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EXERCISE- 5 (A)

Solution 1:
$X^2 - 10x - 24 = 0$
$\Rightarrow x^2 - 12x + 2x - 24 = 0$
$\Rightarrow x (x - 12) + 2 (x - 12) = 0$
$\Rightarrow (x - 12) (x+2) = 0$
Since $x - 12 = 0$ Or $x + 2 = 0$
Then $x = 12$ Or $x = -2$

Solution 3:
$2x^2 - \frac{1}{2}x = 0$
$\Rightarrow x (2x - \frac{1}{2}) = 0$
Since $x = 0$ Or $2x - \frac{1}{2} = 0$
Then $x = 0$ Or $x = \frac{1}{4}$

Solution 4:
$x(x - 5) = 24$
$\Rightarrow x^2 - 5x - 24 = 0$
$\Rightarrow x^2 - 8x + 3x - 24 = 0$
$\Rightarrow x (x - 8) + 3 (x - 8) = 0$
$\Rightarrow (x - 8) (x + 3) = 0$
Since $x - 8 = 0$ Or $x + 3 = 0$
Then $x = 8$ Or $x = -3$

Solution 5:
$\frac{9}{2}x = 5 + x^2$
$\Rightarrow 9x = 10 + 2x^2$
$\Rightarrow 2x^2 - 9x + 10 = 0$
$\Rightarrow 2x^2 - 5x - 4x + 10 = 0$
$\Rightarrow x(2x - 5) - 2(2x - 5) = 0$
$\Rightarrow (2x - 5) (x - 2) = 0$
Since $2x - 5 = 0$ Or $x - 2 = 0$

Then x = $\frac{5}{2}$ Or x = 2

Solution 6:

$\frac{6}{x} = 1 + x$

$\Rightarrow$ 6 = x + x^2

$\Rightarrow$ x^2 + x − 6 = 0

$\Rightarrow$ x^2 + 3x − 2x − 6 = 0

$\Rightarrow$ x (x+3) − 2 (x + 3) = 0

$\Rightarrow$ (x+3) (x − 2) = 0

Since x + 3 = 0 Or x − 2 = 0

Then x = − 3 Or x = 2

Solution 7:

x = $\frac{3x+1}{4x}$

$\Rightarrow$ 4x^2 = 3x +1

$\Rightarrow$ 4x^2 − 3x − 1 = 0

$\Rightarrow$ 4x^2 − 4x + x − 1 = 0

$\Rightarrow$ 4x(x − 1) + 1 (x − 1) = 0

$\Rightarrow$ (x − 1) (4x + 1) = 0

Since x − 1 = 0   Or 4x + 1 = 0

Then x = 1 Or   x = $-\frac{1}{4}$

Solution 8:

x + $\frac{1}{x}$ = 2.5

$\Rightarrow \frac{x^2+1}{x} = \frac{5}{2}$

$\Rightarrow$ 2x^2 + 2 = 5x

$\Rightarrow$ 2x^2 − 5x + 2 = 0

$\Rightarrow$ 2x^2 − 4x − x + 2 = 0

$\Rightarrow$ 2x (x − 2) − 1 (x − 2) = 0

$\Rightarrow$ (x − 2) (2x − 1) = 0

Since x − 2 = 0   Or 2x − 1 = 0

Then x = 2   Or x = $\frac{1}{2}$

Solution 9:
$(2x - 3)^2 = 49$
Taking square root on both sides
$2x - 3 = \pm 7$
When $2x - 3 = 7 \Rightarrow 2x = 10 \Rightarrow x = 5$
And, when $2x - 3 = -7 \Rightarrow 2x = -4 \Rightarrow x = -2$

Solution 10:
$2(x^2 - 6) = 3 (x - 4)$
$\Rightarrow 2x^2 - 12 = 13x - 12$
$\Rightarrow 2x^2 - 3x = 0$
$\Rightarrow x(2x - 3) = 0$
Since $x = 0$ Or $2x - 3 = 0$
Then $x = 0$ Or $x = \frac{3}{2}$

Solution 11:
$(x + 1) (2x + 8) = (x+7) (x+3)$
$\Rightarrow 2x^2 + 8x + 2x + 8 = x^2 + 3x + 7x + 21$
$\Rightarrow 2x^2 + 10x + 8 = x^2 + 10x + 21$
$\Rightarrow x^2 - 13 = 0$
$\Rightarrow x^2 - \left(\sqrt{13}\right)^2 = 0$
$\Rightarrow (x + \sqrt{13}) (x - \sqrt{13}) = 0$
If $x + \sqrt{13} = 0$ Or $x - \sqrt{13} = 0$
$\Rightarrow x = -\sqrt{13}$ Or $x = \sqrt{13}$

Solution 12:
$x^2 - (a + b) x + ab = 0$
$\Rightarrow X^2 - ax - bx + ab = 0$
$\Rightarrow x(x - a) - b (x - a) = 0$
$\Rightarrow x(x - a) (x - b) = 0$
Since $x - a = 0$ Or $x - b = 0$
Then $x = a$ Or $x = b$

Solution 13:
$(x + 3)^2 - 4(x + 3) - 5 = 0$

Let x + 3 = y
Then $y^2 - 4y - 5 = 0$
$\Longrightarrow y^2 - 5y + y - 5 = 0$
$\Longrightarrow y (y - 5) + 1 (y - 5) = 0$
$\Longrightarrow (y - 5) (y + 1) = 0$
If y − 5 = 0   Or   y + 1 = 0
Then y = 5   Or   y = − 1
$\Longrightarrow$ x + 3 = 5   or   x + 3 = -1
$\Longrightarrow$ x = 2   or   x = − 4

Solution 14:
$4(2x - 3)^2 - (2x - 3) - 14 = 0$
Let 2x − 3 = y
Then $4y^2 - y - 14 = 0$
$\Longrightarrow 4y^2 - 8y + 7y - 14 = 0$
$\Longrightarrow 4y(y - 2) + 7 (y - 2) = 0$
$\Longrightarrow (y - 2) (4y + 7) = 0$
If   y − 2 = 0   Or   4y + 7 = 0
$\Longrightarrow y = 2$ or $y = \frac{-7}{4}$
$\Longrightarrow$ 2x − 3 = 2   Or $2x - 3 = \frac{-7}{4}$
$\Longrightarrow$ 2x = 5   Or $2x = \frac{5}{4}$
$\Longrightarrow x = \frac{5}{2}$ Or $x = \frac{5}{8}$

Solution 15:
$\frac{3x - 2}{2x - 3} = \frac{3x - 8}{x + 4}$
$\Longrightarrow (3x - 2) (x + 4) = (2x - 3) (3x - 8)$
$\Longrightarrow 3x^2 + 12x - 2x - 8 = 6x^2 - 16x - 9x + 24$
$\Longrightarrow 3x^2 + 10x - 8 = 6x^2 - 25x + 24$
$\Longrightarrow 3x^2 - 35x + 32 = 0$
$\Longrightarrow 3x^2 - 32x - 3x + 32 = 0$
$\Longrightarrow x(3x - 32) - 1(3x - 32) = 0$
$\Longrightarrow (x - 1) (3x - 32) = 0$
If x − 1 = 0   Or   3x − 32 = 0
$\Longrightarrow$ x = 1   Or $x = \frac{32}{3} = 10\ \frac{2}{3}$

Solution 16:

$$\frac{100}{x} - \frac{100}{x+5} = 1$$

$$\Rightarrow \frac{100(x+5) - 100x}{x(x+5)} = 1$$

$$\Rightarrow \frac{500}{x^2 + 5x} = 1$$

$\Rightarrow x^2 + 5x = 500$

$\Rightarrow x^2 + 5x - 500 = 0$

$\Rightarrow x^2 + 25x - 20(x + 25) = 0$

$\Rightarrow x(x + 25)(x - 20) = 0$

If $x + 25 = 0$ Or $x - 20 = 0$

Then $x = -25$ Or $x = 20$

Solution 17:

$$\frac{x-3}{x+3} + \frac{x+3}{x-3} = 2\frac{1}{2}$$

$$\Rightarrow \frac{(x-3)^2 + (x+3)^2}{(x+3)(x-3)} = \frac{5}{2}$$

$$\Rightarrow \frac{x^2 - 6x + 9 + x^2 + 6x + 9}{(x^2 - 9)} = \frac{5}{2}$$

$\Rightarrow 2(2x^2 + 18) = 5(x^2 - 9)$

$\Rightarrow 4x^2 + 36 = 5x^2 - 45$

$\Rightarrow x^2 - 81 = 0$

$\Rightarrow x^2 - 9^2 = 0$

$\Rightarrow (x + 9)(x - 9) = 0$

If $x + 9 = 0$ Or $x - 9 = 0$

Then $x = -9$ Or $x = 9$

Solution 18:

$$\frac{4}{x+2} - \frac{1}{x+3} = \frac{4}{2x+1}$$

$$\Rightarrow \frac{4(x+3) - 1(x+2)}{(x+2)(x+3)} = \frac{4}{2x+1}$$

$$\Rightarrow \frac{4x + 12 - x - 2}{x^2 + 2x + 3x + 6} = \frac{4}{2x+1}$$

$$\Rightarrow \frac{3x + 10}{x^2 + 5x + 6} = \frac{4}{2x+1}$$

$\Rightarrow (3x + 10)(2x + 1) = 4(x^2 + 5x + 6)$

$\Rightarrow 6x^2 + 3x + 20x + 10 = 4x^2 + 20x + 24$

$\Rightarrow 2x^2 + 3x - 14 = 0$

$\Rightarrow 2x^2 + 7x - 4x - 14 = 0$

$\Rightarrow x\,(2x + 7) - 2(2x + 7) = 0$
$\Rightarrow (2x + 7)\,(\,x - 2) = 0$
If $2x + 7 = 0$ Or $x - 2 = 0$
Then $x = \frac{-7}{2}$ Or $x = 2$

Solution 19:

$\dfrac{5}{x - 2} - \dfrac{3}{x + 6} = \dfrac{4}{x}$

$\Rightarrow \dfrac{5(x + 6) - 3\,(x - 2)}{(x - 2)(x + 6)} = \dfrac{4}{x}$

$\Rightarrow \dfrac{5x + 30 - 3x + 6}{x^2 + 6x - 2x - 12} = \dfrac{4}{x}$

$\Rightarrow \dfrac{2x + 36}{x^2 + 4x - 12} = \dfrac{4}{x}$

$\Rightarrow 4x^2 + 16x - 48 = 2x^2 + 36x$
$\Rightarrow 2x^2 - 20x - 48 = 0$
$\Rightarrow x^2 - 10x - 24 = 0$
$\Rightarrow x^2 - 12x + 2x - 24 = 0$
$\Rightarrow x\,(x - 12) + 2\,(x - 12) = 0$
$\Rightarrow (x - 12)\,(x + 2) = 0$
If $x - 12 = 0$ Or $x + 2 = 0$
Then $x = 12$ Or $x = -2$

Solution 20:

$\left(1 + \dfrac{1}{x+1}\right)\left(1 - \dfrac{1}{x-1}\right) = \dfrac{7}{8}$

$\Rightarrow \left(\dfrac{x + 1 + 1}{x + 1}\right)\left(\dfrac{x - 1 - 1}{x - 1}\right) = \dfrac{7}{8}$

$\Rightarrow \left(\dfrac{x + 2}{x + 1}\right)\left(\dfrac{x - 2}{x - 1}\right) = \dfrac{7}{8}$

$\Rightarrow \dfrac{x^2 - 4}{x^2 - 1} = \dfrac{7}{8}$

$\Rightarrow 8x^2 - 32 = 7x^2 - 7$
$\Rightarrow x^2 = 25$
$\Rightarrow x = \pm\, 5$

Solution 21:

(i) Since solution set is {3,5}
      $\Rightarrow x = 3$ Or $x = 5$
      $\Rightarrow x - 3 = 0$ Or $x - 5 = 0$

$\Rightarrow$ (x – 3) (x – 5) = 0
$\Rightarrow$ x^2 – 5x – 3x + 15 = 0
$\Rightarrow$ x^2 – 8x + 15 = 0   Which is the required equation.

(ii) Since solution set is {−2, 3}

$\Rightarrow$ x = – 2   Or   x = 3
$\Rightarrow$ x + 2 = 0   Or   x – 3 = 0
$\Rightarrow$ (x + 2) (x – 3) = 0
$\Rightarrow$ x^2 – 3x + 2x − 6 = 0
$\Rightarrow$ x^2 – x − 6 = 0   Which is the required equation.

(iii) Since solution set is {5,−4,}

$\Rightarrow$ x = 5   Or   x = −4
$\Rightarrow$ x − 5 = 0   Or x + 4 = 0
$\Rightarrow$ (x − 5) (x + 4) = 0
$\Rightarrow$ x^2 – 5x + 4x − 20 = 0
$\Rightarrow$ x^2 – x − 20 = 0   Which is the required equation.

(iv) Since solution set is {−3, $\frac{-2}{5}$}

$\Rightarrow$ x = −3   Or   x = $\frac{-2}{5}$
$\Rightarrow$ x + 3 = 0   Or 5x + 2 = 0
$\Rightarrow$ (x + 3) (5x + 2) = 0
$\Rightarrow$ $5x^2$ + 2x + 15x + 6 = 0
$\Rightarrow$ $5x^2$ + 17x + 6 = 0   Which is the required equation.

Solution 22:
If a + 1 = 0, then a = −1
Put this value in the given equation x^2 + ax − 6 = 0
x^2 – x – 6 = 0
$\Rightarrow$ x^2 – 3x + 2x − 6 = 0
$\Rightarrow$ x(x – 3) + 2 (x – 3) = 0
$\Rightarrow$ (x – 3) (x + 2) = 0
If x – 3 = 0   Or x + 2 = 0
Then x = 3   Or x = −2.

Solution 23:
If a + 7 =0, then a = -7
and b + 10 =0, then b = − 10
Put these values of a and b in the given equation

$12x^2 = (-7)x - (-10)$
$\Rightarrow 12x^2 + 7x - 10 = 0$
$\Rightarrow 12x^2 + 15x - 8x - 10 = 0$
$\Rightarrow 3x(4x + 5) - 2(4x + 5) = 0$
$\Rightarrow (4x + 5)(3x - 2) = 0$
If $4x + 5 = 0$ Or $3x - 2 = 0$
Then $x = \frac{-5}{4}$ Or $x = \frac{2}{3}$

Solution 24:
$4(2x+3)^2 - (2x + 3) - 14 = 0$
Put $2x + 3 = y$
$4y^2 - y - 14 = 0$
$\Rightarrow 4y^2 - 8y + 7y - 14 = 0$
$\Rightarrow 4y(y - 2) + 7(y - 2) = 0$
$\Rightarrow (y - 2)(4y + 7) = 0$
If $y - 2 = 0$ Or $4y + 7 = 0$
Then $2x + 3 - 2 = 0$ Or $4(2x + 3) + 7 = 0$
$\Rightarrow 2x = -1$ Or $8x = -19$
$\Rightarrow x = \frac{-1}{2}$ Or $x = \frac{-19}{8}$

Solution 25:
Consider the equation, $6x^2 - x - 2 = 0$
Put $x = \frac{2}{3}$ in L.H.S
L.H.S $= 6\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right) - 2$
$= \frac{24}{9} - \frac{2}{3} - 2$
$= \frac{24 - 6 - 18}{9} = 0 = R.H.S.$
Since L.H.S = R.H.S, then $x = \frac{2}{3}$ is a solution of the given equation.

Solution 26:
$x^2 - 3x + 2 = 0$
Put $x = -1$ in L.H.S.
L.H.S. $= (-1)^2 - 3(-1) + 2$
$= 1 + 3 + 2 = 6 \neq$ R.H.S.
Then $x = -1$ is not the solution of the given equation.

Solution 27:
$7x^2 + mx - 3 = 0$
Given $x = \frac{2}{3}$ is the solution of the given equation.
Put given value of x in the given equation
$7 \left(\frac{2}{3}\right)^2 + m \left(\frac{2}{3}\right) - 3 = 0$
$\Rightarrow \frac{28}{9} + \frac{2m}{3} - 3 = 0$
$\Rightarrow 28 + 6m - 27 = 0$
$\Rightarrow 6m = -1$
$\Rightarrow m = \frac{-1}{6}$

Solution 28:
$mx^2 + 7x + n = 0$
Put $x = -3$ in given equation
$m(-3)^2 + 7(-3) + n = 0$
$\Rightarrow 9m - 21 + n = 0$
$9m + n = 21$ ------------ (1)
Put $x = \frac{2}{3}$ in given equation
$m \left(\frac{2}{3}\right)^2 + 7 \left(\frac{2}{3}\right) + n = 0$
$\Rightarrow \frac{4m}{9} + \frac{14}{3} + n = 0$
$\Rightarrow 4m + 9n = -42$ ---------(2)
Solving these equations we get
$m = 3$ and $n = -6$

Solution 29:
$x^2 - (m + 1)x + 6 = 0$
Put $x = 3$ in the given equation
$(3)^2 - (m + 1)(3) + 6 = 0$
$\Rightarrow 9 - 3m - 3 + 6 = 0$
$\Rightarrow -3m = -12$
$\Rightarrow m = 4$
Put this value of m in the given equation, we get
$x^2 - 5x + 6 = 0$
$\Rightarrow x^2 - 3x - 2x + 6 = 0$
$\Rightarrow x(x - 3) - 2(x - 3) = 0$
$\Rightarrow (x - 3)(x - 2) = 0$
If $x - 3 = 0$ Or $x - 2 = 0$

Then x = 3   Or   x = 2
∴ 2 is the other root of the given equation

EXERCISE 7 (B)

Solution 1:
(i) $x^2 - 6x = 27$
⟹ $x^2 - 6x - 27 = 0$
Here a = 1, b = – 6 and c = – 27
Then x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

= $\frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-27)}}{2(1)}$

= $\frac{6 \pm 12}{2} = \frac{6+12}{2}$ and $\frac{6-12}{2}$ = 9 and – 3

(ii) $x^2 - 10x + 21 = 0$
Here a = 1, b = – 10 and c = 21
Then x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

= $\frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(21)}}{2(1)}$

= $\frac{10 \pm 4}{2} = \frac{10+4}{2}$ and $\frac{10-4}{2}$ = 7 and 3

(iii) $x^2 + 6x - 10 = 0$
Here a = 1, b = 6 and c = – 10

Then x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

= $\frac{-(6) \pm \sqrt{(-6)^2 - 4(1)(-10)}}{2(1)}$

= $\frac{-6 \pm \sqrt{76}}{2} = \frac{-6 \pm 2\sqrt{19}}{2}$ and $\frac{-6 - 2\sqrt{19}}{2}$ = $-3 + \sqrt{19}$ and $-3 - \sqrt{19}$

(iv) $x^2 + 2x - 6 = 0$
Here a = 1, b = 2 and c = – 6

Then x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

= $\frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-6)}}{2(1)}$

= $\frac{-2 \pm \sqrt{28}}{2} = \frac{-2 \pm 2\sqrt{7}}{2}$ = $-1 \pm \sqrt{7}$

(v) $3x^2 + 2x - 1 = 0$
Here a = 3, b = 2 and c = −1
Then x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-(2) \pm \sqrt{(2)^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{-2 \pm 4}{6} = \frac{-2 \pm 4}{6} \text{ and } \frac{-2-4}{6} = \frac{1}{3} \text{ and } -1$$

(vi) $2x^2 + 7x + 5 = 0$

Here a = 2, b = 7 and c = 5

Then x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-(7) \pm \sqrt{(7)^2 - 4(2)(5)}}{2(2)}$$

$$= \frac{-7 \pm 3}{4} = \frac{-7+3}{4} \text{ and } \frac{-7-3}{4} = -1 \text{ and } -\frac{5}{2}$$

(vii) $\frac{2}{3}x = -\frac{1}{6}x^2 - \frac{1}{3}$

$\Rightarrow 4x = -x^2 - 2$

$\Rightarrow x^2 + 4x + 2 = 0$

Here a = 1, b = 4 and c = 2

Then x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{8}}{2} = \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$$

(viii) $\frac{1}{15}x^2 + \frac{5}{3} = \frac{2}{3}x$

$\Rightarrow x^2 + 25 = 10x$

$\Rightarrow x^2 - 10x + 25 = 0$

Here a = 1, b = −10 and c = 25

Then x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(25)}}{2(1)}$$

$$= \frac{10 \pm \sqrt{0}}{2} = 5$$

(ix) $x^2 - 6 = 2\sqrt{2}x$

$\Rightarrow x^2 - 2\sqrt{2}x - 6 = 0$

Here a = 1, b = −2$\sqrt{2}$ and c = −6

Then x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-(-2\sqrt{2}) \pm \sqrt{\left(-2\sqrt{2}\right)^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{2\sqrt{2} \pm \sqrt{32}}{2} = \frac{2\sqrt{2} \pm 4\sqrt{2}}{2} = \frac{2\sqrt{2} + 4\sqrt{2}}{2} \text{ and } \frac{2\sqrt{2} - 4\sqrt{2}}{2}$$

$= \frac{6\sqrt{2}}{2}$ and $\frac{-2\sqrt{2}}{2} = 3\sqrt{2}$ and $-\sqrt{2}$

(x) $\frac{4}{x} - 3 = \frac{5}{2x+3}$

$\Rightarrow \frac{4-3x}{x} = \frac{5}{2x+3}$

$\Rightarrow (4 - 3x)(2x + 3) = 5x$

$\Rightarrow 8x + 12 - 6x^2 - 9x = 5x$

$\Rightarrow 6x^2 + 6x - 12 = 0$

$\Rightarrow x^2 + x - 2 = 0$

Here a = 1, b = 1 and c = − 2

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-2)}}{2(1)}$

$= \frac{1 \pm \sqrt{9}}{2} = \frac{-1 \pm 3}{2} = \frac{-1+3}{2}$ and $\frac{-1-3}{2} = 1$ and $- 2$

(xi) $\frac{2x+3}{x+3} = \frac{x+4}{x+2}$

$\Rightarrow (2x + 3)(x + 2) = (x + 3)(x + 4)$

$\Rightarrow 2x^2 + 4x + 3x + 6 = x^2 + 4x + 3x + 12$

$\Rightarrow x^2 - 6 = 0$

Here a = 1, b = 0 and c = − 6

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(0) \pm \sqrt{(0)^2 - 4(1)(-6)}}{2(1)}$

$= \frac{0 \pm \sqrt{24}}{2} = \frac{0 \pm 2\sqrt{6}}{2} = -\sqrt{6}$ and $\sqrt{6}$

(xii) $\sqrt{6}\,x^2 - 4x - 2\sqrt{6} = 0$

Here a = $\sqrt{6}$, b = − 4 and c = − 2 $\sqrt{6}$

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(\sqrt{6})(-2\sqrt{6})}}{2(\sqrt{6})}$

$= \frac{4 \pm \sqrt{64}}{2\sqrt{6}} = \frac{4 \pm 8}{2\sqrt{6}} = \frac{4+8}{2\sqrt{6}}$ and $\frac{4-8}{2\sqrt{6}}$

$= \frac{6}{\sqrt{6}}$ and $\frac{-2}{\sqrt{6}} = \sqrt{6}$ and $\frac{-\sqrt{6}}{3}$

(xiii) $\frac{2x}{x-4} + \frac{2x-5}{x-3} = 8\frac{1}{3}$

$\Rightarrow \frac{2x(x-3) + (x-4)(2x-5)}{(x-4)(x-3)} = \frac{25}{3}$

$\Longrightarrow \dfrac{2x^2 - 6x + 2x^2 - 5x - 8x + 20}{x^2 - 3x - 4x + 12} = \dfrac{25}{3}$

$\Longrightarrow \dfrac{4x^2 - 19x + 20}{x^2 - 7x + 12} = \dfrac{25}{3}$

$\Longrightarrow 25x^2 - 175x + 300 = 12x^2 - 57x + 60$

$\Longrightarrow 13x^2 - 118x + 240 = 0$

Here a = 13, b = − 118 and c = 240

Then x = $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

= $\dfrac{-(-118) \pm \sqrt{(-118))^2 - 4\left(\sqrt{13}\right)(240)}}{2(13)}$

= $\dfrac{118 \pm \sqrt{1444}}{26} = \dfrac{118 \pm 38}{26}$

= $\dfrac{118 + 38}{26}$ and $\dfrac{118 - 38}{26}$ and $\dfrac{40}{13}$

(xiv) $\dfrac{x-1}{x-2} + \dfrac{x-3}{x-4} = 3\dfrac{1}{3}$

$\Longrightarrow \dfrac{(x-1)(x-4) + (x-2)(x-3)}{(x-2)(x-4)} = \dfrac{10}{3}$

$\Longrightarrow \dfrac{x^2 - 4x - x + 4 + x^2 - 3x - 2x + 6}{x^2 - 4x - 2x + 8} = \dfrac{10}{3}$

$\Longrightarrow \dfrac{2x^2 - 10x + 10}{x^2 - 6x + 8} = \dfrac{10}{3}$

$\Longrightarrow 10x^2 - 60x + 80 = 6x^2 - 30x + 30$

$\Longrightarrow 4x^2 - 30x + 50 = 0$

$\Longrightarrow 2x^2 - 15x + 25 = 0$

Here a = 2, b = − 15 and c = 25

Then x = $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

= $\dfrac{-(-15) \pm \sqrt{(-15)^2 - 4(2)(25)}}{2(2)}$

= $\dfrac{15 \pm \sqrt{25}}{4} = \dfrac{15 \pm 5}{4}$

= $\dfrac{15 + 5}{4}$ and $\dfrac{15 - 5}{4} = 5 \; and \; \dfrac{5}{2}$

Solution 2:

(i) $7x^2 − 9x + 2 = 0$

a = 7, b = − 9 and c = 2

∴ Discriminant = $b^2 – 4ac$

= $(−9)^2 – 4(7)(2)$

= 81 – 56 = 25

Since D >0, then equation has two real and unequal roots.

(ii) $6x^2 - 13x + 4 = 0$
      a = 6, b = − 13 and c = 4
      ∴ Discriminant = $b^2 - 4ac$
      = $(-13)^2 - 4(6)(4)$
      = 169 – 96 = 73
      Since 73 is not a perfect square, roots are irrational
      Since D >0, then equation has two real and unequal roots.

(iii) $25x^2 - 10x + 1 = 0$
      a = 25, b = − 10 and c =1
      ∴ Discriminant = $b^2 - 4ac$
        = $(-10)^2 - 4(25)(1)$
        = 100 – 100 = 0
      Since D = 0, then equation has two real and unequal roots.

(iv) $x^2 + 2\sqrt{3}x - 9 = 0$
      a = 1, b = $2\sqrt{3}$ and c = − 9
      ∴ Discriminant = $b^2 - 4ac$
        = $(2\sqrt{3})^2 - 4(1)(-9)$
        = 12 + 36 = 48
       Since 48 is not a perfect square, roots are irrational
       Since D >0, then equation has two real and unequal roots.

(v) $x^2 - ax - b^2 = 0$
      a = 1, b = − a and c = −b^2
      ∴ Discriminant = $b^2 - 4ac$
       = $(-a)^2 - 4(1)(-b)^2$
      = $a^2 + 4b^2$ = a positive value
      Since $a^2 + 4b^2$ is not a perfect square, roots are irrational
      Since D >0, then equation has two real and unequal roots.

(vi) $2x^2 + 8x + 9 = 0$
      a = 2, b = 8 and c = 9
      ∴ Discriminant = $b^2 - 4ac$
      = $(8)^2 - 4(2)(9)$
      = 64 – 72 = − 18 = a negative value
    Since D >0, then equation has no real roots.

Solution 3:
$4x^2 - (p - 2)x + 1 = 0$
Here a = 4, b = − (p − 2) and c = 1

Given: equation has equal roots
Then   d= 0
$\implies b^2 - 4ac = 0$
$\implies [-(p - 2)]^2 - 4(4)(1) = 0$
$\implies p^2 + 4 - 4p - 16 = 0$
$\implies p^2 - 4p - 12 = 0$
$\implies p^2 - 6p + 2p - 12 = 0$
$\implies p(p - 6)(p - 6) = 0$
$\implies (p - 6)(p + 2) = 0$
Then $p - 6 = 0$ Or $p + 2 = 0$
$\implies p = 6$ Or $p = - 2$.

Solution 4:
$3x^2 - 12x + (n - 5) = 0$
Here $a = 3$, $b = - 12$ and $c = n - 5$
Given: equation has equal roots
Then $D = 0$
$\implies b^2 - 4ac = 0$
$\implies [- 12]^2 - 4(3)(n - 5) = 0$
$\implies 144 - 12n + 60 = 0$
$\implies - 12n = - 204$
$\implies n = \frac{- 204}{- 12} = 17$

Solution 5:
$(m - 2)x^2 - (5 + m)x + 16 = 0$
Here $a = m - 2$, $b = - (5 + m)$ and $c = 16$
Given : equation has equal roots
Then $D = 0$
$\implies b^2 - 4ac = 0$
$\implies [- (5 + m)]^2 - 4(m - 2)(16) = 0$
$\implies 25 + m^2 + 10m - 64m + 128 = 0$
$\implies m^2 - 54m + 153 = 0$
$\implies m^2 - 51m - 3m + 153 = 0$
$\implies m(m - 51) - 3(m - 51) = 0$
$\implies (m - 51)(m - 3) = 0$
Then $m - 51 = 0$ Or $m - 3 = 0$
$\implies m = 51$ Or $m = 3$

EXERCISE 7 (C)

Solution 1:
(i) $x^2 - 8x + 5 = 0$
Here a = 1, b = − 8 and c = 5

$\therefore x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(5)}}{2(1)}$

$= \dfrac{8 \pm \sqrt{44}}{2} = \dfrac{8 \pm 2\sqrt{11}}{4} = 4 \pm \sqrt{11} = 4 \pm 3.3 = 7.3$ and 0.7

(ii) $5x^2 + 10x - 3 = 0$
Here a = 5, b = 10 and c = −3

$\therefore x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-10) \pm \sqrt{(10)^2 - 4(5)(-3)}}{2(5)}$

$= \dfrac{-10 \pm \sqrt{160}}{10} = \dfrac{-10 \pm 12.6}{10}$

$= \dfrac{-10 + 12.6}{10}$ and $\dfrac{-10 - 12.6}{10} = 0.26$ and − 2.26 = 0.3 and − 2.3

Solution 2:
(i) $2x^2 - 10x + 5 = 0$
Here a = 2, b = −10 and c = 5

$\therefore x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-10) \pm \sqrt{(10)^2 - 4(2)(5)}}{2(2)}$

$= \dfrac{10 \pm \sqrt{60}}{4} = \dfrac{10 \pm 7.75}{4}$

$= \dfrac{10 + 7.75}{4}$ and $\dfrac{10 - 7.75}{4} = 4.44$ and 0.56

(ii) $4x + \dfrac{6}{x} + 13 = 0$
$\implies 4x^2 + 6 + 13x = 0$
$\implies 4x^2 + 13x + 6 = 0$
$\implies$ Here a = 4, b = 13 and c = 6

$\therefore x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(13) \pm \sqrt{(13)^2 - 4(4)(6)}}{2(4)}$

$= \dfrac{-13 \pm \sqrt{73}}{8} = \dfrac{-13 \pm 8.54}{8}$

$= \frac{-13 + 8.54}{8}$ and $\frac{-13 - 8.54}{8} = -0.56$ and -2.69

(iii) $x^2 - 3x - 9 = 0$

$\Rightarrow$ Here a = 1, b = −3 and c = −9

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-9)}}{2(1)}$

$= \frac{3 \pm \sqrt{45}}{2} = \frac{3 \pm 6.70}{2}$

$= \frac{3 + 6.70}{2}$ and $\frac{3 - 6.70}{2} = 4.85$ and -1.85

Solution 3:

(i) $3x^2 - 12x - 1 = 0$

$\Rightarrow$ Here a = 3, b = −12 and c = −1

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(-1)}}{2(3)}$

$= \frac{12 \pm \sqrt{156}}{6} = \frac{12 \pm 12.489}{6}$

$= \frac{12 + 12.489}{6}$ and $\frac{12 - 12.489}{6} = 4.082$ and -0.082

(ii) $x^2 - 16x + 6 = 0$

$\Rightarrow$ Here a = 1, b = −16 and c = −1

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(6)}}{2(1)}$

$= \frac{16 \pm \sqrt{232}}{2} = \frac{16 \pm 15.231}{2}$

$= \frac{16 + 15.231}{2}$ and $\frac{16 - 15.231}{2}$ = 15.616 and 0.384

(iii) $2x^2 + 11x + 4 = 0$

$\Rightarrow$ Here a = 2, b = 11 and c = 4

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{(-11) \pm \sqrt{(11)^2 - 4(2)(4)}}{2(2)}$

$= \dfrac{-11 \pm \sqrt{89}}{4} = \dfrac{-11 \pm 9.433}{4}$

$= \dfrac{-11 + 9.433}{4}$ and $\dfrac{-11 - 9.433}{4} = -0.392$ and -5.108

Solution 4:
(i) $x^4 - 2x^2 - 3 = 0$

$\Rightarrow x^4 - 3x^2 + x^2 - 3 = 0$

$\Rightarrow x^2(x^2 - 3) + 1(x^2 - 3) = 0$

$\Rightarrow (x^2 - 3)(x^2 + 3) = 0$

If $x^2 - 3 = 0$ Or $x^2 + 1 = 0$

$\Rightarrow x^2 = 3$ Or $x^2 = -1$ (reject)

$\Rightarrow x = \pm \sqrt{3}$

(ii) $x^4 - 10x^2 + 9 = 0$

$\Rightarrow x^4 - 9x^2 - x^2 + 9 = 0$

$\Rightarrow x^2(x^2 - 9) - 1(x^2 - 9) = 0$

$\Rightarrow (x^2 - 9)(x^2 - 1) = 0$

If $x^2 - 9 = 0$ Or $x^2 - 1 = 0$

$\Rightarrow x^2 = 9$ Or $x^2 = 1$

$\Rightarrow x = \pm 3$ Or $x = \pm 1$

Solution 5:
(i) $(x^2 - x)^2 + 5(x^2 - x) + 4 = 0$

Let $x^2 - x = y$

Then $y^2 + 5y + 4 = 0$

$\Rightarrow y^2 + 4y + y + 4 = 0$

$\Rightarrow y(y + 4) + 1(y + 4) = 0$

$\Rightarrow (y + 4)(y + 1) = 0$

If $y + 4 = 0$ Or $y + 1 = 0$

$\Rightarrow x^2 - x + 4 = 0$ Or $x^2 - x + 1 = 0$

$\Rightarrow x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(4)}}{2(1)}$ Or $\dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$

$\Rightarrow 1 = \dfrac{1 \pm \sqrt{-15}}{2}$ (reject)   Or $x = \dfrac{1 \pm \sqrt{-3}}{2}$ (reject)

∴ Given equation has no real solution

(ii) $(x^2 - 3x)^2 - 16(x^2 - 3x) - 36 = 0$

Let $x^2 - 3x = y$

Then $y^2 - 16y - 36 = 0$

$\Rightarrow y^2 - 18y + 2y - 36 = 0$
$\Rightarrow y(y - 18) + 2(y - 18) = 0$
$\Rightarrow (y - 18)(y + 2) = 0$
If $y - 18 = 0$ Or $y + 2 = 0$
$\Rightarrow x^2 - 3x - 18 = 0$ Or $x^2 - 3x + 2 = 0$
$\Rightarrow x^2 - 6x + 3x - 18 = 0$ Or $x^2 - 2x - x + 2 = 0$
$\Rightarrow x(x - 6) + 3(x - 6) = 0$ Or $x(x - 2) - 1(x - 2) = 0$
$\Rightarrow (x - 6)(x + 3) = 0$ Or $(x - 2)(x - 1) = 0$
If $x - 6 = 0$ Or $x + 3 = 0$ Or $x - 2 = 0$ Or $x - 1 = 0$
Then $x = 6$ Or $x = -3$ Or $x = 2$ Or $x = 1$

Solution 6:

(i) $\sqrt{\dfrac{x}{x-3}} + \sqrt{\dfrac{x-3}{x}} = \dfrac{5}{2}$

Let $\sqrt{\dfrac{x}{x-3}} = y$

Then $y + \dfrac{1}{y} = \dfrac{5}{2}$

$\Rightarrow \dfrac{y^2 + 1}{y} = \dfrac{5}{2}$

$\Rightarrow 2y^2 + 2 = 5y$

$\Rightarrow 2y^2 - 5y + 2 = 0$

$\Rightarrow 2y^2 - 4y - y + 2 = 0$

$\Rightarrow 2y(y - 2) - 1(y - 2) = 0$

$\Rightarrow (y - 2)(2y - 1) = 0$

If $y - 2 = 0$ Or $2y - 1 = 0$

Then $y = 2$ Or $y = \dfrac{1}{2}$

$\Rightarrow \sqrt{\dfrac{x}{x-3}} = 2$ Or $\sqrt{\dfrac{x}{x-3}} = \dfrac{1}{2}$

$\Rightarrow \dfrac{x}{x-3} = 4$ Or $\dfrac{x}{x-3} = \dfrac{1}{4}$

$\Rightarrow x = 4$ Or $x = -1$

(ii) $\left(\dfrac{2x-3}{x-1}\right) - 4\left(\dfrac{x-1}{2x-3}\right) = 3$

Let $\dfrac{2x-3}{x-1} = y$

Then $y - \dfrac{4}{y} = 3$

$\Rightarrow \dfrac{y^2 - 4}{y} = 3$

$\Rightarrow y^2 - 4 = 3y$

$\Rightarrow y^2 - 3y - 4 = 0$

$\Rightarrow y^2 - 4y + y - 4 = 0$

$\Rightarrow y(y-4) + 1(y-4) = 0$

$\Rightarrow (y-4)(y+1) = 0$

If $y-4 = 0$ Or $y+1 = 0$

Then $y = 4$ Or $y = -1$

$\Rightarrow \frac{2x-3}{x-1} = 4$ Or $\frac{2x-3}{x-1} = -1$

$\Rightarrow 4x - 4 = 2x - 3$ Or $2x - 3 = -x + 1$

$\Rightarrow 2x = 1$ Or $3x = 4$

$\Rightarrow x = \frac{1}{2}$ Or $x = \frac{4}{3} = 1\frac{1}{3}$

(iii) $\left(\frac{3x+1}{x+1}\right) + \left(\frac{x+1}{3x+1}\right) = \frac{5}{2}$

Let $\frac{3x+1}{x+1} = y$

Then $y + \frac{1}{y} = \frac{5}{2}$

$\Rightarrow \frac{y^2+1}{y} = \frac{5}{2}$

$\Rightarrow 2y^2 + 2 = 5y$

$\Rightarrow 2y^2 - 5y + 2 = 0$

$\Rightarrow 2y^2 - 4y - y + 2 = 0$

$\Rightarrow 2y(y-2) - 1(y-2) = 0$

$\Rightarrow (y-2)(2y-1) = 0$

If $y - 2 = 0$ Or $2y - 1 = 0$

Then $y = 2$ Or $y = \frac{1}{2}$

$\Rightarrow \frac{3x+1}{x+1} = 2$ Or $\frac{3x+3}{x+1} = \frac{1}{2}$

$\Rightarrow 3x + 1 = 2x + 2$ Or $6x + 2 = x + 1$

$\Rightarrow x = 1$ Or $5x = -1$

$\Rightarrow x = 1$ Or $x = \frac{-1}{5}$

(iv) $3\sqrt{\frac{x}{5}} + 3\sqrt{\frac{5}{x}} = 10$

Let $\sqrt{\frac{x}{5}} = y$

Then $3y + \frac{3}{y} = 10$

$\Rightarrow \frac{3y^2+3}{y} = 10$

$\Rightarrow 3y^2 + 3 = 10y$

$\Rightarrow 3y^2 - 10y + 3 = 0$

$\Rightarrow 3y^2 - 9y - y + 3 = 0$

$\Rightarrow 3y(y-3) - 1(y-3) = 0$

$\Rightarrow (y-3)(3y-1) = 0$

If $y - 3 = 0$ Or $3y - 1 = 0$

Then y = 3   Or   y = $\frac{1}{3}$

$\implies \sqrt{\frac{x}{5}} = 3$ or $\sqrt{\frac{x}{5}} = \frac{1}{3}$

$\implies \frac{x}{5} = 9$ Or $\frac{x}{5} = \frac{1}{9}$

$\implies$ x = 45   Or   9x = 5

$\implies$ x = 45   Or   x = $\frac{5}{9}$

Solution 7:

$2x - \frac{1}{x} = 7$

$\implies \frac{2x^2 - 1}{x} = 7$

$\implies 2x^2 - 1 = 7x$

$\implies 2x^2 - 7x - 1 = 0$

Here a = 2, b = − 7 and c = − 1

$\therefore$ x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

= $\frac{-(-7) \pm \sqrt{(-7)^2 - 4\,(2)\,(-1)}}{2(2)}$

= $\frac{7 \pm \sqrt{57}}{4}$ = $\frac{7 \pm 7.55}{4}$

= $\frac{7 + 7.55}{4}$ and $\frac{7 - 7.55}{4}$ = 3.64 and − 0.14

Solution 8:

Consider the given equation:

$5x^2 - 3x - 4 = 0$

Using quadratic formula, we have

$\therefore$ x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\implies$ x = $\frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 5 \times (-4)}}{2 \times 5}$

$\implies$ x = $\frac{3 \pm \sqrt{9+80}}{2 \times 5}$

$\implies$ x = $\frac{3 \pm \sqrt{89}}{10}$

$\implies$ x = $\frac{3 \pm 9.434}{10}$

$\implies$ x = 1.243 Or x = − 0.643

EXERCISE 7 (D)

Solution 1:

$\frac{2X}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$; $x \neq 3$, $x \neq -\frac{3}{2}$

$\implies \frac{2x(2x+3) + 1(x-3) + 3x + 9}{(x-3)(2x+3)} = 0$

$\implies 4x^2 + 6x + x - 3 + 3x + 9 = 0$

$\implies 4x^2 + 10x + 6 = 0$

$\implies 4x^2 + 4x + 6x + 6 = 0$

$\implies 4x(x+1) + 6(x+1) = 0$

$\implies (x+1)(4x+6) = 0$

$\implies x + 1 = 0$ Or $4x + 6 = 0$

$\implies x = -1$ Or $x = \frac{-6}{4} = \frac{-3}{2}$ (reject)

Solution 2:

$(2x + 3)^2 = 81$

$\implies 2x + 3 = \pm 9$

$\implies 2x + 3 = 9$ and $2x + 3 = -9$

$\implies 2x = 6$ and $2x = -12$

$\implies x = 3$ and $x = -6$

Solution 3:

$a^2x^2 - b^2 = 0$

$\implies (ax)^2 - b^2 = 0$

$\implies (ax + b)(ax - b) = 0$

If $ax + b = 0$ and $ax - b = 0$

Then $x = \frac{-b}{a}$ and $x = \frac{b}{a}$

Solution 4:

$X^2 - \frac{11}{4}x + \frac{15}{8} = 0$

$\implies \frac{8x^2 - 22x + 15}{8} = 0$

$\implies 8x^2 - 22x + 15 = 0$

$\implies 8x^2 - 12x - 10x + 15 = 0$

$\implies 4x(2x - 3) - 5(2x - 3) = 0$

$\implies (2x - 3)(4x - 5) = 0$

$\implies 2x - 3 = 0$ Or $4x - 5 = 0$

$\implies x = \frac{3}{2}$ Or $x = \frac{5}{4}$

Solution 5:

$x + \dfrac{4}{x} = -4$

$\Rightarrow \dfrac{x^2 + 4}{x} = -4$

$\Rightarrow x^2 + 4 = -4x$

$\Rightarrow x^2 + 4x + 4 = 0$

$\Rightarrow (x + 2)^2 = 0$

$\Rightarrow x + 2 = 0$

$\Rightarrow x = -2$

Solution 6:

$2x^2 - 5x^2 + 3 = 0$

$\Rightarrow 2x^4 - 3x^2 - 2x^2 + 3 = 0$

$\Rightarrow x^2 (2x^2 - 3) -1(x^2 - 1) = 0$

$(2x^2 - 3)(x^2 - 1) = 0$

If $2x^2 - 3 = 0$ Or $x^2 - 1 = 0$

Then $x^2 = \dfrac{3}{2}$ Or $x^2 = 1$

$\Rightarrow x = \pm \sqrt{\dfrac{3}{2}}$ Or $x = \pm 1$

Solution 7:

$X^4 - 2x^2 - 3 = 0$

$\Rightarrow x^2 - 3x^2 + x^2 - 3 = 0$

$\Rightarrow x^2 (x^2 - 3) +1(x^2 -3) = 0$

$\Rightarrow (x^2 - 3) (x^2 + 1) = 0$

If $x^2 - 3 = 0$ Or $x^2 + 1 = 0$

Then $x^2 = 3$ Or $x^2 = -1$ (reject)

$\Rightarrow x = \pm \sqrt{3}$

Solution 8:

$9 \left(x^2 + \dfrac{1}{x^2}\right) - 9 \left(x + \dfrac{1}{x}\right) - 52 = 0$

Let $x + \dfrac{1}{x} = y$

Squaring on both sides

$X^2 + \frac{1}{x^2} + 2 = y^2$

$\Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$

Putting these values in the given equation

$9(y^2 - 2) - 9y - 52 = 0$

$\Rightarrow 9y^2 - 18 - 9y - 52 = 0$

$\Rightarrow 9y^2 - 9y - 70 = 0$

$\Rightarrow 9y^2 - 30y + 21y - 70 = 0$

$\Rightarrow 3y(3y - 10) + 7(3y - 10) = 0$

$\Rightarrow (3y - 10)(3y + 7) = 0$

$\Rightarrow 3y - 10 = 0$ Or $3y + 7 = 0$

$\Rightarrow y = \frac{10}{3}$ Or $y = \frac{-7}{3}$

$\Rightarrow x + \frac{1}{x} = \frac{10}{3}$ Or $x + \frac{1}{x} = \frac{-7}{3}$

$\Rightarrow \frac{x^2+1}{x} = \frac{10}{3}$ Or $\frac{x^2+1}{x} = \frac{-7}{3}$

$\Rightarrow 3x^2 - 10x + 3 = 0$ Or $3x^2 + 7x + 3 = 0$

$\Rightarrow 3x^2 - 9x - x + 3 = 0$ Or $x = \frac{-7 \pm \sqrt{(-7)^2 - 4(3)(3)}}{2(3)}$

$\Rightarrow 3x(x - 3) - 1(x - 3) = 0$ Or $x = \frac{-7 \pm \sqrt{13}}{6}$

$\Rightarrow (x - 3)(3x - 1) = 0$

$\Rightarrow x = 3$ and $x = \frac{1}{3}$

Solution 9:

$2\left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right) = 11$

Let $x + \frac{1}{x} = y$

Squaring on both sides

$X^2 + \frac{1}{x^2} + 2 = y^2$

$\Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$

Putting these values in the given equation

$2(y^2 - 2) - y = 11$

$\Rightarrow 2y^2 - 4 - y - 11 = 0$

$\Rightarrow 2y^2 - y - 15 = 0$

$\Rightarrow 2y^2 - 6y + 5y - 15 = 0$

$\Rightarrow 2y(y - 3) + 5(y - 3) = 0$

$\Rightarrow (y - 3)(2y + 5) = 0$

$\Rightarrow$ If $y - 3 = 0$ Or $2y + 5 = 0$

$\Rightarrow y = 3$ Or $y = \frac{-5}{2}$

$\Rightarrow$ x + $\frac{1}{x}$ = 3   Or   x + $\frac{1}{x}$ = $\frac{-5}{2}$

$\Rightarrow$ $\frac{x^2+1}{x}$ = 3   Or $\frac{x^2+1}{x}$ = $\frac{-5}{2}$

$\Rightarrow$ x^2 – 3x + 1 = 0   Or $2x^2$ + 5x + 2 = 0

$\Rightarrow$ x = $\frac{-3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$ Or $2x^2$ + 4x + x + 2 = 0

x = $\frac{-3 \pm \sqrt{5}}{2}$ Or   2x (x + 2) + 1 (x + 2) = 0

                              Or   (x + 2) (2x + 1) = 0

                              Then   x = – 2   and   x = $\frac{-1}{2}$

Solution 10:

$\left(x^2 + \frac{1}{x^2}\right) - 3\left(x - \frac{1}{x}\right) - 2 = 0$

Let x $-$ $\frac{1}{x}$ = y

Squaring on both sides

X^2 + $\frac{1}{x^2}$ $-$ 2 = y^2

$\Rightarrow$ x^2 + $\frac{1}{x^2}$ = y^2 + 2

Putting these values in the given equation

$(y^2 + 2) - 3y - 2 = 11$

$\Rightarrow$ y^2 – 3y = 0

$\Rightarrow$ y (y – 3) = 0   Or   y – 3 = 0

If y = 0   Or   y = 3

$\Rightarrow$ x $-$ $\frac{1}{x}$ = 0   Or   x $-$ $\frac{1}{x}$ = 3

$\Rightarrow$ $\frac{x^2-1}{x}$ = 0   Or $\frac{x^2-1}{x}$ = 3

$\Rightarrow$ x^2 – 1 = 0   Or x^2 – 3x – 1 = 0

$\Rightarrow$ (x + 1) (x – 1) = 0   Or $\frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$

$\Rightarrow$ x= – 1 and   x = 1   Or   x = $\frac{3 \pm \sqrt{13}}{2}$

Solution 11:

$(x^2 + 5x + 4)(x^2 + 5x + 6) = 120$

Let $x^2 + 5x = y$

Then (y +4) (y + 6) = 120

$\Rightarrow$ y^2 + 6y + 4y + 24 – 120 = 0

$\Rightarrow y^2 + 10y - 96 = 0$
$\Rightarrow y^2 + 16y - 6y - 96 = 0$
$\Rightarrow y(y + 16) - 6(y + 16) = 0$
$\Rightarrow (y + 16)(y - 16) = 0$
Then y = − 16   Or   y = 6
$\Rightarrow x^2 + 5x + 16 = 0$ Or $x^2 + 5x - 6 = 0$
$\Rightarrow x = \dfrac{-5 \pm \sqrt{(5)^2 - 4(1)(16)}}{2(1)}$ Or $x^2 + 6x - x - 6 = 0$

$x = \dfrac{-5 \pm \sqrt{-39}}{2}$ Or   x(x + 6) − 1(x + 6) = 0
     (reject)   Or   (x + 6)(x − 1) = 0
                                            Then x = − 6   and   x = 1

Solution 12:
Here a = 1, b = − 5 and c = − 10
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-10)}}{2(1)}$

$\Rightarrow x = \dfrac{5 \pm \sqrt{65}}{2} = \dfrac{5 \pm 8.06}{2}$

$\Rightarrow x = \dfrac{13.06}{2}$ and $\dfrac{-3.06}{2}$ = 6.53 and − 1.53

(ii) $3x^2 - x - 7 = 0$
Here a = 3, b = − 1 and c = − 7
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-1-7)}}{2(3)}$

$\Rightarrow x = \dfrac{1 \pm \sqrt{85}}{6} = \dfrac{1 \pm 9.22}{6}$

$\Rightarrow x = \dfrac{10.22}{6}$ and $\dfrac{-8.22}{2}$ = 1.70 and − 1.37

Solution 13:
$\left(\dfrac{x}{x+2}\right)^2 - 7\left(\dfrac{x}{x+2}\right) + 12 = 0; x \neq -2$
Let $\dfrac{x}{x+2} = y$
Then $y^2 - 7y + 12 = 0$

$\Longrightarrow y^2 - 4y - 3y + 12 = 0$

$\Longrightarrow y(y - 4) - 3(y - 3) = 0$

$\Longrightarrow (y - 4)(y - 3) = 0$

Then $y = 4$ and $y = 3$

$\Longrightarrow \dfrac{x}{x+2} = 4$ and $\dfrac{x}{x+2} = 3$

$\Longrightarrow 4x + 8 = x$ and $3x + 6 = x$

$\Longrightarrow x = \dfrac{-8}{3}$ and $x = -3$

Solution 14:

(i) $x^2 - 11x - 12 = 0$

$\Longrightarrow x^2 - 12x + x - 12 = 0$

$\Longrightarrow x(x - 12) + 1(x - 12) = 0$

$\Longrightarrow (x - 12)(x + 1) = 0$

$\Longrightarrow x = 12$ and $x = -1$

Since $x \in N$, then $x = 12$

(ii) $x^2 - 4x - 12 = 0$

$\Longrightarrow x^2 - 6x + 2x - 12 = 0$

$\Longrightarrow x(x - 6) + 2(x - 6) = 0$

$\Longrightarrow (x - 6)(x + 2) = 0$

$\Longrightarrow x = 6$ and $x = -2$

Since $x \in I$, then $x = 6$ and -2

(iii) $2x^2 - 9x + 10 = 0$

$\Longrightarrow 2x^2 - 5x - 4x - 10 = 0$

$\Longrightarrow x(2x - 5) - 2(2x - 5) = 0$

$\Longrightarrow (2x - 5)(x - 2) = 0$

$\Longrightarrow x = \dfrac{5}{2}$ and $x = 2$

Since $x \in Q$, then $x = \dfrac{5}{2}$ and 2.

Solution 15:

$(a + b)^2 x^2 - (a + b)x - 6 = 0; a + b \neq 0$

$\Longrightarrow (a + b)^2 x^2 - 3(a + b)x + 2(a + b)x - 6 = 0$

$\Longrightarrow (a + b)x[(a + b)x - 3] + 2[(a + b)x - 3] = 0$

$\Longrightarrow [(a + b)x - 3][(a + b)x - 2] = 0$

$\Longrightarrow (a + b)x - 3 = 0$ Or $(a + b)x + 2 = 0$

$\Longrightarrow x = \dfrac{3}{a+b}$ Or $x = \dfrac{-2}{a+b}$

Solution 16:

$\dfrac{1}{p} + \dfrac{1}{q} + \dfrac{1}{x} = \dfrac{1}{x+p+q}$

$\Longrightarrow \dfrac{1}{p} + \dfrac{1}{q} + \dfrac{1}{x} - \dfrac{1}{x+p+q} = 0$

$\Longrightarrow \dfrac{q+p}{pq} + \dfrac{x+p+q-x}{x(x+p+q)} = 0$

$\Longrightarrow \dfrac{q+p}{pq} + \dfrac{p+q}{x(x+p+q)} = 0$

$\Longrightarrow (p+q)\left[\dfrac{1}{pq} + \dfrac{1}{x^2+px+qx}\right] = 0$

$\Longrightarrow (p+q)\left[\dfrac{x^2+px+qx+pq}{pq(x^2+px+qx)}\right] = 0$

$\Longrightarrow x^2 + px + qx + pq = 0$

$\Longrightarrow x(x+p) + q(x+p) = 0$

$\Longrightarrow (x+p)(x+q) = 0$

$\Longrightarrow x = -p$ and $x = -q$

Solution 17:

(i) x(x + 1) + (x + 2) (x + 3) = 42

$\Longrightarrow x^2 + x + x^2 + 3x + 2x + 6 - 42 = 0$

$\Longrightarrow 2x^2 + 6x - 36 = 0$

$\Longrightarrow 2x^2 + 12x - 6x - 36 = 0$

$\Longrightarrow 2x(x+6) - 6(x+6) = 0$

$\Longrightarrow (x+6)(2x-6) = 0$

If $x + 6 = 0$ Or $2x - 6 = 0$

Then $x = -6$ Or $x = 3$

(ii) $\dfrac{1}{x+1} - \dfrac{2}{x+2} = \dfrac{3}{x+3} - \dfrac{4}{x+4}$

$\Longrightarrow \dfrac{1(x+2)-2(x+1)}{(x+1)(x+2)} = \dfrac{3(x+4)-4(x+3)}{(x+3)(x+4)}$

$\Longrightarrow \dfrac{-x}{x^2+3x+2} = \dfrac{-x}{x^2+7x+12}$

$\Longrightarrow -x\,[x^2+3x+2 = x^2+7x+12]$

$\Longrightarrow -x\,[-4x = 10]$

X = 0   and $x = \dfrac{-10}{4} = -2.5$

Solution 18:

(i) (m – 3) x² – 4x + 1 = 0

Here a = (m – 3), b = − 4 and c = 1
Given equation has equal roots
Then D = 0
$\Longrightarrow$ $b^2 - 4ac = 0$
$\Longrightarrow$ $(-4)^2 - 4(m - 3)(1) = 0$
$\Longrightarrow$ $16 - 4m + 12 = 0$
$\Longrightarrow$ $- 4m = - 28$
$\Longrightarrow$ m = 7
Put value of m in given equation
$4x^2 - 4x + 1 = 0$
$\Longrightarrow (2x - 1)^2 = 0$
$\Longrightarrow 2x - 1 = 0$
$\Longrightarrow X = \frac{1}{2}$

(ii) $3x^2 + 12x + (m + 7) = 0$
Here a = 3, b = 12 and c = m + 7
Given equation has equal roots
Then D = 0
$\Longrightarrow b^2 - 4ac = 0$
$\Longrightarrow (12)^2 - 4(3)(m + 7) = 0$
$\Longrightarrow 144 - 12m - 84 = 0$
$\Longrightarrow - 12m = - 60$
$\Longrightarrow m = 5$
Put value of m in given equation
$3x^2 - 12x + 12 = 0$
$X^2 + 4x + 4 = 0$
$\Longrightarrow (x + 2)^2 = 0$
$\Longrightarrow x + 2 = 0$
$\Longrightarrow X = - 2$

(iii) $x^2 - (m + 2) x + (m + 5) = 0$
Here a = 1, b = − 4 (m + 2)   and c = m + 5
Given equation has equal roots
Then D = 0
$\Longrightarrow b^2 - 4ac = 0$
$\Longrightarrow [-(m + 2)]^2 - 4(1)(m + 5) = 0$
$\Longrightarrow m^2 + 4m + 4 - 4m - 20 = 0$
$\Longrightarrow m^2 - 16 = 0$
$\Longrightarrow m^2 = 16$
$\Longrightarrow m = \pm 4$
Put value of m in given equation
$x^2 - 6x + 9 = 0$ Or $x^2 + 2x + 1 = 0$
$\Longrightarrow (x - 3)^2 = 0$ Or $(x + 1)^2 = 0$
$\Longrightarrow x - 3 = 0$ Or $x + 1 = 0$

$\Rightarrow$ x = 3   Or   x = − 1

Solution 19:
$Px^2 - 4x + 3 = 0$
Here a = p, b = − 4 and c = 3
Given equation has equal roots
Then D = 0
$\Rightarrow b^2 - 4ac = 0$
$\Rightarrow [-4]^2 - 4(p)(3) = 0$
$\Rightarrow 16 - 12p = 0$
$\Rightarrow - 12p = - 16$
$\Rightarrow p = \frac{-16}{-12} = \frac{4}{3}$

Solution 20:
Consider the given equation:
$X^2 + 2(m - 1)x + (m + 5) = 0$
The nature of the roots of a quadratic equation
$ax^2 + bx + c = 0$, depends entirely on the
value of its discriminant $b^2 - 4ac$.
If a, b and c are real numbers and a ≠ 0,
Then discriminant:
(i) $b^2 - 4ac = 0 \Rightarrow$ the roots are real and equal
(ii) $b^2 - 4ac > 0 \Rightarrow$ the roots are real and unequal.
(iii) $b^2 - 4ac < 0 \Rightarrow$ the roots are imaginary (not equal)
Since the roots of the given equation are real and equal,
We have,
$b^2 - 4ac = 0$
$\Rightarrow (2(m - 1))^2 - 4 \times 1 \times (m + 5) = 0$
$\Rightarrow 4(m^2 + 1 - 2m) - 4(m + 5) = 0$
$\Rightarrow 4m^2 + 4 - 8m - 4m - 20 = 0$
$\Rightarrow 4m^2 - 12m - 16 = 0$
$\Rightarrow m^2 - 3m - 4 = 0$
$\Rightarrow m^2 - 4m + m - 4 = 0$
$\Rightarrow m(m - 4) + 1(m - 4) = 0$
$\Rightarrow m + 1 = 0$ Or m − 4 = 0
$\Rightarrow$ m = − 1   Or m = 4