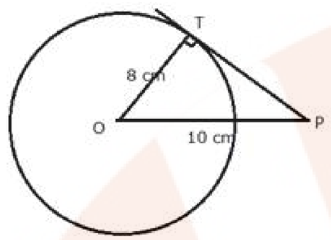
Book Name: Selina Concise

EXERCISE. 18 (A)

Solution 1:



OP = 10 cm; radius OT = 8 cm

$\because$ OT $\perp$ PT

In RT. ΔOTP,

$$OP^2 = OT^2 + PT^2$$

$$10^2 = 8^2 + PT^2$$

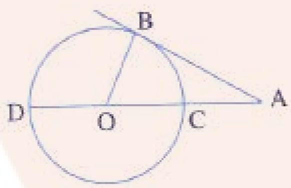
$$PT^2 = 100 - 64$$

$$PT^2 = 36$$

$$PT = 6$$

Length of tangent = 6 cm.

Solution 2:



AB = 15 cm, AC = 7.5 cm

Let 'r' be the radius of the circle.

$\therefore$ OC = OB = r

AO = AC + OC = 7.5 + r

In Δ AOB,

$$AO^2 = AB^2 + OB^2$$

$$(7.5 + r)^2 = 15^2 + r^2$$

$$\Rightarrow \left(\frac{15 + 2r}{2}\right)^2 = 225 + r^2$$

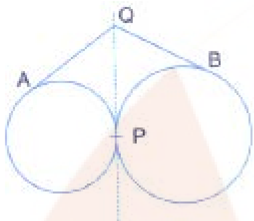
$$\Rightarrow 225 + 4r^2 + 60r = 900 + 4r^2$$

$\Rightarrow 60r = 675$

$\Rightarrow r = 11.25$ cm

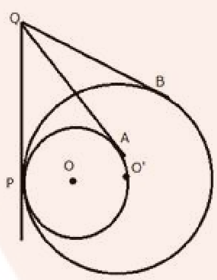
Therefore, r = 11.25 cm

Solution 3:



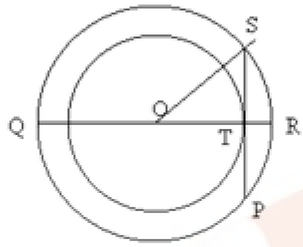
From Q, QA and QP are two tangents to the circle with centre O
Therefore, QA = QP.....(i)
Similarly, from Q, QB and QP are two tangents to the circle with centre O'
Therefore, QB = QP ......(ii)
From (i) and (ii)
QA = QB
Therefore, tangents QA and QB are equal.

Solution 4:



From Q, QA and QP are two tangents to the circle with centre O
Therefore, QA = QP ....(i)
Similarly, from Q, QB and QP are two tangents to the circle with centre O'
Therefore, QB = QP ....(ii)
From (i) and (ii)
QA = QB
Therefore, tangents QA and QB are equal.

Solution 5:



OS = 5 cm
OT = 3 cm
In Rt. Triangle OST
By Pythagoras Theorem,

$$ST^2 = OS^2 - OT^2$$

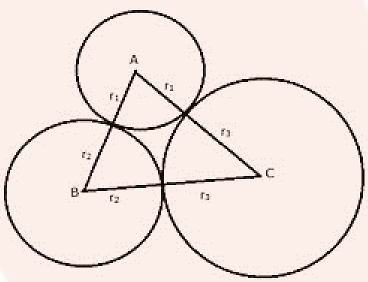
$$ST^2 = 25 - 9$$

$$ST^2 = 16$$

$$ST = 4 \text{ cm}$$

Since OT is perpendicular to SP and OT bisects chord SP
So, SP = 8 cm

Solution 6:



AB = 6 cm, AC = 8 cm and BC = 9 cm
Let radii of the circles having centers A, B and C be r_1, r_2 *and* r_3 respectively.

$$r_1 + r_3 = 8$$

$$r_3 + r_2 = 9$$

$$r_2 + r_1 = 6$$

adding

$r_1 + r_3 + r_3 + r_2 + r_2 + r_1 = 8 + 9 + 6$

$2(r_1 + r_2 + r_3) = 23$

$r_1 + r_2 + r_3 = 11.5$ cm

$r_1 + 9 = 11.5$ (Since $r_2 + r_3 = 9$)

$r_1 = 2.5$ cm

$r_2 + 6 = 11.5$ (Since $r_1 + r_3 = 6$)

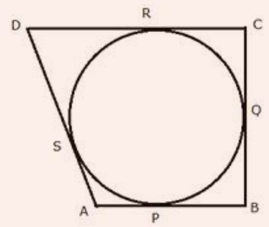
$r_2 = 5.5$ cm

$r_3 + 8 = 11.5$ (Since $r_2 + r_1 = 8$)

$r_3 = 3.5$ cm

Hence, $r_1 = 2.5$ cm, $r_2 = 5.5$ cm and $r_3 = 3.5$ cm

Solution 7:



Let the circle touch the sides AB, BC, CD and DA of quadrilateral ABCD at P, Q, R and S respectively.

Since AP and AS are tangents to the circle from external point A

AP = AS ....(i)

Similarly, we can prove that:

BP = BQ ....(ii)

CR = CQ ....(iii)

DR = DS ....(iv)

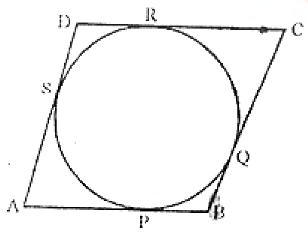
Adding,

AP + BP + CR + DR = AS + DS + BQ + CQ

AB + CD = AD + BC

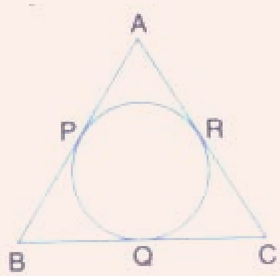
Hence, AB + CD = AD + BC

Solution 8:



From A, AP and AS are tangents to the circle.
Therefore, AP = AS.......(i)
Similarly, we can prove that:
BP = BQ ....(ii)
CR = CQ ....(iii)
DR = DS ....(iv)
Adding,
AP + BP + CR + DR = AS + DS + BQ + CQ
AB + CD = AD + BC
Hence, AB + CD = AD + BC
But AB = CD and BC = AD.......(v) Opposite sides of a ||gm
Therefore, AB + AB = BC + BC
2AB = 2 BC
AB = BC ....(vi)
From (v) and (vi)
AB = BC = CD = DA
Hence, ABCD is a rhombus.

Solution 9:



Since from B, BQ and BP are the tangents to the circle
Therefore, BQ = BP ………..(i)
Similarly, we can prove that
AP = AR …………..(ii)
and CR = CQ ………(iii)
Adding,
AP + BQ + CR = BP + CQ + AR ………(iv)
Adding AP + BQ + CR to both sides
2(AP + BQ + CR) = AP + PQ + CQ + QB + AR + CR
2(AP + BQ + CR) = AB + BC + CA

Therefore, AP + BQ + CR = $\frac{1}{2}$ × (AB + BC + CA)

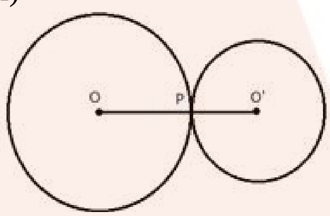
AP + BQ + CR = $\frac{1}{2}$ × perimeter of triangle ABC

Solution 10:

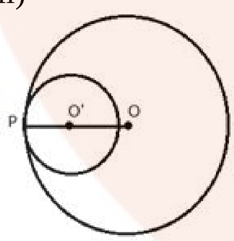
Since, from A, AP and AR are the tangents to the circle
Therefore, AP = AR
Similarly, we can prove that
BP = BQ and CR = CQ
Adding,
AP + BP + CQ = AR + BQ + CR
(AP + BP) + CQ = (AR + CR) + BQ
AB + CQ = AC + BQ
But AB = AC
Therefore, CQ = BQ or BQ = CQ

Solution 11:

Radius of bigger circle = 6.3 cm
and of smaller circle = 3.6 cm
i)

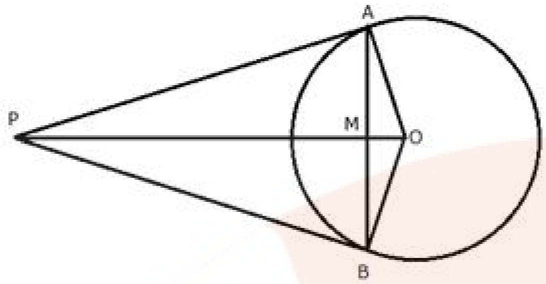


Two circles are touching each other at P externally. O and O' are the centers of the circles. Join OP and $O'P$
OP = 6.3 cm, $O'P$ = 3.6 cm
Adding,
OP + $O'P$ = 6.3 + 3.6 = 9.9 cm
ii)



Two circles are touching each other at P internally. O and O' are the centers of the circles. Join OP and O'P
OP = 6.3 cm, $O'P$ = 3.6 cm
OO' = OP – O'P = 6.3 – 3.6 = 2.7 cm

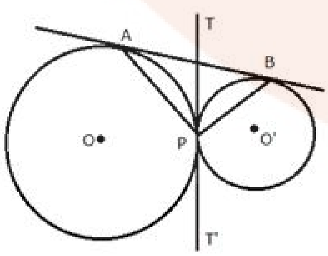
Solution 12:



i) In ΔAOP and ΔBOP
AP = BP (Tangents from P to the circle)
OP = OP (Common)
OA = OB (Radii of the same circle)
$\therefore$ By Side – Side – Side criterion of congruence,
$\Delta AOP \cong \Delta BOP$
The corresponding parts of the congruent triangle are congruent
$\Rightarrow \angle AOP = \angle BOP$ [by c.p.c.t]

ii) In ΔOAM and ΔOBM
OA = OB (Radii of the same circle)
$\angle AOM = \angle BOM$ (Proved $\angle AOP = \angle BOP$)
OM = OM (Common)
$\therefore$ By side – Angle – side criterion of congruence,
$\Delta OAM \cong \angle OBM$
The corresponding parts of the congruent triangles are congruent.
$\Rightarrow AM = MB$
And $\angle OMA = \angle OMB$
But,
$\angle OMA + \angle OMB = 180°$
$\therefore \angle OMA = \angle OMB = 90°$
Hence, OM or OP is the perpendicular bisector of chord AB.

Solution 13:

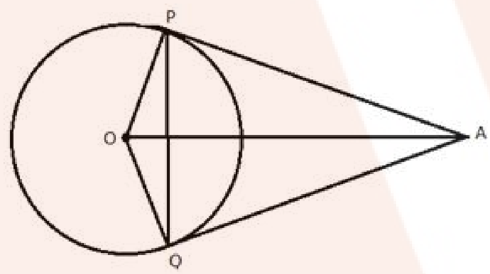


Draw TPT' as common tangent to the circles.

i) TA and TP are the tangents to the circle with centre O.
Therefore, TA = TP ………(i)
Similarly, TP = TB ………..(ii)
From (i) and (ii)
TA = TB
Therefore, TPT' is the bisector of AB.

ii) Now in ΔATP,
$\therefore \angle TAP = \angle TPA$
Similarly in $\Delta BTP, \angle TBP = \angle TPB$
Adding,
$\angle TAP + \angle TBP = \angle APB$
But
$\therefore TAP + \angle TBP + \angle APB = 180°$
$\Rightarrow \angle APB = \angle TAP + \angle TBP = 90°$

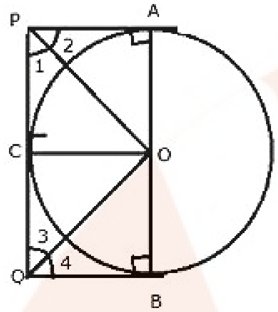
Solution 14:



In quadrilateral OPAQ,
$\angle OPA = \angle OQA = 90°$
$(\because OP \perp PA \text{ and } OQ \perp QA)$
$\therefore \angle POQ + \angle PAQ + 90° + 90° = 360°$
$\Rightarrow \angle POQ + \angle PAQ = 360° - 180° = 180°$ ………(i)

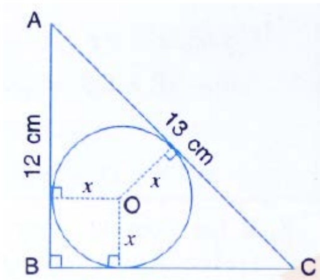
In triangle OPQ,
OP = OQ (Radii of the same circle)
$\therefore OPQ = \angle OQP$
But
$\angle POQ + \angle OPQ + \angle OQP = 180°$
$\Rightarrow \angle POQ + \angle OPQ + \angle OPQ = 180°$
$\Rightarrow \angle POQ + 2\angle OPQ = 180°$ ....(ii)
From (i) and (ii)
$\angle POQ + \angle PAQ = \angle POQ + 2\angle OPQ$
$\Rightarrow \angle PAQ = 2\angle OPQ$

Solution 15:



Join OP, OQ, OA, OB and OC.
In ΔOAP and ΔOCP
OA = OC (Radii of the same circle)
OP = OP (Common)
PA = PC (Tangents from P)
∴ By side – side – side criterion of congruence,
$\Delta OAP \cong \Delta OCP$ (SSS postulate)
The corresponding parts of the congruent triangles are congruent.
$\Rightarrow \angle APO = \angle CPO$ (cpct) …………(i)
Similarly, we can prove that
$\therefore \Delta OCQ \cong \Delta OBQ$
$\Rightarrow \angle CQO = \angle BQO$ …………..(ii)
$\therefore \angle APC = 2\angle CPO$ and $\angle CQB = 2\angle CQO$
But,
$\angle APC + \angle CQB = 180°$
(Sum of interior angles of a transversal)
$\therefore 2\angle CPO + 2\angle CQO = 180°$
$\Rightarrow \angle CPO + \angle CQO = 90°$
Now in ΔPOQ,
$\angle CPO + \angle CQO + \angle POQ = 180°$
$\Rightarrow 90° + \angle POQ = 180°$
$\therefore \angle POQ = 90°$

Solution 16:



In $\Delta ABC, \angle B = 90^\circ$

$OL \perp AB, OM \perp BC$ and $ON \perp AC$

LBNO is a square

LB = BN = OL = OM = ON = x

∴ AL = 12 – x

∴ AL = AN = 12 – x

Since ABC is a right triangle

$AC^2 = AB^2 + BC^2$

$\Rightarrow 13^2 = 12^2 + BC^2$

$\Rightarrow 169 = 144 + BC^2$

$\Rightarrow BC^2 = 25$

$\Rightarrow BC = 5$

$\therefore MC = 5 - X$

But $CM = CN$

$\therefore\ CN = 5 - X$

Now, AC = AN + NC

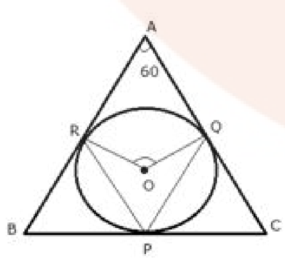
$13 = (12 - x) + (5 - x)$

$13 = 17 - 2x$

$2x = 4$

$x = 2$ cm

Solution 17:



The incircle touches the sides of the triangle ABC and

$OP \perp BC, OQ \perp AC, OR \perp AB$

i) In quadrilateral AROQ,

$\angle ORA = 90^\circ, \angle OQA = 90^\circ, \angle A = 60^\circ$

$\angle QOR = 360^\circ - \left(90^\circ + 90^\circ + 60^\circ\right)$

$\angle QOR = 360^\circ - 240^\circ$

$\angle QOR = 120^\circ$

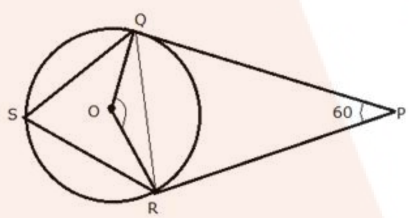
ii) Now arc RQ subtends $\angle QOR$ at the centre and $\angle QPR$ at the remaining part of the circle.

$\therefore \angle QPR = \frac{1}{2} \angle QOR$

$\Rightarrow \angle QPR = \frac{1}{2} \times 120^\circ$

$\Rightarrow \angle QPR = 60^\circ$

Solution 18:



Join QR.

i) In quadrilateral ORPQ,

$OQ \perp OP, OR \perp RP$

$\therefore \angle OQP = 90^\circ, \angle ORP = 90^\circ, \angle QPR = 60^\circ$

$\angle QOR = 360^\circ - \left(90^\circ + 90^\circ + 60^\circ\right)$

$\angle QOR = 360^\circ - 240^\circ$

$\angle QOR = 120^\circ$

ii) In $\angle QOR$,

OQ = QR (Radii of the same circle)

$\therefore$ OQR = $\angle QRO$ ………….(i)

But, $\angle OQR + \angle QRO + \angle QOR = 180^\circ$

$\angle OQR + \angle QRO + 120^\circ = 180^\circ$

$\angle OQR + \angle QRO = 60^\circ$

From (i)

$2\angle OQR = 60^\circ$

$\angle OQR = 30^\circ$

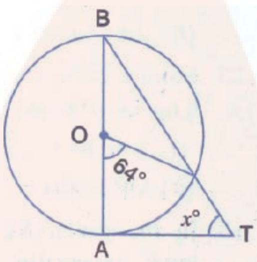
iii) Now arc RQ subtends $\angle QOR$ at the centre and $\angle QSR$ at the remaining part of the circle.

$\therefore \ \angle QSR = \frac{1}{2}\angle QOR$

$\Rightarrow \angle QSR = \frac{1}{2}\times 120^\circ$

$\Rightarrow \angle QSR = 60^\circ$

Solution 19:



In ΔOBC,

OB = OC (Radii of the same circle)

$\therefore \angle OBC = \angle OCB$

But, Ext. $\angle COA = \angle OBC + \angle OCB$

Ext. $\angle COA = 2\angle OBC$

$\Rightarrow 64^\circ = 2\angle OBC$

$\Rightarrow \angle OBC = 32^\circ$

Now in ΔABT

$\angle BAT = 90^\circ \left(OA \perp AT\right)$

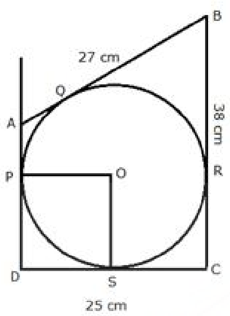
$\angle OBC$ or $\angle ABT = 32^\circ$

$\therefore \angle BAT + \angle ABT + x^\circ = 180^\circ$

$\Rightarrow 90^\circ + 32^\circ + x^\circ = 180^\circ$

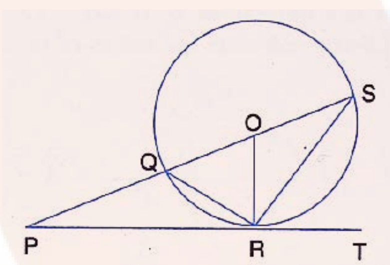
$\Rightarrow x^\circ = 58^\circ$

Solution 20:



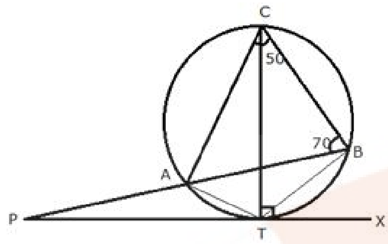
BQ and BR are the tangents from B to the circle.
Therefore, BR =BQ = 27 cm.
Also RC = (38 −; 27) = 11cm
Since CR and CS are the tangents from C to the circle
Therefore, CS = CR = 11 cm
So, DS = (25 − 11) = 14 cm
Now DS and DP are the tangents to the circle
Therefore, DS = DP
Now, $\angle PDS = 90^\circ$ (given)
and $OP \perp AD, OS \perp DC$
therefore, radius = DS = 14 cm

Solution 21:



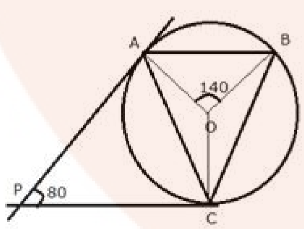
$\angle QRP = \angle OSR = y$ (angles in alternate segment)
But OS = OR (Radii of the same circle)
$\therefore \angle ORS = \angle OSR = y$
$\therefore OQ = OR$ (radii of same circle)
$\therefore \angle OQR = \angle ORQ = 90^\circ - y$ …………(i) (Since $OR \perp PT$)
But in ΔPQR,
Ext $\angle OQR = x + y$ ……(i)
From (i) and (ii)
$x + y = 90^\circ - y$
$\Rightarrow x + 2y = 90^\circ$

Solution 22:



Join AT and BT.
i) TC is the diameter of the circle
$\therefore \angle CBT = 90^\circ$ (Angle in a semi – circle)
(ii) $\angle CBA = 70^\circ$
$\therefore \angle ABT = \angle CBT - \angle CBA = 90° - 70° = 20°$
Now, $\angle ACT = \angle ABT = 20^\circ$ (Angle in the same segment of the circle)
$\therefore \angle TCB = \angle ACB - \angle ACT = 50° - 20° = 30°$
But, $\angle TCB = \angle TAB$ (Angles in the same segment of the circle)
$\therefore \angle TAB$ or $\angle BAT = 30^\circ$
(iii) $\angle BTX = \angle TCB = 30^\circ$ (Angles in the same segment)
$\therefore \angle PTB = 180^\circ - 30^\circ = 150^\circ$
Now in ΔPTB
$\angle APT + \angle PTB + \angle ABT = 180°$
$\Rightarrow \angle APT + 150^\circ + 20^\circ = 180^\circ$
$\Rightarrow \angle APT = 180^\circ - 170^\circ = 10^\circ$

Solution 23:



Join OC.
Therefore, PA and PA are the tangents
$\therefore OA \perp PA$ and $OC \perp PC$
In quadrilateral APCO,
$\angle APC + AOC = 180^\circ$
$\Rightarrow 80^\circ + \angle AOC = 180^\circ$
$\Rightarrow \angle AOC = 100^\circ$

$\angle BOC = 360^\circ - (\angle AOB + \angle AOC)$

$\angle BOC = 360^\circ - (140^\circ + 100^\circ)$

$\angle B = 360^\circ - 240^\circ = 120^\circ$

Now, arc BC subtends $\angle BOC$ at the centre and $\angle BAC$ at the remaining part of the circle

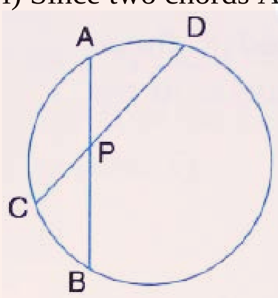
$\therefore \angle BAC = \frac{1}{2}\angle BOC$

$\angle BAC = \frac{1}{2} \times 120^\circ = 60^\circ$

EXERCISE. 18 (B)

Solution 1:

i) Since two chords AB and CD intersect each other at P.

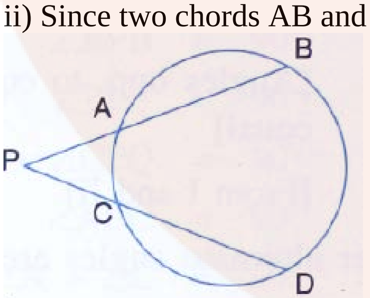


$\therefore AP \times PB = CP \times PD$

$\Rightarrow 4.5 \times PB = 3 \times 9 (3CP = 9cm \Rightarrow CP = 3cm)$

$\Rightarrow PB = \frac{3 \times 9}{4.5} = 6$ cm

ii) Since two chords AB and CD intersect each other at P.



$\therefore AP \times PB = CP \times PD$

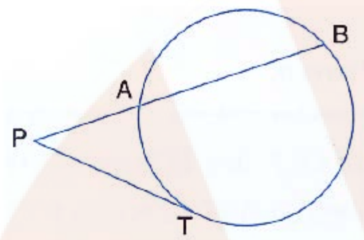
But $5 \times PA = 3 \times AB = 30$ cm

$\therefore 5 \times PA = 30$ cm $\Rightarrow PA = 6$ cm

And $3 \times AB = 30$cm $\Rightarrow AB = 10$cm

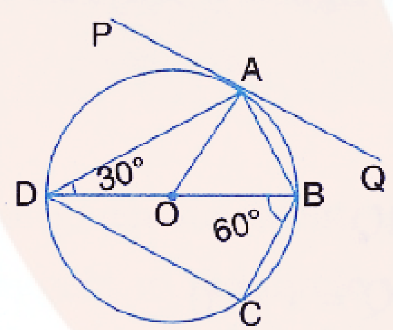
$\Rightarrow BP = PA + AB = 6 + 10 = 16$ cm

Now,
$AP \times PB = CP \times PD$
$\Rightarrow 6 \times 16 = 4 \times PD$
$\Rightarrow PD = \frac{6 \times 16}{4} = 24 \text{ cm}$
CD = PD – PC = 24 – 4 = 20 cm
iii) Since PAB is the secant and PT is the tangent



$\therefore PT^2 = PA \times PB$
$\Rightarrow 12.5^2 = 10 \times PB$
$\Rightarrow PB = \frac{12.5 \times 12.5}{10} = 15.625 \text{ cm}$
$AB = PB - PA = 15.625 - 10 = 5.625 \text{ cm}$

Solution 2:

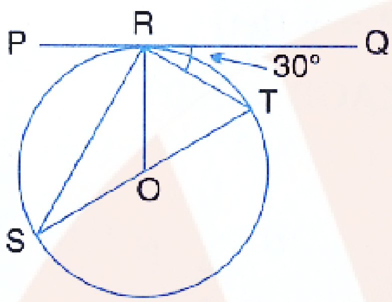


i) PAQ is a tangent and AB is the chord.
$\angle QAB = \angle ADB = 30^\circ$ (angles in the alternate segment)
ii) OA = OD (radii of the same circle)
$\therefore \angle OAD = \angle ODA = 30^\circ$
But, $OA \perp PQ$
$\therefore \angle PAD = \angle OAP - \angle OAD = 90^\circ - 30^\circ = 60^\circ$
iii) BD is the diameter.
$\therefore \angle BCD = 90^\circ$ (angle in a semi – circle)
Now in ΔBCD,
$\angle CDB + \angle CBD + \angle BCD = 180^\circ$

$\Rightarrow \angle CDB + 60^\circ + 90^\circ = 180^\circ$

$\Rightarrow \angle CDB = 180^\circ - 150^\circ = 30^\circ$

Solution 3:



PQ is a tangent and OR is the radius.

$\therefore$ OR $\perp$ PQ

$\therefore \angle ORT = 90^\circ$

$\Rightarrow \angle TRQ = 90^\circ - 30^\circ = 60^\circ$

But in ΔOTR,

OT = OR (Radii of the same circle)

$\therefore \angle OTR = 60^\circ$ Or $\angle STR = 60^\circ$

But,

$\angle PRS = \angle STR = 60^\circ$ (Angle in the alternate segment)

In ΔORT,

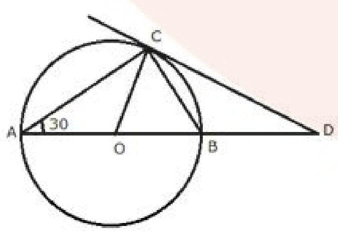
$\angle ORT = 60^\circ$

$\angle OTR = 60^\circ$

$\therefore \angle ROT = 180^\circ - \left(60^\circ + 60^\circ\right)$

$\angle ROT = 180^\circ - 120^\circ = 60^\circ$

Solution 4:



Join OC,

$\angle BCD = \angle BAC = 30^\circ$ (angles in alternate segment)

Arc BC subtends $\angle DOC$ at the centre of the circle and $\angle BAC$ at the remaining part of the circle.

$\therefore \angle BOC = 2\angle BAC = 2\times 30^\circ = 60^\circ$

Now in ΔOCD,

$\angle BOC$ or $\angle DOC = 60^\circ$

$\angle OCD = 90^\circ \left(OC \perp CD\right)$

$\therefore \angle DCO + \angle ODC = 90^\circ$

$\Rightarrow 60^\circ + \angle ODC = 90^\circ$

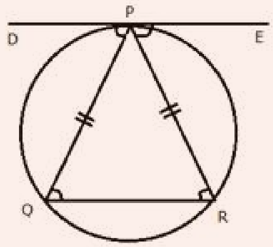
$\Rightarrow \angle ODC = 90^\circ - 60^\circ = 30^\circ$

Now in ΔBCD,

$\because \angle ODC$ or $\angle BDC = \angle BCD = 30^\circ$

$\therefore BC = BD$

Solution 5:



DE is the tangent to the circle at P.

DE ∥ QR (Given)

$\angle EPR = \angle PRQ$ (Alternate angles are equal)

$\angle DPQ = \angle PQR$ (Alternate angles are equal) ….. (i)

Let $\angle DPQ = x$ and $\angle EPR = y$

Since the angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment

$\therefore \angle DPQ = \angle PRQ$ ……….. (ii) (DE is tangent and PQ is chord)

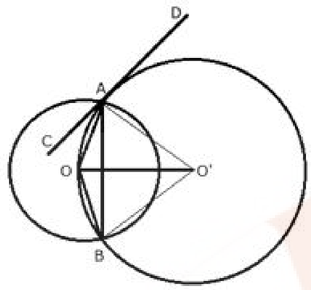
from (i) and (ii)

$\angle PQR = \angle PRQ$

$\Rightarrow PQ = PR$

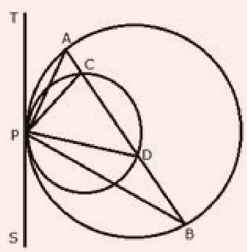
Hence, triangle PQR is an isosceles triangle.

Solution 6:



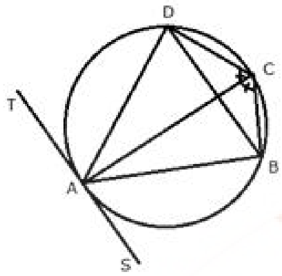
Join OA, OB, O'A, O'B and O'O.
CD is the tangent and AO is the chord.
$\angle OAC = \angle OBA$ (angles in alternate segment)
In ΔOAB,
OA = OB (Radii of the same circle)
$\therefore OAB = \angle OBA$ …..(ii)
From (i) and (ii)
$\angle OAC = \angle OAB$
Therefore, OA is bisector of $\angle BAC$

Solution 7:



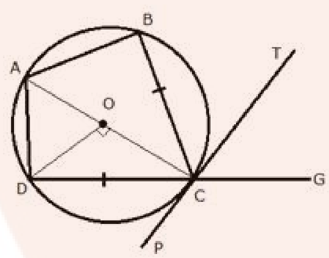
Draw a tangent TS at P to the circles given.
Since TPS is the tangent, PD is the chord.
$\therefore \angle PAB = \angle BPS$ ……..(i) (Angles in alternate segment)
Similarly,
$\angle PCD = \angle DPS$ …… (ii)
Subtracting (i) from (ii)
$\angle PCD - \angle PAB = \angle DPS - \angle BPS$
But in ΔPAC,
Ext. $\angle PCD = \angle PAB + \angle CPA$
$\therefore \angle PAB + \angle CPA - \angle PAB = \angle DPS - \angle BPS$
$\Rightarrow \angle CPA = \angle DPB$

Solution 8:



$\angle ADB = \angle ACB$ …… (i) (Angles in same segement)
Similarly ,
$\angle ABD = \angle ACD$ …….. (ii)
But, $\angle ACB = \angle ACD$ (AC is bisector of $\angle BCD$)
$\therefore \angle ADB = \angle ABD$ (from (i) and (ii))
TAS is a tangent and AB is a chord
$\therefore \angle BAS = \angle ADB$ (angles in alternate segment)
But, $\angle ADB = \angle ABD$
$\therefore \angle BAS = \angle ABD$
But these are alternate angles
Therefore, TS ∥ BD

Solution 9:



Join OC, OD and AC
i)
$\angle BCG + \angle BCD = 180^\circ$ (Linear pair)
$\Rightarrow 180^\circ + \angle BCD = 180^\circ$
$\Rightarrow \angle BCD = 180^\circ - 180^\circ = 72^\circ$
$BC = CD$
$\therefore \angle DCP = \angle BCT$
But, $\angle BCT + \angle BCD + \angle DCP = 180^\circ$
$\therefore \angle BCT + \angle BCT + 72^\circ = 180^\circ$
$2\angle BCT = 180^\circ - 72^\circ$
$\angle BCT = 54^\circ$
ii)

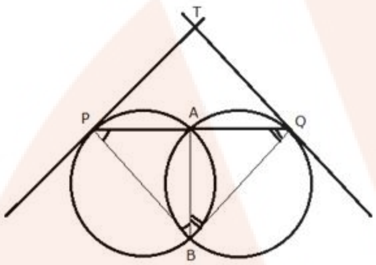
PCT is a tangent and CA is a chord.

$\angle CAD = \angle BCT = 54^\circ$

But arc DC subtends $\angle DOC$ at the centre and $\angle CAD$ at the remaining part of the circle.

$\therefore \angle DOC = 2\angle CAD = 2 \times 54^\circ = 108^\circ$

Solution 10:



Join AB, PB and BQ

TP is the tangent and PA is a chord

$\therefore \angle TPA = \angle ABP$ …… (i) (angles in alternate segment)

Similarly,

$\angle TQA = \angle ABQ$ ……. (ii)

Adding (i) and (ii)

$\angle TPA + \angle TQA = \angle ABP + \angle ABQ$

But, ΔPTQ,

$\angle TPA + \angle TQA + \angle PTQ = 180^\circ$

$\Rightarrow \angle PBQ = 180^\circ - \angle PTQ$

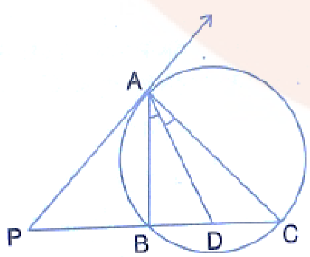
$\Rightarrow \angle PBQ + \angle PTQ = 180^\circ$

But they are the opposite angles of the quadrilateral

Therefore, PBQT are cyclic.

Hence, P, B, Q and T are concyclic.

Solution 11:



i) PA is the tangent and AB is a chord

$\therefore \angle PAB = \angle C$ …….. (i) (angles in the alternate segment)

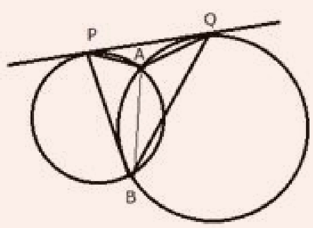
AD is the bisector of $\angle BAC$
$\therefore \angle 1 = \angle 2$ ……….(ii)
In ΔADC,
$\text{Ext.}\angle ADP = \angle C + \angle 1$
$\Rightarrow \text{Ext } \angle ADP = \angle PAB + \angle 2 = \angle PAD$
Therefore, ΔPAD is an isosceles triangle.

ii) In ΔABC,
Ext. $\angle PBA = \angle C + \angle BAC$
$\angle BAC = \angle PBA - \angle C$
$\Rightarrow \angle 1 + \angle 2 = \angle PBA - \angle PAB$
(from (i) part)
$2\angle 1 = \angle PBA - \angle PAB$

$$\angle 1 = \frac{1}{2}(\angle PBA - \angle PAB)$$

$$\Rightarrow \angle CAD = \frac{1}{2}(\angle PBA - \angle PAB)$$

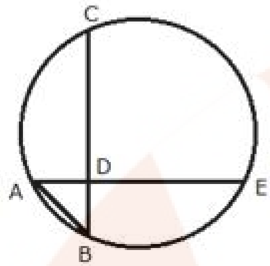
Solution 12:



Join AB.
PQ is the tangent and AB is a chord
$\therefore \angle QPA = \angle PBA$ …………(i) (angles in alternate segment)
Similarly,
$\angle PQA = \angle QBA$ ………… (ii)
Adding (i) and (ii)
$\angle QPA + \angle PQA = \angle PBA + \angle QBA$
But, in ΔPAQ,
$\angle QPA + \angle PQA = 180^\circ - \angle PAQ$ …… (iii)
And $\angle PBA + \angle QBA = \angle PBQ$ ……..(iv)
From (iii) and (iv)
$\angle PBQ = 180^\circ - \angle PAQ$
$\Rightarrow \angle PBQ + \angle PAQ = 180^\circ$
$\Rightarrow \angle PBQ + \angle PBQ = 180^\circ$

Hence $\angle PAQ$ and $\angle PBQ$ are supplementary

Solution 13:



Join AB.

i) In Rt. ΔADB,

$AB^2 = AD^2 + DB^2$

$5^2 = AD^2 + 4^2$

$AD^2 = 25 - 16$

$AD^2 = 9$

$AD = 3$

Chords AE and CB intersect each other at D inside the circle

AD x DE = BD x DC

3 × DE = 4 × 9

DE = 12 cm

ii) If AD = BD ....(i)

We know that:

AD × DE = BD × DC

But AD = BD

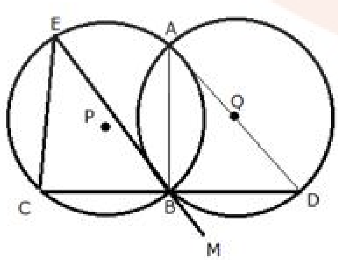
Therefore, DE = DC ....(ii)

Adding (i) and (ii)

AD + DE = BD + DC

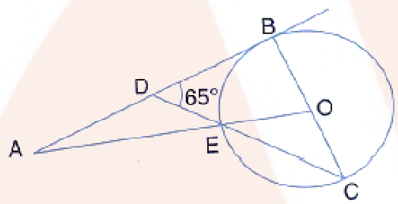
Therefore, AE = BC

Solution 14:



Join AB and AD
EBM is a tangent and BD is a chord.
$\angle DBM = \angle BAD$ (angles in alternate segments)
But, $\angle DBM = \angle CBE$ (Vertically opposite angles)
$\therefore \angle BAD = \angle CBE$
Since in the same circle or congruent circles, if angles are equal, then chords opposite to them are also equal.
Therefore, CE = BD

Solution 15:



AB is a straight line.
$\therefore \angle ADE + \angle BDE = 180^\circ$

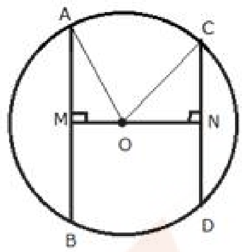
$\Rightarrow \angle ADE + 65^\circ = 180^\circ$
$\Rightarrow \angle ADE = 115^\circ$ ....(i)
AB i.e. DB is tangent to the circle at point B and BC is the diameter.
$\therefore \angle DB\angle = 90^\circ$
In ΔBDC,
$\angle DBC + \angle BDC + \angle DCB = 180^\circ$

$\Rightarrow 90^\circ + 65^\circ + \angle DCB = 180^\circ$

$\Rightarrow \angle DCB = 25^\circ$
Now, OE = OC (radii of the same circle)
$\therefore \angle DCB$ or $\angle OCE = \angle OEC = 25^\circ$
Also,
$\angle OEC = \angle DEC = 25^\circ$
(vertically opposite angles)
In ΔADE,
$\angle ADE + \angle DEA + \angle DAE = 180^\circ$
From (i) and (ii)
$115^\circ + 25^\circ + \angle DAE = 180^\circ$
$\Rightarrow \angle DAE$ or $\angle BAO = 180^\circ - 140^\circ = 40^\circ$
$\therefore \angle BAO = 40^\circ$

EXERCISE. 18 (C)

Solution 1:



Given: A circle with centre O and radius r. $OM \perp AB$ and $ON \perp CD$ Also AB > CD
To prove: OM < ON
Proof: Join OA and OC.
In Rt. ΔAOM,

$$AO^2 = AM^2 + OM^2$$

$$\Rightarrow r^2 = \left(\frac{1}{2}AB\right)^2 + OM^2$$

$$\Rightarrow r^2 = \frac{1}{4}AB^2 + OM^2 \quad \text{..........(i)}$$

Again in Rt. ΔONC,

$$OC^2 = NC^2 + ON^2$$

$$\Rightarrow r^2 = \left(\frac{1}{2}CD\right)^2 + ON^2$$

$$\Rightarrow r^2 = \frac{1}{4}CD^2 + ON^2 \quad \text{..........(ii)}$$

From (i) and (ii)

$$\frac{1}{4}AB^2 + OM^2 = \frac{1}{4}CD^2 + ON^2$$

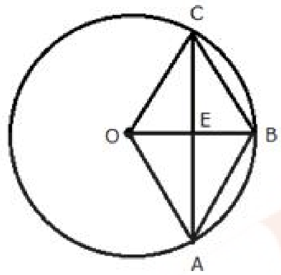
But, AB > CD (given)

$\therefore$ ON > OM

$\Rightarrow$ OM < ON

Hence, AB is nearer to the centre than CD.

Solution 2:



i) Radius = 10 cm
In rhombus OABC,
OC = 10 cm

$\therefore \text{OE} = \frac{1}{2} \times \text{OB} = \frac{1}{2} \times 10 = 5 \text{ cm}$

In Rt. ΔOCE,

$\text{OC}^2 = \text{OE}^2 + \text{EC}^2$

$\Rightarrow 10^2 = 5^2 + \text{EC}^2$

$\Rightarrow \text{EC}^2 = 100 - 25 = 75$

$\Rightarrow \text{EC} = 5\sqrt{3}$

$\therefore \text{AC} = 2 \times \text{EC} = 2 \times 5\sqrt{3} = 10\sqrt{3}$

Area of rhombus = $\frac{1}{2} \times \text{OB} \times \text{AC}$

$= \frac{1}{2} \times 10 \times 10\sqrt{3}$

$= 50\sqrt{3}\ \text{cm}^2 \approx 86.6\ \text{cm}^2\ (\sqrt{3} = 1.73)$

(ii) Area of rhombus = $32\sqrt{3}\ \text{cm}^2$
But area of rhombus OABC = 2 x area of ΔOAB

Area of rhombus OABC $= 2 \times \frac{\sqrt{3}}{4} r^2$

Where r is the side of the equilateral triangle OAB.

$2 \times \frac{\sqrt{3}}{4} r^2 = 32\sqrt{3}$

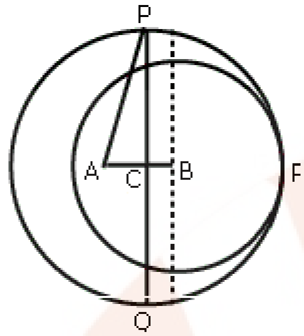
$\Rightarrow \frac{\sqrt{3}}{2} r^2 = 32\sqrt{3}$

$\Rightarrow r^2 = 64$

$\Rightarrow r = 8$

Therefore, radius of the circle = 8 cm

Solution 3:



If two circles touch internally, then distance between their centres is equal to the difference of their radii. So, AB = (5 − 3) cm = 2 cm.

Also, the common chord PQ is the perpendicular bisector of AB. Therefore, AC = CB = $\frac{1}{2}$ AB = 1 cm

In right ΔACP, we have $AP^2 = AC^2 + CP^2$

$\Rightarrow 5^2 = 1^2 + CP^2$

$\Rightarrow CP^2 = 25 −; 1 = 24$

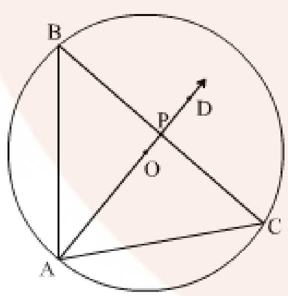
$\Rightarrow CP = \sqrt{24} = 2\sqrt{6}$ cm

Now, PQ = 2 CP

= 2 × $2\sqrt{6}$ cm

= $4\sqrt{6}$ cm

Solution 4:



Given: AB and AC are two equal chords of C (O, *r*).

To prove: Centre, O lies on the bisector of ∠BAC.

Construction: Join BC. Let the bisector of ∠BAC intersects BC in P.

Proof:

In ΔAPB and ΔAPC,

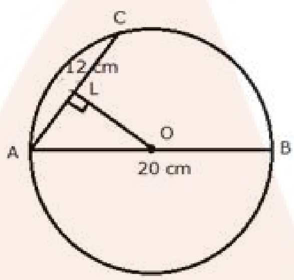
AB = AC (Given)

∠ BAP = ∠CAP (Given)

AP = AP (Common)

$\therefore \Delta APB \cong \Delta APC$ (SAS congruence criterion)
$\Rightarrow$ BP = CP and ∠APB = ∠APC (CPCT)
∠APB + ∠APC = 180° (Linear pair)
$\Rightarrow$ 2 ∠APB = 180° (∠APB = ∠APC)
$\Rightarrow$∠APB = 90°
Now, BP = CP and ∠APB = 90°
∴ AP is the perpendicular bisector of chord BC.
$\Rightarrow$AP passes through the centre, O of the circle.

Solution 5:

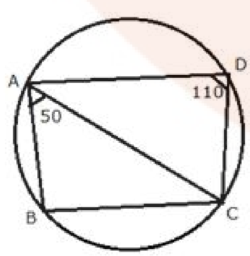


AB is the diameter and AC is the chord.
Draw $OL \perp AC$
Since $OL \perp AC$ and hence it bisects AC, O is the centre of the circle.
Therefore, OA = 10 cm and AL = 6 cm
Now, in Rt. ΔOLA,

$$AO^2 = AL^2 + OL^2$$
$$\Rightarrow 10^2 = 6^2 + OL^2$$
$$\Rightarrow OL^2 = 100 - 36 = 64$$
$$\Rightarrow OL = 8 \text{ cm}$$

Therefore, chord is at a distance of 8 cm from the centre of the circle.

Solution 6:



ABCD is a cyclic quadrilateral in which AD||BC

$$\angle ADC = 110^\circ, \angle BAC = 50^\circ$$
$$\angle B + \angle D = 180^\circ$$

(Sum of opposite angles of a quadrilateral)

$\Rightarrow \angle B + 110^\circ = 180^\circ$

$\Rightarrow \angle B = 70^\circ$

Now in ΔABC,

$\angle BAC + \angle ABC + \angle ACB = 180^\circ$

$\Rightarrow 50^\circ + 70^\circ + \angle ACB = 180^\circ$

$\Rightarrow \angle ACB = 180^\circ - 120^\circ = 60^\circ$

$\because AD \parallel BC$

$\therefore \angle DAC = \angle ACB = 60^\circ$ (alternate angles)

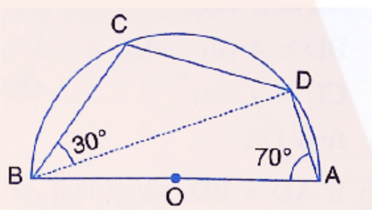
Now in ΔADC,

$\angle DAC + \angle ADC + \angle DCA = 180^\circ$

$\Rightarrow 60^\circ + 110^\circ + \angle DCA = 180^\circ$

$\Rightarrow \angle DCA = 180^\circ - 170^\circ = 10^\circ$

Solution 7:



Since ABCD is a cyclic quadrilateral, therefore, $\angle BCD + \angle BAD = 180°$

(since opposite angles of a cyclic quadrilateral are supplementary)

$\Rightarrow \angle BCD + 70° = 180°$

$\Rightarrow \angle BCD = 180° - 70° = 110°$

In ΔBCD, we have,

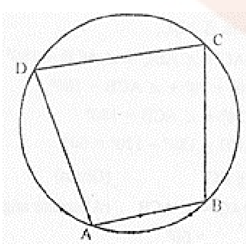
$\angle CBD + \angle BCD + \angle BDC = 180°$

$\Rightarrow 30° + 110° + \angle BDC = 180°$

$\Rightarrow \angle BDC = 180° - 140°$

$\Rightarrow \angle BDC = 40°$

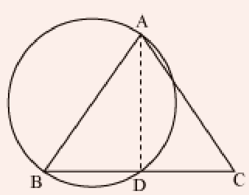
Solution 8:



ABCD is a cyclic quadrilateral.

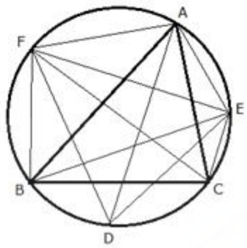
$\therefore \angle A + \angle C = 180^\circ$
$\Rightarrow 3\angle C + \angle C = 180^\circ$
$\Rightarrow 4\angle C = 180^\circ$
$\Rightarrow \angle C = 45^\circ$
$\because \angle A = 3\angle C$
$\Rightarrow \angle A = 3 \times 45^\circ$
$\Rightarrow \angle A = 135^\circ$
Similarly,
$\therefore \angle B + \angle D = 180^\circ$
$\Rightarrow \angle B + 5\angle B = 180^\circ$
$\Rightarrow 6\angle B = 180^\circ$
$\Rightarrow \angle B = 30^\circ$
$\because \angle D = 5\angle B$
$\Rightarrow \angle D = 5 \times 30^\circ$
$\Rightarrow \angle D = 150^\circ$
Hence, $\angle A = 135^\circ, \angle B = 30^\circ, \angle C = 45^\circ, \angle D = 150^\circ$

Solution 9:



Join AD.
AB is the diameter.
$\therefore \angle ADB = 90^\circ$ (Angle in a semi-circle)
But, $\angle ADB + \angle ADC = 180^\circ$ (linear pair)
$\Rightarrow \angle ADC = 90^\circ$
In ΔABD and ΔACD,
$\angle ADB = \angle ADC$ (each 90°)
AB = AC (Given)
AD = AD (Common)
$\Delta ABD \cong \Delta ACD$ (RHS congruence criterion)
$\Rightarrow$ BD = DC (C.P.C.T)
Hence, the circle bisects base BC at D.

Solution 10:



Join ED, EF and DF. Also join BF, FA, AE and EC.

$\angle EBF = \angle ECF = \angle EDF$ …………(i) (angles in the same segment)

In cyclic quadrilateral AFBE,

$\angle EBF + \angle EAF = 180^\circ$ ……………(ii) (sum of opposite angles)

Similarly in cyclic quadrilateral CEAF,

$\angle EAF + \angle ECF = 180^\circ$ …………. (iii)

Adding (ii) and (iii)

$\Rightarrow \angle EDF + \angle ECF + 2\angle EAF = 360^\circ$

$\Rightarrow \angle EDF + \angle EDF + 2\angle EAF = 360^\circ$ (from (i))

$\Rightarrow 2\angle EDF + 2\angle EAF = 360^\circ$

$\Rightarrow \angle EDF + \angle EAF = 180^\circ$

$\Rightarrow \angle EDF + \angle 1 + \angle BAC + \angle 2 = 180^\circ$

But $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$

(angles in the same segment)

$\therefore \angle EDF + \angle 3 + \angle BAC + \angle 4 = 180^\circ$

But $\angle 4 = \frac{1}{2}\angle C, \angle 3 = \frac{1}{2}\angle B$

$\therefore \angle EDF + \frac{1}{2}\angle B + \angle BAC + \frac{1}{2}\angle C = 180^\circ$

$\Rightarrow \angle EDF + \frac{1}{2}\angle B + 2\times\frac{1}{2}\angle A + \frac{1}{2}\angle C = 180^\circ$

$\Rightarrow \angle EDF + \frac{1}{2}(\angle A + \angle B + \angle C) + \frac{1}{2}\angle A = 180^\circ$

$\Rightarrow \angle EDF + \frac{1}{2}(180^\circ) + \frac{1}{2}\angle A = 180^\circ$

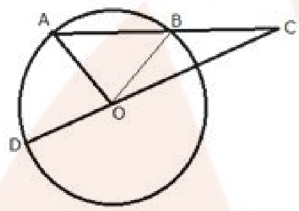
$\Rightarrow \angle EDF + 90^\circ + \frac{1}{2}\angle A = 180^\circ$

$\Rightarrow \angle EDF = 180^\circ - \left(90^\circ + \frac{1}{2}\angle A\right)$

$\Rightarrow \angle EDF = 180^\circ - 90^\circ \frac{1}{2}\angle A$

$\Rightarrow \angle EDF = 90^\circ - \frac{1}{2}\angle A$

Solution 11:



Join OB,

In ΔOBC,

BC = OD = OB (Radii of the same circle)

$\therefore \angle BOC = \angle BCO = 20^\circ$

And Ext. $\angle ABO = \angle BCO + \angle BOC$

$\Rightarrow$ Ext.. $\angle ABO = 20^\circ + 20^\circ = 40^\circ$ …… (i)

In ΔOAB,

$OA = OB$ (radii of the same circle)

$\therefore \angle OAB = \angle OBA = 40^\circ$ (from (i))

$\angle AOB = 180^\circ - \angle OAB - \angle OBA$

$\Rightarrow \angle AOB = 180^\circ - 40^\circ - 40^\circ = 100^\circ$

Since DOC is a straight line

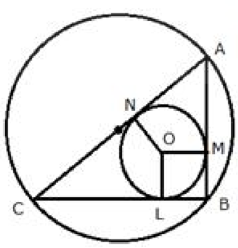
$\therefore \angle AOD + \angle AOB + \angle BOC = 180^\circ$

$\Rightarrow \angle AOD + 100^\circ + 20^\circ = 180^\circ$

$\Rightarrow \angle AOD = 180^\circ - 120^\circ$

$\Rightarrow \angle AOD = 60^\circ$

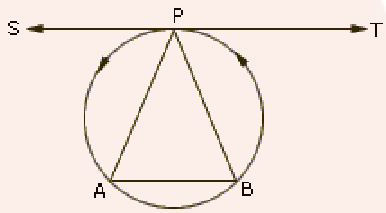
Solution 12:



Join OL, OM and ON.
Let D and d be the diameter of the circumcircle and incircle.
and let R and r be the radius of the circumcircle and incircle.
In circumcircle of ΔABC,

$\angle B = 90^\circ$
Therefore, AC is the diameter of the circumcircle i.e. AC = D
Let radius of the incircle = r
$\therefore OL = OM = ON = r$
Now, from B, BL, BM are the tangents to the incircle.
$\therefore$ BL = BM = r
Similarly,
AM = AN and CL = CN = R
(Tangents from the point outside the circle)
Now,
AB + BC + CA = AM + BM + BL + CL + CA
= AN + r + r + CN + CA
= AN + CN + 2r + CA
= AC + AC + 2r
= 2AC + 2r
= 2D + d

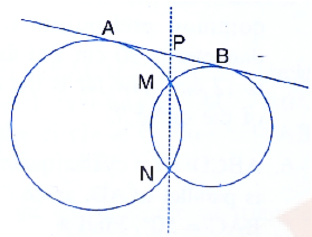
Solution 13:



Join AP and BP.
Since TPS is a tangent and PA is the chord of the circle.
$\angle BPT = \angle PAB$ (angles in alternate segments)
But
$\angle PBA = \angle PAB \left(\because PA = PB\right)$

$\therefore \angle BPT = \angle PBA$
But these are alternate angles
$\therefore TPS \parallel AB$

Solution 14:



From P, AP is the tangent and PMN is the secant for first circle.

$\therefore AP^2 = PM \times PN$ …… (i)

Again from P, PB is the tangent and PMN is the secant for second circle.

$\therefore PB^2 = PM \times PN$ ……..(ii)

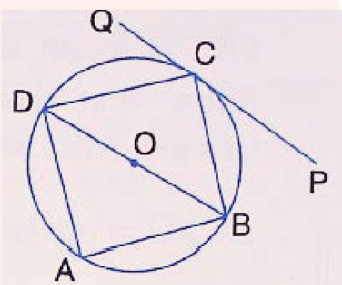
From (i) and (ii)

$AP^2 = PB^2$

$\Rightarrow AP = PB$

Therefore, P is the midpoint of AB.

Solution 15:



i) PQ is tangent and CD is a chord

$\therefore \angle DCQ = \angle DBC$ (angles in the alternate segment)

$\therefore DBC = 40^\circ \left(\because \angle DCQ = 40^\circ\right)$

ii)

$\angle DCQ + \angle DCB + \angle BCP = 180^\circ$

$\Rightarrow 40^\circ + 90^\circ + \angle BCP = 180^\circ \left(\because \angle DCB = 90^\circ\right)$

$\Rightarrow \angle BCP = 180^\circ - 130^\circ = 50^\circ$

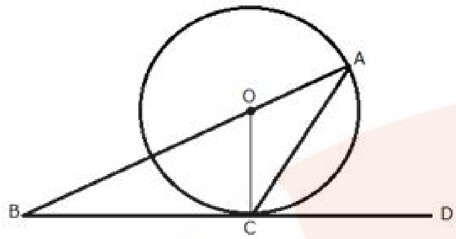
iii) In ΔABD,

$\angle BAD = 90^\circ, \angle ABD = 60^\circ$

$\therefore \angle ADB = 180^\circ - \left(90^\circ + 60^\circ\right)$

$\Rightarrow \angle ADB = 180^\circ - 150^\circ = 30^\circ$

Solution 16:



Join OC.
BCD is the tangent and OC is the radius.
$\therefore OC \perp BD$

$\Rightarrow \angle OCD = 90^\circ$

$\Rightarrow \angle OCA + \angle ACD = 90^\circ$

But in ΔOCA

$OA = OC$ (radii of same circle)

$\therefore \angle OCA = \angle OAC$

Substituting (i)

$\angle OAC + \angle ACD = 90^\circ$

$\Rightarrow \angle BAC + \angle ACD = 90^\circ$

Solution 17:

i) In ΔABC,

$\angle B = 90^\circ$ and BC is the diameter of the circle.
Therefore, AB is the tangent to the circle at B.
Now, AB is tangent and ADC is the secant

$\therefore AB^2 = AD \times AC$

ii) In ΔADB,

$\angle D = 90^\circ$

$\therefore \angle A + \angle ABD = 90^\circ$ ....(i)

But in $\Delta ABC, \angle B = 90^\circ$

$\therefore \angle A + \angle C = 90^\circ$ ....(ii)

From (i) and (ii)

$\angle C = \angle ABD$

Now in ΔABD and ΔCBD

$\angle BDA = \angle BDA = 90^\circ$

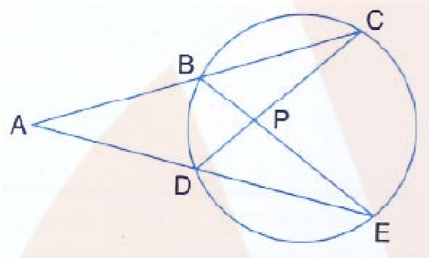
$\angle ABD = \angle BCD$

$\therefore \Delta ABD \sim \Delta CBD$ (AA postulate)

$\therefore \frac{BD}{DC} = \frac{AD}{BD}$

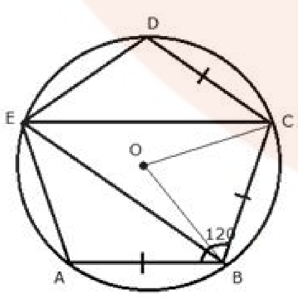
$\Rightarrow BD^2 = AD \times DC$

Solution 18:



In ΔADC and ΔABE,
$\angle ACD = \angle AEB$ (angles in the same segment)
AC = AE (Given)
$\angle A = \angle A$ (common)
$\therefore \Delta ADC \cong \Delta ABE$ (ASA postulate)
$\Rightarrow AB = AD$
But AC = AE
$\therefore AC - AB = AE - AD$
$\Rightarrow BC = DE$
In ΔBPC and ΔDPE
$\angle C = \angle E$ (angles in the same segment)
BC = DE
$\angle CBP = \angle CDE$ (angles in the same segment)
$\therefore \Delta BPC \cong \Delta DPE$ (ASA Postulate)
$\Rightarrow BP = DP$ and $CP = PE$ (cpct)

Solution 19:



i) Join OC and OB.
AB = BC = CD and $\angle ABC = 120^\circ$
$\therefore \angle BCD = \angle ABC = 120^\circ$

OB and OC are the bisectors of $\angle ABC$ and $\angle BCD$ respectively.

$\therefore \angle OBC = \angle BCO = 60^\circ$

In ΔBOC,

$\angle BOC = 180^\circ - (\angle OBC + \angle BOC)$

$\Rightarrow \angle BOC = 180^\circ - (60^\circ + 60^\circ)$

$\Rightarrow \angle BOC = 180^\circ - 120^\circ = 60^\circ$

Arc BC subtends $\angle BOC$ at the centre and $\angle BEC$ at the remaining part of the circle.

$\therefore \angle BEC = \frac{1}{2}\angle BOC = \frac{1}{2} \times 60^\circ = 30^\circ$

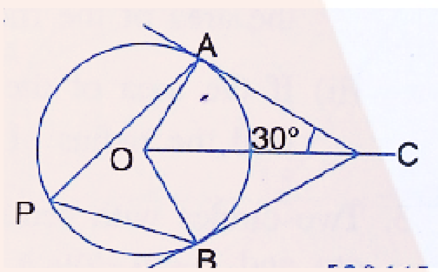
ii) In cyclic quadrilateral BCDE,

$\angle BED + \angle BCD = 180^\circ$

$\Rightarrow \angle BED + 120^\circ = 180^\circ$

$\therefore \angle BED = 60^\circ$

Solution 20:



In the given fig, O is the centre of the circle and CA and CB are the tangents to the circle from C. Also, ∠ACO = 30°

P is any point on the circle. P and PB are joined.

To find: (i) $\angle BCO$

(ii) $\angle AOB$

(iii) $\angle APB$

Proof:

(i) In ΔOAC and OBC

OC = OC (Common)

OA = OB (radius of the circle)

CA = CB (tangents to the circle)

$\therefore \Delta OAC \cong \Delta OBC$ (SSS congruence criterion)

$\therefore \angle ACO = \angle BCO = 30^\circ$

(ii) $\therefore \angle ACB = 30^\circ + 30^\circ = 60^\circ$

$\therefore \angle AOB + \angle ACB = 180^\circ$

$\Rightarrow \angle AOB + 60^\circ = 180^\circ$

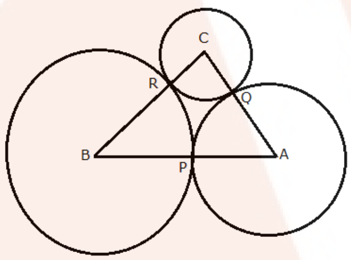
$\Rightarrow \angle AOB = 180^\circ - 60^\circ$

$\Rightarrow \angle AOB = 120^\circ$

(iii) Arc AB subtends $\angle AOB$ at the centre and $\angle APB$ is in the remaining part of the circle.

$\therefore \angle APB = \frac{1}{2}\angle AOB = \frac{1}{2}\times 120^\circ = 60^\circ$

Solution 21:



E

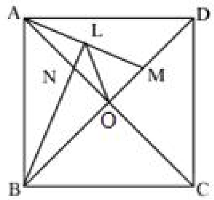
Given: ABC is a triangle with AB = 10 cm, BC= 8 cm, AC = 6 cm. Three circles are drawn with centre A, B and C touch each other at P, Q and R respectively.
We need to find the radii of the three circles.
Let
PA = AQ = x
QC = CR = y
RB = BP = z
∴ x + z = 10 ….. (1)
z + y = 8 ……..(2)
y + x = 6 …….(3)
Adding all the three equations, we have
2 (x + y + z) = 24

$\Rightarrow x + y + z = \frac{24}{2} = 12$ ………. (4)

Subtracting (1) (2) and (3) from (4)
y = 12 – 10 = 2
x = 12 – 8 = 4
z = 12 – 6 = 6
Therefore, radii are 2 cm, 4 cm and 6 cm

Solution 22:

ABCD is a square whose diagonals AC and BD intersect each other at right angles at O.

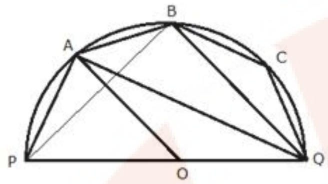


i)
$\therefore \angle AOB = \angle AOD = 90^\circ$
In ΔANB,
$\angle ANB = 180^\circ - (\angle NAB + \angle NBA)$
$\Rightarrow \angle ANB = 180^\circ - \left(45^\circ + \frac{45^\circ}{2}\right)$ (NB is bisector of $\angle ABD$)
$\Rightarrow \angle ANB = 180^\circ - 45^\circ - \frac{45^\circ}{2} = 135^\circ - \frac{45^\circ}{2}$
But, $\angle LNO = \angle ANB$ (vertically opposite angles)
$\therefore \angle LNO = 135^\circ - \frac{45^\circ}{2}$ …… (i)
Now in ΔAMO,
$\angle AMO = 180^\circ - (\angle AOM + \angle OAM)$
$\Rightarrow \angle AMO = 180^\circ - \left(90^\circ + \frac{45^\circ}{2}\right)$ (MA is bisector of $\angle DAO$)
$\Rightarrow \angle AMO = 180^\circ - 90^\circ - \frac{45^\circ}{2} = 90^\circ - \frac{45^\circ}{2}$ …….(ii)
Adding (i) and (ii)
$\angle LNO + \angle AMO = 135^\circ - \frac{45^\circ}{2} + 90^\circ - \frac{45^\circ}{2}$
$\Rightarrow \angle LNO + \angle AMO = 225^\circ - 45^\circ = 180^\circ$
$\Rightarrow \angle ONL + \angle OML = 180^\circ$

ii)
$\angle BAM = \angle BAO + \angle OAM$
$\Rightarrow \angle BAM = 45^\circ + \frac{45^\circ}{2} = 67\frac{1^\circ}{2}$
And
$\Rightarrow \angle BMA = 180^\circ - (\angle AOM + \angle OAM)$
$\Rightarrow \angle BMA = 180^\circ - 90^\circ - \frac{45^\circ}{2} = 90^\circ - \frac{45^\circ}{2} = 67\frac{1^\circ}{2}$
$\therefore \angle BAM = \angle BMA$
iii) In quadrilateral ALOB,

$\because \angle ABO + \angle ALO = 45^\circ + 90^\circ + 45^\circ = 180^\circ$
Therefore, ALOB is a cyclic quadrilateral.

Solution 23:



Join PB.
i) In cyclic quadrilateral PBCQ,
$\angle BPQ + \angle BCQ = 180^\circ$
$\Rightarrow \angle BPQ + 140^\circ = 180^\circ$
$\Rightarrow \angle BPQ = 40^\circ$ ……. (1)

Now in ΔPBQ,
$\angle PBQ + \angle BPQ + \angle BQP = 180^\circ$
$\Rightarrow 90^\circ + 40^\circ + \angle BQP = 180^\circ$
$\Rightarrow \angle BQP = 50^\circ$
In cyclic quadrilateral PQBA,
$\angle PQB + \angle PAB = 180^\circ$
$\Rightarrow 50^\circ + \angle PAB = 180^\circ$
$\Rightarrow \angle PAB = 130^\circ$
ii) Now in ΔPAB,
$\angle PAB + \angle APB + \angle ABP = 180^\circ$
$\Rightarrow 130^\circ + \angle APB + \angle ABP = 180^\circ$
$\Rightarrow \angle APB + \angle ABP = 50^\circ$
But
$\angle APB = \angle ABP \left(\because PA = PB\right)$
$\therefore \angle APB = \angle ABP = 25^\circ$
$\angle BAQ = \angle BPQ = 40^\circ$
$\angle APB = 25^\circ = \angle AQB$ (angles in the same segment)
$\therefore \angle AQB = 25^\circ$ ………. (2)
iii) Arc AQ subtends $\angle AOQ$ at the centre and $\angle APQ$ at the remaining part of the circle.
We have,
$\angle APQ = \angle APB + \angle BPQ$ ……. (3)
From (1), (2) and (3), we have

$\angle APQ = 25^\circ + 40^\circ = 65^\circ$

$\therefore \angle AOQ = 2\angle APQ = 2\times 65^\circ = 130^\circ$

Now in ΔAOQ,

$\angle OAQ = \angle OQA = (\because OA = OQ)$

But

$\angle OAQ + \angle OQA + \angle AOQ = 180^\circ$

$\Rightarrow \angle OAQ + \angle OAQ + 130^\circ = 180^\circ$

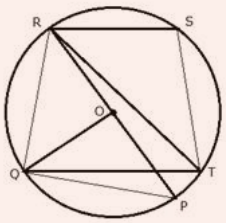
$\Rightarrow 2\angle OAQ = 50^\circ$

$\Rightarrow \angle OAQ = 25^\circ$

$\therefore \angle OAQ = \angle AQB$

But these are alternate angles.
Hence, AO is parallel to BQ.

Solution 24:



Join PQ, RQ and ST.

i)

$\angle POQ + \angle QOR = 180^\circ$

$\Rightarrow 100^\circ + \angle QOR = 180^\circ$

$\Rightarrow \angle QOR = 80^\circ$

Arc RQ subtends $\angle QOR$ at the centre and $\angle QTR$ at the remaining part of the circle.

$\therefore \angle QTR = \frac{1}{2}\angle QOR$

$\Rightarrow \angle QTR = \frac{1}{2}\times 80^\circ = 40^\circ$

ii) Arc QP subtends $\angle QOP$ at the centre and $\angle QRP$ at the remaining part of the circle.

$\therefore \angle QRP = \frac{1}{2}\angle QOP$

$\Rightarrow \angle QRP = \frac{1}{2}\times 100^\circ = 50^\circ$

iii) RS || QT

$\therefore \angle SRT = \angle QTR$ (alternate angles)

But $\angle QTR = 40^\circ$

$\therefore\ \angle SRT = 40^\circ$

Now,

$\angle QRS = \angle QRP + \angle PRT + \angle SRT$

$\Rightarrow \angle QRS = 50^\circ + 20^\circ + 40^\circ = 110^\circ$

iv) Since RSTQ is a cyclic quadrilateral

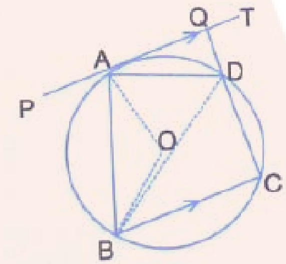
$\therefore \angle QRS + \angle QTS = 180^\circ$ (sum of opposite angles)

$\Rightarrow \angle QRS + \angle QTS + \angle STR = 180^\circ$

$\Rightarrow 110^\circ + 40^\circ + \angle STR = 180^\circ$

$\Rightarrow \angle STR = 30^\circ$

Solution 25:



i) Since PAT ∥ BC

$\therefore \angle PAB = \angle ABC$ (alternate angles) ....(i)

In cyclic quadrilateral ABCD,

Ext $\angle ADQ = \angle ABC$ ………..(ii)

From (i) and (ii)

$\angle PAB = \angle ADQ$

ii) Arc AB subtends $\angle AOB$ at the centre and $\angle ADB$ at the remaining part of the circle.

$\therefore\ \angle AOB = 2\angle ADB$

$\Rightarrow \angle AOB = 2\angle PAB$ (angles in alternate segments)

$\Rightarrow \angle AOB = 2\angle ADQ$ (proved in (i) part)

iii)

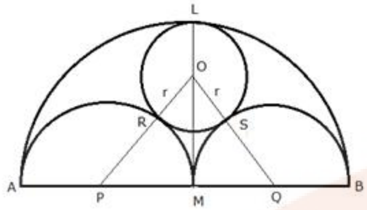
$\because \angle BAP = \angle ADB$ (angles in alternate segments)

But

$\angle BAP = \angle ADQ$ (proved in (i) part)

$\therefore \angle ADQ = \angle ADB$

Solution 26:



Let O, P and Q be the centers of the circle and semicircles.
Join OP and OQ.
OR = OS = r

and AP = PM = MQ = QB = $\frac{AB}{4}$

Now, OP = OR + RP = r + $\frac{AB}{4}$ (since PM=RP=radii of same circle)

Similarly, OQ = OS + SQ = r + $\frac{AB}{4}$

OM = LM -; OL = $\frac{AB}{2} - r$

Now in Rt. ΔOPM,

$OP^2 = PM^2 + OM^2$

$\Rightarrow \left(r + \frac{AB}{4}\right)^2 = \left(\frac{AB}{4}\right)^2 + \left(\frac{AB}{2} - r\right)^2$

$\Rightarrow r^2 + \frac{AB^2}{16} + \frac{rAB}{2} = \frac{AB^2}{16} + \frac{AB^2}{4} + r^2 - rAB$

$\Rightarrow \frac{rAB}{2} = \frac{AB^2}{4} - rAB$

$\Rightarrow \frac{AB^2}{4} = \frac{rAB}{2} + rAB$

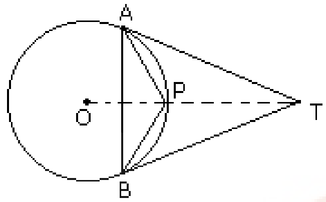
$\Rightarrow \frac{AB^2}{4} = \frac{3rAB}{2}$

$\Rightarrow \frac{AB}{4} = \frac{3}{2}r$

$\Rightarrow AB = \frac{3}{2}r \times 4 = 6r$

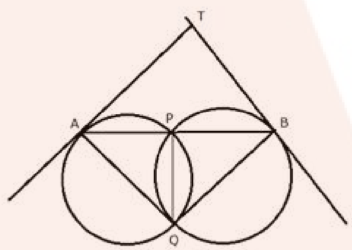
Hence AB = 6 × r

Solution 27:



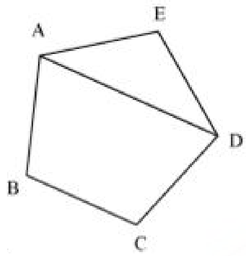
Join PB.
In ΔTAP and ΔTBP,
TA = TB (tangents segments from an external points are equal in length)
Also, $\angle ATP = \angle BTP$. (since OT is equally inclined with TA and TB) TP = TP (common)
$\Rightarrow \Delta TAP \cong \Delta TBP$ (by SAS criterion of congruency)
$\Rightarrow \angle TAP = \angle TBP$ (corresponding parts of congruent triangles are equal)
But $\angle TBP = \angle BAP$ (angles in alternate segments)
Therefore, $\angle TAP = \angle BAP$.
Hence, AP bisects $\angle TAB$.

Solution 28:



Join PQ.
AT is tangent and AP is a chord.
$\therefore \angle TAP = \angle AQP$ (angles in alternate segments) ....(i)
Similarly, $\angle TBP = \angle BQP$ ....(ii)
Adding (i) and (ii)
$\angle TAP + \angle TBP = \angle AQP + \angle BQP$
$\Rightarrow \angle TAP + \angle TBP = \angle AQB$ ....(iii)
Now in ΔTAB,
$\angle ATB + \angle TAP + \angle TBP = 180^\circ$
$\Rightarrow \angle ATB + \angle AQB = 180^\circ$
Therefore, AQBT is a cyclic quadrilateral.
Hence, A, Q, B and T lie on a circle.

Solution 29:

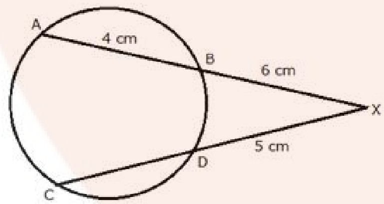


ABCDE is a regular pentagon.

$\therefore \angle BAE = \angle ABC = \angle BCD = \angle CDE = \angle DEA = \left(\frac{5-2}{5}\right)\times 180^\circ = 180^\circ$

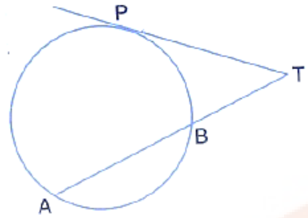
In ΔAED,
AE = ED (Sides of regular pentagon ABCDE)
$\therefore \angle EAD = \angle EDA$
In ΔAED,
$\angle AED + \angle EAD + \angle EDA = 180^o$
$\Rightarrow 108^o + \angle EAD + \angle EAD = 180^o$
$\Rightarrow 2\angle EAD = 180^o - 108^o = 72^o$
$\Rightarrow \angle EAD = 36^o$
$\therefore \angle EDA = 36^o$
$\angle BAD = \angle BAE - \angle EAD = 108^o - 36^o = 72^o$
In quadrilateral ABCD,
$\angle BAD + \angle BCD = 108^o + 72^o = 180^o$
$\therefore$ ABCD is a cyclic quadrilateral

Solution 30:



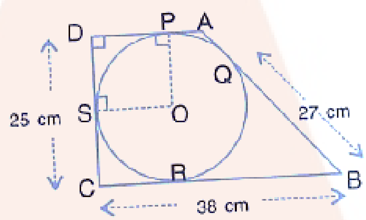
We know that XB.XA = XD.XC
Or, XB.(XB + BA) = XD.(XD + CD)
Or, 6(6 + 4) = 5(5 + CD)
Or, 60 = 5(5 + CD)
Or, $5 + CD = \frac{60}{5} = 12$
Or, CD = 12 − 5 = 7 cm.

Solution 31:



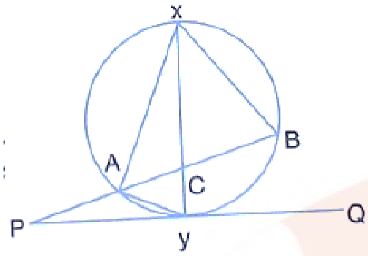
PT is the tangent and TBA is the secant of the circle.
Therefore, $TP^2 = TA \times TB$
$TP^2 = 16 \times (16 - 12) = 16 \times 4 = 64 = (8)^2$
Therefore, TP = 8 cm

Solution 32:



From the figure we see that BQ = BR = 27 cm (since length of the tangent segments from an external point are equal)
As BC = 38 cm
$\Rightarrow$ CR = CB − BR = 38 − 27
= 11 cm
Again,
CR = CS = 11cm (length of tangent segments from an external point are equal)
Now, as DC = 25 cm
$\therefore$ DS = DC − SC
= 25 − 11
= 14 cm
Now, in quadrilateral DSOP,
$\angle PDS = 90°$ (given)
$\angle OSD = 90°$, $\angle OPD = 90°$ (since tangent is perpendicular to the radius through the point of contact)
$\Rightarrow$ DSOP is a parallelogram
$\Rightarrow$ OP $\parallel$ SD and $\Rightarrow$ PD $\parallel$ OS
Now, as OP = OS (radii of the same circle)
$\Rightarrow$ OPDS is a square. $\therefore$ DS = OP = 14cm
$\therefore$ radius of the circle = 14 cm

Solution 33:



In ΔAXB,

$\angle XAB + \angle AXB + \angle ABX = 180°$ [Triangle property]

$\Rightarrow \angle XAB + 50° + 70° = 180°$

$\Rightarrow \angle XAB = 180° - 120° = 60°$

$\Rightarrow \angle XAY = 90°$ [Angle of semi-circle]

$\therefore \angle BAY = \angle XAY - \angle XAB = 90° - 60° = 30°$

and $\angle BXY = \angle BAY = 30°$ [Angle of same segment]

$\angle ACX = \angle BXY + \angle ABX$ [External angle = Sum of two interior angles]

$= 30° + 70°$

$= 100°$

also,

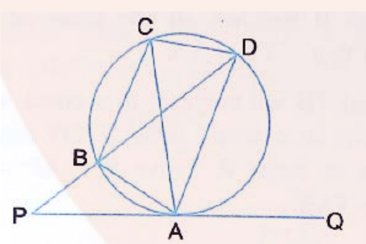
$\angle XYP = 90°$ [Diameter $\perp$ tangent]

$\angle APY = \angle ACX - \angle CYP$

$\angle APY = 100° - 90°$

$\angle APY = 10°$

Solution 34:



PAQ is a tangent and AB is a chord of the circle.

i) $\therefore \angle BAP = \angle ACB = 36^\circ$ (angles in alternate segment)

ii) In ΔAPB

Ext $\angle ABD = \angle APB + \angle BAP$

$\Rightarrow$ Ext $\angle ABD = 42^\circ + 36^\circ = 78^\circ$

iii) $\angle ADB = \angle ACB = 36^\circ$ (angles in the same segment)

Now in ΔPAD

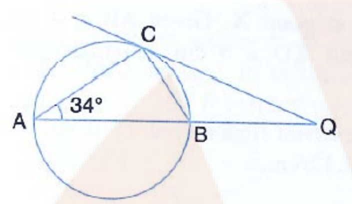
Ext. $\angle QAD = \angle APB + \angle ADB$

$\Rightarrow$ Ext $\angle QAD = 42^\circ + 36^\circ = 78^\circ$

iv) PAQ is the tangent and AD is chord

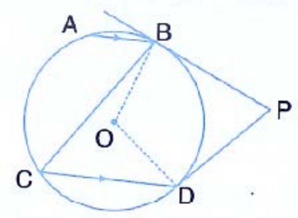
$\therefore$ QAD $= \angle ACD = 78^\circ$ (angles in alternate segment)
And $\angle BCD = \angle ACB + \angle ACD$
$\therefore \angle BCD = 36^\circ + 78^\circ = 114^\circ$

Solution 35:



i) AB is diameter of circle.
$\therefore$ ACB $= 90^\circ$
In ΔABC,
$\angle A + \angle B + \angle C = 180^\circ$
$\Rightarrow 34^\circ + \angle CBA + 90^\circ = 180^\circ$
$\Rightarrow \angle CBA = 56^\circ$
ii) QC is tangent to the circle
$\therefore \angle CAB = \angle QCB$
Angle between tangent and chord = angle in alternate segment
$\therefore \angle QCB = 34^\circ$
ABQ is a straight line
$\Rightarrow \angle ABC + \angle CBQ = 180^\circ$
$\Rightarrow 56^\circ + \angle CBQ = 180^\circ$
$\Rightarrow \angle CBQ = 124^\circ$
Now,
$\angle CQB = 180^\circ - \angle QCB - \angle CBQ$
$\Rightarrow \angle CQB = 180^\circ - 34^\circ - 124^\circ$
$\Rightarrow \angle CQB = 22^\circ$

Solution 36:



i)
$\angle BOD = 2\angle BCD$

$\Rightarrow \angle BOD = 2 \times 55^\circ = 110^\circ$

ii) Since, BPDO is cyclic quadrilateral, opposite angles are supplementary.

$\therefore \angle BOD + \angle BPD = 180^\circ$

$\Rightarrow \angle BPD = 180^\circ - 110^\circ = 70^\circ$

Solution 37:

i) PQ = RQ

$\therefore \angle PRQ = \angle QPR$ (opposite angles of equal sides of a triangle)

$\Rightarrow \angle PRQ + \angle QPR + 68^\circ = 180^\circ$

$\Rightarrow 2\angle PRQ = 180^\circ - 68^\circ$

$\Rightarrow \angle PRQ = \frac{112^\circ}{2} = 56^\circ$

Now, $\angle$ QOP = 2 $\angle$ PRQ (angle at the centre is double)

$\Rightarrow QOP = 2 \times 56^\circ = 112^\circ$

ii) $\angle$ PQC = $\angle$ PRQ (angles in alternate segments are equal)

$\angle$ QPC = $\angle$ PRQ (angles in alternate segments)

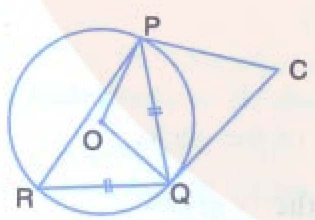
$\therefore \angle PQC = \angle QPC = 56^\circ \left(\because \angle PRQ = 56^\circ \text{ from (i)}\right)$

$\angle PQC + \angle QPC + \angle QCP = 180^\circ$

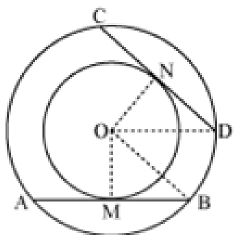
$\Rightarrow 56^\circ + 56^\circ + \angle QCP = 180^\circ$

$\Rightarrow \angle QCP = 68^\circ$

Solution 38:

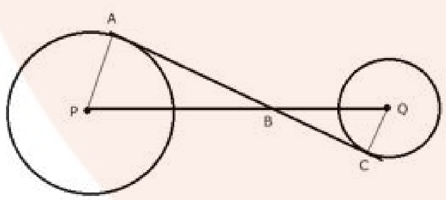


Consider two concentric circles with centres at O. Let AB and CD be two chords of the outer circle which touch the inner circle at the points M and N respectively.



To prove the given question, it is sufficient to prove AB = CD.
For this join OM, ON, OB and OD.
Let the radius of outer and inner circles be *R* and *r* respectively.
AB touches the inner circle at M.
AB is a tangent to the inner circle
$\therefore$ OM $\perp$ AB
$\Rightarrow \text{BM} = \frac{1}{2}\text{AB}$
$\Rightarrow \text{AB} = 2\text{BM}$
Similarly ON $\perp$ CD, and CD = 2DN
Using Pythagoras theorem in ΔOMB and ΔOND
$\text{OB}^2 = \text{OM}^2 + \text{BM}^2,\ \text{OD}^2 = \text{ON}^2 + \text{DM}^2$
$\Rightarrow \text{BM} = \sqrt{\text{R}^2 - \text{r}^2}, \text{DN} = \sqrt{\text{R}^2 - \text{r}^2}$
Now,
$\text{AB} = 2\text{BM} = 2\sqrt{\text{R}^2 - \text{r}^2}, \text{CD} = 2\text{DN} = 2\sqrt{\text{R}^2 - \text{r}^2}$
$\therefore \text{AB} = \text{CD}$
Hence proved.

Solution 39:



Since AC is tangent to the circle with center P at point A.
$\therefore\ \angle\text{PAB}\angle = 90^\circ$
Similarly, $\angle\text{QCB} = 90^\circ$
In ΔPAB and ΔQCB
$\angle\text{PAB} = \angle\text{OCB} = 90^\circ$
$\angle\text{PBA} = \angle\text{QBC}$ (vertically opposite angles)
$\therefore\ \Delta\text{PAB} \sim \Delta\text{QCB}$
$\Rightarrow \frac{\text{PA}}{\text{QC}} = \frac{\text{PB}}{\text{QB}}$ ……… (i)

Also in Rt. ΔPAB,

$$PB = \sqrt{PA^2 + PB^2}$$

$$\Rightarrow PB = \sqrt{6^2 + 8^2} = \sqrt{36+64} = \sqrt{100} = 10 \text{ cm} \quad \text{.......(ii)}$$

From (i) and (ii)

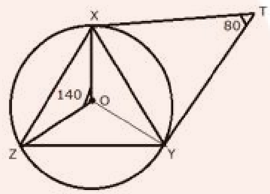
$$\frac{6}{3} = \frac{10}{QB}$$

$$\Rightarrow QB = \frac{3 \times 10}{6} = 5 \text{ cm}$$

Now,

$$PQ = PB + QB = (10+5) \text{ cm} = 15 \text{cm}$$

Solution 40:



In the figure, a circle with centre O, is the circum circle of triangle XYZ.

$\angle XOZ = 140^\circ$

Tangents at X and Y intersect at point T, such that $\angle XTY = 80°$

$\therefore \angle XOY = 180^\circ - 80^\circ = 100^\circ$

But, $\angle XOY + \angle YOZ + \angle ZOX = 360^\circ$ [Angles at a point]

$\Rightarrow 100^\circ + \angle YOZ + 140^\circ = 360^\circ$

$\Rightarrow 240^\circ + \angle YOZ = 360^\circ$

$\Rightarrow \angle YOZ = 360^\circ - 240^\circ$

$\Rightarrow \angle YOZ = 120^\circ$

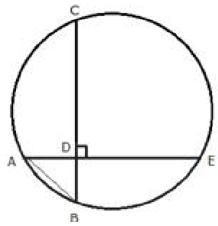
Now arc YZ subtends $\angle YOZ$ at the centre and $\angle YXZ$ at the remaining part of the circle.

$\therefore \angle YOZ = 2\angle YXZ$

$$\Rightarrow \angle YXZ = \frac{1}{2}\angle YOZ$$

$$\Rightarrow \angle YXZ = \frac{1}{2} \times 120^\circ = 60^\circ$$

Solution 41:



From Rt. ΔADB,

$$AD = \sqrt{AB^2 - DB^2} = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3\,\text{cm}$$

Now, since the two chords AE and BC intersect at D,

$AD \times DE = CD \times DB$

$3 \times DE = 9 \times 4$

$$DE = \frac{9 \times 4}{3} = 12$$

Hence, $AE = AD + DE = (3 + 12) = 15$ cm