Chapter 10

Rotational Motion

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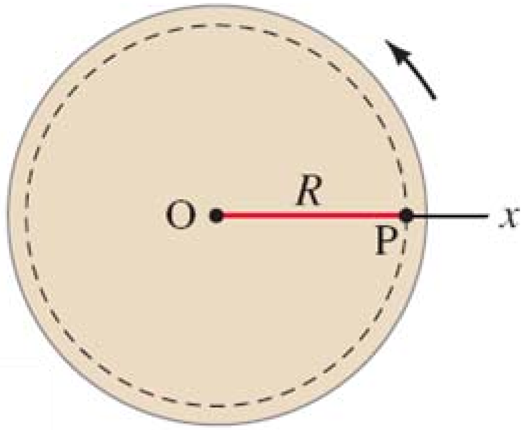
Units of Chapter 10

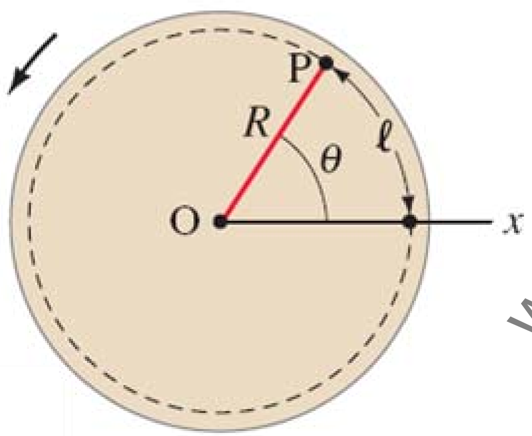
- Angular Quantities
- Vector Nature of Angular Quantities
- Constant Angular Acceleration
- Torque
- Rotational Dynamics; Torque and Rotational Inertia
- Solving Problems in Rotational Dynamics

Units of Chapter 10

- Determining Moments of Inertia
- Rotational Kinetic Energy
- Rotational Plus Translational Motion; Rolling
- Why Does a Rolling Sphere Slow Down?

10-1 Angular Quantities



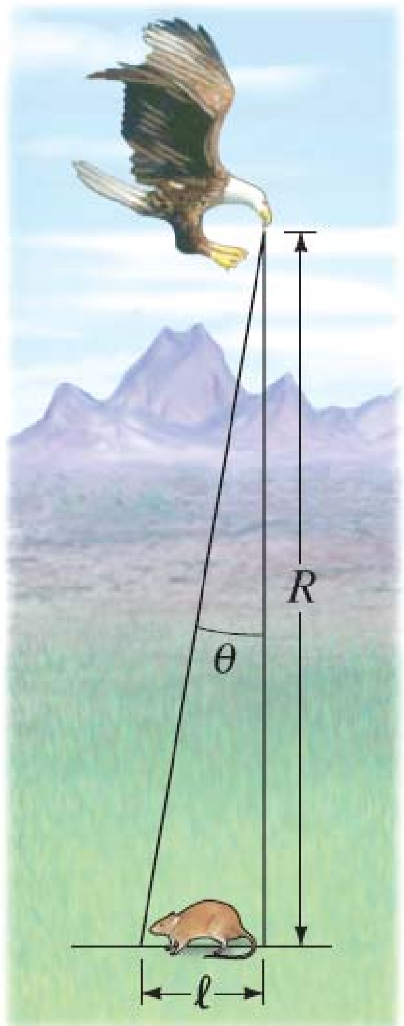


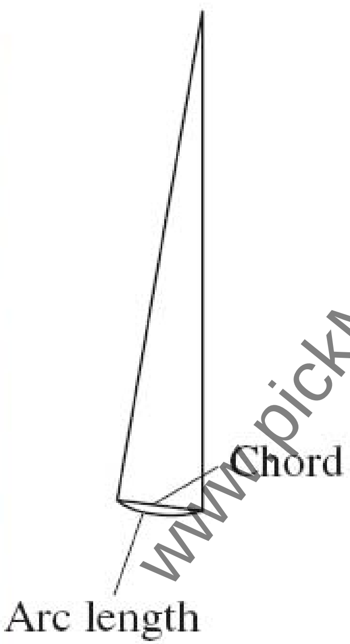
In purely **rotational** motion, all points on the object move in **circles** around the **axis** of rotation (“*O*”). The **radius** of the circle is *R*. All points on a straight line drawn through the axis move through the same **angle** in the same **time**. The angle ***θ*** in **radians** is defined:

$$\theta = \frac{l}{R},$$

where *l* is the **arc length**.

10-1 Angular Quantities

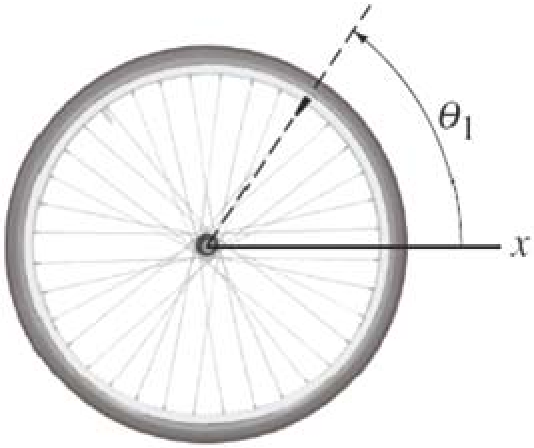


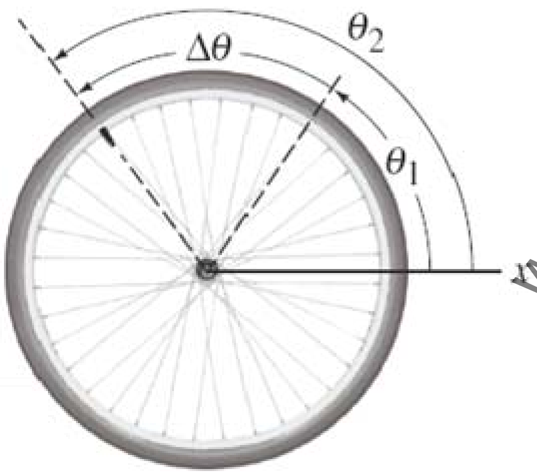


Example 10-1: Birds of prey—in radians.

A particular bird's eye can just distinguish objects that subtend an angle no smaller than about 3 x 10^{-4} rad. (a) How many degrees is this? (b) How small an object can the bird just distinguish when flying at a height of 100 m?

10-1 Angular Quantities





Angular displacement:

$$\Delta\theta = \theta_2 - \theta_1 .$$

The average angular velocity is defined as the total angular displacement divided by time:

$$\overline{\omega} = \frac{\Delta\theta}{\Delta t} .$$

The instantaneous angular velocity:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} .$$

10-1 Angular Quantities

The angular **acceleration** is the rate at which the angular velocity changes with time:

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{\Delta t} = \frac{\Delta\omega}{\Delta t}.$$

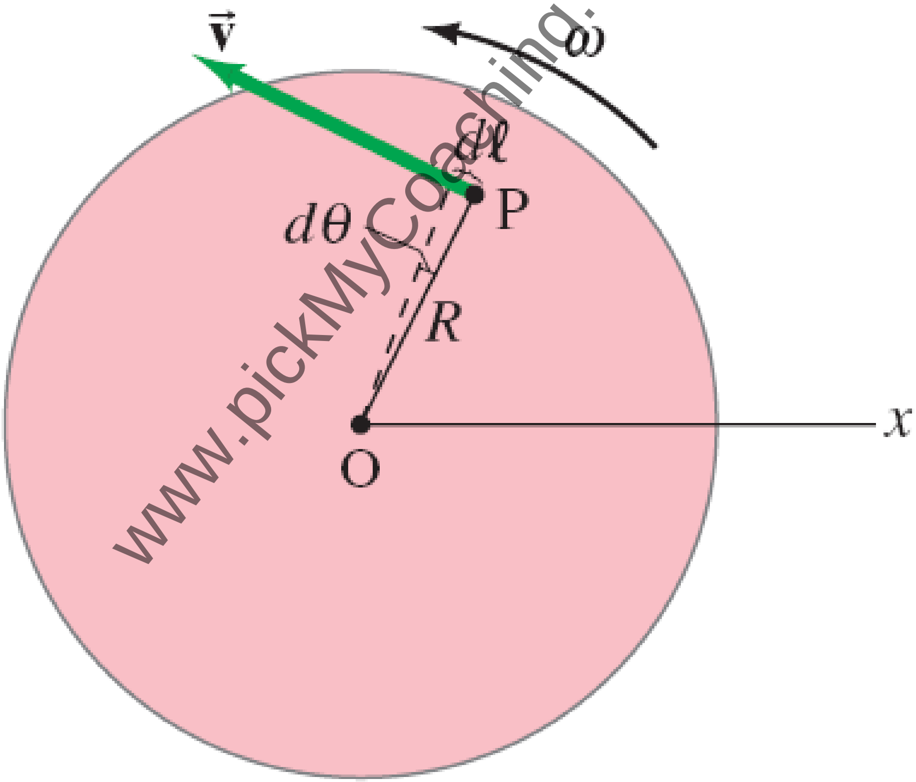
The **instantaneous** acceleration:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}.$$

10-1 Angular Quantities

Every point on a rotating body has an angular velocity *ω* and a linear velocity *v*.

They are related: $v = R\omega.$

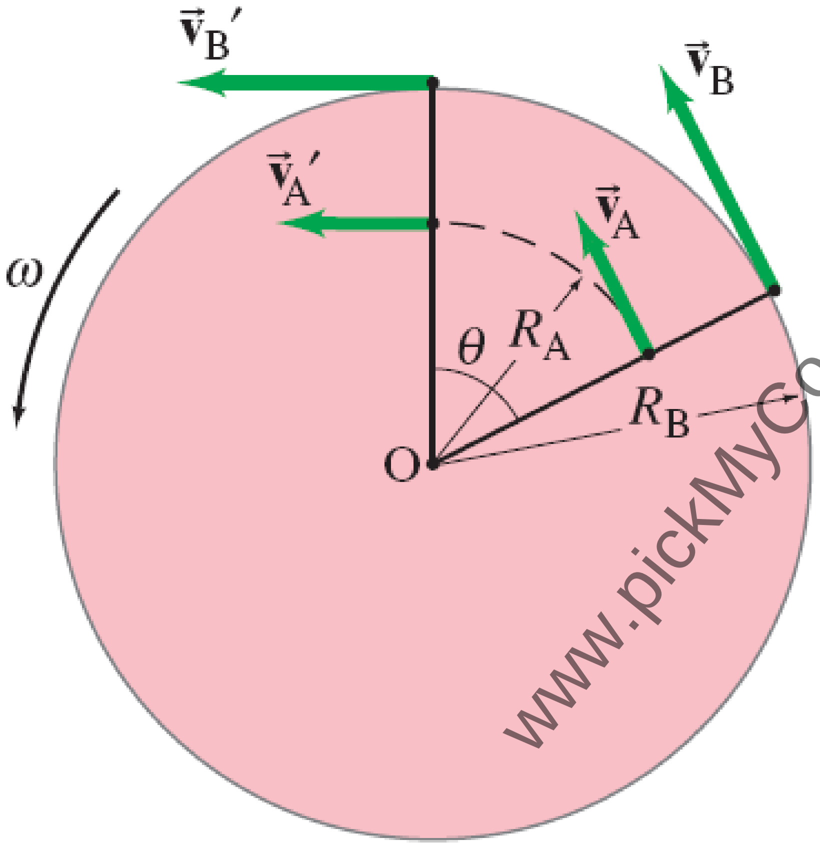


10-1 Angular Quantities

Conceptual Example 10-2: Is the lion faster than the horse?

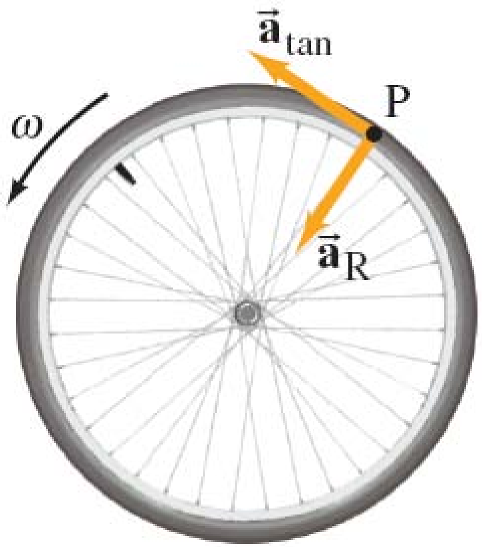
On a rotating carousel or merry-go-round, one child sits on a horse near the outer edge and another child sits on a lion halfway out from the center. (a) Which child has the greater linear velocity? (b) Which child has the greater angular velocity?

10-1 Angular Quantities



Objects farther from the axis of rotation will move faster.

10-1 Angular Quantities



If the angular velocity of a rotating object changes, it has a tangential acceleration:

$$a_{\tan} = \frac{dv}{dt} = R\frac{d\omega}{dt} = R\alpha.$$

Even if the angular velocity is constant, each point on the object has a centripetal acceleration:

$$a_{R} = \frac{v^2}{R} = \frac{(R\omega)^2}{R} = \omega^2 R.$$

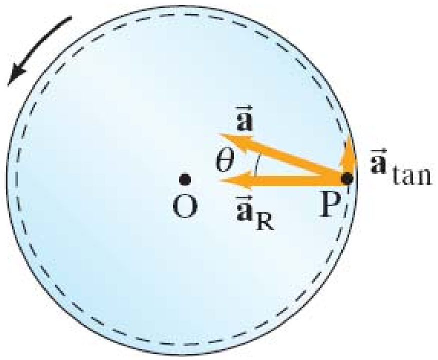
10-1 Angular Quantities

Here is the correspondence between linear and rotational quantities:

TABLE 10–1
Linear and Rotational Quantities

Linear	Type	Rota-tional	Relation (θ in radians)
x	displacement	θ	$x = R\theta$
v	velocity	ω	$v = R\omega$
$a_{\tan}$	acceleration	α	$a_{\tan} = R\alpha$

10-1 Angular Quantities



Example 10-3: Angular and linear velocities and accelerations.

A carousel is initially at rest. At $t = 0$ it is given a constant angular acceleration $\alpha = 0.060\ \text{rad/s}^2$, which increases its angular velocity for 8.0 s. At $t = 8.0$ s, determine the magnitude of the following quantities: (a) the angular velocity of the carousel; (b) the linear velocity of a child located 2.5 m from the center; (c) the tangential (linear) acceleration of that child; (d) the centripetal acceleration of the child; and (e) the total linear acceleration of the child.

10-1 Angular Quantities

The **frequency** is the number of complete **revolutions** per second:

$$f = \frac{\omega}{2\pi}.$$

Frequencies are measured in **hertz**:

$$1\,\mathrm{Hz} = 1\,\mathrm{s}^{-1}.$$

The **period** is the time one revolution takes:

$$T = \frac{1}{f}.$$

10-1 Angular Quantities

Example 10-4: Hard drive.

The platter of the hard drive of a computer rotates at 7200 rpm (rpm = revolutions per minute = rev/min). (a) What is the angular velocity (rad/s) of the platter? (b) If the reading head of the drive is located 3.00 cm from the rotation axis, what is the linear speed of the point on the platter just below it? (c) If a single bit requires 0.50 μm of length along the direction of motion, how many bits per second can the writing head write when it is 3.00 cm from the axis?

10-1 Angular Quantities

Example 10-5: Given ω as function of time.

A disk of radius R = 3.0 m rotates at an angular velocity $\omega = (1.6 + 1.2t)$ rad/s, where t is in seconds. At the instant t = 2.0 s, determine (a) the angular acceleration, and (b) the speed v and the components of the acceleration a of a point on the edge of the disk.

10-2 Vector Nature of Angular Quantities

The **angular velocity** vector points along the **axis of rotation**, with the direction given by the right-hand rule. If the direction of the rotation axis does not change, the **angular acceleration** vector points along it as well.

10-3 Constant Angular Acceleration

The equations of motion for constant angular acceleration are the same as those for linear motion, with the substitution of the angular quantities for the linear ones.

Angular	Linear
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = v_0 t + \frac{1}{2}at^2$
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$v^2 = v_0^2 + 2ax$
$\overline{\omega} = \dfrac{\omega + \omega_0}{2}$	$\overline{v} = \dfrac{v + v_0}{2}$

10-3 Constant Angular Acceleration

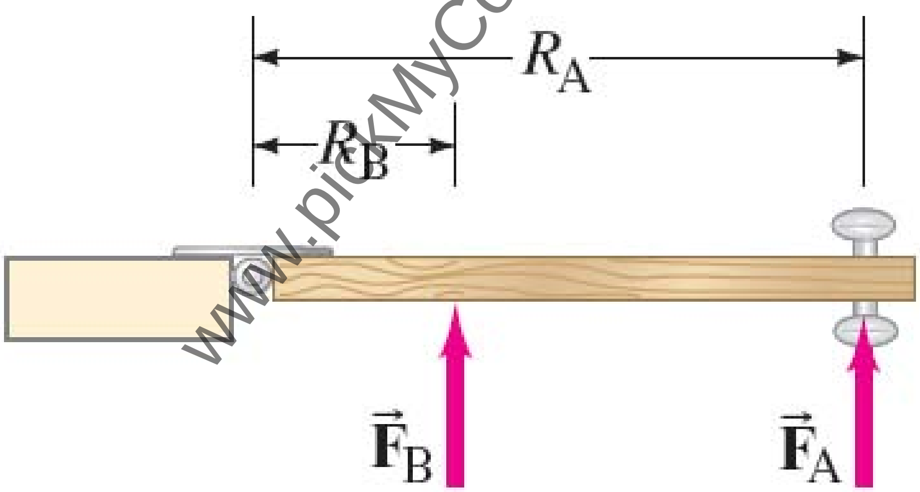
Example 10-6: Centrifuge acceleration.

A centrifuge rotor is accelerated from rest to 20,000 rpm in 30 s. (a) What is its average angular acceleration? (b) Through how many revolutions has the centrifuge rotor turned during its acceleration period, assuming constant angular acceleration?

10-4 Torque

To make an object **start** rotating, a **force** is needed; the **position** and **direction** of the force matter as well.

The **perpendicular** distance from the axis of rotation to the line along which the force acts is called the **lever arm**.

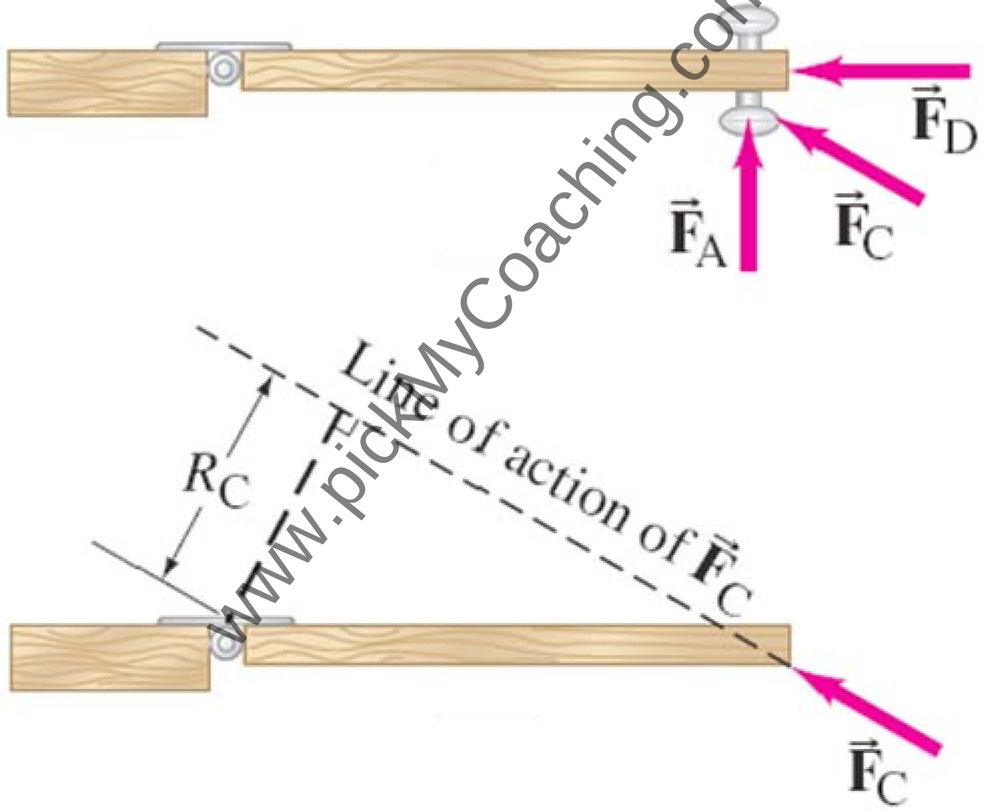


10-4 Torque

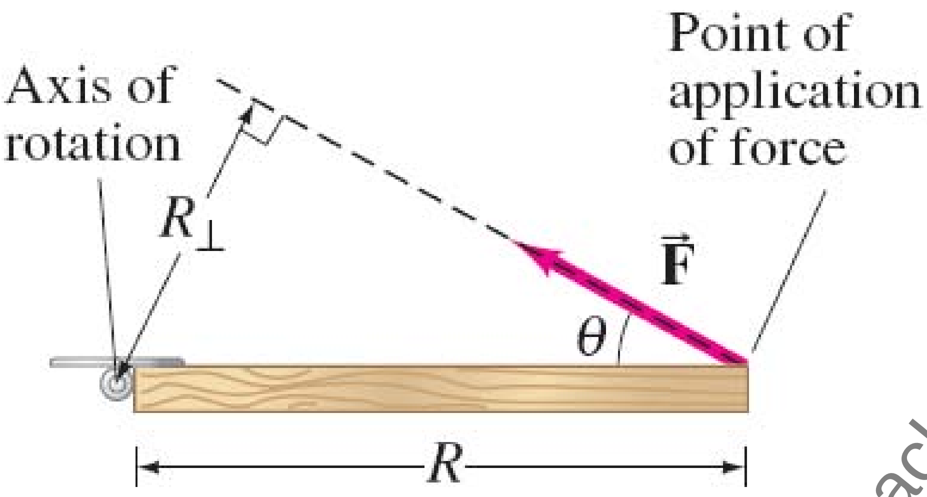
A longer lever arm is very helpful in rotating objects.

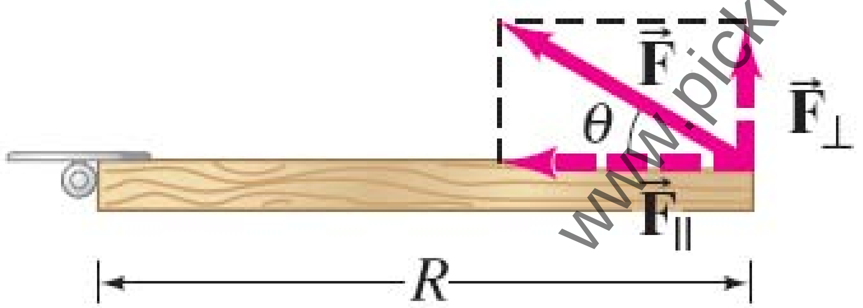
10-4 Torque

Here, the lever arm for F_A is the distance from the **knob** to the **hinge**; the lever arm for F_D is **zero**; and the lever arm for F_C is as shown.



10-4 Torque



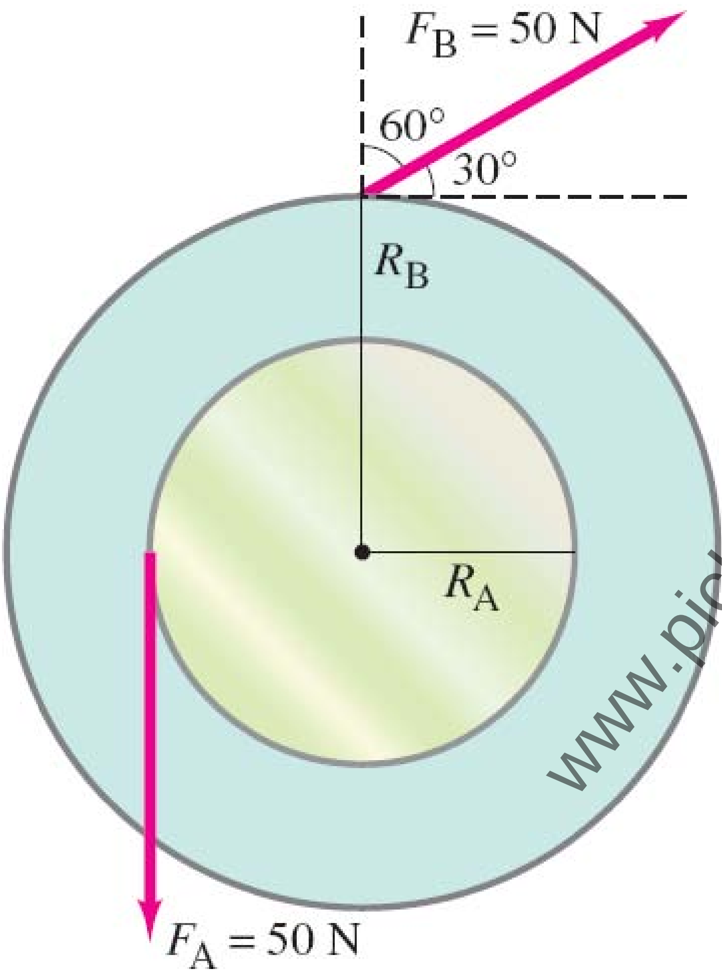


The **torque** is defined as:

$$\tau = R_\perp F.$$

10-4 Torque

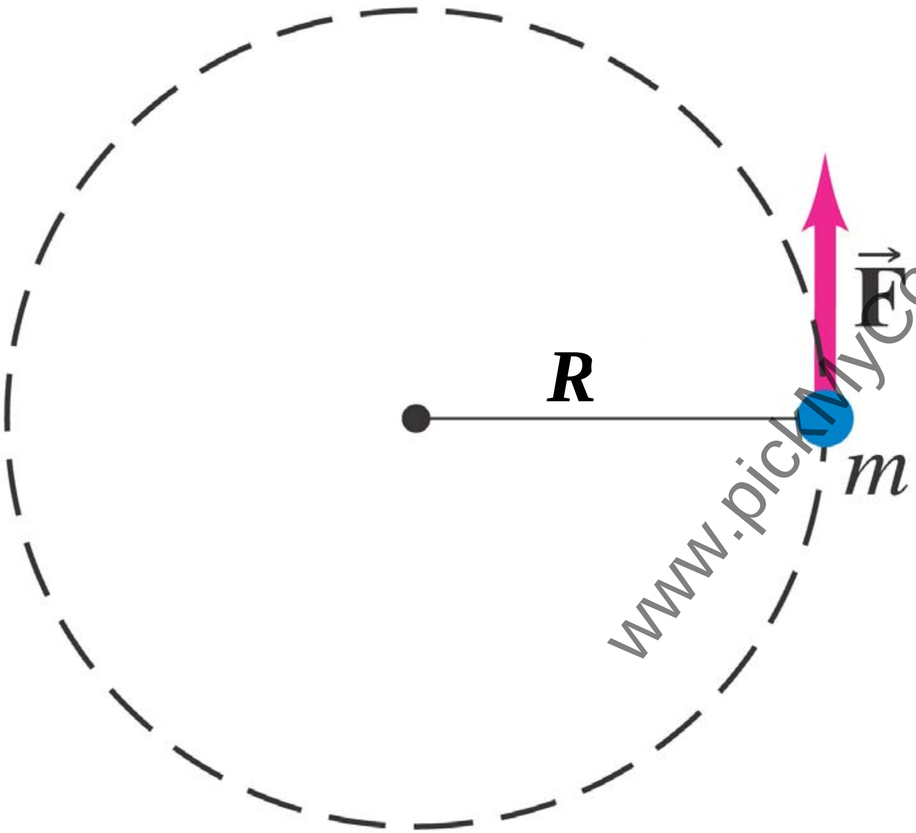
Example 10-7: Torque on a compound wheel.



Two thin disk-shaped wheels, of radii R_A = 30 cm and R_B = 50 cm, are attached to each other on an axle that passes through the center of each, as shown. Calculate the net torque on this compound wheel due to the two forces shown, each of magnitude 50 N.

10-5 Rotational Dynamics; Torque and Rotational Inertia

Knowing that $F = ma$, we see that $\tau = mR^2\alpha$.



This is for a single point mass; what about an **extended** object?

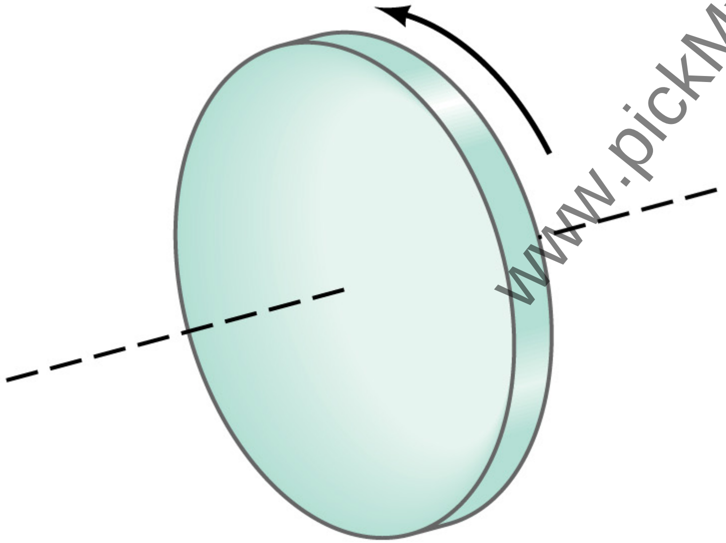
As the angular acceleration is the same for the whole object, we can write:

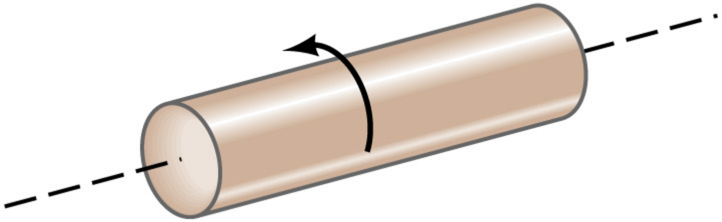
$$\Sigma\tau = \left(\Sigma mR^2\right)\alpha.$$

10-5 Rotational Dynamics; Torque and Rotational Inertia

The quantity $I = \Sigma m_i R_i^2$ is called the **rotational inertia** of an object.

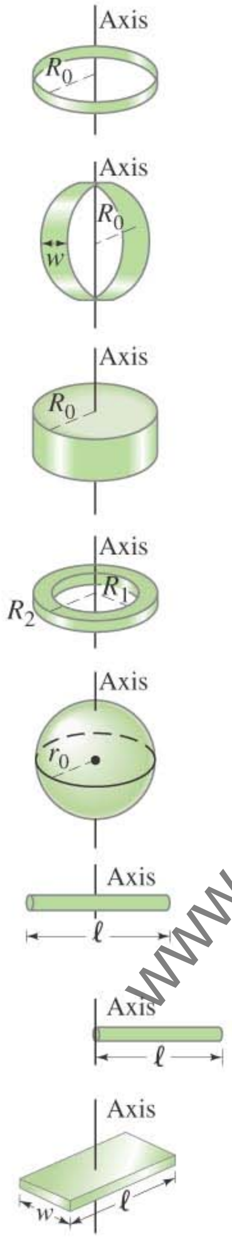
The **distribution** of mass matters here—these two objects have the same mass, but the one on the left has a greater **rotational inertia**, as so much of its mass is far from the axis of rotation.





10-5 Rotational Dynamics; Torque and Rotational Inertia

Object	Location of axis	Moment of inertia
(a) **Thin hoop,** radius R_0	Through center	MR_0^2
(b) **Thin hoop,** radius R_0 width w	Through central diameter	$\frac{1}{2}MR_0^2 + \frac{1}{12}Mw^2$
(c) **Solid cylinder,** radius R_0	Through center	$\frac{1}{2}MR_0^2$
(d) **Hollow cylinder,** inner radius R_1 outer radius R_2	Through center	$\frac{1}{2}M(R_1^2 + R_2^2)$
(e) **Uniform sphere,** radius r_0	Through center	$\frac{2}{5}Mr_0^2$
(f) **Long uniform rod,** length ℓ	Through center	$\frac{1}{12}M\ell^2$
(g) **Long uniform rod,** length ℓ	Through end	$\frac{1}{3}M\ell^2$
(h) **Rectangular thin plate,** length ℓ, width w	Through center	$\frac{1}{12}M(\ell^2 + w^2)$



The **rotational inertia** of an object depends not only on its **mass distribution** but also the location of the **axis** of rotation—compare (f) and (g), for example.

10-6 Solving Problems in Rotational Dynamics

1. Draw a diagram.
2. Decide what the system comprises.
3. Draw a free-body diagram for each object under consideration, including all the forces acting on it and where they act.
4. Find the axis of rotation; calculate the torques around it.

10-6 Solving Problems in Rotational Dynamics

5. Apply Newton's second law for **rotation**. If the **rotational inertia** is not provided, you need to find it **before** proceeding with this step.

6. Apply Newton's second law for **translation** and other laws and principles as needed.

7. **Solve**.

8. **Check** your answer for units and correct order of magnitude.

10-7 Determining Moments of Inertia

If a physical object is available, the moment of inertia can be measured experimentally.

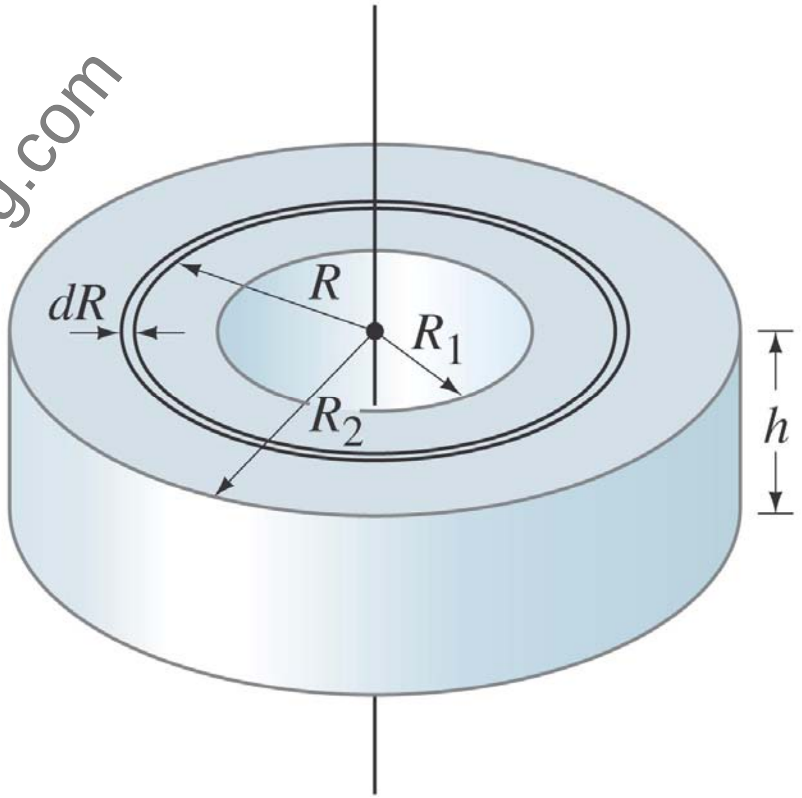
Otherwise, if the object can be considered to be a continuous distribution of mass, the moment of inertia may be calculated:

$$I = \int R^2 \, dm.$$

10-7 Determining Moments of Inertia

Example 10-12: Cylinder, solid or hollow.

(a) Show that the moment of inertia of a uniform hollow cylinder of inner radius R_1, outer radius R_2, and mass M, is $I = ½ M(R_1^2 + R_2^2)$, if the rotation axis is through the center along the axis of symmetry. (b) Obtain the moment of inertia for a solid cylinder.



10-7 Determining Moments of Inertia

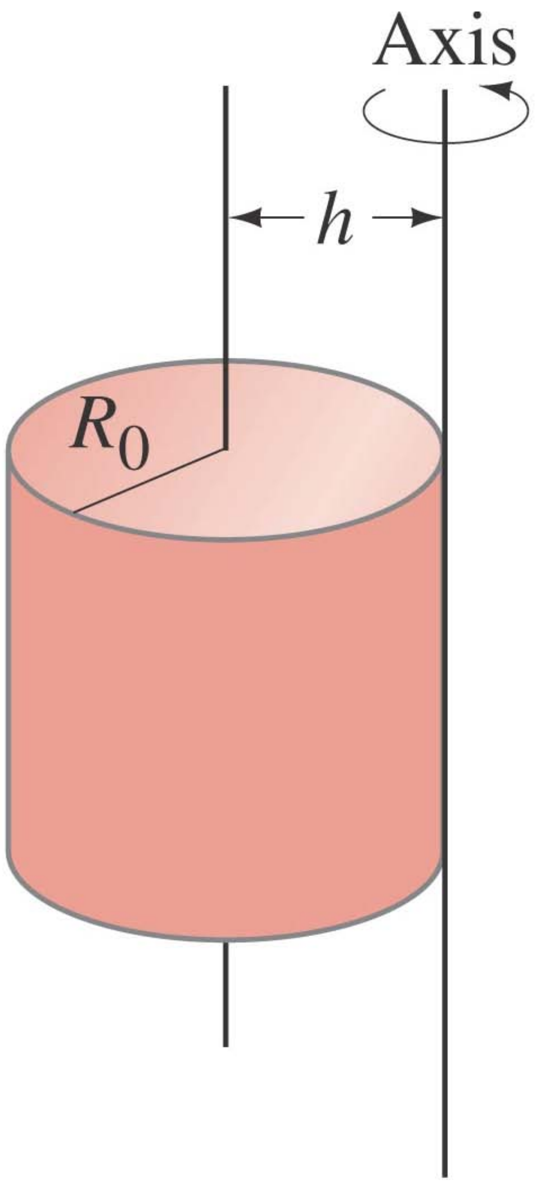
The parallel-axis theorem gives the moment of inertia about any axis parallel to an axis that goes through the center of mass of an object:

$$I = I_{CM} + Mh^2.$$

10-7 Determining Moments of Inertia

Example 10-13: Parallel axis.

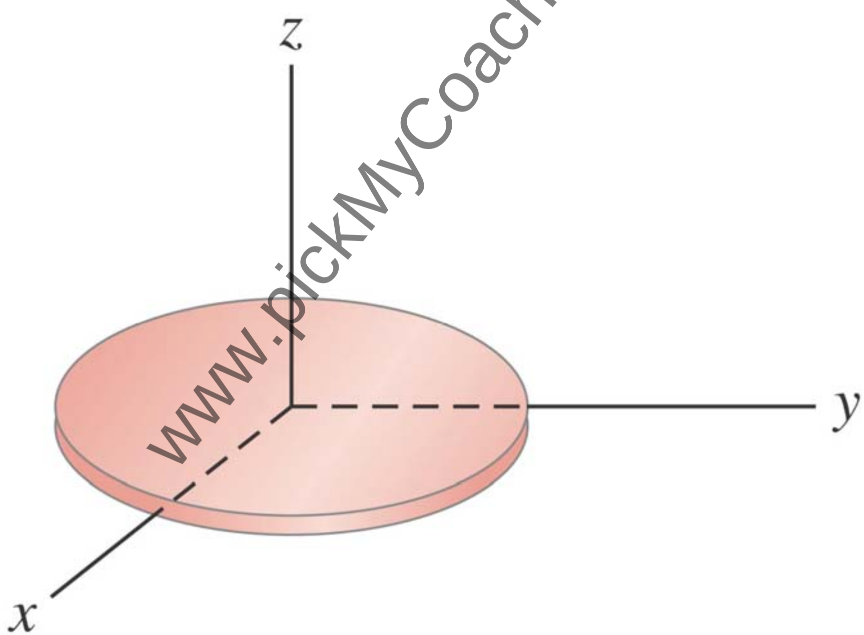
Determine the moment of inertia of a solid cylinder of radius R_0 and mass M about an axis tangent to its edge and parallel to its symmetry axis.



10-7 Determining Moments of Inertia

The perpendicular-axis theorem is valid only for flat objects.

$$I_z = I_x + I_y .$$



10-8 Rotational Kinetic Energy

The **kinetic energy** of a rotating object is given by

$$K = \Sigma\left(\tfrac{1}{2}mv^2\right).$$

By substituting the rotational quantities, we find that the rotational kinetic energy can be written:

$$\text{rotational } K = \tfrac{1}{2}I\omega^2.$$

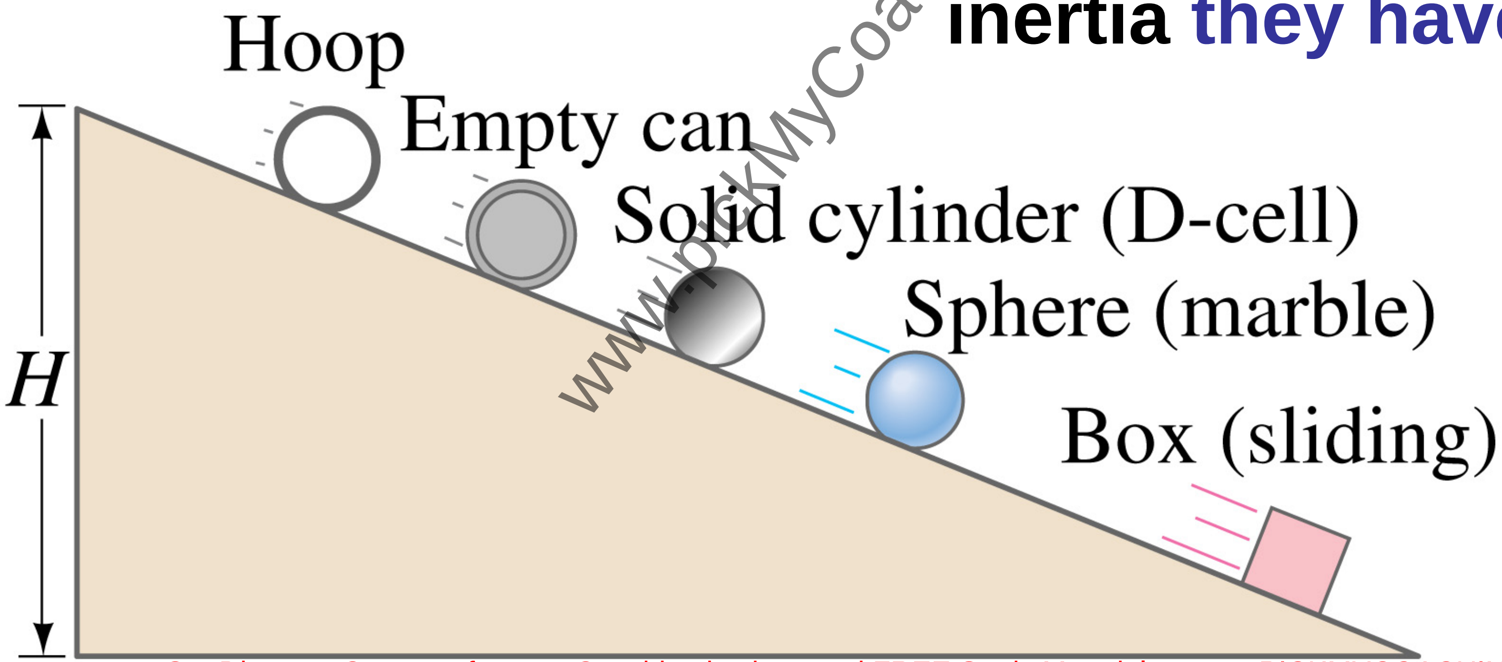
A object that both **translational** and **rotational** motion also has both translational and rotational **kinetic energy**:

$$K = \tfrac{1}{2}Mv_{\mathrm{CM}}^2 + \tfrac{1}{2}I_{\mathrm{CM}}\omega^2.$$

10-8 Rotational Kinetic Energy

When using **conservation** of energy, both **rotational** and **translational** kinetic energy must be taken into account.

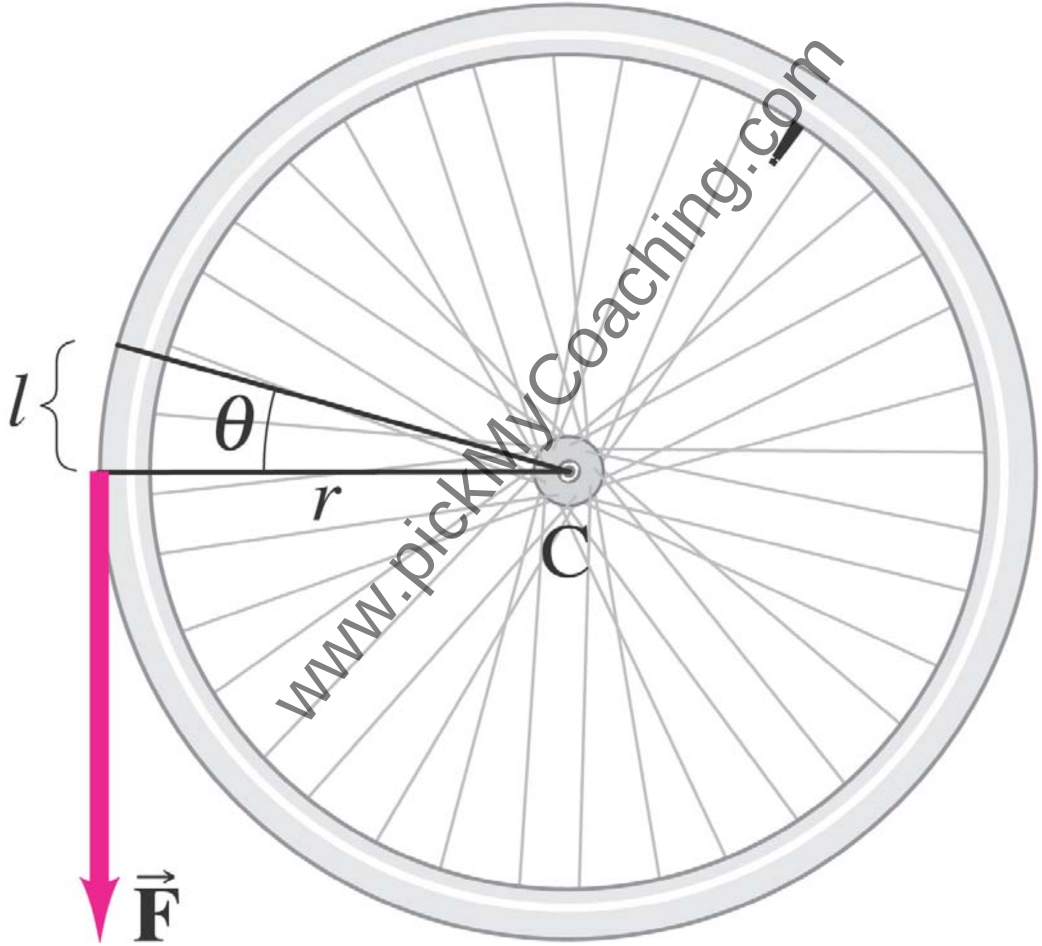
All these objects have the same **potential** energy at the top, but the time it takes them to get down the incline depends on how much **rotational inertia** they have.



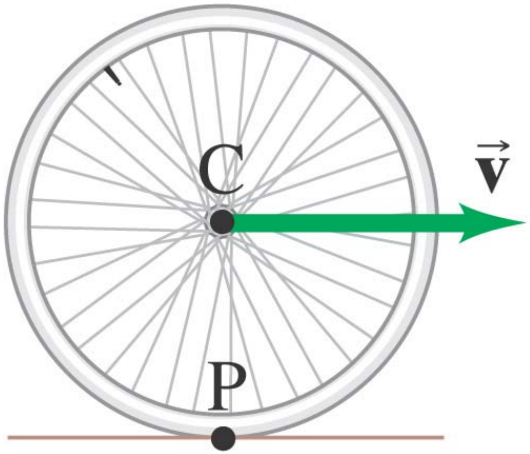
10-8 Rotational Kinetic Energy

The torque does **work** as it moves the wheel through an angle ***θ***:

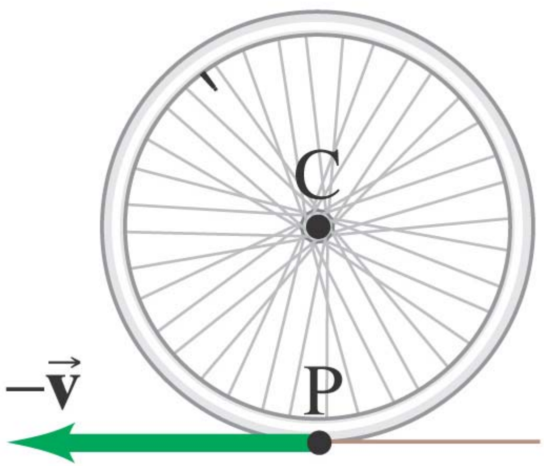
$$W = \tau \Delta \theta.$$



10-9 Rotational Plus Translational Motion; Rolling



In (a), a wheel is **rolling** without slipping. The point P, touching the ground, is instantaneously at **rest**, and the center moves with velocity $\vec{\mathbf{v}}$.



In (b) the same wheel is seen from a **reference frame** where C is at rest. Now point P is moving with velocity $-\vec{\mathbf{v}}$.

The **linear speed** of the wheel is related to its **angular speed**:

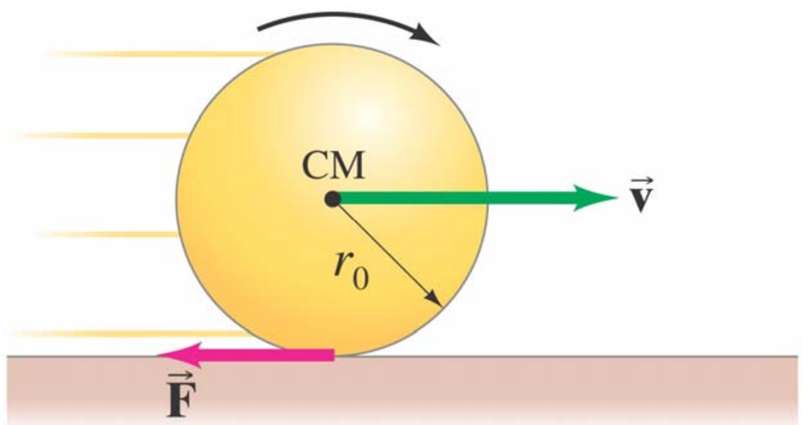
$$v = R\omega.$$

10-10 Why Does a Rolling Sphere Slow Down?

A rolling sphere will slow down and stop rather than roll forever. What force would cause this?

If we say "friction", there are problems:

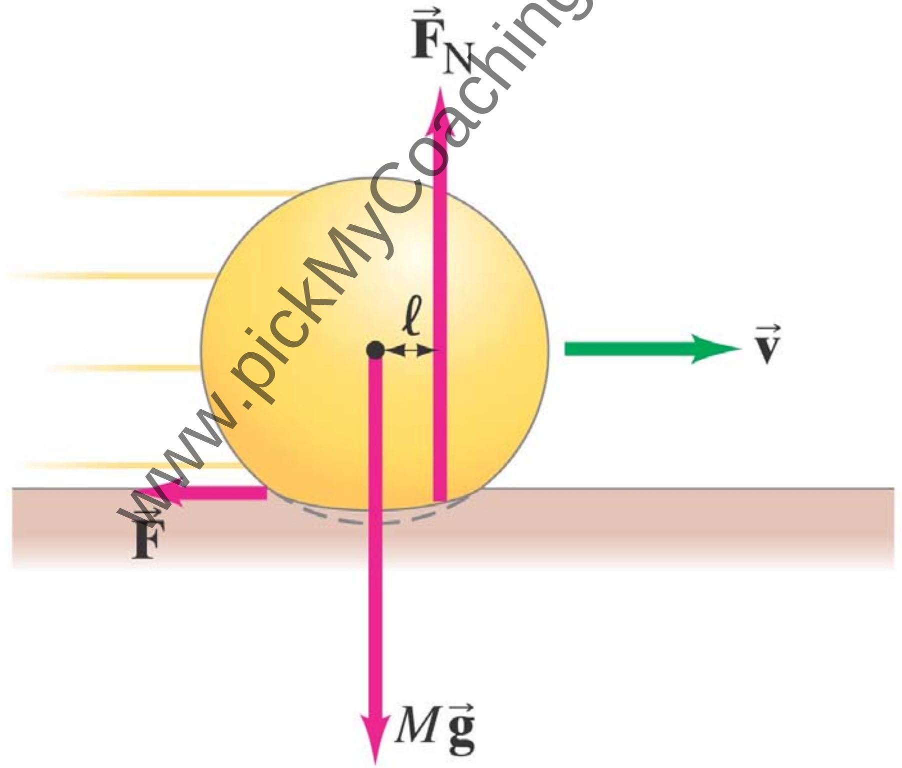
- The frictional force has to act at the point of contact; this means the angular speed of the sphere would increase.



- Gravity and the normal force both act through the center of mass, and cannot create a torque.

10-10 Why Does a Rolling Sphere Slow Down?

The solution: No real sphere is perfectly rigid. The bottom will deform, and the normal force will create a torque that slows the sphere.



Summary of Chapter 10

- Angles are measured in radians; a whole circle is 2π radians.

- Angular velocity is the rate of change of angular position.

- Angular acceleration is the rate of change of angular velocity.

- The angular velocity and acceleration can be related to the linear velocity and acceleration.

- The frequency is the number of full revolutions per second; the period is the inverse of the frequency.

Summary of Chapter 10, cont.

- The equations for rotational motion with constant angular acceleration have the same form as those for linear motion with constant acceleration.
- Torque is the product of force and lever arm.
- The rotational inertia depends not only on the mass of an object but also on the way its mass is distributed around the axis of rotation.
- The angular acceleration is proportional to the torque and inversely proportional to the rotational inertia.

Summary of Chapter 10, cont.

- An object that is rotating has rotational kinetic energy. If it is translating as well, the translational kinetic energy must be added to the rotational to find the total kinetic energy.

- Angular momentum is $L = I\omega$.

- If the net torque on an object is zero, its angular momentum does not change.