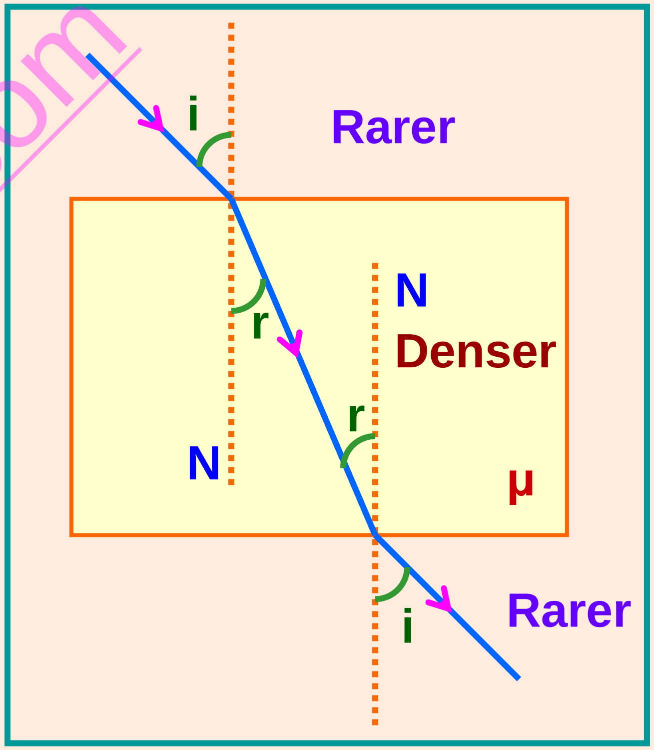
RAY OPTICS - I

Refraction of Light:

Refraction is the phenomenon of change in the path of light as it travels from one medium to another (when the ray of light is incident obliquely).

It can also be defined as the phenomenon of change in speed of light from one medium to another.



Laws of Refraction:

I Law: The incident ray, the normal to the refracting surface at the point of incidence and the refracted ray all lie in the same plane.

II Law: For a given pair of media and for light of a given wavelength, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant. (Snell's Law)

$$\mu = \frac{\sin i}{\sin r}$$

(The constant μ is called refractive index of the medium, i is the angle of incidence and r is the angle of refraction.)

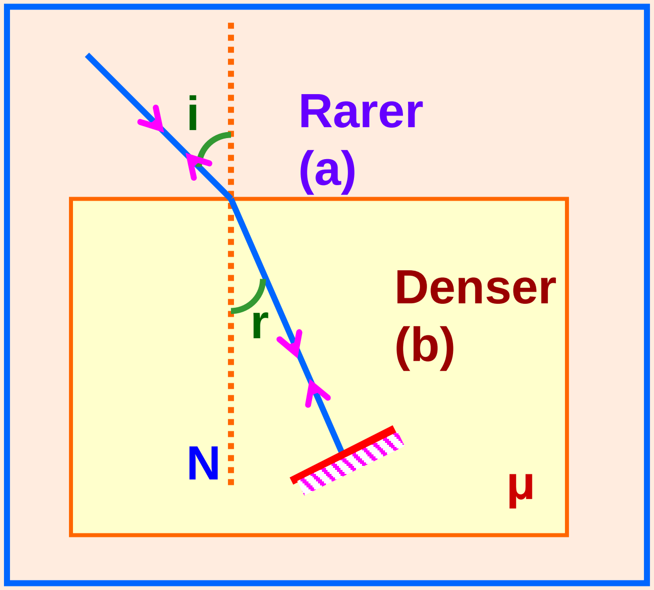
TIPS:

1. μ of optically rarer medium is lower and that of a denser medium is higher.
2. μ of denser medium w.r.t. rarer medium is more than 1 and that of rarer medium w.r.t. denser medium is less than 1. ($\mu_{air} = \mu_{vacuum} = 1$)
3. In refraction, the velocity and wavelength of light change.
4. In refraction, the frequency and phase of light do not change.
5. ${}_a\mu_m = c_a / c_m$ and ${}_a\mu_m = \lambda_a / \lambda_m$

Principle of Reversibility of Light:

$${}_a\mu_b = \frac{\sin i}{\sin r} \qquad {}_b\mu_a = \frac{\sin r}{\sin i}$$

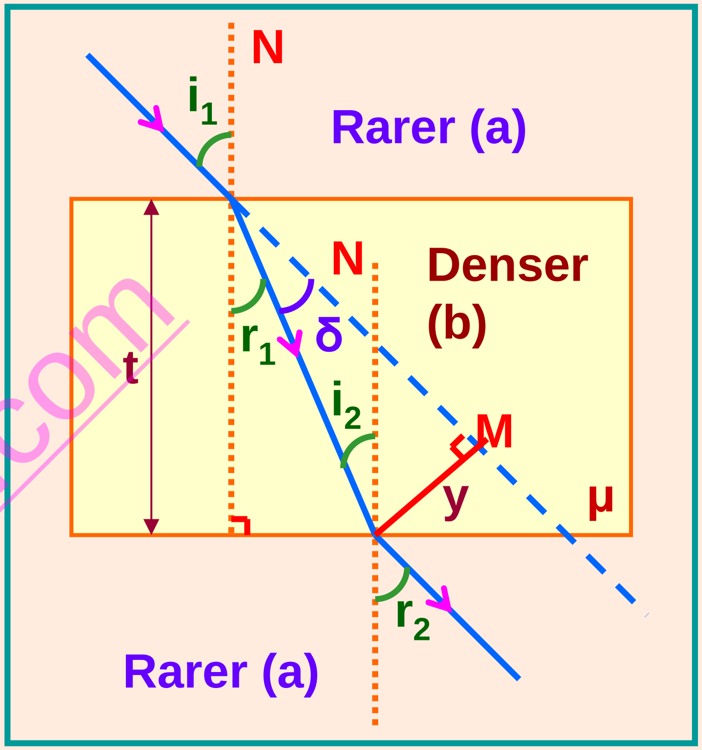
$\boxed{{}_a\mu_b \times {}_b\mu_a = 1}$ or $\boxed{{}_a\mu_b = 1 / {}_b\mu_a}$



If a ray of light, after suffering any number of reflections and/or refractions has its path reversed at any stage, it travels back to the source along the same path in the opposite direction.

A natural consequence of the principle of reversibility is that the image and object positions can be interchanged. These positions are called conjugate positions.

Refraction through a Parallel Slab:



$${}_a\mu_b = \frac{\sin i_1}{\sin r_1} \qquad {}_b\mu_a = \frac{\sin i_2}{\sin r_2}$$

But ${}_a\mu_b \times {}_b\mu_a = 1$

$$\therefore \frac{\sin i_1}{\sin r_1} \times \frac{\sin i_2}{\sin r_2} = 1$$

It implies that $i_1 = r_2$ and $i_2 = r_1$
since $i_1 \neq r_1$ and $i_2 \neq r_2$.

Lateral Shift:

$$\boxed{y = \frac{t \sin \delta}{\cos r_1}} \quad \text{or} \quad \boxed{y = \frac{t \sin(i_1 - r_1)}{\cos r_1}}$$

Special Case:

If i_1 is very small, then r_1 is also very small.
i.e. $\sin(i_1 - r_1) = i_1 - r_1$ and $\cos r_1 = 1$

$$\therefore \boxed{y = t\,(i_1 - r_1)} \quad \text{or} \quad \boxed{y = t\, i_1(1 - 1/{}_a\mu_b)}$$

Refraction through a Compound Slab:

$$_a\mu_b = \frac{\sin i_1}{\sin r_1}$$

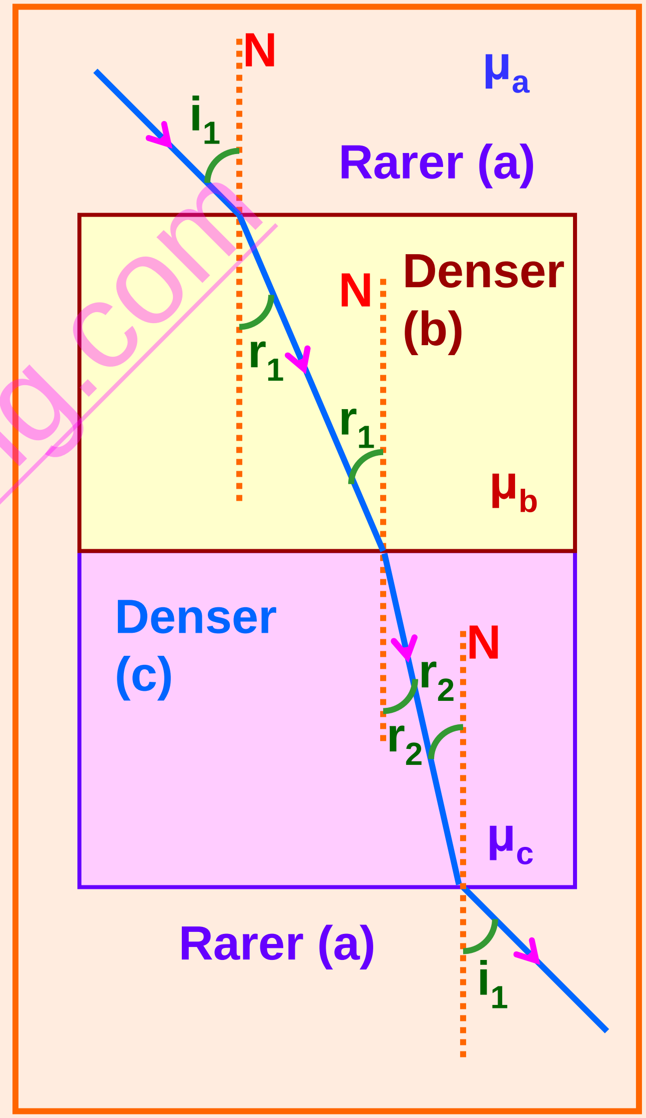
$$_b\mu_c = \frac{\sin r_1}{\sin r_2}$$

$$_c\mu_a = \frac{\sin r_2}{\sin i_1}$$

$$\boxed{_a\mu_b \times {_b\mu_c} \times {_c\mu_a} = 1}$$

or $$\boxed{_a\mu_b \times {_b\mu_c} = {_a\mu_c}}$$

or $$\boxed{_b\mu_c = {_a\mu_c} / {_a\mu_b}}$$

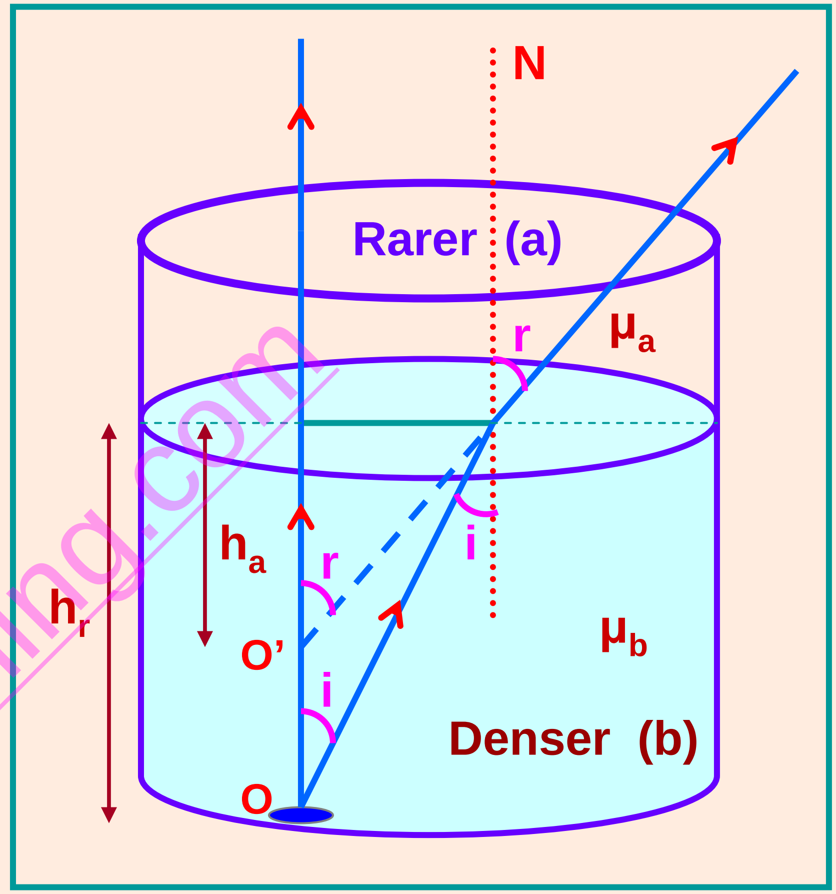


$\mu_c > \mu_b$

Apparent Depth of a Liquid:

$$ {}_b\mu_a = \frac{\sin i}{\sin r} \quad \text{or} \quad {}_a\mu_b = \frac{\sin r}{\sin i} $$

$$ {}_a\mu_b = \frac{h_r}{h_a} = \frac{\text{Real depth}}{\text{Apparent depth}} $$



Apparent Depth of a Number of Immiscible Liquids:

$$ h_a = \sum_{i=1}^{n} h_i / \mu_i $$

Apparent Shift:

Apparent shift = $h_r - h_a = h_r - (h_r / \mu)$

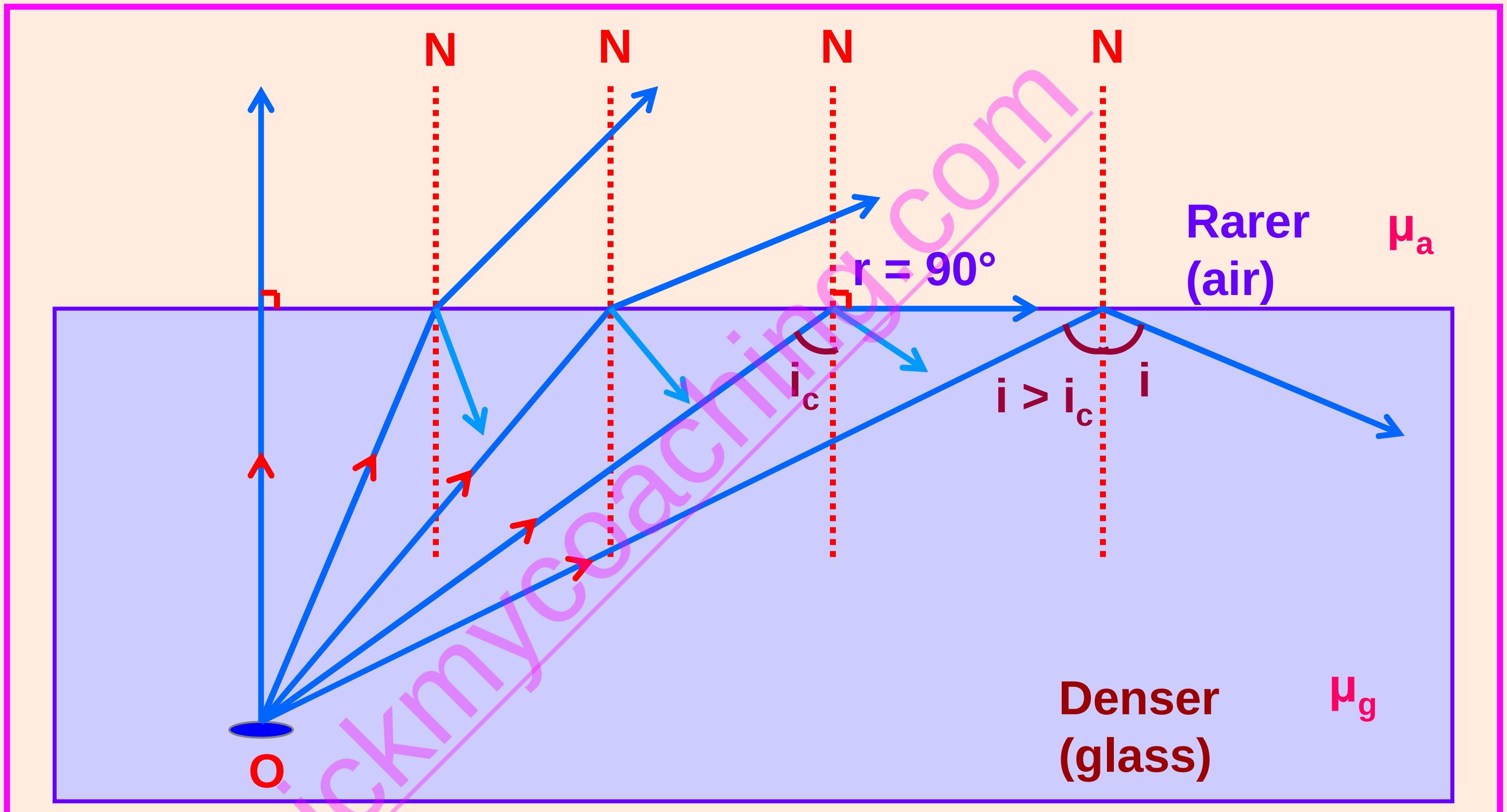
$= h_r [1 - 1/\mu]$

TIPS:

1. If the observer is in rarer medium and the object is in denser medium then $h_a < h_r$. (To a bird, the fish appears to be nearer than actual depth.)
2. If the observer is in denser medium and the object is in rarer medium then $h_a > h_r$. (To a fish, the bird appears to be farther than actual height.)

Total Internal Reflection:

Total Internal Reflection (TIR) is the phenomenon of complete reflection of light back into the same medium for angles of incidence greater than the critical angle of that medium.



Conditions for TIR:

1. **The incident ray must be in optically denser medium.**
2. **The angle of incidence in the denser medium must be greater than the critical angle for the pair of media in contact.**

Relation between Critical Angle and Refractive Index:

Critical angle is the angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90°.

$$ {}_g\mu_a = \frac{\sin i}{\sin r} = \frac{\sin i_c}{\sin 90°} = \sin i_c $$

or $\; {}_a\mu_g = \dfrac{1}{{}_g\mu_a}$ $\;\therefore\;$ $\boxed{{}_a\mu_g = \dfrac{1}{\sin i_c}}$ or $\boxed{\sin i_c = \dfrac{1}{{}_a\mu_g}}$ Also $\boxed{\sin i_c = \dfrac{\lambda_g}{\lambda_a}}$

Red colour has maximum value of critical angle and Violet colour has minimum value of critical angle since,

$$ \boxed{\sin i_c = \frac{1}{{}_a\mu_g} = \frac{1}{a + (b/\lambda^2)}} $$

Applications of T I R:

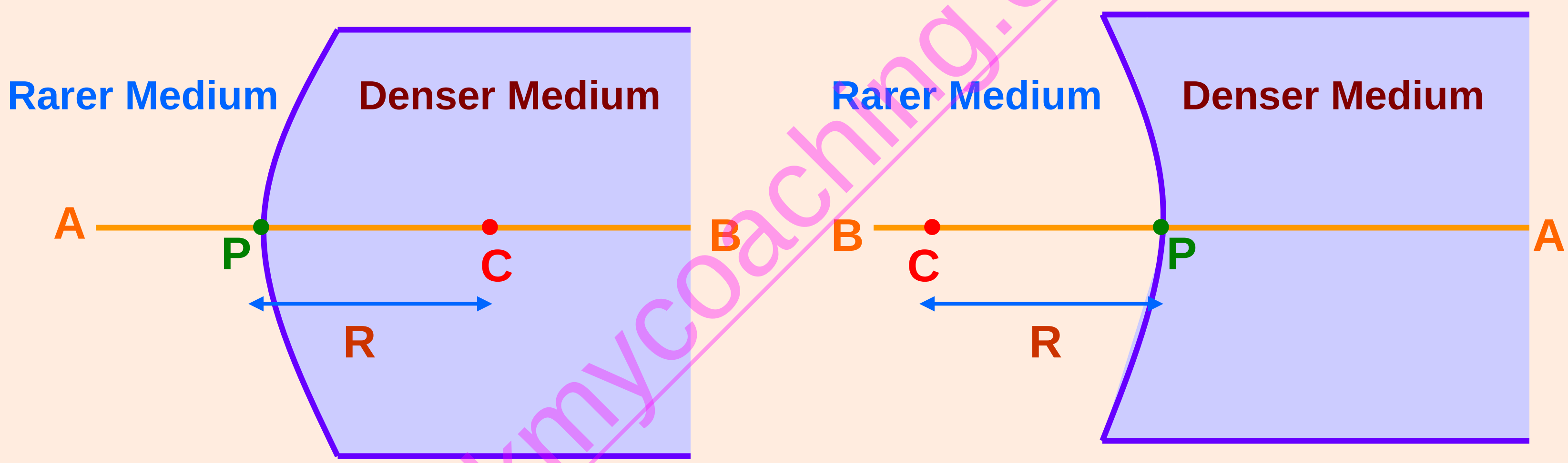
1. **Mirage formation**
2. **Looming**
3. **Totally reflecting Prisms**
4. **Optical Fibres**
5. **Sparkling of Diamonds**

Spherical Refracting Surfaces:

A spherical refracting surface is a part of a sphere of refracting material.

A refracting surface which is convex towards the rarer medium is called convex refracting surface.

A refracting surface which is concave towards the rarer medium is called concave refracting surface.



APCB – Principal Axis
C – Centre of Curvature
P – Pole
R – Radius of Curvature

Assumptions:

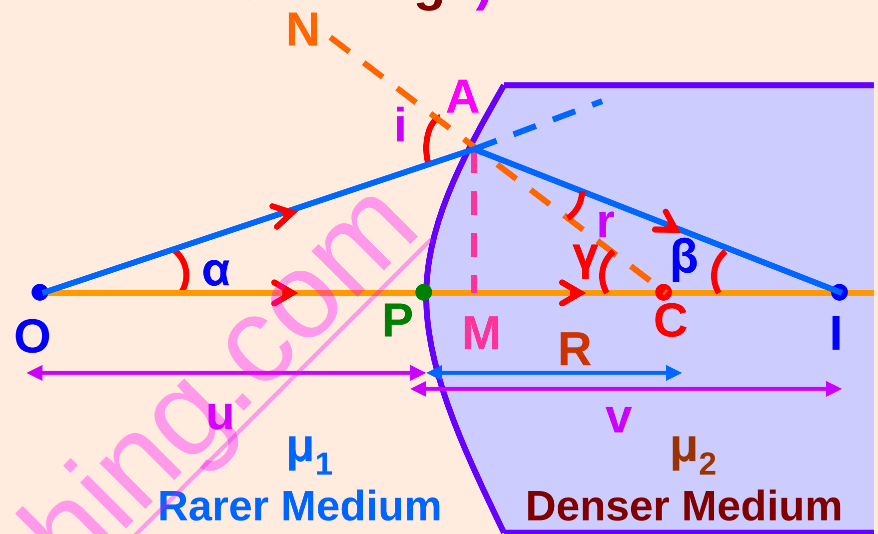
1. Object is the point object lying on the principal axis.
2. The incident and the refracted rays make small angles with the principal axis.
3. The aperture (diameter of the curved surface) is small.

New Cartesian Sign Conventions:

1. The incident ray is taken from left to right.
2. All the distances are measured from the pole of the refracting surface.
3. The distances measured along the direction of the incident ray are taken positive and against the incident ray are taken negative.
4. The vertical distances measured from principal axis in the upward direction are taken positive and in the downward direction are taken negative.

Refraction at Convex Surface:

(From Rarer Medium to Denser Medium - Real Image)



$i = \alpha + \gamma$

$\gamma = r + \beta$ or $r = \gamma - \beta$

$$\tan \alpha = \frac{MA}{MO} \quad \text{or} \quad \alpha = \frac{MA}{MO}$$

$$\tan \beta = \frac{MA}{MI} \quad \text{or} \quad \beta = \frac{MA}{MI}$$

$$\tan \gamma = \frac{MA}{MC} \quad \text{or} \quad \gamma = \frac{MA}{MC}$$

According to Snell's law,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \quad \text{or} \quad \frac{i}{r} = \frac{\mu_2}{\mu_1} \quad \text{or} \quad \boxed{\mu_1 i = \mu_2 r}$$

Substituting for i, r, α, β and γ, replacing M by P and rearranging,

$$\frac{\mu_1}{PO} + \frac{\mu_2}{PI} = \frac{\mu_2 - \mu_1}{PC}$$

Applying sign conventions with values,

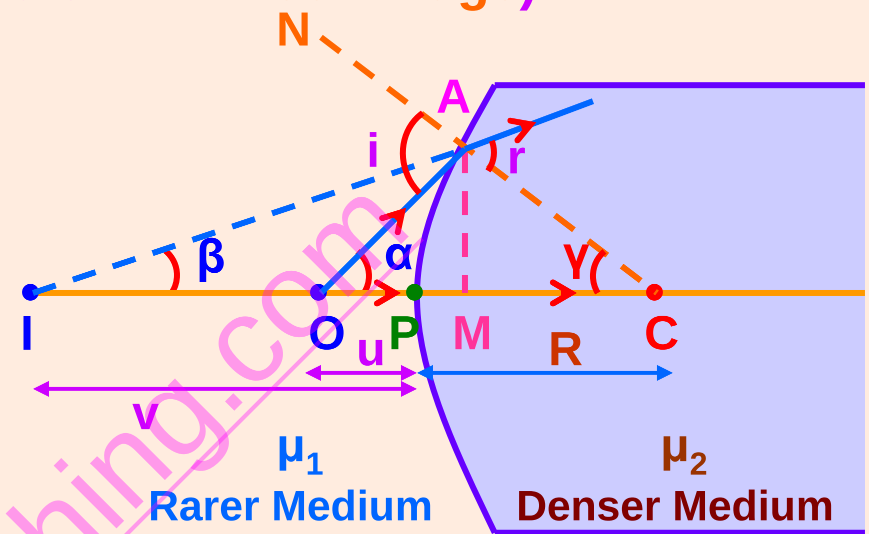
PO = - u, PI = + v and PC = + R

$$\boxed{\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}}$$

Refraction at Convex Surface:

(From Rarer Medium to Denser Medium - Virtual Image)

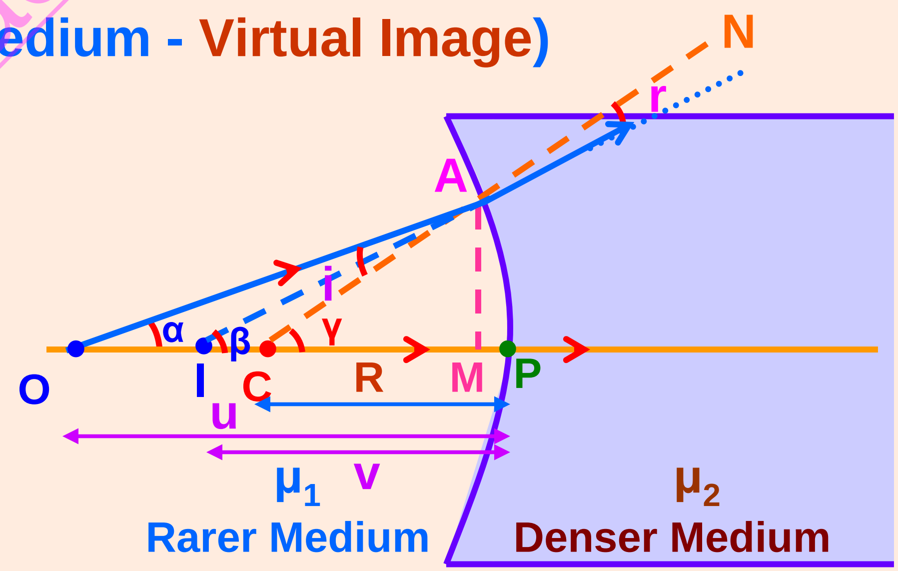
$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$



Refraction at Concave Surface:

(From Rarer Medium to Denser Medium - Virtual Image)

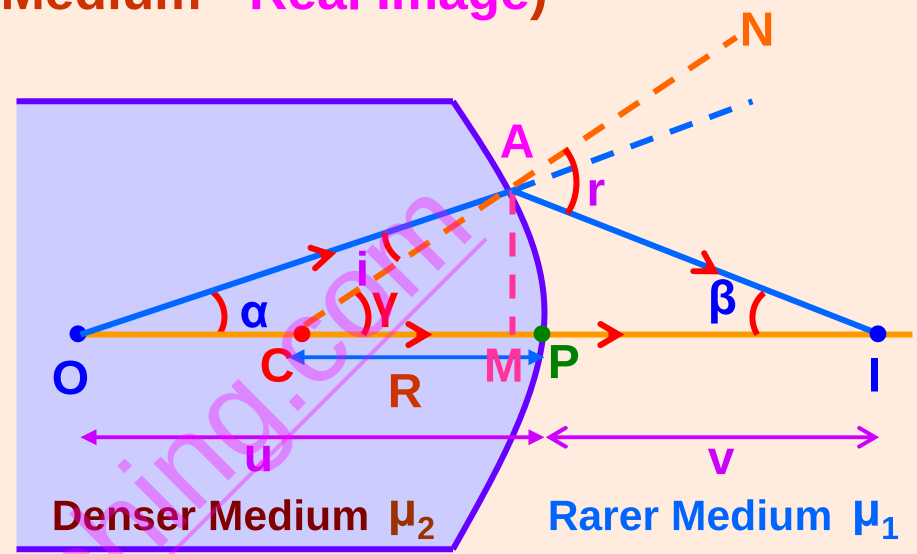
$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$



Refraction at Convex Surface:

(From Denser Medium to Rarer Medium - Real Image)

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$



Refraction at Convex Surface:

(From Denser Medium to Rarer Medium - Virtual Image)

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

Refraction at Concave Surface:

(From Denser Medium to Rarer Medium - Virtual Image)

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

Note:

1. **Expression for 'object in rarer medium' is same for whether it is real or virtual image or convex or concave surface.**

$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

2. **Expression for 'object in denser medium' is same for whether it is real or virtual image or convex or concave surface.**

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

3. **However the values of u, v, R, etc. must be taken with proper sign conventions while solving the numerical problems.**

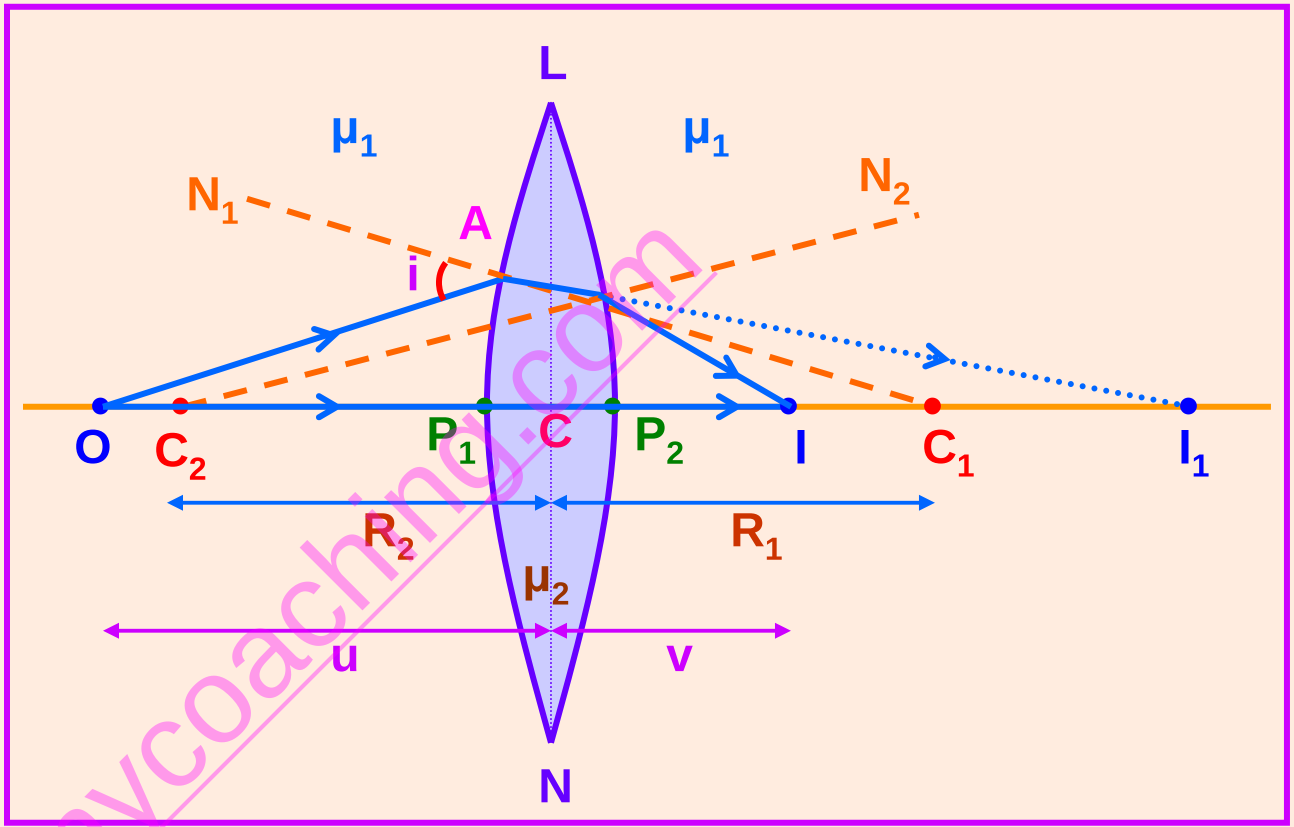
4. **The refractive indices μ_1 and μ_2 get interchanged in the expressions.**

Lens Maker's Formula:

For refraction at LP_1N,

$$\frac{\mu_1}{CO} + \frac{\mu_2}{CI_1} = \frac{\mu_2 - \mu_1}{CC_1}$$

(as if the image is formed in the denser medium)



For refraction at LP_2N,

$$\frac{\mu_2}{-CI_1} + \frac{\mu_1}{CI} = \frac{-(\mu_1 - \mu_2)}{CC_2}$$

(as if the object is in the denser medium and the image is formed in the rarer medium)

Combining the refractions at both the surfaces,

$$\frac{\mu_1}{CO} + \frac{\mu_1}{CI} = (\mu_2 - \mu_1)\left(\frac{1}{CC_1} + \frac{1}{CC_2}\right)$$

Substituting the values with sign conventions,

$$\frac{1}{-u} + \frac{1}{v} = \frac{(\mu_2 - \mu_1)}{\mu_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Since $\mu_2 / \mu_1 = \mu$

$$\frac{1}{-u} + \frac{1}{v} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or

$$\frac{1}{-u} + \frac{1}{v} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

When the object is kept at infinity, the image is formed at the principal focus.

i.e. $u = -\infty$, $v = +f$.

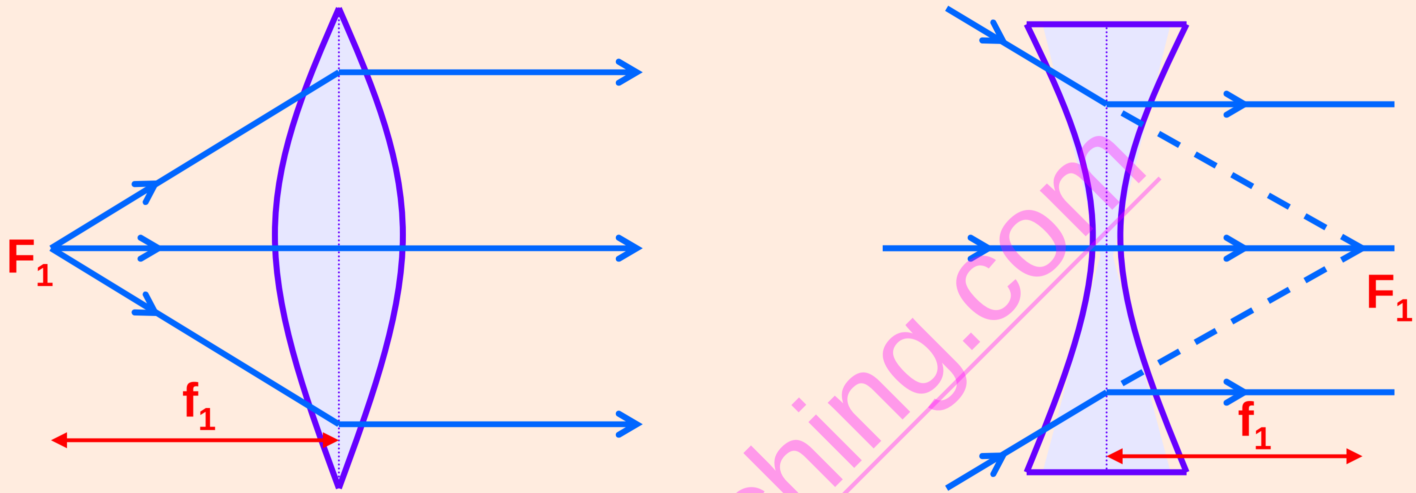
So, $$\boxed{\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

This equation is called 'Lens Maker's Formula'.

Also, from the above equations we get, $$\boxed{\frac{1}{-u} + \frac{1}{v} = \frac{1}{f}}$$

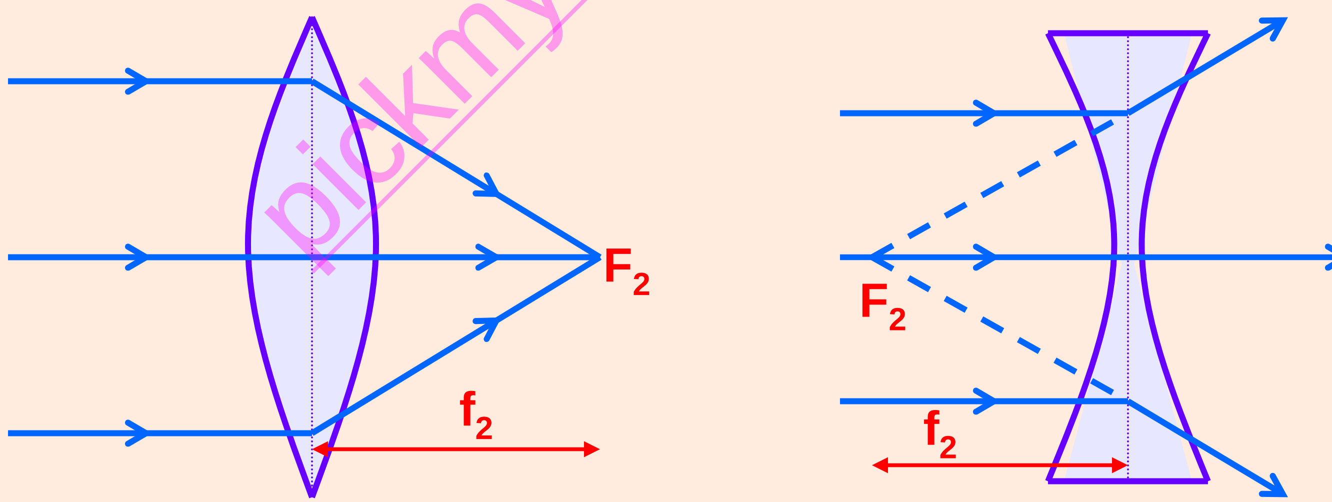
First Principal Focus:

First Principal Focus is the point on the principal axis of the lens at which if an object is placed, the image would be formed at infinity.



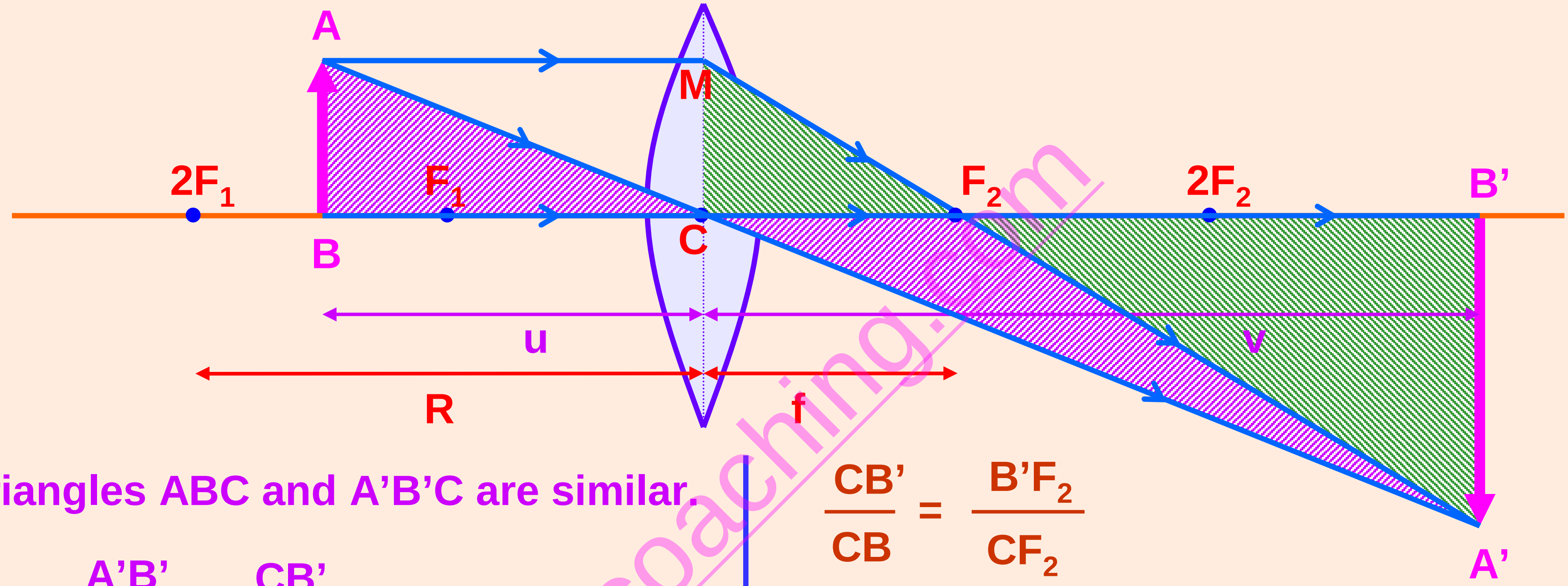
Second Principal Focus:

Second Principal Focus is the point on the principal axis of the lens at which the image is formed when the object is kept at infinity.



Thin Lens Formula (Gaussian Form of Lens Equation):

For Convex Lens:



Triangles ABC and A'B'C are similar.

$$\frac{A'B'}{AB} = \frac{CB'}{CB}$$

Triangles MCF_2 and $A'B'F_2$ are similar.

$$\frac{A'B'}{MC} = \frac{B'F_2}{CF_2}$$

or $$\frac{A'B'}{AB} = \frac{B'F_2}{CF_2}$$

$$\frac{CB'}{CB} = \frac{B'F_2}{CF_2}$$

$$\frac{CB'}{CB} = \frac{CB' - CF_2}{CF_2}$$

According to new Cartesian sign conventions,

$CB = - u$, $CB' = + v$ and $CF_2 = + f$.

$$\therefore \boxed{\frac{1}{v} - \frac{1}{u} = \frac{1}{f}}$$

Linear Magnification:

Linear magnification produced by a lens is defined as the ratio of the size of the image to the size of the object.

$$m = \frac{I}{O}$$

$$\frac{A'B'}{AB} = \frac{CB'}{CB}$$

According to new Cartesian sign conventions,

A'B' = + I, AB = - O, CB' = + v and CB = - u.

$$\frac{+I}{-O} = \frac{+v}{-u} \quad \text{or} \quad \boxed{m = \frac{I}{O} = \frac{v}{u}}$$

Magnification in terms of v and f:

$$\boxed{m = \frac{f - v}{f}}$$

Magnification in terms of v and f:

$$\boxed{m = \frac{f}{f - u}}$$

Power of a Lens:

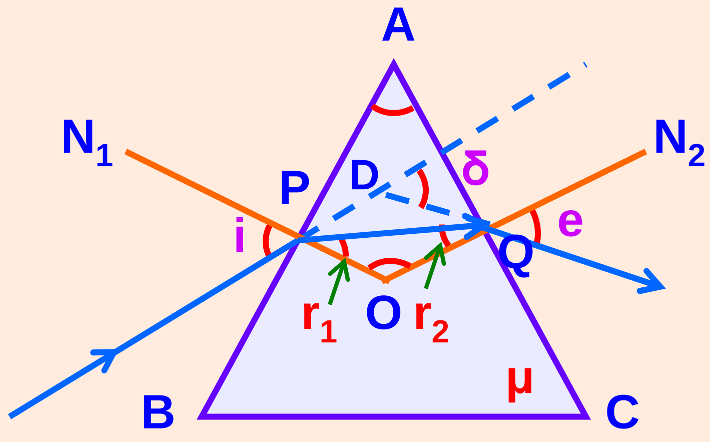
Power of a lens is its ability to bend a ray of light falling on it and is reciprocal of its focal length. When f is in metre, power is measured in Dioptre (D).

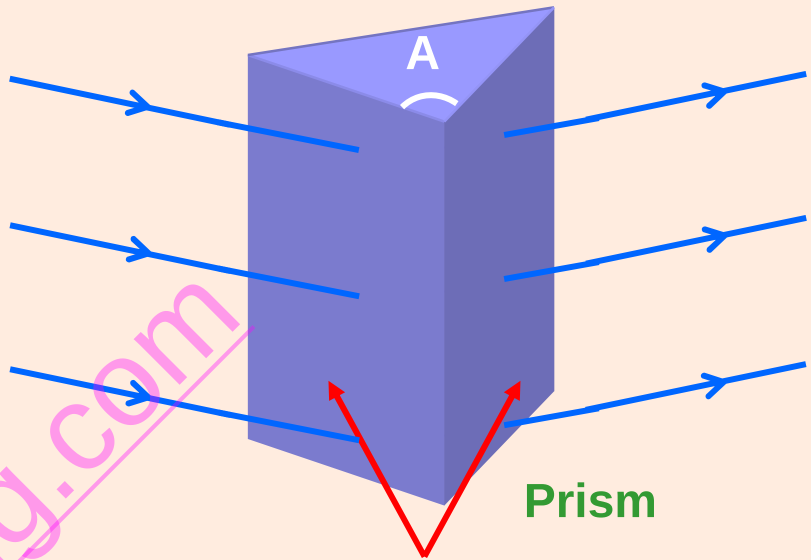
$$\boxed{P = \frac{1}{f}}$$

RAY OPTICS - II

1. **Refraction through a Prism**
2. **Expression for Refractive Index of Prism**
3. **Dispersion**
4. **Angular Dispersion and Dispersive Power**
5. **Blue Colour of the Sky and Red Colour of the Sun**
6. **Compound Microscope**
7. **Astronomical Telescope (Normal Adjustment)**
8. **Astronomical Telescope (Image at LDDV)**
9. **Newtonian Telescope (Reflecting Type)**
10. **Resolving Power of Microscope and Telescope**

Refraction of Light through Prism:





In quadrilateral APOQ,

$A + O = 180°$ …… .(1)

(since N_1 and N_2 are normal)

In triangle OPQ,

$r_1 + r_2 + O = 180°$ …… .(2)

In triangle DPQ,

$\delta = (i - r_1) + (e - r_2)$

$\delta = (i + e) – (r_1 + r_2)$ …….(3)

From (1) and (2),

$A = r_1 + r_2$

From (3),

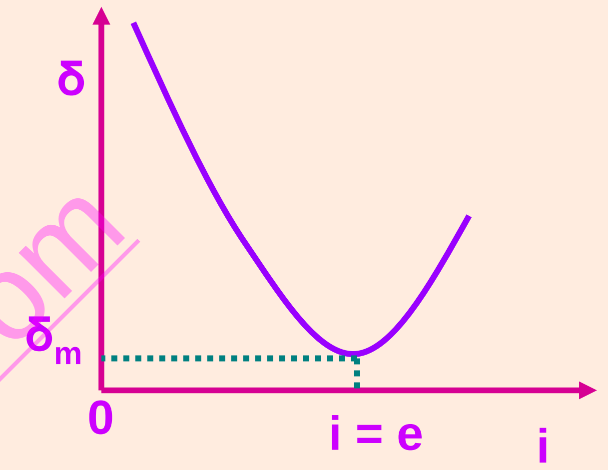
$\delta = (i + e) – (A)$

or $\boxed{i + e = A + \delta}$

Sum of angle of incidence and angle of emergence is equal to the sum of angle of prism and angle of deviation.

Variation of angle of deviation with angle of incidence:

When angle of incidence increases, the angle of deviation decreases.



At a particular value of angle of incidence the angle of deviation becomes minimum and is called 'angle of minimum deviation'.

At δ_m, $i = e$ and $r_1 = r_2 = r$ (say)

After minimum deviation, angle of deviation increases with angle of incidence.

Refractive Index of Material of Prism:

$A = r_1 + r_2$

$A = 2r$

$\boxed{r = A / 2}$

$i + e = A + \delta$

$2i = A + \delta_m$

$\boxed{i = (A + \delta_m) / 2}$

According to Snell's law,

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin i}{\sin r}$$

$$\therefore \quad \boxed{\mu = \frac{\sin \frac{(A + \delta_m)}{2}}{\sin \frac{A}{2}}}$$

Refraction by a Small-angled Prism for Small angle of Incidence:

$$\mu = \frac{\sin i}{\sin r_1} \quad \text{and} \quad \mu = \frac{\sin e}{\sin r_2}$$

If i is assumed to be small, then r_1, r_2 and e will also be very small.
So, replacing sines of the angles by angles themselves, we get

$$\mu = \frac{i}{r_1} \quad \text{and} \quad \mu = \frac{e}{r_2}$$

$$i + e = \mu\,(r_1 + r_2) = \mu\,A$$

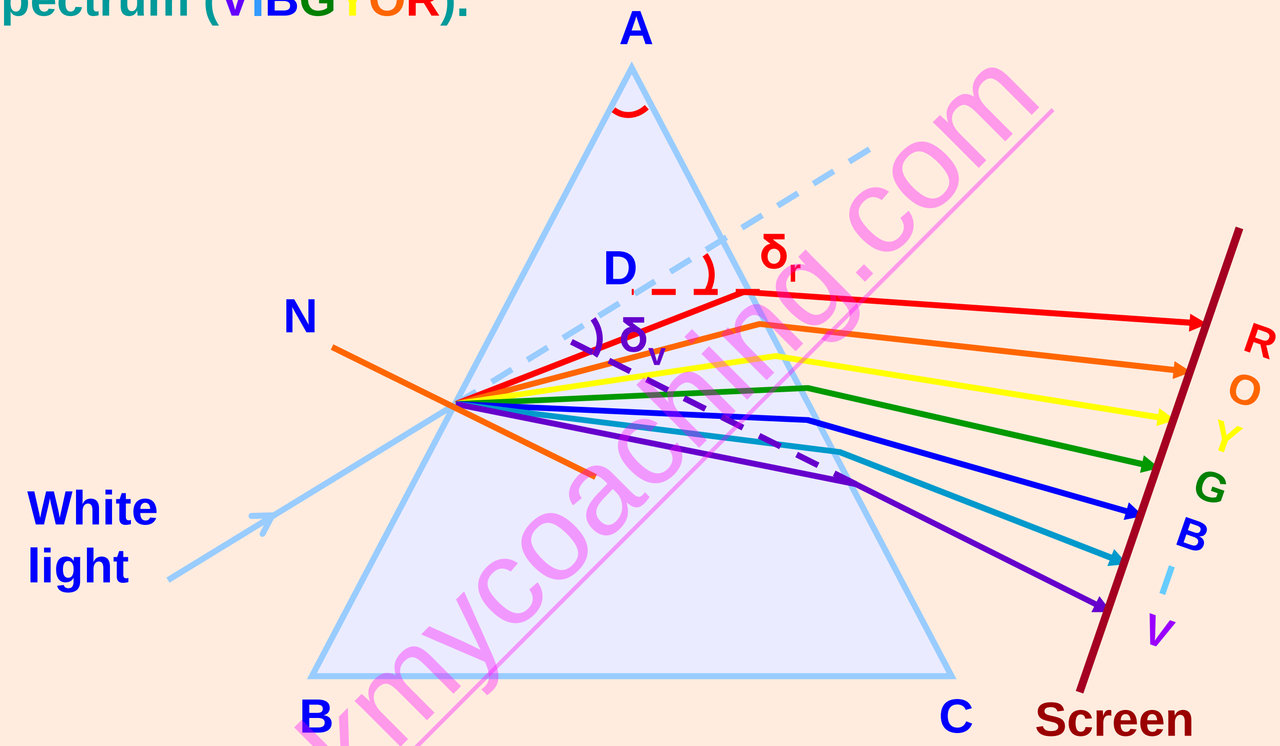
But $i + e = A + \delta$

So, $A + \delta = \mu\,A$

or $\boxed{\delta = A\,(\mu - 1)}$

Dispersion of White Light through Prism:

The phenomenon of splitting a ray of white light into its constituent colours (wavelengths) is called dispersion and the band of colours from violet to red is called spectrum (VIBGYOR).



Cause of Dispersion:

$$\mu_v = \frac{\sin i}{\sin r_v} \quad \text{and} \quad \mu_r = \frac{\sin i}{\sin r_r}$$

Since $\mu_v > \mu_r$, $r_r > r_v$

So, the colours are refracted at different angles and hence get separated.

Dispersion can also be explained on the basis of Cauchy's equation.

$$\mu = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4}$$ (where a, b and c are constants for the material)

Since $\lambda_v < \lambda_r$, $\mu_v > \mu_r$

But $\delta = A(\mu - 1)$

Therefore, $\delta_v > \delta_r$

So, the colours get separated with different angles of deviation.
Violet is most deviated and Red is least deviated.

Angular Dispersion:

1. The difference in the deviations suffered by two colours in passing through a prism gives the angular dispersion for those colours.
2. The angle between the emergent rays of any two colours is called angular dispersion between those colours.
3. It is the rate of change of angle of deviation with wavelength. ($\Phi = d\delta / d\lambda$)

$\Phi = \delta_v - \delta_r$ or $\boxed{\Phi = (\mu_v - \mu_r) A}$

Dispersive Power:

The dispersive power of the material of a prism for any two colours is defined as the ratio of the angular dispersion for those two colours to the mean deviation produced by the prism.

It may also be defined as dispersion per unit deviation.

$$\omega = \frac{\Phi}{\delta}$$ where δ is the mean deviation and $$\delta = \frac{\delta_v + \delta_r}{2}$$

Also $$\omega = \frac{\delta_v - \delta_r}{\delta}$$ or $$\omega = \frac{(\mu_v - \mu_r)A}{(\mu_y - 1)A}$$ or $$\omega = \frac{(\mu_v - \mu_r)}{(\mu_y - 1)}$$

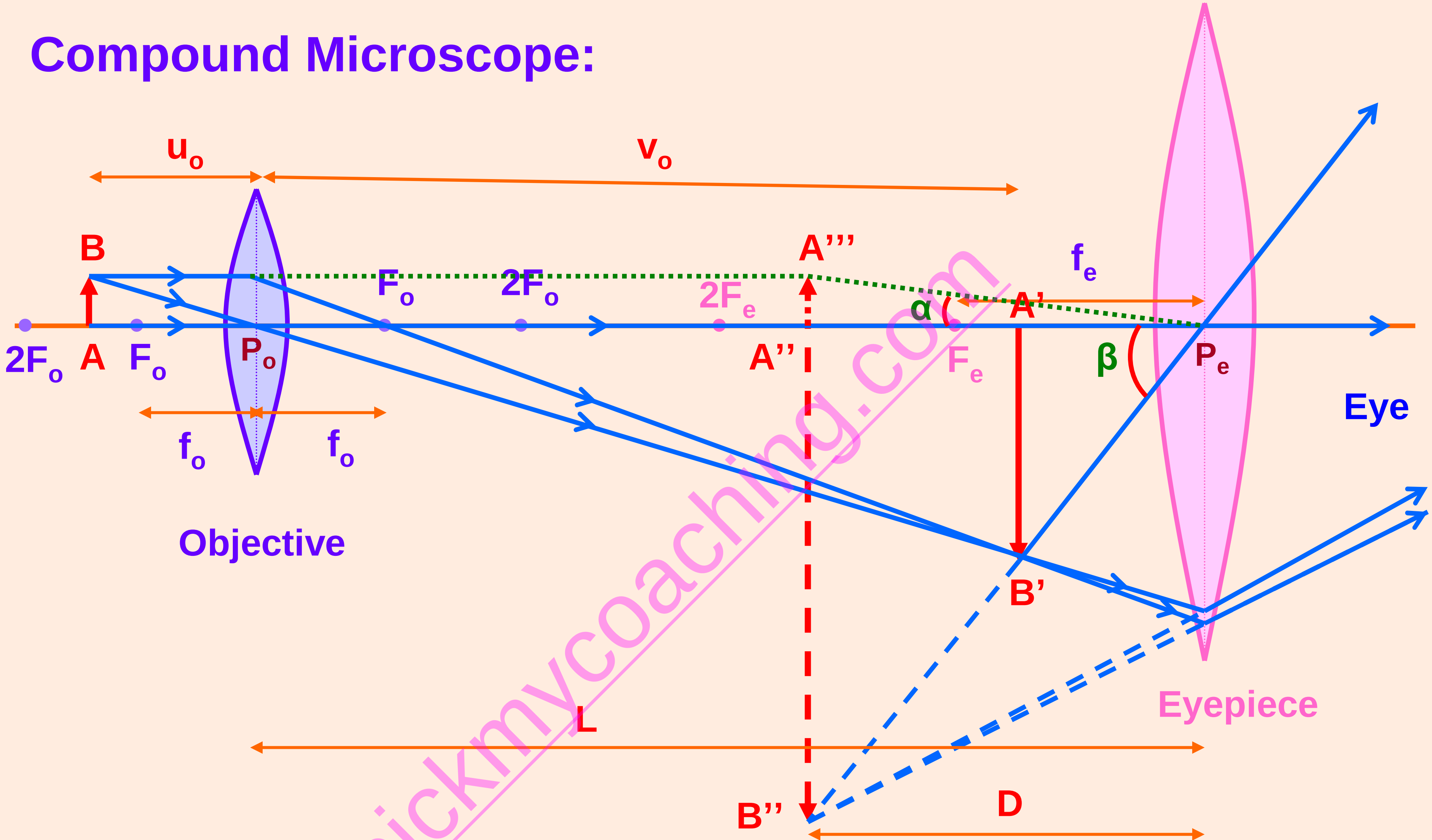
Scattering of Light – Blue colour of the sky and Reddish appearance of the Sun at Sun-rise and Sun-set:

The molecules of the atmosphere and other particles that are smaller than the longest wavelength of visible light are more effective in scattering light of shorter wavelengths than light of longer wavelengths. The amount of scattering is inversely proportional to the fourth power of the wavelength. (Rayleigh Effect)

Light from the Sun near the horizon passes through a greater distance in the Earth's atmosphere than does the light received when the Sun is overhead. The correspondingly greater scattering of short wavelengths accounts for the reddish appearance of the Sun at rising and at setting.

When looking at the sky in a direction away from the Sun, we receive scattered sunlight in which short wavelengths predominate giving the sky its characteristic bluish colour.

Compound Microscope:



Objective: The converging lens nearer to the object.

Eyepiece: The converging lens through which the final image is seen.

Both are of short focal length. Focal length of eyepiece is slightly greater than that of the objective.

Angular Magnification or Magnifying Power (M):

Angular magnification or magnifying power of a compound microscope is defined as the ratio of the angle β subtended by the final image at the eye to the angle α subtended by the object seen directly, when both are placed at the least distance of distinct vision.

$$\boxed{M = \frac{\beta}{\alpha}}$$

Since angles are small,
$\alpha = \tan\alpha$ and $\beta = \tan\beta$

$$M = \frac{\tan\beta}{\tan\alpha}$$

$$M = \frac{A''B''}{D} \times \frac{D}{A''A'''}$$

$$M = \frac{A''B''}{D} \times \frac{D}{AB}$$

$$M = \frac{A''B''}{AB}$$

$$M = \frac{A''B''}{A'B'} \times \frac{A'B'}{AB}$$

$$M = M_e \times M_o$$

$$M_e = 1 - \frac{v_e}{f_e} \quad \text{or} \quad M_e = 1 + \frac{D}{f_e}$$

($v_e = - D = - 25$ cm)

and $M_o = \dfrac{v_o}{- u_o}$ $\therefore$

$$\boxed{M = \frac{v_o}{- u_o}\left(1 + \frac{D}{f_e}\right)}$$

Since the object is placed very close to the principal focus of the objective and the image is formed very close to the eyepiece,
$u_o \approx f_o$ and $v_o \approx L$

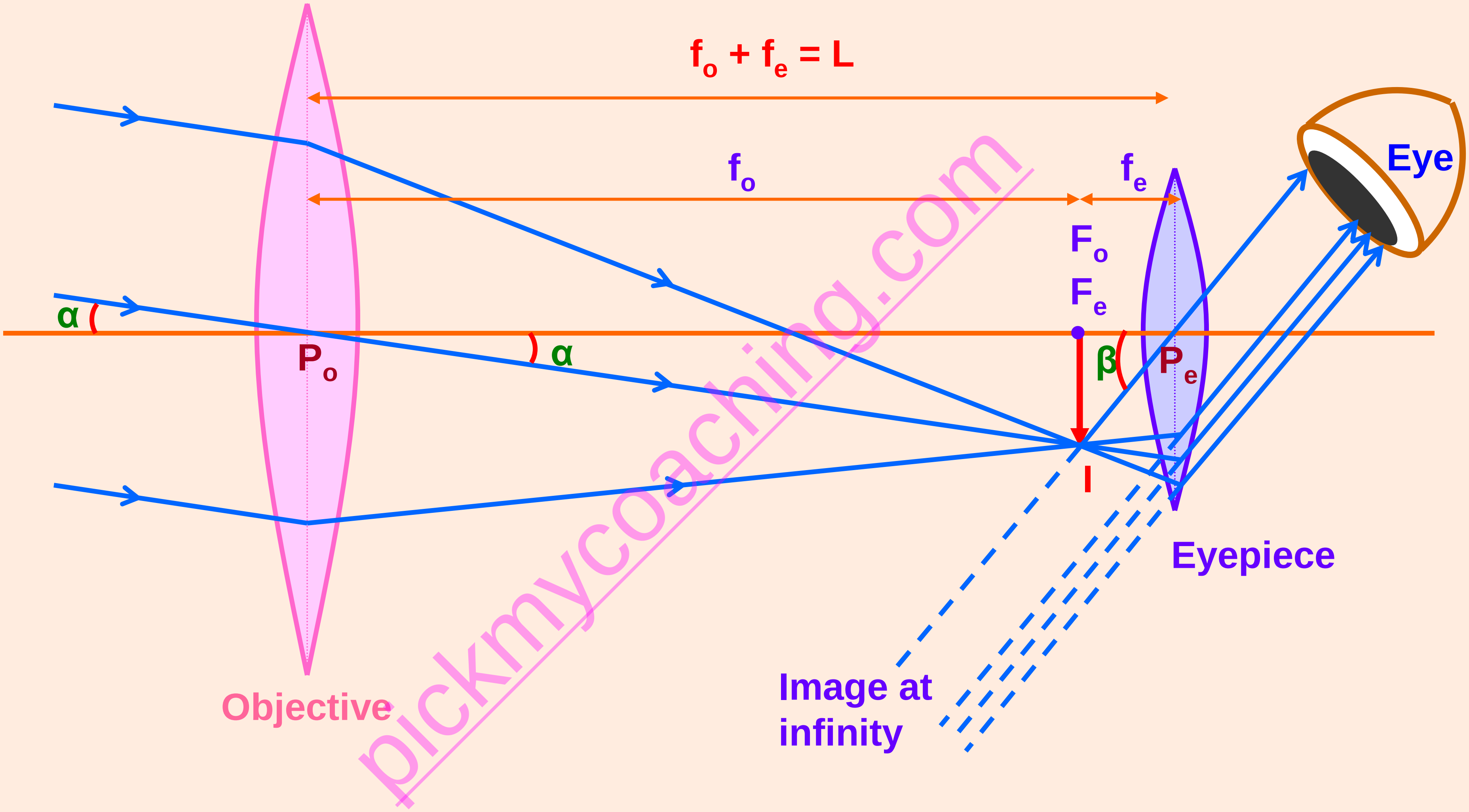
$$\boxed{M = \frac{- L}{f_o}\left(1 + \frac{D}{f_e}\right)}$$

or

$$\boxed{M \approx \frac{- L}{f_o} \times \frac{D}{f_e}}$$

(Normal adjustment i.e. image at infinity)

Astronomical Telescope: (Image formed at infinity – Normal Adjustment)



Focal length of the objective is much greater than that of the eyepiece.

Aperture of the objective is also large to allow more light to pass through it.

Angular magnification or Magnifying power of a telescope in normal adjustment is the ratio of the angle subtended by the image at the eye as seen through the telescope to the angle subtended by the object as seen directly, when both the object and the image are at infinity.

$$\boxed{M = \frac{\beta}{\alpha}}$$

Since angles are small, $\alpha = \tan \alpha$ and $\beta = \tan \beta$

$$M = \frac{\tan \beta}{\tan \alpha}$$

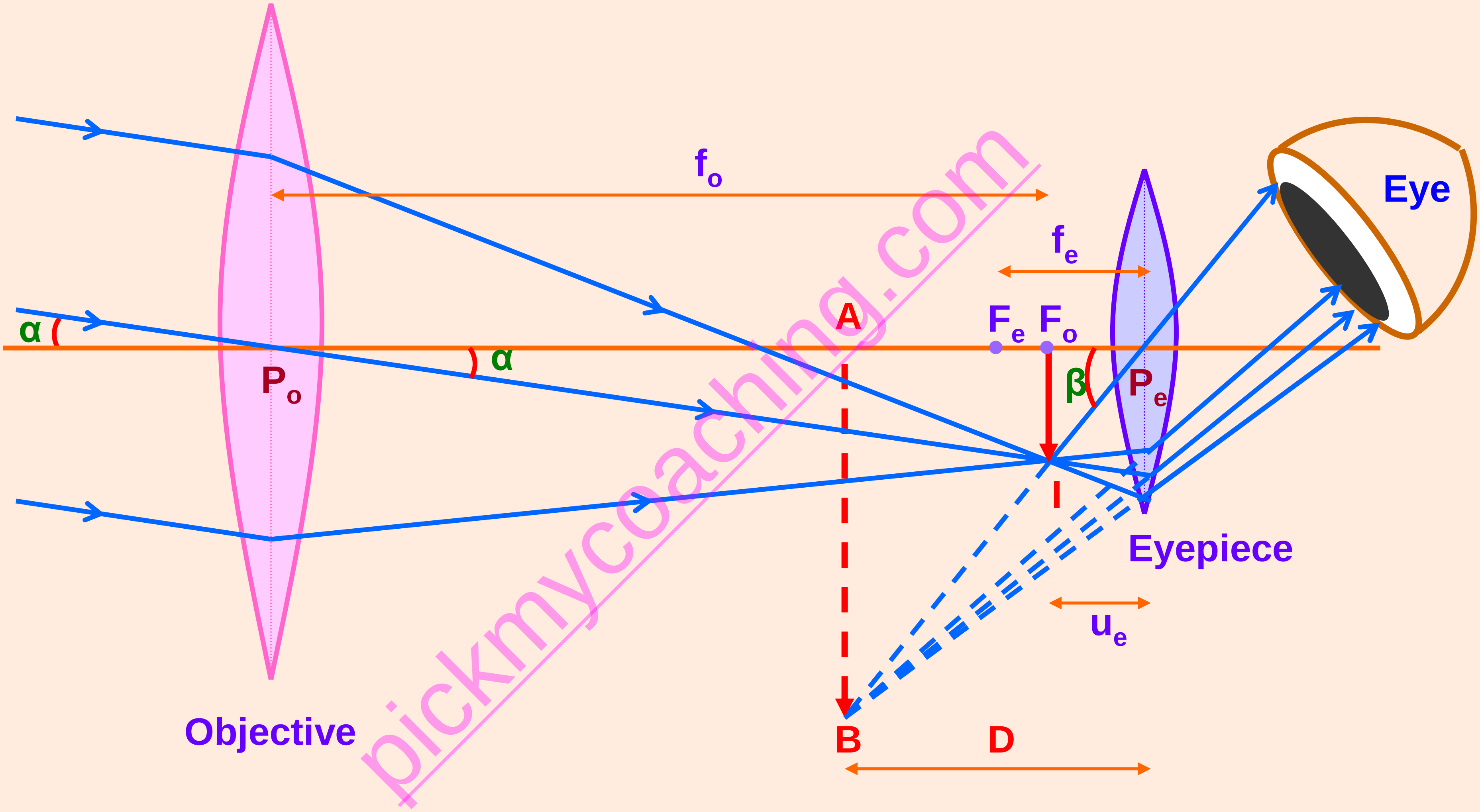
$$M = \frac{F_e I}{P_e F_e} \Big/ \frac{F_e I}{P_o F_e}$$

$$M = \frac{-I}{-f_e} \Big/ \frac{-I}{f_o}$$

$$\boxed{M = \frac{-f_o}{f_e}}$$

($f_o + f_e = L$ is called the length of the telescope in normal adjustment).

Astronomical Telescope: (Image formed at LDDV)



Angular magnification or magnifying power of a telescope in this case is defined as the ratio of the angle β subtended at the eye by the final image formed at the least distance of distinct vision to the angle α subtended at the eye by the object lying at infinity when seen directly.

$$\boxed{M = \frac{\beta}{\alpha}}$$

Since angles are small,
$\alpha = \tan\alpha$ and $\beta = \tan\beta$

$$M = \frac{\tan\beta}{\tan\alpha}$$

$$M = \frac{F_oI}{P_eF_o} \Big/ \frac{F_oI}{P_oF_o}$$

$$M = \frac{P_oF_o}{P_eF_o} \quad \text{or} \quad M = \frac{+f_o}{-u_e}$$

Lens Equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{becomes}$$

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

$$\text{or} \quad \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D}$$

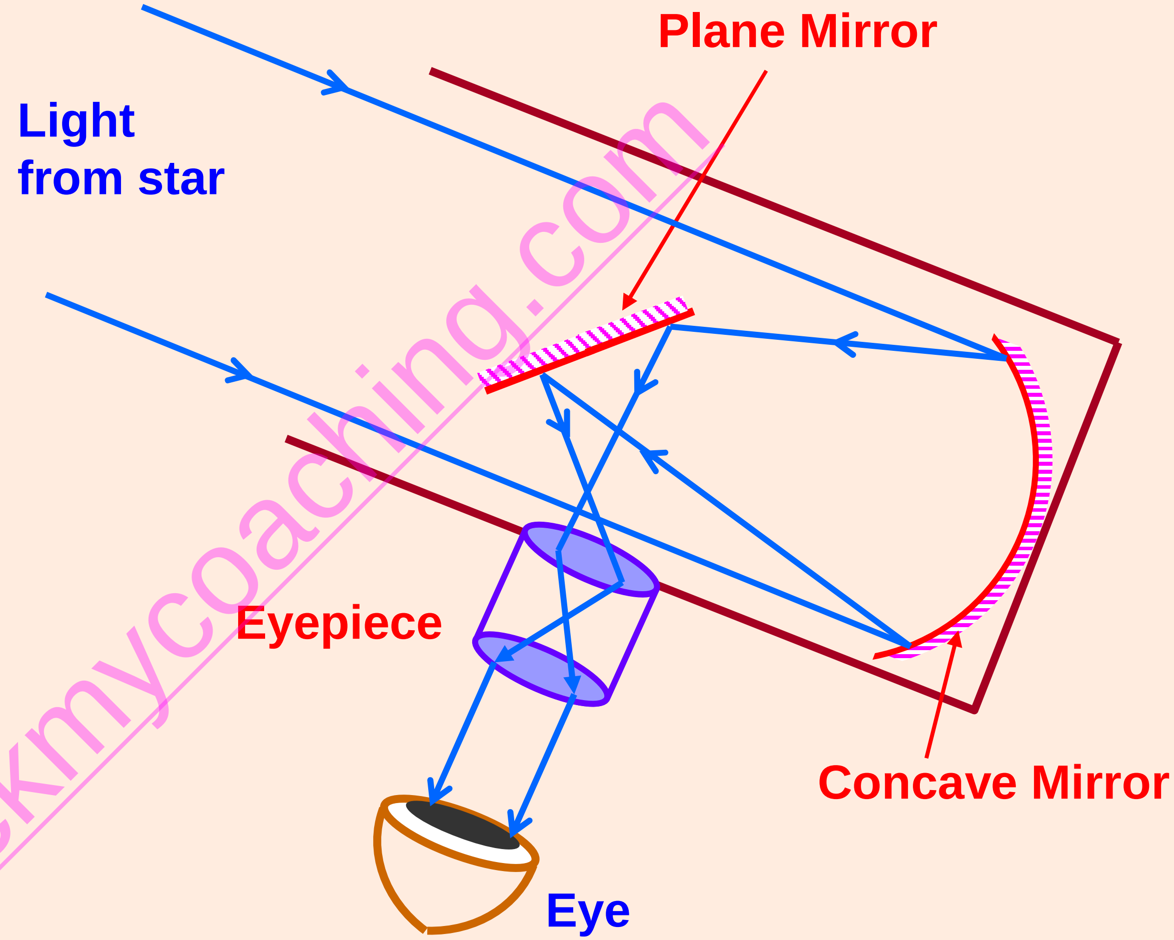
Multiplying by f_o on both sides and rearranging, we get

$$\boxed{M = \frac{-f_o}{f_e}\left(1 + \frac{f_e}{D}\right)}$$

Clearly focal length of objective must be greater than that of the eyepiece for larger magnifying power.

Also, it is to be noted that in this case M is larger than that in normal adjustment position.

Newtonian Telescope: (Reflecting Type)



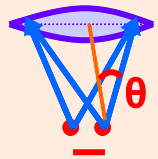
Magnifying Power:

$$M = \frac{f_o}{f_e}$$

Resolving Power of a Microscope:

The resolving power of a microscope is defined as the reciprocal of the distance between two objects which can be just resolved when seen through the microscope.

$$\text{Resolving Power} = \frac{1}{\Delta d} = \boxed{\frac{2\,\mu \sin\theta}{\lambda}}$$

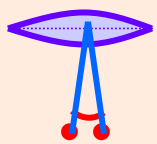


Resolving power depends on i) wavelength λ, ii) refractive index of the medium between the object and the objective and iii) half angle of the cone of light from one of the objects θ.

Resolving Power of a Telescope:

The resolving power of a telescope is defined as the reciprocal of the smallest angular separation between two distant objects whose images are seen separately.

$$\text{Resolving Power} = \frac{1}{d\theta} = \boxed{\frac{a}{1.22\,\lambda}}$$

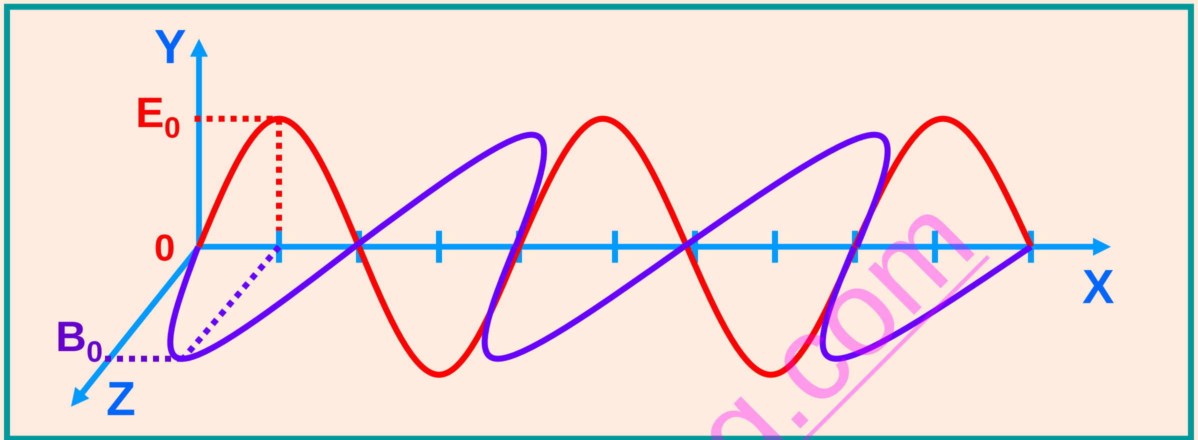


Resolving power depends on i) wavelength λ, ii) diameter of the objective a.

WAVE OPTICS - I

1. Electromagnetic Wave
2. Wavefront
3. Huygens' Principle
4. Reflection of Light based on Huygens' Principle
5. Refraction of Light based on Huygens' Principle
6. Behaviour of Wavefront in a Mirror, Lens and Prism
7. Coherent Sources
8. Interference
9. Young's Double Slit Experiment
10. Colours in Thin Films

Electromagnetic Wave:



1. Variations in both electric and magnetic fields occur simultaneously. Therefore, they attain their maxima and minima at the same place and at the same time.
2. The direction of electric and magnetic fields are mutually perpendicular to each other and as well as to the direction of propagation of wave.
3. The speed of electromagnetic wave depends entirely on the electric and magnetic properties of the medium, in which the wave travels and not on the amplitudes of their variations.

Wave is propagating along X – axis with speed $c = 1 / \sqrt{\mu_0 \varepsilon_0}$

For discussion of optical property of EM wave, more significance is given to Electric Field, E. Therefore, Electric Field is called 'light vector'.

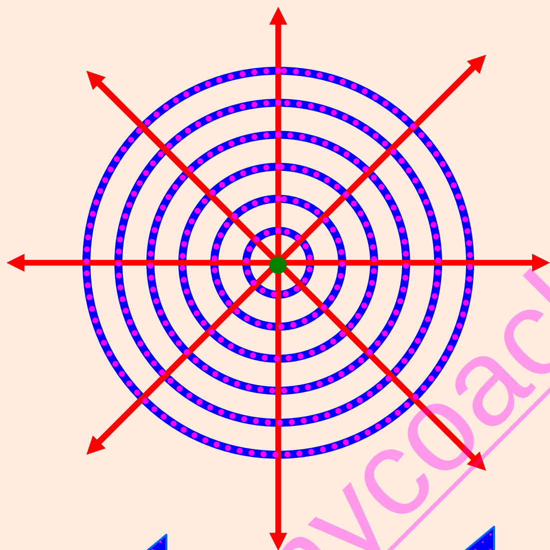
Wavefront:

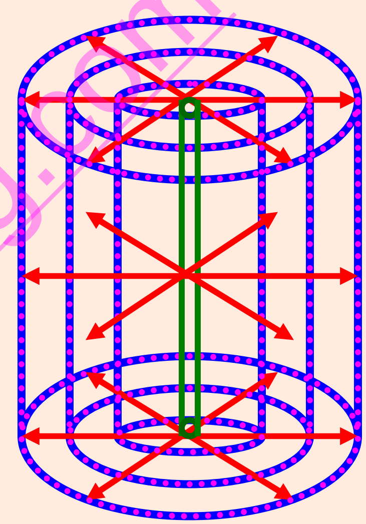
A wavelet is the point of disturbance due to propagation of light.

A wavefront is the locus of points (wavelets) having the same phase of oscillations.

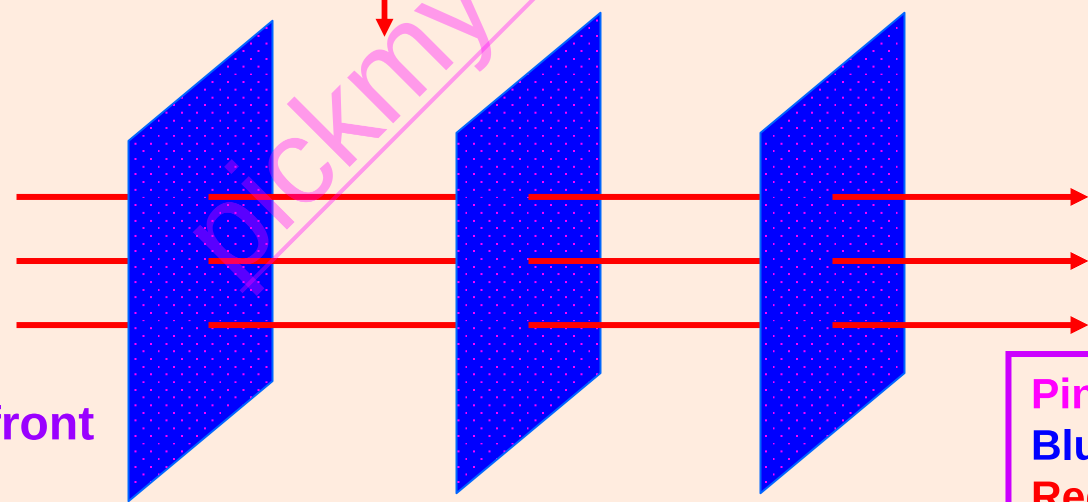
A line perpendicular to a wavefront is called a 'ray'.

Spherical Wavefront from a point source





Cylindrical Wavefront from a linear source



Plane Wavefront

Pink Dots – Wavelets
Blue Envelope– Wavefront
Red Line – Ray

Huygens' Construction or Huygens' Principle of Secondary Wavelets:

(Wavelets - Red dots on the wavefront)

1. Each point on a wavefront acts as a fresh source of disturbance of light.
2. The new wavefront at any time later is obtained by taking the forward envelope of all the secondary wavelets at that time.

Note: Backward wavefront is rejected. Why?

Amplitude of secondary wavelet is proportional to ½ (1+cosθ). Obviously, for the backward wavelet θ = 180° and (1+cos θ) is 0.

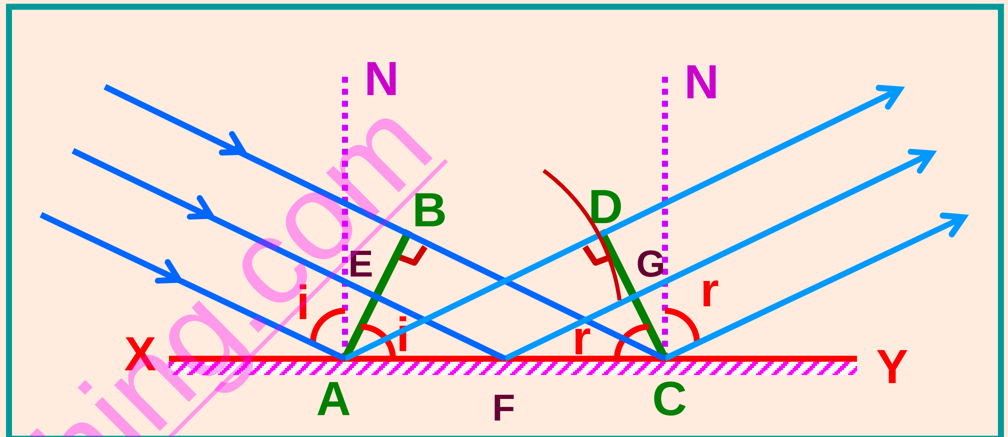
Laws of Reflection at a Plane Surface (On Huygens' Principle):

If c be the speed of light, t be the time taken by light to go from B to C or A to D or E to G through F, then

$$t = \frac{EF}{c} + \frac{FG}{c}$$

$$t = \frac{AF \sin i}{c} + \frac{FC \sin r}{c}$$

$$t = \frac{AC \sin r + AF (\sin i - \sin r)}{c}$$



AB – Incident wavefront
CD – Reflected wavefront
XY – Reflecting surface

For rays of light from different parts on the incident wavefront, the values of AF are different. But light from different points of the incident wavefront should take the same time to reach the corresponding points on the reflected wavefront.

So, t should not depend upon AF. This is possible only if $\sin i - \sin r = 0$.

i.e. $\sin i = \sin r$ or $i = r$

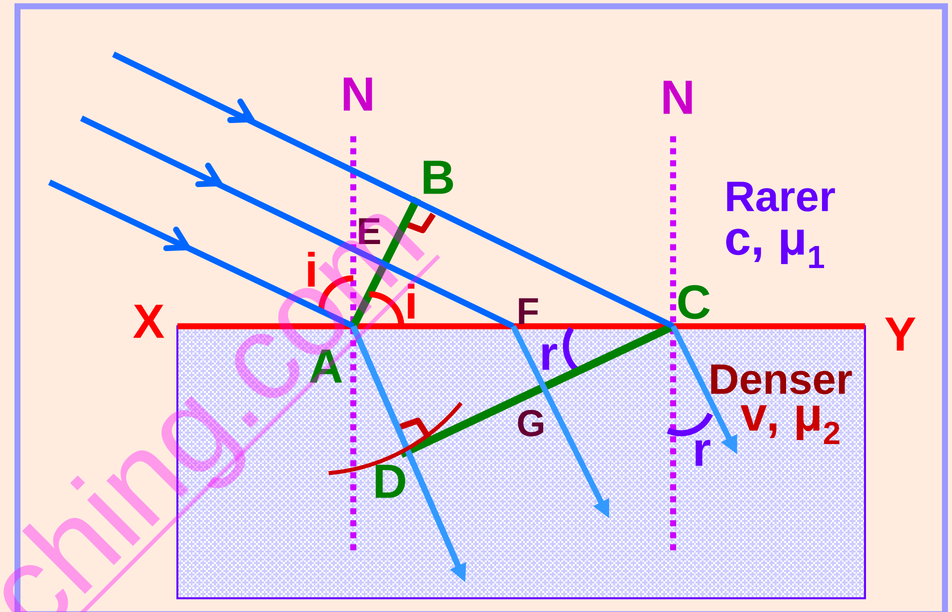
Laws of Refraction at a Plane Surface (On Huygens' Principle):

If c be the speed of light, t be the time taken by light to go from B to C or A to D or E to G through F, then

$$t = \frac{EF}{c} + \frac{FG}{v}$$

$$t = \frac{AF \sin i}{c} + \frac{FC \sin r}{v}$$

$$t = \frac{AC \sin r}{v} + AF\left(\frac{\sin i}{c} - \frac{\sin r}{v}\right)$$



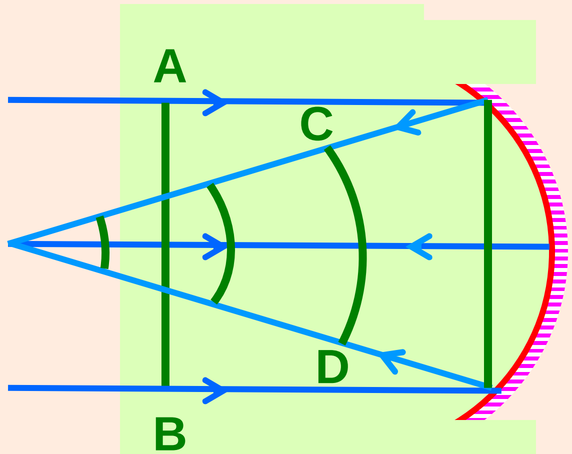
AB – Incident wavefront
CD – Refracted wavefront
XY – Refracting surface

For rays of light from different parts on the incident wavefront, the values of AF are different. But light from different points of the incident wavefront should take the same time to reach the corresponding points on the refracted wavefront.

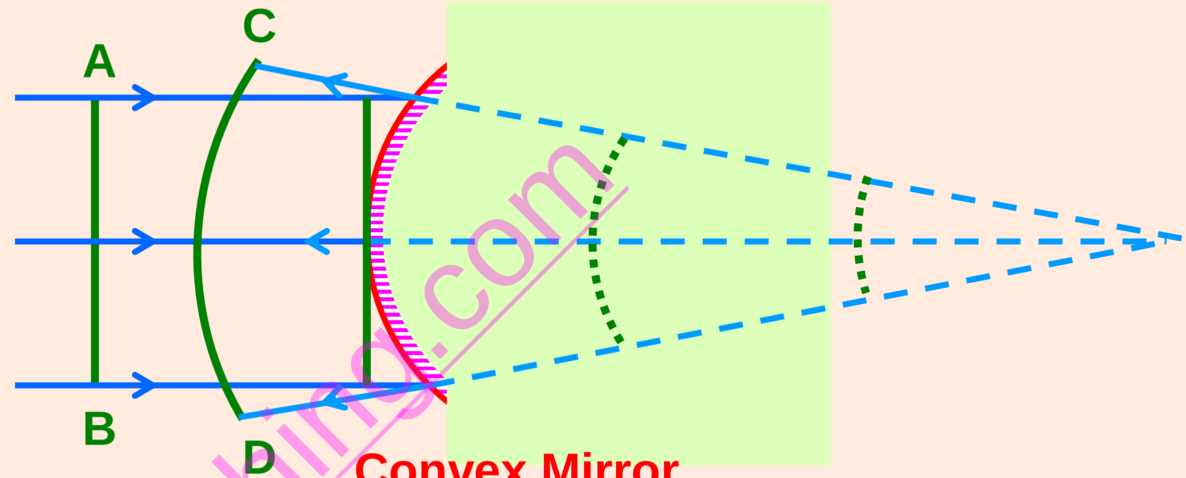
So, t should not depend upon AF. This is possible only

if $\frac{\sin i}{c} - \frac{\sin r}{v} = 0$ or $\frac{\sin i}{c} = \frac{\sin r}{v}$ or $\frac{\sin i}{\sin r} = \frac{c}{v} = \mu$

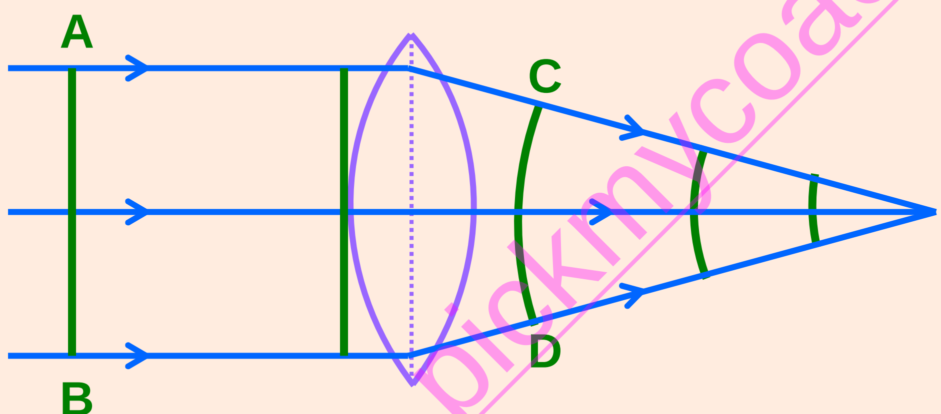
Behaviour of a Plane Wavefront in a Concave Mirror, Convex Mirror, Convex Lens, Concave Lens and Prism:



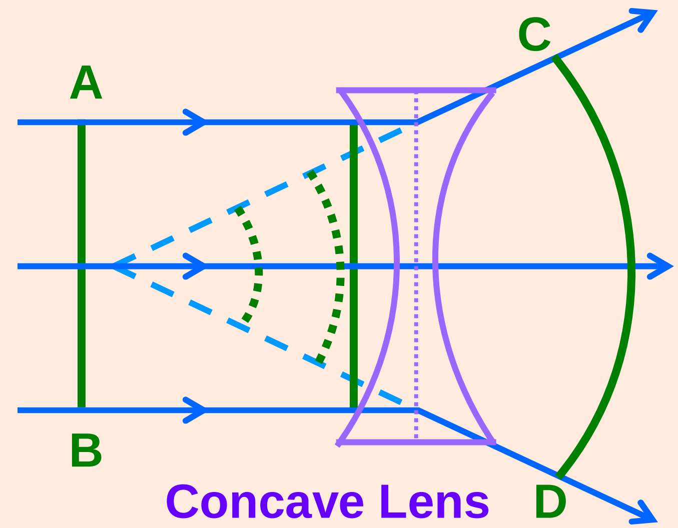
Concave Mirror



Convex Mirror



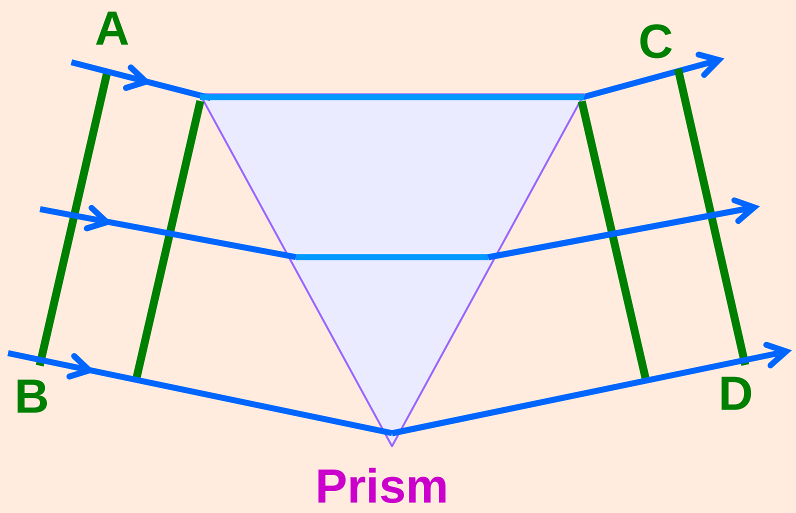
Convex Lens

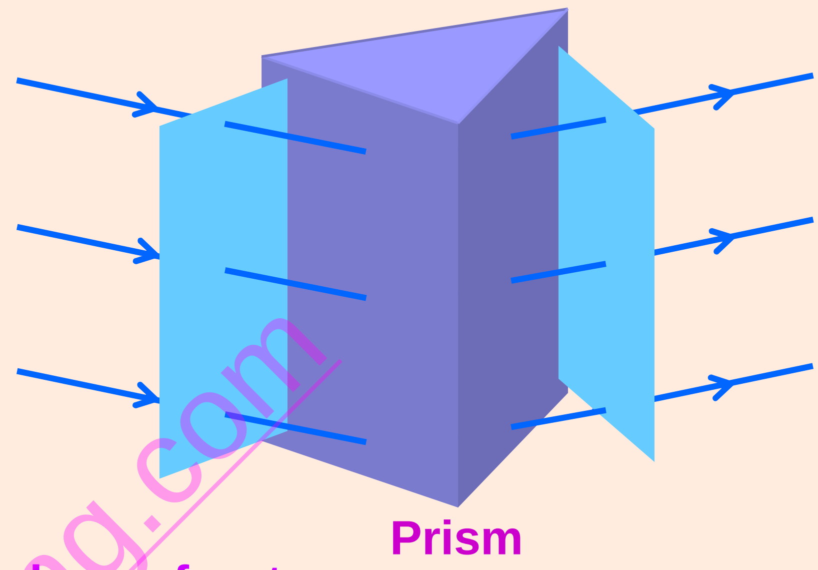


Concave Lens

AB – Incident wavefront

CD – Reflected / Refracted wavefront





AB – Incident wavefront CD –Refracted wavefront

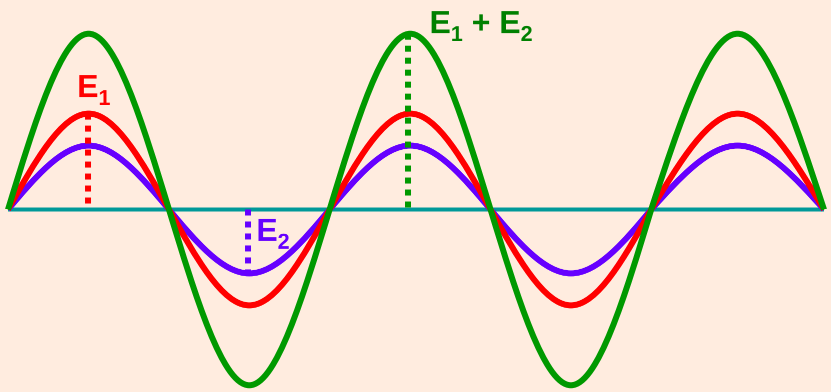
Coherent Sources:

Coherent Sources of light are those sources of light which emit light waves of same wavelength, same frequency and in same phase or having constant phase difference.

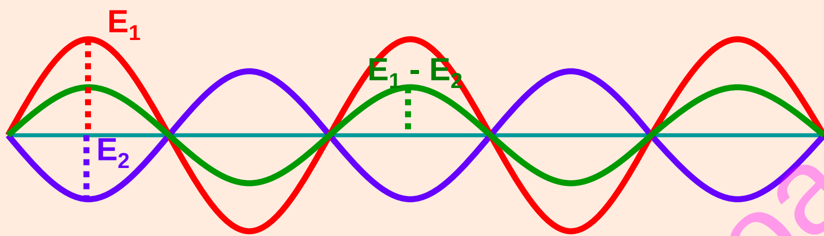
Coherent sources can be produced by two methods:

1. By division of wavefront (Young's Double Slit Experiment, Fresnel's Biprism and Lloyd's Mirror)
2. By division of amplitude (Partial reflection or refraction)

Interference of Waves:

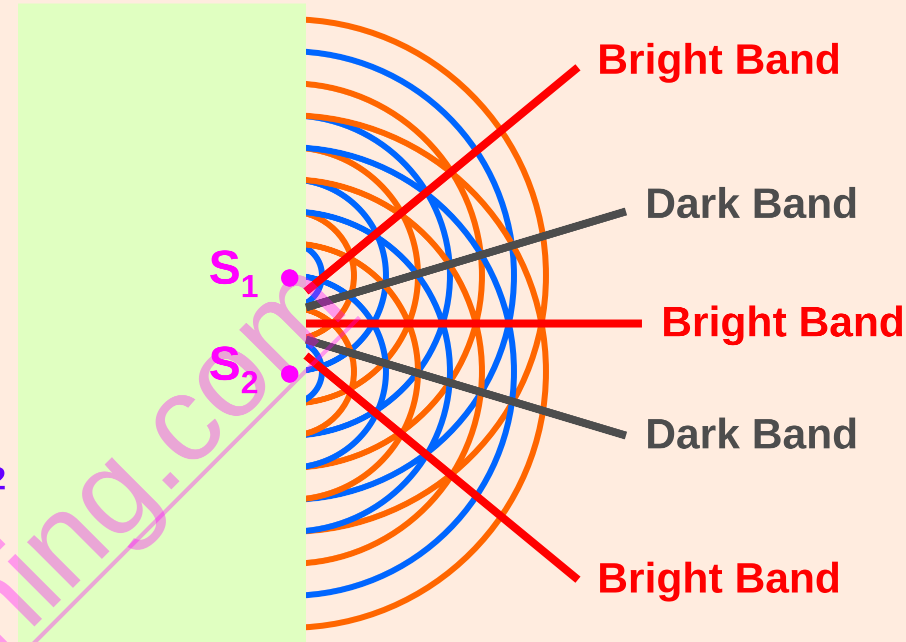


Constructive Interference $E = E_1 + E_2$



Destructive Interference $E = E_1 - E_2$

- 1st Wave (E_1)
- 2nd Wave (E_2)
- Resultant Wave
- Reference Line



- Crest
- Trough
- Bright Band
- Dark Band

The phenomenon of one wave interfering with another and the resulting redistribution of energy in the space around the two sources of disturbance is called interference of waves.

Theory of Interference of Waves:

$E_1 = a \sin \omega t$

$E_2 = b \sin (\omega t + \Phi)$

The waves are with same speed, wavelength, frequency, time period, nearly equal amplitudes, travelling in the same direction with constant phase difference of Φ.

ω is the angular frequency of the waves, a,b are the amplitudes and E_1, E_2 are the instantaneous values of Electric displacement.

Applying superposition principle, the magnitude of the resultant displacement of the waves is $E = E_1 + E_2$

$$E = a \sin \omega t + b \sin (\omega t + \Phi)$$

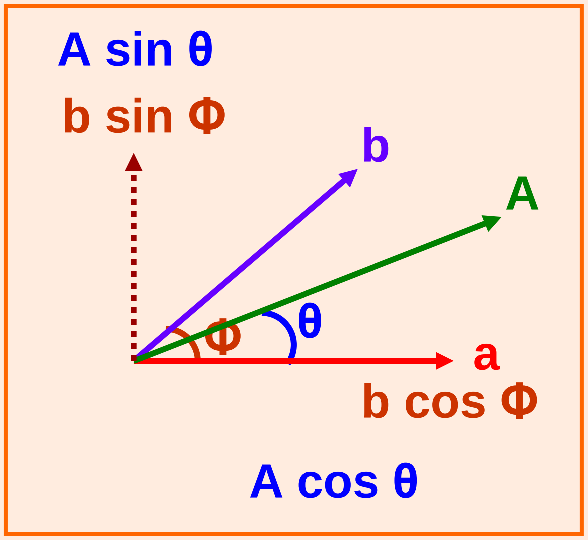
$$E = (a + b \cos \Phi) \sin \omega t + b \sin \Phi \cos \omega t$$

Putting $a + b \cos \Phi = A \cos \theta$

$b \sin \Phi = A \sin \theta$

We get $E = A \sin (\omega t + \theta)$

(where E is the resultant displacement, A is the resultant amplitude and θ is the resultant phase difference)



$$\boxed{A = \sqrt{(a^2 + b^2 + 2ab \cos \Phi)}}$$

$$\boxed{\tan \theta = \frac{b \sin \Phi}{a + b \cos \Phi}}$$

$$\boxed{A = \sqrt{(a^2 + b^2 + 2ab \cos \Phi)}}$$

Intensity I is proportional to the square of the amplitude of the wave.

So, $I \propto A^2$ i.e. $I \propto (a^2 + b^2 + 2ab \cos \Phi)$

Condition for Constructive Interference of Waves:

For constructive interference, I should be maximum which is possible only if $\cos \Phi = +1$.

$\boxed{\text{i.e. } \Phi = 2n\pi}$ where n = 0, 1, 2, 3, …….

Corresponding path difference is $\Delta = (\lambda / 2\pi) \times 2n\pi$

$\boxed{\Delta = n\lambda}$ $\boxed{I_{max} \propto (a + b)^2}$

Condition for Destructive Interference of Waves:

For destructive interference, I should be minimum which is possible only if $\cos \Phi = -1$.

$\boxed{\text{i.e. } \Phi = (2n + 1)\pi}$ where n = 0, 1, 2, 3, …….

Corresponding path difference is $\Delta = (\lambda / 2\pi) \times (2n + 1)\pi$

$\boxed{\Delta = (2n + 1)\lambda / 2}$ $\boxed{I_{min} \propto (a - b)^2}$

Comparison of intensities of maxima and minima:

$$\boxed{I_{max} \ \alpha \ (a + b)^2}$$

$$\boxed{I_{min} \ \alpha \ (a - b)^2}$$

$$\frac{I_{max}}{I_{min}} = \frac{(a + b)^2}{(a - b)^2} = \frac{(a/b + 1)^2}{(a/b - 1)^2}$$

$$\frac{I_{max}}{I_{min}} = \frac{(r + 1)^2}{(r - 1)^2}$$

where $r = a / b$ (ratio of the amplitudes)

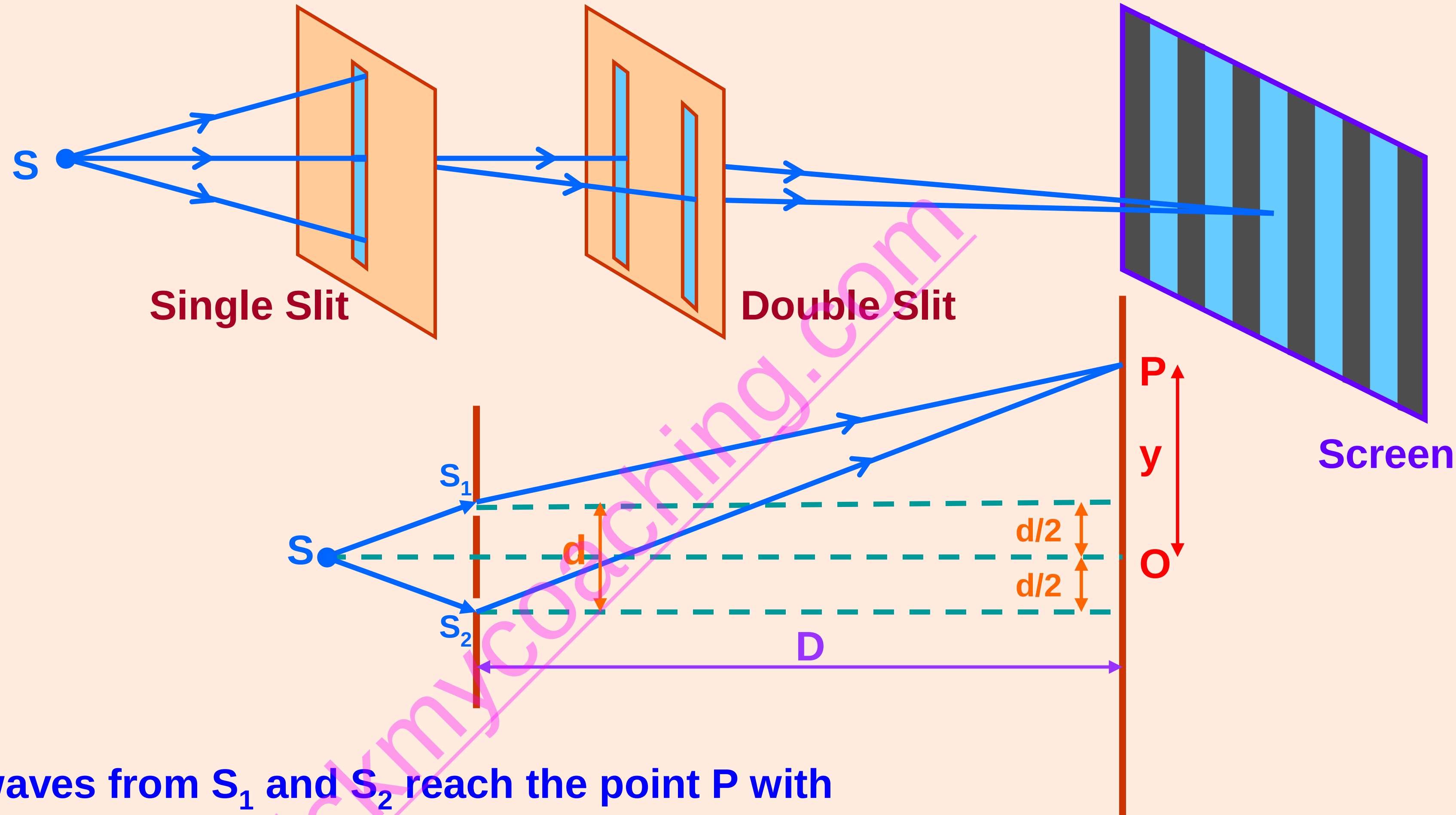
Relation between Intensity (I), Amplitude (a) of the wave and Width (w) of the slit:

$I \ \alpha \ a^2$

$a \ \alpha \ \sqrt{w}$

$$\boxed{\frac{I_1}{I_2} = \frac{(a_1)^2}{(a_2)^2} = \frac{w_1}{w_2}}$$

Young's Double Slit Experiment:



The waves from S_1 and S_2 reach the point P with some phase difference and hence path difference

$\Delta = S_2P - S_1P$

$S_2P^2 - S_1P^2 = [D^2 + \{y + (d/2)\}^2] - [D^2 + \{y - (d/2)\}^2]$

$(S_2P - S_1P)\ (S_2P + S_1P) = 2\ yd$

$\Delta\ (2D) = 2\ yd$

$\boxed{\Delta = yd / D}$

Positions of Bright Fringes:

For a bright fringe at P,

$\Delta = yd / D = n\lambda$

where n = 0, 1, 2, 3, …

$y = n D \lambda / d$

For n = 0, $y_0 = 0$

For n = 1, $y_1 = D \lambda / d$

For n = 2, $y_2 = 2 D \lambda / d$ ……

For n = n, $y_n = n D \lambda / d$

Positions of Dark Fringes:

For a dark fringe at P,

$\Delta = yd / D = (2n+1)\lambda/2$

where n = 0, 1, 2, 3, …

$y = (2n+1) D \lambda / 2d$

For n = 0, $y_0' = D \lambda / 2d$

For n = 1, $y_1' = 3D \lambda / 2d$

For n = 2, $y_2' = 5D \lambda / 2d$ …..

For n = n, $y_n' = (2n+1)D \lambda / 2d$

Expression for Dark Fringe Width:

$\beta_D = y_n - y_{n-1}$

$= n D \lambda / d - (n - 1) D \lambda / d$

$= D \lambda / d$

Expression for Bright Fringe Width:

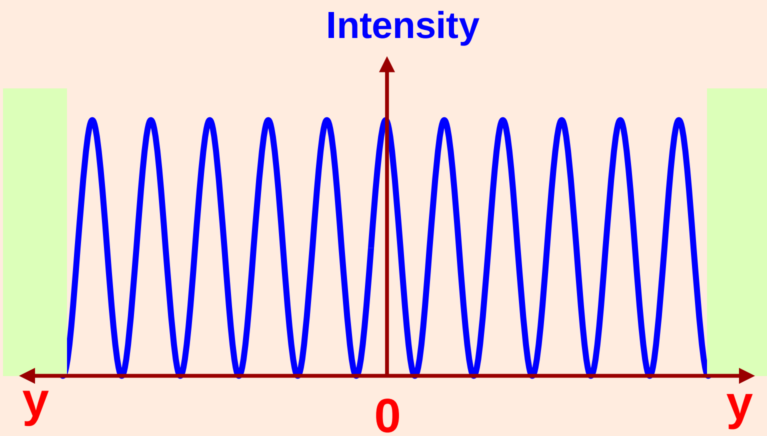
$\beta_B = y_n' - y_{n-1}'$

$= (2n+1) D \lambda / 2d - \{2(n-1)+1\} D \lambda / 2d$

$= D \lambda / d$

The expressions for fringe width show that the fringes are equally spaced on the screen.

Distribution of Intensity:



Suppose the two interfering waves have same amplitude say 'a', then

I_{max} α $(a+a)^2$ i.e. I_{max} α $4a^2$

All the bright fringes have this same intensity.

$I_{min} = 0$

All the dark fringes have zero intensity.

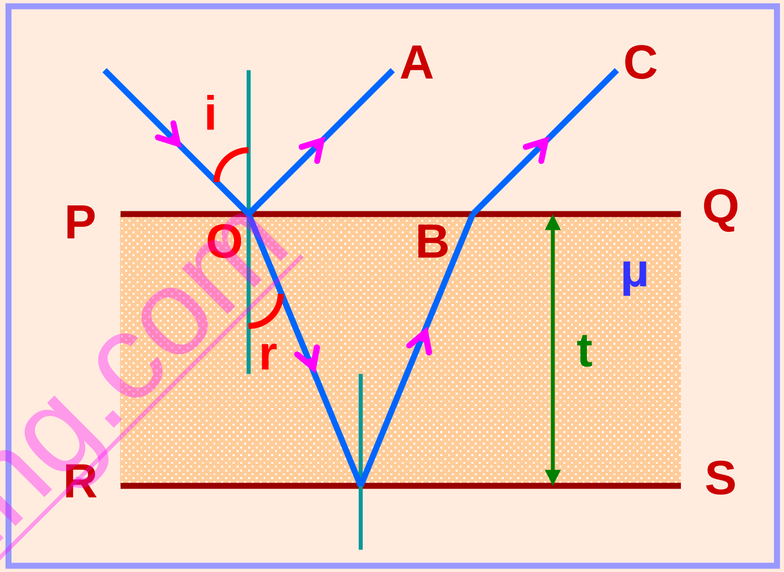
Conditions for sustained interference:

1. The two sources producing interference must be coherent.
2. The two interfering wave trains must have the same plane of polarisation.
3. The two sources must be very close to each other and the pattern must be observed at a larger distance to have sufficient width of the fringe. (D λ / d)
4. The sources must be monochromatic. Otherwise, the fringes of different colours will overlap.
5. The two waves must be having same amplitude for better contrast between bright and dark fringes.

Colours in Thin Films:

It can be proved that the path difference between the light partially reflected from PQ and that from partially transmitted and then reflected from RS is

$$\Delta = 2\mu t \cos r$$



Since there is a reflection at O, the ray OA suffers an additional phase difference of π and hence the corresponding path difference of λ/2.

For the rays OA and BC to interfere constructively (Bright fringe), the path difference must be (n + ½) λ

So, $2\mu t \cos r = (n + \frac{1}{2})\lambda$

For the rays OA and BC to interfere destructively (Dark fringe), the path difference must be nλ

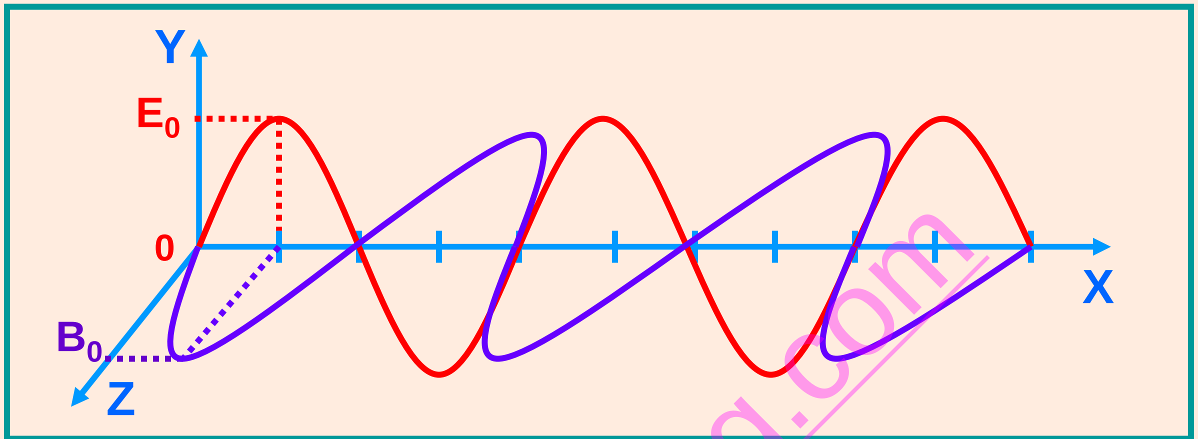
So, $2\mu t \cos r = n\lambda$

When white light from the sun falls on thin layer of oil spread over water in the rainy season, beautiful rainbow colours are formed due to interference of light.

WAVE OPTICS - II

1. Electromagnetic Wave
2. Diffraction
3. Diffraction at a Single Slit
4. Theory of Diffraction
5. Width of Central Maximum and Fresnel's Distance
6. Difference between Interference and Diffraction
7. Polarisation of Mechanical Waves
8. Polarisation of Light
9. Malus' Law
10. Polarisation by Reflection – Brewster's Law
11. Polaroids and their uses

Electromagnetic Wave:



1. Variations in both electric and magnetic fields occur simultaneously. Therefore, they attain their maxima and minima at the same place and at the same time.

2. The direction of electric and magnetic fields are mutually perpendicular to each other and as well as to the direction of propagation of wave.

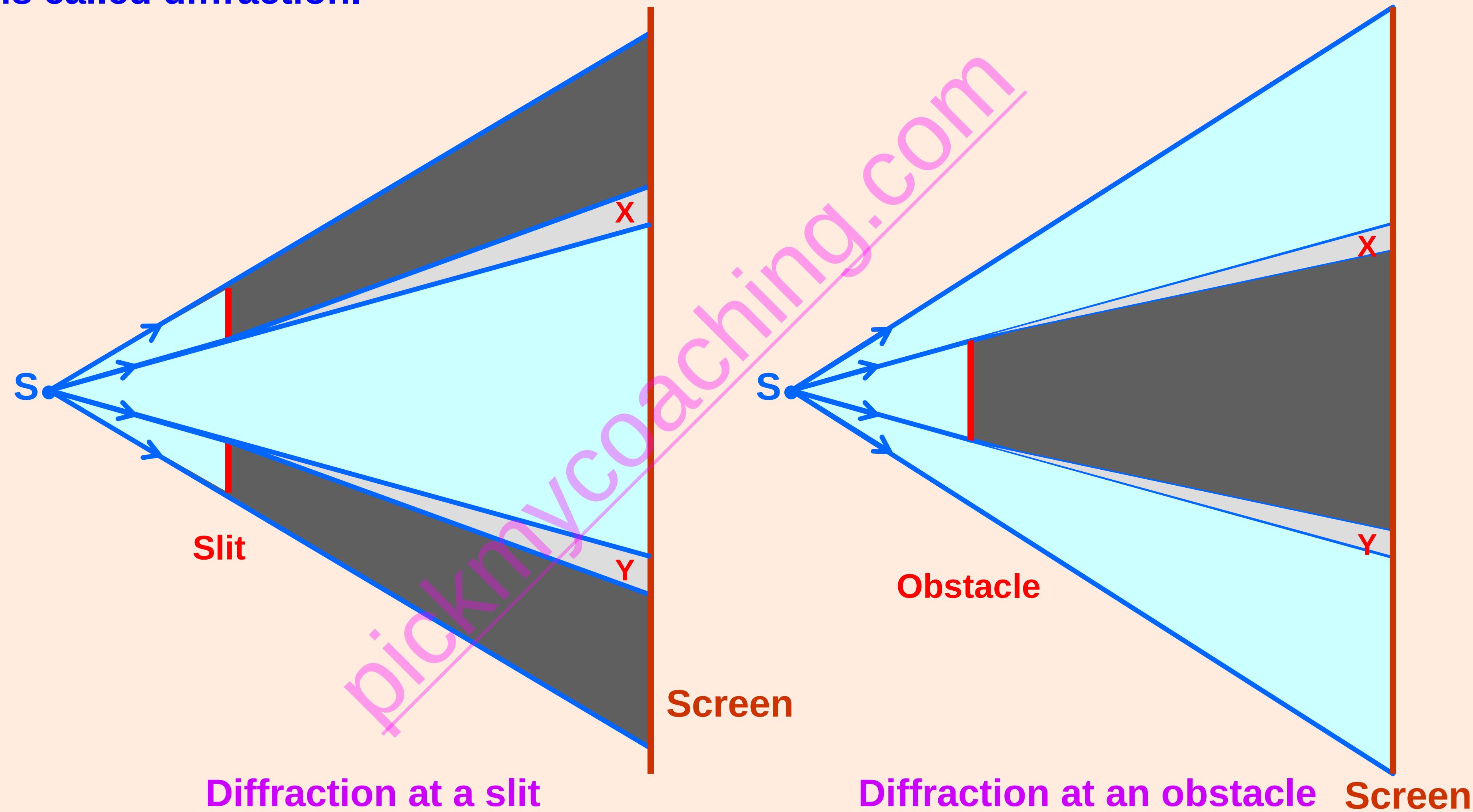
3. The speed of electromagnetic wave depends entirely on the electric and magnetic properties of the medium, in which the wave travels and not on the amplitudes of their variations.

Wave is propagating along X – axis with speed $c = 1 / \sqrt{\mu_0 \varepsilon_0}$

For discussion of EM wave, more significance is given to Electric Field, E.

Diffraction of light:

The phenomenon of bending of light around the corners and the encroachment of light within the geometrical shadow of the opaque obstacles is called diffraction.



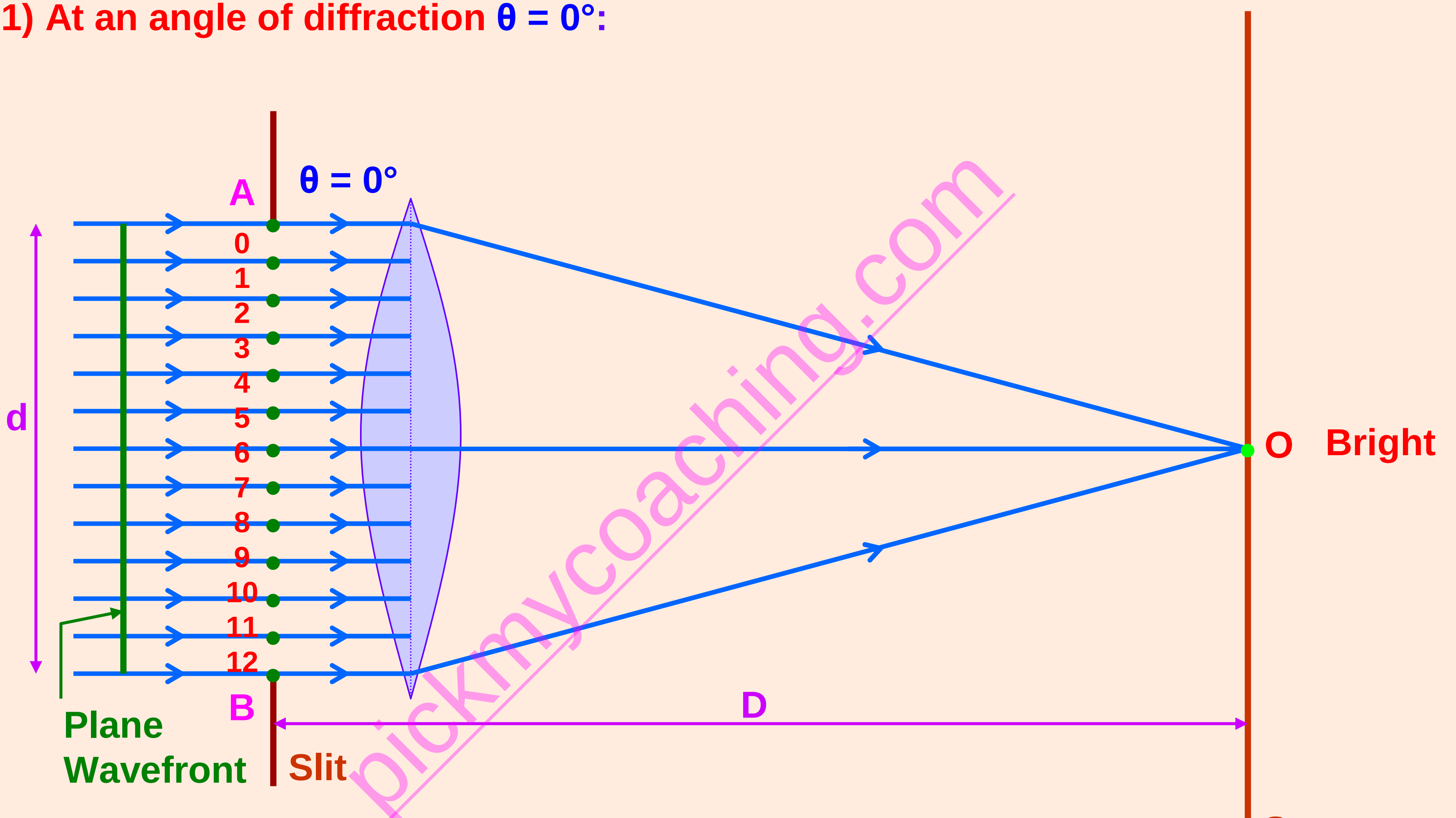
Diffraction at a slit

Diffraction at an obstacle

X & Y – Region of diffraction

Diffraction of light at a single slit:

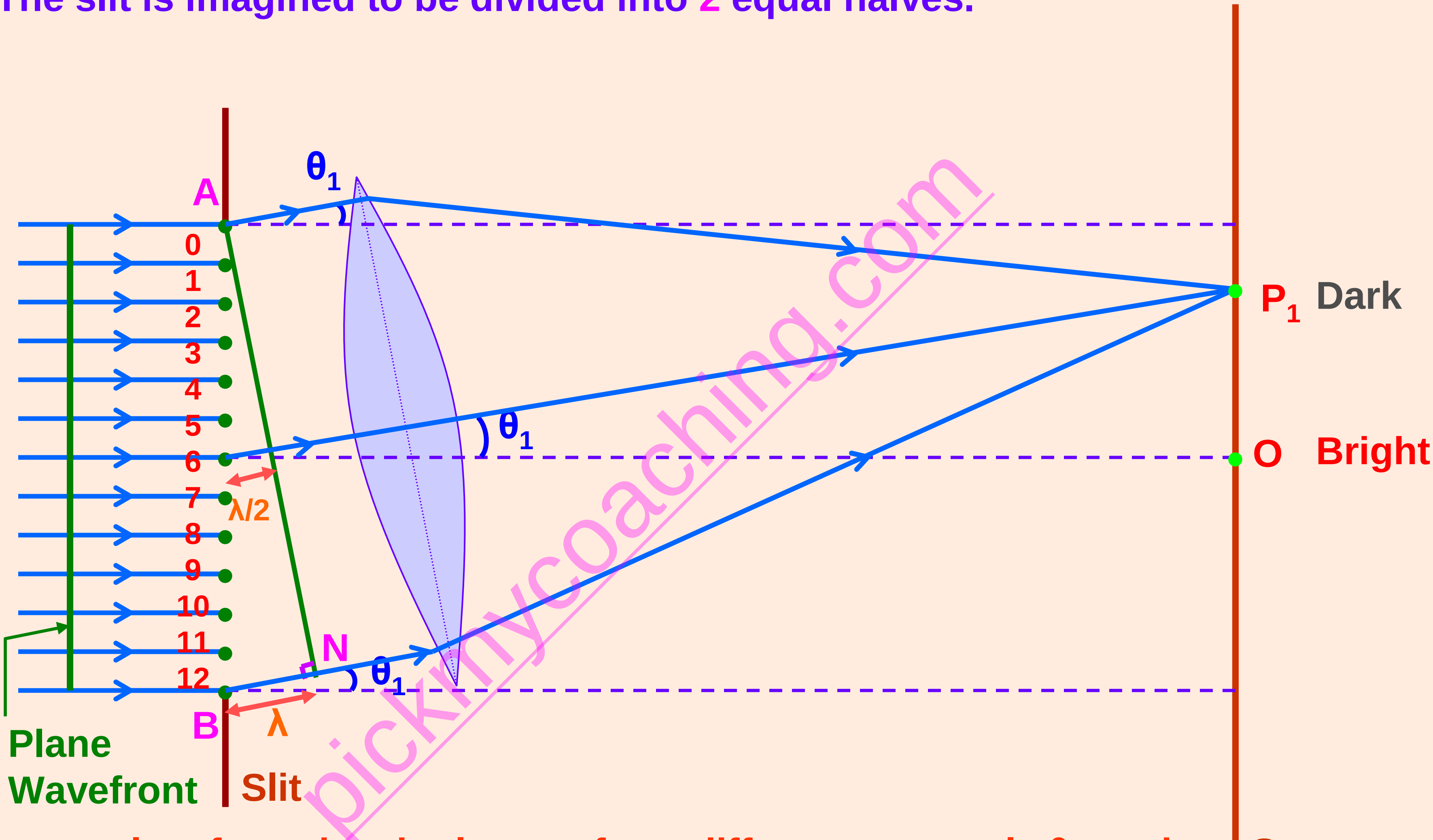
1) At an angle of diffraction θ = 0°:



The wavelets from the single wavefront reach the centre O on the screen in same phase and hence interfere constructively to give Central or Primary Maximum (Bright fringe).

2) At an angle of diffraction $\theta = \theta_1$:

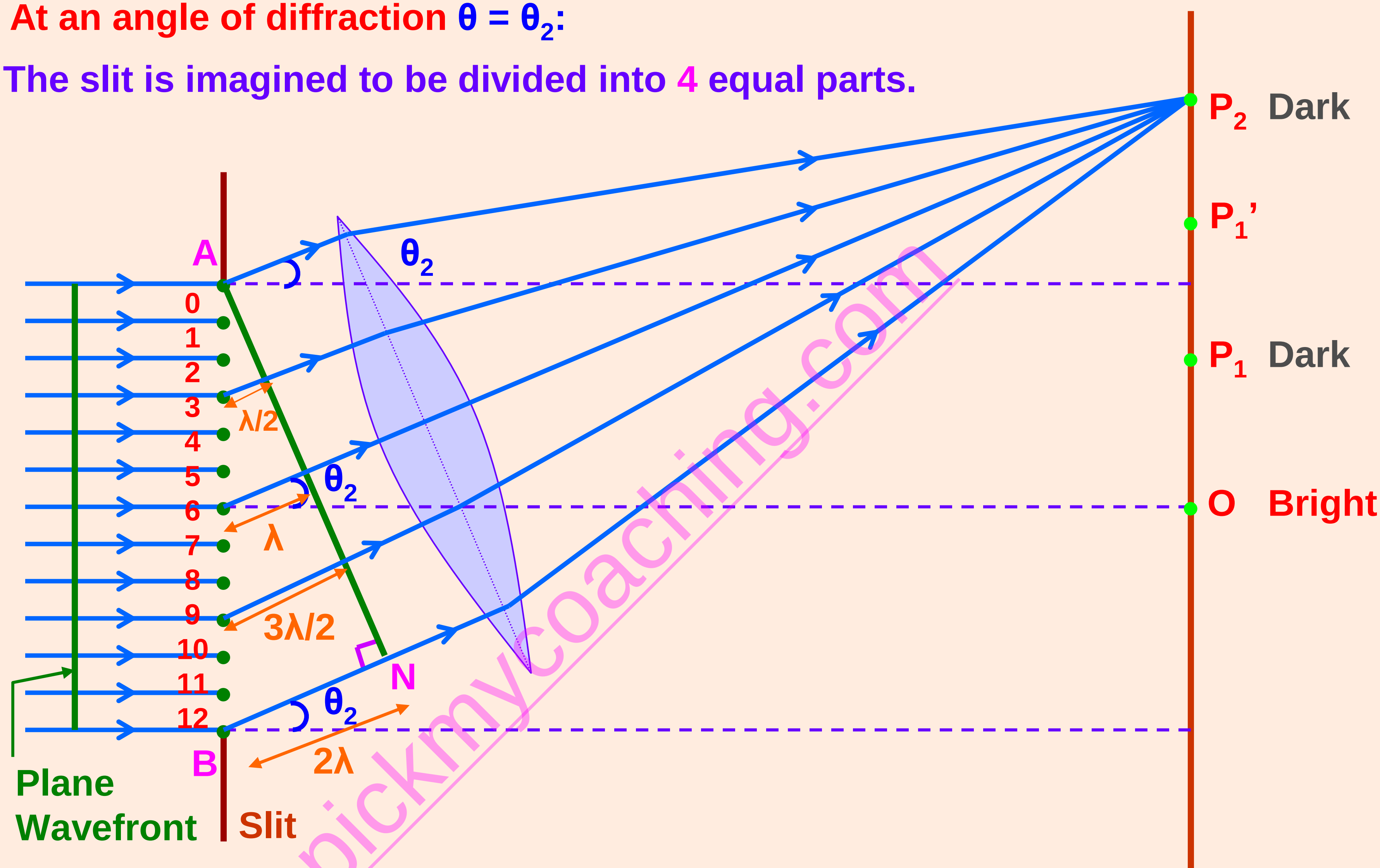
The slit is imagined to be divided into 2 equal halves.



The wavelets from the single wavefront diffract at an angle θ_1 such that BN is λ and reach the point P_1. The pairs (0,6), (1,7), (2,8), (3,9), (4,10), (5,11) and (6,12) interfere destructively with path difference $\lambda/2$ and give First Secondary Minimum (Dark fringe).

3) At an angle of diffraction $\theta = \theta_2$:

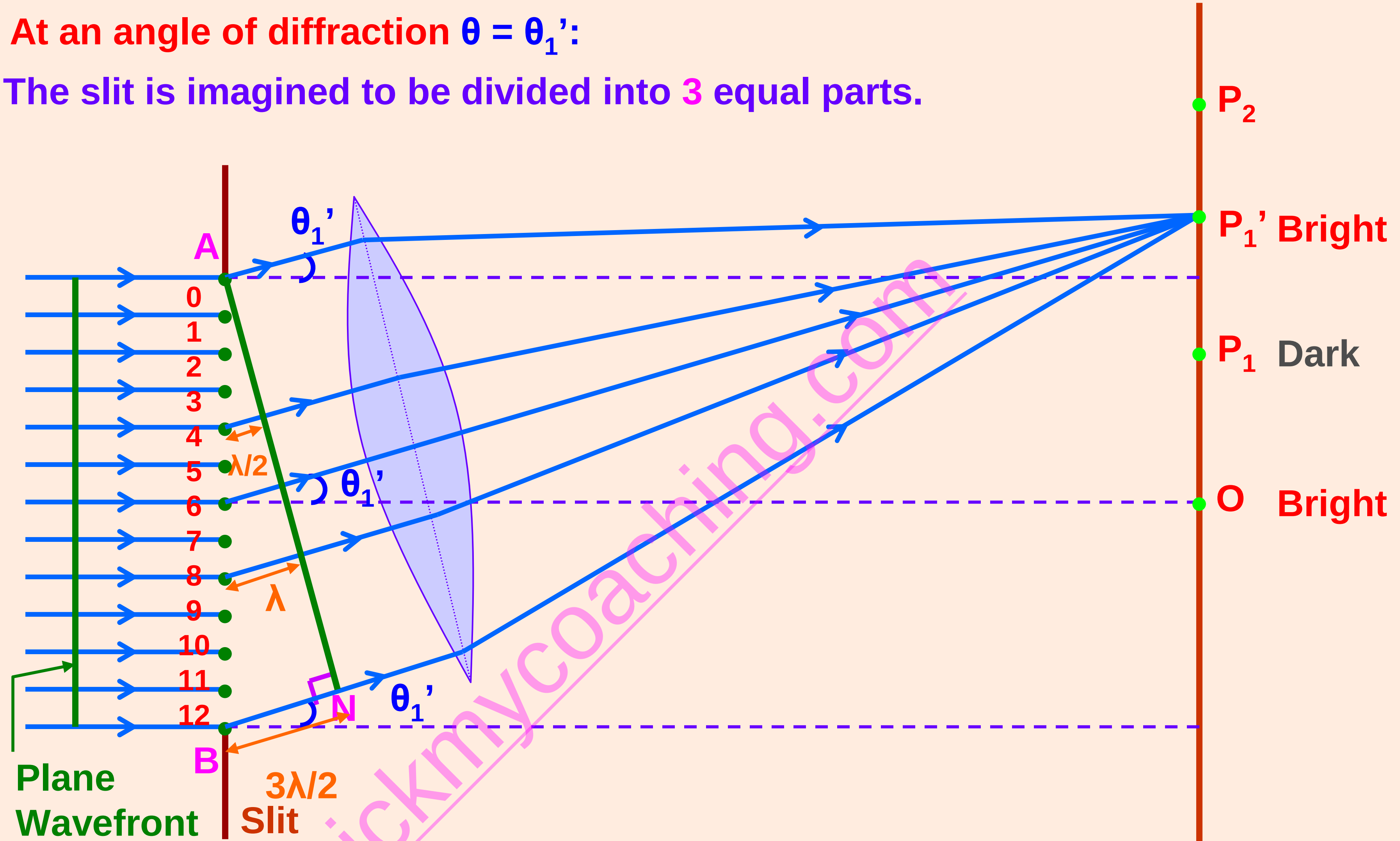
The slit is imagined to be divided into 4 equal parts.



The wavelets from the single wavefront diffract at an angle θ_2 such that BN is 2λ and reach the point P_2. The pairs (0,3), (1,4), (2,5), (3,6), (4,7), (5,8), (6,9), (7,10), (8,11) and (9,12) interfere destructively with path difference $\lambda/2$ and give Second Secondary Minimum (Dark fringe).

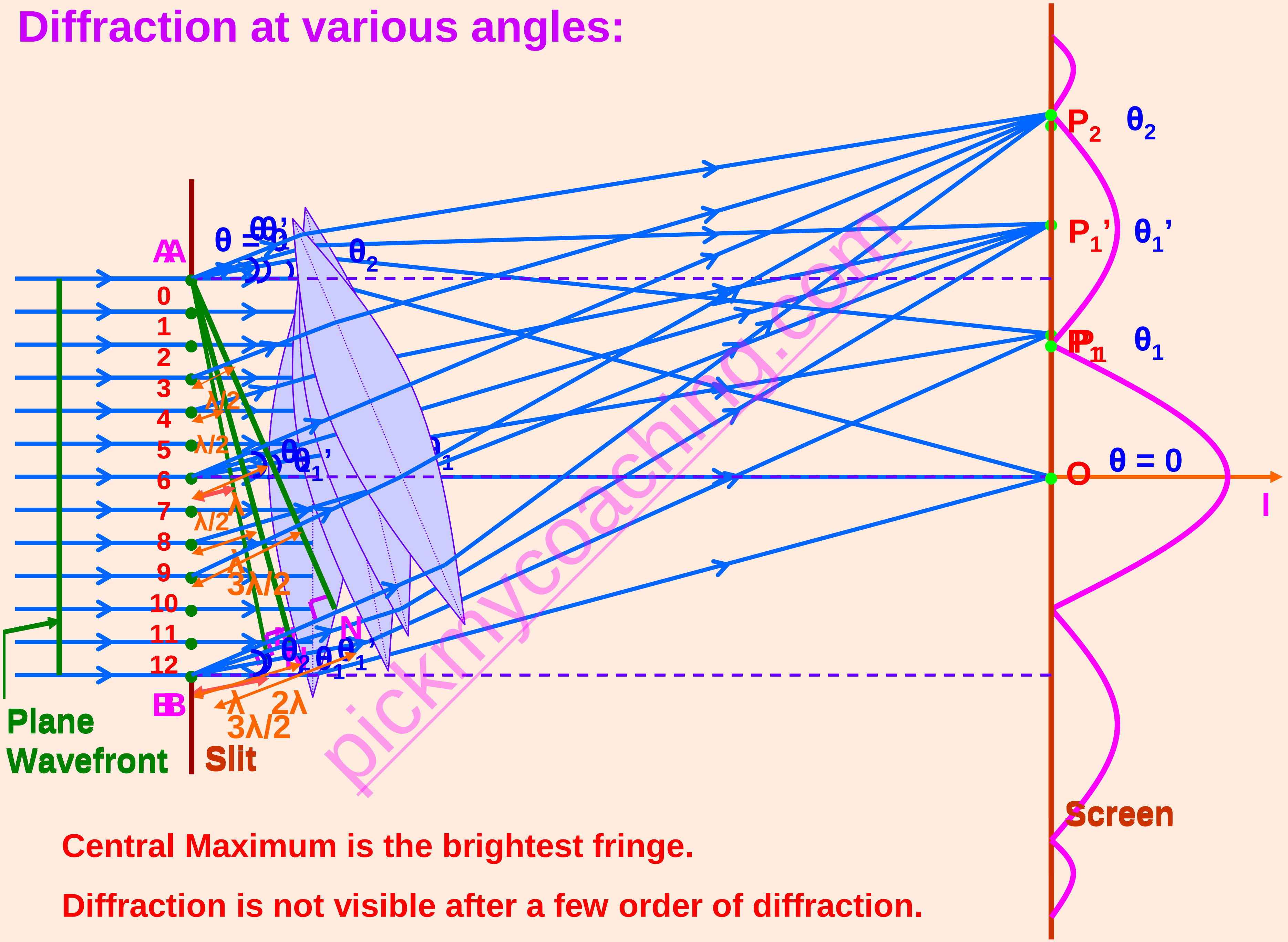
4) At an angle of diffraction $\theta = \theta_1'$:

The slit is imagined to be divided into 3 equal parts.



The wavelets from the single wavefront diffract at an angle θ_1' such that BN is $3\lambda/2$ and reach the point P_1'. The pairs (0,8), (1,9), (2,10), (3,11) and (4,12) interfere constructively with path difference λ and (0,4), (1,5), (2,6), …… and (8,12) interfere destructively with path difference $\lambda/2$. However due to a few wavelets interfering constructively First Secondary Maximum (Bright fringe) is formed.

Diffraction at various angles:



Central Maximum is the brightest fringe.

Diffraction is not visible after a few order of diffraction.

Theory:

The path difference between the 0th wavelet and 12th wavelet is BN.

If 'θ' is the angle of diffraction and 'd' is the slit width, then BN = d sin θ

To establish the condition for secondary minima, the slit is divided into 2, 4, 6, … equal parts such that corresponding wavelets from successive regions interfere with path difference of λ/2.

Or for nth secondary minimum, the slit can be divided into 2n equal parts.

For θ_1, $d \sin \theta_1 = \lambda$

For θ_2, $d \sin \theta_2 = 2\lambda$

For θ_n, $d \sin \theta_n = n\lambda$

Since θ_n is very small,

$d\,\theta_n = n\lambda$

$\theta_n = n\lambda / d \qquad (n = 1, 2, 3, \ldots\ldots)$

To establish the condition for secondary maxima, the slit is divided into 3, 5, 7, … equal parts such that corresponding wavelets from alternate regions interfere with path difference of λ.

Or for nth secondary minimum, the slit can be divided into (2n + 1) equal parts.

For θ_1', $d \sin \theta_1' = 3\lambda/2$

For θ_2', $d \sin \theta_2' = 5\lambda/2$

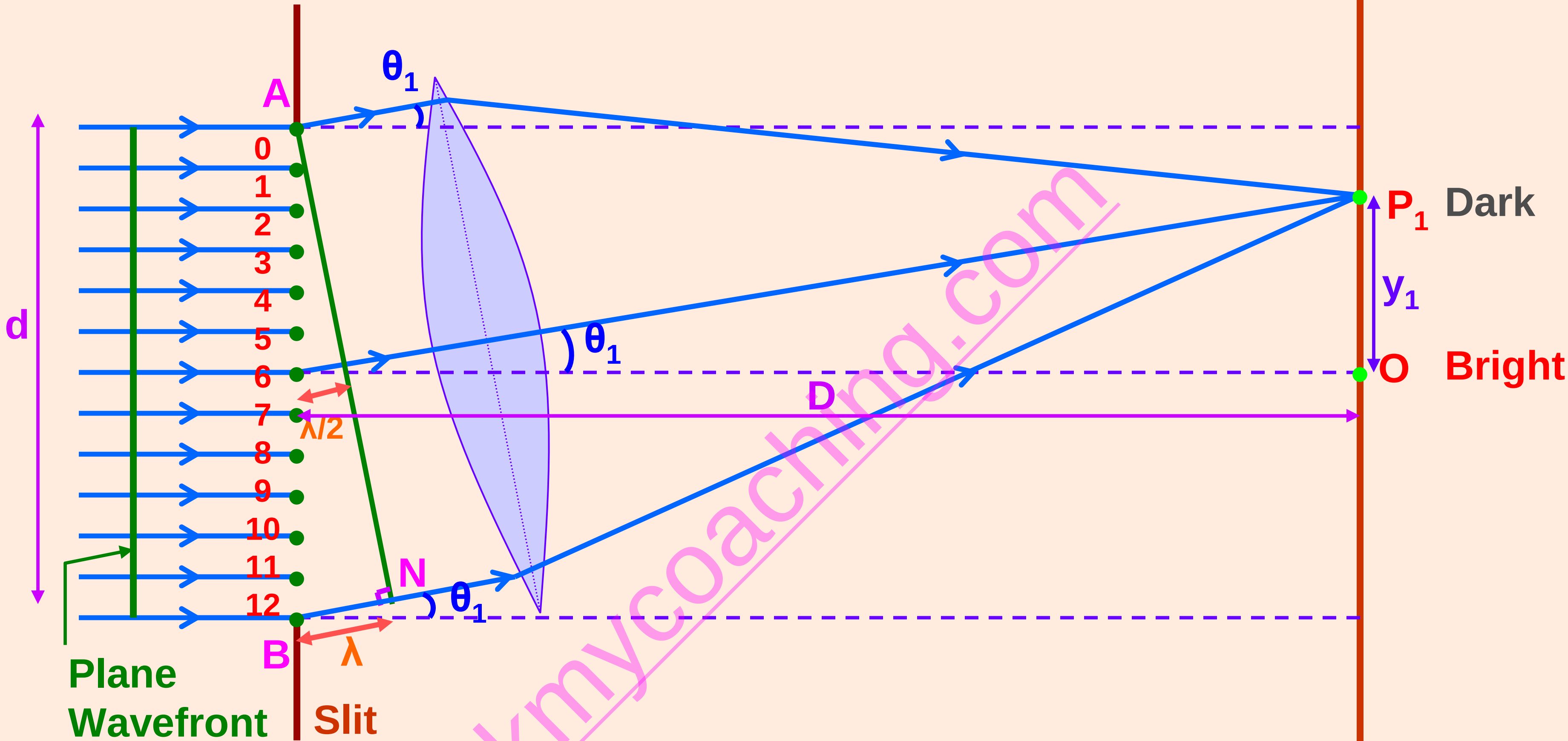
For θ_n', $d \sin \theta_n' = (2n + 1)\lambda/2$

Since θ_n' is very small,

$d\,\theta_n' = (2n + 1)\lambda / 2$

$\theta_n' = (2n + 1)\lambda / 2d \qquad (n = 1, 2, 3, \ldots\ldots)$

Width of Central Maximum:



$\tan \theta_1 = y_1 / D$

or $\theta_1 = y_1 / D$ (since θ_1 is very small)

$d \sin \theta_1 = \lambda$

or $\theta_1 = \lambda / d$ (since θ_1 is very small)

$y_1 = D \lambda / d$

Since the Central Maximum is spread on either side of O, the width is

$$\boxed{\beta_0 = 2D \lambda / d}$$

Fresnel's Distance:

Fresnel's distance is that distance from the slit at which the spreading of light due to diffraction becomes equal to the size of the slit.

$y_1 = D \lambda / d$

At Fresnel's distance, $y_1 = d$ and $D = D_F$

So, $D_F \lambda / d = d$ or $D_F = d^2 / \lambda$

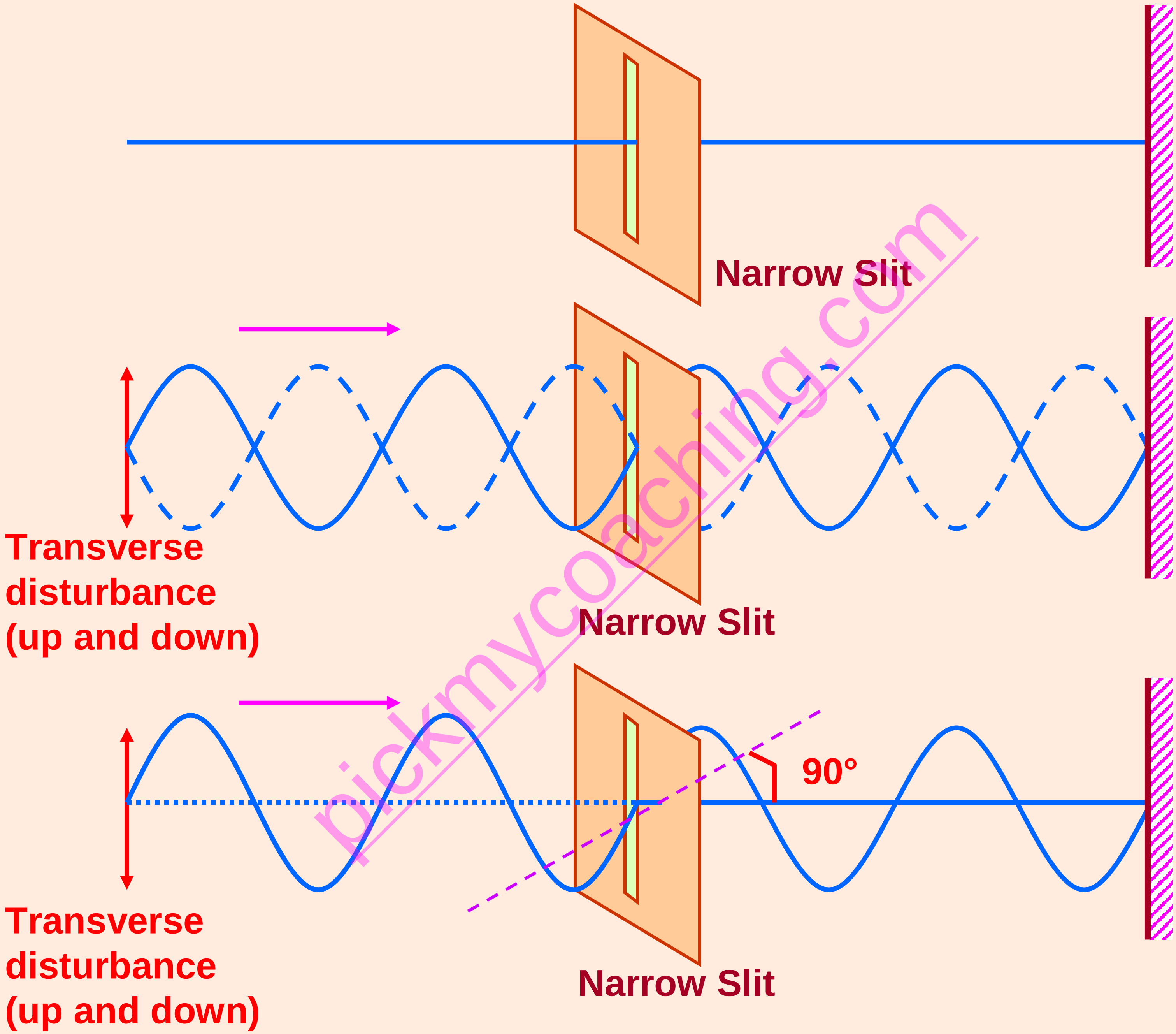
If the distance D between the slit and the screen is less than Fresnel's distance D_F, then the diffraction effects may be regarded as absent.

So, ray optics may be regarded as a limiting case of wave optics.

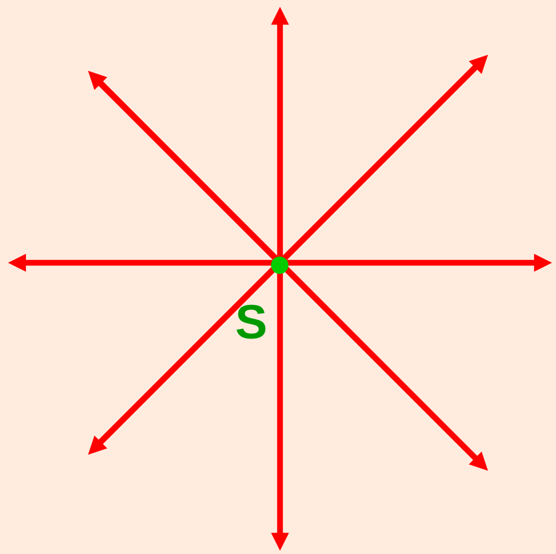
Difference between Interference and Diffraction:

Interference	Diffraction
1. Interference is due to the superposition of two different wave trains coming from coherent sources.	1. Diffraction is due to the superposition of secondary wavelets from the different parts of the same wavefront.
2. Fringe width is generally constant.	2. Fringes are of varying width.
3. All the maxima have the same intensity.	3. The maxima are of varying intensities.
4. There is a good contrast between the maxima and minima.	4. There is a poor contrast between the maxima and minima.

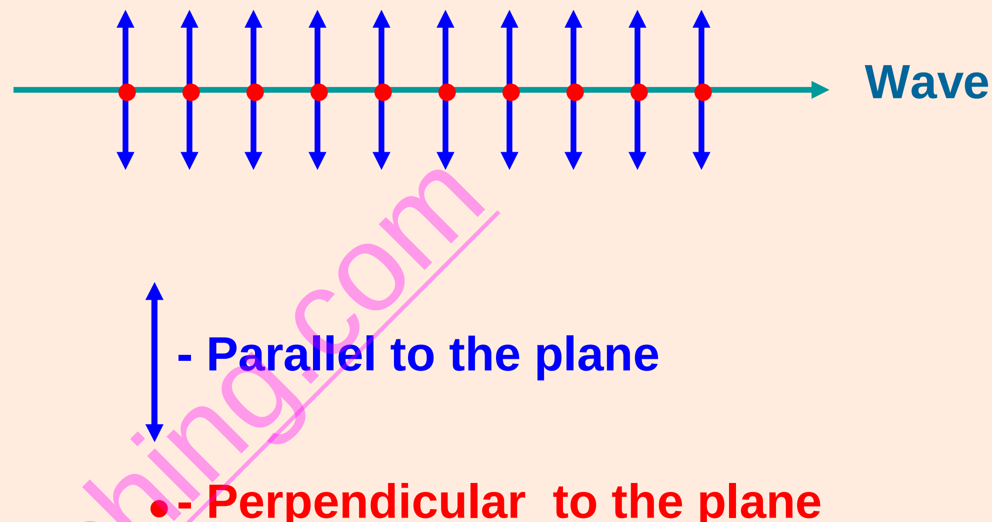
Polarisation of Transverse Mechanical Waves:



Polarisation of Light Waves:



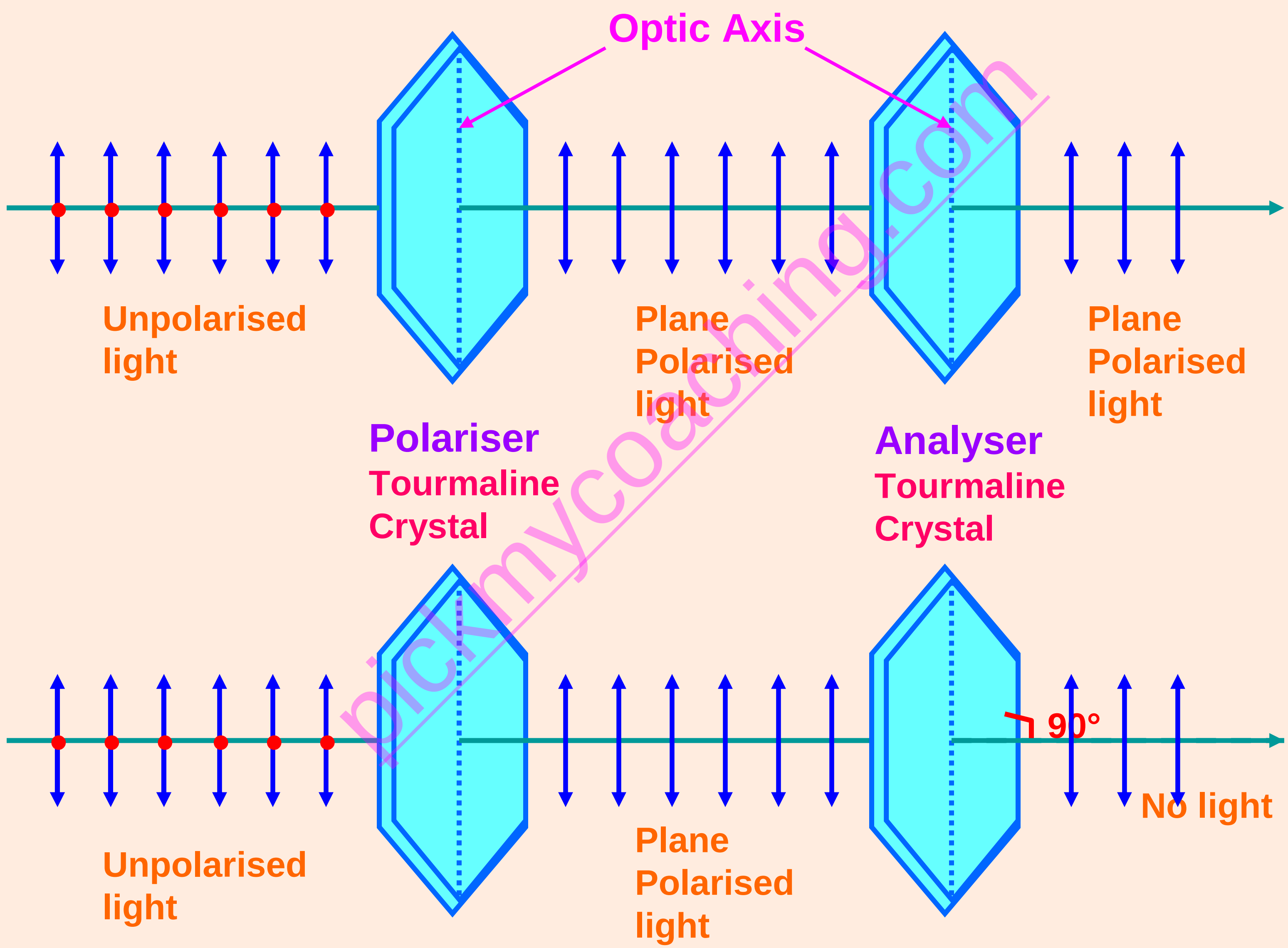
Natural Light

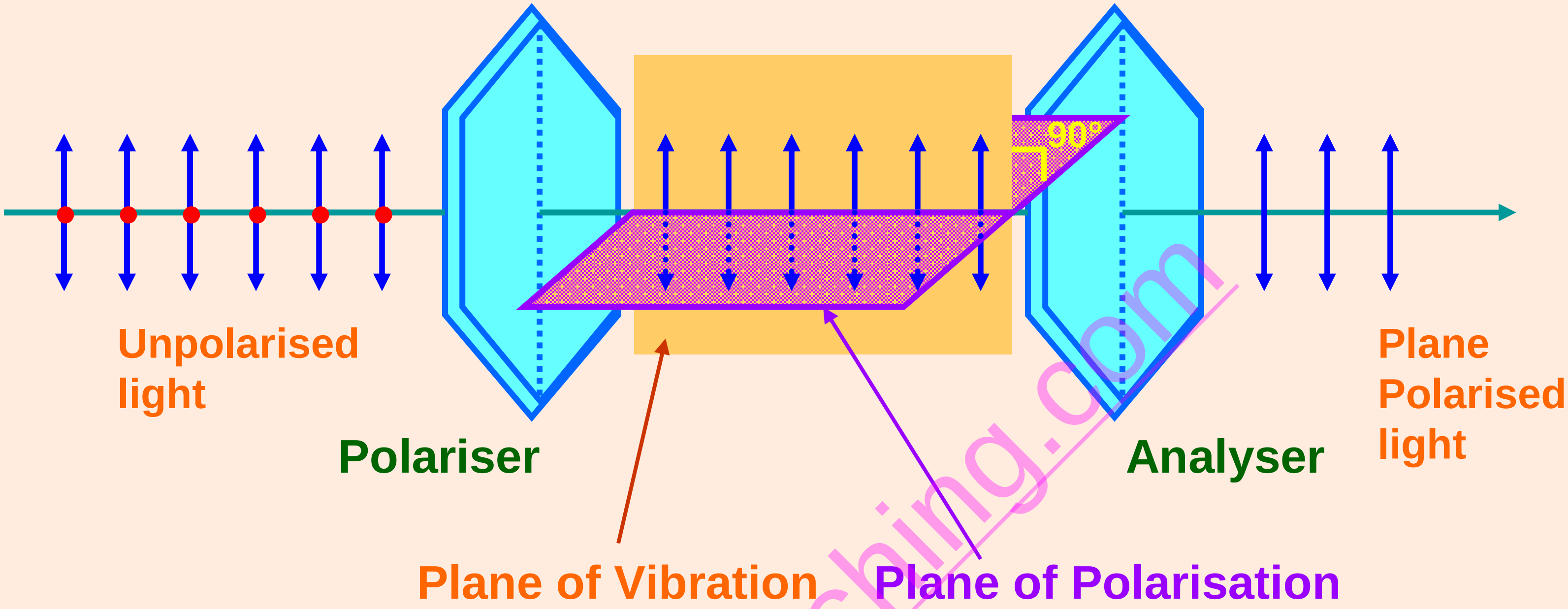


Representation of Natural Light

In natural light, millions of transverse vibrations occur in all the directions perpendicular to the direction of propagation of wave. But for convenience, we can assume the rectangular components of the vibrations with one component lying on the plane of the diagram and the other perpendicular to the plane of the diagram.

Light waves are electromagnetic waves with electric and magnetic fields oscillating at right angles to each other and also to the direction of propagation of wave. Therefore, the light waves can be polarised.





When unpolarised light is incident on the polariser, the vibrations parallel to the crystallographic axis are transmitted and those perpendicular to the axis are absorbed. Therefore the transmitted light is plane (linearly) polarised.

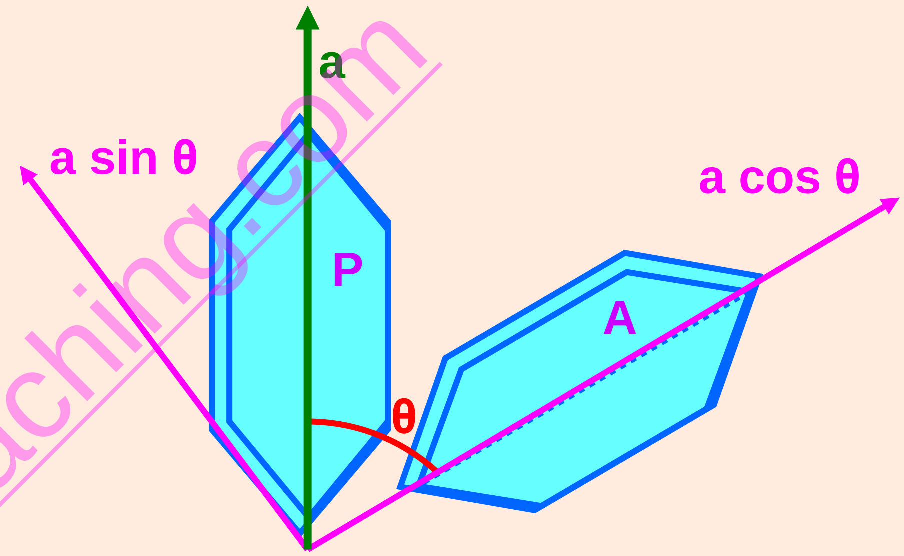
The plane which contains the crystallographic axis and vibrations transmitted from the polariser is called plane of vibration.

The plane which is perpendicular to the plane of vibration is called plane of polarisation.

Malus' Law:

When a beam of plane polarised light is incident on an analyser, the intensity I of light transmitted from the analyser varies directly as the square of the cosine of the angle θ between the planes of transmission of analyser and polariser.

$$\boxed{I \propto \cos^2\theta}$$



If a be the amplitude of the electric vector transmitted by the polariser, then only the component a cos θ will be transmitted by the analyser.

Intensity of transmitted light from the analyser is

$$I = k\,(a\cos\theta)^2$$

or $$I = k\,a^2\cos^2\theta$$

$$\boxed{I = I_0\cos^2\theta}$$

(where $I_0 = k\,a^2$ is the intensity of light transmitted from the polariser)

Case I : When θ = 0° or 180°, $I = I_0$

Case II : When θ = 90°, $I = 0$

Case III: When unpolarised light is incident on the analyser the intensity of the transmitted light is one-half of the intensity of incident light. (Since average value of $\cos^2\theta$ is ½)

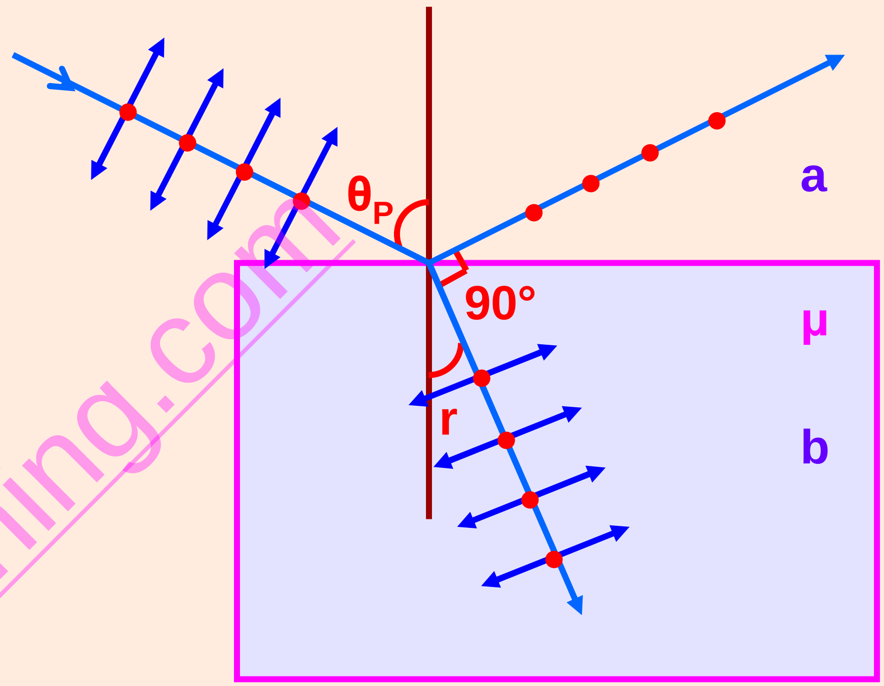
Polarisation by Reflection and Brewster's Law:

The incident light wave is made of parallel vibrations (π – components) on the plane of incidence and perpendicular vibrations (σ – components : perpendicular to plane of incidence).

At a particular angle θ_P, the parallel components completely refracted whereas the perpendicular components partially get refracted and partially get reflected.

i.e. the reflected components are all in perpendicular plane of vibration and hence plane polarised.

The intensity of transmitted light through the medium is greater than that of plane polarised (reflected) light.



$\theta_P + r = 90°$ or $r = 90° - \theta_P$

$${}_a\mu_b = \frac{\sin \theta_P}{\sin r}$$

$${}_a\mu_b = \frac{\sin \theta_P}{\sin (90° - \theta_P)}$$

$$\boxed{{}_a\mu_b = \tan \theta_P}$$

Polaroids:

H – Polaroid is prepared by taking a sheet of polyvinyl alcohol (long chain polymer molecules) and subjecting to a large strain. The molecules are oriented parallel to the strain and the material becomes doubly refracting. When strained with iodine, the material behaves like a dichroic crystal.

K – Polaroid is prepared by heating a stretched polyvinyl alcohol film in the presence of HCl (an active dehydrating catalyst). When the film becomes slightly darkened, it behaves like a strong dichroic crystal.

Uses of Polaroids:

1) **Polaroid Sun Glasses**
2) **Polaroid Filters**
3) **For Laboratory Purpose**
4) **In Head-light of Automobiles**
5) **In Three – Dimensional Motion Picutres**
6) **In Window Panes**
7) **In Wind Shield in Automobiles**