \parallel DB$

Solution 14:



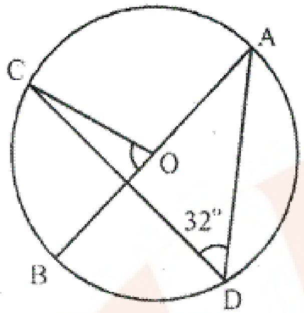
Given – In the figure, ABC, AEQ and CEP are straight line
To prove - $\angle APE + \angle CQE = 180^\circ$
Construction – join EB
Proof – in cyclic quad ABEP,
$\angle APE + \angle ABE = 180^\circ$ .... (1)

Similarly, in cyclic quad BCQE
$\angle CQE + \angle CBE = 180^\circ$ .... (2)

Adding (1) and (2),
$\angle APE + \angle ABE + \angle CQE + \angle CBE = 180^\circ + 180^\circ = 360^\circ$
$\Rightarrow \angle APE + \angle ABE + \angle CBE = 360^\circ$

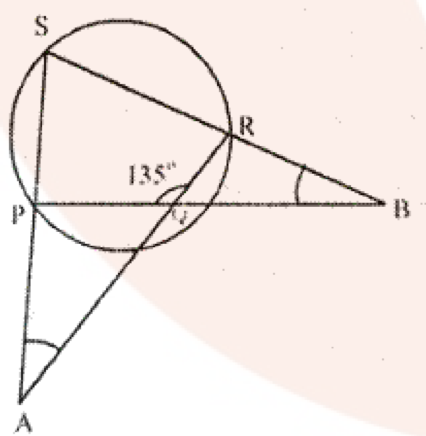
But, $\angle ABE + \angle CBE = 180^\circ$ [Linear pair]
$\therefore \angle APE + \angle CQE + 180^\circ = 360^\circ$
$\Rightarrow \angle APE + \angle CQE = 360^\circ - 180^\circ = 180^\circ$
Hence $\angle APE$ and $\angle CQE$ are supplementary.

Solution 15:



Arc AC subtends $\angle AOC$ at the centre and $\angle ADC$ at the remaining part
Of the circle
$\therefore \angle AOC = 2\angle ADC$
$\Rightarrow \angle AOC = 2 \times 32° = 64°$
Since $\angle AOC$ and $\angle BOC$ are linear pair, we have
$\angle AOC + \angle BOC = 180°$
$\Rightarrow 64° + \angle BOC = 180°$
$\Rightarrow \angle BOC = 180°$
$\Rightarrow \angle BOC = 180° - 64°$
$\Rightarrow \angle BOC = 116°$

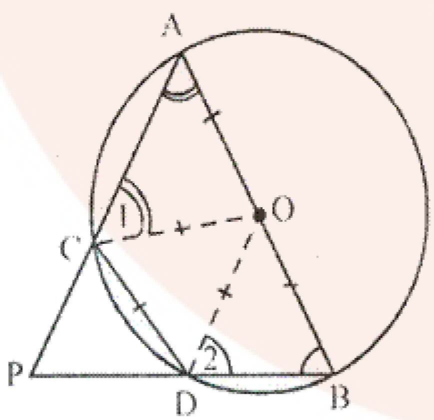
Solution 16:



PQRS is a cyclic quadrilateral in which $\angle PQR = 135°$
Sides SP and RQ are produced to meet at A and
Sides PQ and SR are produced to meet at B.
$\angle A = \angle B = 2:1$

Let $\angle A = 2X$, then $\angle B - X$
Now, in cyclic quad PQRS,
Since, $\angle PQR = 135°, \angle = 180° - 135° = 45°$
[since sum of opposite angles of a cyclic quadrilateral are supplementary]
Since, $\angle PQR$ and $\angle PQA$ are linear pair,
$\angle PQR + \angle PQA = 180°$
$\Rightarrow 135° + \angle PQA = 180°$
$\Rightarrow \angle PQA = 180° - 135° = 45°$
Now, In ΔPBS,
$\angle P = 180° - (45° + x) = 180° - 45° - x = 135° - x$ .....(1)
Again, in ΔPQA,
EXT $\angle P = \angle PQA + \angle = 45° + 2X$ ....(2)
From (1) and (2),
$45° + 2x = 135° - x$
$\Rightarrow 2x + x = 135° - 45°$
$\Rightarrow 3x = 90°$
$\Rightarrow x = 30°$
Hence, $\angle A = 2x = 2 \times 30° = 60°$
And $\angle B = x = 30°$

Solution 17:



Given – In the figure, AB is the diameter of a circle with centre O.
CD is the chord with length equal radius OA.
AC and BD produced meet at point P
To Prove : $\angle APB = 60°$

Construction – join OC and OD

Proof – We have CD = OC = OD [given]

Therefore, ΔOCD is an equilateral triangle

$\therefore \angle OCD = \angle ODC = \angle COD = 60°$

In ΔAOC, OA = OC [radii of the same circle]

$\therefore \angle A = \angle 1$

Similarly, in ΔBOD, OB = OD [radii of the same circle]

$\therefore \angle B = \angle 2$

Now, in cyclic quad ACBD

Since, $\angle ACD + \angle B = 180°$

[Since sum of opposite angles of a cyclic quadrilateral are supplementary]

$\Rightarrow 60° + \angle 1 + \angle B = 180°$

$\Rightarrow \angle 1 + \angle B = 180° - 60°$

$\Rightarrow \angle 1 + \angle B = 120°$

But, $\angle 1 = \angle A$

$\therefore \angle A + \angle B = 120°$ .....(1)

Now, in ΔAPB,

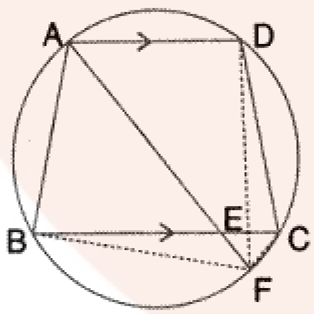
$\angle P + \angle A + \angle B = 180°$ [sum of angles of a triangles]

$\Rightarrow \angle P + 120° = 180°$

$\Rightarrow \angle P = 180° - 120°$ [from (1)]

$\Rightarrow \angle P = 60°$ or $\angle APB = 60°$

Solution 18:



Given – ABCD is a cyclic quadrilateral in which $AD \parallel BC$

Bisector of $\angle A$ meets BC at E and the given circle at F.

DF and BF are joined.

To prove –

(i) EF = FC

(ii) BF = DF

Proof – ABCD is a cyclic quadrilateral and AD ∥ BC

$\because$ AF is the bisector of $\angle A, \angle BAF = \angle DAF$

Also, $\angle DAE = \angle BAE$

$\angle DAE = \angle AEB$ [Alternate angles]

(i) In $\Delta ABE, \angle ABE = 180° - 2\angle AEB$

$\angle CEF = \angle AEB$ [vertically opposite angles]

$\angle ADC = 180° - \angle ABC = 180° - (180° - 2\angle AEB)$

$\angle ADC = 2\angle AEB$

$\angle AFC = 180° - \angle ADC$

$= 180° - 2\angle AEB$ [since ADFC is a cyclic quadrilateral]

$\angle ECF = 180° - (\angle AFC + \angle CEF)$

$= 180° - (180° - 2\angle AEB + \angle AEB)$

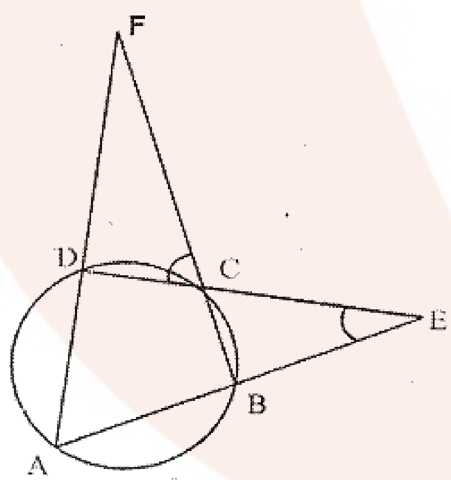
$= \angle AEB$

$\therefore EC = EF$

(ii) $\therefore$ Arc BF = Arc DF [Equal arcs subtends equal angles]

$\Rightarrow BF = DF$ [Equal arcs have equal chords]

Solution 19:



Given – In a circle, ABCD is a cyclic quadrilateral AB and DC

Are produced to meet at E and BC and AD are produced to meet at F.

$\angle DCF : \angle F : \angle E = 3 : 5 : 4$

Let $\angle DCF = 3X, \angle F = 5x, \angle E = 4x$

Now, we have to find, $\angle A, \angle B, \angle C$ and $\angle D$

In cyclic quad. ABCD, BC is produced.

$\therefore \angle A = \angle DCF = 3x$

In ΔCDF,

Ext $\angle CDA = \angle DCF + \angle = 3x + 5x = 8x$

In ΔBCE,

Ext $\angle ABC = \angle BCE + \angle E$ [$\angle BCE = \angle DCF$, Vertically opposite angles]

$= \angle DCF + \angle E$

$= 3x + 4x = 7x$

Now, in cyclic quad ABCD,

Since, $\angle B + \angle = 180°$

[since sum of opposite of a cyclic quadrilateral are supplementary]

$\Rightarrow 7x + 8x = 180°$

$\Rightarrow 15x = 180°$

$\Rightarrow x = \dfrac{180°}{15} = 12°$

$\therefore \angle A = 3x = 3 \times 12° = 36°$

$\angle B = 7x = 7 \times 12° = 84°$

$\angle C = 180° - \angle A = 180° - 36° = 144°$

$\angle D = 8x = 8 \times 12° = 96°$

Solution 20:

In the figure, PQRS is a cyclic quadrilateral in which PR is a diameter

PQ = 7 cm

QR = 3 RS = 6 cm

3 RS = 6 cm $\Rightarrow$ RS = 2 cm

Now in ΔPQR,

$\angle Q = 90°$ [Angles in a semi circle]

$\therefore PR^2 = PQ^2 + QR^2$ [Pythagoras theorem]

$= 7^2 + 6^2$

$= 49 + 36$

$= 85$

Again in right ΔPSQ, $PR^2 = PS^2 + RS^2$

$\Rightarrow 85 = PS^2 + 2^2$

$\Rightarrow PS^2 = 85 - 4 = 81 = (9)^2$

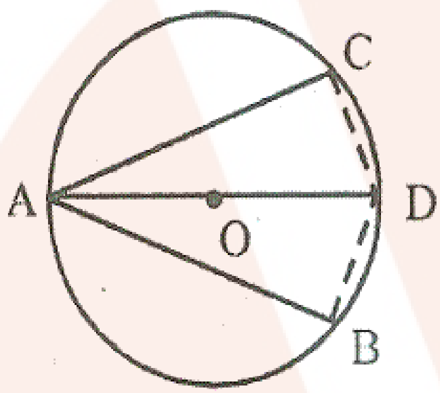
$\therefore PS = 9\,cm$

Now, perimeter of quad $PQRS = PQ + QR + RS + SP$

$= (7 + 9 + 2 + 6)\,cm$

$= 24$

Solution 21:



Given – In a circle with centre O, AB is the diameter and AC and AD are two chords such that AC = AD

To prove: (i) arc BC = arc DB

(ii) AB is the bisector of $\angle CAD$

(iii) If arc AC = 2 arc BC, then find

(a) $\angle BAC$ (b) $\angle ABC$

Construction: Join BC and BD

Proof : In right angled ΔABC and ΔABD

Side AC = AD [Given]

Hyp. AB = AB [Common]

$\therefore$ By right Angle – Hypotenuse – Side criterion of congruence

$\Delta ABC \cong \Delta ABD$

(i) The corresponding parts of the congruent triangle are congruent.

$\therefore$ BC = BD [c.p.c.t]

$\therefore$Arc BC = Arc BD [equal chords have equal arcs]

(ii) $\angle BAC = \angle BAD$

$\therefore$ AB is the bisector of $\angle CAD$

(iii) If Arc AC = 2 arc BC,

Then $\angle ABC = 2\angle BAC$

But $\angle ABC + \angle BAC = 90°$

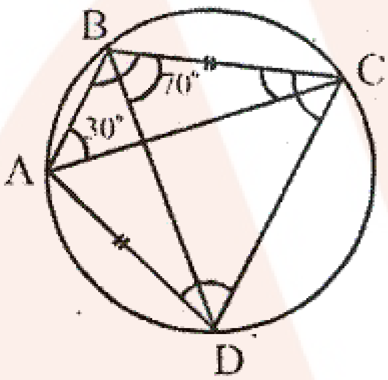
$\Rightarrow 2\angle BAC + \angle BAC = 90°$

$\Rightarrow 3\angle BAC = 90°$

$\Rightarrow \angle BAC = \dfrac{90°}{3} = 30°$

$\angle ABC = 2\angle BAC \Rightarrow \angle ABC = 2 \times 30° = 60°$

Solution 22:



ABCD is a cyclic quadrilateral and AD = BC

$\angle BAC = 30°$, $\angle CBD = 70°$

$\angle DAC = \angle CBD$ [Angles in the same segment]

$\Rightarrow \angle DAC = 70°$ [$\because$ $= 70°$]

$\Rightarrow \angle BAD = \angle BAC + \angle DAC = 30° + 70° = 100°$

Since the sum of opposite angles of cyclic quadrilateral is supplementary

$\angle BAD + \angle BCD = 180°$

$\Rightarrow 100°$. $\angle$3CD $= 180°$ [From (1)]

$\Rightarrow \angle BCD = 180° - 100° = 80°$

Since , AD = BC , $\angle ACD = \angle BDC$ [Equal chords subtends equal angles]

But $\angle ACB = \angle ADB$ [angles in the same segment]

$\therefore \angle ACD + \angle ACB = \angle BDC + \angle ADB$

$\Rightarrow \angle BCD = \angle ADC = 80°$

But in ΔBCD,

$\angle CBD + \angle BCD + \angle BDC = 180°$ [angles oaf a triangle]

$\Rightarrow 70° + 80° + \angle BDC = 180°$

$\Rightarrow 150° + \angle BDC = 180°$

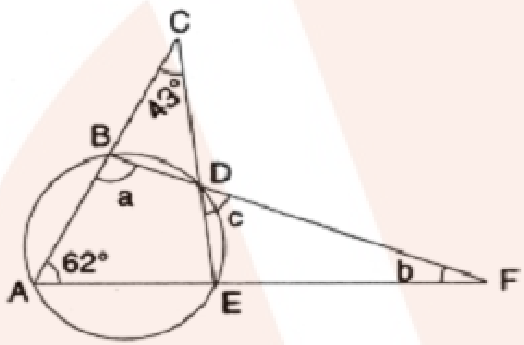
$\therefore \angle BDC = 180° - 150° = 30°$

$\Rightarrow \angle ACD = 30°$ [$\because \angle ACD = \angle BDC$]

$\therefore \angle BCA = \angle BCD - \angle ACD = 80° - 30° = 50°$

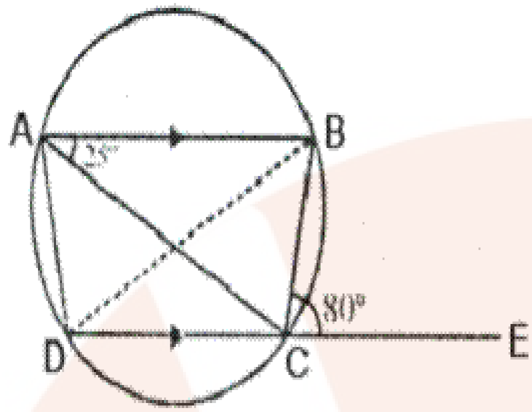
Since the sum of opposite angles of cyclic quadrilateral is supplementary
$\angle ADC + \angle ABC = 180°$
$\Rightarrow 80° + \angle ABC = 180°$
$\Rightarrow \angle ABC = 180° - 80° = 100°$

Solution 23:



Now, $\angle ACE = 43°$ and $\angle CAF = 62°$ [given]
In ΔAEC
$\therefore \angle ACE + \angle CAE + \angle AEC = 180°$
$\Rightarrow 43° + 62° + \angle AEC = 180°$
$\Rightarrow 105° + \angle AEC = 180°$
$\Rightarrow \angle AEC = 180° - 105° = 75°$
Now, $\angle ABD + \angle AED = 180°$
[Opposite angles of a cyclic quad and $\angle AED = \angle AEC$]
$\Rightarrow a + 75° = 180°$
$\Rightarrow a = 180° - 75°$
$\Rightarrow a = 105°$
$\angle EDF = \angle BAF$ [Angles in the alternate segments]
$\therefore c = 62°$
In $\Delta BAF, a + 62° + b = 180°$
$\Rightarrow 105° + 62° + b = 180°$
$\Rightarrow 167° + b = 180°$
$\Rightarrow b = 180° - 167° = 13°$
Hence, $a = 105°$, $b = 13°$ and $c = 62°$

Solution 24:



In the given figure,
ABCD is a cyclic quad in which AB ∥ DC
∴ ABCD is an isosceles trapezium
∴ AD = BC

Ext. $\angle BCE = \angle BAD$ [Exterior angle of a cyclic qud is equal to interior opposite angle]

$\therefore \angle BAD = 80°$ $[\because \angle BCE = 80°]$

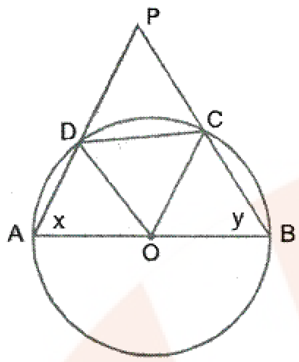
But $\angle BAC = 25°$

$\therefore \angle CAD = \angle BAD - \angle BAC$
$= 80° - 25°$
$= 55°$

(ii) $\angle CBD = \angle CAD$ [Angle of the same segment]
$= 55°$

(iii) $\angle ADC = \angle BCD$ [Angles of the isosceles trapezium]
$= 180° - \angle BCE$
$= 180° - 80°$
$= 100°$

Solution 25:



In a circle, ABCD is a cyclic quadrilateral in which
AB is the diameter and chord CD is equal to the radius of the circle
To prove - $\angle APB = 60°$
Construction – Join OC and OD
Proof – Since chord CD = CO = DO [radii of the circle]
$\therefore \Delta DOC$ is an equilateral triangle
$\therefore \angle DOC = \angle ODC = \angle DCO = 60°$
Let $\angle A = x$ and $\angle B = y$
Since OA =OB = OC = OD [radii of the same circle]
$\therefore \angle ODA = \angle OAD = x$ and
$\angle OCB = \angle OBC = y$
$\therefore \angle AOD = 180° - 2x$ and $\angle BOC = 180° - 2y$
But AOB is a straight line
$\therefore \angle AOD + \angle BOC + \angle COD = 180°$
$\Rightarrow 180° - 2x + 180° - 2y + 60° = 180°$
$\Rightarrow 2x + 2y = 240°$
$\Rightarrow x + y = 120°$
But $\angle A + \angle B + \angle P = 180°$ [Angles of a triangle]
$\Rightarrow 120° + \angle P = 180°$
$\Rightarrow \angle P = 180° - 120°$
$\Rightarrow \angle P = 60°$
Hence $\angle APB = 60°$

Solution 26:

Given – In the figure, CP is the bisector of $\angle ABC$

To prove – DP is the bisector of $\angle ADB$

Proof – Since CP is the bisector of $\angle ACB$

$\therefore \angle ACP = \angle BCP$

But $\angle ACP = \angle ADP$ [Angle in the same segment of the circle]

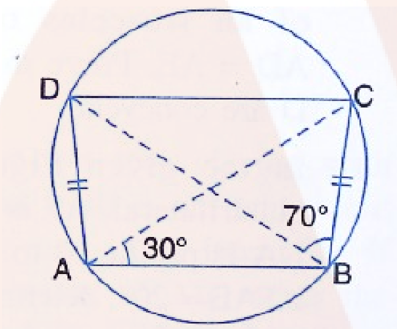
And $\angle BCP = \angle BDP$

But $\angle ACP = \angle BCP$

$\therefore \angle ADP = \angle BDP$

$\therefore$ DP is the bisector of $\angle ADB$

Solution 27:



In the figure, ABCD is a cyclic quadrilateral

AC and BD are its diagonals

$\angle BAC = 30°$ and $\angle CBD = 70°$

Now we have to find the measure of

$\angle BCD, \angle BCA, \angle ABC$ and $\angle ADB$

We have $\angle CAD = \angle CBD = 70°$ [Angles in the same segment]

Similarly, $\angle BAD = \angle BDC = 30°$

$\therefore \angle BAD = \angle BAC + \angle CAD$

$= 30° + 70°$

$= 100°$

(i) Now $\angle BCD + \angle BAD = 180°$ [Opposite angles of cyclic quadrilateral]

$\Rightarrow \angle BCD + \angle BAD\angle = 180°$

$\Rightarrow \angle BCD + 100° = 180°$

$\Rightarrow \angle BCD = 180° - 100°$

$\Rightarrow \angle BCD = 80°$

(ii) Since AD = BC, ABCD is an isosceles trapezium and AB ∥ DC

$\angle BAC = \angle DCA$ [Alternate angles]

$\Rightarrow \angle DCA = 30°$

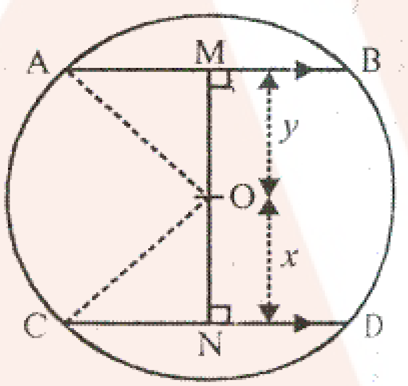
$\therefore \angle ABD = \angle DAC = 30°$ [Angles in the same segment]

$\therefore \angle BCA = \angle BCD - \angle DAC$
$= 80° - 30°$
$= 50°$

(iii) $\angle ABC = \angle ABD + \angle CBD$
$= 30° + 70°$
$= 100°$

(iv) $\angle ADB = \angle BCA = 50°$ [Angles in the same segment]

Solution 28:



Given – AB = 24 cm, cd = 18 cm
$\Rightarrow$ AM = 12 cm, CN = 9 cm
Also, OA = OC = 15 cm
Let MO = y cm, and ON = x cm
In right angled ΔAMO

$(OA)^2 = (AM)^2 + (OM)^2$

$\Rightarrow 15^2 = 12^2 + y^2$

$\Rightarrow y^2 = 15^2 - 12^2$

$\Rightarrow y^2 = 225 - 144$

$\Rightarrow y^2 = 81$

$\Rightarrow y = 9\text{cm}$

In right angled ΔCON

$(OC)^2 = (ON)^2 + (CN)^2$

$\Rightarrow 15^2 = x^2 + 9^2$

$\Rightarrow x^2 = 15^2 - 9^2$

$\Rightarrow x^2 = 225 - 81$

$\Rightarrow x^2 = 144$

$\Rightarrow y = 12\,\text{cm}$

$$\begin{aligned}\text{Now, } MN &= MO + ON \\ &= y + x \\ &= 9\text{ cm} + 12\text{ cm} \\ &= 21\text{ cm}\end{aligned}$$