Book Name: Selina Concise

EXERCISE. 9 (A)

Solution 1:

(i) False

The sum A + B is possible when the order of both the matrices A and B are same.

(ii) True

(iii) False

Transpose of a 2×1 matrix is a 1×2 matrix.

(iv) True

(v) False

A column matrix has only one column and many rows.

Solution 2:

If two matrices are equal, then their corresponding elements are also equal. Therefore, we have:

$x = 3$,

$y + 2 = 1 \Rightarrow y = -1$

$z - 1 = 2 \Rightarrow z = 3$

Solution 3:

If two matrices are equal, then their corresponding elements are also equal.

(i)

$a + 5 = 2 \Rightarrow a = -3$

$-4 = b + 4 \Rightarrow b = -8$

$2 = c - 1 \Rightarrow c = 3$

(ii) $a = 3$

$a - b = -1$

$\Rightarrow b = a + 1 = 4$

$b + c = 2$

$\Rightarrow c = 2 - b = 2 - 4 = -2$

Solution 4:

(i) $A + B = \begin{bmatrix} 8 & -3 \end{bmatrix} + \begin{bmatrix} 4 & -5 \end{bmatrix} = \begin{bmatrix} 8+4 & -3-5 \end{bmatrix} = \begin{bmatrix} 12 & -8 \end{bmatrix}$

(ii) $B - A = \begin{bmatrix} 4 & -5 \end{bmatrix} - \begin{bmatrix} 8 & -3 \end{bmatrix}$

$= \begin{bmatrix} 4-8 & -5+3 \end{bmatrix}$

$= \begin{bmatrix} -4 & -2 \end{bmatrix}$

Solution 5:

(i) $B + C = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 6 \\ -2 \end{bmatrix} = \begin{bmatrix} 1+6 \\ 4-2 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$

(ii) $A - C = \begin{bmatrix} 2 \\ 5 \end{bmatrix} - \begin{bmatrix} 6 \\ -2 \end{bmatrix} = \begin{bmatrix} 2-6 \\ 5+2 \end{bmatrix} = \begin{bmatrix} -4 \\ 7 \end{bmatrix}$

(iii) $A + B - C = \begin{bmatrix} 2 \\ 5 \end{bmatrix} + \begin{bmatrix} 1 \\ 4 \end{bmatrix} - \begin{bmatrix} 6 \\ -2 \end{bmatrix}$

$= \begin{bmatrix} 2+1-6 \\ 5+4+2 \end{bmatrix} = \begin{bmatrix} -3 \\ 11 \end{bmatrix}$

(iv) $A - B + C = \begin{bmatrix} 2 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 6 \\ -2 \end{bmatrix}$

$= \begin{bmatrix} 2-1+6 \\ 5-4-2 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$

Solution 6:

(i) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ 1 & -7 \end{bmatrix} = \begin{bmatrix} 1-1 & 2-2 \\ 3+1 & 4-7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 4 & -3 \end{bmatrix}$

(ii) $\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 3 \\ 6 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2-0 & 3-2 & 4-3 \\ 5-6 & 6+1 & 7-0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 7 & 7 \end{bmatrix}$

(iii) Addition is not possible, because both matrices are not of same order.

Solution 7:

(i) $\begin{bmatrix} 5 & 2 \\ -1 & y-1 \end{bmatrix} - \begin{bmatrix} 1 & x-1 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -3 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 5-1 & 2-x-1 \\ -1-2 & y-1+3 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -3 & 2 \end{bmatrix}$

$$\Rightarrow\begin{bmatrix} 4 & 3-x \\ -3 & y+2 \end{bmatrix}=\begin{bmatrix} 4 & 7 \\ -3 & 2 \end{bmatrix}$$

Equating the corresponding elements, we get,
$3 - x = 7$ and $y + 2 = 2$
Thus, we get, $x = -4$ and $y = 0$.
(ii)

$$\begin{bmatrix} -8 & x \end{bmatrix}+\begin{bmatrix} y & -2 \end{bmatrix}=\begin{bmatrix} -3 & 2 \end{bmatrix}$$

$$\Rightarrow\begin{bmatrix} -8+y & x-2 \end{bmatrix}=\begin{bmatrix} -3 & 2 \end{bmatrix}$$

Equating the corresponding elements, we get,
$-8 + y = -3$ and $x - 2 = 2$
Thus, we get, $x = 4$ and $y = 5$.

Solution 8:

$$M=\begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}$$

$$M^t=\begin{bmatrix} 5 & -2 \\ -3 & 4 \end{bmatrix}$$

$$\text{(i)}M+M^t=\begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}+\begin{bmatrix} 5 & -2 \\ -3 & 4 \end{bmatrix}=\begin{bmatrix} 5+5 & -3-2 \\ -2-3 & 4+4 \end{bmatrix}=\begin{bmatrix} 10 & -5 \\ -5 & 8 \end{bmatrix}$$

$$\text{(i)}M^t-M=\begin{bmatrix} 5 & -2 \\ -3 & 4 \end{bmatrix}-\begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}=\begin{bmatrix} 5-5 & -2+3 \\ -3+2 & 4-4 \end{bmatrix}=\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Solution 9:

We know additive inverse of a matrix is its negative.

Additive inverse of A = $-A=-\begin{bmatrix} 6 & -5 \end{bmatrix}=\begin{bmatrix} -6 & 5 \end{bmatrix}$

Additive inverse of B = $-B=-\begin{bmatrix} -2 & 0 \\ 4 & -1 \end{bmatrix}=\begin{bmatrix} 2 & 0 \\ -4 & 1 \end{bmatrix}$

Additive inverse of C = $-C=-\begin{bmatrix} -7 \\ 4 \end{bmatrix}=\begin{bmatrix} 7 \\ -4 \end{bmatrix}$

Solution 10:

(i) X + B = C – A

$$X+\begin{bmatrix}0 & 2\end{bmatrix}=\begin{bmatrix}-1 & 4\end{bmatrix}-\begin{bmatrix}2 & -3\end{bmatrix}$$

$$X+\begin{bmatrix}0 & 2\end{bmatrix}=\begin{bmatrix}-1 & -2 & 4+ & 3\end{bmatrix}=\begin{bmatrix}-3 & 7\end{bmatrix}$$

$$X=\begin{bmatrix}-3 & 7\end{bmatrix}-\begin{bmatrix}0 & 2\end{bmatrix}=\begin{bmatrix}-3-0 & 7-2\end{bmatrix}=\begin{bmatrix}-3 & 5\end{bmatrix}$$

(ii) A – X = B + C

$$\begin{bmatrix}2 & -3\end{bmatrix}-X=\begin{bmatrix}0 & 2\end{bmatrix}+\begin{bmatrix}-1 & 4\end{bmatrix}$$

$$\begin{bmatrix}2 & -3\end{bmatrix}-X=\begin{bmatrix}0-1 & 2+4\end{bmatrix}$$

$$\begin{bmatrix}2 & -3\end{bmatrix}-X=\begin{bmatrix}-1 & 6\end{bmatrix}$$

$$\begin{bmatrix}2 & -3\end{bmatrix}-\begin{bmatrix}-1 & 6\end{bmatrix}=X$$

$$X=\begin{bmatrix}2+1 & -3-6\end{bmatrix}=\begin{bmatrix}3 & -9\end{bmatrix}$$

Solution 11:

(i) A + X = B

X = B – A

$$X=\begin{bmatrix}3 & -3\\ -2 & 0\end{bmatrix}-\begin{bmatrix}-1 & 0\\ 2 & -4\end{bmatrix}=\begin{bmatrix}3+1 & -3-0\\ -2-2 & 0+4\end{bmatrix}=\begin{bmatrix}4 & -3\\ -4 & 4\end{bmatrix}$$

(ii) A – X = B

X = A – B

$$X=\begin{bmatrix}-1 & 0\\ 2 & -4\end{bmatrix}-\begin{bmatrix}3 & -3\\ -2 & 0\end{bmatrix}=\begin{bmatrix}-1-3 & 0+3\\ 2+2 & -4-0\end{bmatrix}=\begin{bmatrix}-4 & -3\\ 4 & -4\end{bmatrix}$$

(iii) X – B = A

X = A + B

$$X=\begin{bmatrix}-1 & 0\\ 2 & -4\end{bmatrix}+\begin{bmatrix}3 & -3\\ -2 & 0\end{bmatrix}=\begin{bmatrix}-1+3 & 0-3\\ 2-2 & -4+0\end{bmatrix}=\begin{bmatrix}2 & -3\\ 0 & -4\end{bmatrix}$$

EXERCISE. 9 (B)

Solution 1:

(i) $3\begin{bmatrix}5 & -2\end{bmatrix}=\begin{bmatrix}15 & -6\end{bmatrix}$

(ii) $7\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 14 \\ 0 & 7 \end{bmatrix}$

(iii) $2\begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 4 & -6 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} -2+3 & 0+3 \\ 4+5 & -6+0 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 9 & -6 \end{bmatrix}$

(iv) $6\begin{bmatrix} 3 \\ -2 \end{bmatrix} - 2\begin{bmatrix} -8 \\ 1 \end{bmatrix} = \begin{bmatrix} 18 \\ -12 \end{bmatrix} - \begin{bmatrix} -16 \\ 2 \end{bmatrix} = \begin{bmatrix} 18+16 \\ -12-2 \end{bmatrix} = \begin{bmatrix} 34 \\ -14 \end{bmatrix}$

Solution 2:

(i) $3\begin{bmatrix} 4 & x \end{bmatrix} + 2\begin{bmatrix} y & -3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \end{bmatrix}$

$\begin{bmatrix} 12 & 3x \end{bmatrix} + \begin{bmatrix} 2y & -6 \end{bmatrix} = \begin{bmatrix} 10 & 0 \end{bmatrix}$

$\begin{bmatrix} 12+2y & 3x-6 \end{bmatrix} = \begin{bmatrix} 10 & 0 \end{bmatrix}$

Comparing the corresponding elements, we get,

$12 + 2y = 10$ and $3x - 6 = 0$

Simplifying, we get, $y = -1$ and $x = 2$.

(ii) $x\begin{bmatrix} -1 \\ 2 \end{bmatrix} - 4\begin{bmatrix} -2 \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -8 \end{bmatrix}$

$\begin{bmatrix} -x \\ 2x \end{bmatrix} - \begin{bmatrix} -8 \\ 4y \end{bmatrix} = \begin{bmatrix} 7 \\ -8 \end{bmatrix}$

$\begin{bmatrix} -x+8 \\ 2x-4y \end{bmatrix} = \begin{bmatrix} 7 \\ -8 \end{bmatrix}$

Comparing corresponding the elements, we get,

$-x + 8 = 7$ and $2x - 4y = -8$

Simplifying, we get,

$x = 1$ and $y = \frac{5}{2} = 2.5$

Solution 3:

(i) $2A - 3B + C$

$= 2\begin{vmatrix} 2 & 1 \\ 3 & 0 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 \\ 5 & 2 \end{vmatrix} + \begin{vmatrix} -3 & -1 \\ 0 & 0 \end{vmatrix}$

$= \begin{bmatrix} 4 & 2 \\ 6 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 3 \\ 15 & 6 \end{bmatrix} + \begin{bmatrix} -3 & -1 \\ 0 & 0 \end{bmatrix}$

$$=\begin{bmatrix} 4-3-3 & 2-3-1 \\ 6-15+0 & 0-6+0 \end{bmatrix}$$

$$=\begin{bmatrix} -2 & -2 \\ -9 & -6 \end{bmatrix}$$

(ii) A + 2C – B

$$=\begin{vmatrix} 2 & 1 \\ 3 & 0 \end{vmatrix}+2\begin{vmatrix} -3 & -1 \\ 0 & 0 \end{vmatrix}-\begin{vmatrix} 1 & 1 \\ 5 & 2 \end{vmatrix}$$

$$=\begin{vmatrix} 2 & 1 \\ 3 & 0 \end{vmatrix}+\begin{bmatrix} -6 & -2 \\ 0 & 0 \end{bmatrix}-\begin{vmatrix} 1 & 1 \\ 5 & 2 \end{vmatrix}$$

$$=\begin{vmatrix} 2-6-1 & 1-2-1 \\ 3+0-5 & 0+0-2 \end{vmatrix}$$

$$=\begin{bmatrix} -5 & -2 \\ -2 & -2 \end{bmatrix}$$

Solution 4:

$$\begin{bmatrix} 4 & -2 \\ 4 & 0 \end{bmatrix}+3A=\begin{bmatrix} -2 & -2 \\ 1 & -3 \end{bmatrix}$$

$$3A=\begin{bmatrix} -2 & -2 \\ 1 & -3 \end{bmatrix}-\begin{bmatrix} 4 & -2 \\ 4 & 0 \end{bmatrix}$$

$$3A=\begin{bmatrix} -2-4 & -2+2 \\ 1-4 & -3-0 \end{bmatrix}$$

$$3A=\begin{bmatrix} -6 & 0 \\ -3 & -3 \end{bmatrix}$$

$$A=\frac{1}{3}\begin{bmatrix} -6 & 0 \\ -3 & -3 \end{bmatrix}=\begin{bmatrix} -2 & 0 \\ -1 & -1 \end{bmatrix}$$

Solution 5:

(i) $2\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}+\begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}=\begin{bmatrix} 2 & 8 \\ 4 & 6 \end{bmatrix}+\begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}=\begin{bmatrix} 2-4 & 8-1 \\ 4-3 & 6-2 \end{bmatrix}=\begin{bmatrix} -2 & 7 \\ 1 & 4 \end{bmatrix}$

(ii) $C+B=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$C=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}-B=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}-\begin{bmatrix}-4 & -1\\ -3 & -2\end{bmatrix}=\begin{bmatrix}0+4 & 0+1\\ 0+3 & 0+2\end{bmatrix}=\begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix}$$

Solution 6:

$$2\begin{bmatrix}3 & x\\ 0 & 1\end{bmatrix}+3\begin{bmatrix}1 & 3\\ y & 2\end{bmatrix}=\begin{bmatrix}z & -7\\ 15 & 8\end{bmatrix}$$

$$\begin{bmatrix}6 & 2x\\ 0 & 2\end{bmatrix}+\begin{bmatrix}3 & 9\\ 3y & 6\end{bmatrix}=\begin{bmatrix}z & -7\\ 15 & 8\end{bmatrix}$$

$$\begin{bmatrix}9 & 2x+9\\ 3y & 8\end{bmatrix}=\begin{bmatrix}z & -7\\ 15 & 8\end{bmatrix}$$

Comparing the corresponding elements, we get,

$2x + 9 = -7 \Rightarrow 2x = -16 \Rightarrow x = -8$

$3y = 15 \Rightarrow y = 5$

$z = 9$

Solution 7:

$$A=\begin{bmatrix}-3 & 6\\ 0 & -9\end{bmatrix}$$

$$A^t=\begin{bmatrix}-3 & 0\\ 6 & -9\end{bmatrix}$$

(i) $2A + 3A^t$

$$=2\begin{bmatrix}-3 & 6\\ 0 & -9\end{bmatrix}+3\begin{bmatrix}-3 & 0\\ 6 & -9\end{bmatrix}$$

$$=\begin{bmatrix}-6 & 12\\ 0 & -18\end{bmatrix}+\begin{bmatrix}-9 & 0\\ 18 & -27\end{bmatrix}$$

$$=\begin{bmatrix}-15 & 12\\ 18 & -45\end{bmatrix}$$

(ii) $2A^t - 3A$

$$=2\begin{bmatrix}-3 & 0\\ 6 & -9\end{bmatrix}-3\begin{bmatrix}-3 & 6\\ 0 & -9\end{bmatrix}$$

$$=\begin{bmatrix}-6 & 0\\ 12 & -18\end{bmatrix}-\begin{bmatrix}-9 & 18\\ 0 & -27\end{bmatrix}$$

$$=\begin{bmatrix}3 & -18\\ 12 & 9\end{bmatrix}$$

(iii) $\frac{1}{2}A-\frac{1}{3}A^t$

$$=\frac{1}{2}\begin{bmatrix}-3 & 6\\ 0 & -9\end{bmatrix}-\frac{1}{3}\begin{bmatrix}-3 & 0\\ 6 & -9\end{bmatrix}$$

$$=\begin{bmatrix}\frac{-3}{2} & 3\\ 0 & \frac{-9}{2}\end{bmatrix}-\begin{bmatrix}-1 & 0\\ 2 & -3\end{bmatrix}$$

$$=\begin{bmatrix}\frac{-1}{2} & 3\\ -2 & \frac{-3}{2}\end{bmatrix}$$

(iv) $A^t-\frac{1}{3}A$

$$=\begin{bmatrix}-3 & 0\\ 6 & -9\end{bmatrix}-\frac{1}{3}\begin{bmatrix}-3 & 6\\ 0 & -9\end{bmatrix}$$

$$=\begin{bmatrix}-3 & 0\\ 6 & -9\end{bmatrix}-\begin{bmatrix}-1 & 2\\ 0 & -3\end{bmatrix}$$

$$=\begin{bmatrix}-2 & -2\\ 6 & -6\end{bmatrix}$$

Solution 8:

(i) $X + 2A = B$

$X = B - 2A$

$$X=\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}-2\begin{bmatrix}1 & 1\\ -2 & 0\end{bmatrix}$$

$$X=\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}-\begin{bmatrix}2 & 2\\ -4 & 0\end{bmatrix}$$

$$X=\begin{bmatrix}0 & -3\\ 5 & 1\end{bmatrix}$$

(ii) $3X + B + 2A = O$

$3X = -2A - B$

$$3x=-2\begin{bmatrix}1 & 1\\ -2 & 0\end{bmatrix}-\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}$$

$$3x=\begin{bmatrix}-2 & -2\\ 4 & 0\end{bmatrix}-\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}$$

$$3x=\begin{bmatrix}-4 & -1\\ 3 & -1\end{bmatrix}$$

$$x=\begin{bmatrix}\frac{-4}{3} & \frac{-1}{3}\\ 1 & \frac{-1}{3}\end{bmatrix}$$

(iii) $3A - 2X = X - 2B$

$3A + 2B = X + 2X$

$3X = 3A + 2B$

$$3x=3\begin{bmatrix}1 & 1\\ -2 & 0\end{bmatrix}+2\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}$$

$$3x=\begin{bmatrix}3 & 3\\ -6 & 0\end{bmatrix}+\begin{bmatrix}4 & -2\\ 2 & 2\end{bmatrix}$$

$$3x=\begin{bmatrix}7 & 1\\ -4 & 2\end{bmatrix}$$

$$x=\begin{bmatrix}\frac{7}{3} & \frac{1}{3}\\ \frac{-4}{3} & \frac{2}{3}\end{bmatrix}$$

Solution 9:

3M + 5N

$$= 3\begin{bmatrix} 0 \\ 1 \end{bmatrix} + 5\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 3 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

Solution 10:

$$\text{(i) } M - 2I = 3\begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix}$$

$$M = 3\begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix} + 2I$$

$$M = 3\begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} -3 & 0 \\ 12 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$M = \begin{bmatrix} -1 & 0 \\ 12 & 5 \end{bmatrix}$$

$$\text{(ii) } 5M + 3I = 4\begin{bmatrix} 2 & -5 \\ 0 & -3 \end{bmatrix}$$

$$5M = 4\begin{bmatrix} 2 & -5 \\ 0 & -3 \end{bmatrix} - 3I$$

$$5M = 4\begin{bmatrix} 2 & -5 \\ 0 & -3 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$5M = \begin{bmatrix} 8 & -20 \\ 0 & -12 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$5M = \begin{bmatrix} 5 & -20 \\ 0 & -15 \end{bmatrix}$$

$$M = \frac{1}{5}\begin{bmatrix} 5 & -20 \\ 0 & -15 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 0 & -3 \end{bmatrix}$$

Solution 11:

$$\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$$

$$\Rightarrow 2M = \begin{bmatrix} 9 & 6 \\ 0 & -9 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 2 & -12 \end{bmatrix}$$

$$\Rightarrow M = \begin{bmatrix} 4 & 1 \\ 1 & -6 \end{bmatrix}$$

EXERCISE. 9 (C)

Solution 1:

(i) $\begin{bmatrix} 3 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 6+0 \end{bmatrix} = \begin{bmatrix} 6 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -2 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} -2+2 & 3-8 \end{bmatrix} = \begin{bmatrix} 0 & -5 \end{bmatrix}$

(iii) $\begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -6+12 \\ -3-3 \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \end{bmatrix}$

(iv) $\begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} -1 & 3 \end{bmatrix}$

The number of columns in the first matrix is not equal to the number of rows in the second matrix. Thus, the product is not possible.

Solution 2:

(i) $AB = \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 0+6 & 0+4 \\ 5-6 & -5-4 \end{bmatrix}$

$= \begin{bmatrix} 6 & 4 \\ -1 & -9 \end{bmatrix}$

$\text{(ii)} BA = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}$

$= \begin{bmatrix} 0-5 & 2+2 \\ 0+10 & 6-4 \end{bmatrix}$

$= \begin{bmatrix} -5 & 4 \\ 10 & 2 \end{bmatrix}$

$\text{(iii) } AI = \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0+0 & 0+2 \\ 5-0 & 0-2 \end{bmatrix}$

$\begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} = A$

$\text{(iv) } IB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 21 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & -1+0 \\ 0+3 & 0+2 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} = B$

$\text{(v) } A^2 = \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}$

$= \begin{bmatrix} 0+10 & 0-4 \\ 0-10 & 10+4 \end{bmatrix}$

$= \begin{bmatrix} 10 & -4 \\ -10 & 14 \end{bmatrix}$

$\text{(vi)} B^2 = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1-3 & -1-2 \\ 3+6 & -3+4 \end{bmatrix}$

$$=\begin{bmatrix}-2 & -3\\ 9 & 1\end{bmatrix}$$

$$B^2A=\begin{bmatrix}-2 & -3\\ 9 & 1\end{bmatrix}\begin{bmatrix}0 & 2\\ 5 & -2\end{bmatrix}$$

$$=\begin{bmatrix}0-15 & -4+6\\ 0+5 & 18-2\end{bmatrix}$$

$$=\begin{bmatrix}-15 & 2\\ 5 & 16\end{bmatrix}$$

Solution 3:

$$M=\begin{bmatrix}2 & 1\\ 1 & -2\end{bmatrix}$$

$$M^2=\begin{bmatrix}2 & 1\\ 1 & -2\end{bmatrix}\begin{bmatrix}2 & 1\\ 1 & -2\end{bmatrix}=\begin{bmatrix}4+1 & 2-2\\ 2-2 & 1+4\end{bmatrix}=\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}$$

$$M^3=MM^2=\begin{bmatrix}2 & 1\\ 1 & -2\end{bmatrix}\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}=\begin{bmatrix}10+0 & 0+5\\ 5-0 & 0-10\end{bmatrix}=\begin{bmatrix}10 & 5\\ 5 & -10\end{bmatrix}$$

$$M^5=M^2.M^3=\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}\begin{bmatrix}10 & 5\\ 5 & -10\end{bmatrix}=\begin{bmatrix}50+0 & 25+0\\ 0+25 & 0-50\end{bmatrix}=\begin{bmatrix}50 & 25\\ 25 & -50\end{bmatrix}$$

Solution 4:

(i) $\begin{bmatrix}4 & 3x\\ x & -2\end{bmatrix}\begin{bmatrix}5\\ 1\end{bmatrix}=\begin{bmatrix}y\\ 8\end{bmatrix}$

$$\begin{bmatrix}20+3x\\ 5x-2\end{bmatrix}=\begin{bmatrix}y\\ 8\end{bmatrix}$$

Comparing the corresponding elements, we get,

$5x-2=8 \Rightarrow x=2$

$20+3x=y \Rightarrow y=20+6=26$

(ii) $\begin{bmatrix}x & 0\\ -3 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & y\end{bmatrix}=\begin{bmatrix}2 & 2\\ -3 & -2\end{bmatrix}$

$$\begin{bmatrix} x+0 & x+0 \\ -3+0 & -3+y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$$

$$\begin{bmatrix} x & x \\ -3 & -3+y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$x = 2$

$-3 + y = -2 \Rightarrow y = 1$

Solution 5:

$$\text{(i) } AB = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 1+12 & 2+9 \\ 2+16 & 4+12 \end{bmatrix} = \begin{bmatrix} 13 & 11 \\ 18 & 16 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} 13 & 11 \\ 18 & 16 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 52+11 & 39+22 \\ 72+16 & 54+32 \end{bmatrix} = \begin{bmatrix} 63 & 61 \\ 88 & 86 \end{bmatrix}$$

$$\text{(ii)} BC = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4+2 & 3+4 \\ 16+3 & 12+6 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 19 & 18 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 6 & 7 \\ 19 & 18 \end{bmatrix} = \begin{bmatrix} 6+57 & 7+54 \\ 12+76 & 14+72 \end{bmatrix} = \begin{bmatrix} 63 & 61 \\ 88 & 86 \end{bmatrix}$$

Solution 6:

$$\text{(i) } AB = \begin{bmatrix} 0 & 4 & 6 \\ 3 & 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 2 \\ -5 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 0-4-30 & 0+8-36 \\ 0-0+5 & 3+0+6 \end{bmatrix}$$

$$= \begin{bmatrix} -34 & -28 \\ 5 & 9 \end{bmatrix}$$

$$\text{(ii) } BA = \begin{bmatrix} 0 & 1 \\ -1 & 2 \\ -5 & -6 \end{bmatrix}\begin{bmatrix} 0 & 4 & 6 \\ 3 & 0 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} 0+3 & 0+0 & 0-1 \\ 0+6 & -4+0 & -6-2 \\ 0-18 & -20-0 & -30+6 \end{bmatrix}$$

$$=\begin{bmatrix} 3 & 0 & -1 \\ 6 & -4 & -8 \\ -18 & -20 & -24 \end{bmatrix}$$

(iii) Product AA (=A^2) is not possible as the number of columns of matrix A is not equal to its number of rows.

Solution 7:

(i) $AB=\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$

$$=\begin{bmatrix} 2-6+1 & 1-4+1 \\ 4+3+3 & 2+2+3 \end{bmatrix}$$

$$=\begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$$

(ii) Product BA is possible

$$BA=\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} 2+2 & -4+1 & 2+3 \\ 3+4 & -6+2 & 3+6 \\ 1+2 & -2+1 & 1+3 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & -3 & 5 \\ 7 & -4 & 9 \\ 3 & -1 & 4 \end{bmatrix}$$

Solution 8:

$$M^2=\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$=\begin{bmatrix}1+4 & 2+2\\ 2+2 & 4+1\end{bmatrix}$$

$$=\begin{bmatrix}5 & 4\\ 4 & 5\end{bmatrix}$$

$$2M+3I=2\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}+3\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}2 & 4\\ 4 & 2\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}5 & 4\\ 4 & 5\end{bmatrix}$$

Hence, $M^2 = 2M + 3I$.

Solution 9:

$$BA=\begin{bmatrix}0 & -b\\ 1 & 0\end{bmatrix}\begin{bmatrix}a & 0\\ 0 & 2\end{bmatrix}$$

$$=\begin{bmatrix}0+0 & 0-2b\\ a+0 & 0+0\end{bmatrix}$$

$$=\begin{bmatrix}0 & -2b\\ a & 0\end{bmatrix}$$

$$M^2=\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & -1-1\\ 1+1 & -1+1\end{bmatrix}$$

$$=\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}$$

Given, $BA = M^2$

$$\begin{bmatrix}0 & -2b\\ a & 0\end{bmatrix}=\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}$$

Comparing the corresponding elements, we get,

$a = 2$

$-2b = -2 \Longrightarrow b = 1$

Solution 10:

(i) $A-B=\begin{bmatrix}4 & 1\\ 2 & 3\end{bmatrix}-\begin{bmatrix}1 & 0\\ -2 & 1\end{bmatrix}=\begin{bmatrix}3 & 1\\ 4 & 2\end{bmatrix}$

(ii) $A^2=\begin{bmatrix}4 & 1\\ 2 & 3\end{bmatrix}\begin{bmatrix}4 & 1\\ 2 & 3\end{bmatrix}$

$=\begin{bmatrix}16+2 & 4+3\\ 8+6 & 2+9\end{bmatrix}$

$=\begin{bmatrix}18 & 7\\ 14 & 11\end{bmatrix}$

(iii) $AB=\begin{bmatrix}4 & 1\\ 2 & 3\end{bmatrix}\begin{bmatrix}1 & 0\\ -2 & 1\end{bmatrix}$

$=\begin{bmatrix}4-2 & 0+1\\ 2-6 & 0+3\end{bmatrix}$

$=\begin{bmatrix}2 & 1\\ -4 & 3\end{bmatrix}$

(iv) $A^2-AB+2B$

$=\begin{bmatrix}18 & 7\\ 14 & 11\end{bmatrix}-\begin{bmatrix}2 & 1\\ -4 & 3\end{bmatrix}+2\begin{bmatrix}1 & 0\\ -2 & 1\end{bmatrix}$

$=\begin{bmatrix}16 & 6\\ 18 & 8\end{bmatrix}+\begin{bmatrix}2 & 0\\ -4 & 2\end{bmatrix}$

$=\begin{bmatrix}18 & 6\\ 14 & 10\end{bmatrix}$

Solution 11:

(i) $A+B=\begin{bmatrix}1 & 4\\ 1 & -3\end{bmatrix}+\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}=\begin{bmatrix}2 & 6\\ 0 & -4\end{bmatrix}$

$(A+B)^2=\begin{bmatrix}2 & 6\\ 0 & -4\end{bmatrix}\begin{bmatrix}2 & 6\\ 0 & -4\end{bmatrix}$

$=\begin{bmatrix}4+0 & 12-24\\ 0+0 & 0+16\end{bmatrix}$

$=\begin{bmatrix}4 & -12\\ 0 & 16\end{bmatrix}$

(ii) $A^2 = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}$

$= \begin{bmatrix} 1+4 & 4-12 \\ 1-3 & 4+9 \end{bmatrix}$

$\begin{bmatrix} 5 & -8 \\ -2 & 13 \end{bmatrix}$

$B^2 = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1-2 & 2-2 \\ -1+1 & -2+1 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$A^2 + B^2 = \begin{bmatrix} 5 & -8 \\ -2 & 13 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 4 & -8 \\ -2 & 12 \end{bmatrix}$

(iii) Clearly, $(A + B)^2 \neq A^2 + B^2$

Solution 12:

$B^2 = B + \frac{1}{2}A$

$\frac{1}{2}A = B^2 - B$

$A = 2\left(B^2 - B\right)$

$B^2 = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 4+0 & 2+1 \\ 0+0 & 0+1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$

$B^2 - B = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}$

$\therefore A = 2\left(B^2 - B\right)$

$= 2\begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 0 & 0 \end{bmatrix}$

Solution 13:

$A = \begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix}$

$A^2 = \begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix}\begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix}$

$= \begin{bmatrix} 1+a & -1+b \\ -a+ab & a+b^2 \end{bmatrix}$

It is given that $A^2 = I$.

$\therefore \ = \begin{bmatrix} 1+a & -1+b \\ -a+ab & a+b^2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Comparing the corresponding elements, we get,

$1 + a = 1$

Therefore, $a = 0$

$-1 + b = 0$

Therefore, $b = 1$

Solution 14:

(i) $B + C = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 4 & 3 \end{bmatrix}$

$A(B+C) = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ 4 & 3 \end{bmatrix}$

$$=\begin{bmatrix}6+4 & 14+3\\ 0 & 0\end{bmatrix}=\begin{bmatrix}10 & 17\\ 0 & 0\end{bmatrix}$$

$$AB=\begin{bmatrix}2 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}2 & 3\\ 4 & 1\end{bmatrix}=\begin{bmatrix}4+4 & 6+1\\ 0 & 0\end{bmatrix}=\begin{bmatrix}8 & 7\\ 0 & 0\end{bmatrix}$$

$$AC=\begin{bmatrix}2 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}1 & 4\\ 0 & 2\end{bmatrix}=\begin{bmatrix}2+0 & 8+2\\ 0 & 0\end{bmatrix}=\begin{bmatrix}2 & 10\\ 0 & 0\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}8 & 7\\ 0 & 0\end{bmatrix}+\begin{bmatrix}2 & 10\\ 0 & 0\end{bmatrix}=\begin{bmatrix}10 & 17\\ 0 & 0\end{bmatrix}$$

Hence, $A(B+C)=AB+AC$

$$\text{(ii)}B-A=\begin{bmatrix}2 & 3\\ 4 & 1\end{bmatrix}-\begin{bmatrix}2 & 1\\ 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 2\\ 4 & 1\end{bmatrix}$$

$$(B-A)C=\begin{bmatrix}0 & 2\\ 4 & 1\end{bmatrix}\begin{bmatrix}1 & 4\\ 0 & 2\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0+4\\ 4+0 & 16+2\end{bmatrix}=\begin{bmatrix}0 & 4\\ 4 & 18\end{bmatrix}$$

$$BC=\begin{bmatrix}2 & 3\\ 4 & 1\end{bmatrix}\begin{bmatrix}1 & 4\\ 0 & 2\end{bmatrix}=\begin{bmatrix}2+0 & 8+6\\ 4+0 & 16+2\end{bmatrix}=\begin{bmatrix}2 & 14\\ 4 & 18\end{bmatrix}$$

$$AC=\begin{bmatrix}2 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}1 & 4\\ 0 & 2\end{bmatrix}=\begin{bmatrix}2+0 & 8+2\\ 0 & 0\end{bmatrix}=\begin{bmatrix}2 & 10\\ 0 & 0\end{bmatrix}$$

$$BC-AC=\begin{bmatrix}2 & 14\\ 4 & 18\end{bmatrix}-\begin{bmatrix}2 & 10\\ 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 4\\ 4 & 18\end{bmatrix}$$

Hence, $(B-A)C=BC-AC$

Solution 15:

$$A^2=\begin{bmatrix}1 & 4\\ 2 & 1\end{bmatrix}\begin{bmatrix}1 & 4\\ 2 & 1\end{bmatrix}=\begin{bmatrix}1+8 & 4+4\\ 2+2 & 8+1\end{bmatrix}=\begin{bmatrix}9 & 8\\ 4 & 9\end{bmatrix}$$

$$BC=\begin{bmatrix}-3 & 2\\ 4 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}=\begin{bmatrix}-3+0 & 0+4\\ 4+0 & 0+0\end{bmatrix}=\begin{bmatrix}-3 & 4\\ 4 & 0\end{bmatrix}$$

$$A^2+BC=\begin{bmatrix}9 & 8\\ 4 & 9\end{bmatrix}+\begin{bmatrix}-3 & 4\\ 4 & 0\end{bmatrix}=\begin{bmatrix}6 & 12\\ 8 & 9\end{bmatrix}$$

Solution 16:

(i) $\begin{bmatrix} 2 & 5 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 \\ 14 \end{bmatrix}$

$\begin{bmatrix} 2x+5y \\ 5x+2y \end{bmatrix} = \begin{bmatrix} -7 \\ 14 \end{bmatrix}$

Comparing the corresponding elements, we get,

$2x + 5y = -7$ ...(1)

$5x + 2y = 14$ ...(2)

Multiplying (1) with 2 and (2) with 5, we get,

$4x + 10y = -14$ ...(3)

$25x + 10y = 70$ ...(4)

Subtracting (3) from (4), we get,

$21x = 84 \Rightarrow x = 4$

From (2), $2y = 14 - 5x = 14 - 20 = -6 \Rightarrow y = -3$

(ii) $\begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

$\begin{bmatrix} -6 & -4 \\ -4 & -4 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

$\begin{bmatrix} -10 \\ -8 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

Comparing the corresponding elements, we get,

$x = -10$ and $y = -8$

(iii) $\begin{bmatrix} x+y & x-4 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} -7 & -11 \end{bmatrix}$

$\begin{bmatrix} -x-y+2x-8 & -2x-2y+2x-8 \end{bmatrix} = \begin{bmatrix} -7 & -11 \end{bmatrix}$

$\begin{bmatrix} -y+x-8 & -2y-8 \end{bmatrix} = \begin{bmatrix} -7 & -11 \end{bmatrix}$

Comparing the corresponding elements, we get,

$-2y - 8 = -11 \Rightarrow -2y = -3 \Rightarrow y = \frac{3}{2}$

$-y + x - 8 = -7$

$\Rightarrow -\frac{3}{2} + x - 8 = -7$

$\Rightarrow x = 1 + \frac{3}{2} = \frac{5}{2}$

Solution 17:

We know, the product of two matrices is defined only when the number of columns of first matrix is equal to the number of rows of the second matrix.

(i) Let the order of matrix M be a × b.

$$M_{a\times b} \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 1 & 2 \end{bmatrix}_{1\times 2}$$

Clearly, the order of matrix M is 1 × 2.

$$\text{Let } M = \begin{bmatrix} a & b \end{bmatrix}$$

$$M \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a+0 & a+2b \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$a = 1$ and $a + 2b = 2 \Longrightarrow 2b = 2 - 1 = 1 \Longrightarrow b = \frac{1}{2}$

$$\therefore\ M = \begin{bmatrix} a & b \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \end{bmatrix}$$

(ii) Let the order of matrix M be a x b.

$$\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}_{2\times 2} \times M_{a\times b} = \begin{bmatrix} 13 \\ 5 \end{bmatrix}_{2\times 1}$$

Clearly, the order of matrix M is 2 x 1.

$$\text{Let } M = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \times M = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} a+4b \\ 2a+b \end{bmatrix} = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$a + 4b = 13$ ....(1)

$2a + b = 5$ ....(2)

Multiplying (2) by 4, we get,

$8a + 4b = 20$ ....(3)

Subtracting (1) from (3), we get,

$7a = 7 \Longrightarrow a = 1$

From (2), we get,

$b = 5 - 2a = 5 - 2 = 3$

$\therefore M = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Solution 18:

$A^2 = \begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4+0 & 2x+x \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 4 & 3x \\ 0 & 1 \end{bmatrix}$

Given, $A^2 = B$

$\begin{bmatrix} 4 & 3x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix}$

Comparing the two matrices, we get,

$3x = 36 \Rightarrow x = 12$

Solution 19:

$\begin{bmatrix} p & q \end{bmatrix}\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 25 \end{bmatrix}$

$\begin{bmatrix} p^2 + q^2 \end{bmatrix} = \begin{bmatrix} 25 \end{bmatrix}$

$\therefore p^2 + q^2 = 25$

Since, p and q are positive integers, and $(3)^2 + (4)^2 = 9 + 16 = 25$.

Hence, p = 3 and q = 4 or p = 4 and q = 3

Solution 20:

AB = BA = B

We know that AI = IA = I, where I is the identity matrix.

Hence, B is the identity matrix.

Solution 21:

$AB = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = \begin{bmatrix} 3a+0 & 3b+0 \\ 0+0 & 0+4c \end{bmatrix} = \begin{bmatrix} 3a & 3b \\ 0 & 4c \end{bmatrix}$

$$A+B=\begin{bmatrix}3 & 0\\ 0 & 4\end{bmatrix}+\begin{bmatrix}a & b\\ 0 & c\end{bmatrix}=\begin{bmatrix}3+a & b\\ 0 & 4+c\end{bmatrix}$$

Given, AB = A+B

$$\therefore \begin{bmatrix}3a & 3b\\ 0 & 4c\end{bmatrix}=\begin{bmatrix}3+a & b\\ 0 & 4+c\end{bmatrix}$$

Comparing the corresponding elements, we get,

$3a = 3 + a$

$\Longrightarrow 2a = 3$

$\Longrightarrow a = \frac{3}{2}$

$3b = b \Longrightarrow b = 0$

$4c = 4 + c \Longrightarrow 3c = 4 \Longrightarrow c = \frac{4}{3}$

Solution 22:

$$\text{(i) } P^2=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}=\begin{bmatrix}1+4 & 2-2\\ 2-2 & 4+1\end{bmatrix}=\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}$$

$$Q^2=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}=\begin{bmatrix}1+0 & 0+0\\ 2+2 & 0+1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 4 & 1\end{bmatrix}$$

$$P^2-Q^2=\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}-\begin{bmatrix}1 & 0\\ 4 & 1\end{bmatrix}=\begin{bmatrix}4 & 0\\ -4 & 4\end{bmatrix}$$

$$P+Q=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}+\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}=\begin{bmatrix}2 & 2\\ 4 & 0\end{bmatrix}$$

$$P-Q=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}-\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}=\begin{bmatrix}0 & 2\\ 0 & -2\end{bmatrix}$$

$$(P+Q)(P-Q)=\begin{bmatrix}2 & 2\\ 4 & 0\end{bmatrix}\begin{bmatrix}0 & 2\\ 0 & -2\end{bmatrix}=\begin{bmatrix}0+0 & 4-4\\ 0+0 & 8-0\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 8\end{bmatrix}$$

Clearly, it can be said that:

$(P + Q)(P - Q) = P^2 - Q^2$ not true for matrix algebra.

Solution 23:

$$\text{(i) } AB=\begin{bmatrix}2 & 1\\ 4 & 2\end{bmatrix}\begin{bmatrix}3 & 4\\ -1 & -2\end{bmatrix}=\begin{bmatrix}6-1 & 8-2\\ 12-2 & 16-4\end{bmatrix}=\begin{bmatrix}5 & 6\\ 10 & 12\end{bmatrix}$$

$$ABC=\begin{bmatrix}5&6\\10&12\end{bmatrix}\begin{bmatrix}-3&1\\0&-2\end{bmatrix}=\begin{bmatrix}-15+0&5-12\\-30+0&10-24\end{bmatrix}=\begin{bmatrix}-15&-7\\-30&-14\end{bmatrix}$$

$$\text{(ii) } AC=\begin{bmatrix}2&1\\4&2\end{bmatrix}\begin{bmatrix}-3&1\\0&-2\end{bmatrix}=\begin{bmatrix}-6+0&2-2\\-12+0&4-4\end{bmatrix}=\begin{bmatrix}-6&0\\-12&0\end{bmatrix}$$

$$ACB=\begin{bmatrix}-6&0\\-12&0\end{bmatrix}\begin{bmatrix}3&4\\-1&-2\end{bmatrix}=\begin{bmatrix}-18-0&-24-0\\-36-0&-48-0\end{bmatrix}=\begin{bmatrix}-18&-24\\-36&-48\end{bmatrix}$$

Hence, ABC = ACB.

Solution 24:

$$\text{(i) } CA=\begin{bmatrix}-2&-3\\0&1\end{bmatrix}\begin{bmatrix}1&2\\3&4\end{bmatrix}=\begin{bmatrix}-2-9&-4-12\\0+3&0+4\end{bmatrix}=\begin{bmatrix}-11&-16\\3&4\end{bmatrix}$$

$$CA+B=\begin{bmatrix}-11&-16\\3&4\end{bmatrix}+\begin{bmatrix}6&1\\1&1\end{bmatrix}=\begin{bmatrix}-5&-15\\4&5\end{bmatrix}$$

$$\text{(ii)} CB=\begin{bmatrix}-2&-3\\0&1\end{bmatrix}\begin{bmatrix}6&1\\1&1\end{bmatrix}=\begin{bmatrix}-12-3&-2-3\\0+1&0+1\end{bmatrix}=\begin{bmatrix}-15&-5\\1&1\end{bmatrix}$$

$$A+CB=\begin{bmatrix}1&2\\3&4\end{bmatrix}+\begin{bmatrix}-15&-5\\1&1\end{bmatrix}=\begin{bmatrix}-14&-3\\4&5\end{bmatrix}$$

Thus, CA + B $\neq$ A + CB

Solution 25:

Let the order of the matrix X be a × b

AX=B

$$\begin{bmatrix}2&1\\1&3\end{bmatrix}_{2\times2}\times X_{a\times b}=\begin{bmatrix}3\\-11\end{bmatrix}_{2\times1}$$

Clearly, the order of matrix X is 2 x 1.

$$\text{Let } X=\begin{bmatrix}x\\y\end{bmatrix}$$

$$\begin{bmatrix}2&1\\1&3\end{bmatrix}\times\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}3\\-11\end{bmatrix}$$

$$\begin{bmatrix}2x+y\\x+3y\end{bmatrix}=\begin{bmatrix}3\\-11\end{bmatrix}$$

Comparing the two matrices, we get,

2x + y = 3 … (1)
x + 3y = − 11 … (2)
Multiplying (1) with 3, we get,
6x + 3y = 9 … (3)
Subtracting (2) from (3), we get,
5x = 20
x = 4
From (1), we have:
y = 3 − 2x = 3 − 8 = −5

$$\therefore x = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$$

Solution 26:

$$A - 2I = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix}$$

$$A - 3I = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$$

$$(A - 2I)(A - 3I) = \begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 4-4 \\ 1-1 & 2+2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

Solution 27:

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 2 & 0 \\ 1 & 1 \\ -1 & -2 \end{bmatrix}$$

$$(i)A^t.A = \begin{bmatrix} 2 & 0 \\ 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 2+0 & -2-0 \\ 2+0 & 1+1 & -1-2 \\ -2-0 & -1-2 & 1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & -2 \\ 2 & 2 & -3 \\ -2 & -3 & 5 \end{bmatrix}$$

$$(ii)A.A^t = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+1 & 0+1+2 \\ 0+1+2 & 0+1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 3 \\ 3 & 5 \end{bmatrix}$$

Solution 28:

$$M^2 = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 16-1 & 4+2 \\ -4-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$6M - M^2 = 6\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 24 & 6 \\ -6 & 12 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$$= 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 9I$$

Hence Proved.

Solution 29:

$$PQ=\begin{bmatrix}2 & 6\\ 3 & 9\end{bmatrix}\begin{bmatrix}3 & x\\ y & 2\end{bmatrix}=\begin{bmatrix}6+6y & 2x+12\\ 9+9y & 3x+18\end{bmatrix}$$

PQ = Null matrix

$$\therefore \begin{bmatrix}6+6y & 2x+12\\ 9+9y & 3x+18\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

Comparing the corresponding elements, we get,

$2x + 12 = 0$

Therefore $x = -6$

$6 + 6y = 0$

Therefore $y = -1$

Solution 30:

$$\begin{bmatrix}2\cos 60^{o} & -2\sin 30^{o}\\ -\tan 45^{o} & \cos 0^{o}\end{bmatrix}\begin{bmatrix}\cot 45^{o} & \operatorname{cosec} 30^{o}\\ \sec 60^{o} & \sin 90^{o}\end{bmatrix}$$

$$=\begin{bmatrix}2\times\frac{1}{2} & -2\times\frac{1}{2}\\ -1 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1-2 & 2-1\\ -1+2 & -2+1\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$$

Solution 31:

(i) True.

Addition of matrices is commutative.

(ii) False.

Subtraction of matrices is commutative.

(iii) True.

Multiplication of matrices is associative.

(iv) True.

Multiplication of matrices is distributive over addition.

(v) True.
Multiplication of matrices is distributive over subtraction.
(vi) True.
Multiplication of matrices is distributive over subtraction.
(vii) False.
Laws of algebra for factorization and expansion are not applicable to matrices.
(viii) False.
Laws of algebra for factorization and expansion are not applicable to matrices.

EXERCISE 9 (D)

Solution 1:

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4\begin{bmatrix} 2 \\ y \end{bmatrix}$$

$$\begin{bmatrix} 6x & -2 \\ -2x & +4 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

$$\begin{bmatrix} 6x-10 \\ -2x+14 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

Comparing the corresponding elements, we get,
$6x - 10 = 8$
$\Rightarrow 6x = 18$
$\Rightarrow x = 3$
$-2x + 14 = 4y$
$\Rightarrow 4y = -6 + 14 = 8$
$\Rightarrow y = 2$

Solution 2:

$$\begin{bmatrix} 3x & 8 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix} - 3\begin{bmatrix} 2 & -7 \end{bmatrix} = 5\begin{bmatrix} 3 & 2y \end{bmatrix}$$

$$\begin{bmatrix} 3x+24 & 12x+56 \end{bmatrix} - \begin{bmatrix} 6 & -21 \end{bmatrix} = \begin{bmatrix} 15 & 10y \end{bmatrix}$$

$$\begin{bmatrix} 3x+24-6 & 12x+56+21 \end{bmatrix} = \begin{bmatrix} 15 & 10y \end{bmatrix}$$

$$\begin{bmatrix} 3x+18 & 12x+77 \end{bmatrix} = \begin{bmatrix} 15 & 10y \end{bmatrix}$$

Comparing the corresponding elements, we get,

3x + 18 = 15
3x = −3
x = −1
12x + 77 = 10y
10y = −12 + 77 = 65
y = 6.5

Solution 3:

$$\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 25 \end{bmatrix}$$

$$x^2 + y^2 = 25$$

and

$$-2x^2 + y^2 = -2$$

(i) x, y Î W (whole numbers)
It can be observed that the above two equations are satisfied when x = 3 and y = 4.
(ii) x, y Î Z (integers)
It can be observed that the above two equations are satisfied when x = ±3 and y = ± 4.

Solution 4:

(i) let the order of matrix X be a × b

$$\therefore \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}_{2\times 2} \times X_{a\times b} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}_{2\times 1}$$

⇒ a = 2 and b = 1
∴ The order of the matrix X = a × b = 2 × 1

(ii) Let X = $\begin{bmatrix} x \\ y \end{bmatrix}$

$$\therefore \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x + y \\ -3x + 4y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

⇒ 2x + y = 7 and – 3x + 4y = 6
On solving the above simultaneous equations
In x and y, we have, x = 2 and y = 3

$$\therefore \text{ The matrix } X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Solution 5:

$$\begin{bmatrix} \cos 45^o & \sin 30^o \\ \sqrt{2}\cos 0^o & \sin 0^o \end{bmatrix}\begin{bmatrix} \sin 45^o & \cos 90^o \\ \sin 90^o & \cot 45^o \end{bmatrix}$$

$$= \begin{vmatrix} \frac{1}{\sqrt{2}} & \frac{1}{2} \\ \sqrt{2} & 0 \end{vmatrix}\begin{vmatrix} \frac{1}{\sqrt{2}} & 0 \\ 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \frac{1}{2}+\frac{1}{2} & 0+\frac{1}{2} \\ 1+0 & 0+0 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0.5 \\ 1 & 0 \end{vmatrix}$$

Solution 6:

Let the order of matrix M be a × b.

3A × M = 2B

$$3\begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix}_{2\times 2} \times M_{a\times b} = 2\begin{bmatrix} -5 \\ 6 \end{bmatrix}_{2\times 1}$$

Clearly, the order of matrix M is 2 × 1

$$\text{let } M = \begin{bmatrix} x \\ y \end{bmatrix}$$

then,

$$3\begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = 2\begin{bmatrix} -5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -3 \\ 12 & -9 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 0-3y \\ 12x-9y \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} -3y \\ 12x-9y \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$-3y = -10$

$\Longrightarrow y = \frac{10}{3}$

$\Rightarrow 12x - 9y = 12$

$\Rightarrow 12x - 30 = 12$

$\Rightarrow x = \frac{7}{2}$

$$\therefore\ M = \begin{bmatrix} \frac{7}{2} \\ \frac{10}{3} \end{bmatrix}$$

Solution 7:

$$\begin{bmatrix} a & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

$$\begin{bmatrix} a+1 & 2+b \\ 7 & -1-c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$a + 1 = 5 \Rightarrow a = 4$

$2 + b = 0 \Rightarrow b = -2$

$-1 - c = 3 \Rightarrow c = -4$

Solution 8:

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

(i)

$$BA = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 4+1 \\ 1+4 & 2+2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$A(BA)=\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}4 & 5\\5 & 4\end{bmatrix}$$

$$=\begin{bmatrix}4+10 & 5+8\\8+5 & 10+4\end{bmatrix}$$

$$=\begin{bmatrix}14 & 13\\13 & 14\end{bmatrix}$$

(ii)

$$AB=\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}2+2 & 1+4\\4+1 & 2+2\end{bmatrix}$$

$$=\begin{bmatrix}4 & 5\\5 & 4\end{bmatrix}$$

$$(AB)B=\begin{bmatrix}4 & 5\\5 & 4\end{bmatrix}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}8+5 & 4+10\\10+4 & 5+8\end{bmatrix}$$

$$=\begin{bmatrix}13 & 14\\14 & 13\end{bmatrix}$$

Solution 9:

$$\begin{bmatrix}x & 3x\\y & 4y\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}5\\12\end{bmatrix}$$

$$\begin{bmatrix}2x+3x\\2y+4y\end{bmatrix}=\begin{bmatrix}5\\12\end{bmatrix}$$

$$\begin{bmatrix}5x\\6y\end{bmatrix}=\begin{bmatrix}5\\12\end{bmatrix}$$

Comparing the corresponding elements, we get,

$5x = 5 \Rightarrow x = 1$

$6y = 12 \Rightarrow y = 2$

Solution 10:

$$X=\begin{bmatrix}-3 & 4\\ 2 & -3\end{bmatrix}\begin{bmatrix}2\\ -2\end{bmatrix}$$

$$=\begin{bmatrix}-6-8\\ 4+6\end{bmatrix}$$

$$=\begin{bmatrix}-14\\ 10\end{bmatrix}$$

$$\text{Given, } 2X-3Y=\begin{bmatrix}10\\ -8\end{bmatrix}$$

$$2\begin{bmatrix}-14\\ 10\end{bmatrix}-3Y=\begin{bmatrix}10\\ -8\end{bmatrix}$$

$$3Y=2\begin{bmatrix}-14\\ 10\end{bmatrix}-\begin{bmatrix}10\\ -8\end{bmatrix}$$

$$3Y=\begin{bmatrix}-28\\ 20\end{bmatrix}-\begin{bmatrix}10\\ -8\end{bmatrix}$$

$$3Y=\begin{bmatrix}-38\\ 28\end{bmatrix}$$

$$Y=\frac{1}{3}\begin{bmatrix}-38\\ 28\end{bmatrix}$$

Solution 11:

Given, A + X = 2B + C

$$\begin{bmatrix}2 & -1\\ 2 & 0\end{bmatrix}+X=2\begin{bmatrix}-3 & 2\\ 4 & 0\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}$$

$$\begin{bmatrix}2 & -1\\ 2 & 0\end{bmatrix}+X=\begin{bmatrix}-6 & 4\\ 8 & 0\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}$$

$$\begin{bmatrix}2 & -1\\ 2 & 0\end{bmatrix}+X=\begin{bmatrix}-5 & 4\\ 8 & 2\end{bmatrix}$$

$$X=\begin{bmatrix}-5 & 4\\ 8 & 2\end{bmatrix}-\begin{bmatrix}2 & -1\\ 2 & 0\end{bmatrix}$$

$$X=\begin{bmatrix}-7 & 5\\ 6 & 2\end{bmatrix}$$

Solution 12:

$A = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4+0 & 24+12 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix}$

Given, $A^2 = B$

$\therefore \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$

Comparing the corresponding elements, we get,

$x = 36$

Solution 13:

Transpose of a matrix is the matrix obtained on

Interchanging its rows and columns

Thus, if $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, then $A^t = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

Identify matrix of order 2 is $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Two matrices can be multiplied together if and only

If the number of columns in the first matrix is equal

To the number of rows in B square matrices

Since the matrices A^t and B are square matrices

The condition of compatibility for multiplication of matrices is satisfied.

Let us compute A^tB.

$A^tB = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \times \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$

$\Rightarrow A^tB = \begin{bmatrix} 8-1 & -4+3 \\ 20-3 & -10+9 \end{bmatrix}$

$\Rightarrow A^tB = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix}$

Since I is identity matrix for multiplication

We have $BI = B = IB$

Two matrices are compatible for addition, only

When they have the same order
Both A^tB and BI are of the same order
Hence matrix addition is possible
Thus,

$$A^tB + BI = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7+4 & -1-2 \\ 17-1 & -1+3 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & -3 \\ 16 & 2 \end{bmatrix}$$

Solution 14:

$$\text{Given } 2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

On comparing, corresponding elements,
$8 + y = 0 \Rightarrow y = ?8$
$2x + 1 = 5 \Rightarrow 2x = 5 - 1 \Rightarrow 2x = 4 \Rightarrow x = 2$
Hence, $x = 2$, $y = -8$

Solution 15:

$$A^2 = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 16-12 & -8+6 \\ 24-18 & -12+9 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$$

$$BC = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0+2 & 0-2 \\ -2-1 & 3+1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$$

$$A^2 - A + BC = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}+\begin{bmatrix}2 & -2\\-3 & 4\end{bmatrix}$$

$$=\begin{bmatrix}2 & -2\\-3 & 4\end{bmatrix}$$

Solution 16:

$$A=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}, B=\begin{bmatrix}2 & 3\\-1 & 0\end{bmatrix}$$

$$A^2=A\times A=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}\times\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1\times1+0\times2 & 1\times0+0\times1\\2\times1+1\times2 & 2\times0+1\times1\end{bmatrix}=\begin{bmatrix}1 & 0\\4 & 1\end{bmatrix}$$

$$AB=A\times B=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}\times\begin{bmatrix}2 & 3\\-1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}12\times+0\times(-1) & 1\times3+0\times0\\2\times2+1\times(-1) & 2\times3+1\times0\end{bmatrix}$$

$$=\begin{bmatrix}2 & 3\\3 & 6\end{bmatrix}$$

$$B^2=B\times B=\begin{bmatrix}2 & 3\\-1 & 0\end{bmatrix}\times\begin{bmatrix}2 & 3\\-1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}2\times2+3\times(-1) & 2\times3+3\times0\\(-1)\times2+0\times(-1) & -1\times3+0\times0\end{bmatrix}$$

$$=\begin{bmatrix}1 & 6\\-2 & -3\end{bmatrix}$$

$$\therefore\ A^2+AB+B^2=\begin{bmatrix}1 & 0\\4 & 1\end{bmatrix}+\begin{bmatrix}2 & 3\\3 & 6\end{bmatrix}+\begin{bmatrix}1 & 6\\-2 & -3\end{bmatrix}$$

$$=\begin{bmatrix}4 & 9\\5 & 4\end{bmatrix}$$

Solution 17:

$$3A-2C=6B$$

$$3\begin{bmatrix}3 & a\\-4 & 8\end{bmatrix}-2\begin{bmatrix}-1 & 4\\3 & b\end{bmatrix}=6\begin{bmatrix}c & 4\\-3 & 0\end{bmatrix}$$

$$\begin{bmatrix} 9 & 3a \\ -12 & 24 \end{bmatrix} - \begin{bmatrix} -2 & 8 \\ 6 & 2b \end{bmatrix} = \begin{bmatrix} 6c & 24 \\ -18 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 11 & 3a-8 \\ -18 & 24-2b \end{bmatrix} = \begin{bmatrix} 6c & 24 \\ -18 & 0 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$3a - 8 = 24 \Longrightarrow 3a = 32 \Longrightarrow a = \frac{32}{3} = 10\frac{2}{3}$

$24 - 2b = 0 \Longrightarrow 2b = 24 \Longrightarrow b = 12$

$11 = 6c \Longrightarrow c = \frac{11}{6} = 1\frac{5}{6}$

Solution 18:

$$A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix} B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}\begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -2q \\ p & 0 \end{bmatrix}$$

$$C^2 = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$$

$$BA = C^2 \Rightarrow \begin{bmatrix} 0 & -2q \\ p & 0 \end{bmatrix} = \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$$

By comparing,

$-2q = -8 \Longrightarrow q = 4$

And $p = 8$

Solution 19:

$$AB = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 18-2 \\ -6+4 \end{bmatrix} = \begin{bmatrix} 16 \\ -2 \end{bmatrix}$$

$$\therefore\ AB + 2C - 4D = \begin{bmatrix} 16 \\ -2 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} - \begin{bmatrix} 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solution 20:

$$\begin{bmatrix} 4\sin 30^\circ & 2\cos 60^\circ \\ \sin 90^\circ & 2\cos 0^\circ \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \times \frac{1}{2} & 2 \times \frac{1}{2} \\ 1 & 2 \times 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 8+5 & 10+4 \\ 4+10 & 5+8 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$$

Solution 21:

Given that $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

We need to find $A^2 - 5A + 7I$

$$A^2 = A \times A$$

$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$$

$$7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$