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1

Mechanics

Force System

When a member of forces simultaneously acting on the body, it is known as force system. A force system is a collection of forces acting at specified locations. Thus, the set of forces can be shown on any free body diagram makes-up a force system.

Truss

It is a rigid structure composed of number of straight members pin jointed to each other. It can sustain static or dynamic load without any relative motion to each other.

Types of Truss

1. **Plane Truss** It is defined as a truss in which members are essentially lies in a single plane.
2. **Rigid Truss** Rigid means there is no deformation take place due to internal strain in members.
3. **Simple Truss** This type of trusses built a basic triangle by adding different members are known as **simple truss**.

Classification of a Truss (based on joints)

Truss can be classified on the basis of joints (j) and members (m) in the structure. It can be easily understood with the help of following hierarchical approach.

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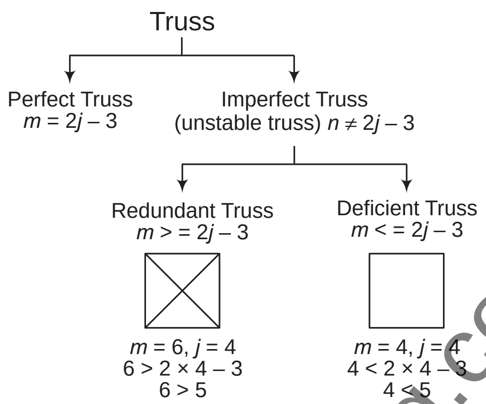


Classification of truss

where m = number of members and j = number of joints

Key Points

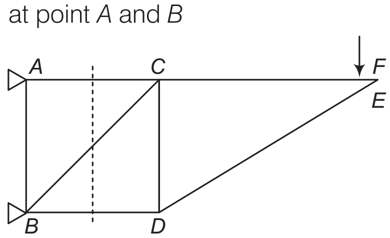
- When truss collapse under loading, then truss is known as unstable or imperfect truss.
- When truss is not collapse under the loading, then truss is known perfect truss.

Analysis of a Framed Structure (Section Method)

1. This method is used when the forces in the few members of a truss is required to found out in a truss structure.

To find out force in AC, BC and BD

- First cut a section which passes through AC, BC and BD members.
- Find out reaction at point A and B



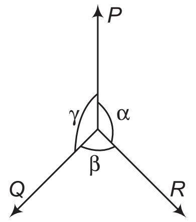
Framed structure using section method

- Find out forces F_{CA}, F_{CB} and F_{DB} in members CA, CB and DB respectively by taking moment about A and B.

2. **Analytical Method** In this method, the free body diagram of each joint is separately analysed to find magnitude of stresses in the truss members.

Lami's Theorem

If a body is in equilibrium under three concurrent forces, the each force is proportional to the sine of the angle between other two.



Three concurrent forces P, Q and R

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\alpha} = \frac{R}{\sin\gamma}$$

Friction Force

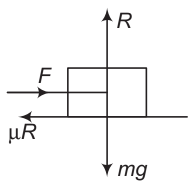
It is resistant force which acts in opposite direction at the surface in body which tend to move or its move.

Normal force $R = mg$

If $\mu\, mg > F$, the body will not move.

$\mu\, mg = F$ the body will tend to move.

$\mu\, mg < F$ the body will move.



Friction force on a body

Angle of Friction

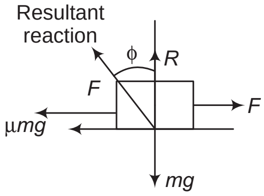
It is defined as the angle between normal reaction and resultant reaction when the body is in condition of just sliding.

$$\tan\phi = \frac{\mu\, mg}{R}$$

$$= \frac{\mu\, mg}{mg} = \mu \qquad (\because R = mg)$$

$$\phi = \tan^{-1}\mu$$

μ = coefficient of friction



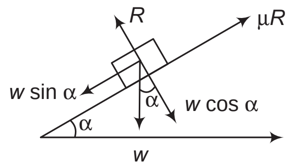
Angle of friction due to resultant reaction

Angle of Repose (α)

It is defined as angle of inclined plane with horizontal at which body is in condition of just sliding.

$$\alpha = \phi$$

Angle of friction is equal to angle of repose.



Angle of Repose (α)

Plane Motion

When all parts of the body move in a parallel planes then a rigid body said to perform plane motion.

Key Points

- The motion of rigid body is said to be translation, if every line in the body remains parallel to its original position at all times.
- In translation motion, all the particles forming a rigid body move along parallel paths.
- If all particles forming a rigid body move along parallel straight line, it is known as rectilinear translation.
- If all particles forming a rigid body does not move along a parallel straight line but they move along a curve path, then it is known as curvilinear translation.

Straight Line Motion

It defines the three equations with the relationship between velocity, acceleration, time and distance travelled by the body. In straight line motion, acceleration is constant.

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

where, u = initial velocity
v = final velocity
a = acceleration of body
t = time
s = distance travelled by body

Distance travelled in nth second

$$s_n = u + \frac{1}{2}a(2n - 1)$$

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Projectile Motion

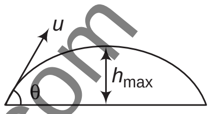
Projectile motion defines that motion in which velocity has two components, one in horizontal direction and other one in vertical direction. Horizontal component of velocity is constant during the flight of the body as no acceleration in horizontal direction.

Let the block of mass is projected at angle θ from horizontal direction

$$\text{Maximum height } h_{max} = \frac{u^2 \sin^2 \theta}{2g}$$

$$\text{Time of flight } T = \frac{2u \sin \theta}{g}$$

$$\text{Range } R = \frac{u^2 \sin 2\theta}{g}$$



Projectile motion

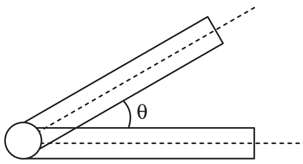
where, u = initial velocity

Key Points

- At maximum height vertical component of velocity becomes zero.
- When a rigid body move in circular paths centered on the same fixed axis, then the particle located on axis of rotation have zero velocity and zero acceleration.
- Projectile motion describe the motion of a body, when the air resistance is negligible.

Rotational Motion with Uniform Acceleration

Uniform acceleration occurs when the speed of an object changes at a constant rate. The acceleration is the same over time. So, the rotation motion with uniform acceleration can be defined as the motion of a body with the same acceleration over time. Let the rod of block rotated about a point in horizontal plane with ω angular velocity.



Rotational motion

$$\text{Angular velocity } \omega = \frac{d\theta}{dt} \text{ (change in angular displacement per unit time)}$$

$$\text{Angular acceleration } \alpha = \frac{d\omega}{dt} \Rightarrow \alpha = \frac{d^2\theta}{dt^2}$$

where θ = angle between displacement.

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In case of angular velocity, the various equations with the relationships between velocity, displacement and acceleration are as follows.

$$\theta = \omega t$$
$$\alpha = 0$$
$$\omega = \omega_0 + \alpha t$$
$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

where ω_0 = initial angular velocity
ω = final angular velocity
α = angular acceleration
θ = angular displacement

Angular displacement in nth second

$$\theta_n = \omega_0 + \frac{1}{2}\alpha\,(2n - 1)$$

Relation between Linear and Angular Quantities

There are following relations between linear and angular quantities in rotational motion.

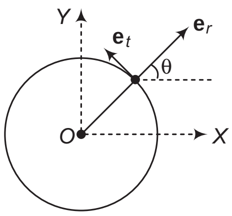
$$|\mathbf{e}_r| = |\mathbf{e}_t| = 1$$

$\mathbf{e}_r$ and $\mathbf{e}_t$ are radial and tangential unit vector.

Linear velocity $v = r\omega\,\mathbf{e}_t$

Linear acceleration (Net)

$$a = -\omega^2 r\,\mathbf{e}_r + \frac{dv}{dt}\mathbf{e}_t$$



Position of radial and tangential vectors

Tangential acceleration $a_t = \frac{dv}{dt}$ (rate of change of speed)

Centripetal acceleration $a_r = \omega^2 r = \frac{v^2}{r}$ $(\because\ v = r\omega)$

Net acceleration $a = \sqrt{a_r^2 + a_t^2}$

$$= \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

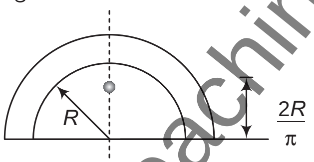
where a_r = centripetal acceleration
a_t = tangential acceleration

Centre of Mass of Continuous Body

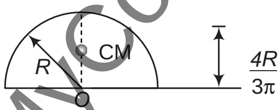
Centre of mass of continuous body can be defined as

- Centre of mass about x, $x_{CM} = \dfrac{\int x\,dm}{\int dm} = \dfrac{\int x\,dm}{M}$
- Centre of mass about y, $y_{CM} = \dfrac{\int y\,dm}{\int dm} = \dfrac{\int y\,dm}{M}$
- Centre of mass about z, $z_{CM} = \dfrac{\int z\,dm}{\int dm} = \dfrac{\int z\,dm}{M}$

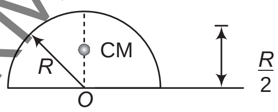
- CM of uniform rectangular, square or circular plate lies at its centre.
- CM of semicircular ring



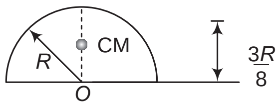
- CM of semicircular disc



- CM of hemispherical shell



- CM of solid hemisphere



Law of Conservation of Linear Momentum

The product of mass and velocity of a particle is defined as its **linear momentum** (p).

$$p = mv$$

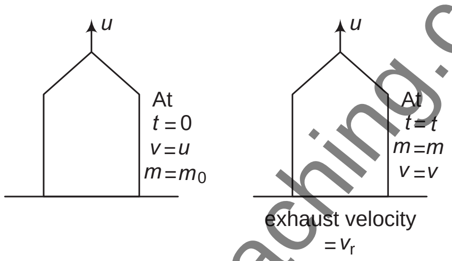
$$p = \sqrt{2Km}$$

$$F = \frac{dp}{dt}$$

where, K = kinetic energy of the particle
F = net external force applied to body
P = momentum

Rocket Propulsion

Let m_0 be the mass of the rocket at time $t = 0$, m its mass at any time t and v its velocity at that moment. Initially, let us suppose that the velocity of the rocket is u.



Rocket propulsion

- Thrust force on the rocket $F_t = v_r\left(-\frac{dm}{dt}\right)$

where, $-\frac{dm}{dt}$ = rate at which mass is ejecting

v_r = relative velocity of ejecting mass (exhaust velocity)

- Weight of the rocket $w = mg$
- Net force on the rocket $F_{\text{net}} = F_t - w = v_r\left(\frac{-dm}{dt}\right) - mg$
- Net acceleration of the rocket

$$a = \frac{F}{m}$$

$$\frac{dv}{dt} = \frac{v_r}{m}\left(\frac{-dm}{dt}\right) - g$$

$$v = u - gt + v_r \ln\frac{m_0}{m}$$

where, m_0 = mass of rocket at time $t = 0$
m = mass of rocket at time t

Impulse

The product of constant force F and time t for which it acts is called the impulse (J) of the force and this is equal to the change in linear momentum which it produces.

$$\text{Impulse } J = F\,t$$

$$\Rightarrow \qquad \Delta p = p_f - p_i$$

where, F = constant force

P = linear momentum

Instantaneous Impulse *e.g.*, bat and ball contact

$$J = \int F \cdot dt \;\Rightarrow\; \Delta p = p_f - p_i$$

Key Points

- The relation between impulse and linear momentum can be understood by the following equation.

$$Ft = m\,(v - u)$$

where, F = force, t = time, m = mass, v = initial velocity, u = final velocity

- Rotation about a fixed point gives the three dimensional motion of a rigid body attached at a fixed point.

Collision

A Collision is an isolated event in which two or more moving bodies exert forces on each other for a relatively short time.

Collision between two bodies may be classified in two ways

- Head-on collision
- Oblique collision.

Head-on Collision

Let the two balls of masses m_1 and m_2 collide directly with each other with velocities v_1 and v_2 in direction as shown in figure. After collision the velocity become v_1' and v_2' along the same line.

Before collision After collision

$$v_1' = \left(\frac{m_1 - em_2}{m_1 + m_2}\right) v_1 + \left(\frac{m_2 + em_2}{m_1 + m_2}\right) v_2$$

$$v_2' = \left(\frac{m_2 - em_2}{m_1 + m_2}\right) v_2 + \left(\frac{m_1 + em_1}{m_1 + m_2}\right) v_1$$

where, m_1 = mass of body 1
m_2 = mass of body 2
v_1 = velocity of body 1
v_2 = velocity of body 2
v_1' = velocity of body 1 after collision
v_2' = velocity of body 2 after collision

where e = coefficient restitution

$$e = \frac{\text{Separation speed}}{\text{Approach speed}}$$

$$e = \frac{v_1' - v_2'}{v_2 - v_1}$$

- In case of head-on elastic collision

$$e = 1$$

- In case of head-on inelastic collision

$$0 < e < 1$$

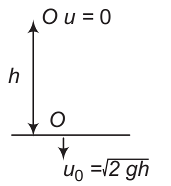
- In case of head-on perfectly inelastic collision

$$e = 0$$

If e is coefficient of restitution between ball and ground, then after nth collision with the floor, the speed of ball will remain $e^n v_0$ and it will go upto a height e^{2n} h.

$$v_n = e^n v_0 = e^n \sqrt{2gh}$$

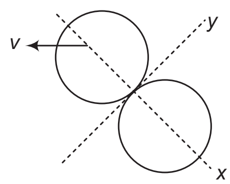
$$h_0 = e^{2n} h$$



Collision of a ball with floor

Oblique Collision

In case of oblique collision linear momentum of individual particle do change along the common normal direction. No component of impulse act along common tangent direction. So, linear momentum or linear velocity remains unchanged along tangential direction. Net momentum of both the particle remain conserved before and after collision in any direction.



Oblique collision

Moment of Inertia

Momentum of inertia can be defined as

r = distance of the body of mass, m from centre of axis.

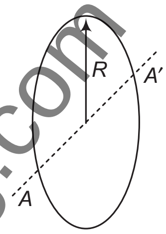
$$I = \sum_i m_i r_i^2$$

$$I = \int r^2 \, dm$$

- Very thin circular loop (ring)

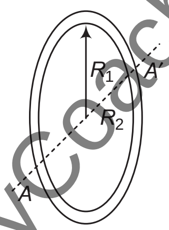
$$I = MR^2$$

where, M = mass of the body
R = radius of the ring
I = moment of inertia



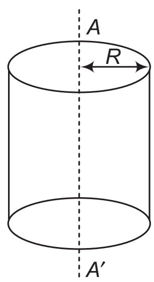
Thin circular ring

- Uniform circular loop $I = M\left(\dfrac{R_1^2 + R_2^2}{2}\right)$



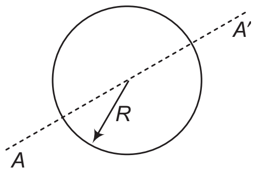
Uniform circular loop

- Uniform solid cylinder $I = \dfrac{MR^2}{2}$



Uniform solid cylinder

- Uniform solid sphere

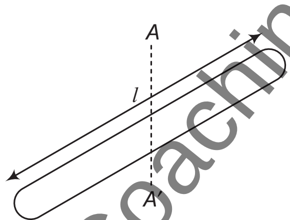


Uniform solid sphere

$$I = \frac{2}{5} MR^2$$

- Uniform thin rod

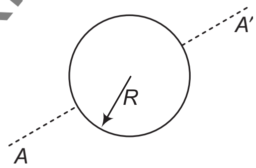
(*AA*') moment of inertia about the centre and perpendicular axis to the rod moment of inertia about the one corner point and perpendicular (BB') axis to the rod.



Uniform thin rod

$$I = \frac{Ml^2}{12} \Rightarrow I = \frac{1}{3} Ml^2$$

- Very thin spherical shell

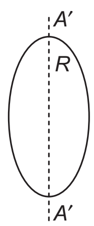


Thin sperical shell

$$I = \frac{2}{3} MR^2$$

- Thin circular sheet

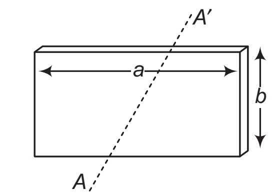
$$I = \frac{MR^2}{4}$$



Thin circular sheet

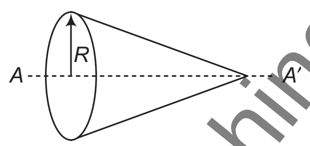
- Thin rectangular sheet

$$I = M\left(\frac{a^2 + b^2}{12}\right)$$



Thin rectangular sheet

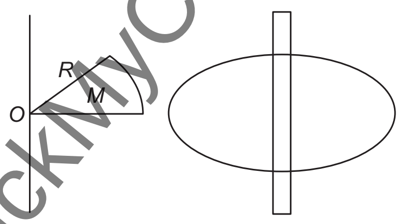
- Uniform right cone



Uniform right cone

$$I = \frac{3}{10} MR^2$$

- Uniform cone as a disc



A part of uniform cone as a disc

Suppose the given section is $\frac{1}{n}$th part of the disc, then mass of disc will be nM.

Inertia of the disc,

$$I_{\text{disc}} = \frac{1}{2}(nM)\,R^2$$

Inertia of the section,

$$I_{\text{section}} = \frac{1}{n} I_{\text{disc}} = \frac{1}{2} MR^2$$

Torque and Angular Acceleration of a Rigid Body

For a rigid body, net torque acting

$$\tau = I\alpha$$

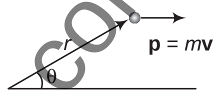
where, α = angular acceleration of rigid body
I = moment of inertia about axis of rotation

- Kinetic energy of a rigid body rotating about fixed axis

$$KE = \frac{1}{2} I\omega^2 \qquad (\omega = \text{angular velocity})$$

- Angular moment of a particle about same point

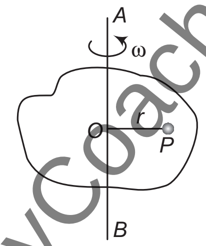
$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$
$$\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$$



Angular moment

where L = angular displacement

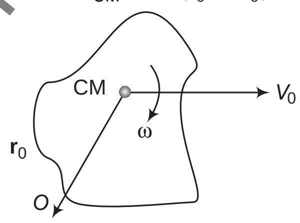
- Angular moment of a rigid body rotating about a fixed axis. $L = I\omega$



Angular moment of a rigid body

- Angular moment of a rigid body in combined rotation and translation

$$\mathbf{L} = \mathbf{L}_{CM} + M(\mathbf{r}_0 \times \mathbf{v}_0)$$



Combined rotation and translation in a rigid body

- Conservation of angular momentum

$$\tau = \frac{d\mathbf{L}}{dt}$$

$$\frac{d\mathbf{L}}{d\mathbf{t}} = \mathbf{r} \times \mathbf{F} + \mathbf{v} \times \mathbf{p}$$

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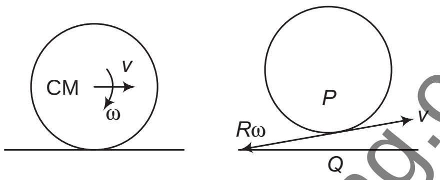
- Kinetic energy of rigid body in combined translational and rotational motion

$$K = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I_{CM} \omega^2$$

Uniform Pure Rolling

Pure rolling means no relative motion or no slipping at point of contact between two bodies.

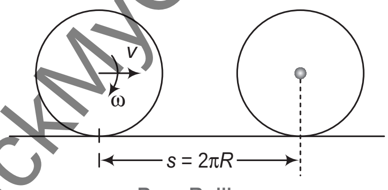


Uniform Pure Rolling

If $\quad v_P = v_Q \Rightarrow$ no slipping
$\quad v = R\omega$

if $\quad v_p > v_Q \Rightarrow$ forward slipping
$\quad v > R\omega$

if $\quad v_P < v_Q \Rightarrow$ backward slipping
$\quad v < R\omega$



Pure Rolling

No slipping $s = 2\pi R$
Forward slipping $s > 2\pi R$
Backward slipping $s < 2\pi R$

Accelerated Pure Rolling

A pure rolling is equivalent to pure translation and pure rotation. It follows a uniform rolling and accelerated pure rolling can be defined as

$$F + f = Ma$$
$$(F - f) \cdot R = I\alpha$$

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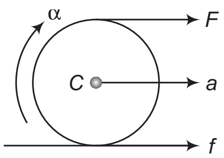

Accelerated pure rolling

F = force acting on a body, f = friction on that body

Angular Impulse

The angular impulse of a torque in a given time interval is defined as

$$\int_{t_1}^{t_2} \tau \cdot dt$$

$$\int_{t_1}^{t_2} \tau \cdot dt = L_2 - L_1$$

where, L_2 and L_1 are the angular momentum at time t_2 and t_1 respectively.

Key Points

- A force, whose line of action does not pass through centre of mass, works as force to produce translational acceleration.
- Different types of collisions are examined, whether they possess kinetic energy or not.
- The radial component of the force, which goes through the axis of rotation, has no contribution to torque.