Book Name: Selina concise

EXERCISE. 22 (A)

Solution 1:

Slant height (ℓ) = 17 cm
Radius (r) = 8 cm
But,

$$l^2 = r^2 + h^2$$
$$\Rightarrow h^2 = l^2 - r^2$$
$$\Rightarrow h^2 = 17^2 - 8^2$$
$$\Rightarrow h^2 = 289 - 64 = 225 = (15)^2$$
$$\therefore h = 15$$

Now, volume of cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 8 \times 8 \times 15 \text{ cm}^3$$
$$= \frac{7040}{7} \text{ cm}^3$$
$$= 1005.71 \text{ cm}^3$$

Solution 2:

Curved surface area = 12320 cm^2
Radius of base (r) = 56 cm
Let slant height = ℓ

$$\therefore \pi r \ell = 12320$$
$$\Rightarrow \frac{22}{7} \times 56 \times \ell = 12320$$
$$\Rightarrow \ell = \frac{12320 \times 7}{56 \times 22}$$
$$\Rightarrow \ell = 70 \text{ cm}$$

Height of the cone =

$$= \sqrt{\ell^2 - r^2}$$

$= \sqrt{(70)^2 - (56)^2}$

$= \sqrt{4900 - 3136}$

$= \sqrt{1764}$

= 42 cm

Solution 3:

Circumference of the conical tent = 66 m

and height (h) = 12 m

$\therefore$ Radius $= \frac{c}{2\pi} = \frac{66 \times 7}{2 \times 22} = 10.5$ m

Therefore, volume of air contained in it $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 12\,m^3$

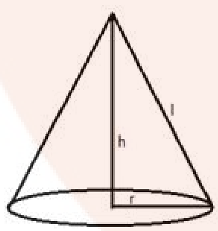
$= 1386\ m^3$

Solution 4:

The ratio between radius and height = 5:12

Volume = 5212 cubic cm

Let radius (r) = 5x, height (h) = 12x and slant height = ℓ



$\ell^2 = r^2 + h^2$

$\Rightarrow \ell^2 = (5x)^2 + (12x)^2$

$\Rightarrow \ell^2 = 25x^2 + 144x^2$

$\Rightarrow \ell^2 = 169\ x^2$

$\Rightarrow \ell = 13x$

Now volume $= \frac{1}{3}\pi r^2 h$

$\Rightarrow \frac{1}{3}\pi r^2 h = 2512$

$$\Rightarrow \frac{1}{3}(3.14)(5x)^2(12x) = 2512$$

$$\Rightarrow \frac{1}{3}(3.14)(300x^3) = 2512$$

$$\therefore\ x^3 = \frac{2512\times 3}{3.14\times 300} = \frac{2512\times 3\times 100}{314\times 300} = 8$$

$\Rightarrow x = 2$

$\therefore$ Radius = 5x = 5 × 2 = 10 cm

Height = 12x = 12 × 2 = 24 cm

Slant height = 13x = 13 × 2 = 26 cm

Solution 5:

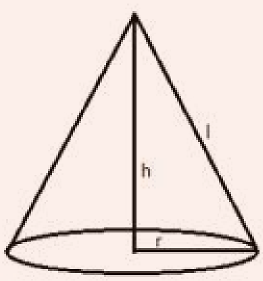
Let radius of cone y = r

Therefore, radius of cone x = 3r

Let volume of cone y = V

then volume of cone x = 2V

Let h_1 be the height of x and h2 be the height of y.



Therefore, Volume of cone = $\frac{1}{3}\pi r^2 h$

Volume of cone x = $\frac{1}{3}\pi(3r)^2 h_1 = \frac{1}{3}\pi 9r^2 h_1 = 3\pi r^2 h_1$

Volume of cone y = $\frac{1}{3}\pi r^2 h_2$

$$\therefore\ \frac{2V}{v} = \frac{3\pi r^2 h_1}{\frac{1}{3}\pi r^2 h_2}$$

$$\Rightarrow \frac{2}{1} = \frac{3h_1\times 3}{h_2} = \frac{9h_1}{h_2}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{2}{1} \times \frac{1}{9} = \frac{2}{9}$$

$\therefore\ h_1 : h_2 = 2 : 9$

Solution 6:

Let radius of each cone = r
Ratio between their slant heights = 5:4
Let slant height of the first cone = 5x
and slant height of second cone = 4x
Therefore, curved surface area of the first cone =
$\pi r\ell = \pi r \times (5x) = 5\pi rx$
curved surface area of the second cone = $\pi r\ell = \pi r \times (4x) = 4\pi rx$
Hence, ratio between them = $5\pi rx : 4\pi rx = 5 : 4$

Solution 7:

Let slant height of the first cone = ℓ
then slant height of the second cone = 2ℓ
Radius of the first cone = r_1
Radius of the second cone = r_2
Then, curved surface area of first cone = $\pi\ r_1\ \ell$
curved surface area of second cone = $\pi\ r_2 (2\ell) = 2\pi\ r_2\ \ell$
According to given condition:

$\pi r_1\ l = 2(2\pi r_2\ l)$

$\pi\ \pi r_1\ l = 4\pi r_2\ l$

$r_1 = 4r_2$

$\frac{r_1}{r_2} = \frac{4}{1}$

$\therefore\ r_1 : r_2 = 4 : 1$

Solution 8:

Diameter of the cone = 16.8 m

Therefore, radius (r) = 8.4 m
Height (h) = 3.5 m

(i) Volume of heap of wheat = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 8.4 \times 8.4 \times 3.5$

= 258.72 m^3

(ii) Slant height (ℓ) = $\sqrt{r^2 + h^2}$

$= \sqrt{(8.4)^2 + (3.5)^2}$

$= \sqrt{70.56 + 12.25}$

$= \sqrt{82.81}$

$= 9.1m$

Therefore, cloth required or curved surface area = $\pi r \ell$

$= \frac{22}{7} \times 8.4 \times 9.1$

$= 240.24\ m^2$

Solution 9:

Diameter of the tent = 48 m
Therefore, radius (r) = 24 m
Height (h) = 7 m
Slant height (ℓ) = $\sqrt{r^2 + h^2}$

$= \sqrt{(24)^2 + (7)^2}$

$= \sqrt{576 + 49}$

$= \sqrt{625}$

$= 25\ m$

Curved surface area = $\pi r \ell$

$= \frac{22}{7} \times 24 \times 25$

$= \frac{13200}{7}\ m^2$

Canvas required for stitching and folding

$= \frac{13200}{7} \times \frac{10}{100}$

$= \frac{1320}{7}\ m^2$

Total canvas required (area)

$= \frac{13200}{7} + \frac{1320}{7}$

$= \frac{14520}{7}\ m^2$

Length of canvas

$= \frac{\frac{14520}{7}}{\frac{3}{2}}$

$= \frac{14520}{7} \times \frac{2}{3}$

$= \frac{9680}{7}$

$= 1382.86\ m$

Rate = Rs 24 per meter

Total cost $= \frac{9680}{7} \times$ Rs 24 = Rs 33,188.64

Solution 10:

Height of solid cone (h) = 8 cm

Radius (r) = 6 cm

Volume of solid cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 6 \times 6 \times 8$

$= 96\pi\ cm^3$

Height of smaller cone = 2 cm

and radius = $\frac{1}{2}$ cm

Volume of smaller cone

$= \frac{1}{3} \times \pi \times \frac{1}{2} \times \frac{1}{2} \times 2$

$= \frac{1}{6}\pi cm^3$

Number of cones so formed

$$= \frac{96\pi}{\frac{1}{6}\pi}$$

$$= 96\pi \times \frac{6}{\pi}$$

$$= 576$$

Solution 11:

Total surface area of cone $= 90\pi \ cm^2$

slant height (l) = 13 cm

(i) Let r be its radius, then

Total surface area $= \pi r\ell + \pi r^2 = \pi r(\ell + r)$

$\therefore \pi r(\ell + r) = 90\pi$

$\Rightarrow r(13 + r) = 90$

$\Rightarrow r^2 + 13r - 90 = 0$

$\Rightarrow r^2 + 18r - 5r - 90 = 0$

$\Rightarrow r(r + 18) - 5(r + 18) = 0$

$\Rightarrow (r + 18)(r - 5) = 0$

Either r+18 = 0, then r = -18 which is not possible

or r – 5 = 0, then r = 5

Therefore, radius = 5 cm

(ii) Now

$h = \sqrt{\ell^2 - r^2}$

$= \sqrt{13^2 - 5^2}$

$= \sqrt{169 - 25}$

$= \sqrt{144}$

h = 12 cm

Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3.14 \times 5 \times 5 \times 12$

$= 314 \text{ cm}^3$

Solution 12:

Area of the base, $\pi r^2 = 38.5\ cm^2$

Volume of the solid, v = 154 cm^3

Curved surface area of the solid = $\pi r^2 h$

Volume, v = $\frac{1}{3}\pi r^2 h$

$$\Rightarrow 154 = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow h = \frac{154 \times 3}{\pi r^2}$$

$$\Rightarrow h = \frac{154 \times 3}{38.5} = 12\,cm$$

Area = 38.5

$$\pi r^2 = 38.5$$

$$\Rightarrow r^2 = \frac{38.5}{3.14}$$

$$\Rightarrow r = \sqrt{\frac{38.5}{3.14}} = 3.5$$

Curved surface area of solid = πrl

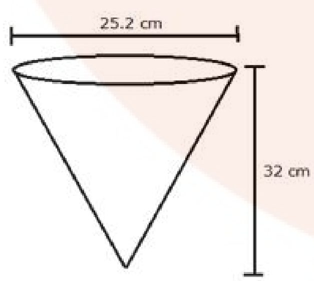
$$= \pi r\sqrt{r^2 + h^2}$$

$$= \pi \times 3.5 \times \sqrt{3.5^2 + 12^2}$$

$$= \pi \times 3.5 \times 12.5$$

$$= 137.44\ cm^2$$

Solution 13:



Volume of vessel = volume of water = $\frac{1}{3}\pi r^2 h$

diameter = 25.2 cm, therefore radius = 12.6 cm

height = 32 cm

Volume of water in the vessel = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 12.6 \times 12.6 \times 32$

$= 5322.24 \text{ cm}^3$

On submerging six equal solid cones into it, one-fourth of the water overflows.
Therefore, volume of the equal solid cones submerged
= Volume of water that overflows

$= \frac{1}{4} \times 5322.24$

$= 1330.56 \text{ cm}^3$

Now, volume of each cone submerged

$= \frac{1330.56}{6} = 221.76 \text{ cm}^3$

Solution 14:

(i) Let r be the radius of the base of the conical tent, then area of the base floor = $\pi r^2 m^2$

$\pi r^2 = 154$

$\frac{22}{7} \times r^2 = 154$

$\Rightarrow r^2 = \frac{154 \times 7}{22} = 49$

$\Rightarrow r = 7$

Hence, radius of the base of the conical tent i.e. the floor = 7m

(ii) Let h be the height of the conical tent, then the volume =

$\frac{1}{3}\pi r^2 h m^3$

$\therefore \frac{1}{3}\pi r^2 h = 1232$

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times h = 1232$

$\Rightarrow h = \frac{1232 \times 3}{22 \times 7} = 24$

Hence, radius of the base of the conical tent i.e. the floor = 7 m

(iii) Let l be the slant height of the conical tent, then $= \ell = \sqrt{h^2 + r^2}$ m

$\therefore \ell = \sqrt{h^2 + r^2} = \sqrt{(24)^2 + (7)^2} = \sqrt{576 + 49} = \sqrt{625} = 25$ m

The area of the canvas required to make the tent = $\pi r \ell m^2$

$\therefore \pi r \ell = \frac{22}{7} \times 7 \times 25 \text{ m}^2 = 550 \text{ m}^2$

Length of the canvas required to cover the conical tent of its width 2 m = $\frac{550}{2} = 275$ m

EXERCISE. 22 (B)

Solution 1:

Surface area of the sphere = 2464 cm^2

Let radius = r, then

$4\pi r^2 = 2464$

$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 2464$

$\Rightarrow r^2 = \frac{2462 \times 7}{4 \times 22} = 196$

$\Rightarrow r = 14\,\text{cm}$

Volume = $\frac{4}{3}\pi r^3$

$\therefore \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14 = 11498.67 \text{ cm}^3$

Solution 2:

Volume of the sphere = 38808 cm^3

Let radius of sphere = r

$\therefore \frac{4}{3}\pi r^3 = 38808$

$\Rightarrow \frac{4}{3} \times \frac{22}{7} \times r^3 = 38808$

$\Rightarrow r^3 = \frac{38808 \times 7 \times 3}{4 \times 22} = 9261$

$\Rightarrow r = 21$ cm

$\therefore$ diameter = 2r = 21 × 2 cm = 42 cm

Surface area = $4\pi r^2 = 4\times\frac{22}{7}\times 21\times 21\ \text{cm}^3 = 5544\ \text{cm}^3$

Solution 3:

Let the radius of spherical ball = r

$\therefore$ Volume = $\frac{4}{3}\pi r^3$

Radius of smaller ball = $\frac{r}{2}$

$\therefore$ Volume of smaller ball = $\frac{4}{3}\pi\left(\frac{r}{2}\right)^3 = \frac{4}{3}\pi\frac{r^3}{8} = \frac{\pi r^3}{6}$

Therefore, number of smaller balls made out of the given ball =

$$\frac{\frac{4}{3}\pi r^3}{\frac{\pi r^3}{6}} = \frac{4}{3}\times 6 = 8$$

Solution 4:

Diameter of bigger ball = 8 cm

Therefore, Radius of bigger ball = 4 cm

Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\times 4\times 4\times 4 = \frac{265\pi}{3}\ \text{cm}^3$

Radius of small ball = 1 cm

Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\times 1\times 1\times 1 = \frac{4\pi}{3}\ \text{cm}^3$

Number of balls = $\frac{\frac{256\pi}{3}}{\frac{4\pi}{3}} = \frac{256\pi}{3}\times\frac{3}{4\pi} = 64$

Solution 5:

Radius of metallic sphere = 2 mm $=\frac{1}{5}$ cm

Volume $=\frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}=\frac{88}{21\times125}$ cm^3

Volume of 8 spheres = $\frac{88\times8}{21\times125}=\frac{704}{21\times125}$ cm^3 …… (i)

Let radius of new sphere = R

$\therefore$ Volume $=\frac{4}{3}\pi R^3 = \frac{4}{3}\times\frac{22}{7}R^3=\frac{88}{21}R^3$ ……….. (ii)

From (i) and (ii)

$\frac{88}{21}R^3=\frac{704}{21\times125}$

$\Rightarrow R^3=\frac{704}{21\times125}\times\frac{21}{88}=\frac{8}{125}$

$\Rightarrow R=\frac{2}{5}=0.4\text{ cm}=4\text{ mm}$

Solution 6:

Volume of first sphere = 27 × volume of second sphere

Let radius of first sphere = r_1

and radius of second sphere = r_2

Therefore, volume of first sphere = $\frac{4}{3}\pi r_1^3$

and volume of second sphere = $\frac{4}{3}\pi r_2^3$

(i) Now, according to the question

$=\frac{4}{3}\pi r_1^3 = 27\times\frac{4}{3}\pi r_2^3$

$r_1^3=27r_2^3=\left(3r_2\right)^3$

$\Rightarrow r_1=3r_2$

$\Rightarrow \frac{r_1}{r_2}=\frac{3}{1}$

$\therefore\ r_1:r_2=3:1$

(ii) Surface area of first sphere = $4\pi r_1^2$

and surface area of second sphere = $4\pi r_2^2$

Ratio in surface area = $\frac{4\pi r_1^{2}}{4\pi r_2^{2}} = \frac{r_1^{2}}{r_2^{2}} = \frac{3^2}{1^2} = \frac{9}{1} = 9:1$

Solution 7:

Let r be the radius of the sphere.

Surface area = $4\pi r^2$ and volume = $\frac{4}{3}\pi r^3$

According to the condition:

$4\pi r^2 = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{r^3}{r^2} = 4\pi \times \frac{3}{4\pi}$

$\Rightarrow r = 3 \text{ cm}$

Diameter of sphere = 2 × 3 cm = 6 cm

Solution 8:

Diameter of sphere = $3\frac{1}{2}$ cm = $\frac{7}{2}$ cm

Therefore, radius of sphere = $\frac{7}{4}$ cm

Total curved surface area of each hemispheres =

$2\pi r^2 + \pi r^2 = 3\pi r^2$

$= 3 \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4}$

$= 28.88 \text{ cm}^2$

Solution 9:

External radius (R) = 14 cm

Internal radius (r) = $\frac{21}{2}$ cm

(i) Internal curved surface area =

$2\pi r^2$

$= 2\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}$

$= 693\ \text{cm}^3$

(ii) External curved surface area =

$2\pi R^2$

$= 2\times\frac{22}{7}\times 14\times 14$

$= 1232\ \text{cm}^2$

(iii) Total surface area =

$2\pi R^2 + 2\pi r^2 + \pi\left(R^2 - r^2\right)$

$= 693 + 1232 + \frac{22}{7}\left((14)^2 - \left(\frac{21}{2}\right)^2\right)$

$= 1925 + \frac{22}{7}\left(196 - \frac{441}{4}\right)$

$= 1925 + \frac{22}{7}\times\frac{343}{4}$

$= 1925 + 269.5$

$= 2194.5\ \text{cm}^3$

(iv) Volume of material used =

$\frac{2}{3}\pi\left(R^3 - r^3\right)$

$= \frac{2}{3}\times\frac{22}{7}\left((14)^3 - \left(\frac{21}{2}\right)^3\right)$

$= \frac{44}{21}(2744 - 1157.625)$

$= \frac{44}{21}\times 1586.375$

$= 3323.83\ \text{cm}^3$

Solution 10:

Let the radius of the sphere be 'r_1'.

Let the radius of the hemisphere be 'r_2'

TSA of sphere = $4\pi r_1^2$

TSA of hemisphere = $3\pi r_2^2$

TSA of sphere = TSA of hemi-sphere

$$4\pi r_1^2 = 3\pi r_2^2$$

$$\Rightarrow r_2^2 = \frac{4}{3} r_1^2$$

$$\Rightarrow r_2 = \frac{2}{\sqrt{3}} r_1$$

Volume of sphere, $v_1 = \frac{4}{3}\pi r_1^3$

Volume of hemisphere, $V_2 = \frac{2}{3}\pi r_2^3$

$$v_2 = \frac{2}{3}\pi r_2^3$$

$$\Rightarrow v_2 = \frac{2}{3}\pi\left(\frac{r_1 2}{\sqrt{3}}\right)^3$$

$$\Rightarrow v_2 = \frac{2}{3}\pi \frac{r_2^3 8}{3\sqrt{3}}$$

Dividing v_1 by v_2

$$\frac{v_1}{v_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{2}{3}\pi \frac{8}{3\sqrt{3}} r_1^3}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{\frac{4}{3}}{\frac{2}{3}\frac{8}{3\sqrt{3}}}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{4}{3} \times \frac{9\sqrt{3}}{16}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{3\sqrt{3}}{4}$$

Solution 11:

Let radius of the larger sphere be 'R'

Volume of single sphere

= Vol. of sphere 1 + Vol. of sphere 2 + Vol. of sphere 3

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$$

$$\Rightarrow \frac{4}{3}\pi R^3 = \frac{4}{3}\pi r 6^3 + \frac{4}{3}\pi r 8^3 + \frac{4}{3}\pi 10^3$$

$$\Rightarrow R^3 = \left[6^3 + 8^3 + 10^3\right]$$

$$\Rightarrow R^3 = 1728$$

$$\Rightarrow R = 12$$

Surface area of the sphere

$$= 4\pi R^2$$

$$= 4\pi 12^2$$

$$= 1785.6 \text{ cm}^2$$

Solution 12:

Let the radius of the sphere be 'r'.

Total surface area the sphere, S = $4\pi r^2$

New surface area of the sphere, S'

$$= 4\pi r^2 + \frac{21}{100} \times 4\pi r^2$$

$$= \frac{121}{100} 4\pi r^2$$

(i) let the new radius be r_1

$$S^I = 4p\, r_1^2$$

$$S^I = \frac{121}{100} 4\pi r^2$$

$$\Rightarrow 4\pi r_1^2 = \frac{121}{100} 4\pi r^2$$

$$\Rightarrow r_1^2 = \frac{121}{100} r^2$$

$$\Rightarrow r_1 = \frac{11}{10} r$$

$$\Rightarrow r_1 = r + \frac{r}{10}$$

$$\Rightarrow r_1 - r = \frac{r}{10}$$

$\Rightarrow$ change in radius = $\frac{r}{10}$

Percentage change in radius = $\frac{change\ in\ radius}{original\ radius} \times 100$

$= \frac{r/_{10}}{r} \times 100$

= 10

Percentage change in radius = 10%

(ii) Let the volume of the sphere be V

Let the new volume of the sphere be V'.

$v = \frac{4}{3}\pi r^3$

$v^I = \frac{4}{3}\pi r_1^3$

$\Rightarrow v^I = \frac{4}{3}\pi\left(\frac{11r}{10}\right)^3$

$\Rightarrow v^I = \frac{4}{3}\pi\frac{1331}{1000}r^3$

$\Rightarrow v^I = \frac{4}{3}\pi r^3\frac{1331}{1000}$

$\Rightarrow v^I = \frac{1331}{1000}v$

$\Rightarrow v^I = v + \frac{1331}{1000}v$

$\Rightarrow v^I - v = \frac{331}{1000}v$

$\therefore$ Change in volume = $\frac{331}{1000}v$

Percentage change in volume = $\frac{change\ in\ volume}{original\ volume} \times 100$

$= \frac{\frac{331}{1000}v}{v} \times 100$

$= \frac{331}{10}$

$= 33.1$

Percentage change in volume = 33.1 %

EXERCISE. 22 (C)

Solution 1:

Radius of sphere = 8 cm

$\therefore$ Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times 8\times 8\times 8 = \frac{45056}{21}\text{ cm}^3$ …….(i)

Height of the cone = 32 cm

Let radius = r

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times r^2\times 32 = \frac{704}{21}r^2\text{cm}^3$ ……...(ii)

From (i) and (ii)

$$\frac{704}{21}r^2 = \frac{45056}{21}$$

$$\Rightarrow r^2 = \frac{45056}{21}\times\frac{21}{704}$$

$$\Rightarrow r^2 = 64$$

$$\Rightarrow r = 8\text{ cm}$$

Solution 2:

External diameter = 8 cm

Therefore, radius (R) = 4 cm

Internal diameter = 4 cm

Therefore, radius (r) = 2 cm

Volume of metal used in hollow sphere =

$\frac{4}{3}\pi(R^3 - r^3) = \frac{4}{3}\times\frac{22}{7}\times(4^3 - 2^3) = \frac{88}{21}(64-8) = \frac{88}{21}\times 56\text{ cm}^3$ …… (i)

Diameter of cone = 8 cm

Therefore, radius = 4 cm

Let height of cone = h

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times 4\times 4\times h = \frac{352}{21}h$ ….. (ii)

From (i) and (ii)

$$\frac{352}{21}h = \frac{88}{21}\times 56$$

$$\Rightarrow h = \frac{88\times 56\times 21}{21\times 352} = 14\text{ cm}$$

Height of the cone = 14 cm.

Solution 3:

Internal radius = 3cm
External radius = 5 cm
Volume of spherical shell

$$= \frac{4}{3}\pi\left(5^3 - 3^3\right)$$

$$= \frac{4}{3} \times \frac{22}{7}(125 - 27)$$

$$= \frac{4}{3} \times \frac{22}{7} \times 98$$

Volume of solid circular cone

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times r^2 \times 32$$

Vol. of Cone = Vol. of sphere

$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times r^2 \times 32 = \frac{4}{3} \times \frac{22}{7} \times 98$$

$$\Rightarrow r^2 = \frac{4 \times 98}{32}$$

$$\therefore \ r = \frac{7}{2} = 3.5 \ \text{cm}$$

Solution 4:

Let the radius of the smaller cone be 'r' cm.
Volume of larger cone

$$= \frac{1}{3}\pi \times 20^2 \times 9$$

Volume of smaller cone

$$= \frac{1}{3}\pi \times r^2 \times 108$$

Volume of larger cone = 3 × Volume of smaller cone

$$\frac{1}{3}\pi \times 20^2 \times 9 = \frac{1}{3}\pi \times r^2 \times 108 \times 3$$

$$\Rightarrow r^2 = \frac{20^2 \times 9}{108 \times 3}$$

$$\Rightarrow r = \frac{20}{6} = \frac{10}{3}$$

Solution 5:

Volume of rectangular block = 49 × 44 × 18 cm^3 = 38808 cm^3 ……… (i)

Let r be the radius of sphere

$$\therefore \text{Volume} = = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times r^3 = \frac{88}{21} r^3 \quad \text{…… (ii)}$$

From (i) and (ii)

$$\frac{88}{21} r^3 = 38808$$

$$\Rightarrow r^3 = 38808 \times \frac{21}{88} = 441 \times 21$$

$$\Rightarrow r^3 = 9261$$

$$\Rightarrow r = 21 \text{ cm}$$

Radius of sphere = 21 cm

Solution 6:

Radius of hemispherical bowl = 9 cm

$$\text{Volume } = \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi 9^3 \times \frac{2}{3}\pi \times 729 = 486\pi \text{ cm}^3$$

Diameter each of cylindrical bottle = 3 cm

Radius = $\frac{3}{2}$ cm, and height = 4 cm

$$\therefore \text{volume of bottle} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times \left(\frac{3}{2}\right)^2 \times 4 = 3\pi$$

$$\therefore \text{No of bottles} = \frac{486\pi}{3\pi} = 162$$

Solution 7:

Diameter of the hemispherical bowl = 7.2 cm

Therefore, radius = 3.6 cm

Volume of sauce in hemispherical bowl = $\frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times (3.6)^3$

Radius of the cone = 4.8 cm

Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (4.8)^2 \times h$

Now, volume of sauce in hemispherical bowl = volume of cone

$\Rightarrow \frac{2}{3}\pi \times (3.6)^3 = \frac{1}{3}\pi \times (4.8)^2 \times h$

$\Rightarrow h = \frac{2 \times 3.6 \times 3.6 \times 3.6}{4.8 \times 4.8}$

$\Rightarrow h = 4.05$ cm

Height of the cone = 4.05 cm

Solution 8:

Radius of a solid cone (r) = 5 cm

Height of the cone = 8 cm

$\Rightarrow$ Volume of a cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 5 \times 5 \times 8 \text{ cm}^3$

$= \frac{200\pi}{3} \text{cm}^3$

Radius of each sphere = 0.5 cm

$\therefore$ Volume of one sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \Pi \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \text{cm}^3$

$= \frac{\Pi}{6} \text{ cm}^3$

Number of spheres = $\frac{Total\ volume}{Volume\ of\ one\ sphere}$

$= \frac{\frac{200\pi}{3}}{\frac{\pi}{6}} \times \frac{6}{\pi}$

$$= \frac{200\pi}{3} \times \frac{6}{\pi}$$
$$= 400$$

Solution 9:

Total area of solid metallic sphere = 1256 cm^2

(i)Let radius of the sphere is r then

$$4\pi r^2 = 1256$$
$$4 \times \frac{22}{7} r^2 = 1256$$
$$\Rightarrow r^2 = \frac{1256 \times 7}{4 \times 22}$$
$$\Rightarrow r^2 = \frac{157 \times 7}{11}$$
$$\Rightarrow r^2 = \frac{1099}{11}$$
$$\Rightarrow r = \sqrt{99.909} = 9.995 \text{ cm}$$
$$\Rightarrow r = 10 \text{ cm}$$

(ii) Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 10 \times 10 \times 10 = \frac{88000}{21} \text{ cm}^3$

volume of right circular cone =

$$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (2.5)^2 \times 8 = \frac{1100}{21} \text{ cm}^3$$

Number of cones

$$= \frac{88000}{21} \div \frac{1100}{21}$$
$$= \frac{88000}{21} \times \frac{21}{1100}$$
$$= 80$$

Solution 10:

Volume of the whole cone of metal A

$$= \frac{1}{3}\pi r^2 h$$

$=\frac{1}{3}\times\pi\times 6^2\times 10$

$=120\pi$

Volume of the cone with metal B

$=\frac{1}{3}\pi r^2 h$

$=\frac{1}{3}\times\pi\times 3^2\times 4$

$=12\pi$

Final Volume of cone with metal A=120π − 12π =108π

$\frac{Volume\ of\ cone\ with\ metal\ A}{Volume\ of\ cone\ with\ metal\ B}$

$=\frac{108\pi}{12\pi}=\frac{9}{1}$

Solution 11:

Let the number of small cones be 'n'

Volume of sphere

$=\frac{4}{3}\pi\left(8^3-6^3\right)$

$=\frac{4}{3}\times\pi\times 2\times 148$

Volume of small spheres

$=\frac{1}{3}\pi r^2 h$

$=\frac{1}{3}\times\pi\times 3^2\times 4$

Volume of sphere = n × Volume of small sphere

$\Rightarrow\frac{4}{3}\times\pi\times 2\times 148=n\times\frac{1}{3}\times\pi\times 2^2\times 8$

$\Rightarrow n=\frac{4\times 2\times 148\times 3}{4\times 8\times 3}$

$\Rightarrow n=37$

The number of cones = 37 cm.

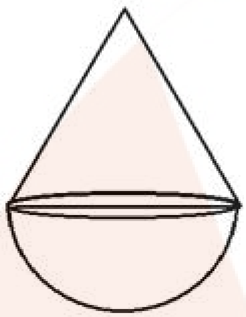
EXERCISE. 22 (D)

Solution 1:

Height of cone = 15 cm

and radius of the base = $\frac{7}{2}$ cm

Therefore, volume of the solid = volume of the conical part + volume of hemispherical part.



$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 (h + 2r)$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\left(15 + 2 \times \frac{7}{2}\right)$$

$$= \frac{847}{3}$$

$$= 282.33 \text{ cm}^3$$

Solution 2:

Radius of hemispherical part (r) = 3.5 m = $\frac{7}{2}$m

Therefore, Volume of hemisphere = $\frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{539}{6} \text{ m}^3$$

Volume of conical part = $\frac{2}{3} \times \frac{539}{6}\text{m}^3$ (2/3 of hemisphere)

Let height of the cone = h

Then,

$$\frac{1}{3}\pi r^2 h = \frac{2\times 539}{3\times 6}$$

$$\Rightarrow \frac{1}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times h = \frac{2\times 539}{3\times 6}$$

$$\Rightarrow h = \frac{2\times 539\times 2\times 2\times 7\times 3}{3\times 6\times 22\times 7\times 7}$$

$$\Rightarrow h = \frac{14}{3}\text{m} = 4\frac{2}{3}\text{ m} = 4.67\text{ m}$$

Height of the cone = 4.67 m

Surface area of buoy = $2\pi r^2 + \pi r\ell$

But $\ell = \sqrt{r^2 + h^2}$

$$\ell = \sqrt{\left(\frac{7}{2}\right)^2 + \left(\frac{14}{3}\right)^2}$$

$$= \sqrt{\frac{49}{4}+\frac{196}{9}} = \sqrt{\frac{1225}{36}} = \frac{35}{6}\text{m}$$

Therefore, Surface area =

$$= \left(2\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right) + \left(\frac{22}{7}\times\frac{7}{2}\times\frac{35}{6}\right)\text{m}^2$$

$$= \frac{77}{1}+\frac{385}{6} = \frac{847}{6}$$

$$= 141.17\text{ m}^2$$

Surface Area = 141.17 m^2

Solution 3:

(i) Total surface area of cuboid = 2(ℓb + bh + ℓh)

= 2 (42 × 30 + 30 × 20 + 20 × 42)

= 2 (1260 + 600 + 840)

= 2 × 2700

= 5400 cm^2

Diameter of the cone = 14 cm

$\Rightarrow$ Radius of the cone = $\frac{14}{2} = 7\,\text{cm}$

Area of circular base $= \pi r^2 = \frac{22}{7}\times 7\times 7 = 154\text{ cm}^2$

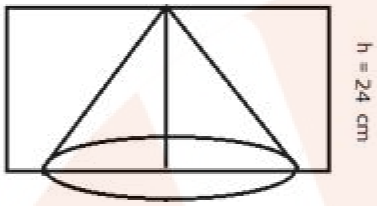
Area of curved surface area of cone =

$$\pi r\ell = \frac{22}{7}\times 7\times\sqrt{7^2+24^2} = 22\sqrt{49+576} = 22\times 25 = 550\ \text{cm}^2$$

Surface area of remaining part = 5400 + 550 – 154 = 5796 cm^2

(ii) Dimensions of rectangular solids = (42 × 30 × 20) cm

volume = (42 × 30 × 20) = 25200 cm^3



Radius of conical cavity (r) =7 cm

height (h) = 24 cm

Volume of cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\times\frac{22}{7}\times 7\times 7\times 24$$

$$= 1232\ \text{cm}^3$$

Volume of remaining solid = (25200 - 1232) = 23968 cm^3

(iii) Weight of material drilled out

=1232 × 7 g = 8624g = 8.624 kg

Solution 4:

The diameter of the largest hemisphere that can be placed on a face of a cube of side 7 cm will be 7 cm.

Therefore, radius = $r = \frac{7}{2}$ cm

Its curved surface area $= 2\pi r^2$

$$= 2\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$$

$$= 77\ \text{cm}^2 \ \ldots\ldots\ldots\ldots(i)$$

Surface area of the top of the resulting solid = Surface area of the top face of the cube − Area of the base of the hemisphere

$$= (7\times 7) - \left(\frac{22}{7}\times\frac{49}{4}\right)$$

$$= 49 - \frac{77}{2}$$

$= \frac{98-77}{2}$

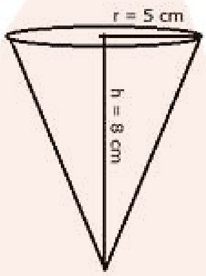
$= \frac{21}{2}$

$= 10.5\ cm^2$ …….. (ii)

Surface area of the cube = 5 × (side)^2 = 5 × 49 = 245 cm^2 ………. (iii)

Total area of resulting solid = 245 + 10.5 + 77 = 332.5 cm^2

Solution 5:



Height of cone = 8 cm

Radius = 5 cm

Volume = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 8\ cm^3$

$= \frac{4400}{21} cm^3$

Therefore, volume of water that flowed out =

$= \frac{1}{4} \times \frac{4400}{21}\ \ cm^3$

$= \frac{1100}{21}\ \ cm^3$

Radius of each ball = 0.5 cm = $\frac{1}{2}$ cm

Volume of a ball = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\ cm^3$

$= \frac{11}{21}\ cm^3$

Therefore, No. of balls = $= \frac{1100}{21} \div \frac{11}{21} = 100$

Hence, number of lead balls = 100

Solution 6:

Let r be the radius of the bowl.

$\therefore\ 2\pi r = 198$

$\Rightarrow r = \frac{198 \times 7}{2 \times 22}$

$\Rightarrow r = 31.5\ \text{cm}$

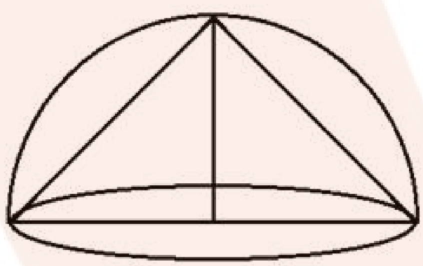
Capacity of the bowl =

$\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (31.5)^3$

$= 65488.5\ \text{cm}^3$

Solution 7:



For the volume of cone to be largest, h = r cm

Volume of the cone

$\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times r^2 \times r$

$= \frac{1}{3}\pi r^3$

Solution 8:

Let the height of the solid cones be 'h'

Volume of solid circular cones

$$v_1 = \frac{1}{3}\pi r_1^2 h$$

$$v_2 = \frac{1}{3}\pi r_2^2 h$$

Volume of sphere

$$= \frac{4}{3}\pi R^3$$

Volume of sphere = Volume of cone 1 + volume of cone 2

$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi r_1^2 h + \frac{1}{3}\pi r_2^2 h$$

$$\Rightarrow 4R^3 = r_1^2 h + r_2^2 h$$

$$\Rightarrow h\left(r_1^2 + r_2^2\right) = 4R^3$$

$$\Rightarrow h = \frac{R^3}{\left(r_1^2 + r_2^2\right)}$$

Solution 9:

Volume of the solid hemisphere

$$= \frac{4}{3}\pi R^3$$

$$= \frac{4}{3}\pi 14^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14$$

Volume of 1 cone

$$\frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 8$$

No of cones formed

$$= \frac{Volume\, of\ sphere}{volume\, of\ 1 cone}$$

$$= \frac{\frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14}{\frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 8}$$

$$= 28$$

Solution 10:

Let the radius of base be 'r' and the height be 'h'

Volume of cone, V_c

$$= \frac{1}{3}\pi r^2 h$$

Volume of hemisphere, V_h

$$= \frac{2}{3}\pi r^3$$

$$\frac{V_c}{V_h} = \frac{\frac{1}{3}\pi r^2 r}{\frac{2}{3}\pi r^3}$$

$$\Rightarrow \frac{V_c}{V_h} = \frac{1}{2}$$

<u>EXERCISE. 22 (E)</u>

Solution 1:

Height of the cylinder (h) = 10 cm

and radius of the base (r) = 6 cm

Volume of the cylinder = $\pi r^2 h$

Height of the cone = 10 cm

Radius of the base of cone = 6 cm

Volume of the cone = $\frac{1}{3}\pi r^2 h$

Volume of the remaining part

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h$$

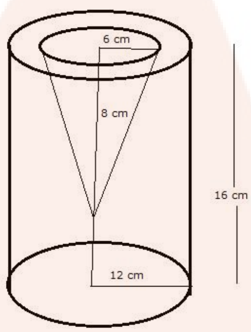
$=\frac{2}{3}\pi r^2 h$

$=\frac{2}{3}\times\frac{22}{7}\times 6\times 6\times 10$

$=\frac{5280}{7}$

$=754\frac{2}{7}\text{ cm}^3$

Solution 2:



Radius of solid cylinder (R) = 12 cm
and Height (H) = 16 cm

$\therefore$ Volume = $\pi R^2 H$

$=\frac{22}{7}\times 12\times 12\times 16$

$=\frac{50688}{7}\text{ cm}^3$

Radius of cone (r) = 6 cm, and height (h) = 8 cm.

$\therefore$ Volume = $\frac{1}{3}\pi r^2 h$

$=\frac{1}{3}\times\frac{22}{7}\times 6\times 6\times 8$

$=\frac{2112}{7}\text{ cm}^3$

(i) Volume of remaining solid

$=\frac{50688}{7}-\frac{2112}{7}$

$=\frac{48567}{7}$

$= 6939.43^2 \text{ cm}^3$

(ii) Slant height of cone $\ell = \sqrt{h^2 + r^2}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10 \text{ cm}$

Therefore, total surface area of remaining solid = curved surface area of cylinder + curved surface area of cone + base area of cylinder + area of circular ring on upper side of cylinder

$= 2\pi Rh + \pi r\ell + \pi R^2 + \pi\left(R^2 - r^2\right)$

$= \left(2 \times \frac{22}{7} \times 12 \times 16\right) + \left(\frac{22}{7} \times 6 \times 10\right) + \left(\frac{22}{7} \times 12 \times 12\right) + \left(\frac{22}{7}\left(12^2 - 6^2\right)\right)$

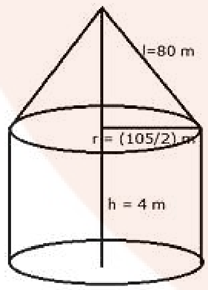
$= \frac{8448}{7} + \frac{1320}{7} + \frac{3168}{7} + \frac{22}{7}(144 - 36)$

$= \frac{8448}{7} + \frac{1320}{7} + \frac{3168}{7} + \frac{2376}{7}$

$= \frac{15312}{7}$

$= 2187.43 \text{ cm}^3$

Solution 3:



Radius of the cylindrical part of the tent (r) = $\frac{105}{2}$ m

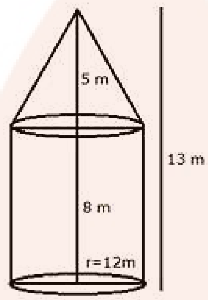
Slant height (ℓ) = 80 m

Therefore, total curved surface area of the tent $= 2\pi rh + \pi r\ell$

$= \left(2 \times \frac{22}{7} \times \frac{105}{2} \times 4\right) + \left(\frac{22}{7} \times \frac{105}{2} \times 80\right)$

= 1320 + 13200
= 14520 m^2
Width of canvas used = 1.5 m
Length of canvas = $\frac{14520}{1.5} = 9680$ m
Total cost of canvas at the rate of Rs 15 per meter
= 9680 × 15 = Rs. 145200

Solution 4:



Height of the cylindrical part = H = 8 m
Height of the conical part = h = (13- 8)m = 5 m
Diameter = 24 m ⟶ radius = r = 12 m
Slant height of the cone = l
$\ell = \sqrt{r^2 + h^2}$
$\ell = \sqrt{12^2 + 5^2}$
$\ell = \sqrt{169}$ = 13 m
Slant height of cone = 13 m
(i) Total surface are of the tent = $2\pi rh + \pi r\ell = \pi r(2h + \ell)$
$= \frac{22}{7} \times 12 \times (2 \times 8 + 13)$
$= \frac{264}{7}(16 + 13)$
$= \frac{264}{7} \times 29$
$= \frac{7656}{7} m^2$
$= 1093.71\ m^2$
(ii) Area of canvas used in stitching = total area

Total area of canvas = $\frac{7656}{7}+\frac{\text{Total area of canvas}}{10}$

$\Rightarrow$ Total area of canvas $-\frac{\text{Total area of canvas}}{10}=\frac{7656}{7}$

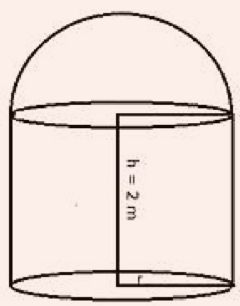
$\Rightarrow$ Total area of canvas $\left(1-\frac{1}{10}\right)=\frac{7656}{7}$

$\Rightarrow$ Total area of canvas $\times\frac{9}{10}=\frac{7656}{7}$

$\Rightarrow$ Total area of canvas $=\frac{7656}{7}\times\frac{10}{9}$

$\Rightarrow$ Total area of canvas $=\frac{76560}{63}=1215.23\ \text{m}^2$

Solution 5:



Diameter of cylindrical boiler = 3.5 m

$\therefore$ Radius (r) = $\frac{3.5}{2}=\frac{35}{20}=\frac{7}{4}$m

Height (h) = 2 m

Radius of hemisphere (R) = $\frac{7}{4}$m

Total volume of the boiler = $\pi r^2 h+\frac{2}{3}\pi r^3$

$=\pi r^2\left(h+\frac{2}{3}r\right)$

$=\frac{22}{7}\times\frac{7}{4}\times\frac{7}{4}\left(2+\frac{2}{3}\times\frac{7}{4}\right)$

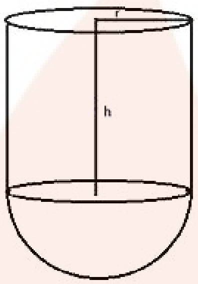
$=\frac{77}{8}\left(2+\frac{7}{6}\right)$

$= \frac{77}{8} \times \frac{19}{6}$

$= \frac{1463}{48}$

$= 30.48 \text{ m}^3$

Solution 6:



Diameter of the base = 3.5 m

Therefore, radius = $\frac{3.5}{2} \text{m} = 1.75\text{m} = \frac{7}{4} \text{m}$

Height of cylindrical part = $4\frac{2}{3} = \frac{14}{3} \text{m}$

(i) Capacity (volume) of the vessel = $\pi r^2 h + \frac{2}{3}\pi r^3 = \pi r^2 \left(h + \frac{2}{3} r \right)$

$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \left(\frac{14}{3} + \frac{2}{3} \times \frac{7}{4} \right)$

$= \frac{77}{8} \left(\frac{14}{3} + \frac{7}{6} \right)$

$= \frac{77}{8} \left(\frac{28 + 7}{6} \right)$

$= \frac{77}{8} \times \frac{35}{6}$

$= \frac{2695}{48}$

$= 56.15 \text{ m}^3$

(ii) Internal curved surface area = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times \frac{7}{4}\left(\frac{14}{3} + \frac{7}{4}\right)$

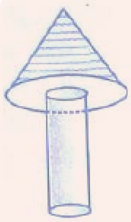
$= 11\left(\frac{56+21}{12}\right)$

$= 11 \times \frac{77}{12}$

$= \frac{847}{12}$

$= 70.58\ m^2$

Solution 7:



Height of the cone = 24 cm
Height of the cylinder = 36 cm
Radius of the cone = twice the radius of the cylinder = 10 cm
Radius of the cylinder = 5 cm

Slant height of the cone = $\sqrt{r^2 + h^2}$

$= \sqrt{10^2 + 24^2}$

$= \sqrt{100 + 576}$

$= \sqrt{676}$

$= 26$ cm

Now, the surface area of the toy = curved area of the conical point + curved area of the cylinder

$= \pi r\ell + \pi r^2 + 2\pi RH$

$= \pi\left[r\ell + r^2 + 2RH\right]$

$= 3.14\left[10 \times 26 + (10)^2 + 2 \times 5 \times 36\right]$

$= 3.14[260 + 100 + 360]$

$= 3.14[720]$

$= 2260.8\ cm^2$

Solution 8:

Diameter of cylindrical container = 42 cm
Therefore, radius (r) = 21 cm
Dimensions of rectangular solid = 22cm × 14cm × 10.5cm
Volume of solid = $22 \times 14 \times 10.5 \text{ cm}^3$ ……….(i)
Let height of water = h
Therefore, volume of water in the container = $\pi r^2 h$

$$= \frac{22}{7} \times 21 \times 21 \times h \text{ cm}^3 = 22 \times 63h \text{ cm}^3 \quad \text{……. (ii)}$$

From (i) and (ii)

$$22 \times 63h = 22 \times 14 \times 10.5$$

$$\Rightarrow h = \frac{22 \times 14 \times 10.5}{22 \times 63}$$

$$\Rightarrow h = \frac{7}{3}$$

$$\Rightarrow h = 2\frac{1}{3} \text{ or } 2.33 \text{ cm}$$

Solution 9:

Diameter of spherical marble = 1.4 cm
Therefore, radius = 0.7 cm

Volume of one ball = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.7)^3 \text{ cm}^3$ ……….(i)

Diameter of beaker = 7 cm

Therefore, radius = $\frac{7}{2}$ cm

Height of water = 5.6 cm

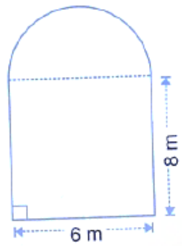
Volume of water $= \pi r^2 h = \pi \times \left(\frac{7}{2} \times \frac{7}{2} \times 5.6\right) \text{ cm}^3 = \pi \times \frac{49 \times 56}{4 \times 10} \text{ cm}^3$

No. of balls dropped

$$= \frac{\pi \times 49 \times 56 \times 3}{4 \times 10 \times 4\pi \times (0.7)^3}$$

$$= 150$$

Solution 10:



Breadth of the tunnel = 6 m
Height of the tunnel = 8 m
Length of the tunnel = 35 m
Radius of the semi-circle = 3 m

Circumference of the semi-circle = $\pi r = \frac{22}{7} \times 3 = \frac{66}{7}$ m

Internal surface area of the tunnel

$= 35\left(8+8+\frac{66}{7}\right)$

$= 35\left(16+\frac{66}{7}\right)$

$= 35\left(\frac{112+66}{7}\right)$

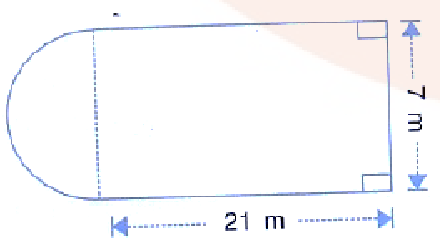
$= 35 \times \frac{178}{7}$

$= 890 \text{ m}^2$

Rate of plastering the tunnel = Rs 2.25 per m^2

Therefore, total expenditure = Rs. $890 \times \frac{225}{100}$ = Rs $890 \times \frac{9}{4}$ = Rs $\frac{8010}{4}$ = Rs. 2002.50

Solution 11:



Length = 21 m

Depth of water = 2.4 m
Breadth = 7 m

Therefore, radius of semicircle = $\frac{7}{2}$ m

Area of cross-section = area of rectangle + Area of semicircle

$= l \times b + \frac{1}{2}\pi r^2$

$= 21 \times 7 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 147 + \frac{77}{4}$

$= \frac{588 + 77}{4}$

$= \frac{665}{4}\ m^2$

Therefore, volume of water filled in gallons

$= \frac{665}{4} \times 2.4\ \ m^3$

$= 665 \times 0.6$

$= 399\ m^3$

$= 399 \times 100^3 cm^3$

$= \frac{399 \times 100 \times 100 \times 100}{1000}$ gallons

$= \frac{399 \times 100 \times 100 \times 100}{1000 \times 4.5}$

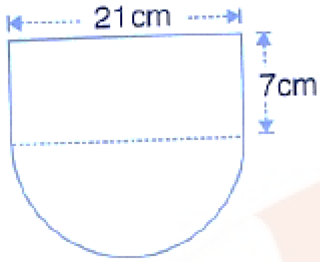
$= \frac{399 \times 100 \times 100 \times 100 \times 10}{1000 \times 45}$ gallons

$= \frac{1330000}{15}$ gallons

$= \frac{266000}{3}$ gallons

$= 88666.67$ gallons

Solution 12:



Length = 21 cm, Breadth = 7 cm

Radius of semicircle $=\dfrac{21}{2}$ cm

Area of cross section of the water channel $=l\times b+\dfrac{1}{2}\pi r^2$

$=21\times 7+\dfrac{1}{2}\times\dfrac{22}{7}\times\dfrac{21}{2}\times\dfrac{21}{2}$

$=147+\dfrac{693}{4}$

$=\dfrac{588+693}{4}$

$=\dfrac{1281}{4}\text{ cm}^2$

Flow of water in one minute at the rate of 20 cm per second

$\Rightarrow$ Length of the water column = 20 × 60 = 1200 cm

Therefore, volume of water =

$=\dfrac{1281}{4}\times 1200\text{ cm}^3$

$=384300\text{ cm}^2$

$=\dfrac{384300}{100\times100\times100}\text{m}^3$

$=0.3843\text{ m}^3$

$=0.4\text{ m}^3$

Solution 13:

Diameter of the base of the cylinder = 7 cm

Therefore, radius of the cylinder = $\dfrac{7}{2}$ cm

Volume of the cylinder $\pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 8 = 308\ \text{cm}^3$

Diameter of the base of the cone $= \frac{7}{2}$ cm

Therefore, radius of the cone $= \frac{7}{4}$ cm

Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 8 = \frac{77}{3}\ \text{cm}^3$

On placing the cone into the cylindrical vessel, the volume of the remaining portion where the water is to be filled

$= 308 - \frac{77}{3}$

$= \frac{924 - 77}{3}$

$= \frac{847}{3}$

$= 282.33\ \text{cm}^3$

Height of new cone = $1\frac{3}{4} = \frac{7}{4}$cm

Radius = 2 cm

Therefore, volume of new cone

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 2 \times 2 \times \frac{7}{4} = \frac{22}{3}\ \text{cm}^3$

Volume of water which comes down = $\frac{77}{3} - \frac{22}{3}\text{cm}^3 = \frac{55}{3}\ \text{cm}^3$ …….(i)

Let h be the height of water which is dropped down.

Radius $= \frac{7}{2}$ cm

$\therefore$Volume = $\pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h = \frac{77}{2} h$ …… (ii)

From (i) and (ii)

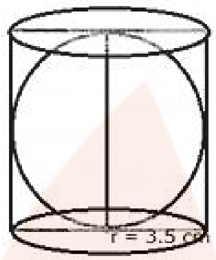
$\frac{77}{2} h = \frac{55}{3}$

$\Rightarrow h = \frac{55}{3} \times \frac{22}{77}$

$\Rightarrow h = \frac{10}{21}$

Drop in water level $=\frac{10}{21}$ cm

Solution 14:



Radius of the base of the cylindrical can = 3.5 cm

(i) When the sphere is in can, then total surface area of the can = Base area + curved surface area

$=\pi r^2+2\pi rh$

$=\left(\frac{22}{7}\times3.5\times3.5\right)+\left(2\times\frac{22}{7}\times3.5\times7\right)$

$=\frac{77}{2}+154$

$=38.5=154$

$=192.5\ \text{cm}^2$

(ii) Let depth of water = x cm

When sphere is not in the can, then volume of the can = volume of water + volume of sphere

$\Rightarrow \pi r^2h+\pi r^2x\times+\frac{4}{3}\pi r^3$

$\Rightarrow \pi r^2h+\pi r^2\left(x+\frac{4}{3}r\right)$

$\Rightarrow h=x+\frac{4}{3}r$

$\Rightarrow x=h-\frac{4}{3}r$

$\Rightarrow x=7-\frac{4}{3}\times\frac{7}{2}$

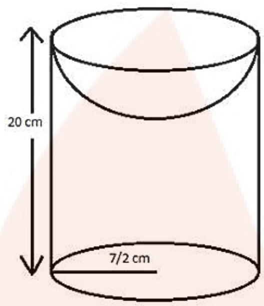
$\Rightarrow x=7-\frac{14}{3}$

$\Rightarrow x=\frac{21-14}{3}$

$$\Rightarrow x = \frac{7}{3}$$

$$\Rightarrow x = 2\frac{1}{3}\text{cm}$$

Solution 15:



Let the height of the water level be 'h', after the solid is turned upside down.
Volume of water in the cylinder

$$= \pi\left(\frac{7}{2}\right)^2 10$$

Volume of the hemisphere

$$= \frac{2}{3}\pi\left(\frac{7}{2}\right)^3$$

Volume of water in the cylinder
= Volume of water level − Volume of the hemisphere

$$\pi\left(\frac{7}{2}\right)^2 10 = \pi\left(\frac{7}{2}\right)^2 h - \frac{2}{3}\pi\left(\frac{7}{2}\right)^2$$

$$\Rightarrow 10 = h - \frac{7}{3}$$

$$\Rightarrow h = 10 + \frac{7}{3}$$

$$\Rightarrow h = 12\frac{1}{3}\text{ cm}$$

The height of water when the hemisphere is facing downwards is $12\frac{1}{3}$ cm

EXERCISE. 22 (F)

Solution 1:

Let the number of solid metallic spheres be 'n'

Volume of 1 sphere

$= \frac{4}{3}\pi(3)^3$

Volume of metallic cone

$= \frac{1}{3}\pi 6^2 \times 45$

$n = \frac{\text{Volume of metal cone}}{\text{Volume of 1 sphere}}$

$\Rightarrow n = \frac{\frac{1}{3}\pi 6^2 \times 45}{\frac{4}{3}\pi(3)^3}$

$\Rightarrow n = \frac{6 \times 6 \times 45}{4 \times 3 \times 3 \times 3}$

$\Rightarrow n = 15$

The least number of spheres needed to form the cone is 15

Solution 2:

Radius of largest sphere that can be formed inside the cylinder should be equal to the radius of the cylinder.

Radius of the largest sphere = 7 cm

Volume of sphere

$= \frac{4}{3}\pi 7^3$

$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$

$= \frac{4312}{3}$

$= 1437 \text{ cm}^3$

Solution 3:

Let the number of cones be 'n'.

Volume of the cylinder = $\pi \times 6^2 \times 15$

Volume of 1 cone $= \frac{1}{3}\pi \times 3^2 \times 12$

$$n = \frac{\text{volume of cylinder}}{\text{Volume of 1 cone}}$$

$$= \frac{\pi \times 6^2 \times 15}{\frac{1}{3}\pi \times 3^2 \times 12}$$

$= 15$

Number of cones required = 15

Solution 4:

Volume of the solid

$$= \frac{1}{3}\pi r^2 r + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3} \times \pi \times 8^3 + \frac{2}{3} \times \pi \times 8^3$$

$$= \pi 8^3$$

$$= 512\pi \text{ cm}^3$$

Solution 5:

Diameter of a sphere = 6 cm

Radius = 3 cm

$\therefore$ Volume = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3 = \frac{792}{7} \text{ cm}^3$ ……(i)

Diameter of cylindrical wire = 0.2 cm

Therefore, radius of wire = $\frac{0.2}{2} = 0.1 = \frac{1}{10}$ cm

Let length of wire = h

$\therefore$ Volume = $\pi r^2 h = \frac{22}{7} \times \frac{1}{10} \times \frac{1}{10} \times h \text{ cm}^3 = \frac{22h}{700} \text{ cm}^3$ ……(ii)

From (i) and (ii)

$$\frac{22h}{700}=\frac{792}{7}$$
$$\Rightarrow h=\frac{792}{7}\times\frac{700}{22}$$
$$\Rightarrow h=3600\ \text{cm}=36\ \text{m}$$

Hence, length of the wire = 36 m

Solution 6:

Let edge of the cube = a
volume of the cube = a × a × a = a^3

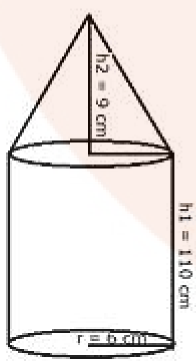
The sphere, which exactly fits in the cube, has radius $=\frac{a}{2}$

Therefore, volume of sphere = $\frac{4}{3}\pi r^3=\frac{4}{3}\times\frac{22}{7}\times\left(\frac{a}{2}\right)^3=\frac{4}{3}\times\frac{22}{7}\times\frac{a^3}{8}=\frac{11}{21}a^3$

Volume of cube : volume of sphere

$$=a^3:\frac{11}{21}a^3$$
$$=1:\frac{11}{21}$$
$$=21:11$$

Solution 7:



Radius of the base of poles (r) = 6 cm
Height of the cylindrical part (h_1) = 110 cm
Height of the conical part (h_2) = 9 cm

Total volume of the iron pole = $\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2 = \pi r^2\left(h_1 + \frac{1}{3}h_2\right)$

$= \frac{355}{113} \times 6 \times 6\left(110 + \frac{1}{3} \times 9\right)$

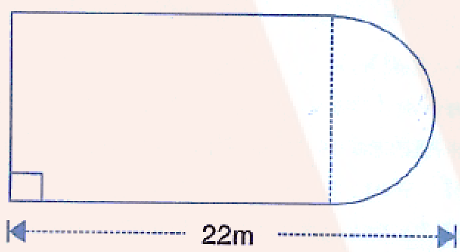
$= \frac{355}{113} \times 36 \times 113$

$= 12780\ \text{cm}^3$

Weight of 1 cm^3 = 8 gm

Therefore, total weight = 12780 × 8 = 102240 gm = 102.24 kg

Solution 8:



Length of the platform = 22 m

Circumference of semicircle = 11 m

$\therefore$ Radius = $\frac{c \times 2}{2 \times \pi} = \frac{11 \times 7}{22} = \frac{7}{2}$ m

Therefore, breadth of the rectangular part = $\frac{7}{2} \times 2 = 7$ m

And length $= 22 - \frac{7}{2} = \frac{37}{2} = 18.5\text{m}$

Now area of platform = $l \times b + \frac{1}{2}\pi r^2$

$= 18.5 \times 7 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\,\text{m}^2$

$= 129.5 + \frac{77}{4}\,\text{m}^2$

$= 148.75\ \text{m}^2$

Height of the platform = 1.5 m

$\therefore$ Volume = 148.75 × 1.5 = 223.125 m^3

Rate of construction = Rs 4 per m^3

Total expenditure = Rs 4 × 223.125 = Rs 892.50

Solution 9:

Side of square = 7 m

Radius of semicircle = $\frac{7}{2}$ m

Length of the tunnel = 80 m

Area of cross section of the front part = $a^2 + \frac{1}{2}\pi r^2$

$= 7\times 7 + \frac{1}{2}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$

$= 49 + \frac{77}{4} m^2$

$= \frac{196+77}{4}$

$= \frac{273}{4} \quad m^2$

(i) therefore, volume of tunnel = area × length

$= \frac{273}{4}\times 80$

$= 5460 \ m^3$

(ii) Circumference of the front of tunnel

$= 2\times 7 + \frac{1}{2}\times 2\pi r$

$= 14 + \frac{22}{7}\times\frac{7}{2}$

$= 14 + 11$

$= 25$ m

Therefore, surface area of the inner part of the tunnel

= 25 × 80

= 2000 m^2

(iii) Area of floor = l × b = 7 × 80 = 560 m^2

Solution 10:

Diameter of cylindrical tank = 2.8 m

Therefore, radius = 1.4 m

Height = 4.2 m

Volume of water filled in it = $\pi r^2 h$

$= \frac{22}{7} \times 1.4 \times 1.4 \times 4.2 \ \ m^3$

$= \frac{181.104}{7} \ \ m^3$

$= 25.872 \ \ m^3$ …….(i)

Diameter of pipe = 7 cm

Therefore, radius (r) = $\frac{7}{2}$

Let length of water in the pipe = h_1

∴ Volume = $\pi r^2 h_1$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h_1$

$= \frac{77}{2} \ \ h_1 cm^3$ ………(ii)

From (i) and (ii)

$\frac{77}{2} \ \ h_1 cm^3 = 25.872 \times 100^3 \ cm^3$

$\Rightarrow h_1 = \frac{25.872 \times 100^3 \times 2}{77}$

$\Rightarrow h_1 = \frac{25.872 \times 100^3 \times 2}{77 \times 100}$

$\Rightarrow h_1 = 0.672 \times 100^2 \, m$

$\Rightarrow h_1 = 6720 \ m$

Therefore, time taken at the speed of 4 m per second

$= \frac{6720}{4 \times 60}$ minutes = 28 minutes

Solution 11:

Rate of flow of water = 9 km/hr

Water flow in 1 hour 15 minutes

i.e. in $\frac{5}{4} hr = 9 \times \frac{5}{4} = \frac{45}{4} km = \frac{45}{4} \times 1000 = 11250 \ m$

Area of cross-section = $25 \ cm^2 = \frac{25}{10000} m^2 = \frac{1}{400} \ m^2$

Therefore, volume of water = $\frac{1}{400} \times 11250 = 28.125\,m^3$

Dimensions of water tank = 7.5m × 5m × 4m

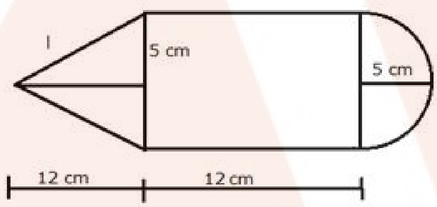
Area of tank = $l \times b = 7.5 \times 5 = 37.5\ m^2$

Let h be the height of water then,

$37.5 \times h = 28.125$

$h = \frac{28.125}{37.5} = 0.75\ m = 75cm$

Solution 12:



Diameter = 10 cm

Therefore, radius (r) = 5 cm

Height of the cone (h) = 12 cm

Height of the cylinder = 12 cm

$\therefore \ell = \sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13\ cm$

(i) Total surface area of the solid

$= \pi r\ell + 2\pi rh + 2\pi r^2$

$= \pi r(\ell + 2h + 2r)$

$= \frac{22}{7} \times 5[13 + (2 \times 12) + (2 \times 5)]$

$= \frac{110}{7}[13 + 24 + 10]$

$= \frac{110}{7} \times 47$

$= \frac{5170}{7}$

$= 738.57\ \ cm^2$

(ii) Total volume of the solid

$= \frac{1}{3}\pi r^2 h + \pi r^2 h + \frac{2}{3}\pi r^3$

$= \pi r^2 \left[\frac{1}{3}h + h + \frac{2}{3}r\right]$

$= \frac{22}{7} \times 5 \times 5\left[\frac{1}{3} \times 12 + 12 + \frac{2}{3} \times 5\right]$

$= \frac{550}{7}\left[4 + 12 + \frac{10}{3}\right]$

$= \frac{550}{7}\left[16 + \frac{10}{3}\right]$

$= \frac{550}{7} \times \frac{58}{3}$

$= \frac{31900}{21}$

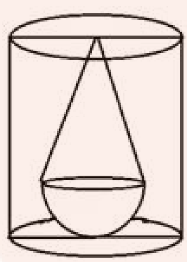
$= 1519.0476 \text{ cm}^3$

(iii) Total weight of the solid = 1.7 kg

$\therefore$ Density = $\frac{1.7 \times 1000}{1519.0476}$ gm / cm^3 $= 1.119$ gm / cm^3

$\Rightarrow$ Density = 1.12 gm / cm^3

Solution 13:



Radius of cylinder = 3 cm
Height of cylinder = 6 cm
Radius of hemisphere = 2 cm
Height of cone = 4 cm
Volume of water in the cylinder when it is full =
$\pi r^2 h = \pi \times 3 \times 3 \times 6 = 54\pi \text{ cm}^3$
Volume of water displaced = volume of cone + volume of hemisphere

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2 (h + 2r)$

$= \frac{1}{3}\pi \times 2 \times 2(4 + 2 \times 2)$

$= \frac{1}{3}\pi \times 4 \times 8$

$= \frac{32}{3}\pi cm^3$

Therefore, volume of water which is left

$= 54\pi - \frac{32}{3}\pi$

$= \frac{130}{3}\pi cm^3$

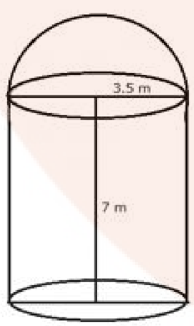
$= \frac{130}{3} \times \frac{22}{7} cm^3$

$= \frac{2860}{21} cm^3$

$= 136.19 cm^3$

$= 136 cm^3$

Solution 14:



Radius of the cylinder = 3.5 m
Height = 7 m
(i) Total surface area of container excluding the base = Curved surface area of the cylinder + area of hemisphere

$= 2\pi rh + 2\pi r^2$

$$= \left(2 \times \frac{22}{7} \times 3.5 \times 7\right) + \left(2 \times \frac{22}{7} \times 3.5 \times 3.5\right)$$

$$= 154 + 77 \text{ m}^2$$

$$= 231 \text{ m}^2$$

(ii) Volume of the container = $\pi r^2 h + \frac{2}{3}\pi r^3$

$$= \left(\frac{22}{7} \times 3.5 \times 3.5 \times 7\right) + \left(\frac{2}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5\right)$$

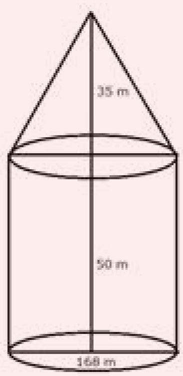
$$= \frac{539}{2} + \frac{539}{6}$$

$$= \frac{1617 + 539}{6}$$

$$= \frac{2156}{6}$$

$$= 359.33 \text{ m}^3$$

Solution 15:



Total height of the tent = 85 m
Diameter of the base = 168 m
Therefore, radius (r) = 84 m
Height of the cylindrical part = 50 m
Then height of the conical part = (85 - 50) = 35 m

Slant height (l) = $\sqrt{r^2 + h^2} = \sqrt{84^2 + 35^2} = \sqrt{7056 + 1225} = \sqrt{8281} = 91$ cm

Total surface area of the tent = $2\pi rh + \pi r\ell = \pi(2h + \ell)$

$$= \frac{22}{7} \times 84(2 \times 50 + 91)$$

$$= 264(100 + 91)$$

$= 264 \times 191$

$= 50424 \text{ m}^2$

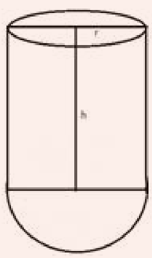
Since 20% extra is needed for folds and stitching, total area of canvas needed

$= 50424 \times \frac{120}{100}$

$= 60508.8$

$= 60509 \text{ m}^2$

Solution 16:



Volume of water filled in the test tube = $\frac{5159}{6}\text{cm}^3$

Volume of water filled up to 4 cm = $\frac{4235}{6}$ cm^3

Let r be the radius and h be the height of test tube.

$\therefore \frac{2}{3}\pi r^3 + \pi r^2 h = \frac{5159}{6}$

$\Rightarrow \pi r^2\left(\frac{2}{3}r + h\right) = \frac{5159}{6}$

$\Rightarrow \frac{\pi r^2}{3}(2r + 3h) = \frac{5159}{6}$

$\Rightarrow \pi r^2(2r + 3h) = \frac{5159}{2}$ ……(i)

And

$\frac{2}{3}\pi r^3 + \pi r^2(h - 4) = \frac{4235}{6}$

$\Rightarrow \pi r^2\left(\frac{2}{3}r + h - 4\right) = \frac{4235}{6}$

$$\Rightarrow \frac{\pi r^2}{3}(2r+3h-12)=\frac{4235}{6}$$

$$\Rightarrow \pi r^2(2r+3h-12)=\frac{4235}{2} \quad \text{........(ii)}$$

Dividing (i) by (ii)

$$\frac{2r+3h}{2r+3h-12}=\frac{5259}{4235} \quad \text{.......(iii)}$$

Subtracting (ii) from (i)

$$\pi r^2(12)=\frac{5159}{2}-\frac{4235}{2}=\frac{924}{2}$$

$$\Rightarrow 12\times\frac{22}{7}\times r^2=\frac{924}{2}$$

$$\Rightarrow r^2=\frac{924\times 7}{2\times 12\times 22}=\frac{7\times 7}{2\times 2}$$

$$\Rightarrow r^2=\frac{49}{4}$$

$$\Rightarrow r=\frac{7}{2}=3.5\,\text{cm}$$

Subtracting the value of r in (iii)

$$\frac{2\times\frac{7}{2}+3h}{2\times\frac{7}{2}+3h-12}=\frac{5159}{4235}$$

$$\Rightarrow \frac{7+3h}{7+3h-12}=\frac{5159}{4235}$$

$$\Rightarrow \frac{7+3h}{7+3h-12}=\frac{469}{385}$$

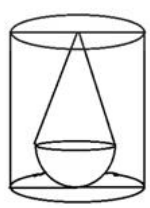
$$\Rightarrow 2695+1155h=1407h-2345$$

$$\Rightarrow 252h=5040$$

$$\Rightarrow h=20$$

Hence, height = 20 cm and radius = 3.5 cm

Solution 17:



Diameter of hemisphere = 7 cm
Diameter of the base of the cone = 7 cm
Therefore, radius (r) = 3.5 cm
Height (h) = 8 cm
Volume of the solid =

$$\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 (h + 2r)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 (8 + 2 \times 3.5)$$

$$= \frac{77}{6}(8 + 7)$$

$$= \frac{385}{2}$$

$$= 192.5 \text{ cm}^3$$

Now, radius of cylindrical vessel (R) = 7 cm
Height (H) = 10 cm
∴ Volume = $\pi R^2 H$

$$= \frac{22}{7} \times 7 \times 7 \times 10$$

$$= 1540 \text{ cm}^3$$

Volume of water required to fill = 1540 − 192.5 = 1347.5 cm^3