4

Motion in a Plane

Earlier, we have studied the motion of an object along a straight line. In this chapter, we will study the motion of an object in two dimensions (in plane) and in three dimensions (in space). As a simple case of motion in a plane, we shall discuss motion with constant acceleration and treat it in details as projectile and circular motion.

Chapter Checklist

- Scalar Quantity
- Vector Quantity
- Position and Displacement Vectors
- Scalar Product
- Vector Product
- Resultant Vector
- Resolution of Vectors
- Motion in a Plane
- Projectile Motion
- Uniform Circular Motion

Topic 1 Scalars and Vectors

A study of motion will involve the introduction of a variety of quantities that are used to describe the physical world. *e.g.,* Distance, speed, displacement, velocity, acceleration, force, mass, momentum, work, power, energy, etc.

In order to describe the motion of an object to be in two-dimensional and in three-dimensional, we need to understand the concept of vectors first.

Many quantities that have both magnitude and direction need a special mathematical language *i.e.* the language of vectors to describe such quantities.

On the basis of magnitude and direction, all the physical quantities are classified into two groups as scalars and vectors.

Scalar Quantity

They are those physical quantities which have only magnitudes but no direction. It is specified completely by a single number, alongwith the proper unit.

e.g., Temperature, mass, length, time, work, etc.

- *The rules for combining scalars are of ordinary algebraic. Scalars can be added, subtracted, multiplied and divided just as the ordinary numbers.*
- *The quantities with same units can be added or subtracted, but the quantities of different units can be multiplied or divided to make sense in scalars.*

The position vector provides two informations such as

(*i*) It tells us about the minimum distance of an object from the origin O.

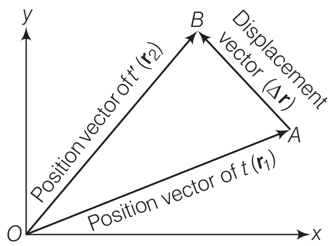
(*ii*) It tells us about the direction of the object w.r.t. origin.

Displacement Vector

The vector which tells how much and in which direction an object has changed its position in a given interval of time is called **displacement vector**.

Displacement vector is the straight line joining the initial and final positions and does not depend on the actual path undertaken by the object between the two positions.

Consider an object moving in the xy-plane. Suppose it is at point A at any instant t and at point B at any later instant t'. Then, vector **AB** is the displacement vector of the object in time t to t'.



Displacement vector representation

If the coordinates of A be (x_1, y_1) and B be (x_2, y_2) then the position vector of the object at point A, $\mathbf{r}_1 = x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}}$ and the position vector of the object at point B, $\mathbf{r}_2 = x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}}$

$\therefore$ The displacement vector for AB can be given as

$$\Delta r = \mathbf{r}_2 - \mathbf{r}_1$$

$$\boxed{\text{Displacement vector, } \Delta r = (x_2 - x_1)\hat{\mathbf{i}} + (y_2 - y_1)\hat{\mathbf{j}}}$$

(*i*) Magnitude of the displacement vector is given by

$$|\Delta \mathbf{r}| = \Delta r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The magnitude of displacement is either less or equal to the path length of an object between two points.

(*ii*) Magnitude of vectors for three-dimensional is given by $\Delta \mathbf{r} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

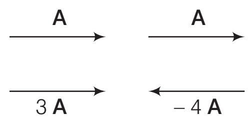
Multiplication of Vectors by Real Numbers (or Scalar)

When we multiply a vector **A** by a real number λ, then we get a new vector along the direction of vector **A**. Its magnitude becomes λ times the magnitude of the given vector.

Similarly, if we multiply vector **A** with a negative real number $-\lambda$, then we get a vector whose magnitude is λ times the magnitude of vector **A** but direction is opposite to that of vector **A**.

Hence, $\boxed{\lambda(\mathbf{A}) = \lambda\mathbf{A}}$ and $\boxed{-\lambda(\mathbf{A}) = -\lambda\mathbf{A}}$

e.g., (*i*) Consider a vector **A** is multiplied by a real number $\lambda = 3$ or -4, we get $3\mathbf{A}$ or $-4\mathbf{A}$



Multiplication of vectors by real numbers

(*ii*) If we multiply a constant velocity vector by time, we will get a displacement vector in the direction of velocity vector.

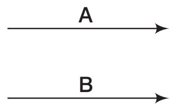
Resultant Vector

The resultant of two or more vectors is defined as the single vector which produces the same effect as two or more vectors (given vectors) combinedly produces.

There are two cases

Case I **When two vectors are acting in the same direction.**

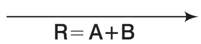
Consider the vectors **A** and **B** are acting in the same direction as shown below



Then, the resultant of these two vectors is given by a vector having direction as same as that of **A** or **B** and the magnitude of the resultant vector will be equal to the sum of respective vectors (*i.e.* $A + B$).

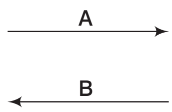
Thus, $\boxed{\text{Resultant vector, } \mathbf{R} = \mathbf{A} + \mathbf{B}}$

= A vector in direction of **A** or **B** with magnitude $(A + B)$



Case II **When two vectors are acting in mutually opposite directions.**

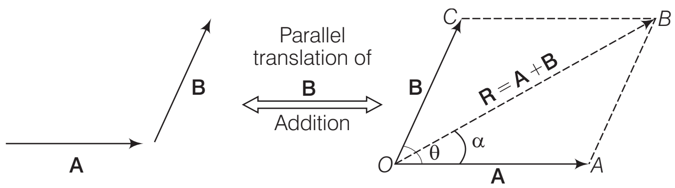
Consider **A** and **B** are acting in mutually opposite direction as shown below.



2. Parallelogram Law of Addition of Two Vectors

This law states that if two vectors acting on a particle at the same time be represented in magnitude and direction by two adjacent sides of a parallelogram drawn from a point, their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point.

If two vectors **A** and **B** are taken as two adjacent sides of a parallelogram which is to be added the diagonal with same initial point as that of **A** and **B**. Give the resultant (**R** = **A** + **B**) of **A** and **B**. *The figure is shown below*



Parallelogram law of addition of two vectors

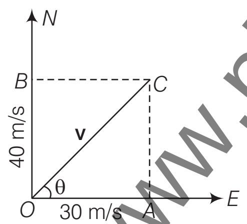
The resultant vector form in this method is also same as that formed in triangle law of addition. Hence, the resultant of **A** and **B** is given by

$$\boxed{\text{Resultant vector, } \mathbf{R} = \mathbf{A} + \mathbf{B}}$$

Example 2

A body is simultaneously given two velocities of 30 m/s due East and 40 m/s due North, respectively. Find the resultant velocity.

Solution Le the body be starting from point *O* as shown.



$\mathbf{v}_A = 30$ m/s due East

$\mathbf{v}_B = 40$ m/s due North

According to parallelogram law, **OC** is the resultant velocity.

Its magnitude is given by

$$v = \sqrt{v_A^2 + v_B^2} = \sqrt{(30)^2 + (40)^2}$$
$$= \sqrt{900 + 1600} = 50 \text{ m/s}$$

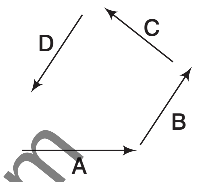
Since, the resultant velocity **v** makes an angle θ with the East direction. Then,

$$\theta = \tan^{-1}\frac{CA}{OA} = \tan^{-1}\left(\frac{40}{30}\right) = 53°\,8'$$

3. Polygon Law of Addition of Vectors

This law states that when the number of vectors are represented in both magnitude and direction by the sides of an open polygon taken in an order, then their resultant is represented in both magnitude and direction by the closing side of the polygon taken in opposite order.

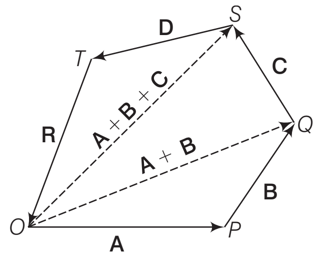
Consider the number of vectors **A**, **B**, **C** and **D** be acting in different directions as shown.



Vectors acting in different directions

According to the polygon law of vectors addition, the closing side *OT* of the polygon taken in the reverse order represents the resultant vector **R**.

Then, their resultant vector (**R**) is given by



Polygon law of addition of vectors

$$\mathbf{R} = \mathbf{OP} + \mathbf{PQ} + \mathbf{QS} + \mathbf{ST}$$

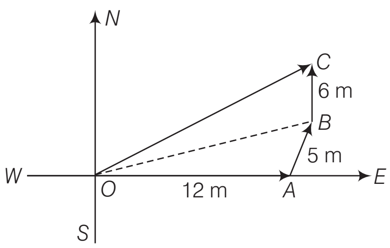
$$\boxed{\text{Resultant vector, } \mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D}}$$

In order to obtain resultant vector polygon is formed as shown in figure. It is known as **polygon law of vector addition.**

Example 3

A particle has a displacement of 12 m *towards East and* 5 m *towards the North and then* 6 m *vertically upwards. Find the magnitude of the sum of these displacements.*

Solution Suppose initially the particle is at origin *O*. Then, its displacement vectors are



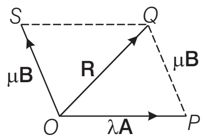
OA = 12 m, **AB** = 5 m, **BC** = 6 m

From triangle law of vector addition, we have

$$\mathbf{OQ} = \mathbf{OP} + \mathbf{PQ}$$

But **OP** and **PQ** are two component vectors of **R** in the direction of A and B respectively.

Let $\mathbf{OP} = \lambda\mathbf{A}$ and $\mathbf{PQ} = \mu\mathbf{B}$, where λ and μ are two real numbers. This is also illustrated in the figure as shown below.

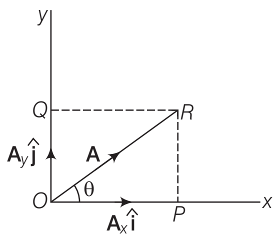


Now, the resultant vector becomes

$$\mathbf{R} = \lambda\mathbf{A} + \mu\mathbf{B}$$

Rectangular Components of a Vector in a Plane

When a vector is splitted into two component vectors at right angle to each other. Then, the component vectors are called **rectangular components.** The resultant vector is given by



Resultant vector **A** of two vectors $\mathbf{A}_x$ and $\mathbf{A}_y$

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$$

where $\boxed{\text{Magnitude of vector, } A = \sqrt{A_x^2 + A_y^2}}$

We can also find the angle (θ) between them.

From $\quad \tan\theta = \left(\dfrac{A_y}{A_x}\right)$

$\Rightarrow \quad \boxed{\text{Angle, } \theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right)}$

Where, A_y and A_x are the splitted vectors component of **A** in the direction of $\hat{\mathbf{j}}$ and $\hat{\mathbf{i}}$, respectively.

Worked out Problem

The greatest and the least resultant of two forces acting at a point are 29 N and 5 N, respectively. If each force is increased by 3 N. Find the resultant of two new forces acting at right angle to each other.

Step I List what is given and what you want to know.

Let P and Q be the two forces.

Greatest resultant $R_1 = P + Q = 29$ N

Least resultant $R_2 = P - Q = 5$ N

Now forces, $P' = P + 3, Q' = Q + 3$

Step II Write the formula used.

$$R' = ?\ \theta = ?$$

$$R' = \sqrt{P'^2 + Q'^2} \Rightarrow \tan\theta = \frac{Q'}{P'}$$

Step III Solve the forces P and Q.

$$P + Q = 29 \quad \text{...(i)}$$

$$P - Q = 5 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$P = 17\text{ N}, Q = 12\text{ N}$$

Step IV Now, find the new forces P' and Q'.

$$P' = P + 3 = 17 + 3 = 20 \text{ N} \Rightarrow Q' = 12 + 3 = 15 \text{ N}$$

Step V Substitute the values in Step III and solve.

$$R' = \sqrt{(20)^2 + (15)^2} = \sqrt{400 + 225} = 25 \text{ N}$$

$$\tan\theta = \frac{15}{20} = 0.75 \Rightarrow \theta = \tan^{-1}(0.75) = 36°52'$$

Thus, the resultant of two new forces P' and Q' are 25 N and angle between them is 36°52′.

Example 5

A person walks in the following pattern 3.1 km North, then 2.4 km West and finally 5.2 km South.

(i) Sketch the vector diagram that represents this motion.

(ii) How far the person will be from its initial point?

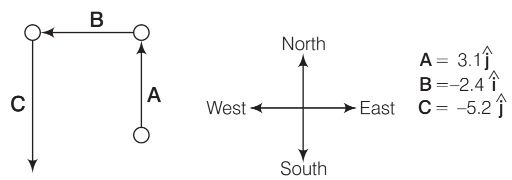
(iii) In what direction would a bird fly in a straight line from the same starting point to the same final point?

Solution

Label the displacement vectors **A**, **B** *and* **C** *(and denote the result of their vector sum as* **r***) now, choose East as the* $\hat{\mathbf{i}}$ *direction (+x direction) and North as the* $\hat{\mathbf{j}}$ *direction (+y direction). All distances are understood to be in kilometers.*

The vectors **B** *and* **C** *are taken as negative because according to our assumption, person walks in –x and –y directions.*

(*i*) The vector diagram representing the motion is shown below.



(ii) The final point is represented by

$$\mathbf{r} = \mathbf{A} + \mathbf{B} + \mathbf{C} = 3.1\,\hat{\mathbf{j}} - 2.4\,\hat{\mathbf{i}} - 5.2\,\hat{\mathbf{j}} = -2.4\hat{\mathbf{i}} - 2.1\hat{\mathbf{j}}$$

whose magnitude is

$$|\mathbf{r}| = \sqrt{(-2.4)^2 + (-2.1)^2} \approx 3.2 \text{ km}$$

Hence, the person will be 3.2 km away from the initial point.

(iii) There are two possibilities for the angle.

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{-3.1}{-2.4}\right) = 41° \text{ or } 221°$$

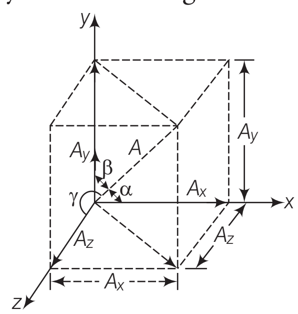
$\because$ r is in the third quadrant, hence $\theta = 221°$

Resolution of a Space Vector

(In Three Dimensions)

Similarly, we can resolve a general vector **A** into three components along x, y, and z-axes in three dimensions (*i.e.* space).

Let α, β and λ are the angles between **A** and the x, y and z-axes, respectively as shown in figure.



While resolving we have,

$$A_x = A\cos\alpha,\; A_y = A\cos\beta,\; A_z = A\cos\gamma$$

$\therefore$ Resultant vector, $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$

Magnitude of vector **A** is $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Position vector r is given by $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}$

Remember that

$$\cos\alpha = \frac{A_x}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = l$$

$$\cos\beta = \frac{A_y}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = m$$

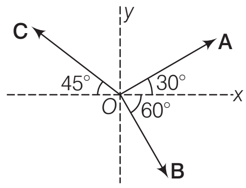
$$\cos\gamma = \frac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = n$$

Here, l, m and n are known as **direction cosines of A**.

- *The angles α, β and γ are angles in space. They are between pairs of lines, which are not coplanar.*

Worked out Problem

The figure shows three vectors **OA, OB** *and* **OC** *which are equal in magnitude (say, F). Determine the direction of* **OA + OB − OC**

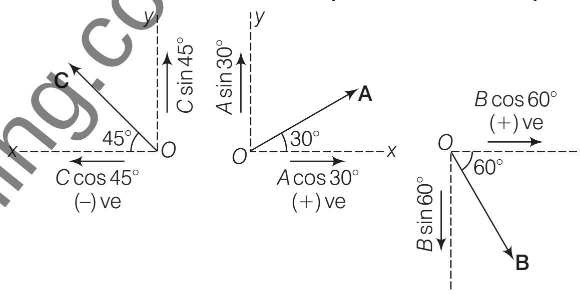


Step **I** List what are given and what do you want to know.

$|OA| = |OB| = |OC| = F$

Angles are 30°, 45° and 60°.

Step **II** Resolve all the vector components individually.



Step **III** Write the formula used for resultant vector.

$$\mathbf{R}_x = \mathbf{R}_{x_1} + \mathbf{R}_{x_2} + \mathbf{R}_{x_3}$$

Step **IV** Sum of vectors in x-direction (*i.e.* $\mathbf{R}_x$) and sum of vectors in y-direction (*i.e.* $\mathbf{R}_y$)

$$\mathbf{R}_x = A\cos 30° + B\cos 60° - C\cos 45°$$

$$= \frac{F\sqrt{3}}{2} + \frac{F}{2} - \frac{F}{\sqrt{2}} \qquad (\because A = B = C = F)$$

$$= \frac{F}{2\sqrt{2}}(\sqrt{6} + \sqrt{2} - 2)$$

$$\mathbf{R}_y = A\sin 30° + C\cos 45° - B\sin 60°$$

$$= \frac{F}{2} + \frac{F}{\sqrt{2}} - \frac{F\sqrt{3}}{2}$$

$$= \frac{F}{2\sqrt{2}}(\sqrt{2} + 2 - \sqrt{6})$$

Step **V** To find the resultant vector (*i.e.* a single equivalent vector), get the magnitude and direction of the resultant vector.

Determination of magnitude,

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{\left[\frac{F}{2\sqrt{2}}(\sqrt{6} + \sqrt{2} - 2)\right]^2 + \left[\frac{F}{2\sqrt{2}}(\sqrt{6} + \sqrt{2} - 2)\right]^2}$$

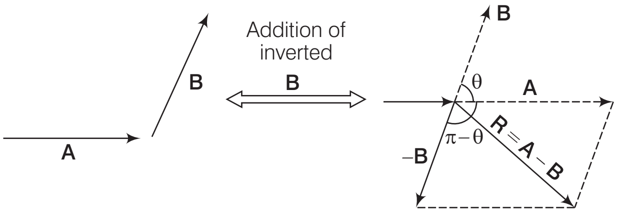
$$= \sqrt{F^2(1.02) + F^2(0.94)}$$

$$= F\sqrt{1.96}$$

$$\Rightarrow R = 1.4\,F$$

Subtraction of Vectors (Analytical Method)

There are two vectors **A** and **B** at an angle θ. If we have to subtract **B** from **A** then first invert the vector **B** and then add with **A** as shown in figure.



Subtraction of vectors

The resultant vector is $\mathbf{R} = \mathbf{A} + (-\mathbf{B}) = \mathbf{A} - \mathbf{B}$

The magnitude of resultant in this case is

$$R = |\mathbf{R}| = \sqrt{A^2 + B^2 + 2AB\cos(\pi - \theta)}$$

$$\boxed{\text{Resultant, } R = \sqrt{A^2 + B^2 - 2AB\cos\theta}}$$

Here, θ = angle between **A** and **B**

Regarding the magnitude of R

(*i*) When $\theta = 0°$, then $R = A - B$ (minimum)

(*ii*) When $\theta = 90°$, then $R = \sqrt{A^2 + B^2}$

(*iii*) When $\theta = 180°$, then $R = A + B$ (maximum)

Example 13

Two vectors of magnitude 3 units and 4 units are at angle 60° between them. Find the magnitude of their difference?

Solution Let the vectors are **A** and **B**

Given, $|\mathbf{A}| = 3$ unit, $|\mathbf{B}| = 4$ unit and $\theta = 60°$

The magnitude of resultant of **A** and **B**

$$R = |\mathbf{R}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$= \sqrt{3^2 + 4^2 - 2 \times 3 \times 4\cos 60°}$$

$$R = \sqrt{25 - 12} = \sqrt{13} = 3.61$$

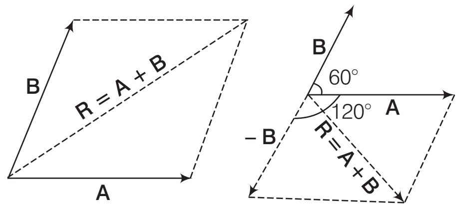
Worked out Problem

*Consider vectors **A** and **B** having equal magnitude of 5 units and are inclined each other by 60°. Find the magnitude of sum and difference of these vectors.*

Step **I** Write the given quantity and quantity to be known.

$$A = 5 \text{ units}$$
$$B = 5 \text{ units}$$
$$\theta = 60°$$
$$A + B = ? \quad \text{and} \quad A - B = ?$$

Step **II** Draw the diagrams to show the construction of the sum and difference.



Step **III** Find the magnitude of the resultant vectors of the sum.

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{5^2 + 5^2 + 2 \times 5 \times 5 \times \cos 60°} = 5\sqrt{3} \text{ unit}$$

Step **IV** Find the magnitude of the resultant vector of the difference.

$$R = \sqrt{A^2 + (-B)^2 + 2AB\cos\theta}$$

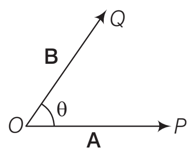
$$R = \sqrt{5^2 + 5^2 + 2 \times 5 \times 5\cos 120°}$$

$$R = 5 \text{ unit}$$

Dot Product *or* Scalar Product

It is defined as the product of the magnitudes of vectors **A** and **B** and the cosine angle between them. It is represented by

$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$$



Case **I** When the two vectors are parallel, then $\theta = 0°$. We have

$$\mathbf{A} \cdot \mathbf{B} = AB\cos 0° = AB$$

Case **II** When the two vectors are mutually perpendicular, then $\theta = 90°$.

We have

$$\mathbf{A} \cdot \mathbf{B} = AB\cos 90° = 0$$

Case **III** When the two vectors are antiparallel, then $\theta = 180°$.

We have $\mathbf{A} \cdot \mathbf{B} = AB\cos 180° = -AB$

Dot Product of Two Vectors in Terms of Their Components

It is defined as the product of the magnitude of one vector and the magnitude of the component of other vector in the direction of first vector.

$$= \frac{1}{2} A \times B \sin\theta$$

$$= \frac{1}{2} AB \sin\theta = \frac{1}{2}|\mathbf{A} \times \mathbf{B}|$$

Properties of Cross Product

(*i*) $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$

(*ii*) $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$

(*iii*) $(\mathbf{a}\times\mathbf{b}) + (\mathbf{c}\times\mathbf{d}) = (\mathbf{a} \times \mathbf{c}) + (\mathbf{a}\times \mathbf{d}) + (\mathbf{b}\times\mathbf{c}) + (\mathbf{b}\times\mathbf{d})$

(*iv*) $m\mathbf{a} \times \mathbf{b} = \mathbf{a} \times m\mathbf{b}$

(*v*) $(\mathbf{b} + \mathbf{c})\mathbf{a} = \mathbf{b} \times \mathbf{a} + \mathbf{c} \times \mathbf{a}$

(*vi*) $\mathbf{a} \times \mathbf{a} = 0$

(*vii*) $\mathbf{a} \times (\mathbf{b} - \mathbf{c}) = \mathbf{a} \times \mathbf{b} - \mathbf{a} \times \mathbf{c}$

(*viii*) $|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 - |\mathbf{a} \cdot \mathbf{b}|^2$

(*ix*) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \cdot \mathbf{a})\, \mathbf{b} - (\mathbf{b} \cdot \mathbf{a})\, \mathbf{c}$

Example 9 Cross Product of Two Vectors

The vector **A** *has a magnitude of 5 unit,* **B** *has a magnitude of 6 unit and the cross product* **A** *and* **B** *has the magnitude of 15 unit. Find the angle between* **A** *and* **B**.

Solution If the angle between **A** and **B** is θ, then cross product will have a magnitude,

$$|\mathbf{A} \times \mathbf{B}| = AB \sin\theta$$

$$\Rightarrow \quad 15 = 5 \times 6 \sin\theta$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

Representation of Unit Vectors in a Circle

Unit vectors along three axes of cartesian coordinate system (*i.e.* $\hat{i}, \hat{j}, \hat{k}$) can be represented on a circle in such a way that if we rotate our eyes in anti-clockwise product of two consecutive vector will produce the third unit vector.

e.g., $\hat{i} \times \hat{j} = \hat{k}$

and $\hat{j} \times \hat{i} = -\hat{k}$

and similarly for other possibilities.

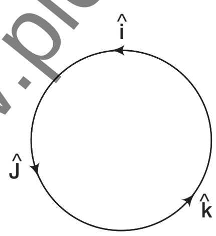
> **Scalar Product of Vectors**
> The scalar product of vectors produce pseudo scalars, such as volume, power etc. The vector product of two vectors produces a pseudo vector. It is also called axial vector. The direction of this vector is perpendicular to the plane containing the multiple vectors.

EXAM Practice

Very Short Answer Type Questions [1 Mark]

1. *When do we say two vectors are orthogonal?*

Sol. If the dot product of two vectors is zero, then the vectors are orthogonal.



Representation of unit vectors

2. *Can the walking on a road be an example of resolution of vectors?* [HOTS]

Sol. Yes, when a man walks on the road, he presses the road along an oblique direction. The horizontal component of the reaction helps the man to walk on the road.

3. *Three vectors not lying in a plane can never end up to give a null vector. Is it true?* [NCERT]

Sol. Yes, because they cannot be represented by the three sides of a triangle taken in the same order.

4. *Under what condition the three vectors give zero resultant?*

Sol. When all the three vectors are in same plane and forms a triangle and sum of two sides of the triangle is greater than the third side or the sum of any two vectors is equal to third one then all the three vectors are collinear will produce their resultant zero.

5. *The total path length is always equal to the magnitude of the displacement vector of a particle. Why?* [NCERT]

Sol. It is only true if the particle moves along a straight line in the same direction, otherwise the statement is false.

6. *When the sum of the two vectors are maximum and minimum?* [HOTS]

Sol. The sum of two vectors is maximum, when both the vectors are in the same direction and is minimum when they act in opposite direction.

Short Answer Type I Questions [2 Marks]

19. *Can two equal vectors* **a** *and* **b** *at different locations in space necessarily have identical physical effects?*

Sol. Yes, because the two bodies while falling under gravity will have same acceleration. Though, their locations may be different with respect to a common origin. [2]

20. *We can order events in time and there is no sense of time, distinguishing past, present and future. Is time a vector?*

Sol. We know that time always flows on and on *i.e.* from past to present and then to future.

Therefore, a direction can be assigned to time. Since, the direction of time is unique and it is unspecified or unstated. That is why time cannot be a vector though it has a direction. [2]

21. *Explain the property of two vectors* **A** *and* **B** *if* $|\mathbf{A}+\mathbf{B}| = |\mathbf{A}-\mathbf{B}|$.

Sol. As we know that

$$|\mathbf{A}+\mathbf{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

and $$|\mathbf{A}-\mathbf{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

But as per question, we have

$$\sqrt{A^2 + B^2 + 2AB\cos\theta} = \sqrt{A^2 + B^2 - 2\,AB\cos\theta} \quad [1]$$

Squaring both sides, we have $(4\,AB\cos\theta) = 0$

$\Rightarrow \quad \cos\theta = 0 \quad$ or $\quad \theta = 90°$

Hence, the two vectors **A** and **B** are perpendicular to each other. [1]

22. *Two forces whose magnitudes are in the ratio 3 : 5 give a resultant of 28 N. If the angle of their inclination is 60°. Find the magnitude of each force.*

Sol. Let A and B be the two forces.

Then, $A = 3x$, $B = 5x$, $R = 28$ N and $\theta = 60°$

Thus, $\dfrac{A}{B} = \dfrac{3}{5}$

Now, $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ [1]

$\Rightarrow \quad 28 = \sqrt{9x^2 + 25x^2 + 30\,x^2\cos 60°} = 7x \Rightarrow x = 4$

$\therefore$ Forces are $A = 12$ N, $B = 20$ N [1]

23. *Determine that vector which when added to the resultant of* $\mathbf{A} = 3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$ *and* $\mathbf{B} = 2\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} - 3\,\hat{\mathbf{k}}$ *gives unit vector along y-direction.*

Sol. We are given $\mathbf{A} = 3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$ and $\mathbf{B} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 9\hat{\mathbf{k}}$

Thus, the resultant vector is given by

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = (3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}}) + (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$$
$$= 5\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\,\hat{\mathbf{k}} \quad [1]$$

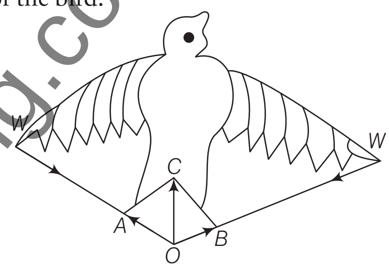
But the unit vector along y-direction $= \hat{\mathbf{j}}$

$\therefore$ Required vector $= \hat{\mathbf{j}} - (5\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}})$

$$= -5\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\,\hat{\mathbf{k}} \quad [1]$$

24. *Can a flight of a bird an example of composition of vectors. Why?* [HOTS]

Sol. Yes, the flight of a bird is an example of composition of vectors. As the bird flies, it strikes the air with its wings W, W along WO. According to Newton's third law of motion, air strikes the wings in opposite directions with the same force in reaction. The reactions are **OA** and **OB**. From law of parallelogram vectors, **OC** is the resultant of **OA** and **OB**. This resultant upwards force **OC** is responsible for the flight of the bird. [1+1]



25. *The dot product of two vectors vanishes when vectors are orthogonal and has maximum value when vectors are parallel to each other. Explain.*

Sol. We know that $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$, when vectors are orthogonal $\theta = 90°$. So, $\mathbf{A}\cdot\mathbf{B} = AB\cos 90° = 0$, when vectors are parallel then $\theta = 0°$. So, $\mathbf{A}\cdot\mathbf{B} = AB\cos 0° = AB$ (maximum) [1 + 1]

26. *Suppose you have two forces* **F** *and* **F**. *How would you combine them in order to have resultant force of magnitudes?*

(i) *Zero.* (ii) **F**.

Sol. (*i*) If they act at opposite direction, resultant is zero. [1]

(*ii*) For the resultant to be **F**,

$$F^2 = F^2 + F^2 + 2F^2\cos\theta$$

$\Rightarrow \quad \cos\theta = \dfrac{-1}{2} \Rightarrow \theta = 120°$ [1]

27. *The angle between vectors* **A** *and* **B** *is 60°. What is the ratio of* $\mathbf{A}\cdot\mathbf{B}$ *and* $|\mathbf{A}\times\mathbf{B}|$ *?*

Sol. The dot product $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$ and cross product $|\mathbf{A}\times\mathbf{B}| = AB\sin\theta$ [1]

$\therefore$ Ratio is $\dfrac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{A}\times\mathbf{B}|} = \dfrac{AB\cos\theta}{AB\sin\theta} = \cot\theta$

$$= \cot 60° = \frac{1}{\sqrt{3}}$$

28. *Two forces* 5 kg-wt. *and* 10 kg-wt. *are acting with an inclination of* $120°$ *between them. Find the angle when the resultant makes with* 10 kg-wt.

Sol. Given, $A = 5$ KG-wt, $B = 10$ kg-wt, $\theta = 120°$ then $\beta = ?$

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

$$= \frac{10\sin 120°}{5 + 10\cos 120°} = \frac{5\sin 60°}{10 - 5\cos 60°} \quad [1]$$

$$= \frac{5\times\sqrt{3}/2}{10 - 5/2} = \frac{1}{\sqrt{3}} = \tan 30°$$

$\therefore \quad \beta = 30°$ [1]

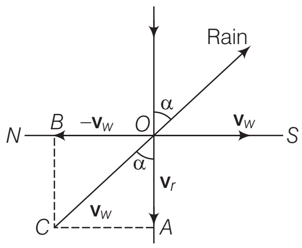
Try Yourself

1. Rain is falling vertically with a speed of 35 m/s. Winds start blowing after sometime with a speed of 12 m/s in East to West direction. Should a boy waiting at a bus stop hold his umbrella.
2. We can order events in time and there is a sense of time, distinguish past, present and future. Is time a vector?
3. What is a zero vector? Explain the need of a zero vector.
4. Is the working of a sling based on the parallelogram law of vector addition, why?

Short Answer Type II Questions [3 Marks]

29. *Rain is falling vertically with a speed of* 30 m/s. *A woman rides a bicycle with a speed of* 10 m/s *in the North to South direction. What is the direction in which she should hold her umbrella?* [NCERT]

Sol Figure shows vectorially the situations,



[1]

Velocity of rain falling vertically downward

$$\mathbf{v}_r = 30 \text{ m/s}$$

Velocity of woman riding a bicycle

$$\mathbf{v}_w = 10 \text{ m/s (North to South)}$$

To protect herself from rain, the woman should hold her umbrella in the direction of relative velocity of the rain with respect to the woman *i.e.* $\mathbf{v}_{rw}$.

The relative velocity of rain with respect to the woman *i.e.*

$$\mathbf{v}_{rw} = \mathbf{v}_r - \mathbf{v}_w$$

$$\therefore \quad |\mathbf{v}_{rw}| = \sqrt{(30)^2 + (10)^2}$$

$$= \sqrt{900 + 100} = \sqrt{1000} = 10\sqrt{10} \text{ m/s} \quad [1]$$

If $\mathbf{v}_{rw}$ makes an angle α with the vertical, then

$$\tan\alpha = \frac{v_w}{v_r} = \frac{10}{30} = \frac{1}{3} = 0.3333$$

$$\Rightarrow \quad \alpha = 18°\ 26'$$

Hence, woman should hold her umbrella at an angle of 18° 26′ with the vertical towards South. [1]

30. *There are two displacement vectors, one of magnitude* 3 m *and the other of* 4 m. *How would the two vectors be added so that the magnitude of the resultant vector be (i)* 7 m *(ii)* 1m and *(iii)* 5 m

Sol. The magnitude of resultant **R** of two vectors **A** and **B** is given by $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$$= \sqrt{3^2 + 4^2 + 2\times 3\times 4\cos\theta} \quad [1½]$$

(*i*) R is 7 m if $\theta = 0°$
(*ii*) R is 1 m if $\theta = 180°$
(*iii*) R is 5 m if $\theta = 90°$ [½ × 3]

31. *Show that vectors* $\mathbf{A} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ *and* $\mathbf{B} = -6\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ *are parallel.*

Sol. The given vectors are

$$\mathbf{A} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}} \text{ and } \mathbf{B} = -6\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

Then, the vectors are parallel if $\mathbf{A}\times\mathbf{B} = 0$ [1]

$$\therefore \mathbf{A}\times\mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -3 & -1 \\ -6 & 9 & 3 \end{vmatrix}$$

$$= \hat{\mathbf{i}}(-9+9) - \hat{\mathbf{j}}(6-6) + \hat{\mathbf{k}}(18-18) = 0 \quad [1]$$

But $|\mathbf{A}\times\mathbf{B}| = 0$ or $AB\sin\theta = 0$ $\quad [\because \mathbf{A}\neq 0 \text{ and } \mathbf{B}\neq 0]$

$\therefore \quad \sin\theta = 0$ or $\theta = 0$

Hence, the vectors **A** and **B** are parallel. [1]

32. *A man can swim at the rate of* 5 km/h *in still water. A river* 1 km *wide flows at the rate of* 3km/h. *A swimmer wishes to cross the river straight.*

(i) Along what direction must he strike?
(ii) What should be his resultant velocity?
(iii) How much time he would take to cross? [HOTS]

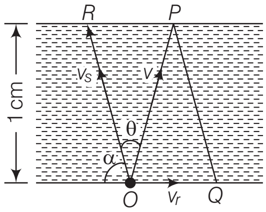
Sol. Width of the river, $d = 1$ km

Velocity of swimmer, $v_s = 5$ km/h

Velocity of water flowing in river,

$v_r = 3$ km/h along OQ. [1]

(i) The swimmer wants to cross the river straight if the resultant velocity of the river flow and swimmer acts perpendicular to the direction of the river flow *i.e.* along OP. This will be so if the swimmer moves making an angle α with upstream *i.e.* along OR. [1/2]



But, $\alpha + \theta = 90°$ or $\theta = 90° - \alpha$

From ΔOPR, we have

$$\sin\theta = \sin(90° - \alpha) = \cos\alpha = \frac{RP}{RO} = \frac{3}{5} = 0.6$$

$\therefore \cos\alpha = \cos 53° 8' \Rightarrow \alpha = 53° 8'$ upstream [1/2]

(ii) The resultant velocity along OP is given by

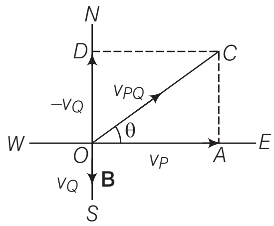
$$v = \sqrt{v_s^2 - v_r^2} = \sqrt{5^2 - 3^2} = 4 \text{ km/h}$$ [1/2]

(iii) Time taken by swimmer to cross the river

$$t = \frac{d}{v} = \frac{1}{4} = 0.25 \text{ h} = 15 \text{ min}$$ [1/2]

33. *The velocity of a particle P due East is 4 m/s and that of θ is 3 m/s due South. What is the velocity of P w.r.t. θ?*

Sol. From the figure, [1]



$\mathbf{v}_P = \mathbf{OA} = 4$ M/S due East

$\mathbf{v}_Q = \mathbf{OB} = 3$ M/S due South

$\mathbf{v}_{PQ} = \mathbf{v}_P - \mathbf{v}_Q = \mathbf{v}_P + (-\mathbf{v}_Q) = \mathbf{OA} + \mathbf{OD}$ [1]

$$\therefore \quad |\mathbf{v}_{PQ}| = \sqrt{4^2 + 3^2} = 5 \text{ M/S}$$

$$\tan\theta = \frac{AC}{OA} = \frac{OD}{OA} = \frac{3}{4} = 0.75$$

$\theta = 36°52'$ North of East [1]

34. *Can you associate vectors with?*

(i) The length of a wire bent into a loop

(ii) A plane area

(iii) A sphere [NCERT]

Sol. (i) We cannot associate a vector with the length of a wire bent into a loop.

(ii) We can associate a vector with a plane area. Such a vector is called **area vector** and its direction is represented by outward drawn normal to the area.

(iii) We cannot associate a vector with volume of sphere, however, a vector can be associated with the area of sphere. [1 × 3]

Try Yourself

1. It is easier to pull than to push a lawn roller. Why?
2. A vector **A** of magnitude A is turned through an angle α. Calculate the change in the magnitude of vector. [**Ans.** $2A\sin\alpha/2$] [2]
3. State parallelogram law of vector addition. Show that the resultant of two vectors **A** and **B** inclined at an angle θ is $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$
4. Determine a unit vector which is perpendicular to both $\mathbf{A} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ $\left[\textbf{Ans. } \frac{3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 3\hat{\mathbf{k}}}{\sqrt{27}}\right]$

Value Based Questions [4 Marks]

35. *From a school, a group of boys went for a picnic in a village. They went through fields and enjoyed the beauty of nature. While walking, they saw a well which they had never seen in the city. They were very excited and started drawing water from well. They planned to have a competition in which they decided that the who would draw more water would become winner. A villager who was listening to them, went to them and told them about the importance of water. He also explained that they use the water of this for irrigating their fields and also for drinking.*

(i) What values were possessed by the villager?

(ii) If the two boys raising the bucket, pull it an angle θ to each other and each exerts a force of 20 N their effective pull is 30 N. What is the angle between their arms?

(iii) What are the other means of irrigation in villages?

Sol. (*i*) The villager was nature loving and he know the importance of natural resources. [1]

(*ii*) Given, **A** = 20 N, **B** = 30 N, $R = 30$ N, $\theta = ?$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$ [1]

$$30 = \sqrt{20^2 + 20^2 + 2 \times 20 \times 20 \cos\theta}$$

$$\cos\theta = \frac{30^2 - 20^2 - 20^2}{2 \times 20^2} = 0.125 \Rightarrow \theta = 82° \, 49'$$ [1]

(*iii*) Canals, lakes and ponds are other means of irrigation in villages. [1]

36. *Rahul and Shyam are two friends. They were playing cricket in a field. Rahul was balling and Shyam hits the ball with his bat. Now, Rahul is going to search the ball, he finds a big stone in the field then he called Shyam and both decided to remove the stone from the field. Rahul pushed stone in a particular direction and Shyam pushed the stone in such a way the making an angle of 60° with pushing force exerted by Rahul.*

(i) What are the values shown by Rahul and Shyam?

(ii) What will be net force exerted on the stone by Rahul and Shyam?

(iii) What value in daily life we learn from this incidence?

Sol. (*i*) The value shown by Rahul and Shyam was their attitude of working in a team and helping nature. [1]

(*ii*) The net force exerted on the stone by both of them will be $F_R = \sqrt{F^2 + F^2 + 2F^2 \cos 60°}$

$$= \sqrt{2F^2 + 2F^2 \times 1/2} = \sqrt{2F^2 + F^2} = \sqrt{3}F$$

where F is force by Rahul. [1]

(*iii*) We should work in a team and we should help one another. [1]

Long Answer Type Questions [5 Marks]

37. *State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful*

(i) Adding any two scalars.

(ii) Adding a scalar to a vector of the same dimensions.

(iii) Multiplying any vector by any scalar.

(iv) Multiplying any two scalars.

(v) Adding any two vectors.

(vi) Adding a component of a vector to the same vector. [NCERT]

Sol. (*i*) No, adding any two scalars is not meaningful because only the scalars of same dimensions *i.e.* having same unit can be added. [1/2]

(*ii*) No, adding a scalar to a vector of the same dimensions is not meaningful because a scalar cannot be added to a vector. [1/2]

(*iii*) Yes, multiplying any vector by any scalar is meaningful. When a vector is multiplied by a scalar we get a vector, whose magnitude is equal to the product of magnitude of vector and the scalar and direction remains the same as the direction of the given vector. [1]

e.g., A body of mass 4 kg is moving with a velocity 20 m/s towards East then, product of velocity and mass gives the momentum of the body which is also a vector quantity.

$$\mathbf{p} = m\mathbf{v} = 4 \text{ kg} \times (20 \text{ m/s}) \text{ (East)}$$

$$= 80 \text{ kg-m/s, East}$$

(*iv*) Yes, multiplying any two scalars is meaningful. Density ρ and volume V both are scalar quantities. When density is multiplied by volume, then we get $\rho \times V = m$, mass of the body, which is a scalar quantity. [1]

(*v*) No, adding any two vectors is not meaningful because only vectors of same dimensions *i.e.* having same unit can be added. [1]

(*vi*) Yes, adding a component of a vector to the same vector is meaningful because both vectors are of same dimensions. [1]

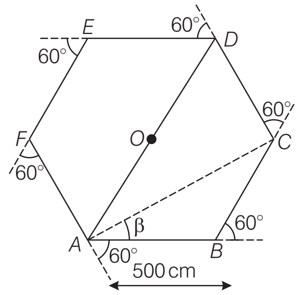
38. *On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.*

As motorist is taking turn to his left at an angle 60° after every 500 m therefore, he is moving on a regular hexagon.

Sol. The distance after which motorist take a turn = 500 m

As motorist takes a turn at an angle of 60° each time, therefore motorist is moving on a regular hexagonal path. Let the motorist starts from point A and reaches at point D at the end of third turn and at initial point A at the end of sixth turn and at point C at the end of eight turn. [1/2]

Displacement of the motorist at the third turn = AD [1/2]



$$= AO + OD = 500 + 500 = 1000 \text{ m}$$

Topic 2 Motion in a Plane

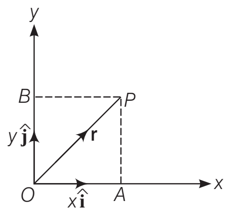
Here, we will discuss how to describe motion of an object in two dimensions.

Position, Displacement and Velocity Vectors

Position Vector

A vector that extends from a reference point to the point at which particle is located is called **position vector.**

Let r be the position vector of a particle P located in a plane with reference to the origin O in xy - plane as shown in figure.



Representation of position vector

$$\mathbf{OP} = \mathbf{OA} + \mathbf{OB}$$

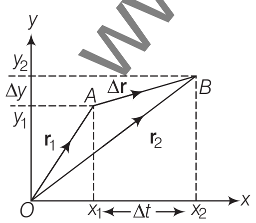
$$\boxed{\text{Position vector, } \mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}}$$

In three dimensions, the position vector is represented as

$$r = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

Displacement

Consider a particle moving in xy-plane with a uniform velocity $\mathbf{v}$ and point O as an origin for measuring time and position of the particle. Let, the particle be at position A and B at timings t_1 and t_2 respectively. The position vectors are $\mathbf{OA} = \mathbf{r}_1$ and $\mathbf{OB} = \mathbf{r}_2$.



Representation of displacement vector

Then, the displacement of the object in time interval $(t_2 - t_1)$ is **AB**. From triangle law of vector addition, we have

$$\mathbf{OA} + \mathbf{AB} = \mathbf{OB}$$

$$\Rightarrow \quad \mathbf{AB} = \mathbf{OB} - \mathbf{OA}$$

$$\mathbf{AB} = \mathbf{r}_2 - \mathbf{r}_1 \quad \ldots\text{(i)}$$

If the coordinates of the particle at points A and B are (x_1, y_1) and (x_2, y_2), then

$$\therefore \quad \mathbf{r}_1 = x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}}$$

$$\text{and} \quad \mathbf{r}_2 = x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}}$$

Substituting the values of $\mathbf{r}_1$ and $\mathbf{r}_2$ in Eq. (i), we have

$$\mathbf{AB} = (x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}}) - (x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}})$$

$$\boxed{\text{Displacement, } \mathbf{AB} = (x_2 - x_1)\,\hat{\mathbf{i}} + (y_2 - y_1)\,\hat{\mathbf{j}}}$$

Similarly, in three dimensions the displacement can be represented as

$$\Delta\mathbf{r} = (x_2 - x_1)\,\hat{\mathbf{i}} + (y_2 - y_1)\hat{\mathbf{j}} + (z_2 - z_1)\,\hat{\mathbf{k}}$$

Velocity

It is defined as the capacity of body to cover a distance in a certain interval of time and in a particular direction.

It is of two types

(*i*) Average velocity

(*ii*) Instantaneous velocity

Average Velocity

It is defined as the ratio of the displacement and the corresponding time interval.

$$\text{Thus, average velocity} = \frac{\text{displacement}}{\text{time taken}}$$

$$\boxed{\text{Average velocity, } \mathbf{v}_{AV} = \frac{\Delta\mathbf{r}}{\Delta t} = \frac{\mathbf{r}_2 - \mathbf{r}_2}{t_2 - t_1}}$$

Velocity can be expressed in the component form as

$$\mathbf{v}_{av} = \frac{\Delta x}{\Delta t}\hat{\mathbf{i}} + \frac{\Delta y}{\Delta t}\hat{\mathbf{j}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$$

where v_x and v_y are the components of velocity along x-direction and y-direction respectively.

The magnitude of $\mathbf{v}_{av}$ is given by $v_{AV} = \sqrt{v_x^2 + v_y^2}$

and the direction of $\mathbf{v}_{AV}$ is given by angle θ

$$\tan\theta = \frac{v_y}{v_x}$$

$$\Rightarrow \quad \boxed{\text{Direction of average velocity, } \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)}$$

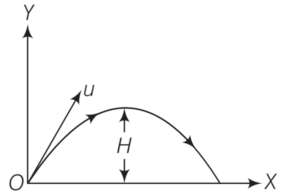
Since, $v_y = u_y + a_y t \Rightarrow 0 = u\sin\theta - g\dfrac{T}{2}$

$$\boxed{\text{Total time of flight, } T = \frac{2u\sin\theta}{g}}$$

For a projectile, time of ascent equals time of descent.

Maximum Height of a Projectile

It is defined as the maximum vertical height attained by an object above the point of projection during its flight. It is denoted by H.



Maximum height of motion

Let us consider the vertical upward motion of the object from O to H.

We have,

$$u_y = u\sin\theta,\ a_y = -g,\ y_0 = 0,\ y = H,\ t = \frac{T}{2} = \frac{u\sin\theta}{g},$$

Using this relation, $y = y_0 + u_y t + \dfrac{1}{2}a_y t^2$

$$\text{We have } H = 0 + u\sin\theta\,\frac{u\sin\theta}{g} + \frac{1}{2}(-g)\left(\frac{u\sin\theta}{g}\right)^2$$

$$= \frac{u^2}{g}\sin^2\theta - \frac{1}{2}\frac{u^2\sin^2\theta}{g}$$

$$\boxed{\text{Maximum height, } H = \frac{u^2\sin^2\theta}{2g}}$$

Horizontal Range of a Projectile

The horizontal range of the projectile is defined as the horizontal distance covered by the projectile during its time of flight. It is denoted by R.

If the object having uniform velocity $u\cos\theta$ (*i.e.* horizontal component) and the total time of flight T, then the horizontal range covered by the objective.

$$\therefore\ R = u\cos\theta \times T = u\cos\theta \times 2u\frac{\sin\theta}{g}$$

$$\boxed{\text{Horizontal range, } R = \frac{u^2\sin 2\theta}{g}}$$

$(\because \sin 2\theta = 2\sin\theta\cos\theta)$

The horizontal range will be maximum, if

$$\sin 2\theta = \text{maximum} = 1$$
$$\sin 2\theta = \sin 90^\circ$$

or $\theta = 45^\circ$

$$\therefore\ \boxed{\text{Maximum horizontal range, } R_m = \frac{u^2}{g}}$$

Worked out Problem

A soccer player kicks a ball at an angle of 30° with an initial speed of 20 m/s. Assuming that the ball travels in a vertical plane. Calculate (i) the time at which the ball reaches the highest point, (ii) the maximum height reached, (iii) the horizontal range of the ball and (iv) the time for which the ball is in the air. $g = 10\ \text{M/S}^2$.

Solution

Step I Write what you know and what you want to know.

$\theta = 30^\circ, u = 20\ \text{m/s}, g = 10\ \text{m/s}^2$

(i) $t = ?$ (ii) $H = ?$

(iii) $R = ?$ (iv) $T = ?$

Step II List all the formulae used.

(i) $t = \dfrac{T}{2} = \dfrac{u\sin\theta}{g}$ (ii) $H = \dfrac{u^2\sin^2\theta}{2g}$

(iii) $R = \dfrac{u^2\sin 2\theta}{g}$ (iv) $T = \dfrac{2u\sin\theta}{g}$

Step III Substitute the values in step II and solve.

(i) $t = \dfrac{T}{2} = \dfrac{20\times\sin 30^\circ}{10} = 2\times\dfrac{1}{2} = 1\text{s}$

(ii) $H = \dfrac{(20)^2\times\sin^2 30^\circ}{2\times 10} = 5\text{ m}$

(iii) $R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{(20)^2\times\sin 2\times 30^\circ}{10} = 34.64\text{ m}$

(iv) $T = \dfrac{2\times 20\times\sin 30^\circ}{10} = 2\text{ s}$

Example 6 *A Pace Bowler*

A cricket ball is thrown at a speed of 28 ms^{-1} in a direction 30° above the horizontal. Calculate

(i) the maximum height

(ii) the time taken by the ball to return to the same level and

(iii) the distance from the thrower to the point where the ball returns to the same level. [NCERT]

Solution (*i*) The maximum height attained by the ball is

$$H_m = \frac{(v_0\sin\theta_0)^2}{2g}$$

$$= \frac{(28\sin 30^\circ)^2}{2(9.8)} = \frac{14\times 14}{2\times 9.8} = 10.0\text{ m}$$

EXAM Practice

Very Short Answer Type Questions [1 Mark]

1. *Can a body move on a curved path without having acceleration?*

Sol. No, a body cannot move on a curved path without acceleration because while moving on a curved path, the velocity of the body changes with time.

2. *A particle cannot accelerate if its velocity is constant, why?*

Sol. When the particle is moving with a constant velocity, there is no change in velocity with time and hence, its acceleration is zero.

3. *The magnitude and direction of the acceleration of a body both are constant. Will the path of the body be necessarily be a straight line?*

Sol. No, the acceleration of a body remains constant, the magnitude and direction of the velocity of the body may change.

4. *Give a few examples of motion in two dimensions.*

Sol. A ball dropped from an aircraft flying horizontally, a gun shot fired at some angle with the horizontal, etc.

5. *A man moving in rain holds his umbrella inclined to the vertical even though the rain drops are falling vertically downwards. Why?*

Sol. The man experiences the velocity of rain relative to himself. To protect himself from the rain, the man should hold umbrella in the direction of relative velocity of rain with respect to man.

6. *A football is kicked into the air vertically upwards. What is its (i) acceleration and (ii) velocity at the highest point?* [NCERT Exemplar]

Sol. (*i*) Acceleration at the highest point $= -g$

(*ii*) Velocity at the highest point $= 0$.

7. *Can there be motion in two dimensions with an acceleration only in one dimension?*

Sol. Yes, in a projectile motion the acceleration acts vertically downwards, while the projectile follows a parabolic path.

8. *A stone is thrown vertically upwards and then it returns to the thrower. Is it a projectile?*

Sol. No, it is not a projectile. Because a projectile should have two component velocities in two mutually perpendicular directions but in this case, the body has velocity only in one direction while going up or coming down.

9. *At what point in its trajectory does a projectile have its*

(i) minimum speed and

(ii) maximum speed?

Sol. (*i*) Projectile has minimum speed at the highest point of its trajectory.

(*ii*) Projectile has maximum speed at the point of projection.

10. *A stone tied at the end of string is whirled in a circle. If the string breaks, the stone flies away tangentially. Why?*

Sol. When a stone is going around a circular path, the instantaneous velocity of stone is acting as tangent to the circle. When the string breaks, the centripetal force stops to act. Due to inertia, the stone continues to move along the tangent to circular path. So, the stone flies off tangentially to the circular path.

11. *The direction of the oblique projectile becomes horizontal at the maximum height. What is the cause of it?*

Sol. At the maximum height of projectile, the vertical component velocity becomes zero and only horizontal component velocity of projectile is there.

12. *Two bodies are projected at an angle θ and $(\pi/2 - \theta)$ to the horizontal with the same speed. Find the ratio of their time of flight.*

Sol. The times of flights are $T_1 = \dfrac{2u\sin\theta}{g}$

and $$T_2 = \frac{2u\sin\left(\frac{\pi}{2}-\theta\right)}{g} = \frac{2u\cos\theta}{g}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{\sin\theta}{\cos\theta} = \tan\theta.$$

13. *A body is moving on a circular path with a constant speed. What is the nature of its acceleration?*

Sol. The nature of its acceleration is centripetal, which is perpendicular to motion at every point and acts along the radius and directed towards the centre of the curved circular path.

14. *Is the rocket in flight is an illustration of projectile?*

Sol. No, because it is propelled by combustion of fuel and does not move under the effect of gravity alone.

Value Based Questions [4 Marks]

48. *In a flood hit areas of Uttarakhand, helicopter was dropping ration, medicines and other items for the victims. The helicopter was flying at a height of 49 m above the ground. Students of nearby school were helping the authorities to evacuate the victims. They saw that a child was drowning. They rushed towards the child with life boat and saved the child.*

(i) What is the time taken by the objects dropped from helicopter to reach the ground?

(ii) What values are shown by students?

Sol. (*i*) Time taken by the object

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 49}{9.8}} = 3.16\,\text{s} \quad [\because h = 49 \text{ m}] \quad [2]$$

(*ii*) Students are helpful and concerned for others. They are brave and have the ability to act quickly. [2]

49. *Hatrick and Peterson were good friends. They went to a international trade fair at Pragati Maidan. Peterson saw a shop, where people were firing at balloons. He also asked the shopkeeper for a gun. Peterson fired three shots but none of the shots hit the target and he was quite confused. Hatrick was watching Peterson. He told Peterson to fire at the balloon by taking aim just above the balloon. Peterson acted on his advice of Hatrick and then he successfully hited all the balloons.*

(i) What values are shown by Hatrick?

(ii) Hatrick asked Peterson to take aim just above the balloon. Why?

Sol. (*i*) Hatrick is a good friend. He was concerned with his friend Peterson. He has high degree of general awareness. [2]

(*ii*) Hatrick knew that the bullet falls under gravity, when fired. It follows a parabolic path. Peterson was firing the bullet by keeping the gun in the line of the sight and hence missing the target. Thus, the bullet has to be fired by keeping the gun tilted above the line of sight to hit the target. [2]

50. *A foreigner arrived at Mumbai airport at around 1 AM in the morning. She found a taxi waiting outside. She asked the driver to drop her at the nearby hotel. The taxi driver obliged and drove the foreigner round and round in the city and dropped her at a hotel at around 1:30 AM in the morning.*

The hotel was only few kilometre away but he charged her one thousand rupees. While driver was arguing with foreigner, a man from the hotel came out to help her. When he heard that driver was charging one thousand rupees, he scolded him and asked him to charge genuinely and not to spoil their country's name. Driver apologised to foreigner and refunded her eight hundred rupees.

(i) What does this show about the driver?

(ii) The hotel of the foreigner was at a distance of 10 km away from airport on a straight road and dishonest cabman took her along a circuitous path 23 km long and reached the hotel in 28 min. What was the average speed and magnitude of average velocity?

(iii) When is average speed equal to average velocity?

Sol. (*i*) The dishonest cabman had put the reputation of the city at stake because of his greed for money. But he realised his mistake and apologised to foreigner. [1½]

(*ii*) Actual length of path travelled = 23 km

Displacement = 10 km

Time taken = 28 min = 28/60 h [1/2]

Average speed of taxi

$$= \frac{\text{actual path length}}{\text{time taken}}$$

$$= \frac{23}{28/60} = 49.3 \text{ km/h} \quad [1]$$

Magnitude of average velocity

$$= \frac{\text{displacement}}{\text{time}} = \frac{10}{28/60} = 21.4 \text{ km/h} \quad [1]$$

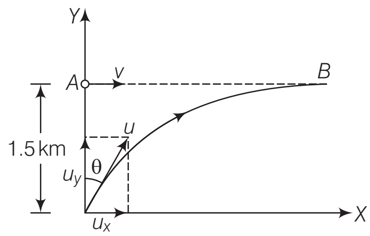
(*iii*) When an object is moving in a straight line. [1]

Long Answer Type Questions [5 Marks]

51. *A fighter plane flying horizontally at an altitude of 1.5 km with speed 720 km h^{-1} passes directly over head an anticraft gun. At what angle from the vertical should the gun be fired from the shell with muzzle speed 600 ms^{-1} to hit the plane.*

At what minimum altitude should the pilot fly the plane to avoid being hit? (Take g = 10 ms^{-2}). [NCERT]

Sol. From the figure, let O be the position of gun and A be the position of plane.



Misconception *of* Concepts

Concept Projectile Motion of a Particle

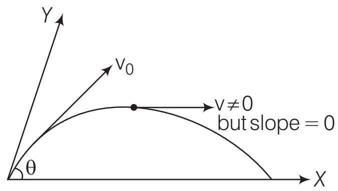
Misconception *Projectile motion is not a free motion because a projectile is not simply dropped but thrown with a certain velocity.*

Clarification The projectile motion is a free motion and a projectile is a freely falling body. In a free motion, the acceleration of the body is always equal to gravity (i.e. g) and the body feels weightlessness.

Concept Projectile Fired at an Angle with Vertical Projection

Misconception *Direction of velocity vector is always along the tangent to the path, therefore its magnitude must be given by its slope.*

Clarification The slope of a tangent to the path is not a measure of magnitude of velocity at that point. At the highest position the slope of the tangent is zero but the velocity is not equal to zero.



Concept Relative Velocity

Misconception *The students are usually confused with the term velocity of swimmer.*

They believe that it is with respect to river if stated as velocity of swimmer w.r.t. still water otherwise it is with respect to ground.

Clarification Velocity of swimmer is always with respect to water, irrespective of the fact whether the water is stationary or flowing.

Concept Path of a Projectile

Misconception *When an object is travelling with a horizontal velocity, and is thrown upward, the path the object follows is along a line.*

Clarification When an object is travelling with a horizontal velocity, and is thrown upward, the object actually makes a parabolic path.

Concept Projectile Motion with Horizontal Projection

Misconception *In projectile motion, keeping v_i fixed but increasing the launch angle by 5 degree will increase the range.*

Clarification This statement is true if a projectile starts from 30° angle, but false if it starts from 60° angle. In general, changing the angle so that it gets further from 45° decreases the range.

Concept Resolution of Vector Components

Misconception *Each component of a vector is always a scalar.*

Clarification No, each component of a vector is also a vector.

Concept Scalar Quantity

Misconception *A scalar quantity is one that does not vary from one point to another in space.*

Clarification Gravitational potential being a scalar quantity vary from point to point in space.

Concept Uniform Circular Motion

Misconception *A body in a uniform horizontal circular motion possesses a variable velocity.*

Clarification KE of a body $= \frac{1}{2}mv^2 = \frac{1}{2}m(\mathbf{v}\cdot\mathbf{v})$, which is scalar. In uniform circular motion, speed of body v remains constant, hence KE of body remains constant in uniform circular motion.

Concept Projectile Motion

Misconception *A stone is thrown vertically upwards and then it returns to the thrower. So, it is a projectile motion.*

Clarification It is not a projectile motion because a projectile should have two component velocities in two mutually perpendicular directions but in this case, the stone has velocity only in one direction while going up or coming downwards.

Concept Determine the Dimension of Scalar Quantity

Misconception *A scalar quantity is one that must be dimensionless.*

Clarification Density being a scalar quantity has Dimensions.

Revision MAP

Motion in a Plane

Physical Quantity

Position and Displacement Vectors

- **Position vector** A vector which gives position of an object with reference to the origin of a coordinate system. It is denoted by **OA** = **r**
- **Displacement vector** The vector which tells how much and in which direction an object has changed its position in a given interval of time.

$\Delta \mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$

$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

$|\Delta r| = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Scalar Quantity

- They are those physical quantities which have only magnitudes but no direction.

Vector Quantity

- They are those physical quantities which have both magnitudes and directions.
- It is denoted by bold face or by arrows.

Types of Vectors

- **Polar vectors** Vectors which have a starting point or a point of application.
- **Axial vectors** Vectors which represent the rotational effect and acts along the axis of rotation are called axial vectors.

Terms Related to Vectors

- **Modulus of a vectors** The magnitude of a vector. It is denoted by | **A** |.
- **Unit vector** A vector having magnitude equal to unity but having a specific direction

$\hat{\mathbf{A}} = \dfrac{\mathbf{A}}{|\mathbf{A}|}$

- **Null vector** A vector with magnitude zero and having an arbitrary direction. It is also known as zero vector.
- **Equal vector** Two vectors are said to be equal if they have equal magnitude and same direction.
  It is denoted by **A** = **B**
- **Negative vectors** Two vectors are said to be negative of each other if their magnitudes are equal but opposite in direction.
  It is denoted by **A** = – **B**
- **Collinear vector** Two or more vectors are said to be collinear when they act along the same lines or parallel lines.
- **Coplanar vector** The vectors lying in the same plane.

Addition of Vectors (Analytical Method)

- $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$
- $\tan\beta = \dfrac{B\sin\theta}{A + B\cos\theta}$
- If $\theta = 0$, then $R = A + B$ (maximum)
- If $\theta = 90°$, then $R = \sqrt{A^2 + B^2}$
- If $\theta = 180°$, then $R = A - B$ (minimum)

Addition of Vectors (Graphical Method)

- Two vectors can be added if both of them are of same nature.
- Triangle law of addition **R = A + B**
- Parallelogram law of addition of vectors **R = A + B**
- Polygon law of addition of vectors **R = A + B + C + D**

Subtraction of Two Vectors

- Two vectors can be subtracted if both of them are of different nature.
- **R = A + (– B) = A – B**

Subtraction of Two (Analytical Method)

- $R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$
- $\tan\beta = B\sin\theta/(A - B\cos\theta)$
- If $\theta = 0°$, then **R** = **A** – **B** (minimum)
- If $\theta = 90°$, then **R** = $\sqrt{\mathbf{A}^2 + \mathbf{B}^2}$
- If $\theta = 180°$, then **R** = **A** + **B** (maximum)

Product of Two Vectors

Dot or Scalar Product

- The product of magnitudes of vectors **A** and **B** and the cosine angle between them **A · B** = AB cos θ
- If θ = 0 **A · B** = AB, two vectors are parallel.
- If θ = 90° **A · B** = 0, two vectors are mutually perpendicular to each other.
- If θ = 180° **A · B** = –AB, two vectors are mutually perpendicular to each other.

Vector or Cross Product

- The product of the magnitudes of vectors **A** and **B** and the sine angle

Projectile Motion

- It is a form of motion in which an object or particle is thrown with some initial velocity near the earth's surface and it moves along a curved path under the action of gravity alone.

Motion along Horizontal Direction

- $x = x_0 + u_n t + 1/2\ a_x t^2$
  and $x = u\cos\theta\ t$

Time of Flight

- $T = 2u\sin\theta/g$

Motion along Vertical Direction

- $y = y_0 + u_n t + 1/2\ a_y t^2$
- $y = x\tan\theta - (1/2 \times g/u^2\cos^2\theta)\ x^2$

Maximum Height of a Projectile

- $H = u^2\sin^2\theta/2g$

Horizontal Range of a Projectile

- $R = u^2\sin 2\theta/2g$
- If θ = 45°, $R_{max} = u^2/g$

Uniform Circular Motion

When an object follows a circular path at a constant speed.

Relation between ω, f and T

Relation between v and ω

Centripetal Acceleration

- $a = v^2/r$

Practice Exercise

(Based on the Complete Chapter)

1. When the component of a vector **A** along the direction of **B** is zero, what can you conclude about the two vectors?

2. Displacement vector is fundamentally a position vector. Comment on this statement.

3. Is it necessary to mention the direction of vector having zero magnitude?

4. Does the nature of a vector change when it is multiplied by a scalar? Explain with example.

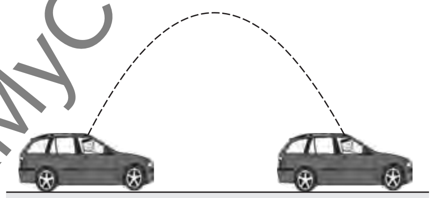
5. Draw the conclusion about **B** if $A - B = A + B$.

6. For what angle between **P** and **Q**, the value of $P + Q$ is maximum?

7. A bullet P is fired from a gun when the angle of elevation of the gun is 30° another bullet Q is fired from the gun when the angle of elevation is 60°. The vertical height attained in the second case is x times the vertical height attained in the first case. What is the value of x? [**Ans.** 3]

8. Can there be two vectors where the resultant is equal to either of them?

9. Suppose you are driving in a convertible car with the top down, the car is moving to the right at a constant velocity. As the figure illustrates, you point a toy rifle straight upward and trigger it. In the absence of air resistance, where would the bullet land (i) behind you, (ii) ahead of you or (iii) in the barrel of the rifle?



10. A rabbit runs across a parking lot on which a set of coordinates axes has strangely enough, been drawn. The coordinates (in meters) of the rabbit's position as functions of time t (in seconds) are given by

$$x = -0.31t^2 + 7.2t + 28 \text{ and } y = 0.22t^2 - 9.1t + 30$$

At $t = 15$ s, what is the rabbit's position vector **r** in unit vector notation and in magnitude angle notation?

(*i*) Draw a graph for rabbit's path for $t = 0$ to 25 s.

(*ii*) Find the rabbit's velocity **v** at time $t = 15$ s.

(*iii*) Find the rabbit's acceleration **a** at time $t = 15$ s. [**Ans.** (*b*) $\mathbf{v} = 3.3$ m/s, $\theta = -130°$
(*c*) $a = 0.76$ m/s^2, $\theta = -135°$]

11. When a large star becomes a supernova, its core may be compressed so tightly that it becomes a neutron star, with a radius of about 20 km. If a neutron star rotates once every second, (*a*) what is the speed of a particle on the star's equator and (*b*) what is the magnitude of the particle's centripetal acceleration? (*c*) If the neutron star rotates faster, do the answer to (*a*) and (*b*) increase, decrease or remain the same? [**Ans.** (a) $a = 1.3 \times 10^5$ m/s, (b) 7.9×10^5 m/s^2 and (c) increase]

Appendix

(Some Mathematical Concepts Applied in Physics)

Binomial Theorem

Binomial theorem states that if n is any integer, positive or negative or a fraction and x is any real number, then

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

where, $2! = 2 \times 1, 3! = 3 \times 2 \times 1$

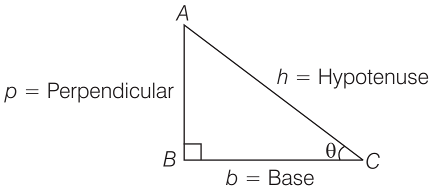
In general, $n! = n(n-1)(n-2)\cdots 3 \times 2 \times 1$

If $|x| << 1$, then $(1+x)^n \approx 1 + nx$

Trigonometric Ratio

In a right angled triangle ABC, given below

$$\angle ABC = 90° \text{ and } \angle ACB = \theta$$



Thus, trigonometric ratios are

(i) $\text{sine}\,\theta = \sin\theta = \dfrac{p}{h}$

(ii) $\text{cosine}\,\theta = \cos\theta = \dfrac{b}{h}$

(iii) $\text{tangent}\,\theta = \dfrac{p}{b} = \tan\theta = \dfrac{\sin\theta}{\cos\theta}$

(iv) $\text{cosecant}\,\theta = \dfrac{h}{p} = \text{cosec}\,\theta = \dfrac{1}{\sin\theta}$

(v) $\text{Secant}\,\theta = \dfrac{h}{b} = \sec\theta$

(vi) $\text{cotangent}\,\theta = \dfrac{b}{p} = \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Fundamental Trigonometric Ratios

(i) $\sin^2\theta + \cos^2\theta = 1$ (ii) $1 + \tan^2\theta = \sec^2\theta$

(iii) $1 + \cot^2\theta = \text{cosec}^2\,\theta$ (iv) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

(v) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ (vi) $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$

(vii) $\sec\theta = \dfrac{1}{\cos\theta}$

$\sin(-\theta) = -\sin\theta$	$\text{cosec}(-\theta) = -\text{cosec}\,\theta$
$\cos(-\theta) = \cos\theta$	$\sec(-\theta) = \sec\theta$
$\tan(-\theta) = -\tan\theta$	$\cot(-\theta) = -\cot\theta$
$\sin(90° - \theta) = \cos\theta$	$\text{cosec}(90° - \theta) = \sec\theta$
$\cos(90° - \theta) = \sin\theta$	$\sec(90° - \theta) = \text{cosec}\,\theta$
$\tan(90° - \theta) = \cot\theta$	$\cot(90° - \theta) = \tan\theta$
$\sin(90° + \theta) = \cos\theta$	$\text{cosec}(90° + \theta) = \sec\theta$
$\cos(90° + \theta) = -\sin\theta$	$\sec(90° + \theta) = -\text{cosec}\,\theta$
$\tan(90° + \theta) = -\cot\theta$	$\cot(90° + \theta) = -\tan\theta$
$\sin(180° - \theta) = \sin\theta$	$\text{cosec}(180° - \theta) = \text{cosec}\,\theta$
$\cos(180° - \theta) = -\cos\theta$	$\sec(180° - \theta) = -\sec\theta$
$\tan(180° - \theta) = -\tan\theta$	$\cot(180° - \theta) = -\cot\theta$
$\sin(180° + \theta) = -\sin\theta$	$\text{cosec}(180° + \theta) = -\text{cosec}\,\theta$
$\cos(180° + \theta) = -\cos\theta$	$\sec(180° + \theta) = -\sec\theta$
$\tan(180° + \theta) = \tan\theta$	$\cot(180° + \theta) = \cot\theta$
$\sin(270° - \theta) = -\cos\theta$	$\text{cosec}(270° - \theta) = -\sec\theta$
$\cos(270° - \theta) = -\sin\theta$	$\sec(270° - \theta) = -\text{cosec}\,\theta$
$\tan(270° - \theta) = \cot\theta$	$\cot(270° - \theta) = \tan\theta$
$\sin(270° + \theta) = -\cos\theta$	$\text{cosec}(270° + \theta) = -\sec\theta$
$\cos(270° + \theta) = \sin\theta$	$\sec(270° + \theta) = \text{cosec}\,\theta$
$\tan(270° + \theta) = -\cot\theta$	$\cot(270° + \theta) = -\tan\theta$
$\sin(360° - \theta) = -\sin\theta$	$\text{cosec}(360° - \theta) = -\text{cosec}\,\theta$
$\cos(360° - \theta) = \cos\theta$	$\sec(360° - \theta) = \sec\theta$
$\tan(360° - \theta) = -\tan\theta$	$\cot(360° - \theta) = -\cot\theta$

Some Important Trigonometrical Formulae

(i) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

(ii) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

(iii) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

(iv) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

(v) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(vi) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

(vii) $\sin 2A = 2\sin A \cos A = \dfrac{2\tan A}{1 + \tan^2 A}$

(viii) $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$

$$= 2\cos^2 A - 1 = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

(ix) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

(x) $\sin(A+B)+\sin(A-B) = 2\sin A\cos B$

(xi) $\sin(A+B)-\sin(A-B) = 2\cos A\sin B$

(xii) $\cos(A+B)+\cos(A-B) = 2\cos A\cos B$

(xiii) $\cos(A+B)-\cos(A-B) = -2\sin A\sin B$

(xiv) $\sin C+\sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(xv) $\sin C-\sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

(xvi) $\cos C+\cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(xvii) $\cos C-\cos D = -2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

Values of Trigonometrical Ratios of Some Standard Angles

Angle θ	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	0	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$-\infty$	0

Logarithm

The logarithm of any number to a given base is the power to which base must be raised to obtain that number.

e.g., We can write $2^3 = 8$ in the form of logarithm as $\log_2^8 = 3$.

Thus, if $N = a^x$, then $\log_a N = x$

Formulae for Logarithm

(i) $\log_a mn = \log_a m + \log_a n$ (product formula)

(ii) $\log_a m/n = \log_a m - \log_a n$ (quotient formula)

(iii) $\log_a m^n = n\log_a m$ (power formula)

(iv) $\log_a m = \log_b m \times \log_a b$ (base change formula)

Systems of Logarithm

There are two systems in logarithm

(i) **Neperian log or natural log** Here, the base is e,

where $e = 1 + \dfrac{1}{1!} + \dfrac{1}{2!} + \dfrac{1}{3!} + \cdots = 2.718$ (approx)

This logarithm is used in all theoretical calculations.

(ii) **Common logarithm** Here, the base of the log is 10. In all numerical calculations, we use common log.

Relation between natural log and common log

$$\log_e m = \log_{10} m \times \log_e 10$$

where, $\log_e 10 = 2.3026\log_{10} 10$ ($\because \log_{10} 10 = 1$)

$= 2.3026 \times 1 = 2.3026$

$\therefore \quad \log_e m = 2.3026\log_{10} m$

Some Important Results on Differentiation

(i) Let c be a constant. Then, $\dfrac{d}{dx}(c) = 0$

(ii) $\dfrac{d}{dx}(cy) = c\cdot\dfrac{dy}{dx}$

(iii) $\dfrac{d}{dx}(x^n) = nx^{n-1}$

(iv) Let $y = u \pm v$, where u and v are functions of x.

Then, $\dfrac{dy}{dx} = \dfrac{du}{dx} \pm \dfrac{dv}{dx}$

(v) Product rule

Let $y = uv$, then

$$\frac{dy}{dx} = FF\frac{d}{dx}(SF) + SF\frac{d}{dx}(FF) = u\frac{dv}{dx} + v\frac{du}{dx}$$

(vi) Quotient rule

Let $y = \dfrac{u}{v}$, then

$$\frac{dy}{dx} = \frac{\text{Den}\dfrac{d}{dx}(\text{Num}) - (\text{Num})\dfrac{d}{dx}(\text{Den})}{(\text{Den})^2} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

(vii) **Chain rule** Let y be a function of u and u be a function of x. Then, $\dfrac{dy}{dx} = \dfrac{dy}{du}\cdot\dfrac{du}{dx}$

Let $y = u^n$. Then, $\dfrac{d}{dx}(u^n) = nu^{n-1}\cdot\dfrac{du}{dx}$

(viii) $\dfrac{d}{dx}(\log_e x) = \dfrac{1}{x}$

(ix) $\frac{d}{dx}(\log_a x) = \frac{1}{x}\log_e a$

(x) $\frac{d}{dx}(e^x) = e^x$

(xi) $\frac{d}{dx}(a^x) = a^x \log_e a$

(xii) $\frac{d}{dx}(\sin x) = \cos x$

(xiii) $\frac{d}{dx}(\cos x) = -\sin x$

(xiv) $\frac{d}{dx}(\tan x) = \sec^2 x$

(xv) $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

(xvi) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(xvii) $\frac{d}{dx}(\text{cosec } x) = -\text{coseec } x \cot x$

Integration

Integration is the reverse process of differentiation. It is the process of finding a function whose derivative is given. If derivative of function $f(x)$ with respect to x is $f'(x)$, the integration of $f'(x)$ with respect to x is $f(x)$. Symbolically, we can say

If $\frac{d}{dx}[f(x)] = f'(x)$, then $\int f'(x)\,dx = f(x)$

Theorems of Integration

Theorem 1. The integral of the product of a constant and a function of x is equal to the product of the constant and integral of that function. Mathematically,

$$\int cu\,dx = c\int u dx$$

Theorem 2. The integral of sum or difference of a number of functions equal to the sum or difference of their integrals. Mathematically.

$$\int (u \pm v \pm w)\,dx = \int u dx \pm \int v dx \pm \int w dx,$$

where, u, v and w are functions of x.

Definite Integral

When an integral is defined between two definite limits a and b, it is said to be a **definite integral** It is given by

$$\int_a^b f(x)\,dx = [\phi(x)]_a^b = \phi(b) - \phi(a)$$

Comparative Study of Integration and Differentiation

	Differentiation	Integration
1.	$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n\,dx = \frac{x^{n+1}}{(n+1)} + C$, provided $n \neq -1$. Here, C is constant of integration.
2.	$\frac{d}{dx}(x) = 1$	$\int dx = x + C$
3.	$\frac{d}{dx}(\log_e x) = \frac{1}{x}$	$\int \frac{dx}{x} = \log_e x + C$
4.	$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x\,dx = \sin x + C$
5.	$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x\,dx = -\cos x + C$
6.	$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x\,dx = -\tan x + C$
7.	$\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$	$\int \text{cosec}^2 x \cdot dx = -\cot x + C$
8.	$\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$	$\int \sec x \cdot \tan x\,dx = \sec x + C$
9.	$\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cdot \cot x$	$\int \text{cosec } x \cdot \cot x\,dx = -\text{cosec } x + C$
10.	$\frac{d}{dx}(ax+b)^n = na(ax+b)^{n-1}$	$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$
11.	$\frac{d}{dx}\log_e(ax+b) = \frac{a}{(ax+b)}$	$\int \frac{dx}{(ax+b)} = \frac{1}{a}\log_e(ax+b) + C$
12.	$\frac{d}{dx}(e^x) = e^x$	$\int e^x \cdot dx = e^x + C$
13.	$\frac{d}{dx}(a^x) = a^x \cdot \log_e a$	$\int a^x \cdot dx = \frac{a^x}{\log_e a} = a^x \cdot \log_a e + C$
14.	$y = u \pm v \pm w$; $\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$	$\int (u \pm v \pm w)\,dx = \int u \cdot dx \pm \int v dx \pm \int w dx + C$

Expansion Formulae

(a) Exponential expansion

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

(b) Logarithmic expansion

$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots (x < 1)$$

(c) Trigonometry expansion (when θ is in radian).

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots$$

Sample Question Paper 1

(Fully Solved)

Physics

A sample question paper for CBSE Class XI

Time : 3 hrs

Max. Marks : 70

General Instructions

1. All questions are compulsory.
2. The question paper consists of 26 questions divided into five sections *A, B, C, D* and *E*. *Section A* comprises of 5 questions of 1 mark each, *Section B* comprises of 5 questions of 2 marks each, *Section C* comprises of 12 questions of 3 marks each, *Section D* comprises of 1 question of 4 marks and *Section E* comprises of 3 questions of 5 marks each.
3. All questions in *Section A* are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in some questions. You have to attempt only one of the alternative in all such questions.
5. Use of calculator is not permitted. You may ask for logarithmic table, if required.

Section A

1. Write the dimensional formula of impulse and name of the physical quantity having same dimension.
2. Write the relation between two angles for which horizontal ranges will be equal.
3. On what factors does the two vectors **a** and **b** to be perpendicular to each other.
4. Write the condition for conservation of mechanical energy of a system.
5. What is the ratio of escape speed of the earth and escape speed of the moon?

Section B

6. What is the heat associated with adiabatic process and what is the change in internal energy for isothermal process?
7. A ball is thrown vertically upwards. Draw its
   (i) velocity-time curve.
   (ii) acceleration-time curve.

8. Estimate will be angle of projection of a projectile for which range R and maximum height H are equal.

9. Obtain the equation, $\omega = \omega_0 + \alpha t$.

or

Why is the weight of a body at the poles more than the weight at the equator? Explain.

10. What is Kepler's law of periods? Express it mathematically.

Section C

11. Obtain the expression which shows dependence of speed of sound in a gas on its temperature.

12. Calculate the rms speed of a oxygen molecule at 37°C. Atomic mass of oxygen is 16.

13. State Bernoulli's principle and express it mathematically.

14. Show that velocity of sound in a gaseous medium does not depend on the pressure of the gas.

15. Determine a unit vector which is perpendicular to both $\mathbf{A} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\mathbf{B} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$.

16. A projectile is fired at an angle α with the horizontal with velocity u. Derive the expression for
(i) horizontal range.
(ii) maximum height attained.

17. Define impulse and impulse-momentum theorem. Why does one feel more pain when he/she punch on a hard wall than when he/she punch on soft muddy ground?

18. Briefly explain conservation of mechanical energy for a vibrating simple pendulum.

19. Prove the relation, $\tau = \frac{d\mathbf{L}}{dt}$.

Also, explain law of conservation of angular momentum.

20. Show that the escape velocity from the surface of the earth is given by $\sqrt{\frac{2GM}{R}}$, where R is the radius of the earth.

21. (i) Write the Hooke's law.
(ii) A steel wire of length 5 m and diameter 0.10 mm is stretched by 10 kg weight. Find the increase in its length if the Young's modulus of steel wire is 2.5×10^{11} N/m^2.

or

Calculate the work done for adiabatic expansion of a gas.

22. A source and an observer can approach one another with velocity 5 ms^{-1}. If the original frequency is 1500 Hz, calculate apparent frequency when
(i) only source is approaching
(ii) only observer is approaching
Take speed of sound in air 340 m/s.

Section D

23. Ashutosh and Asif were travelling by a bus. All of sudden, driver applied powerful brake to stop the bus within a very short distance.

Ashutosh and Asif experienced a forward jerk. Asif was holding a handle in the bus but Ashutosh was not, he was just about to fall in the bus, at the instant Asif saved him by pulling his arm.
(i) What values were shown by Asif?
(ii) When a bus suddenly stops, in which direction a passenger will experience a jerk and why?

Section E

24. (i) Derive an expression for the acceleration of a body sliding down a rough inclined plane.

(ii) A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m by applying a force F at the free end of the rope. Find the force by the rope on the block.

or

A satellite is to be placed in equatorial geostationary orbit around the earth for communication.

(i) Calculate height of such a satellite.

(ii) Find out the minimum number of satellites that are needed to cover entire earth so that atleast one satellite is visible from any point on the equator.

[$M = 6 \times 10^{24}$ kg, $R = 6400$ km, $T = 24$ h, $G = 6.67 \times 10^{-11}$ SI units]

25. A progressive wave is given by

$$y(x, t) = 8 \sin (400t - 0.2x)$$

where, x in metre, y in cm and t in second.

What is the

(i) direction of propagation (ii) wavelength

(iii) frequency (iv) wave speed

(v) phase difference between two points 0.4 m apart?

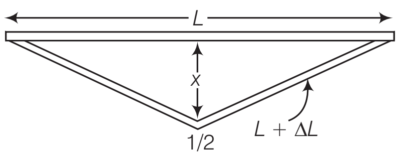
or

A tunnel is made along the diameter of the earth and a small stone is dropped in it. Show that stone will execute SHM around the centre of the earth. What is its time period?

26. Calculate the stress developed inside a tooth cavity filled with copper when hot tea at temperature of 57°C is drunk. You can take body (tooth) temperature to be 37°C and $\alpha = 1.7 \times 10^{-5}$/°C, bulk modulus for copper is 140×10^9 N-m^2.

or

A rail track made of steel having length 10 m is clamped on a railway line at its two ends. On a summer day due to rise in temperature by 20°C, it is deformed as shown in figure. Find x (displacement of the centre) if $\alpha_{steel} = 1.2 \times 10^{-5}$/°C.



Answers with Solutions

1. Dimensional formula for impulse will be

$$= [F][T]$$
$$= [MLT^{-2}]\,[T] = [MLT^{-1}]$$

Linear momentum will have the same dimension.

2. We know that Range $(R) = \dfrac{u^2 \sin 2\theta}{g}$

For angle of projection θ and $(90° - \theta)$, ranges will be equal.

3. Condition for two vectors a and b to be perpendicular on each other is $a \cdot b = 0$.

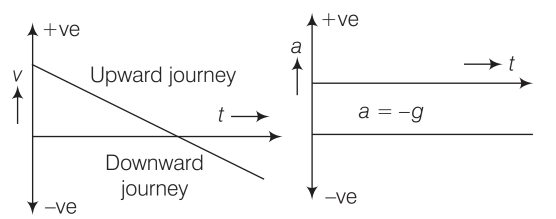
4. For mechanical energy of a system to be conserved, no dissipative force should be present.

5. $\dfrac{(V_{escape})_{Earth}}{(V_{escape})_{Moon}} \approx 5$

6. For adiabatic process, $\Delta Q = 0$ [1]

and for isothermal process, $\Delta U = 0$ [1]

7. Let us assume vertical upward direction as the positive direction.



Velocity-time curve

Acceleration-time curve

[2]

8. Given, $R = H$

$$\Rightarrow \quad \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2 \theta}{2g}$$ [1]