Book Name: Selina Concise

EXERCISE- 8 (A)

Solution 1:

By remainder theorem we know that when a polynomial f (x) is divided by x − a, then the remainder is f(a).

$$\text{(i)} f(x) = x^4 - 3x^2 + 2x + 1$$

$$\text{Remainder} = f(1) = (1)^4 - 3(1)^2 + 2(1) + 1 = 1 - 3 + 2 + 1 = 1$$

$$\text{(ii)} f(x) = x^3 + 3x^2 - 12x + 4$$

$$\text{Remainder} = f(2) = (2)^3 + 3(2)^2 - 12(2) + 4$$

$$= 8 + 12 - 24 + 4$$

$$= 0$$

$$\text{(iii)} f(x) = x^4 + 1$$

$$\text{Remainder} = f(-1) = (-1)^4 + 1 = 1 + 1 = 2$$

$$\text{(iv)} f(x) = 4x^3 - 3x^2 + 2x - 4$$

$$\text{Remainder} = f\left(\frac{-1}{2}\right)$$

$$= 4\left(\frac{-1}{2}\right)^3 - 3\left(\frac{-1}{2}\right)^2 + 2\left(\frac{-1}{2}\right) - 4$$

$$= \frac{-1}{2} - \frac{3}{4} - 1 - 4$$

$$= \frac{-2 - 3 - 20}{4}$$

$$= \frac{-25}{4} = -6\frac{1}{4}$$

$$\text{(v)} f(x) = 4x^3 + 4x^2 - 27x + 16$$

$$\text{Remainder} = f\left(\frac{3}{2}\right)$$

$$= 4\left(\frac{3}{2}\right)^3 + 4\left(\frac{3}{2}\right)^2 - 27\left(\frac{3}{2}\right) + 16$$

$$= \frac{27}{2} + 9 - \frac{81}{2} + 16$$

$$= -27 + 25$$

$$= -2$$

$(vi) f(x) = 2x^3 + 9x^2 - x - 15$

$\text{Remainder} = f\left(\frac{-3}{2}\right)$

$= 2\left(\frac{-3}{2}\right)^3 + 9\left(\frac{-3}{2}\right)^2 - \left(\frac{-3}{2}\right) - 15$

$= \frac{-27}{4} + \frac{81}{4} + \frac{3}{2} - 15$

$= \frac{27}{2} + \frac{3}{2} - 15$

$= \frac{30}{2} - 15 = 15 - 15 = 0$

Solution 2:

(x − a) is a factor of a polynomial f(x) if the remainder, when f(x) is divided by (x − a), is 0, i.e., if f (a) = 0.

$(i) f(x) = 5x^2 + 15x - 50$

$f(2) = 5(2)^2 + 15(2) - 50 = 20 + 30 - 50 = 0$

Hence, $x - 2$ is a factor of $5x^2 + 15x - 50$

$(ii) f(x) = 3x^2 - x - 2$

$f\left(\frac{-2}{3}\right) = 3\left(\frac{-2}{3}\right)^2 - \left(\frac{-2}{3}\right) - 2 = \frac{4}{3} + \frac{2}{3} - 2 = 2 - 2 = 0$

Hence, $3x + 2$ is a factor of $3x^2 - x - 2$

$(iii) f(x) = x^3 + 3x^2 + 3x + 1$

$f(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1 = -1 + 3 - 3 + 1 = 0$

Hence, $x + 1$ is a factor of $x^3 + 3x^2 + 3x + 1$

Solution 3:

By remainder theorem we know that when a polynomial f (x) is divided by x - a, then the remainder is f(a).

Let $f(x) = 2x^3 + 3x^2 - 5x - 6$

(i) $f(-1) = 2(-1)^3 + 3(-1)^2 - 5(-1) - 6 = -2 + 3 + 5 - 6 = 0$

Thus, (x + 1) is a factor of the polynomial f(x).

(ii)

$$f\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3+3\left(\frac{1}{2}\right)^2-5\left(\frac{1}{2}\right)-6$$
$$=\frac{1}{4}+\frac{3}{4}-\frac{5}{2}-6$$
$$=-\frac{5}{2}-5=\frac{-15}{2}\neq 0$$
Thus, $(2x - 1)$ is not a factor of the polynomial $f(x)$.
(iii) $f(-2) = 2(-2)^3 + 3(-2)^2 - 5(-2) - 6 = -16 + 12 + 10 - 6 = 0$
Thus, $(x + 2)$ is a factor of the polynomial $f(x)$.
(iv)
$$f\left(\frac{2}{3}\right)=2\left(\frac{3}{2}\right)^3+3\left(\frac{3}{2}\right)^2-5\left(\frac{2}{3}\right)-6$$
$$=\frac{16}{27}+\frac{4}{3}-\frac{10}{3}-6$$
$$=\frac{16}{27}-2-6$$
$$=\frac{16}{27}-8\neq 0$$
Thus, $(3x - 2)$ is not a factor of the polynomial $f(x)$.
(v)
$$f\left(\frac{3}{2}\right)=2\left(\frac{3}{2}\right)^3+3\left(\frac{3}{2}\right)^2-5\left(\frac{3}{2}\right)-6$$
$$=\frac{27}{4}+\frac{27}{4}-\frac{15}{2}-6$$
$$=\frac{27}{2}-\frac{15}{2}-6$$
$$=6-6=0$$
Thus, $(2x - 3)$ is a factor of the polynomial $f(x)$.

Solution 4:
(i) $2x + 1$ is a factor of $f(x) = 2x^2 + ax - 3$.

$$\therefore f\left(\frac{-1}{2}\right)=0$$

$$\Rightarrow 2\left(\frac{-1}{2}\right)^2+a\left(\frac{-1}{2}\right)-3=0$$

$$\Rightarrow \frac{1}{2}-\frac{a}{2}=3$$

$$\Rightarrow 1-a=6$$

$$\Rightarrow a=-5$$

(ii) $3x - 4$ is a factor of $g(x) = 3x^2 + 2x - k$.

$$\therefore f\left(\frac{4}{3}\right)=0$$

$$\Rightarrow 3\left(\frac{4}{3}\right)^2+2\left(\frac{4}{3}\right)-k=0$$

$$\Rightarrow \frac{16}{3}+\frac{8}{3}-k=0$$

$$\Rightarrow \frac{24}{3}=k$$

$$\Rightarrow k=8$$

Solution 5:

Let $f(x) = x^3 + ax^2 + bx - 12$

$x - 2 = 0 \Longrightarrow x = 2$

$x - 2$ is a factor of $f(x)$. So, remainder = 0

$$\therefore (2)^3+a(2)^2+b(2)-12=0$$

$$\Rightarrow 8+4a+2b-12=0$$

$$\Rightarrow 4a+2b-4=0$$

$$\Rightarrow 2a+b-2=0 \qquad \text{..........(1)}$$

$x + 3 = 0 \Longrightarrow x = -3$

$x + 3$ is a factor of $f(x)$. So, remainder = 0

$$\therefore (-3)^3+a(-3)^2+b(-3)-12=0$$

$$\Rightarrow -27+9a-3b-12=0$$

$$\Rightarrow 9a-3b-39=0$$

$$\Rightarrow 3a-b-13=0 \qquad \text{..........(2)}$$

Adding (1) and (2), we get,

$5a - 15 = 0$

$\Rightarrow a = 3$

Putting the value of a in (1), we get,

$6 + b - 2 = 0$

$\Rightarrow b = -4$

Solution 6:

Let $f(x) = (3k + 2)x^3 + (k - 1)$

$2x + 1 = 0 \Rightarrow x = \frac{-1}{2}$

Since, $2x + 1$ is a factor of f(x), remainder is 0.

$$\therefore (3k+2)\left(\frac{-1}{2}\right)^3 + (k-1) = 0$$

$$\Rightarrow \frac{-(3k+2)}{8} + (k-1) = 0$$

$$\Rightarrow \frac{-3k-2+8k-8}{8} = 0$$

$$\Rightarrow 5k - 10 = 0$$

$$\Rightarrow k = 2$$

Solution 7:

$f(x) = 2x^5 - 6x^4 - 2ax^3 + 6ax^2 + 4ax + 8$

$x - 2 = 0 \Rightarrow x = 2$

Since, $x - 2$ is a factor of f(x), remainder = 0.

$2(2)^5 - 6(2)^4 - 2a(2)^3 + 6a(2)^2 + 4a(2) + 8 = 0$

$64 - 96 - 16a + 24a + 8a + 8 = 0$

$-24 + 16a = 0$

$16a = 24$

$a = 1.5$

Solution 8:

Let $f(x) = x^3 + (3m + 1)\, x^2 + nx - 18$

$x - 1 = 0 \Rightarrow x = 1$

$x - 1$ is a factor of f(x). So, remainder = 0

$\therefore (1)^3 + (3m+1)(1)^2 + n(1) - 18 = 0$

$\Rightarrow 1 + 3m + 1 + n - 18 = 0$

$\Rightarrow 3m + n - 16 = 0 \qquad \ldots\ldots\ldots..(1)$

$x + 2 = 0 \Rightarrow x = -2$

$x + 2$ is a factor of f(x). So, remainder = 0

$\therefore (-2)^3 + (3m+1)(-2)^2 + n(-2) - 18 = 0$

$\Rightarrow -8 + 12m + 4 - 2n - 18 = 0$

$\Rightarrow 12m - 2n - 22 = 0$

$\Rightarrow 6m - n - 11 = 0 \qquad \ldots\ldots\ldots..(2)$

Adding (1) and (2), we get,

$9m - 27 = 0$

$m = 3$

Putting the value of m in (1), we get,

$3(3) + n - 16 = 0$

$9 + n - 16 = 0$

$n = 7$

Solution 9:

Let $f(x) = x^3 + 2x^2 - kx + 4$

$x - 2 = 0 \Rightarrow x = 2$

On dividing f(x) by $x - 2$, it leaves a remainder k.

$\therefore f(2) = k$

$(2)^3 + 2(2)^2 - k(2) + 4 = k$

$8 + 8 - 2k + 4 = k$

$20 = 3k$

$k = \frac{20}{3} = 6\frac{2}{3}$

Solution 10:

Let $f(x) = ax^3 + 9x^2 + 4x - 10$

$x + 3 = 0 \Rightarrow x = -3$

On dividing f(x) by $x + 3$, it leaves a remainder 5.

$\therefore f(-3) = 5$

$a(-3)^3 + 9(-3)^2 + 4(-3) - 10 = 5$

$-27a + 81 - 12 - 10 = 5$

$54 = 27a$

$a = 2$

Solution 11:

Let $f(x) = x^3 + ax^2 + bx + 6$

$x - 2 = 0 \Longrightarrow x = 2$

Since, x - 2 is a factor, remainder = 0

$\therefore f(2) = 0$

$(2)^3 + a(2)^2 + b(2) + 6 = 0$

$8 + 4a + 2b + 6 = 0$

$2a + b + 7 = 0 \qquad \text{..........(i)}$

$x - 3 = 0 \Longrightarrow x = 3$

On dividing f(x) by $x - 3$, it leaves a remainder 3.

$\therefore f(3) = 3$

$(3)^3 + a(3)^2 + b(3) + 6 = 3$

$27 + 9a + 3b + 6 = 3$

$3a + b + 10 = 0 \qquad \text{..........(ii)}$

Subtracting (i) from (ii), we get,

$a + 3 = 0$

$a = -3$

Substituting the value of a in (i), we get,

$-6 + b + 7 = 0$

$b = -1$

Solution 12:

Let $f(x) = 2x^3 + ax^2 + bx - 2$

$2x - 3 = 0 \quad x = \frac{3}{2}$

On dividing f(x) by $2x - 3$, it leaves a remainder 7.

$$\therefore 2\left(\frac{3}{2}\right)^3 + a\left(\frac{3}{2}\right)^2 + b\left(\frac{3}{2}\right) - 2 = 7$$

$$\frac{27}{4} + \frac{9a}{4} + \frac{3b}{2} = 9$$

$$\frac{27 + 9a + 6b}{4} = 9$$

$$27 + 9a + 6b = 36$$

$$9a + 6b - 9 = 0$$

$$3a + 2b - 3 = 0 \qquad ....(i)$$

$x + 2 = 0 \Rightarrow x = -2$

On dividing f(x) by x + 2, it leaves a remainder 0.

$$\therefore 2(-2)^3 + a(-2)^2 + b(-2) - 2 = 0$$

$$-16 + 4a - 2b - 2 = 0$$

$$4a - 2b - 18 = 0 \qquad ....(ii)$$

Adding (i) and (ii), we get,

$7a - 21 = 0$

$a = 3$

Substituting the value of a in (i), we get,

$$3(3) + 2b - 3 = 0$$

$9 + 2b - 3 = 0$

$2b = -6$

$b = -3$

Solution 13:

Let the number k be added and the resulting polynomial be f(x).

So, $f(x) = 3x^3 - 5x^2 + 6x + k$

It is given that when f(x) is divided by $(x - 3)$, the remainder is 8.

$$\therefore f(3) = 8$$

$$3(3)^3 - 5(3)^2 + 6(3) + k = 8$$

$$81 - 45 + 18 + k = 8$$

$$54 + k = 8$$

$$k = -46$$

Thus, the required number is -46

Solution 14:

Let the number to be subtracted be k and the resulting polynomial be f(x).

So, $f(x) = x^3 + 3x^2 - 8x + 14 - k$

It is given that when f(x) is divided by (x − 2), the remainder is 10.

$\therefore f(2) = 10$

$(2)^3 + 3(2)^2 - 8(2) + 14 - k = 10$

$8 + 12 - 16 + 14 - k = 10$

$18 - k = 10$

$k = 8$

Thus, the required number is 8.

Solution 15:

Let $f(x) = 2x^3 - 7x^2 + ax - 6$

$x - 2 = 0 \Rightarrow x = 2$

When f(x) is divided by (x − 2), remainder = f (2)

$\therefore f(2) = 2(2)^3 - 7(2)^2 + a(2) - 6$

$= 16 - 28 + 2a - 6$

$= 2a - 18$

Let $g(x) = x^3 - 8x^2 + (2a + 1)x - 16$

When g(x) is divided by (x − 2), remainder = g(2)

$\therefore g(2) = (2)^3 - 8(2)^2 + (2a + 1)(2) - 16$

$= 8 - 32 + 4a + 2 - 16$

$= 4a - 38$

By the given condition, we have:

f (2) = g (2)

$2a - 18 = 4a - 38$

$4a - 2a = 38 - 18$

$2a = 20$

$a = 10$

Thus, the value of a is 10.

EXERCISE. 8(B)

Solution 1:

(i) Let $f(x) = x^3 - 2x^2 - 9x + 18$

$x - 2 = 0 \Longrightarrow x = 2$

$\therefore$ Remainder = f (2)

$= (2)^3 - 2(2)^2 - 9(2) + 18$

$= 8 - 8 - 18 + 18$

$= 0$

Hence, $(x - 2)$ is a factor of $f(x)$.

Now, we have:

$$\begin{array}{r|l} & x^2 - 9 \\ \hline x-2 & x^3 - 2x^2 - 9x + 18 \\ & x^3 - 2x^2 \\ \hline & \quad\quad -9x + 18 \\ & \quad\quad -9x + 18 \\ \hline & \quad\quad\quad 0 \end{array}$$

$\therefore\ x^3 - 2x^2 - 9x + 18 = (x - 2)(x^2 - 9) = (x - 2)(x + 3)(x - 3)$

(ii) Let $f(x) = 2x^3 + 5x^2 - 28x - 15$

$x + 5 = 0 \Longrightarrow x = -5$

$\therefore$ Remainder = f (−5)

$= 2(-5)^3 + 5(-5)^2 - 28(-5) - 15$

$= -250 + 125 + 140 - 15$

$= -265 + 265$

$= 0$

Hence, $(x + 5)$ is a factor of $f(x)$.

Now, we have:

$$\begin{array}{r|l} & 2x^2 - 5x - 3 \\ \hline x+5 & 2x^3 + 5x^2 - 28x - 15 \\ & 2x^3 + 10x^2 \\ \hline & \quad -5x^2 - 28x \\ & \quad -5x^2 - 25x \\ \hline & \quad\quad -3x - 15 \\ & \quad\quad -3x - 15 \\ \hline & \quad\quad\quad 0 \end{array}$$

$\therefore 2x^3 + 5x^2 - 28x - 15 = (x + 5)(2x^2 - 5x - 3)$

$= (x + 5)\ [2x^2 - 6x + x - 3]$
$= (x + 5)\ [2x(x - 3) + 1(x - 3)]$
$= (x + 5)\ (2x + 1)\ (x - 3)$

(iii) Let $f(x) = 3x^3 + 2x^2 - 3x - 2$

$3x + 2 = 0 \Rightarrow x = \frac{-2}{3}$

$\therefore$ Remainder = $f\left(\frac{-2}{3}\right)$

$= 3\left(\frac{-2}{3}\right)^3 + 2\left(\frac{-2}{3}\right)^2 - 3\left(\frac{-2}{3}\right) - 2$

$= \frac{-8}{9} + \frac{8}{9} + 2 - 2$

$= 0$

Hence, (3x + 2) is a factor of f(x).
Now, we have:

$$
\begin{array}{r|l}
 & x^2 - 1 \\
\hline
3x + 2 & 3x^3 + 2x^2 - 3x - 2 \\
 & 3x^3 + 2x^2 \\
\hline
 & -3x - 2 \\
 & -3x - 2 \\
\hline
 & 0
\end{array}
$$

$\therefore 3x^3 + 2x^2 - 3x - 2 = (3x + 2)(x^2 - 1) = (3x + 2)(x + 1)(x - 1)$

(iv) $f(x) = 2x^3 + 5x^2 - 11x - 14$

$2x + 7 = 0 \Rightarrow x = \frac{-7}{2}$

$\therefore$ Remainder = $f\left(\frac{-7}{2}\right)$

$= 2\left(\frac{-7}{2}\right)^3 + 5\left(\frac{-7}{2}\right)^2 - 11\left(\frac{-7}{2}\right) - 14$

$= \frac{-343}{4} + \frac{245}{4} + \frac{77}{2} - 14$

$= \frac{-49}{2} + \frac{77}{2} - 14$

$= \frac{28}{2} - 14$

$= 14 - 14 = 0$

Hence, (2x + 7) is a factor of f(x).

Now, we have:

$$\begin{array}{r|l} & x^2 - x - 2 \\ \hline 2x+7 & 2x^3 + 5x^2 - 11x - 14 \\ & 2x^3 + 7x^2 \\ \hline & \quad -2x^2 - 11x \\ & \quad -2x^2 - 7x \\ \hline & \qquad\quad -4x - 14 \\ & \qquad\quad -4x - 14 \\ \hline & \qquad\qquad 0 \end{array}$$

$$\begin{aligned} \therefore 2x^3 + 5x^2 - 11x - 14 &= (2x+7)(x^2 - x - 2) \\ &= (2x+7)(x^2 - 2x + x - 2) \\ &= (2x+7)[x(x-2) + (x-2)] \\ &= (2x+7)(x-2)(x+1) \end{aligned}$$

Solution 2:

(i)

For $x = 2$, the value of the given

expression $3x^3 + 2x^2 - 19x + 6$

$= 3(2)^3 + 2(2)^2 - 19(2) + 6$

$= 24 + 8 - 38 + 6$

$= 0$

$\Rightarrow x - 2$ is a factor of $3x^3 + 2x^2 - 19x + 6$

Now let us do long division.

$$\begin{array}{r|l} & 3x^2+8x-3 \\ \hline x-2 & 3x^3+2x^2-19x+6 \\ & 3x^3-6x^2 \\ \hline & 8x^2-19x \\ & 8x^2-16x \\ \hline & -3x+6 \\ & -3x+6 \\ \hline & 0 \end{array}$$

Thus we have,

$3x^3+2x^2-19x+6=(x-2)(3x^2+8x-3)$

$=(x-2)(3x^2+9x-x-3)$

$=(x-2)(3x(x+3)-(x-3))$

$=(x-2)(3x-1)(x+3)$

(ii) Let f (x) = $2x^3 + x^2 - 13x + 6$

For x = 2,

$f(x) = f(2) = 2(2)^3 + (2)^2 - 13(2) + 6 = 16 + 4 - 26 + 6 = 0$

Hence, (x − 2) is a factor of f(x).

$$\begin{array}{r|l} & 2x^2+5x-3 \\ \hline x-2 & 2x^3+x^2-13x+6 \\ & 2x^3-4x^2 \\ \hline & 5x^2-13x \\ & 5x^2-10x \\ \hline & -3x+6 \\ & -3x+6 \\ \hline & 0 \end{array}$$

$\therefore\ 2x^3+x^2-13x+6=(x-2)(2x^2+5x-3)$

$=(x-2)(2x^2+6x-x-3)$

$=(x-2)[2x(x+3)-(x-3)]$

$=(x-2)[2x(x+3)-(x+3)]$

(iii) $f(x) = 3x^3 + 2x^2 - 23x - 30$

For x = −2,

$f(x) = f(-2) = 3(-2)^3 + 2(-2)^2 - 23(-2) - 30$

$= -24 + 8 + 46 - 30 = -54 + 54 = 0$

Hence, (x + 2) is a factor of f(x).

$$\begin{array}{r|l} & 3x^2-4x-15 \\ \hline x+2 & 3x^3+2x^2-23x-30 \\ & 3x^3+6x^2 \\ \hline & -4x^2-23x \\ & -4x^2-8x \\ \hline & -15x-30 \\ & -15x-30 \\ \hline & 0 \end{array}$$

$\therefore 3x^3+2x^2-23x-30=(x+2)(3x^2-4x-15)$

$=(x+2)(3x^2+5x-9x-15)$

$=(x+2)[x(3x+5)-3(3x+5)]$

$=(x+2)(3x+5)(x-3)$

(iv) $f(x) = 4x^3 + 7x^2 - 36x - 63$

For $x = 3$,

$f(x) = f(3) = 4(3)^3 + 7(3)^2 - 36(3) - 63$

$= 108 + 63 - 108 - 63 = 0$

Hence, $(x + 3)$ is a factor of $f(x)$.

$$\begin{array}{r|l} & 4x^2-5x-21 \\ \hline x+3 & 4x^3+7x^2-36x-63 \\ & 4x^3+12x^2 \\ \hline & -5x^2-36x \\ & -5x^2-15x \\ \hline & -21x-63 \\ & -21x-63 \\ \hline & 0 \end{array}$$

$\therefore 4x^3+7x^2-36x-63=(x+3)(4x^2-5x-21)$

$=(x+3)(4x^2-12x+7x-21)$

$=(x+3)[4x(x-3)+7(x-3)]$

$=(x+3)(4x+7)(x-3)$

(v) $f(x) = x^3 + x^2 - 4x - 4$

For $x = -1$,

$f(x) = f(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4$

$= -1 + 1 + 4 - 4 = 0$

Hence, $(x + 1)$ is a factor of $f(x)$.

$$\begin{array}{r|l} & x^2-4 \\ \hline x+1 & x^3+x^2-4x-4 \\ & x^3+x^2 \\ \hline & -4x-4 \\ & -4x-4 \\ \hline & 0 \end{array}$$

$\therefore x^3+x^2-4x-4=(x+1)(x^2-4)$

$=(x+1)(x+2)(x-2)$

Solution 3:

Let $f(x) = 3x^3 + 10x^2 + x - 6$

For $x = -1$,

$f(x) = f(-1) = 3(-1)^3 + 10(-1)^2 + (-1) - 6 = -3 + 10 - 1 - 6 = 0$

Hence, $(x + 1)$ is a factor of $f(x)$.

$$\begin{array}{r|l} & 3x^2+7x-6 \\ \hline x+1 & 3x^3+10x^2+x-6 \\ & 3x^3+3x^2 \\ \hline & 7x^2+x \\ & 7x^2+7x \\ \hline & -6x-6 \\ & -6x-6 \\ \hline & 0 \end{array}$$

$\therefore\ 3x^3+10x^2+x-6=(x+1)(3x^2+7x-6)$

$=(x+1)(3x^2+9x-2x-6)$

$=(x+1)[3x(x+3)-2(x+3)]$

$=(x+1)(x+3)(3x-2)$

Now, $3x^3 + 10x^2 + x - 6 = 0$

$\Rightarrow (x+1)(x+3)(3x-2)=0$

$\Rightarrow x=-1,-3,\dfrac{2}{3}$

Solution 4:

f (x) = $2x^3 - 7x^2 - 3x + 18$

For x = 2,

$f(x) = f(2) = 2(2)^3 - 7(2)^2 - 3(2) + 18$

$= 16 - 28 - 6 + 18 = 0$

Hence, (x − 2) is a factor of f(x).

$$
\begin{array}{r|l}
 & 2x^2 - 3x - 9 \\
\hline
x-2 & 2x^3 - 7x^2 - 3x + 18 \\
 & \underline{2x^3 - 4x^2} \\
 & \quad -3x^2 - 3x \\
 & \quad \underline{-3x^2 + 6x} \\
 & \qquad -9x + 18 \\
 & \qquad \underline{-9x + 18} \\
 & \qquad\quad 0
\end{array}
$$

$\therefore\ 2x^3 - 7x^2 - 3x + 18 = (x-2)(2x^2 - 3x - 9)$

$= (x-2)(2x^2 - 6x + 3x - 9)$

$= (x-2)[2x(x-3) + 3(x-3)]$

$= (x-2)(x-3)(2x+3)$

$\text{Now}, f(x) = 0$

$\Rightarrow 2x^3 - 7x^2 - 3x + 18 = 0$

$\Rightarrow (x-2)(x-3)(2x+3) = 0$

$\Rightarrow x = 2, 3, \frac{-3}{2}$

Solution 5:

$f(x) = x^3 + 3x^2 + ax + b$

Since, (x − 2) is a factor of f (x), f (2) = 0

$\Rightarrow (2)^3 + 3(2)^2 + a(2) + b = 0$

$\Rightarrow 8 + 12 + 2a + b = 0$

$\Rightarrow 2a + b + 20 = 0$ ...(i)

Since, (x + 1) is a factor of f (x), f (−1) = 0

$\Rightarrow (-1)^3 + 3(-1)^2 + a(-1) + b = 0$

$\Rightarrow -1 + 3 - a + b = 0$

$\Rightarrow -a + b + 2 = 0$ .... (ii)

Subtracting (ii) from (i), we get,
$3a + 18 = 0$
$\Rightarrow a = -6$
Substituting the value of a in (ii), we get,
$b = a - 2 = -6 - 2 = -8$
$\therefore f(x) = x^3 + 3x^2 - 6x - 8$
Now, for $x = -1$,
$f(x) = f(-1) = (-1)^3 + 3(-1)^2 - 6(-1) - 8 = -1 + 3 + 6 - 8 = 0$
Hence, $(x + 1)$ is a factor of f (x).

$$\begin{array}{r|l} & x^2+2x-8 \\ \hline x+1 & x^3+3x^2-6x-8 \\ & \underline{x^3+x^2} \\ & \quad 2x^2-6x \\ & \quad \underline{2x^2+2x} \\ & \qquad -8x-8 \\ & \qquad \underline{-8x-8} \\ & \qquad\quad 0 \end{array}$$

$$\therefore x^3+3x^2-6x-8=(x+1)(x^2+2x-8)$$
$$=(x+1)(x^2+4x-2x-8)$$
$$=(x+1)[x(x+4)-2(x+4)]$$
$$=(x+1)(x+4)(x-2)$$

Solution 6:

Let $f(x) = 4x^3 - bx^2 + x - c$
It is given that when f(x) is divided by $(x + 1)$, the remainder is 0.
$f(-1) = 0$
$4(-1)^3 - b(-1)^2 + (-1) - c = 0$
$-4 - b - 1 - c = 0$
$b + c + 5 = 0$ ...(i)
It is given that when f(x) is divided by $(2x - 3)$, the remainder is 30.

$\therefore f\left(\frac{3}{2}\right)=30$

$4\left(\frac{3}{2}\right)^3-b\left(\frac{3}{2}\right)^2+\left(\frac{3}{2}\right)-c=30$

$\frac{27}{2}-\frac{9b}{4}+\frac{3}{2}-c=30$

$54-9b+6-4c-120=0$

$9b+4c+60=0$ ....(ii)

Multiplying (i) by 4 and subtracting it from (ii), we get,
$5b + 40 = 0$
$b = -8$
Substituting the value of b in (i), we get,
$c = -5 + 8 = 3$
Therefore, $f(x) = 4x^3 + 8x^2 + x - 3$
Now, for $x = -1$, we get,
$f(x) = f(-1) = 4(-1)^3 + 8(-1)^2 + (-1) - 3 = -4 + 8 - 1 - 3 = 0$
Hence, $(x + 1)$ is a factor of $f(x)$.

$$\begin{array}{r|l} & 4x^2+4x-3 \\ \hline x+1 & 4x^3+8x^2+x-3 \\ & 4x^3+4x^2 \\ \hline & \quad 4x^2+x \\ & \quad 4x^2+4x \\ \hline & \qquad -3x-3 \\ & \qquad -3x-3 \\ \hline & \qquad 0 \end{array}$$

$\therefore 4x^3+8x^2+x-3=(x+1)(4x^2+4x-3)$

$=(x+1)(4x^2+6x-2x-3)$

$=(x+1)[2x(2x+3)-(2x+3)]$

$=(x+1)(2x+3)(2x-1)$

Solution 7:

$f(x) = x^2 + px + q$
It is given that $(x + a)$ is a factor of $f(x)$.

$\therefore f(-a)=0$

$\Rightarrow (-a)^2+p(-a)+q=0$

$\Rightarrow a^2-pa+q=0$

$\Rightarrow a^2=pa-q \qquad \text{..........(i)}$

$g(x) = x^2 + mx + n$

It is given that $(x + a)$ is a factor of $g(x)$.

$\therefore g(-a)=0$

$\Rightarrow (-a)^2+m(-a)+n=0$

$\Rightarrow a^2-ma+n=0$

$\Rightarrow a^2=ma-n \qquad \text{...........(ii)}$

From (i) and (ii), we get,

$pa - q = ma - n$

$n - q = a(m - p)$

$a=\dfrac{n-q}{m-p}$

Hence, proved.

Solution 8:

Let $f(x) = ax^3 + 3x^2 - 3$

When $f(x)$ is divided by $(x - 4)$, remainder $= f(4)$

$f(4) = a(4)^3 + 3(4)^2 - 3 = 64a + 45$

Let $g(x) = 2x^3 - 5x + a$

When $g(x)$ is divided by $(x - 4)$, remainder $= g(4)$

$g(4) = 2(4)^3 - 5(4) + a = a + 108$

It is given that $f(4) = g(4)$

$64a + 45 = a + 108$

$63a = 63$

$a = 1$

Solution 9:

Let $f(x) = x^3 - ax^2 + x + 2$

It is given that $(x - a)$ is a factor of $f(x)$.

$\therefore$ Remainder $= f(a) = 0$

$a^3 - a^3 + a + 2 = 0$
$a + 2 = 0$
$a = -2$

Solution 10:

Let the number to be subtracted from the given polynomial be k.
Let $f(y) = 3y^3 + y^2 - 22y + 15 - k$
It is given that f(y) is divisible by (y + 3).
Remainder = f(−3) = 0
$3(-3)^3 + (-3)^2 - 22(-3) + 15 - k = 0$
$-81 + 9 + 66 + 15 - k = 0$
$9 - k = 0$
$k = 9$

EXERCISE. 8 (C)

Solution 1:

Let $f(x) = x^3 - 7x^2 + 14x - 8$
$f(1) = (1)^3 - 7(1)^2 + 14(1) - 8 = 1 - 7 + 14 - 8 = 0$
Hence, (x − 1) is a factor of f (x).

$$\begin{array}{r|l} & x^2 - 6x + 8 \\ \hline x - 1 & x^3 - 7x^2 + 14x - 8 \\ & x^3 - x^2 \\ \hline & -6x^2 + 14x \\ & -6x^2 + 6x \\ \hline & 8x - 8 \\ & 8x - 8 \\ \hline & 0 \end{array}$$

$\therefore x^3 - 7x^2 + 14x - 8 = (x-1)(x^2 - 6x + 8)$
$= (x-1)(x^2 - 2x - 4x + 8)$
$= (x-1)[x(x-2) - 4(x-2)]$
$= (x-1)(x-2)(x-4)$

Solution 2:

Let f (x) = $2x^3 + 7x^2 - 8x - 28$

For x = 2,

$f(x) = f(2) = 2(2)^3 + 7(2)^2 - 8(2) - 28 = 16 + 28 - 16 - 28 = 0$

Hence, (x − 2) is a factor of f(x).

$$
\begin{array}{r|l}
 & 2x^2 + 11x + 14 \\
\hline
x-2 & 2x^3 + 7x^2 - 8x - 28 \\
 & \underline{2x^3 - 4x^2} \\
 & \quad 11x^2 - 8x \\
 & \quad \underline{11x^2 - 22x} \\
 & \qquad 14x - 28 \\
 & \qquad \underline{14x - 28} \\
 & \qquad\quad 0
\end{array}
$$

$$\therefore 2x^3 + 7x^2 - 8x - 28 = (x-2)(2x^2 + 11x + 14)$$

$$= (x-2)(2x^2 + 4x + 7x + 14)$$

$$= (x-2)[2x(x+2) + 7(x+2)]$$

$$= (x-2)(x+2)(2x+7)$$

Solution 3:

Let $f(x) = x^3 + 3x^2 - mx + 4$

According to the given information,

$f(2) = m + 3$

$(2)^3 + 3(2)^2 - m(2) + 4 = m + 3$

$8 + 12 - 2m + 4 = m + 3$

$24 - 3 = m + 2m$

$3m = 21$

$m = 7$

Solution 4:

Let the required number be k.

Let $f(x) = 3x^3 - 8x^2 + 4x - 3 - k$

According to the given information,

$f(-2) = 0$

$3(-2)^3 - 8(-2)^2 + 4(-2) - 3 - k = 0$

$-24 - 32 - 8 - 3 - k = 0$

$-67 - k = 0$

$k = -67$

Thus, the required number is -67.

Solution 5:

Let $f(x) = x^3 + (a + 1)x^2 - (b - 2)x - 6$

Since, $(x + 1)$ is a factor of $f(x)$.

$\therefore$ Remainder $= f(-1) = 0$

$(-1)^3 + (a + 1)(-1)^2 - (b - 2)(-1) - 6 = 0$

$-1 + (a + 1) + (b - 2) - 6 = 0$

$a + b - 8 = 0$ ...(i)

Since, $(x - 2)$ is a factor of $f(x)$.

$\therefore$ Remainder $= f(2) = 0$

$(2)^3 + (a + 1)(2)^2 - (b - 2)(2) - 6 = 0$

$8 + 4a + 4 - 2b + 4 - 6 = 0$

$4a - 2b + 10 = 0$

$2a - b + 5 = 0$ ...(ii)

Adding (i) and (ii), we get,

$3a - 3 = 0$

$a = 1$

Substituting the value of a in (i), we get,

$1 + b - 8 = 0$

$b = 7$

$\therefore f(x) = x^3 + 2x^2 - 5x - 6$

Now, $(x + 1)$ and $(x - 2)$ are factors of $f(x)$. Hence, $(x + 1)(x - 2) = x^2 - x - 2$ is a factor of $f(x)$.

$$
\begin{array}{r|l}
 & x+3 \\
\hline
x^2-x-2 & x^3+2x^2-5x-6 \\
 & \underline{x^2-x^2-2x} \\
 & \quad 3x^2-3x-6 \\
 & \quad \underline{3x^2-3x-6} \\
 & \qquad 0
\end{array}
$$

$\therefore$ f(x) = $x^3 + 2x^2 - 5x - 6 = (x + 1)(x - 2)(x + 3)$

Solution 6:

Let f(x) = $x^2 + ax + b$

Since, (x − 2) is a factor of f(x).

$\therefore$ Remainder = f(2) = 0

$(2)^2 + a(2) + b = 0$

$4 + 2a + b = 0$

$2a + b = -4$ ...(i)

It is given that:

$a + b = 1$ ...(ii)

Subtracting (ii) from (i), we get,

$a = -5$

Substituting the value of a in (ii), we get,

$b = 1 - (-5) = 6$

Solution 7:

Let f(x) = $x^3 + 6x^2 + 11x + 6$

For x = −1

$f(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6$

$= -1 + 6 - 11 + 6 = 12 - 12 = 0$

Hence, (x + 1) is a factor of f(x).

$$\begin{array}{r|l} & x^2+5x+6 \\ \hline x+1 & x^3+6x^2+11x+6 \\ & x^3+x^2 \\ \hline & 5x^2+11x \\ & 5x^2+5x \\ \hline & 6x+6 \\ & 6x+6 \\ \hline & 0 \end{array}$$

$\therefore x^3+6x^2+11x+6=(x+1)(x^2+5x+6)$

$=(x+1)(x^2+2x+3x+6)$

$=(x+1)[x(x+2)+3(x+2)]$

$=(x+1)(x+2)(x+3)$

Solution 8:

Let $f(x) = mx^3 + 2x^2 - 3$

$g(x) = x^2 - mx + 4$

It is given that f (x) and g (x) leave the same remainder when divided by (x − 2). Therefore, we have:

$f(2) = g(2)$

$m(2)^3 + 2(2)^2 - 3 = (2)^2 - m(2) + 4$

$8m + 8 - 3 = 4 - 2m + 4$

$10m = 3$

$m = \frac{3}{10}$

Solution 9:

Let $f(x) = px^3 + 4x^2 - 3x + q$

It is given that f(x) is completely divisible by $(x^2 - 1) = (x + 1)(x - 1)$.

Therefore, $f(1) = 0$ and $f(-1) = 0$

$f(1) = p(1)^3 + 4(1)^2 - 3(1) + q = 0$

$p + q + 1 = 0$ ...(i)

$f(-1) = p(-1)^3 + 4(-1)^2 - 3(-1) + q = 0$

$-p + q + 7 = 0$ ...(ii)

Adding (i) and (ii), we get,

2q + 8 = 0
q = −4
Substituting the value of q in (i), we get,
p = −q − 1 = 4 − 1 = 3
∴ $f(x) = 3x^3 + 4x^2 - 3x - 4$
Given that f(x) is completely divisible by $(x^2 - 1)$

$$\begin{array}{r|l} & 3x+4 \\ \hline x^2-1 & 3x^3+4x^2-3x-4 \\ & \underline{3x^3 \qquad\quad -3x} \\ & \quad 4x^2 \qquad\quad -4 \\ & \quad \underline{4x^2 \qquad\quad -4} \\ & \qquad\quad 0 \end{array}$$

$$\therefore\ 3x^3+4x^2-3x-4=(x^2-1)(3x-4)$$
$$=(x-1)(x+1)(3x+4)$$

Solution 10:

Let the required number be k.
Let $f(x) = x^2 + x + 3 + k$
It is given that f(x) is divisible by (x + 3).
∴ Remainder = 0
f (−3) = 0
$(-3)^2 + (-3) + 3 + k = 0$
9 − 3 + 3 + k = 0
9 + k = 0
k = −9
Thus, the required number is − 9.

Solution 11:

It is given that when the polynomial $x^3 + 2x^2 - 5ax - 7$ is divided by (x − 1), the remainder is A.
∴ $(1)^3 + 2(1)^2 - 5a(1) - 7 = A$
1 + 2 − 5a − 7 = A
− 5a − 4 = A ...(i)
It is also given that when the polynomial $x^3 + ax^2 - 12x + 16$ is divided by (x + 2), the remainder is B.
∴ $x^3 + ax^2 - 12x + 16 = B$
$(-2)^3 + a(-2)^2 - 12(-2) + 16 = B$

$-8 + 4a + 24 + 16 = B$
$4a + 32 = B$ ...(ii)
It is also given that $2A + B = 0$
Using (i) and (ii), we get,
$2(-5a - 4) + 4a + 32 = 0$
$-10a - 8 + 4a + 32 = 0$
$-6a + 24 = 0$
$6a = 24$
$a = 4$

Solution 12:

Let $f(x) = (a - 1)x^3 + (a + 1)x^2 - (2a + 1)x - 15$
It is given that $(3x + 5)$ is a factor of f(x).

$$\therefore \text{Remainder} = 0$$

$$f\left(\frac{-5}{3}\right) = 0$$

$$(a-1)\left(\frac{-5}{3}\right)^3 + (a+1)\left(\frac{-5}{3}\right)^2 - (2a+1)\left(\frac{-5}{3}\right) - 15 = 0$$

$$(a-1)\left(\frac{-125}{27}\right) + (a+1)\left(\frac{25}{9}\right) - (2a+1)\left(\frac{-5}{3}\right) - 15 = 0$$

$$\frac{-125(a-1) + 75(a+1) + 45(2a+1) - 405}{27} = 0$$

$$-125a + 125 + 75a + 75 + 90a + 45 - 405 = 0$$

$$40a - 160 = 0$$

$$401 = 160$$

$$a = 4$$

$\therefore f(x) = (a - 1)x^3 + (a + 1)x^2 - (2a + 1)x - 15$
$= 3x^3 + 5x^2 - 9x - 15$

$$\begin{array}{r|l} & x^2 - 3 \\ 3x+5 & 3x^3 + 5x^2 - 9x - 15 \\ & \underline{3x^3 + 5x^2} \\ & \qquad\quad -9x - 15 \\ & \qquad\quad \underline{-9x - 15} \\ & \qquad\qquad 0 \end{array}$$

$$\therefore 3x^3 + 5x^2 - 9x - 15 = (3x+5)(x^2 - 3)$$
$$= (3x+5)(x+\sqrt{3})(x-\sqrt{3})$$

Solution 13:

If $(x - 3)$ divides $f(x) = x^3 - px^2 + x + 6$, then,
Remainder $= f(3) = 3^3 - p(3)^2 + 3 + 6 = 36 - 9p$
If $(x - 3)$ divides $g(x) = 2x^3 - x^2 - (p + 3)x - 6$, then
Remainder $= g(3) = 2(3)^3 - (3)^2 - (p + 3)(3) - 6 = 30 - 3p$
Now, $f(3) = g(3)$
$\Rightarrow 36 - 9p = 30 - 3p$
$\Rightarrow -6p = -6$
$\Rightarrow p = 1$

Solution 14:

$f(x) = 2x^3 + x^2 - 13x + 6$
Factors of constant term 6 are $\pm 1, \pm 2, \pm 3, \pm 6$.
Putting $x = 2$, we have:
$f(2) = 2(2)^3 + 2^2 - 13(2) + 6 = 16 + 4 - 26 + 6 = 0$
Hence $(x - 2)$ is a factor of $f(x)$.

$$\begin{array}{r|l} & 2x^2 + 5x - 3 \\ \hline x - 2 & 2x^3 + x^2 - 13x + 6 \\ & 2x^3 - 4x^2 \\ \hline & \quad 5x^2 - 13x \\ & \quad 5x^2 - 10x \\ \hline & \qquad -3x + 6 \\ & \qquad -3x + 6 \\ \hline & \qquad 0 \end{array}$$

$$2x^3 + x^2 - 13x + 6 = (x-2)(2x^2 + 5x - 3)$$
$$= (x-2)(2x^2 + 6x - x - 3)$$
$$= (x-2)(2x(x+3) - 1(x+3))$$
$$= (x-2)(2x-1)(x+3)$$