Book Name: Selina Concise

EXERCISE- 4 (A)

Solution 1:

(i) $x < -y \Rightarrow -x > y$

The given statement is true.

(ii) $-5x \geq 15 \Rightarrow \frac{-5x}{5} \geq \frac{15}{5}\ x \leq -3$

The given statement is false

(iii) $2x \leq -7 \Rightarrow \frac{2x}{-4} \geq \frac{-7}{-4}$

The given statement is true

(iv) $7 > 5 \Rightarrow \frac{1}{7} < \frac{1}{5}$

The given statement is true.

Solution 2:

(i) $a < b \Rightarrow a - c < b - c$

The given statement is true.

(ii) If $a > b \Rightarrow a + c > b + c$

The given statement is true.

(iii) If $a < b \Rightarrow ac < bc$

The given statement is false.

(iv) If $a > b \Rightarrow \frac{a}{c} > \frac{b}{c}$

The given statement is false.

(v) If $a - c > b - d \Rightarrow a + d > b + c$

The given statement is true.

(vi) If $a < b \Rightarrow a - c < b - c$ (Since, $c > O$)

The given statement is false.

Solution 3:

(i) $5x + 3 \leq 2x + 18$

$5x - 2x \leq 18 - 3$

$3x \leq 15$

$X \leq 5$

Since, $x \in N$, therefore solution set is $\{1,2,3,4,5\}$

(ii) $3x - 2 < 19 - 4x$
$3x + 4x < 19 + 2$
$7x < 21$
$X < 3$
Since, $x \in N$, therefore solution set is {1,2}.

Solution 4:

(i) $x + 7 \leq 11$
$X \leq 11 - 7$
$X \leq 4$
Since, the replacement set = W (set of whole numbers)
$\Longrightarrow$ Solution set = {0,1,2,3,4}

(ii) $3x - 1 > 8$
$3x > 8 + 1$
$X > 3$
Since, the replacement set = W (Set of whole numbers)
$\Longrightarrow$ Solution set = {4, 5, 6……}

(iii) $8 - x > 5$
$-X > 5 - 8$
$-X > -3$
$X < 3$
Since, the replacement set = W (Set of whole numbers)
$\Longrightarrow$ Solution set = {0, 1, 2 ……}

(iv) $7 - 3x \geq -\frac{1}{2}$
$-3x \geq -\frac{1}{2} - 7$
$-3x \geq -\frac{15}{2}$
$X \leq \frac{5}{2}$
Since, the replacement set = W (set of whole numbers)
$\therefore$ Solution set = {0, 1, 2}

(v) $x - \frac{3}{2} < \frac{3}{2} - x$
$x + x < \frac{3}{2} + \frac{3}{2}$
$2x < 3$
$X < \frac{3}{2}$
Since, the replacement set = W (set of whole numbers)
$\therefore$ Solution set = {0, 1}

(vi) $18 \leq 3x - 2$

$18 + 2 \leq 3x$

$20 \leq 3x$

$X \geq \frac{20}{3}$

Since, the replacement set = W (set of whole numbers)

$\therefore$ Solution set = {7, 8, 9….}

Solution 5:

$3 - 2x \geq x - 12$

$-\ 2x - x \geq -12 - 3$

$-\ 3x \geq -15$

$X \leq 5$

Since, $x \in N$, therefore,

Solution set = {1, 2, 3, 4, 5}

Solution 6:

$25 - 4x \leq 16$

$-\ 4x \leq 16 - 25$

$-\ 4x \leq -9$

$X \geq \frac{9}{4}$

$X \geq 2.25$

(i) The smallest value of x, when x is a real number, is 2.25.

(ii) The smallest value of x, when x is an integer, is 3.

Solution 7:

(i) $-4x \geq -16$

$X \leq 4$

Since, the replacement set of real numbers.

$\therefore$ solution set = { $x : x \in R$ and $x \leq 4$}

(ii) $8 - 3x \leq 20$

$-\ 3x \leq 20 - 8$

$-\ 3x \leq 12$

$X \geq -4$

Since the replacement set of real numbers.

$\therefore$ solution set = { $x : x \in R$ and $x \leq -4$}

(iii) $5 + \frac{x}{4} > \frac{x}{5} + 9$

$\frac{x}{4} - \frac{x}{5} > 9 - 5$
$\frac{x}{20} > 4$
$X > 80$
Since the replacement set of real numbers.
∴ solution set = { x:x ∈ R and x > 80}

(iv) $\frac{x+3}{8} < \frac{x-3}{5}$
$5x + 15 < 8x - 24$
$5x - 8x < -24 - 15$
$-3x < -39$
$X > 13$
Since the replacement set of real numbers.
∴ solution set = { x:x ∈ R and x > 13}

Solution 8:

$5 - 2x < 5\frac{1}{2} - \frac{5}{3}x$
$-2x + \frac{5}{3}x < \frac{11}{2} - 5$
$\frac{-x}{3} < \frac{1}{2}$
$-x < \frac{3}{2}$
$X > \frac{-3}{2}$
$X > -1.5$
Thus, the required smallest value of x is -1.

Solution 9:

$2(x - 1) \leq 9 - x$
$2x - 2 \leq 9 - x$
$2x + x \leq 9 + 2$
$3x \leq 11$
$x \leq \frac{11}{3}$
$X \leq 3.67$
Since, x ∈ W, thus the required largest value of x is 3.

Solution 10:

$12 + 1\frac{5}{6} \times \leq 5 + 3x$
$\frac{11}{6}X - 3X \leq 5 - 12$
$\frac{-7}{6}X \leq -7$

$X \geq 6$
$\therefore$ solution set = $\{x : x \in R \text{ and } x \geq 6\}$

Solution 11:

$-5 \leq 2x - 3 < x + 2$

$\Rightarrow -5 \leq 2x - 3$ and $2x - 3 < x + 2$
$\Rightarrow -5 + 3 \leq 2x$ and $2x - x < 2 + 3$
$\Rightarrow -2 \leq 2x$ and $x < 5$
$\Rightarrow X \geq -1$ and $x < 5$

Since, $x \in$ {integers}
$\therefore$ Solution set = { -1, 0, 1, 2, 3, 4}

Solution 12:

$- \ 1 \leq 3 + 4x < 23$

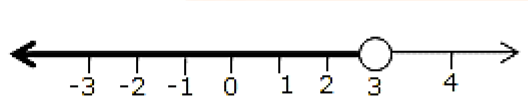
$\Longrightarrow -1 \leq 3 + 4x$ and $3 + 4x < 23$
$\Longrightarrow -4 \leq 4x$ and $4x < 20$
$\Longrightarrow x \geq -1$ and $x < 5$

Since, $x \in$ { Whole numbers}
$\therefore$ solution set = { 0, 1, 2, 3, 4}

EXERCISE 4(B)

Solution 1:

(i) $2x - 1 < 5$
$2x < 6$
$X < 3$
Solution on number line is:

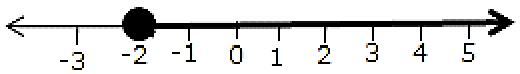


(ii) $3x + 1 \geq -5$

$3x \geq -6$

$X \geq -2$

Solution on number line is:



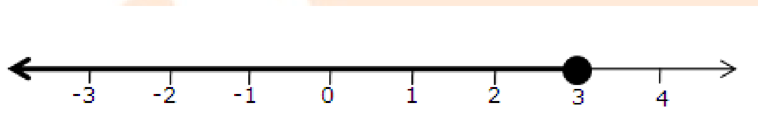
(iii) $2(2x - 3) \leq 6$

$2x - 3 \leq 3$

$2x \leq 6$

$X \leq 3$

Solution on number line is:

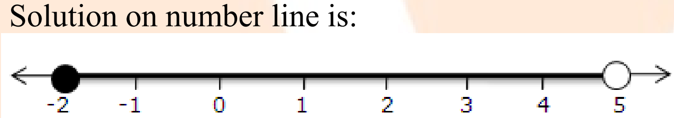


(iv) $-4 < x < 4$

Solution on number line is:

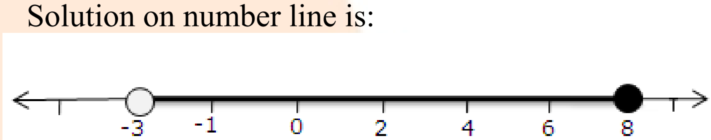
(v) $-2 \leq x < 5$

Solution on number line is:



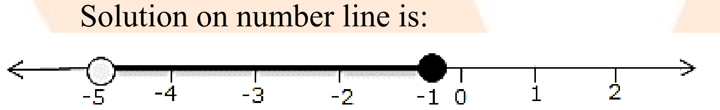
(vi) $8 \geq x > -3$

Solution on number line is:



(vii) $-5 < x \leq -1$

Solution on number line is:



Solution 2:

(i) $x \leq -1, x \in R$

(ii) $x \geq 2, x \in R$

(iii) $-4 \leq x < 3, x \in R$

(iv) $-1 < x \leq 5, x \in R$

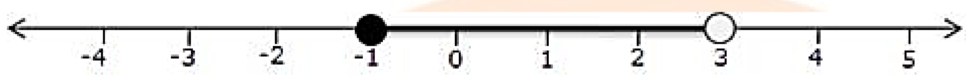
Solution 3:

(i) $-4 \le 3x - 1 < 8$

$-4 \le 3x - 1$ and $3x - 1 < 8$

$-1 \le x$ and $x < 3$

The solution set on the real number line is:



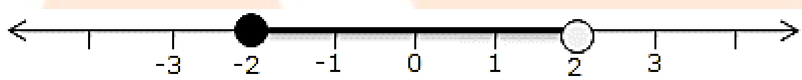
(ii) $x - 1 < 3 - x \le 5$

$X - 1 < 3 - x$ and $3 - x \le 5$

$2x < 4$ and $-x \le 2$

$X < 2$ and $x \ge -2$

The solution set on the real number line is:



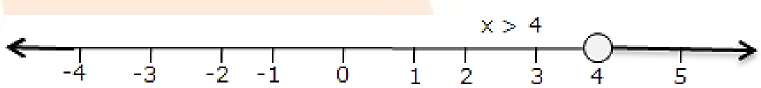
Solution 4:

(i) $4x - 1 > x + 11$

$3x > 12$

$X > 4$

The solution on number line is:

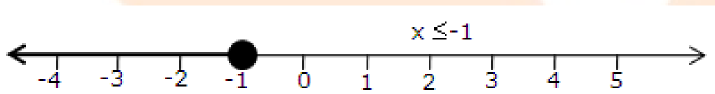


(ii) $7 - x \le 2 - 6x$

$5x \le -5$

$X \le -1$

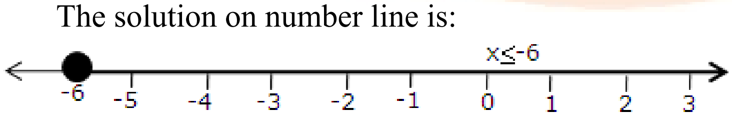
The solution on number line is:



(iii) $x + 3 \le 2x + 9$

$-6 \le x$

The solution on number line is:



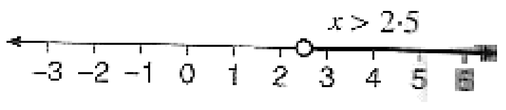
(iv) $2 - 3x > 7 - 5x$

$2x > 5$

$X > \frac{5}{2}$

$X > 2.5$

The solution on number line is:

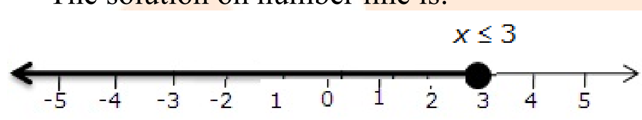


(v) $1 + x \geq 5x - 11$

$12 \geq 4x$

$3 \geq x$

The solution on number line is:



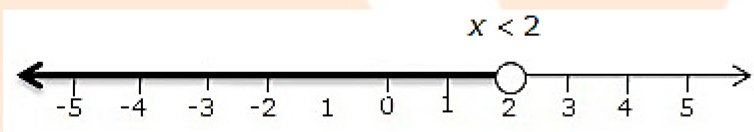
(vi) $\frac{2x+5}{3} > 3x - 3$

$2x + 5 > 9x - 9$

$-7x > -14$

$X < 2$

The solution on number line is:



Solution 5:

– $1 < 3 - 2x \leq 7$

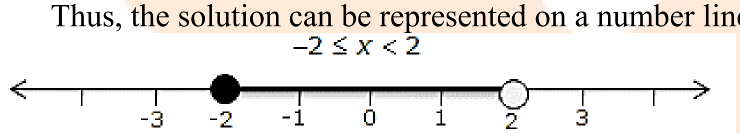
– $1 < 3 - 2x$ and $3 - 2x \leq 7$

$2x < 4$ and $-2x \leq 4$

$X < 2$ and $x \geq -2$

Solution set = $\{ -2 \leq x < 2, x \in R\}$

Thus, the solution can be represented on a number line as:



Solution 6:

– $3 < x - 2 \leq 9 - 2x$

– $3 < x - 2$ and $x - 2 \leq 9 - 2x$

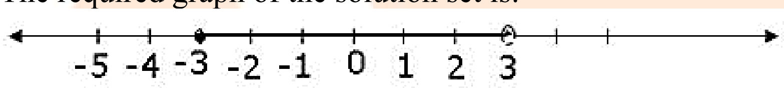
– $1 < x$ and $3x \leq 11$

– $1 < x \leq \frac{11}{3}$

Since, $x \in N$
∴ Solution set = {1, 2, 3}

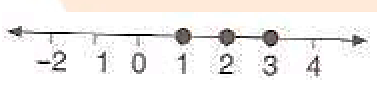
Solution 7:

$-\ 2\frac{2}{3} \le x + \frac{1}{3}$ and $x + \frac{1}{3} < 3\frac{1}{3}$
$\Rightarrow -\frac{8}{3} \le x + \frac{1}{3}$ and $x + \frac{1}{3} < \frac{10}{3}$
$\Rightarrow -\frac{8}{3} - \frac{1}{3} \le x$ and $x < \frac{10}{3} - \frac{1}{3}$
$\Rightarrow -\frac{9}{3} \le x$ and $x < \frac{9}{3}$
$\Rightarrow -3 \le x$ and $x < 3$
∴ $-3 \le x$ and $x < 3$
The required graph of the solution set is:



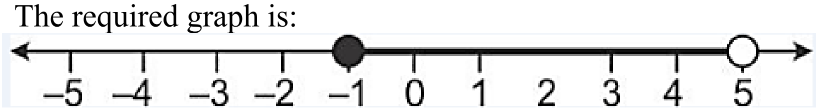
Solution 8:

$-\ 2 \le \frac{1}{2} - \frac{2x}{3} < 1\ \frac{5}{6}$
$-\ 2 \le \frac{1}{2} - \frac{2x}{3}$ and $\frac{1}{2} - \frac{2x}{3} < 1\ \frac{5}{6}$
$\frac{-5}{2} \le -\frac{2x}{3}$ and $\frac{-2x}{3} < \frac{8}{6}$
$\frac{15}{4} \ge x$ and $x > -2$
$3.75 \ge x$ and $x > -2$
Since, $x \in N$
∴ Solution set = {1, 2, 3}
The required graph of the solution set is:



Solution 9:

$-\ 5 \le 2x - 3 < x + 2$
$-\ 5 \le 2x - 3$ and $2x - 3 < x + 2$
$-\ 2 \le 2x$ and $x < 5$
$-\ 1 \le x$ and $x < 5$
∴ Required range is $-1 \le x < 5$
The required graph is:

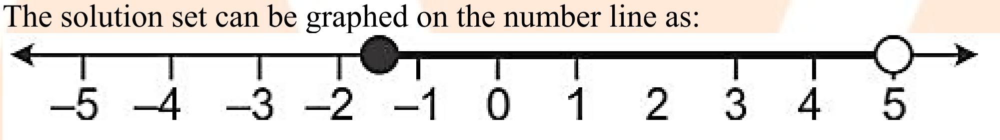


Solution 10:

$5x - 3 \leq 5 + 3x \leq 4x + 2$
$5x - 3 \leq 5 + 3x$ and $5 + 3x \leq 4x + 2$
$2x \leq 8$ and $-x \leq -3$
$X \leq 4$ and $x \geq 3$
Thus, $3 \leq x \leq 4$
Hence, $a = 3$ and $b = 4$

Solution 11:

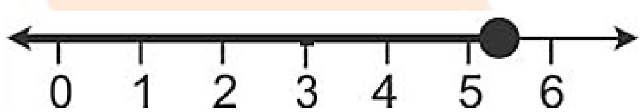
$2x - 3 < x + 2 \leq 3x + 5$
$2x - 3 < x + 2$ and $x + 2 \leq 3x + 5$
$X < 5$ and $-3 \leq 2x$
$X < 5$ and $-1.5 \leq x$
Solution set = $\{-1.5 \leq x < 5\}$
The solution set can be graphed on the number line as:



Solution 12:

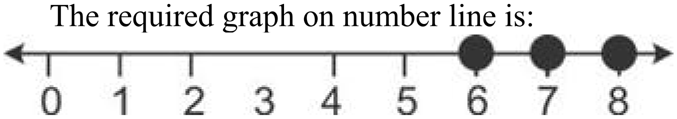
(i) $2x - 9 < 7$ and $3x + 9 \leq 25$

$2x < 16$ and $3x \leq 16$
$X < 8$ and $x \leq 5\frac{1}{3}$
$\therefore$ Solution set = $\{ x \leq 5\frac{1}{3}, x \in R\}$
The required graph on number line is:



(ii) $2x - 9 \leq 7$ and $3x + 3x + 9 > 25$

$2x \leq 16$ and $3x > 16$
$X \leq 8$ and $x > 5\frac{1}{3}$
$\therefore$ Solution set = $\{ 5\frac{1}{3} < x \leq 8, x \in I\} = \{6, 7, 8\}$
The required graph on number line is:



(iii) $x + 5 \geq 4(x - 1)$ and $3 - 2x < -7$

$9 \geq 3x$ and $-2x < -10$
$3 \geq x$ and $x > 5$
$\therefore$ solution set = Empty set

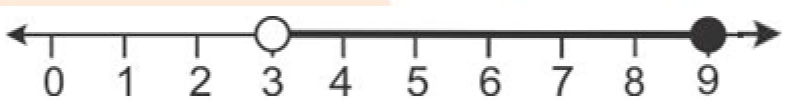
Solution 13:

(i) $3x - 2 > 19$ or $3 - 2x \geq -7$

$3x > 21$ or $-2x \geq -10$
$X > 7$ or $x \leq 5$
Graph of solution set of $x > 7$ or $x \leq 5$ = Graph of points which belong to $x > 7$ or $x \leq 5$ or both.
Thus, the graph of the solution set is:

(ii) $5 > p - 1 > 2$ or $7 \leq 2p - 1 \leq 17$

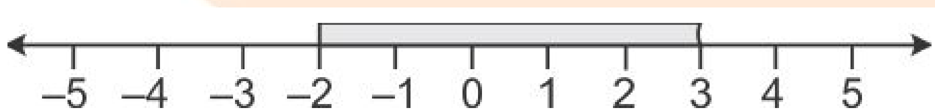
$6 > p > 3$ or $8 \leq 2p \leq 18$
$6 > p > 3$ or $4 \leq p \leq 9$
Graph of solution set of $6 > p > 3$ or $4 \leq p \leq 9$
= Graph of points which belong to $6 > p > 3$ or $4 \leq p \leq 9$ or both
= Graph of points which belong to $3 < p \leq 9$
Thus, the graph of the solution set is:



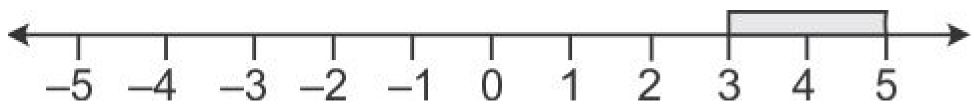
Solution 14:

(i) $A = \{ x \in R: -2 \leq x < 5\}$
$B = \{x \in R: -4 \leq x < 3\}$

(ii) $A \cap B = \{ x \in R:- 2 \leq x < 5\}$
It can be represented on number line as:



$B' = \{ x \in R: 3 < x -4\}$
$A \cap B' = \{ x \in R: 3 \leq x < 5\}$
It can be represented on number line as:



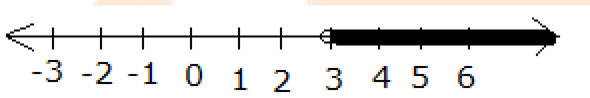
Solution 15:

(i) x > 3 and 0 < X < 6

Both the given in equations are true in the range where their graphs on the real number lines overlap.

The graphs of the given in equations can be drawn as:

x > 3



0 < x < 6

From both graphs, it is clear that their common range is

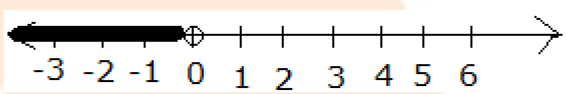
3 < x < 6

(ii) x < 0 and -3 ≤ x < 1

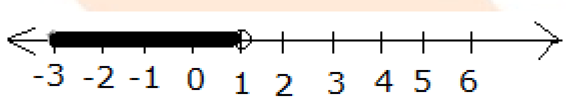
Both the given in equations are true in the range where their graphs on the real number lines overlap.

The graphs of the given in equations can be drawn as:

X < 0



− 3≤ x < 1



From both graphs, it is clear that their common range is

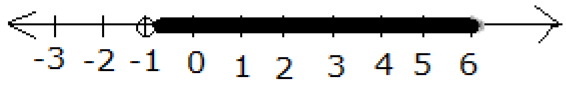
-3 ≤ x < 0

(iii) -1 < x ≤ 6 and -2 ≤ x ≤ 3

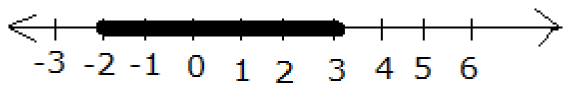
Both the given in equations are true in the range where their graphs on the real number lines overlap.

The graphs of the given in equations can be drawn as:

− 1 < x ≤ 6



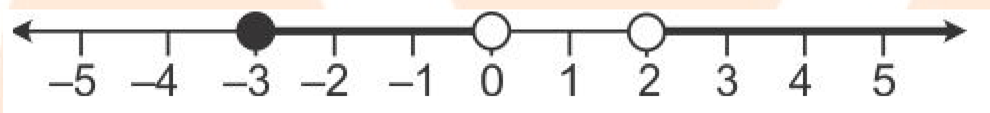
– 2 ≤ x ≤ 3



From both graphs, it is clear that their common range is
$1 < x \leq 3$

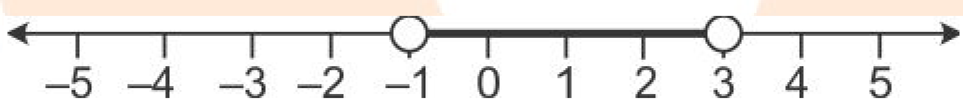
Solution 16:

Graph of solution set of $-3 \leq x < 0$ or $x > 2$
= Graph of points which belong to $-3 \leq x < 0$ or $x > 2$ or both
Thus, the required graph is:

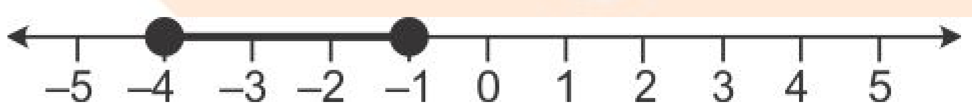


Solution 17:

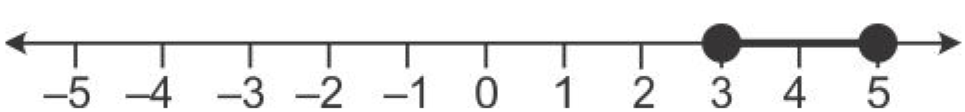
(i) $A \cap B = \{x: -1 < x < 3, x \in R\}$
It can be represented on a number line as:



(ii) Numbers which belong to B but do not belong to A = B - A
$A' \cap B = \{x: -4 \leq x \leq -1, x \in R\}$
It can be represented on a number line as:



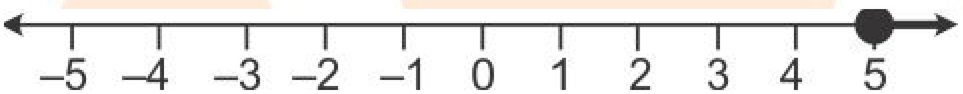
(iii) $A - B = \{x: 3 \leq x \leq 5, x \in R\}$
It can be represented on a number line as:



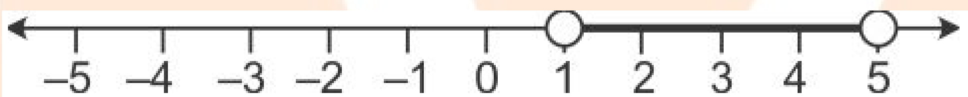
Solution 18:

P = { X : 7X – 2 > 4X + 1, X ∈ R }
7x – 2 > 4x + 1
7x – 4x > 1 + 2
3x > 3
X > 1
and
Q = {x: 9x – 45 ≥ 5 (x – 5), x ∈ R}
9x – 45 ≥ 5x – 25
9x – 5x ≥ – 25 + 45
4x ≥ 20
X ≥ 5

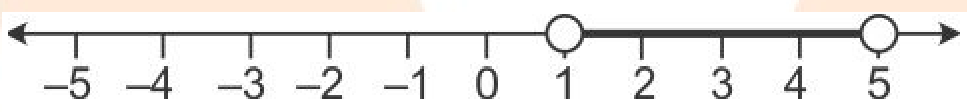
(i) P ∩ Q = {x : x ≥ 5, x ∈ R}



(ii) P – Q = { X : 1 < X < 5, X ∈ R}



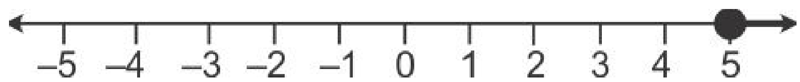
(iii) P ∩ Q' = {x : 1 < x < 5, x ∈ R}



Solution 19:

P = {X : 7X – 4 > 5X + 2, X ∈ R}
7X – 4 > 5X + 2
7X – 5X > 2 + 4
2X > 6
X > 3

Q = { X:X – 19 ≥ 1 – 3X, X ∈ R}
X – 19 ≥ 1 – 3X
X + 3X ≥ 1 + 19
4X ≥ 20
X ≥ 5
P ∩ Q = { X:X ≥ 5, X ∈ R}



Solution 20:

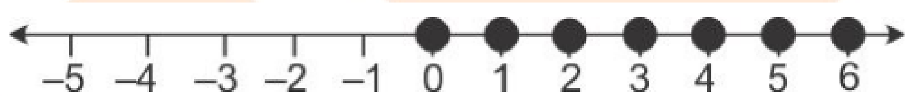
$-\frac{1}{3} \le \frac{X}{2} + 1\frac{2}{3} < 5\frac{1}{6}$

$-\frac{1}{3} - \frac{5}{3} \le \frac{X}{2} < \frac{31}{6} - \frac{5}{3}$

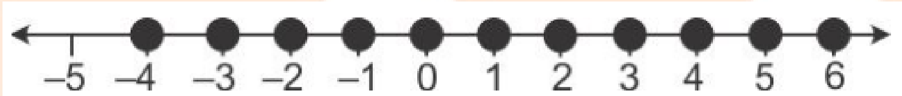
$-\frac{6}{3} \le \frac{X}{2} < \frac{21}{6}$

$-4 \le X < 7$

(i) If x ∈ W, range of value of x is {0, 1, 2, 3, 4, 5, 6}



(ii) If x ∈ Z, range of values of x is {-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6}.

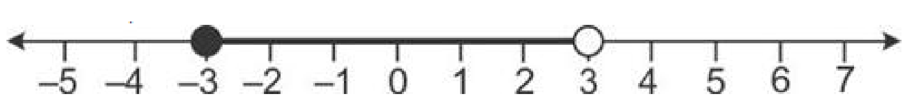


(iii) If x ∈ R, range of values of x is $-4 \le x < 7$.

Solution 21:

A = {x: $-8 < 5x + 2 \le 17$, x ∈ I}
= {x: $-10 < 5x \le 15$, x ∈ I}
= {x: $-2 < x \le 3$, x ∈ I}
It can be represented on number line as:

B = {x: $-2 \le 7 + 3x < 17$, x ∈ R}
= {x: $-9 \le 3x < 10$, x ∈ R}
= {x: $-3 \le x < 3.33$, x ∈ R}
It can be represented on number line as:



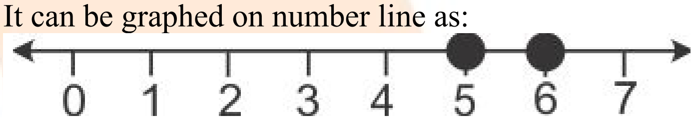
A ∩ B = {-1, 0, 1, 2, 3}

Solution 22:

$2x - 5 \leq 5x + 4$ and $5x + 4 < 11$
$2x - 5x \leq 4 - 5$ and $5x < 11 - 4$
$3x \leq - 1$ and $5x < 7$
$x \geq - 1$ and $x < \frac{7}{5}$
$x \geq - 1$ and $x < 1.4$
Since $x \in I$, the solution set is $\{- 3, -2 -1, 0, 1\}$
And the number line representation is

Solution 23:

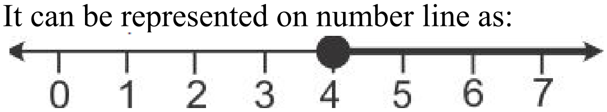
$3 \geq \frac{x-4}{2} + \frac{x}{3} \geq 2$
$3 \geq \frac{3x-12+2x}{6} \geq 2$
$18 \geq 5x - 12 \geq 12$
$30 \geq 5x \geq 24$
$6 \geq x \geq 4.8$
Solution set = {5,6}
It can be graphed on number line as:



Solution 24:

A = {x: 11x - 5 > 7x + 3, x ∈ R}
= {x : 4x > 8, x ∈ R}
= {x : x > 2, x ∈ R}

B = {x : 18x - 9 ≥ 15 + 12x, x ∈ R}
= {x : 6x ≥ 24, x ∈ R}
= {x : x ≥ 4, x ∈ R}
A ∩ B = {x : x ≥ 4, x ∈ R}
It can be represented on number line as:



Solution 25:

7X + 3 ≥ 3X – 5
4X ≥ – 8
X ≥ - 2

$\boldsymbol{\frac{X}{4} - 5 \le \frac{5}{4} - X}$
$\frac{X}{4} - X \le \frac{5}{4} + 5$
$\frac{5X}{4} \le \frac{25}{4}$
X ≤ 5

Since, x ∈ N
∴ Solution set = {1, 2, 3, 4, 5}

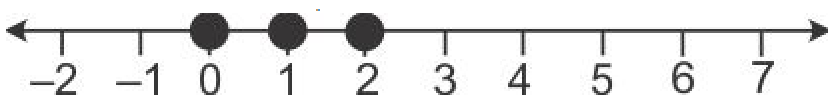
Solution 26:

(i) $\boldsymbol{\frac{x}{2} + 5 \le \frac{x}{3} + 6}$
$\boldsymbol{\frac{x}{2} - \frac{x}{3} \le 6 - 5}$
$\boldsymbol{\frac{x}{6} \le 1}$
$\boldsymbol{x \le 6}$
Since, x is a positive odd integer
∴ Solution set = {1, 3, 5}

(ii) $\frac{2x+3}{3} \ge \frac{3x-1}{4}$
8x + 12 ≥ 9x – 3
–X ≥ – 15
X ≤ 15
Since, x is a positive even integer
∴ Solution set = {2, 4, 6, 8, 10, 12, 14}

Solution 27:

$-2\frac{1}{2} + 2x \le \frac{4x}{5} \le \frac{4}{3} + 2x$
$-2\frac{1}{2} \le \frac{4x}{5} - 2x \le \frac{4}{3}$
$-\frac{5}{2} \le -\frac{6x}{5} \le \frac{4}{3}$
$\frac{25}{12} \ge x \ge -\frac{10}{9}$
2.083 ≥ x ≥ – 1.111
Since, x ∈ W
∴ Solution set = {0, 1, 2}
The solution set can be represented on number line as:



Solution 28:

Let the required integers be x, x + 1 and x + 2.
According to the given statement,

$$\frac{1}{3}x + \frac{1}{4}(x+1) + \frac{1}{5}(x+2) \leq 20$$

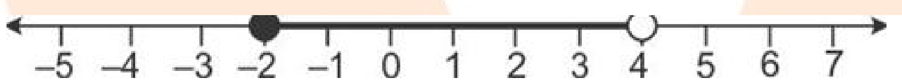
$$\frac{20x + 15x + 15 + 12x + 24}{60} \leq 20$$

$47x + 39 \leq 1200$
$47x \leq 1161$
$X \leq 24.702$
Thus, the largest value of the positive integer x is 24.
Hence, the required integers are 24, 25 and 26.

Solution 29:

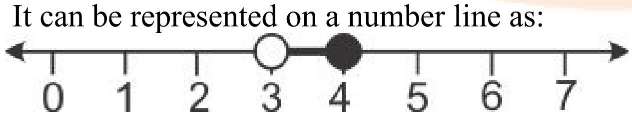
$2y - 3 < y + 1 \leq 4y + 7,\ y \in R$
$\Rightarrow 2y - 3 - y < y + 1 - y \leq 4y + 7 - y$
$\Rightarrow y - 3 < 1 \leq 3y + 7$
$\Rightarrow y - 3 < 1$ and $1 \leq 3y + 7$
$\Rightarrow y < 4$ and $3y \geq -6 \Rightarrow y \geq -2$
$\Rightarrow -2 \leq y < 4$

The graph of the given equation can be represented on a number line as:



Solution 30:

$3z - 5 \leq z + 3 < 5z - 9$
$3z - 5 \leq z + 3$ and $z + 3 < 5z - 9$
$2z \leq 8$ and $12 < 4z$
$Z \leq 4$ and $3 < z$
Since, $z \in R$
$\therefore$ Solution set = $\{3 < z \leq 4,\ x \in R\}$
It can be represented on a number line as:



Solution 31:

$$-\ 3 < -\frac{1}{2} - \frac{2X}{3} \leq \frac{5}{6}$$

Multiply by 6, we get

$\Rightarrow -18 < -3 - 4x \leq 5$

$\Rightarrow -15 < -4x \leq 8$

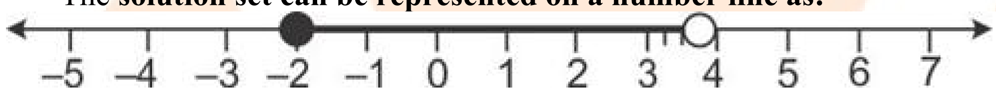
Dividing by – 4, We get

$\Rightarrow \frac{-15}{-4} > x \geq \frac{8}{-4}$

$\Rightarrow -2 \leq x < \frac{15}{4}$

$\Rightarrow x \in \left(-2, \frac{15}{4}\right)$

The **solution set can be represented on a number line as:**



Solution 32:

$4x - 19 < \frac{3x}{5} - 2 \leq \frac{-2}{5} + x, x \in R$

$\Rightarrow 4X - 19 + 2 < \frac{3X}{5} - 2 + 2 \leq \frac{-2}{5} + X + 2, X \in R$

$\Rightarrow 4X - 17 < \frac{3X}{5} \leq X + \frac{8}{5}, X \in R$

$\Rightarrow 4X - \frac{3X}{5} < 17 \text{ and } \frac{-8}{5} \leq x - \frac{3x}{5}, x \in R$

$\Rightarrow \frac{20X - 3X}{5} < 17 \text{ and } \frac{-8}{5} \leq \frac{5X - 3X}{5}, X \in R$

$\Rightarrow \frac{17x}{5} < 17 \text{ and } \frac{-8}{5} \leq \frac{2x}{5}, x \in R$

$\Rightarrow \frac{x}{5} < 1 \text{ and } -4 \leq x, x \in R$

$\Rightarrow x < 5 \text{ and } -4 \leq x, x \in R$

The solution set can be represented on a number line as: