Exercise 14.1

Question 1:

Find the common factors of the terms

(i) 12x, 36

(ii) 2y, 22xy

(iii) 14pq, $28p^2q^2$

(iv) 2x, $3x^2$, 4

(v) 6abc, $24ab^2$, $12a^2b$

(vi) $16x^3$, $-4x^2$, 32x

(vii) 10pq, 20qr, 30rp

(viii) $3x^2y^3$, $10x^3y^2$, $6x^2y^2z$

(i) $12x = 2 \times 2 \times 3 \times x$

$36 = 2 \times 2 \times 3 \times 3$

(ii) $2y = 2 \times y$

$22xy = 2 \times 11 \times x \times y$

(iii) $14pq = 2 \times 7 \times p \times q$

$28p^2q^2 = 2 \times 2 \times 7 \times p \times p \times q \times q$

The common factors are 2, 7, p, q.

And, $2 \times 7 \times p \times q = 14pq$

(iv) $2x = 2 \times x$

$3x^2 = 3 \times x \times x$

$4 = 2 \times 2$

The common factor is 1.

(v) $6abc = 2 \times 3 \times a \times b \times c$

$24ab^2 = 2 \times 2 \times 2 \times 3 \times a \times b \times b$

$12a^2b = 2 \times 2 \times 3 \times a \times a \times b$

The common factors are 2, 3, a, b.

And, $2 \times 3 \times a \times b = 6ab$

(vi) $16x^3 = 2 \times 2 \times 2 \times 2 \times x \times x \times$

$x \; -4x^2 = -1 \times 2 \times 2 \times x \times x$

$32x = 2 \times 2 \times 2 \times 2 \times 2 \times x$

The common factors are 2, 2, x.

And, $2 \times 2 \times x = 4x$

(vii) $10pq = 2 \times 5 \times p \times q$

$20qr = 2 \times 2 \times 5 \times q \times r$

$30rp = 2 \times 3 \times 5 \times r \times p$

The common factors are 2, 5.

And, $2 \times 5 = 10$

(viii) $3x^2y^3 = 3 \times x \times x \times y \times y \times y$

$10x^3y^2 = 2 \times 5 \times x \times x \times x \times x \times y \times y$

$6x^2y^2z = 2 \times 3 \times x \times x \times y \times y \times z$

$x \times x \times y \times y = x^2y^2$

Question 2:

(i) $7x - 42$

(ii) $6p - 12q$

(iii) $7a^2 + 14a$

(iv) $-16z + 20z^3$

(v) $20l^2m + 30\,alm$

(vi) $5x^2y - 15xy^2$

(vii) $10a^2 - 15b^2 + 20c^2$

(viii) $-4a^2 + 4ab - 4\,ca$

(ix) $x^2yz + xy^2z + xyz^2$

(x) $ax^2y + bxy^2 + cxyz$

Answer:

(i) $7x = 7 \times x$

$42 = 2 \times 3 \times 7$

The common factor is 7.

$\therefore 7x - 42 = (7 \times x) - (2 \times 3 \times 7) = 7 (x - 6)$

(ii) $6p = 2 \times 3 \times p$

$12q = 2 \times 2 \times 3 \times q$

$\therefore 6p - 12q = (2 \times 3 \times p) - (2 \times 2 \times 3 \times q) = 2 \times 3 [p - (2 \times q)]$

$= 6 (p - 2q)$

(iii) $7a^2 = 7 \times a \times a$

$14a = 2 \times 7 \times a$

$\therefore 7a^2 + 14a = (7 \times a \times a) + (2 \times 7 \times a) = 7 \times a [a + 2] = 7a (a + 2)$

(iv) $16z = 2 \times 2 \times 2 \times 2 \times z$

$20z^3 = 2 \times 2 \times 5 \times z \times z \times z$

$\therefore -16z + 20z^3 = - (2 \times 2 \times 2 \times 2 \times z) + (2 \times 2 \times 5 \times z \times z \times z)$

$= (2 \times 2 \times z) [- (2 \times 2) + (5 \times z \times z)]$

$= 4z (- 4 + 5z^2)$

(v) $20l^2m = 2 \times 2 \times 5 \times l \times l \times m$

$30alm = 2 \times 3 \times 5 \times a \times l \times m$

$\therefore 20l^2m + 30alm = (2 \times 2 \times 5 \times l \times l \times m) + (2 \times 3 \times 5 \times a \times l \times m) = (2 \times 5 \times l \times m) [(2 \times l) + (3 \times a)]$

$= 10lm (2l + 3a)$

(vi) $5x^2y = 5 \times x \times x \times y$

$15xy^2 = 3 \times 5 \times x \times y \times y$

The common factors are 5, x, and y.

$\therefore 5x^2y - 15xy^2 = (5 \times x \times x \times y) - (3 \times 5 \times x \times y \times y) = 5 \times x \times y\,[x - (3 \times y)]$

$= 5xy\,(x - 3y)$

(vii) $10a^2 = 2 \times 5 \times a \times a$

$15b^2 = 3 \times 5 \times b \times b$

$20c^2 = 2 \times 2 \times 5 \times c \times c$

The common factor is 5.

$10a^2 - 15b^2 + 20c^2 = (2 \times 5 \times a \times a) - (3 \times 5 \times b \times b) + (2 \times 2 \times 5 \times c \times c)$

$= 5\,[(2 \times a \times a) - (3 \times b \times b) + (2 \times 2 \times c \times c)]$

$= 5\,(2a^2 - 3b^2 + 4c^2)$

(viii) $4a^2 = 2 \times 2 \times a \times a$

$4ab = 2 \times 2 \times a \times b$

$4ca = 2 \times 2 \times c \times a$

$\therefore -4a^2 + 4ab - 4ca = -(2 \times 2 \times a \times a) + (2 \times 2 \times a \times b) - (2 \times 2 \times c \times a) = 2 \times 2 \times a\,[-(a) + b - c]$

$= 4a\,(-a + b - c)$

(ix) $x^2yz = x \times x \times y \times z$

$xy^2z = x \times y \times y \times z$

$xyz^2 = x \times y \times z \times z$

$\therefore x^2yz + xy^2z + xyz^2 = (x \times x \times y \times z) + (x \times y \times y \times z) + (x \times y \times z \times z)$

$= x \times y \times z\,[x + y + z]$

$= xyz\,(x + y + z)$

(x) $ax^2y = a \times x \times x \times y$

$bxy^2 = b \times x \times y \times y$

$cxyz = c \times x \times y \times z$

The common factors are x and y.

$ax^2y + bxy^2 + cxyz = (a \times x \times x \times y) + (b \times x \times y \times y) + (c \times x \times y \times z)$

$= (x \times y)\ [(a \times x) + (b \times y) + (c \times z)]$

$= xy\ (ax + by + cz)$

(i) $x^2 + xy + 8x + 8y$

(ii) $15xy - 6x + 5y - 2$

(iii) $ax + bx - ay - by$

(iv) $15pq + 15 + 9q + 25p$

(v) $z - 7 + 7xy - xyz$

(i) $x^2 + xy + 8x + 8y = x \times x + x \times y + 8 \times x + 8 \times y$

$= x\ (x + y) + 8\ (x + y)$

$= (x + y)\ (x + 8)$

(ii) $15xy - 6x + 5y - 2 = 3 \times 5 \times x \times y - 3 \times 2 \times x + 5 \times y - 2$

$= 3x\ (5y - 2) + 1\ (5y - 2)$

$= (5y - 2)\ (3x + 1)$

(iii) $ax + bx - ay - by = a \times x + b \times x - a \times y - b \times y$

$= x\ (a + b) - y\ (a + b)$

$= (a + b)\ (x - y)$

(iv) $15pq + 15 + 9q + 25p = 15pq + 9q + 25p + 15$

$= 3 \times 5 \times p \times q + 3 \times 3 \times q + 5 \times 5 \times p + 3 \times 5$

$= 3q\ (5p + 3) + 5\ (5p + 3)$

$= (5p + 3)\ (3q + 5)$

(v) $z - 7 + 7xy - xyz = z - x \times y \times z - 7 + 7 \times x \times y$

$= z\ (1 - xy) - 7\ (1 - xy)$

$= (1 - xy)\ (z - 7)$

Exercise 14.2

Question 1:

Factorise the following expressions.

(i) $a^2 + 8a + 16$

(ii) $p^2 - 10p + 25$

(iii) $25m^2 + 30m + 9$

(iv) $49y^2 + 84yz + 36z^2$

(v) $4x^2 - 8x + 4$

(vi) $121b^2 - 88bc + 16c^2$

(vii) $(l + m)^2 - 4lm$ (Hint: Expand $(l + m)^2$ first)

(viii) $a^4 + 2a^2b^2 + b^4$

(i) $a^2 + 8a + 16 = (a)^2 + 2 \times a \times 4 + (4)^2$

$= (a + 4)^2$ $[(x + y)^2 = x^2 + 2xy + y^2]$

(ii) $p^2 - 10p + 25 = (p)^2 - 2 \times p \times 5 + (5)^2$

$= (p - 5)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

(iii) $25m^2 + 30m + 9 = (5m)^2 + 2 \times 5m \times 3 + (3)^2$

$= (5m + 3)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

(iv) $49y^2 + 84yz + 36z^2 = (7y)^2 + 2 \times (7y) \times (6z) + (6z)^2$

$= (7y + 6z)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

(v) $4x^2 - 8x + 4 = (2x)^2 - 2\,(2x)\,(2) + (2)^2$

$= (2x - 2)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$= [(2)\,(x - 1)]^2 = 4(x - 1)^2$

(vi) $121b^2 - 88bc + 16c^2 = (11b)^2 - 2\,(11b)\,(4c) + (4c)^2$

$= (11b - 4c)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

(vii) $(l + m)^2 - 4lm = l^2 + 2lm + m^2 - 4lm$

$= l^2 - 2lm + m^2$

$= (l - m)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

(viii) $a^4 + 2a^2b^2 + b^4 = (a^2)^2 + 2\,(a^2)\,(b^2) + (b^2)^2$

$= (a^2 + b^2)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$

Question 2:

Factorise

(i) $4p^2 - 9q^2$

(ii) $63a^2 - 112b^2$

(iii) $49x^2 - 36$

(iv) $16x^5 - 144x^3$

(v) $(l + m)^2 - (l - m)^2$

(vi) $9x^2y^2 - 16$

(vii) $(x^2 - 2xy + y^2) - z^2$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2$

(i) $4p^2 - 9q^2 = (2p)^2 - (3q)^2$

$= (2p + 3q)(2p - 3q)$ $[a^2 - b^2 = (a - b)(a + b)]$ (ii) $63a^2 - 112b^2 = 7(9a^2 - 16b^2)$

$= 7[(3a)^2 - (4b)^2]$

$= 7(3a + 4b)(3a - 4b)$ $[a^2 - b^2 = (a - b)(a + b)]$

(iii) $49x^2 - 36 = (7x)^2 - (6)^2$

$= (7x - 6)(7x + 6)$ $[a^2 - b^2 = (a - b)(a + b)]$ (iv) $16x^5 - 144x^3 = 16x^3(x^2 - 9)$

$= 16\,x^3\,[(x)^2 - (3)^2]$

$= 16\,x^3(x - 3)(x + 3)$ $[a^2 - b^2 = (a - b)(a + b)]$

(v) $(l + m)^2 - (l - m)^2 = [(l + m) - (l - m)]\,[(l + m) + (l - m)]$ [Using identity $a^2 - b^2 = (a - b)(a + b)$]

$= (l + m - l + m)(l + m + l - m)$

$= 2m \times 2l$

$= 4ml$

$= 4lm$

(vi) $9x^2y^2 - 16 = (3xy)^2 - (4)^2$

$= (3xy - 4)(3xy + 4)$ $[a^2 - b^2 = (a - b)(a + b)]$

(vii) $(x^2 - 2xy + y^2) - z^2 = (x - y)^2 - (z)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$

$= (x - y - z)(x - y + z)$ [$a^2 - b^2 = (a - b)(a + b)$]

(viii) $25a^2 - 4b^2 + 28bc - 49c^2 = 25a^2 - (4b^2 - 28bc + 49c^2) = (5a)^2 - [(2b)^2 - 2 \times 2b \times 7c + (7c)^2]$

$= (5a)^2 - [(2b - 7c)^2]$

[Using identity $(a - b)^2 = a^2 - 2ab + b^2$] $= [5a + (2b - 7c)]\,[5a - (2b - 7c)]$

[Using identity $a^2 - b^2 = (a - b)(a + b)$] $= (5a + 2b - 7c)(5a - 2b + 7c)$

(i) $ax^2 + bx$

(ii) $7p^2 + 21q^2$

(iii) $2x^3 + 2xy^2 + 2xz^2$

(iv) $am^2 + bm^2 + bn^2 + an^2$

(v) $(lm + l) + m + 1$

(vi) $y(y + z) + 9(y + z)$

(vii) $5y^2 - 20y - 8z + 2yz$

(viii) $10ab + 4a + 5b + 2$

(ix) $6xy - 4y + 6 - 9x$

Answer:

(i) $ax^2 + bx = a \times x \times x + b \times x = x(ax + b)$

(ii) $7p^2 + 21q^2 = 7 \times p \times p + 3 \times 7 \times q \times q = 7(p^2 + 3q^2)$

(iii) $2x^3 + 2xy^2 + 2xz^2 = 2x(x^2 + y^2 + z^2)$

(iv) $am^2 + bm^2 + bn^2 + an^2 = am^2 + bm^2 + an^2 + bn^2$

$= m^2(a + b) + n^2(a + b)$

$= (a + b)(m^2 + n^2)$

(v) $(lm + l) + m + 1 = lm + m + l + 1$

$= m(l + 1) + 1(l + 1)$

$= (l + l)(m + 1)$

(vi) $y(y + z) + 9(y + z) = (y + z)(y + 9)$

(vii) $5y^2 - 20y - 8z + 2yz = 5y^2 - 20y + 2yz - 8z$

$= 5y(y - 4) + 2z(y - 4)$

$= (y - 4)(5y + 2z)$

(viii) $10ab + 4a + 5b + 2 = 10ab + 5b + 4a + 2$

$= 5b(2a + 1) + 2(2a + 1)$

$= (2a + 1)(5b + 2)$

(ix) $6xy - 4y + 6 - 9x = 6xy - 9x - 4y + 6$

$= 3x(2y - 3) - 2(2y - 3)$

$= (2y - 3)(3x - 2)$

(i) $a^4 - b^4$

(ii) $p^4 - 81$

(iii) $x^4 - (y + z)^4$

(iv) $x^4 - (x - z)^4$

(v) $a^4 - 2a^2b^2 + b^4$

Answer:

(i) $a^4 - b^4 = (a^2)^2 - (b^2)^2$

$= (a^2 - b^2)(a^2 + b^2)$

$= (a - b)(a + b)(a^2 + b^2)$

(ii) $p^4 - 81 = (p^2)^2 - (9)^2$

$= (p^2 - 9)(p^2 + 9)$

$= [(p)^2 - (3)^2](p^2 + 9)$

$= (p - 3)(p + 3)(p^2 + 9)$

(iii) $x^4 - (y + z)^4 = (x^2)^2 - [(y + z)^2]^2$

$= [x^2 - (y + z)^2][x^2 + (y + z)^2]$

$= [x - (y + z)][x + (y + z)][x^2 + (y + z)^2]$

$= (x - y - z)(x + y + z)[x^2 + (y + z)^2]$

(iv) $x^4 - (x - z)^4 = (x^2)^2 - [(x - z)^2]^2$

$= [x^2 - (x - z)^2][x^2 + (x - z)^2]$

$= [x - (x - z)] [x + (x - z)] [x^2 + (x - z)^2]$

$= z(2x - z) [x^2 + x^2 - 2xz + z^2]$

$= z(2x - z) (2x^2 - 2xz + z^2)$

(v) $a^4 - 2a^2b^2 + b^4 = (a^2)^2 - 2 (a^2) (b^2) + (b^2)^2$

$= (a^2 - b^2)^2$

$= [(a - b) (a + b)]^2$

$= (a - b)^2 (a + b)^2$

(i) $p^2 + 6p + 8$

(ii) $q^2 - 10q + 21$

(iii) $p^2 + 6p - 16$

Answer:

(i) $p^2 + 6p + 8$

It can be observed that, $8 = 4 \times 2$ and $4 + 2 = 6$

$\therefore p^2 + 6p + 8 = p^2 + 2p + 4p +$

$8 = p(p + 2) + 4(p + 2)$

$= (p + 2) (p + 4)$

(ii) $q^2 - 10q + 21$

It can be observed that, $21 = (-7) \times (-3)$ and $(-7) + (-3) = -10$

$\therefore q^2 - 10q + 21 = q^2 - 7q - 3q +$

$21 = q(q - 7) - 3(q - 7)$

$= (q - 7) (q - 3)$

(iii) $p^2 + 6p - 16$

It can be observed that, $16 = (-2) \times 8$ and $8 + (-2) = 6$

$p^2 + 6p - 16 = p^2 + 8p - 2p - 16$

$= p(p + 8) - 2(p + 8)$

$= (p + 8) (p - 2)$

Exercise 14.3

Question 1:

Carry out the following divisions.

(i) $28x^4 \div 56x$

(ii) $-36y^3 \div 9y^2$

(iii) $66pq^2r^3 \div 11qr^2$

(iv) $34x^3y^3z^3 \div 51xy^2z^3$

(v) $12a^8b^8 \div (-6a^6b^4)$

Answer:

(i) $28x^4 = 2 \times 2 \times 7 \times x \times x \times x \times x \times x$

$56x = 2 \times 2 \times 2 \times 7 \times x$

$$28x^4 \div 56x = \frac{2\times2\times7\times x\times x\times x\times x}{2\times2\times2\times7\times x} = \frac{x^3}{2} = \frac{1}{2}x^3$$

(ii) $36y^3 = 2 \times 2 \times 3 \times 3 \times y \times y \times$

$y\ 9y^2 = 3 \times 3 \times y \times y$

$$-36y^3 \div 9y^2 = \frac{-2\times2\times3\times3\times y\times y\times y}{3\times3\times y\times y} = -4y$$

(iii) $66\ pq^2\ r^3 = 2 \times 3 \times 11 \times p \times q \times q \times r \times r \times$

$r\ 11qr^2 = 11 \times q \times r \times r$

$$66pq^2r^3 \div 11qr^2 = \frac{2\times3\times11\times p\times q\times q\times r\times r\times r}{11\times q\times r\times r} = 6pqr$$

(iv) $34\ x^3y^3z^3 = 2 \times 17 \times x \times x \times x \times x \times y \times y \times y \times z \times z \times$

$z\ 51\ xy^2z^3 = 3 \times 17 \times x \times y \times y \times z \times z \times z$

$$34\,x^3y^3z^3 \div 51\,xy^2z^3 = \frac{2\times17\times x\times x\times x\times y\times y\times y\times z\times z\times z}{3\times17\times x\times y\times y\times z\times z\times z}$$

$$= \frac{2}{3}x^2y$$

(v) $12a^8b^8 = 2 \times 2 \times 3 \times a^8 \times b^8$

$6a^6b^4 = 2 \times 3 \times a^6 \times b^4$

$$12a^8b^8 \div \left(-6a^6b^4\right) = \frac{2\times 2\times 3\times a^8\times b^8}{-2\times 3\times a^6\times b^4} = -2a^2b^4$$

Question 2:

Divide the given polynomial by the given monomial.

(i) $(5x^2 - 6x) \div 3x$

(ii) $(3y^8 - 4y^6 + 5y^4) \div y^4$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$

(iv) $(x^3 + 2x^2 + 3x) \div 2x$

(v) $(p^3q^6 - p^6q^3) \div p^3q^3$

(i) $5x^2 - 6x = x(5x - 6)$

$$\left(5x^2-6x\right)\div 3x = \frac{x\left(5x-6\right)}{3x} = \frac{1}{3}\left(5x-6\right)$$

(ii) $3y^8 - 4y^6 + 5y^4 = y^4(3y^4 - 4y^2 + 5)$

$$\left(3y^8-4y^6+5y^4\right)\div y^4 = \frac{y^4\left(3y^4-4y^2+5\right)}{y^4} = 3y^4-4y^2+5$$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) = 8x^2y^2z^2(x + y + z)$

$$8\left(x^3y^2z^2+x^2y^3z^2+x^2y^2z^3\right)\div 4x^2y^2z^2 = \frac{8x^2y^2z^2\left(x+y+z\right)}{4x^2y^2z^2} = 2\left(x+y+z\right)$$

(iv) $x^3 + 2x^2 + 3x = x(x^2 + 2x + 3)$

$$\left(x^3+2x^2+3x\right)\div 2x = \frac{x\left(x^2+2x+3\right)}{2x} = \frac{1}{2}\left(x^2+2x+3\right)$$

(v) $p^3q^6 - p^6q^3 = p^3q^3(q^3 - p^3)$

$$\left(p^3q^6-p^6q^3\right)\div p^3q^3 = \frac{p^3q^3\left(q^3-p^3\right)}{p^3q^3} = q^3-p^3$$

Question 3:

Work out the following divisions.

(i) $(10x - 25) \div 5$

(ii) (10x – 25) ÷ (2x – 5)

(iii) 10y(6y + 21) ÷ 5(2y + 7)

(iv) $9x^2y^2(3z – 24) ÷ 27xy(z – 8)$

(v) 96abc(3a – 12)(5b – 30) ÷ 144(a – 4) (b – 6)

Answer:

(i) $(10x-25)\div 5 = \frac{2\times 5\times x-5\times 5}{5}=\frac{5(2x-5)}{5}=2x-5$

(ii) $(10x-25)\div(2x-5)=\frac{2\times 5\times x-5\times 5}{(2x-5)}=\frac{5(2x-5)}{2x-5}=5$

(iii) $10y(6y+21)\div 5(2y+7)=\frac{2\times 5\times y[2\times 3\times y+3\times 7]}{5(2y+7)}$

$=\frac{2\times 5\times y\times 3(2y+7)}{5(2y+7)}=6y$

(iv) $9x^2y^2(3z-24)\div 27xy(z-8)=\frac{9x^2y^2[3\times z-2\times 2\times 2\times 3]}{27xy(z-8)}$

$=\frac{xy\times 3(z-8)}{3(z-8)}=xy$

(v) 96 abc(3a – 12) (5b – 30) ÷ 144 (a – 4) (b – 6)

$=\frac{96abc(3\times a-3\times 4)(5\times b-2\times 3\times 5)}{144(a-4)(b-6)}$

$=\frac{2abc\times 3(a-4)\times 5(b-6)}{3(a-4)(b-6)}=10abc$

Question 4:

Divide as directed.

(i) 5(2x + 1) (3x + 5) ÷ (2x + 1)

(ii) 26xy(x + 5) (y – 4) ÷ 13x(y – 4)

(iii) 52pqr (p + q) (q + r) (r + p) ÷ 104pq(q + r) (r + p)

(iv) 20(y + 4) (y^2 + 5y + 3) ÷ 5(y + 4)

(v) x(x + 1) (x + 2) (x + 3) ÷ x(x + 1)

Answer:

(i) $5(2x+1)(3x+5) \div (2x+1) = \frac{5(2x+1)(3x+1)}{(2x+1)}$ = 5(3x + 1)

(ii) $26xy(x+5)(y-4) \div 13x(y-4) = \frac{2 \times 13 \times xy(x+5)(y-4)}{13x(y-4)}$ = 2y (x + 5)

(iii) 52pqr (p + q) (q + r) (r + p) ÷ 104pq (q + r) (r + p)

$$= \frac{2 \times 2 \times 13 \times p \times q \times r \times (p+q) \times (q+r) \times (r+p)}{2 \times 2 \times 2 \times 13 \times p \times q \times (q+r) \times (r+p)}$$

$$= \frac{1}{2} r(p+q)$$

(iv) 20(y + 4) (y^2 + 5y + 3) = 2 × 2 × 5 × (y + 4) (y^2 + 5y + 3)

$$20(y+4)(y^2+5y+3) \div 5(y+4) = \frac{2 \times 2 \times 5 \times (y+4) \times (y^2+5y+3)}{5 \times (y+4)}$$

$$= 4(y^2+5y+3)$$

(v) $x(x+1)(x+2)(x+3) \div x(x+1) = \frac{x(x+1)(x+2)(x+3)}{x(x+1)}$

= (x + 2) (x + 3)

Question 5:

Factorise the expressions and divide them as directed.

(i) (y^2 + 7y + 10) ÷ (y + 5)

(ii) (m^2 – 14m – 32) ÷ (m + 2)

(iii) ($5p^2$ – 25p + 20) ÷ (p – 1)

(iv) 4yz(z^2 + 6z – 16) ÷ 2y(z + 8)

(v) 5pq(p^2 – q^2) ÷ 2p(p + q)

(vi) 12xy($9x^2$ – $16y^2$) ÷ 4xy(3x + 4y)

(vii) $39y^3$($50y^2$ – 98) ÷ $26y^2$(5y + 7)

Answer:

(i) $(y^2 + 7y + 10) = y^2 + 2y + 5y + 10$

$= y(y + 2) + 5(y + 2)$

$= (y + 2)(y + 5)$

$$\left(y^2+7y+10\right)\div\left(y+5\right)=\frac{\left(y+5\right)\left(y+2\right)}{\left(y+5\right)}=y+2$$

(ii) $m^2 - 14m - 32 = m^2 + 2m - 16m - 32$

$= m(m + 2) - 16(m + 2)$

$= (m + 2)(m - 16)$

$$\left(m^2-14m-32\right)\div\left(m+2\right)=\frac{\left(m+2\right)\left(m-16\right)}{\left(m+2\right)}=m-16$$

(iii) $5p^2 - 25p + 20 = 5(p^2 - 5p + 4)$

$= 5[p^2 - p - 4p + 4]$

$= 5[p(p - 1) - 4(p - 1)]$

$= 5(p - 1)(p - 4)$

$$\left(5p^2-25p+20\right)\div\left(p-1\right)=\frac{5\left(p-1\right)\left(p-4\right)}{\left(p-1\right)}=5\left(p-4\right)$$

(iv) $4yz(z^2 + 6z - 16) = 4yz[z^2 - 2z + 8z - 16]$

$= 4yz[z(z - 2) + 8(z - 2)]$

$= 4yz(z - 2)(z + 8)$

$$4yz\left(z^2+6z-16\right)\div 2y\left(z+8\right)=\frac{4yz\left(z-2\right)\left(z+8\right)}{2y\left(z+8\right)}=2z\left(z-2\right)$$

(v) $5pq(p^2 - q^2) = 5pq(p - q)(p + q)$

$$5pq\left(p^2-q^2\right)\div 2p\left(p+q\right)=\frac{5pq\left(p-q\right)\left(p+q\right)}{2p\left(p+q\right)}=\frac{5}{2}q\left(p-q\right)$$

(vi) $12xy(9x^2 - 16y^2) = 12xy[(3x)^2 - (4y)^2] = 12xy(3x - 4y)(3x + 4y)$

$$12xy\left(9x^2-16y^2\right)\div 4xy\left(3x+4y\right)=\frac{2\times 2\times 3\times x\times y\times\left(3x-4y\right)\times\left(3x+4y\right)}{2\times 2\times x\times y\times\left(3x+4y\right)}$$

$$=3\left(3x-4y\right)$$

(vii) $39y^3(50y^2 - 98) = 3 \times 13 \times y \times y \times y \times 2[(25y^2 - 49)] = 3 \times 13 \times 2 \times y \times y \times y \times [(5y)^2 - (7)^2]$

$= 3 \times 13 \times 2 \times y \times y \times y (5y - 7) (5y + 7)$

$26y^2(5y + 7) = 2 \times 13 \times y \times y \times (5y + 7)$ $39y^3(50y^2 - 98) \div 26y^2 (5y + 7)$

Question 3:

Find and correct the errors in the statement: $2x + 3y = 5xy$

Answer:

L.H.S = $2x + 3y \neq$ R.H.S.

The correct statement is $2x + 3y = 2x + 3y$

Question 4:

Find and correct the errors in the statement: $x + 2x + 3x = 5x$

Answer:

L.H.S = $x + 2x + 3x = 1x + 2x + 3x = x(1 + 2 + 3) = 6x \neq$ R.H.S.

The correct statement is $x + 2x + 3x = 6x$

Question 5:

Find and correct the errors in the statement: $5y + 2y + y - 7y = 0$

Answer:

L.H.S. = $5y + 2y + y - 7y = 8y - 7y = y \neq$ R.H.S The correct statement is $5y + 2y + y - 7y = y$ Question 6:

Find and correct the errors in the statement: $3x + 2x = 5x^2$

Answer:

L.H.S. = $3x + 2x = 5x \neq$ R.H.S

The correct statement is $3x + 2x = 5x$

Question 7:

Find and correct the errors in the statement: $(2x)^2 + 4(2x) + 7 = 2x^2 + 8x + 7$

Answer:

L.H.S = $(2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7 \neq$ R.H.S

The correct statement is $(2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7$

Question 8:

Find and correct the errors in the statement: $(2x)^2 + 5x = 4x + 5x = 9x$

Answer:

L.H.S = $(2x)^2 + 5x = 4x^2 + 5x \neq$ R.H.S.

The correct statement is $(2x)^2 + 5x = 4x^2 + 5x$

Question 9:

Find and correct the errors in the statement: $(3x + 2)^2 = 3x^2 + 6x + 4$

Answer:

L.H.S. = $(3x + 2)^2 = (3x)^2 + 2(3x)(2) + (2)^2$ $[(a + b)^2 = a^2 + 2ab + b^2] = 9x^2 + 12x + 4 \neq$ R.H.S

The correct statement is $(3x + 2)^2 = 9x^2 + 12x + 4$

Question 10:

Find and correct the errors in the following mathematical statement. Substituting $x = -3$ in

(a) $x^2 + 5x + 4$ gives $(-3)^2 + 5(-3) + 4 = 9 + 2 + 4 = 15$

(b) $x^2 - 5x + 4$ gives $(-3)^2 - 5(-3) + 4 = 9 - 15 + 4 = -2$

(c) $x^2 + 5x$ gives $(-3)^2 + 5(-3) = -9 - 15 = -24$

Answer:

(a) For $x = -3$,

$x^2 + 5x + 4 = (-3)^2 + 5(-3) + 4 = 9 - 15 + 4 = 13 - 15 = -2$

(b) For $x = -3$,

$x^2 - 5x + 4 = (-3)^2 - 5(-3) + 4 = 9 + 15 + 4 = 28$

(c) For $x = -3$,

$x^2 + 5x = (-3)^2 + 5(-3) = 9 - 15 = -6$

Question 11:

Find and correct the errors in the statement: $(y - 3)^2 = y^2 - 9$

Answer:

L.H.S = $(y - 3)^2 = (y)^2 - 2(y)(3) + (3)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$ = $y^2 - 6y + 9 \neq$ R.H.S

The correct statement is $(y - 3)^2 = y^2 - 6y + 9$

Question 12:

Find and correct the errors in the statement: $(z + 5)^2 = z^2 + 25$

Answer:

L.H.S = $(z + 5)^2 = (z)^2 + 2(z)(5) + (5)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$ = $z^2 + 10z + 25 \neq$ R.H.S

The correct statement is $(z + 5)^2 = z^2 + 10z + 25$

Question 13:

Find and correct the errors in the statement: $(2a + 3b)(a - b) = 2a^2 - 3b^2$

Answer:

L.H.S. = $(2a + 3b)(a - b) = 2a \times a + 3b \times a - 2a \times b - 3b \times b = 2a^2 + 3ab - 2ab - 3b^2 = 2a^2 + ab - 3b^2 \neq$ R.H.S.

The correct statement is $(2a + 3b)(a - b) = 2a^2 + ab - 3b^2$

Question 14:

Find and correct the errors in the statement: $(a + 4)(a + 2) = a^2 + 8$

Answer:

L.H.S. = $(a + 4)(a + 2) = (a)^2 + (4 + 2)(a) + 4 \times 2 = a^2 + 6a + 8 \neq$ R.H.S

The correct statement is $(a + 4)(a + 2) = a^2 + 6a + 8$

Question 15:

Find and correct the errors in the statement: $(a - 4)(a - 2) = a^2 - 8$

Answer:

L.H.S. = $(a - 4)(a - 2) = (a)^2 + [(-4) + (-2)](a) + (-4)(-2)$ = $a^2 - 6a + 8 \neq$ R.H.S.

The correct statement is $(a - 4)(a - 2) = a^2 - 6a + 8$

Question 16:

Find and correct the errors in the statement: $\frac{3x^2}{3x^2} = 0$

Answer:

L.H.S = $\frac{3x^2}{3x^2} = \frac{3 \times x \times x}{3 \times x \times x} = 1 \neq \text{R.H.S.}$

The correct statement is $\frac{3x^2}{3x^2} = 1$

Question 17:

Find and correct the errors in the statement: $\frac{3x^2+1}{3x^2} = 1+1 = 2$

Answer:

$\frac{3x^2+1}{3x^2} = \frac{3x^2}{3x^2} + \frac{1}{3x^2} = 1 + \frac{1}{3x^2} \neq \text{R.H.S.}$

The correct statement is $\frac{3x^2+1}{3x^2} = 1 + \frac{1}{3x^2}$

Question 18:

Find and correct the errors in the statement: $\frac{3x}{3x+2} = \frac{1}{2}$

Answer:

L.H.S = $\frac{3x}{3x+2} \neq \text{R.H.S.}$

The correct statement is $\frac{3x}{3x+2} = \frac{3x}{3x+2}$

Question 19:

Find and correct the errors in the statement: $\frac{3}{4x+3} = \frac{1}{4x}$

Answer:

L.H.S. = $\frac{3}{4x+3}$ ≠ R.H.S.

The correct statement is $\frac{3}{4x+3}=\frac{3}{4x+3}$

Question 20:

Find and correct the errors in the statement: $\frac{4x+5}{4x}=5$

Answer:

L.H.S. = $\frac{4x+5}{4x}=\frac{4x}{4x}+\frac{5}{4x}=1+\frac{5}{4x}\neq \text{R.H.S}$

The correct statement is $\frac{4x+5}{4x}=1+\frac{5}{4x}$

Question 21:

Find and correct the errors in the statement: $\frac{7x+5}{5}=7x$

Answer:

L.H.S. = $\frac{7x+5}{5}=\frac{7x}{5}+\frac{5}{5}=\frac{7x}{5}+1\neq \text{R.H.S.}$

The correct statement is $\frac{7x+5}{5}=\frac{7x}{5}+1$