2 POLYNOMIALS

EXERCISE 2.1

Q.1. *Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.*

(i) $4x^2 - 3x + 7$ *(ii)* $y^2 + \sqrt{2}$ *(iii)* $3\sqrt{t} + t\sqrt{2}$ *(iv)* $y + \frac{2}{y}$ *(v)* $x^{10} + y^3 + t^{50}$

Sol. (i) Polynomial in one variable, x **Ans.**

(ii) Polynomial in one variable, y. **Ans.**

(iii) $3\sqrt{t} + t\sqrt{2}$ is not a polynomial as power of t in $\sqrt{t}$ is not a whole number. **Ans.**

(iv) $y + \frac{2}{y}$ is not a polynomial as power of y in second term, i.e., $\frac{1}{y} = y^{-1}$ is not a whole number. **Ans.**

(v) $x^{10} + y^3 + t^{50}$ is not a polynomial in one variable but a polynomial in three variables x, y and t. **Ans.**

Q.2. *Write the coefficients of x^2 in each of the following :*

(i) $2 + x^2 + x$ *(ii)* $2 - x^2 + x^3$ *(iii)* $\frac{\pi}{2}x^2 + x$ *(iv)* $\sqrt{2}x - 1$

Sol. (i) In $2 + x^2 + x$, coefficient of x^2 is 1. **Ans.**

(ii) In $2 - x^2 + x^3$, coefficient of x^2 is -1. **Ans.**

(iii) $\frac{\pi}{2}x^2 + x$, coefficient of x^2 is $\frac{\pi}{2}$. **Ans.**

(iv) $\sqrt{2}x - 1$, x^2 is not present hence no coefficient. **Ans.**

Q.3. *Give one example each of a binomial of degree 35, and of a monomial of degree* 100.

Sol. $x^{35} + 5$ is a binomial of degree 35.

$2y^{100}$ is a monomial of degree 100. **Ans.**

Q.4. *Write the degree of each of the following polynomials :*

(i) $5x^3 + 4x^2 + 7x$ *(ii)* $4 - y^2$ *(iii)* $5t - \sqrt{7}$ *(iv)* 3

Sol. (i) Degree is 3 as x^3 is the highest power. **Ans.**

(ii) Degree is 2 as y^2 is the highest power. **Ans.**

(iii) Degree is 1 as t is the highest power. **Ans.**

(iv) Degree is 0 as x^0 is the highest power. **Ans.**

Q.5. *Classify the following as linear, quadratic and cubic polynomials :*

(i) $x^2 + x$ *(ii)* $x - x^3$ (iii) $y + y^2 + 4$

(iv) $1 + x$ *(v)* $3t$ (vi) r^2 (vii) $7x^3$

Sol. (i) $x^2 + x$ is quadratic. (ii) $x - x^3$ is cubic.

(iii) $y + y^2 + 4$ is quadratic. (iv) $1 + x$ is linear.

(v) $3t$ is linear. (vi) r^2 is quadratic.

(vii) $7x^3$ is cubic.

EXERCISE 2.2

Q.1. *Find the value of the polynomial* $5x - 4x^2 + 3$ *at*

(i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$

Sol. $p(x) = 5x - 4x^2 + 3$

(i) At $x = 0$, $p(0) = 5 \times 0 - 4 \times 0^2 + 3 = 3$ **Ans.**

(ii) At $x = -1$, $p(-1) = 5 \times (-1) - 4 \times (-1)^2 + 3 = -5 - 4 + 3 = -6$ **Ans.**

(iii) At $x = 2$, $p(2) = 5 \times 2 - 4 \times (2)^2 + 3 = 10 - 16 + 3 = -3$ **Ans.**

Q.2. *Find* $p(0)$, $p(1)$ *and* $p(2)$ *for each of the following polynomials :*

(i) $p(y) = y^2 - y + 1$ (ii) $p(t) = 2 + t + 2t^2 - t^3$

(iii) $p(x) = x^3$ (iv) $p(x) = (x - 1)(x + 1)$

Sol. (i) $p(y) = y^2 - y + 1$

$p(0) = 0^2 - 0 + 1 = 1$

$p(1) = 1^2 - 1 + 1 = 1$

$p(2) = 2^2 - 2 + 1 = 3$. **Ans.**

(ii) $p(t) = 2 + t + 2t^2 - t^3$

$p(0) = 2 + 0 + 2 \times 0^2 - 0^3 = 2$

$p(1) = 2 + 1 + 2 \times 1^2 - 1^3 = 4$

$p(2) = 2 + 2 + 2 \times 2^2 - 2^3 = 4 + 8 - 8 = 4$.

(iii) $p(x) = x^3$

$p(0) = 0$

$p(1) = 1$

$p(2) = 8$. **Ans.**

(iv) $p(x) = (x - 1)(x + 1)$

$p(0) = (-1)(1) = -1$

$p(1) = (1 - 1)(1 + 1) = 0$

$p(2) = (2 - 1)(2 + 1) = 3$ **Ans.**

Q.3. *Verify whether the following are zeroes of the polynomial, indicated against them.*

(i) $p(x) = 3x + 1$, $x = -\frac{1}{3}$ (ii) $p(x) = 5x - \pi$, $x = \frac{4}{5}$

(iii) $p(x) = x^2 - 1$, $x = 1, -1$

(iv) $p(x) = (x + 1)(x - 2)$, $x = -1, 2$

(v) $p(x) = x^2$, $x = 0$ (vi) $p(x) = lx + m$, $x = -\frac{m}{l}$

(vii) $p(x) = 3x^2 - 1$, $x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(viii) $p(x) = 2x + 1$, $x = \frac{1}{2}$

Sol. (i) Yes. $3x + 1 = 0$, for $x = -\frac{1}{3}$. **Ans.**

(ii) No. $5x - \pi = 5 \times \frac{4}{5} - \pi = 4 - \pi \neq 0$ **Ans.**

(iii) Yes. $x^2 - 1 = 1^2 - 1 = 0$ for $x = 1$

and $x^2 - 1 = (-1)^2 - 1 = 0$ for $x = 1$ **Ans.**

(iv) Yes. $(x + 1)(x - 2) = 0$ for $x = -1$, or, $x = 2$.

(v) Yes. $x^2 = 0$ for $x = 0$

(vi) Yes. $lx + m = 0$ for $x = -\dfrac{m}{l}$

(vii) $3x^2 - 1 = 3\dfrac{1}{3} - 1 = 0$ for $x = \dfrac{-1}{\sqrt{3}}$

and $3x^2 - 1 = 3\cdot\dfrac{4}{3} - 1 = 3 \neq 0$

Thus, for $\dfrac{-1}{\sqrt{3}}$ is a zero but $-\dfrac{2}{\sqrt{3}}$ is not a zero of the polynomial **Ans.**

(viii) No. $2x + 1 \neq 0$ for $x = \dfrac{1}{2}$.

Q.4. *Find the zero of the polynomial in each of the following cases :*

(i) $p(x) = x + 5$ *(ii)* $p(x) = x - 5$

(iii) $p(x) = 2x + 5$ *(iv)* $p(x) = 3x - 2$

(v) $p(x) = 3x$ *(vi)* $p(x) = ax,\ a \neq 0$

(vii) $p(x) = cx + d,\ c \neq 0,\ c,\ d$ are real numbers.

Sol. (i) $x + 5 = 0,\ x = -5$, so, -5 is the zero of $x + 5$ **Ans.**

(ii) $x - 5 = 0,\ x = 5$ so, 5 is the zero of $x - 5$ **Ans.**

(iii) $2x + 5 = 0, \Rightarrow 2x = -5$,

$\Rightarrow x = \dfrac{-5}{2}$, so $-\dfrac{5}{2}$ is the zero of $2x + 5$ **Ans.**

(iv) $3x - 2 = 0 \Rightarrow 3x = 2$

$\Rightarrow x = \dfrac{2}{3}$, so $\dfrac{2}{3}$ is the zero of $3x - 2$ **Ans.**

(v) $3x = 0, \Rightarrow x = 0$, so 0 is the zero of $3x$ **Ans.**

(vi) $ax = 0\ (a \neq 0) \Rightarrow x = \dfrac{0}{a} = 0$, so, is the zero of ax **Ans.**

(vii) $cx + d = 0\ (c \neq 0)$

$\Rightarrow cx = -d$

$\Rightarrow x = \dfrac{-d}{c}$, so, $\dfrac{-d}{c}$ is the 0 of $cx + d$ **Ans.**

EXERCISE 2.3

Q.1. *Find the remainder when* $x^3 + 3x^2 + 3x + 1$ *is divided by*

(i) $x + 1$ *(ii)* $x - \dfrac{1}{2}$ *(iii)* x

(iv) $x + \pi$ *(v)* $5 + 2x$

Sol. $p(x) = x^3 + 3x^2 + 3x + 1$

(i) When $p(x)$ is divided by $x + 1$,

i.e., $x + 1 = 0,\ x = -1$ is to be substituted in $p(x)$.

$p(-1) = (-1)^3 + 3\,(-1)^2 + 3\,(-1) + 1$

$= -1 + 3 - 3 + 1 = 0$

Remainder = 0. **Ans.**

(ii) When $p(x)$ is divided by $x - \dfrac{1}{2}$ remainder is $p\left(\dfrac{1}{2}\right)$.

$$p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1$$

$$= \frac{1}{8}+\frac{3}{4}+\frac{3}{2}+1 = \frac{1+6+12+8}{8}$$

$$\therefore \text{ Remainder } = \frac{27}{8} = 3\frac{3}{8} \quad \textbf{Ans.}$$

(iii) When $p(x)$ is divided by x, then remainder is $p(0)$.

$x = 0$, substitute in $p(x)$

$p(0) = 0^3 + 3 \times 0^2 + 3 \times 0 + 1 = 1.$

$\therefore$ Remainder $= 1$ **Ans.**

(iv) Wgeb $p(x)$ is divided by $x + \pi$, then, remainder is $p(-\pi)$.

$x = -\pi$ to be substituted in $p(x)$

$p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1.$

$\therefore$ Remainder $= -\pi^3 + 3\pi^2 - 3\pi + 1$ **Ans.**

(v) When $p(x)$ is divided by $(5 + 2x)$, then remainder is $p\left(\frac{-5}{2}\right)$.

$$p\left(\frac{-5}{2}\right) = \left(\frac{-5}{2}\right)^3 + 3\left(\frac{-5}{2}\right)^2 + 3\left(\frac{-5}{2}\right) + 1$$

$$= \frac{-125}{8}+\frac{75}{4}-\frac{15}{2}+1 = \frac{-125+150-60+8}{8}$$

$$\text{Remainder} = \frac{-35+8}{8} = \frac{-27}{8} \quad \textbf{Ans.}$$

Q.2. *Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.*

Sol. $p(x) = x^3 - ax^2 + 6x - a$

When $p(x)$ is divided by $x - a$, the remainder is $p(a)$.

Substitute $x = a$ in $p(x)$

$p(a) = a^3 - a^3 + 6a - a = 5a$ **Ans.**

Q.3. *Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.*

Sol. $7 + 3x = 0$

$\Rightarrow \quad 3x = -7$

$\Rightarrow \quad x = \frac{-7}{3}$

Substitute $x = \frac{-7}{3}$ in $p(x) = 3x^3 + 7x$

$$p\left(\frac{-7}{3}\right) = 3\left(\frac{-7}{3}\right)^3 + 7\left(\frac{-7}{3}\right) = \frac{-343}{9} - \frac{49}{3} = \frac{-343-147}{9} = \frac{-490}{9}.$$

So, remainder $= \frac{-490}{9}$ which is different from 0.

Therefore, $(3x + 7)$ is not a factor of the polynomial $3x^3 + 7x$. **Ans.**

EXERCISE 2.4

Q.1. *Determine which of the following polynomials has $(x + 1)$ a factor :*

(i) $x^3 + x^2 + x + 1$ *(ii)* $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$ *(iv)* $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

Sol. To have $(x + 1)$ as a factor, substituting $x = -1$ must give $p(-1) = 0$.

(i) $x^3 + x^2 + x + 1$

$= (-1)^3 + (-1)^2 + (-1) + 1 = -1 + 1 - 1 + 1 = 0$

Therefore, $x + 1$ is a factor of $x^3 + x^2 + x + 1$ **Ans.**

(ii) $x^4 + x^3 + x^2 + x + 1$

$= (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1 = 1 - 1 + 1 - 1 + 1 = 1$

Remainder is not 0. Therefore $(x + 1)$ is not its factor. **Ans.**

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

$= (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 = 1$. Remaidner is not 0

Therefore, $(x + 1)$ is not its factor. **Ans.**

(iv) $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

$= (-1)^3 - (-1)^2 - (2+\sqrt{2})(-1) + \sqrt{2}$

$= -1 - 1 + 2+\sqrt{2}+\sqrt{2} = 2\sqrt{2}$

Remainder not 0, therefore $(x + 1)$ is not a factor. **Ans.**

Q.2. *Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases :*

(i) $p(x) = 2x^3 + x^2 - 2x - 1,\ g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1,\ g(x) = x + 2$

(iii) $p(x) = x^3 - 4x^2 + x + 6,\ g(x) = x - 3$

Sol. (i) $g(x) = x + 1$. $x = -1$ to be substituted in

$p(x) = 2x^3 + x^2 - 2x - 1$

$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1 = -2 + 1 + 2 - 1 = 0.$

So, $g(x)$ is a factor of $p(x)$. **Ans.**

(ii) $g(x) = x + 2$, substitute $x = -2$ in $p(x)$

$p(x) = x^3 + 3x^2 + 3x + 1$

$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1 = -8 + 12 - 6 + 1 = -1.$

So, $g(x)$ is not a factor of $p(x)$ **Ans.**

(iii) $g(x) = x - 3$ substitute $x = 3$ in (x).

$p(x) = x^3 - 4x^2 + x + 6$

$p(3) = (3)^3 - 4(3)^2 + 3 + 6 = 27 - 36 + 3 + 6 = 0.$

Therefore, $g(x)$ is a factor of $x^3 - 4x^2 + x + 6$. **Ans.**

Q.3. *Find the value of k, if x – 1 is a factor of p(x) in each of the following cases :*

(i) $p(x) = x^2 + x + k$ (ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$ *(iv)* $p(x) = kx^2 - 3x + k$

Sol. $(x - 1)$ is a factor, so we substitute $x = 1$ in each case and solve for k by making $p(1)$ equal to 0.

(i) $p(x) = x^2 + x + k$

$p(1) = 1 + 1 + k = 0 \Rightarrow k = -2$ **Ans.**

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

$p(1) = 2 \times 1^2 + k \times 1 + \sqrt{2} = 0$

$\Rightarrow \quad 2 + k + \sqrt{2} = 0$

$\Rightarrow \quad k = -2 - \sqrt{2} = -(2 + \sqrt{2})$ **Ans.**

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

$p(1) = k - \sqrt{2} + 1 = 0$

$\Rightarrow \quad k = \sqrt{2} - 1$ **Ans.**

(iv) $p(x) = kx^2 - 3x + k$

$p(1) = k - 3 + k = 0 \Rightarrow 2k - 3 = 0$

$\Rightarrow \quad k = \dfrac{3}{2}$ **Ans.**

Q.4. *Factorise :*

(i) $12x^2 - 7x + 1$ *(ii)* $2x^2 + 7x + 3$ *(iii)* $6x^2 + 5x - 6$ *(iv)* $3x^2 - x - 4$

Sol. (i) $12x^2 - 7x + 1$

$= 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1) = (4x - 1)(3x - 1)$ **Ans.**

(ii) $2x^2 + 7x + 3$

$= 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3) = (2x + 1)(x + 3)$ **Ans.**

(iii) $6x^2 + 5x - 6$

$= 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3) = (3x - 2)(2x + 3)$ **Ans.**

(iv) $3x^2 - x - 4$

$= 3x^2 - 4x + 3x - 4 = x(3x - 4) + 1(3x - 4) = (x + 1)(3x - 4)$ **Ans.**

Q.5. *Factorise :*

(i) $x^3 - 2x^2 - x + 2$ *(ii)* $x^3 - 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + 32x + 20$ *(iv)* $2y^3 + y^2 - 2y - 1$

Sol. (i) $p(x)\ x^3 - 2x^2 - x + 2$

Let us guess a factor $(x - a)$ and choose value of a arbitrarily as 1.

Now, putting this value in $p(x)$.

$1 - 2 - 1 + 2 = 0$

So $(x - 1)$ is a factor of $p(x)$

Now, $x^3 - 2x^2 - x + 2 = x^3 - x^2 - x^2 + x - 2x + 2$

$= x^2(x - 1) - x(x - 1) - 2(x - 1)$

$= (x - 1)(x^2 - x - 2)$

$= (x - 1)(x^2 - 2x + x - 2)$

$= (x - 1)\{x(x - 2) + 1(x - 2)\}$

$= (x - 1)(x + 1)(x - 2)$ **Ans.**

To factorise it

$x^2 - 2x + x - 2 = x(x - 2) + 1(x - 2) = (x + 1)(x - 2)$.

After factorisation $(x - 1)(x + 1)(x - 2)$.

(ii) $p(x) = x^3 - 3x^2 - 9x - 5$

Take a factor $(x - a)$. a should be a factor of 5, i.e., ± 1 or ± 5.

For $(x - 1)$, $a = 1$

$p(1) = (1)^3 - (-3)\ 1^2 - 9 \times 1 - 5$

$= 1 - 3 - 9 - 5 = -16$.

So, $(x - 1)$ is not a factor of $p(x)$.

For $a = 5$

$p(5) = (5)^3 - 3(5)^2 - 9\ (5) - 5$
$= 125 - 75 - 45 - 5 = 0.$

Therefore, $(x - 5)$ is a factor of $x^3 - 3x^2 - 9x - 5$.

Now, $x^3 - 3x^2 - 9x - 5 = x^3 - 5x^2 + 2x^2 - 10x + x - 5$
$= x^2\ (x - 5) + 2x\ (x - 5) + 1\ (x - 5)$
$= (x - 5)\ (x^2 + 2x + 1)$
$= (x - 5)\ (x + 1)^2$
$= (x - 5)\ (x + 1)\ (x + 1)$

So, $x^3 - 3x^2 - 9x - 5 = (x - 5)\ (x + 1)\ (x + 1)$. **Ans.**

(iii) $p(x) = x^3 + 13x^2 + 32x + 20$

Let a factor be $(x - a)$. a should be a factor of 20 which are ± 1, ± 2, ± 4, ± 5, ± 10.

For $x - 1 = 0 \Rightarrow x = 1$

Now, $p(1) = 1 + 13 + 32 + 20$
$= 66 \neq 0$

Hence, $(x - 1)$ is not a factor of $p(x)$.

Again, for $x + 1 = 0 \quad \Rightarrow x = -1$

Now, $p\ (-1) = -1 + 13 - 32 + 20$
$= -33 + 33 = 0$

Hence, $(x + 1)$ is a factors of $p(x)$.

Now, $x^3 + 13x^2 + 32x + 20 = x^3 + x^2 + 12x^2 + 20x + 20$
$= x^2(x + 1) + 12\ x\ (x + 1) + 20\ (x + 1)$
$= (x + 1)\ (x^2 + 12x + 20)$
$= (x + 1)\ (x^2 + 10x + 2x + 20)$
$= (x + 1)\ \{x\ (x + 10) + 2\ (x + 10)\}$
$= (x + 2)\ (x + 1)\ (x + 10)$ **Ans.**

(iv) $p(y) = 2y^3 + y^2 - 2y - 1$

factors of -2 are ± 1, ± 2.

$p(1)\ 2 \times 1^3 + 1^2 - 2 \times 1 - 1$
$= 2 + 1 - 2 - 1 = 0.$

Therefore, $(y - 1)$ is a factor of $p(y)$.

Now, $2y^3 + y^2 - 2y - 1 = 2y^3 - 2y^2 + 3y^2 - 3y + y - 1$
$= 2y^2\ (y - 1) + 3y\ (y - 1) + 1\ (y - 1)$
$= (y - 1)\ (2y^2 + 3y) + 1)$
$= (y - 1)\ (2y^2 + 2y + y + 1)$
$= (y - 1)\ \{2y\ (y + 1) + 1\ (y + 1)$
$= (y - 1)\ (y + 1)\ (2y + 1)$

Therefore, $2y^3 + y^2 - 2y - 1 = (y - 1)\ (2y + 1)\ (y + 1)$. **Ans.**

EXERCISE 2.5

Q.1. *Use suitable identities to find the following products :*

(i) $(x + 4)\ (x + 10)$ *(ii)* $(x + 8)\ (x - 10)$

(iii) $(3x + 4)\ (3x - 5)$ *(iv)* $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$ *(v)* $(3 - 2x)\ (3 + 2x)$

Sol. (i) Using identity $(x + a)\ (x + b) = x^2 + (a + b)\ x + ab$

$(x + 4)\ (x + 10) = x^2 + (4 + 10)x + 4 \times 10 = x^2 + 14x + 40$ **Ans.**

(ii) Using the same identity as in (i) above

$(x + 8)\ (x - 10) = x^2 + (8 - 10)x + 8 \times (-10) = x^2 - 2x - 80$ **Ans.**

(iii) Using the same identity

$(3x + 4)(3x - 5)\ 3x \times 3x + (-1)(3x) - 20 = 9x^2 - 3x - 20$. **Ans.**

(iv) Using $(x + y)(x - y) = x^2 - y^2$

$$\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right) = (y^2)^2 - \left(\frac{3}{2}\right)^2 = y^4 - \frac{9}{4} \quad \textbf{Ans.}$$

(v) Using the same identity as in (iv)

$(3 - 2x)(3 + 2x) = 3^2 - (2x)^2$

$= 9 - 4x^2$ **Ans.**

Q.2. *Evaluate the following products without multiplying directly :*

(i) 103×107 *(ii)* 95×96 *(iii)* 104×96

Sol. (i) $103 \times 107 = (100 + 3)(100 + 7)$

$= (100)^2 + (3 + 7) \times 100 + 3 \times 7$

$= 10000 + 1000 + 21 = 11021$ **Ans.**

(ii) $95 \times 96 = (100 - 5)(100 - 4)$

$= (100)^2 - (5 + 4) \times 100 + 5 \times 4$

$= 10000 - 900 + 20 = 9120$ **Ans.**

(iii) $104 \times 96 = (100 + 4)(100 - 4) = 100^2 - 4^2$

$= 10000 - 16 = 9984$ **Ans.**

Q.3. *Factorise the following using appropriate identities :*

(i) $9x^2 + 6xy + y^2$ *(ii)* $4y^2 - 4y + 1$ *(iii)* $x^2 - \frac{y^2}{100}$

Sol. (i) $9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)y + (y)^2$

$= (3x + y)^2$ [Using $a^2 + 2ab + b^2 = (a + b)^2$]

$= (3x + y)(3x + y)$ **Ans.**

(ii) $4y^2 - 4y + 1$

$= (2y)^2 - 2(2y)(1) + (1)^2$

$= (2y - 1)^2 = (2y - 1)(2y - 1)$ [Using $a^2 - 2ab + b^2 = (a - b)^2$] **Ans.**

(iii) $x^2 - \frac{y^2}{100} = x^2 - \left(\frac{y}{10}\right)^2$

$= \left(\frac{x + y}{10}\right)\left(\frac{x - y}{10}\right)$ [Using $a^2 - b^2 = (a + b)(a - b)$] **Ans.**

Q.4. *Expand each of the following, using suitable identities :*

(i) $(x + 2y + 4z)^2$ *(ii)* $(2x - y + z)^2$ *(iii)* $(-2x + 3y + 2z)^2$

(iv) $(3a - 7b - c)^2$ *(v)* $(-2x + 5y - 3z)^2$ *(vi)* $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

Sol. (i) $(x + 2y + 4z)^2 = x^2 + (2y)^2 + (4z)^2 + 2x \times 2y + 2 \times 2y \times 4z + 2 \times 4z \times x$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$ **Ans.**

(ii) $(2x - y + z)^2 = (2x)^2 + (-y)^2 + (z)^2 + 2 \times (2x)(-y)$

$+ 2(-y)(z) + 2(z) \times 2x$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$ **Ans.**

(iii) $(-2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(2z)(-2x)$
$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx$ **Ans.**

(iv) $(3a - 7b - c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(-c)(3a)$
$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac$ **Ans.**

(v) $(-2x + 5y - 3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-3z)(-2x)$
$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$ **Ans.**

(vi) $$\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2 = \left(\frac{1}{4}a\right)^2 + \left(\frac{-1}{2}b\right)^2 + (1)^2 + 2\left(\frac{1}{4}a\right)\left(\frac{-1}{2}b\right) + 2\left(\frac{-1}{2}b\right)\times 1 + 2(1)\times\frac{1}{4}a$$
$$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$$
$$= \frac{a^2}{16} + \frac{b^2}{4} + 1 - \frac{ab}{4} - b + \frac{a}{2}$$ **Ans.**

Q.5. *Factorise :*

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}\,xy + 4\sqrt{2}\,yz - 8xz$

Sol. (i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$
$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$
$= (2x + 3y - 4z)^2 = (2x + 3y - 4z)(2x + 3y - 4z)$ **Ans.**

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}\,xy + 4\sqrt{2}\,yz - 8xz$
$= (\sqrt{2}\,x)^2 + (-y)^2 + (-2\sqrt{2}\,z)^2 + 2(\sqrt{2}\,x)(-y) + 2(-y)(-2\sqrt{2}\,z) + 2(\sqrt{2}\,x)(-2\sqrt{2}\,z)$
$= (\sqrt{2}\,x - y - 2\sqrt{2}\,z)^2$
$= (\sqrt{2}\,x - y - 2\sqrt{2}\,z)(\sqrt{2}\,x - y - 2\sqrt{2}\,z)$ **Ans.**

Q.6. *Write the following cubes in expanded form :*

(i) $(2x + 1)^3$ *(ii)* $(2a - 3b)^3$ *(iii)* $\left[\frac{3}{2}x + 1\right]^3$ *(iv)* $\left[x - \frac{2}{3}y\right]^3$

Sol. (i) $(2x + 1)^3 = (2x)^3 + 1^3 + 3(2x)(1)(2x + 1)$
$= 8x^3 + 1 + 6x(2x + 1) = 8x^3 + 12x^2 + 6x + 1$ **Ans.**

(ii) $(2a - 3b)^3 = (2a)^3 - (3b)^3 - 3 \times 2a \times 3b\,(2a - 3b)$
$= 8a^3 - 27b^3 - 18ab(2a - 3b)$
$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$ **Ans.**

(iii) $$\left[\frac{3}{2}x + 1\right]^3 = \left(\frac{3}{2}x\right)^3 + 1^3 + 3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x + 1\right)$$
$$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x\left(\frac{3}{2}x + 1\right)$$
$$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$$ **Ans.**

$$\text{(iv)} \quad \left[x-\frac{2}{3}y\right]^3 = x^3-\left(\frac{2}{3}y\right)^3 \quad 3(x)\left(\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$$

$$= x^3-\frac{8}{27}y^3-2xy\left(x-\frac{2}{3}y\right)$$

$$= x^3-\frac{8}{27}y^3-2x^2y+\frac{4}{3}xy^2 \quad \textbf{Ans.}$$

Q.7. *Evaluate the following using suitable identities :*

(i) $(99)^3$ (ii) $(102)^3$ *(iii)* $(998)^3$

Sol. (i) $(99)^3 = (100-1)^3 = (100)^3 + (-1)^3 + 3(100)(-1)(100-1)$

$= 1000000 - 1 - 300(100-1)$

$= 1000000 - 1 - 30000 + 300 = 970299$

(ii) $(102)^3 = (100+2)^3 = 100^3 + 2^3 + 3(100)(2)(100+2)$

$= 1000000 + 8 + 600(100+2)$

$= 1000000 + 8 + 60000 + 1200 = 1061208$ **Ans.**

(iii) $(998)^3 = (1000-2)^3 = (1000)^3 + (-2)^3 + 3(1000)(-2)(998)$

$= (1000)^3 - 8 - 6000(998)$

$= 1000000000 - 8 - 5988000 = 994011992$ **Ans.**

Q.8. *Factorise each of the following :*

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$ *(ii)* $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

Sol. (i) $8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + b^3 + 3(2a)(b)(2a+b)$

$= (2a+b)^3 = (2a+b)(2a+b)(2a+b)$ **Ans.**

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

$= (2a)^3 + (-b)^3 + 3(2a)(-b)(2a-b)$

$= (2a-b)^3 = (2a-b)(2a-b)(2a-b)$ **Ans.**

(iii) $27 - 125a^3 - 135a + 225a^2$

$= 3^3 + (-5a)^3 + 3 \times (3)(-5a)(3-5a)$

$= (3-5a)^3 = (3-5a)(3-5a)(3-5a)$ **Ans.**

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

$= (4a)^3 + (-3b)^3 + 3(4a) \times (-3b)(4a-3b)$

$= (4a-3b)^3 = (4a-3b)(4a-3b)(4a-3b)$ **Ans.**

(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

$$= (3p)^3+\left(-\frac{1}{6}\right)^3+3(3p)\left(-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)$$

$$= \left(3p-\frac{1}{6}\right)^3 = \left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right) \quad \textbf{Ans.}$$

Q.9. *Verify :* (i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Sol. (i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

R.H.S. $= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$
$= x^3 - x^2y + xy^2 + yx^2 - xy^2 + y^3 = x^3 + y^3 =$ L.H.S.

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

R.H.S. $= x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$
$= x^3 + x^2y + xy^2 - yx^2 - xy^2 - y^3 = x^3 - y^3 =$ L.H.S.

Q.10. *Factorise each of the following :*

(i) $27y^3 + 125z^3$ (ii) $64m^3 - 343n^3$

Sol. (i) $27y^3 + 125z^3 = (3y)^3 + (5z)^3 = (3y + 5z)[(3y)^2 - (3y)(5z) + (5z)^2]$
$= (3y + 5z)(9y^2 - 15yz + 25z^2)$ **Ans.**

(ii) $64m^3 - 343n^3 = (4m)^3 - (7n)^3$
$= (4m - 7n)[(4m)^2 + (4m)(7n) + (7n)^2]$
$= (4m - 7n)(16m^2 + 28mn + 49n^2)$

Q.11. *Factorise :* $27x^3 + y^3 + z^3 - 9xyz$

Sol. $27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + y^3 + z^3 - 3(3x)yz$
$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$ **Ans.**

Q.12. *Verify that :*

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

Sol. To verify :

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

R.H.S. $= \frac{1}{2}(x + y + z)[x^2 + y^2 - 2xy + y^2 + z^2 - 2yz + z^2 + x^2 - 2zx]$
$= \frac{1}{2}(x + y + z)[2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx]$
$= (x + y + z)[x^2 + y^2 + z^2 - xy - yz - zx]$
$= x[x^2 + y^2 + z^2 - xy - yz - zx]$
$+ y(x^2 + y^2 + z^2 - xy - yz - zx)$
$+ z(x^2 + y^2 + z^2 - xy - yz - zx)$
$= x^3 + xy^2 + xz^2 - x^2y - xyz - zx^2 + yx^2 + y^3 + yz^2 - xy^2$
$- y^2z - zxy + zx^2 + zy^2 + z^3 - zxy - yz^2 - z^2x$
$= x^3 + y^3 + z^3 - 3xyz =$ L.H.S. **Hence verified.**

Q.13. *If* $x + y + z = 0$, *show that* $x^3 + y^3 + z^3 = 3xyz$.

Sol. $x + y + z = 0$
$(x + y + z)^3 = x^3 + y^3 + z^3 - 3xyz = 0$
$\Rightarrow x^3 + y^3 + z^3 = 3xyz$. **Proved.**

Q.14. *Without actually calculating the cubes, find the value of each of the following :*

(i) $(-12)^3 + (7)^3 + (5)^3$ (ii) $(28)^3 + (-15)^3 + (-13)^3$

Sol. From the above question, we have $x^3 + y^3 + z^3 = 3xyz$, if $x + y + z = 0$

(i) Here $-12 + 7 + 5 = 0$
$(-12)^3 + (7)^3 + (5)^3$
$3(-12)(7)(5) = -1260$ **Ans.**

(ii) Here $28 + (-15) + (-13) = 0$

So, $(28)^3 + (-15)^3 + (-13)^3$

$= 3 \times 28 (-15) (-13) = 16380$ **Ans.**

Q.15. *Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given :*

$\boxed{\text{Area} : 25a^2 - 35a + 12}$ $\boxed{\text{Area} : 35y^2 + 13y - 12}$

(i) *(ii)*

Sol. (i) Area $= 25a^2 - 35a + 12$

$= 25a^2 - 20a - 15a + 12$

$= 5a (5a - 4) - 3 (5a - 4)$

$= (5a - 4) (5a - 3)$

So, one possible answer is length = $(5a - 4)$, breadth = $(5a - 3)$

Therefore $p\left(\frac{3}{5}\right)$ gives zero value and $(5a - 3)$ is a factor.

Second factor $(5a - 4)$, lenght = $(5a - 3)$; breadth = $(5a - 4)$.

(ii) Area $= 35y^2 + 13y - 12$

$= 35y^2 + 28y - 15y - 12$

$= 7y (5y + 4) - 3 (5y + 4)$

$= (5y + 4) (7y - 3)$

So, $(5y + 4)$ may be taken as breadth and $(7y - 3)$ as length. **Ans.**

Q.16. *What are the possible expressions for the dimensions of the cuboids whose volumes are given below?*

$\boxed{\text{Volume} : 3x^2 - 12x}$ $\boxed{\text{Volume} : 12ky^2 + 8ky - 20k}$

(i) *(ii)*

Sol. (i) $abc = 3x^2 - 12x = 3x (x - 4)$

3, x $(x - 4)$ are the three factors so they can be three dimensions.

(ii) $abc = 12ky^2 + 8ky - 20k$

$= 4k (3y^2 + 2y - 5)$

$= 4k \{3y (y - 1) + 5 (y - 1)\}$

$= 4k (y - 1) (3y + 5)$

$4k$, $(y - 1)$ and $(3y + 5)$ are the three factors, so they can be three dimensions **Ans.**