Assignments in Mathematics Class X (Term II)

4. QUADRATIC EQUATIONS

IMPORTANT TERMS, DEFINITIONS AND RESULTS

- An equation of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$, is called a quadratic equation in x.
- A real number α is called a root of the quardratic equation $ax^2 + bx + c = 0$, $a \neq 0$, if $a\alpha^2 + b\alpha + c = 0$. Any quardratic equation can have at most two roots.

Note : If α is a root of $ax^2 + bx + c = 0$, then we say that

(i) $x = \alpha$ satisfies the equation $ax^2 + bx + c = 0$

or (ii) $x = \alpha$ is a solution of the equation $ax^2 + bx + c = 0$

- The roots of a quadratic equation $ax^2 + bx + c = 0$ are called the zeros of the polynomial $ax^2 + bx + c$.
- Solving a quadratic equation means finding its roots.
- If $ax^2 + bx + c$ can be factorised as $(x - \alpha)(x - \beta)$, then $ax^2 + bx + c = 0$ is equivalent to $(x - \alpha)(x - \beta) = 0$

  Thus, $(x - \alpha)(x - \beta) = 0$ $x - \alpha = 0$ or $x - \beta = 0$

  i.e., $x = \alpha$ or $x = \beta$.

  Here α and β are called the roots of the equation $ax^2 + bx + c = 0$.
- To solve a quadratic equation by factorisation :
  (a) Clear fractions and brackets, if necessary.
  (b) Transfer all the terms to L.H.S. and combine like terms.
  (c) Write the equation in the standard form, i.e., $ax^2 + bx + c = 0$.
  (d) Factorise the L.H.S.
  (e) Put each factor equal to zero and solve.
  (f) Check each value by substituting it in the given equation.
- The roots of a quadratic equation can also be found by using the method of completing the square.
- The roots of the quadratic equation $ax^2 + bx + c = 0$, $a, b, c \in$ R and $a \neq 0$ are given by

  $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (\textit{Shridharacharya's formula})$$

  The expression $b^2 - 4ac$ is called the **discriminant** of the quadratic equation $ax^2 + bx + c = 0$.
- The discriminant, usually denoted by D, decides the *nature of roots* of a quadratic equation.
  (i) If $D > 0$, the equation has real roots and roots are unequal, i.e., unequal-real roots.
  If D is a perfect square, the equation has unequal-rational roots.
  (ii) If $D = 0$, the equation has real and equal roots and each root is $\dfrac{-b}{2a}$
  (iii) If $D < 0$, the equation has no real roots.
- (i) If $-p \geq 5$, then $p \leq -5$
  (ii) If $-p \geq -5$, then $p \leq -5$
  (iii) If $p^2 \geq 4$, then either $p \leq -2$ or $p \geq -5$ **(Important)**
  (iv) If $p^2 \leq 4$, then p lies between –2 and 2, i.e., $-2 \leq p \leq 2$ **(Important)**
- Quadratic equations can be applied to solve word problems involving various situations.
  To solve problems leading to quadratic equations, following steps may be used :
  1. Represent the unknown quantity in the problem by a variable (letter).
  2. Translate the problem into an equation involving this variable.
  3. Solve the equation for the variable.
  4. Check the result by satisfying the conditions of the original problem.
  5. A root of the quadratic equation, which does not satisfy the conditions of the problem, must be rejected.

SUMMATIVE ASSESSMENT

MULTIPLE CHOICE QUESTIONS [1 Mark]

A. Important Questions

1. If $p(x) = 0$ is a quadratic equation, then $p(x)$ is a polynomial of degree :

(a) one (b) two
(c) three (d) four

2. Which of the following is a quadratic equation in x?

(a) $x + 12 = 0$ (b) $7x = 2x^2$
(c) $x^2 + \dfrac{1}{x^2} = 2$ (d) $6 - x(x^2 + 2) = 0$

3. Which of the following is a root of the equation $2x^2 - 5x - 3 = 0$?

(a) $x = 3$ (b) $x = 4$
(c) $x = 1$ (d) $x = -3$

4. Which of the following is a root of the equation $3x^2 - 2x - 1 = 0$?

(a) $x = -1$ (b) $x = -2$
(c) $x = 1$ (d) $x = -1$

5. $x = \sqrt{2}$ is a solution of the equation :

(a) $x^2 + \sqrt{2}x - 4 = 0$ (b) $x^2 - \sqrt{2}x - 4 = 0$
(c) $3x^2 + 5x + 2 = 0$ (d) (a) and (b) both

6. Which of the following is a quadratic equation in x?

(a) $x + \frac{2}{x} = x^2$ (b) $x^2 - \frac{1}{x^2} = 5$
(c) $x^2 - 3x - \sqrt{x} + 4 = 0$
(d) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

7. For what value of k, $x = 2$ is a solution of $kx^2 + 2x - 3 = 0$?

(a) $k = -\frac{1}{2}$ (b) $k = \frac{1}{2}$
(c) $k = -\frac{1}{4}$ (d) $k = \frac{1}{4}$

8. Which of the following is a root of the equation $2x^2 - x + \frac{1}{8}$?

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 4

9. If $x = 2$ is a root of the equation $3x^2 - 2kx + 5 = 0$, then k is equal to :

(a) $\frac{4}{17}$ (b) $\frac{17}{4}$ (c) $\frac{1}{17}$ (d) $\frac{1}{4}$

10. Which of the following is a quadratic equation in x?

(a) $\sqrt{x} + \frac{1}{\sqrt{x}} = 4$ (b) $x^2 + \frac{2}{x^2} = 3$
(c) $x + \frac{1}{x^2} = 3$ (d) $\frac{4}{3}x + x^2 = 0$

11. Which of the following is a quadratic equation?

(a) $x^2 + 2x + 1 = (4 - x)^2 + 3$
(b) $-2x^2 = (5 - x)\left(2x - \frac{2}{5}\right)$
(c) $(k+1)x^2 + \frac{3}{2}x = 7, \text{where } k = -1$
(d) $x^3 - x^2 = (x - 1)^3$

12. Which of the following is not a quadratic equation?

(a) $2(x - 1)^2 = 4x^2 - 2x + 1$
(b) $2x - x^2 = x^2 + 5$
(c) $\left(\sqrt{2}x + \sqrt{3}\right)^2 + x^2 = 3x^2 - 5x$
(d) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

13. Which of the following equations has 2 as a root?

(a) $x^2 - 4x + 5 = 0$ (b) $x^2 + 3x - 12 = 0$
(c) $2x^2 - 7x + 6 = 0$ (d) $3x^2 - 6x - 2 = 0$

14. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is :

(a) 2 (b) -2
(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

15. Which of the following equations has the sum of its roots as 3?

(a) $2x^2 - 3x + 6 = 0$ (b) $-x^2 + 3x - 3 = 0$
(c) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$ (d) $3x^2 - 3x + 3 = 0$

16. The roots of $4x^2 + 4\sqrt{3}x + 3 = 0$ are :

(a) real and equal (b) real and unequal
(c) not real (d) none of these

17. Discriminant of $x^2 + px + 2q = 0$ is :

(a) $p - 8q$ (b) $p^2 + 8q$
(c) $p^2 - 8q$ (d) $q^2 - 8p$

18. If the equation $x^2 + 4x + k = 0$ has real and distinct roots, then :

(a) $k < 4$ (b) $k > 4$
(c) $k > 4$ (d) $k < 4$

19. The quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ has two distinct real roots, if D is equal to :

(a) $b^2 - 4ac > 0$ (b) $b^2 - 4ac = 0$
(c) $b^2 - 4ac < 0$ (d) none of these

20. If $ax^2 + bx + c = 0$ has equal roots, then c is equal to :

(a) $-\frac{b}{2a}$ (b) $\frac{b}{2a}$
(c) $\frac{-b^2}{4a}$ (d) $\frac{b^2}{4a}$

21. If the equation $9x^2 + 6kx + 4 = 0$ has equal roots, then the roots are :

(a) $\pm\frac{2}{3}$ (b) $\pm\frac{3}{2}$
(c) 0 (d) ± 3

22. The roots of the equation $2x^2 + 5x + 5 = 0$ are:

(a) real and distinct (b) not real
(c) real and equal (d) real and unequal

23. If the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + c^2 + d^2 = 0$ has equal roots, then :

(a) $ad = cd$ (b) $ad = bc$
(c) $ad = \sqrt{bc}$ (d) $ab = \sqrt{cd}$

24. If the roots of the equation $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$ are equal, then :

(a) $2b = a + c$ (b) $b^2 = ac$
(c) $b = \dfrac{2ac}{a+c}$ (d) $b = ac$

25. If $x = 1$ is a common root of the equations $ax^2 + ax + 3 = 0$ and $x^2 + x + b = 0$, then ab is equal to:

(a) 3 (b) 3.5 (c) 6 (d) – 3

26. Values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is :
(a) 0 only (b) 4
(c) 8 only (d) 0, 8

27. Which constant must be added and subtracted to solve the quadratic equation $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$ by the method of completing the square?

(a) $\frac{1}{8}$ (b) $\frac{1}{64}$ (c) $\frac{1}{4}$ (d) $\frac{9}{64}$

28. The quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$ has :

(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

29. Which of the following equations has two distinct real roots?

(a) $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$ (b) $x^2 + x - 5 = 0$
(c) $x^2 + 3x + 2\sqrt{2} = 0$ (d) $5x^2 - 3x + 1 = 0$

30. Which of the following equations has no real roots?

(a) $x^2 + 4x + 3\sqrt{2} = 0$ (b) $x^2 + 4x - 3\sqrt{2} = 0$
(c) $x^2 - 4x - 3\sqrt{2} = 0$ (d) $3x^2 + 4\sqrt{3}x + 4 = 0$

31. $(x^2 + 1)^2 - x^2 = 0$ has :

(a) four real roots (b) two real roots
(c) no real roots (d) one real root

32. If $kx^2 - 5x + 3 = 0$ and $2x^2 - kx + 1 = 0$ have equal discriminants, then the value of k is :

(a) 1 (b) 2 (c) – 2 (d) 3

33. The sum and product of the roots of $2x^2 - 5x + 4$ are in the ratio :

(a) 5 : 2 (b) 5 : 4
(c) 10 : 4 (d) none of these

34. The quadratic equation $2(p + q)^2x^2 + 2(p + q)x + 1 = 0$ has :

(a) equal roots (b) no real roots
(c) real but not equal roots
(d) none of these

35. If $p^2x^2 + (p^2 + q^2)x + q^2 = 0$ has equal roots, then $p^2 - q^2$ is equal to :

(a) $-2q^2$ (b) $2q^2$ (c) 0 (d) – 1

36. If $x = -7$ and $x = -5$ are roots of $x^2 + mx + n = 0$, then the values of m and n are :

(a) 12 and 36 (b) 12 and 35
(c) – 12 and – 35 (d) none of these

37. A boy said, "the product of my age 5 years before and after 5 years is 75," then the present age of the boy is :

(a) 8 years (b) 10 years
(c) 12 years (d) 15 years

38. If the price of a pen is increased by Rs 2, a person can buy 1 pen less for Rs 40, then the original price of one pen is :

(a) Rs 10 (b) Rs 8
(c) Rs 6 (d) Rs 4

39. If the sum of the squares of three consecutive integers is 29, then one of the integer is :

(a) 1 (b) 2 (c) 5 (d) 6

B. Questions From CBSE Examination Papers

1. The roots of the equation $x^2 - \sqrt{3}x - x + \sqrt{3} = 0$ are : **[2011 (T-II)]**

(a) $\sqrt{3}$, 1 (b) $-\sqrt{3}$, 1
(c) $-\sqrt{3}$, –1 (d) $\sqrt{3}$, –1

2. The roots of the quadratic equation $\sqrt{3}x^2 - 2x - \sqrt{3} = 0$ are : **[2011 (T-II)]**

(a) $-\sqrt{3}, \sqrt{\frac{1}{3}}$ (b) 2, 3

(c) $\frac{\sqrt{3}}{2}, -\frac{2}{\sqrt{3}}$ (d) $\sqrt{3}, \frac{-1}{\sqrt{3}}$

3. Which of the following is not a quadratic equation : **[2011 (T-II)]**

(a) $(x-2)^2 + 1 = 2x - 3$

(b) $x(x+1) + 8 = (x+2)(x-2)$

(c) $x(2x+3) = x^2 + 1$

(d) $(x+2)^3 = x^3 - 4$

4. The roots of the quadratic equation $x^2 + 7x + 12 = 0$ are : **[2011 (T-II)]**

(a) $-4, -3$ (b) $4, -3$

(c) $4, 3$ (d) $-4, 3$

5. The quadratic equation whose roots are real and equal is : **[2011 (T-II)]**

(a) $2x^2 - 4x + 3 = 0$ (b) $x^2 - 4x + 4 = 0$

(c) $3x^2 - 5x + 2 = 0$ (d) $x^2 - 2\sqrt{2}x - 6 = 0$

6. Which of the following equations has two distinct real roots? **[2011 (T-II)]**

(a) $2x^2 - 3\sqrt{2}x + 9/4 = 0$

(b) $x^2 + x - 5 = 0$

(c) $x^2 + 3x + 2\sqrt{2} = 0$

(d) $5x^2 - 3x + 1 = 0$

7. If 8 is a root of the equation $x^2 - 10x + k = 0$, then the value of k is : **[2011 (T-II)]**

(a) 2 (b) 8 (c) -8 (d) 16

8. If the discriminant of $3x^2 + 2x + a = 0$, is double the discriminant of $x^2 - 4x + 2 = 0$, then the value of a is: **[2011 (T-II)]**

(a) 2 (b) -2 (c) 1 (d) -1

9. The positive root of $\sqrt{3x^2 + 6} = 9$ is : **[2011 (T-II)]**

(a) 3 (b) 4 (c) 5 (d) 7

10. One of the roots of the quadratic equation $6x^2 - x - 2 = 0$ is : **[2011 (T-II)]**

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $-\frac{2}{3}$ (d) -1

11. For what value of k will $\frac{7}{3}$ be a root of $3x^2 - 13x - k = 0$? **[2011 (T-II)]**

(a) 14 (b) $\frac{3}{7}$ (c) $\frac{-7}{2}$ (d) -14

12. If $x^2 + 2kx + 4 = 0$ has a root $x = 2$, then the value of k is? **[2011 (T-II)]**

(a) -1 (b) -2 (c) 2 (d) -4

13. The value of k for which the equation $2x^2 - (k-1)x + 8 = 0$ will have real and equal roots are : **[2011 (T-II)]**

(a) 9 and -7 (b) only 9

(c) only -7 (d) -9 and -7

14. Which of the following is a solution of the quadratic equation $x^2 - b^2 = a(2x - a)$? **[2011 (T-II)]**

(a) $a + b$ (b) $2b - a$ (c) ab (d) $\frac{a}{b}$

15. If one root of the equation $2x^2 - 10x + p = 0$ is 2, then the value of p is : **[2011 (T-II)]**

(a) -3 (b) -6 (c) 9 (d) 12

16. Which of the following is a root of the equation $2x^2 - 5x - 3 = 0$? **[2011 (T-II)]**

(a) $x = 3$ (b) $x = 4$

(c) $x = 1$ (d) $x = -3$

17. If $\frac{1}{2}$ is the root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is : **[2011 (T-II)]**

(a) 2 (b) -2 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

18. The roots of the equation $ax^2 + x + b = 0$ are equal if : **[2011 (T-II)]**

(a) $b^2 = 4a$ (b) $b^2 < 4a$

(c) $b^2 > 4a$ (d) $ab = \frac{1}{4}$

19. If the equation $9x^2 + 6kx + 4 = 0$ has equal roots, then the value of k is : **[2011 (T-II)]**

(a) ± 2 (b) $\pm\frac{3}{2}$ (c) 0 (d) ± 3

20. The roots of the equation $3x^2 - 4x + 3 = 0$ are : **[2011 (T-II)]**

(a) real and unequal (b) real and equal

(c) imaginary (d) none of these

21. If the equation $kx^2 + 4x + 1 = 0$ has real and distinct roots, then : **[2011 (T-II)]**

(a) $k < 4$ (b) $k > 4$

(c) $k \leq 4$ (d) $k \geq 4$

22. Value of k for which quadratic equation $2x^2 - kx + k = 0$ has equal roots is : **[2011 (T-II)]**

(a) -4 (b) 4 (c) 8 (d) -8

23. If $r = 3$ is a root of quadratic equation $kr^2 - kr - 3 = 0$, value of k is : **[2011 (T-II)]**

(a) $\frac{1}{2}$ (b) 2 (c) -2 (d) $-\frac{1}{2}$

24. The value of k, for which the quadratic equation $4x^2 + 4\sqrt{3}\,x + k = 0$ has equal roots is :

(a) $k = 2$ (b) $k = -2$

(c) $k = -3$ (d) $k = 3$

25. For what value of k the equation $kx^2 - 6x - 2 = 0$ has equal roots ? **[2011 (T-II)]**

(a) $\frac{7}{2}$ (b) $\frac{-9}{2}$ (c) -3 (d) $\frac{-7}{2}$

26. The value of p for which the quadratic equation $x(x-4) + p = 0$ has real roots, is : **[2011 (T-II)]**

(a) $p < 4$ (b) $p \geq 4$

(c) $p = 4$ (d) none of these

27. The value of k for which $3x^2 + 2x + k = 0$ has real roots is : **[2011 (T-II)]**

(a) $k > \frac{1}{3}$ (b) $k \leq \frac{1}{3}$

(c) $k \geq \frac{1}{3}$ (d) $k < \frac{1}{3}$

28. For the quadratic equation $x^2 - 2x + 1 = 0$ the value of $x + \frac{1}{x}$ is : **[2011 (T-II)]**

(a) -1 (b) 1 (c) 2 (d) -2

SHORT ANSWER TYPE QUESTIONS [2 Marks]

A. Important Questions

1. Find the roots of the quadratic equation $x^2 - 3x = 0$.

2. Are $x = 0$, $x = 1$ the solution of the equation $x^2 + x + 1 = 0$?

3. Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$.

4. If $4a^2x^2 - 4abx + k = 0$ has equal roots, then find the value of k.

5. Find the discriminant of the quadratic equation $bx^2 - 2\sqrt{ac}x + b = 0$

6. If $kx^2 - 2\sqrt{5}x + 4 = 0$ has real and equal roots, then find the value of k.

7. If 1 is a root of $x^2 - (\sqrt{3}+1)x + k = 0$ and $4x^2 + 4kx + a = 0$ has equal roots, then find the value of a.

8. If 2 is a root of the equation $x^2 + bx + 12 = 0$ and the equation has equal roots, find the value of b.

9. If one of the roots of the equation $3a^2x^2 + 8abx + 4b^2 = 0$ is $\frac{-2b}{a}$, then find the other root.

10. If $x = \frac{c}{b}$ is a root of $abx^2 + (b^2 - ac)x - k = 0$, then find the value of k.

11. State whether the quadratic equation $(x - \sqrt{2})^2 - 2(x+1) = 0$ has two distinct real roots. Justify your answer.

12. Write whether the following statement is true or false. Justify your answer.

If the coefficient of x^2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots.

13. If $b = 0$, $c < 0$, is it true that the roots of the quadratic equation $x^2 + bx + c = 0$ are numerically equal and opposite in sign? Justify. **[HOTS]**

14. Does a quadratic equation with integral coefficient has always integral roots. Justify your answer. **[HOTS]**

15. Is the following statement 'true' or 'false'? Justify your answer. If in a quadratic equation the coefficient of x is zero, then the quadratic equation has no real roots. **[HOTS]**

16. Does $(x-1)^2 + 2(x+1) = 0$ have a real root? Justify your answer. **[HOTS]**

Solve the following quadratic equations by factorisation (17-26)*:*

17. $x^2 - \left(\sqrt{2}+1\right)x + \sqrt{2} = 0$

18. $x^2 - \left(\sqrt{3}+1\right)x + \sqrt{3} =$

19. $4x^2 + 4bx - (a^2 - b^2) = 0$

20. $ax^2 + (4a^2 - 3b)x - 12ab = 0$

21. $a^2x^2 + (a^2 + b^2)x + b^2 = 0,\ a \neq 0$

22. $\frac{m}{n}x^2 + \frac{n}{m} = 1 - 2x$

23. $(a + b)^2x^2 - 4abx - (a - b)^2 = 0$

24. $a(x^2 + 1) - x(a^2 + 1) = 0$

25. $x^2 - x - a(a + 1) = 0$

26. $(x-3)(x-4) = \frac{34}{(33)^2}$

Solve the following quadratic equations by completing the squares (27-32)*:*

27. $4x^2 + 4\sqrt{3}x + 3 = 0$

28. $\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

29. $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

30. $x^2 - \left(\sqrt{2} + 1\right)x + \sqrt{2} = 0$

31. $a^2x^2 - 3abx + 2b^2 = 0$

32. $y^2 + \frac{1}{2}y - 1 = 0$

Solve the following equations by using the formula (33-38)*:*

33. $x^2 - 2ax + 3x - 6a = 0$

34. $(2p + q)x = x^2 + 2pq$

35. $a^2b^2x^2 + b^2x - a^2x - 1 = 0$

36. $a^2 - b^2 = 4ax - 4x^2$

37. $(a - b)x^2 + (b - c)x + (c - a) = 0$

38. $x^2 - 2(m + n)x + (m + n)^2 = 1$

Find the values of k for which the roots are real and equal in each of the following equations (39-42)*:*

39. $(k + 1)x^2 + 2(k + 3)x + (k + 8) = 0$

40. $x^2 - 2kx + 7k - 12 = 0$

41. $5x^2 - 4x + 2 + k(4x^2 - 2x - 1) = 0$

42. $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$

43. Show that the equation $x^2 + ax - 4 = 0$ has real and distinct roots for all real values of a.

44. For what value of k, $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$, is a perfect square.

45. If the roots of the equations $ax^2 + 2bx + c = 0$ and $bx^2 - 2\sqrt{ac}x + b = 0$ are simultaneously real, then prove that $b^2 = ac$.

B. Questions From CBSE Examination Papers

1. Find the values of k for which roots of the equation $x^2 - 8kx + 2k = 0$ are equal. **[2011 (T-II)]**

2. Find the values of p such that the quadratic equation $(p - 12)x^2 - 2(p - 12)x + 2 = 0$ has equal roots. **[2011 (T-II)]**

3. Find the roots of the quadratic equation $3x^2 - 14x + 8 = 0$. **[2011 (T-II)]**

4. Find the value(s) of k for which the equation $x^2 - 2x + k = 0$ has equal roots. **[2011 (T-II)]**

5. For what value(s) of k, the equation $x^2 - 2kx - k = 0$ will have equal roots? **[2011 (T-II)]**

6. Solve the equation : $10ax^2 + 15ax - 6x - 9 = 0$, $a \neq 0$. **[2011 (T-II)]**

7. Write all the values of k for which the quadratic equation $2x^2 + 2kx + 8 = 0$, has equal roots. Also, find the roots. **[2011 (T-II)]**

8. Find the value of k for which the equation : $kx(x - 2) + 6 = 0$ has equal roots. **[2011 (T-II)]**

9. Solve : $2x - \frac{3}{x} = 1$ **[2011 (T-II)]**

10. Find the roots of the following quadratic equation by factorisation method. **[2011 (T-II)]**

$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

11. One root of the equation $2x^2 - 8x - m = 0$ is $\frac{5}{2}$. Find the other root and the value of m. **[2011 (T-II)]**

12. For what value of k the equation $4x^2 - 2(k + 1)x + (k + 1) = 0$ has real and equal roots? **[2011 (T-II)]**

13. Using quadratic formula, determine the roots of following equation : **[2011 (T-II)]**

$x - \frac{1`}{x} = 3$

14. Find the values of k for which the following quadratic equation has two equal roots.
$2x^2 + kx + 3 = 0$ **[2011 (T-II)]**

15. For what value of k, the quadratic equation $9x^2 + 8kx + 16 = 0$ has equal roots? **[2011 (T-II)]**

16. Find the roots of the quadratic equation $3x^2 - 2\sqrt{6}x + 2 = 0$ **[2011 (T-II)]**

17. Find the roots of the following quadratic equation: **[2011 (T-II)]**

$(x + 3)(x - 1) = 3\left(x - \frac{1}{3}\right)$

18. Find the value of k such that the quadratic equation $x(x - 2k) + 6 = 0$ has real and equal roots. **[2011 (T-II)]**

19. Find the value of k such that the quadratic equation $x^2 - 2kx + (7k - 12) = 0$ has real and equal roots. **[2011 (T-II)]**

20. Find the nature of roots of the quadratic equation $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + \frac{1}{\sqrt{2}} = 0$ **[2011 (T-II)]**

21. Find the roots of $6x^2 - \sqrt{2}x - 2 = 0$ **[2011 (T-II)]**

22. Find the roots of the quadratic equation $2x^2 - 5x + 3 = 0$ **[2011 (T-II)]**

23. Find the roots of the quadratic equation $6x^2 + 5x - 6 = 0$ **[2011 (T-II)]**

24. If –4 is a root of the quadratic equation $x^2 + px - 4 = 0$ and the equation $2x^2 + px + k = 0$ has equal roots, find the value of k. **[2011 (T-II)]**

25. If $4a^2x^2 - 4abx + k = 0$ has equal roots of x, then find the value of k. **[2011 (T-II)]**

26. For what value of k does $(k - 12)\,x^2 + 2\,(k - 12)\,x + 2 = 0$ have equal roots? **[2008C]**

27. Solve for x : $4x^2 - 2(a^2 + b^2)x + a^2b^2 = 0$. **[2004]**

28. Solve for x : $4x^2 - 4a^2x + (a^4 - b^4) = 0$. **[2004]**

29. Using quadratic formula, solve the following quadratic equation for x :
$p^2x^2 + (p^2 - q^2)x - q^2 = 0$. **[2004]**

30. Using quadratic formula, solve the following quadratic equation for x :
$x^2 - 2ax + (a^2 - b^2) = 0$. **[2004]**

31. Using quadratic formula, solve the following quadratic equation for x :
$x^2 - 4ax + 4a^2 - b^2 = 0$ **[2004]**

32. Solve for x : $16x^2 - 8a^2x + (a^4 - b^4) = 0$ **[2004]**

33. Solve for x : $36x^2 - 12ax + (a^2 - b^2) = 0$. **[2004C]**

SHORT ANSWER TYPE QUESTIONS [3 Marks]

A. Important Questions

Solve the following quadratic equations by factorisation (1–6) *:*

1. $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}, \ (x \neq 0)$

2. $\dfrac{1}{2 - \dfrac{1}{2 - \dfrac{1}{2-y}}} = y, \ (y \neq 2)$ **[HOTS]**

3. $\frac{a}{ax-1} + \frac{b}{bx-1} = a + b$

4. $\frac{x-a}{x-b} + \frac{x-b}{x-a} = \frac{a}{b} + \frac{b}{a}$

5. $\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$

6. $\left(\frac{x}{x+1}\right)^2 - 5\left(\frac{x}{x+1}\right) + 6 = 0, (x \neq 0)$

7. If a, b, c, are real numbers such that $ac \neq 0$ then show that at least one of the equations $ax^2 + bx + c = 0$ and $-ax^2 + bx + c = 0$ has a real root.

8. If the equation $(1 + m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2(1 + m^2)$.

9. If p, q, r and s are real numbers such that $pr = 2(q + s)$, then show that at least one of the equations $x^2 + px + q = 0$ and $x^2 + rx + s = 0$ has real roots.

10. Prove that the equation $x^2(a^2 + b^2) + 2x(ac + bd) + (c^2 + d^2) = 0$ has no real root, if $ad \neq bc$

11. If the roots of the equation $x^2 + 2cx + ab = 0$ are real and unequal, prove that the equation $x^2 - 2(a + b)x + a^2 + b^2 + 2c^2 = 0$ has no real roots.

12. The sum of two numbers is 8 and 15 times the sum of their reciprocals is also 8. Find the numbers.

13. The product of two successive integral multiples of 5 is 300. Determine the multiples.

14. The sum of the squares of two numbers is 233 and one of the numbers is 3 less than twice the other number. Find the numbers.

15. Find two consecutive positive integers, the sum of whose squares is 365.

16. The difference of two numbers is 8 and the sum of their squares is 274. Find the numbers.

17. Find three consecutive positive numbers such that the square of the middle number exceeds the difference of the squares of the other two by 60.

18. If a number is added to three times its reciprocal, the result is $5\frac{3}{5}$. Find the number.

19. The length of a rectangle is 3 cm more than its width and its area is 40 cm^2. Find the dimensions of the rectangle.

20. The denominator of a fraction is 3 more than its numerator. The sum of the fraction and its reciprocal is $2\frac{9}{10}$ Find the fraction.

21. A two-digit number is 4 times the sum of its digits and also equal to twice the product of its digits. Find the number.

B. Questions From CBSE Examination Papers

1. The sum of the squares of two consecutive natural numbers is 421. Find the numbers. **[2011 (T-II)]**

2. Solve for x by using quadratic formula $36x^2 - 12ax + (a^2 - b^2) = 0$ **[2011 (T-II)]**

3. The sum of a number and its reciprocal is $\frac{10}{3}$. Find the number. **[2011 (T-II)]**

4. Solve the following equation :

$$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0;\ x \neq 3, -\frac{3}{2}$$

[2011 (T-II]

5. Solve the equation :

$$\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}\ x \neq -4, 7$$

[2011 (T-II]

6. If one root of the quadratic equation $x^2 - 5x + 6k = 0$ is reciprocal of other, find the value of k. Also find the roots. **[2011 (T-II)]**

7. Solve the given equation for x : **[2011 (T-II)]**

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{x}=\frac{1}{a+b+x}$$

8. Solve : $\frac{1}{x}-\frac{1}{x-2}=3$ **[2011 (T-II)]**

9. Find the roots of the following quadratic equation by the factorisation method. **[2011 (T-II)]**

$$4\sqrt{3}x^2+5x-2\sqrt{3}=0$$

10. Find the roots of the following quadratic equation.

$$-x^2+7x-10=0$$

[2011 (T-II)]

11. Find two natural numbers, which differ by 3 and whose squares have the sum 149. **[2011 (T-II)]**

12. Solve for x : $\frac{4}{x}-3=\frac{5}{2x+3}, x \neq 0, \frac{-3}{2}$ **[2011 (T-II)]**

13. The sum of two natural numbers is 8. Determine the numbers, if the sum of their reciprocals is 8/15. **[2011 (T-II)]**

14. An express train takes 1 hour less than a passenger train to travel 132 km between stations A and B (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/hour more than that of the passenger train, find the average speed of the two trains. **[2011 (T-II)]**

15. If two pipes function simultaneously, a reservoir will be filled in twelve hours. First pipe fills the reservoir 10 hours faster than the second pipe. How many hours will the second pipe take to fill the reservoir? **[2011 (T-II)]**

16. Solve the equation for x : **[2011 (T-II)]**

$$\frac{x+3}{x-2}-\frac{1-x}{x}=\frac{17}{4}$$

17. Solve for x :

$$a(a^2+b^2)x^2+b^2x-a=0$$

18. Solve for x : $\frac{1}{x-3}-\frac{1}{x+5}=\frac{1}{6}$ **[2011 (T-II)]**

19. Solve for x : $\frac{x+1}{x-1}+\frac{x-2}{x+2}=3; x \neq 1, -2$ **[2011 (T-II)]**

20. If the equations $5x^2+(9+4p)x+2p^2=0$ and $5x + 9 = 0$ are satisfied by the same value of x, find the value of p. **[2011 (T-II)]**

21. Solve the following quadratic equation :

$x^2 - 3x - 10 = 0$ **[2011 (T-II)]**

22. Solve for x : $\frac{x}{x+1}+\frac{x+1}{x}=\frac{34}{15}$ **[2011 (T-II)]**

23. Find the value of k for which the quadratic equation $(k + 4)\, x^2 + (k + 1)\, x + 1 = 0$ has equal roots. **[2011 (T-II)]**

24. The sum of the reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age. **[2011 (T-II)]**

25. Solve for x : $\frac{1}{x+1}+\frac{2}{x+2}=\frac{4}{x+4}; x \neq -1, -2, -4$ **[2011 (T-II)]**

26. Solve for x : $9x^2-3(a+b)x+ab=0$ **[2011 (T-II)]**

27. Find two positive numbers whose squares have the difference 48 and the sum of the numbers is 12. **[2011 (T-II)]**

28. Solve for x : **[2011 (T-II)]**
$\left(\frac{4x-3}{2x+1}\right)-10\left(\frac{2x+1}{4x-3}\right)=3, x \neq -\frac{1}{2}, \frac{3}{4}$

29. Find the roots of the quadratic equation : $a^2b^2x^2+b^2x-a^2x-1=0$ **[2011 (T-II)]**

30. The length of a rectangular plot is greater than thrice its breadth by 2 m. The area of the plot is 120 sq. m. Find the length and breadth of the plot. **[2011 (T-II)]**

31. Solve for x : $\sqrt{7}x^2-x-13\sqrt{7}=0$ **[2011 (T-II)]**

32. Solve the following quadratic equation for x : $p^2x+(p^2-q^2)x-q^2=0$ **[2011 (T-II)]**

33. Solve for x : **[2004]**
$2\left(\frac{2x-1}{x+3}\right)-3\left(\frac{x+3}{2x-1}\right)=5$; given that $x \neq -3, x \neq \frac{1}{2}$.

LONG ANSWER TYPE QUESTIONS **[4 Marks]**

A. Important Questions

1. The difference of the squares of two numbers is 45. The square of the smaller number is 4 times the larger number. Find the numbers.
2. Out of a number of *Saras* birds, one fourth the number are moving about in lotus plants, $\frac{1}{9}$th coupled (along) with $\frac{1}{4}$ as well as 7 times the square root of the number move on a hill, 56 birds remain in Vakula trees. What is the total number of birds? (*Mahavira around* 850 *A.D.*)
3. A train, travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed of the train.
4. If Zeba were younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?
5. At present Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Asha and Nisha.
6. In the centre of a rectangular lawn of dimensions 50 m × 40 m, a rectangular pond has to be constructed so that the area of the grass surrounding the pond would be 1184 m^2. Find the length and breadth of the pond.
7. A sailor can row a boat 8 km downstream and return back to the starting point in 1 hour 40 minutes. If the speed of the stream is 2 km/hr, find the speed of the boat in still water.
8. The length of a rectangle is thrice as long as the side of a square. The side of the square is 4 cm more than the width of the rectangle. If their areas being equal, find their dimensions.
9. Out of a group of swans, 7/2 times the square root of the total number are playing on the shore of a pond. The two remaining ones are swinging in water. Find the total number of swans.
10. Is it possible to design a rectangular mango grove whose length is twice its breadth and the area is 800 m^2? If so, find its length and breadth.
11. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.
12. Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2? If so, find its length and breadth.

B. Questions From CBSE Examination Papers

1. Three consecutive positive integers are taken such that the sum of the square of the first and the product of the other two is 154. Find the integers. **[2011 (T-II)]**
2. The speed of a boat in still water is 11 km/hr. It can go 12 km upstream and return downstream to the original point in 2 hrs 45 min. Find the speed of the stream. **[2011 (T-II)]**
3. The product of the digits of a two digit positive number is 24. If 18 is added to the number, then the digits of the number are interchanged. Find the number. **[2011 (T-II)]**
4. Two taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank. **[2011 (T-II)]**

5. A person on tour has Rs 360 for his daily expenses. If he extends his tour for four days, he has to cut down his daily expenses by Rs 3. Find the original duration of the tour. **[2011 (T-II)]**

6. A train travels at a uniform speed for a distance of 63 km and then travels a distance of 72 km at an aveage speed of 6 km/h more than its original speed. If it takes 3 hours to complete the total journey, what is the original speed of the train? **[2011 (T-II)]**

7. Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares. **[2011 (T-II)]**

8. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers. **[2011 (T-II)]**

9. Two pipes running together can fill a tank in 6 minutes. If one pipe takes 5 minutes more than the other to fill the tank, find the time in which each pipe would fill the tank separately. **[2011 (T-II)]**

10. A motorboat whose speed is 18 km/hr in still water takes 1 hr more to go 24 km upstream than to return downstream to the same spot. Find the speed of stream. **[2011 (T-II)]**

11. Some students planned a picnic.The budget for food was Rs 480. But 8 of them failed to go, the cost of food for each member increased by Rs 10. How many students attended the picnic? **[2011 (T-II)]**

12. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train was 10 km/hr less than that of the fast train, find the speeds of the trains. **[2011 (T-II)]**

13. A person has a rectangular garden whose area is 100 sq m. He fences three sides of the garden with 30 m barbed wire. On the fourth side, the wall of his house is constructed; find the dimensions of the garden. **[2011 (T-II)]**

14. Two pipes can together fill a tank in $3\frac{1}{13}$ minutes. If one pipe takes 3 minutes more than the other to fill it, find the time in which each pipe can fill the tank. **[2011 (T-II)]**

15. By increasing the speed of a bus by 10 km/hr, it takes one and half hours less to cover a journey of 450 km. Find the original speed of the bus. **[2011 (T-II)]**

16. A factory produces certain pieces of pottery in a day. It was observed on a particular day that the cost of production of each piece (in rupees) was 3 more than twice the number of articles produced in the day. If the total cost of production on that day was Rs 90, find the number of pieces produced and cost of each piece. **[2011 (T-II)]**

17. The hypotenuse of a right triangle is $3\sqrt{5}$ cm. If the smaller side is tripled and the larger side is doubled, the new hypotenuse will be 15 cm. Find the length of each side. **[2011 (T-II]**

18. A man bought a certain number of toys for Rs 180. He kept one for his own use and sold the rest for one rupee each more than he gave for them. Besides getting his own toy for nothing, he made a profit of Rs 10. Find the number of toys, he initially bought. **[2011 (T-II)]**

19. The product of Tanay's age (in years) five years ago and his age ten years later is 16. Determine Tanay's present age. **[2011 (T-II)]**

20. A plane left 30 minutes later than the schedule time and in order to reach its destination 1500 km away in time, it has to increase its speed by 250 km/hr from its usual speed. Find its usual speed. **[2011 (T-II)]**

21. A train travels 300 km at a uniform speed. If the speed of the train had been 5 km/hour more, it would have taken 2 hours less for the same journey. Find the usual speed of the train. **[2011 (T-II)]**

22. The sum of two natural numbers is 8. Determine the numbers, if the sum of their reciprocals is $\frac{8}{15}$. **[2011 (T-II)]**

23. A two digit number is four times the sum of its digits. It is also equal to three times the product of its digits. Find the number. **[2011 (T-II)]**

24. The speed of a boat in still water is 15 km/hr. It can go 30 km upstream and return downstream to the original point in 4 hours 30 minutes. Find the speed of the stream. **[2011 (T-II)]**

25. The sum of ages of father and his son is 45 years. 5 years ago, the product of their ages was 124. Determine their present ages. **[2011 (T-II)]**

26. The numerator of a fraction is 3 less than its denominator. If 2 is addd to both numerator as well as denominator, the sum of new and original fraction is $\frac{29}{20}$, find the frction. **[2011 (T-II)]**

27. The hypotenuse of a right angled triangle is 6 cm more than twice its shortest side. If third side is 2 cm less than the hypotenuse, find the sides of this triangle. **[2011 (T-II)]**

28. A takes 6 days less tahn the time taken by B to finish a piece of work. If both A and B together can finish it in 4 days, find the time taken by B to finish the work. **[2011 (T-II)]**

29. The differene of squares of two natural numbers is 45. The square of the smaller number is four times the larger number. Find the numbers.

30. The denominator of a fraction is one more than twice the numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction.

[2011 (T-II)]

31. In a class test, the sum of the marks obtained by P in Mathematics and Science is 28. Had he got 3 more marks in Maths and 4 marks less in Science, the product of marks obtained in the two subjects would have been 180. Find the marks obtained in the two subjects separately. **[2008]**

32. A peacock is sitting on the top of a pillar which is 9 m high. From a point 27 m away from the bottom of the pillar, a snake is coming to its hole at the base of the pillar. Seeing the snake the peacock pounces on it. If their speeds are equal, at what distane from the hole is the snake caught? **[2008]**

33. In a class test, the sum of Kamal's marks in Maths and English is 40. Had he got 3 marks more in Maths and 4 marks less in English, the product of their marks would have been 360. Find his marks in two subjects. **[2008C]**

34. A person on tour has Rs 4200 for his expenses. If he extends his tour for 3 days, he has to cut down his daily expenses by Rs 70. Find the duration of the tour. **[2008C]**

35. In a class test the sum of Gagan's, marks in Mathematics and English is 45. If he had 1 more mark in Maths and 1 less in English, the product of marks would have been 500. Find the original marks obtained by Gagan in Maths and English separately. **[2008C]**

36. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars? **[2009]**

37. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the values of p and k. **[2009]**

38. A trader bought a number of articles for Rs 900, five articles were found damaged. He sold each of the remaining articles at Rs 2 more than what he paid for it. He got a profit of Rs 80 on the whole transaction. Find the number of articles he bought. **[2009]**

39. Two years ago a man's age was three times the square of his son's age. Three years hence his age will be four times his son's age. Find their present ages. **[2009]**

FORMATIVE ASSESSMENT

Activity

Objective : To find solution of quadratic equation $x^2 + bx + c = 0$ by completing the square.

Materials Required : Coloured paper, a pair of scissors, geometry box and fevistick.

Procedure :

Case I : Let us find the solution of $x^2 + 6x + 8 = 0$.

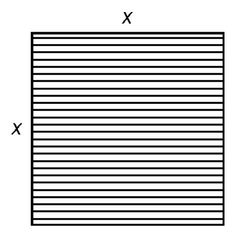

Figure 1

1. Make a square of dimension $x \times x$ (Here, $x = 5$ cm) as shown in figure 1. Here, area of square is equal to 1st term of polynomial $x^2 + bx + c$, i.e., x^2.
2. Add two strips of dimensions $3 \times x$ to figure 1 (2nd term of polynomial is +ve) to obtain figure 2. Here, area of each strip is equal to $\frac{1}{2}$ of 2nd term of polynomial $x^2 + bx + c$, i.e., $\frac{1}{2} \times bx$.

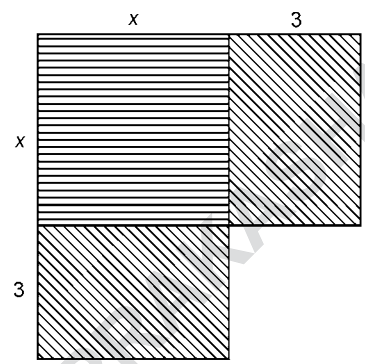

Figure 2

3. Add and subtract a square of dimension 3×3 to figure 2. We will get an arrangement as shown in figure 3.

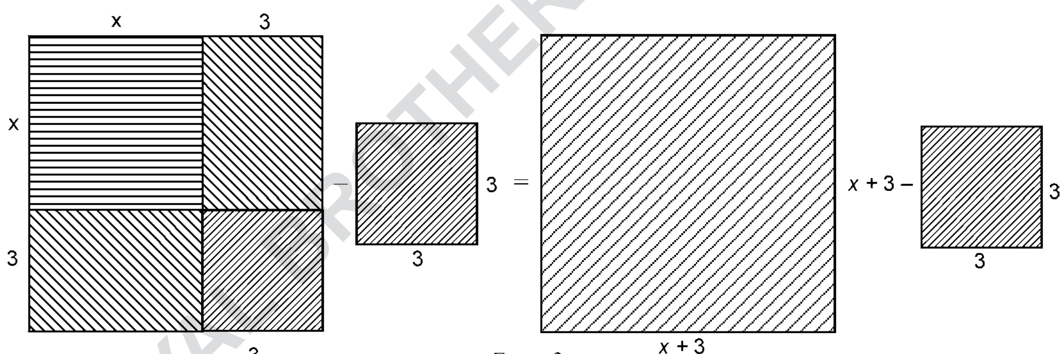

Figure 3

4. From above three figures, we observe that

$x^2 + 6x = x^2 + 3x + 3x + 9 - 9 = (x + 3)^2 - 9$

$\Rightarrow x^2 + 6x + 8 = (x + 3)^2 - 9 + 8 = (x + 3)^2 - 1$

$\therefore\ x^2 + 6x + 8 = 0 \Rightarrow (x + 3)^2 - 1 = 0$

$\Rightarrow (x + 3)^2 = 1 \ \Rightarrow x + 3 = \pm 1$

$\Rightarrow x = -4, -2.$

Case II : Let us find the solution of $x^2 - 6x + 8 = 0$.

1. Make a grid of dimension $x \times x$ [$x = 10$] as shown in figure 4.

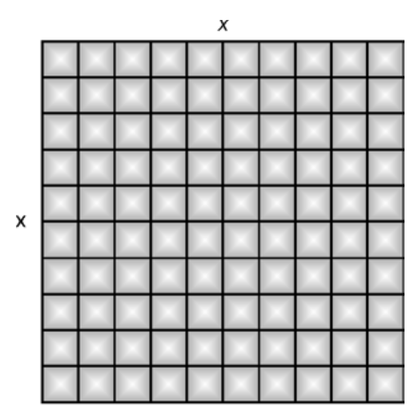

Figure 4

2. Cut out 60 strips (exactly shown in figure 5) from the grid.

[$\because$ 2nd term of $x^2 - 6x + 8$ is –ve]

Area of 60 strips = Area of two rectangles of dimensions $3 \times x$.

[$\because$ Here, $3x = \frac{1}{2}$ of $6x$, i.e., 2nd term of $x^2 - 6x + 8$]

3. From figure 5, we observe that

Area of unshaded portion

= Area of square ABCD – Area of square EFGD

$= (x - 3)^2 - 3^2$

$\Rightarrow x^2 - 6x = (x - 3)^2 - 9$

$\Rightarrow x^2 - 6x + 8 = (x - 3)^2 - 9 + 8 = (x - 3)^2 - 1$

$\Rightarrow x^2 - 6x + 8 = 0 \Rightarrow (x - 3)^2 - 1 = 0$

$\Rightarrow (x - 3)^2 = 1 \Rightarrow x - 3 = \underline{+}\ 1$

$\Rightarrow x = 4, 2.$

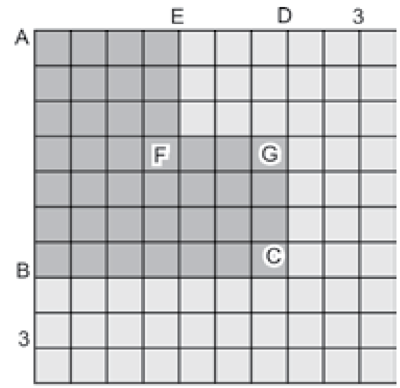

Figure 5

Exercise 4.1

Question 1:

Check whether the following are quadratic equations:

(i) $(x+1)^2 = 2(x-3)$ (ii) $x^2 - 2x = (-2)(3-x)$

(iii) $(x-2)(x+1) = (x-1)(x+3)$ (iv) $(x-3)(2x+1) = x(x+5)$

(v) $(2x-1)(x-3) = (x+5)(x-1)$ (vi) $x^2 + 3x + 1 = (x-2)^2$

(vii) $(x+2)^3 = 2x(x^2-1)$ (viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$

Answer:

(i) $(x+1)^2 = 2(x-3) \Rightarrow x^2 + 2x + 1 = 2x - 6 \Rightarrow x^2 + 7 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(ii) $x^2 - 2x = (-2)(3-x) \Rightarrow x^2 - 2x = -6 + 2x \Rightarrow x^2 - 4x + 6 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(iii) $(x-2)(x+1) = (x-1)(x+3) \Rightarrow x^2 - x - 2 = x^2 + 2x - 3 \Rightarrow 3x - 1 = 0$

It is not of the form $ax^2 + bx + c = 0$.

Hence, the given equation is not a quadratic equation.

(iv) $(x-3)(2x+1) = x(x+5) \Rightarrow 2x^2 - 5x - 3 = x^2 + 5x \Rightarrow x^2 - 10x - 3 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(v) $(2x-1)(x-3)=(x+5)(x-1) \Rightarrow 2x^2-7x+3=x^2+4x-5 \Rightarrow x^2-11x+8=0$ It is of the form $ax^2+bx+c=0$.

Hence, the given equation is a quadratic equation.

(vi) $x^2+3x+1=(x-2)^2 \Rightarrow x^2+3x+1=x^2+4-4x \Rightarrow 7x-3=0$

It is not of the form $ax^2+bx+c=0$.

Hence, the given equation is not a quadratic equation.

(vii) $(x+2)^3=2x(x^2-1) \Rightarrow x^3+8+6x^2+12x=2x^3-2x \Rightarrow x^3-14x-6x^2-8=0$ It is not of the form $ax^2+bx+c=0$.

Hence, the given equation is not a quadratic equation.

(viii) $x^3-4x^3-x+1=(x-2)^3 \Rightarrow x^3-4x^2-x+1=x^3-8-6x^2+12x \Rightarrow 2x^2-13x+9=0$

It is of the form $ax^2+bx+c=0$.

Hence, the given equation is a quadratic equation.

Question 2:

Represent the following situations in the form of quadratic equations.

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Answer:

(i) Let the breadth of the plot be x m.

Hence, the length of the plot is $(2x + 1)$ m.

Area of a rectangle = Length × Breadth

$\therefore 528 = x(2x + 1)$

$\Rightarrow 2x^2 + x - 528 = 0$

(ii) Let the consecutive integers be x and $x + 1$.

It is given that their product is 306.

$\therefore x(x+1) = 306 \Rightarrow x^2 + x - 306 = 0$

(iii) Let Rohan's age be x.

Hence, his mother's age = $x + 26$

3 years hence,

Rohan's age = $x + 3$

Mother's age = $x + 26 + 3 = x + 29$

It is given that the product of their ages after 3 years is 360.

$\therefore (x+3)(x+29) = 360$

$\Rightarrow x^2 + 32x - 273 = 0$

(iv) Let the speed of train be x km/h.

Time taken to travel 480 km = $\frac{480}{x}$ hrs

In second condition, let the speed of train = $(x-8)$ km/h

It is also given that the train will take 3 hours to cover the same distance.

Therefore, time taken to travel 480 km = $\left(\frac{480}{x}+3\right)$hrs

Speed × Time = Distance

$$(x-8)\left(\frac{480}{x}+3\right)=480$$

$$\Rightarrow 480+3x-\frac{3840}{x}-24=480$$

$$\Rightarrow 3x-\frac{3840}{x}=24$$

$$\Rightarrow 3x^2-24x+3840=0$$

$$\Rightarrow x^2-8x+1280=0$$

Exercise 4.2

Question 1:

Find the roots of the following quadratic equations by factorisation:

(i) $x^2 - 3x - 10 = 0$ (ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ (iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

Answer:

(i) $x^2 - 3x - 10$

$$= x^2 - 5x + 2x - 10$$
$$= x(x-5) + 2(x-5)$$
$$= (x-5)(x+2)$$

Roots of this equation are the values for which $(x-5)(x+2) = 0$

$\therefore$ $x - 5 = 0$ or $x + 2 = 0$

i.e., $x = 5$ or $x = -2$

(ii) $2x^2 + x - 6$

$$= 2x^2 + 4x - 3x - 6$$
$$= 2x(x+2) - 3(x+2)$$
$$= (x+2)(2x-3)$$

Roots of this equation are the values for which $(x+2)(2x-3) = 0$

$\therefore$ $x + 2 = 0$ or $2x - 3 = 0$

i.e., $x = -2$ or $x = \frac{3}{2}$

(iii) $\sqrt{2}x^2+7x+5\sqrt{2}$

$=\sqrt{2}x^2+5x+2x+5\sqrt{2}$

$=x\left(\sqrt{2}x+5\right)+\sqrt{2}\left(\sqrt{2}x+5\right)$

$=\left(\sqrt{2}x+5\right)\left(x+\sqrt{2}\right)$

Roots of this equation are the values for which $\left(\sqrt{2}x+5\right)\left(x+\sqrt{2}\right)$ = 0

∴ $\sqrt{2}x+5$ = 0 or $x+\sqrt{2}$ = 0

i.e., x = $\frac{-5}{\sqrt{2}}$ or x = $-\sqrt{2}$

(iv) $2x^2-x+\frac{1}{8}$

$=\frac{1}{8}\left(16x^2-8x+1\right)$

$=\frac{1}{8}\left(16x^2-4x-4x+1\right)$

$=\frac{1}{8}\left(4x\left(4x-1\right)-1\left(4x-1\right)\right)$

$=\frac{1}{8}\left(4x-1\right)^2$

Roots of this equation are the values for which $\left(4x-1\right)^2$ = 0

Therefore, $\left(4x-1\right)=0 \text{ or } \left(4x-1\right)=0$

i.e., $x=\frac{1}{4} \text{ or } x=\frac{1}{4}$

(v) $100x^2 - 20x + 1$

$= 100x^2 - 10x - 10x + 1$

$= 10x(10x-1) - 1(10x-1)$

$= (10x-1)^2$

Roots of this equation are the values for which $(10x-1)^2 = 0$

Therefore, $(10x-1) = 0$ or $(10x-1) = 0$

i.e., $x = \frac{1}{10}$ or $x = \frac{1}{10}$

Question 2:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. Find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. Find out the number of toys produced on that day.

Answer:

(i) Let the number of John's marbles be x.

Therefore, number of Jivanti's marble = $45 - x$

After losing 5 marbles,

Number of John's marbles = $x - 5$

Number of Jivanti's marbles = $45 - x - 5 = 40 - x$

It is given that the product of their marbles is 124.

$\therefore (x-5)(40-x)=124$

$\Rightarrow x^2-45x+324=0$

$\Rightarrow x^2-36x-9x+324=0$

$\Rightarrow x(x-36)-9(x-36)=0$

$\Rightarrow (x-36)(x-9)=0$

Either $x-36$ = 0 or x − 9 = 0

i.e., x = 36 or x = 9

If the number of John's marbles = 36,

Then, number of Jivanti's marbles = 45 − 36 = 9

If number of John's marbles = 9,

Then, number of Jivanti's marbles = 45 − 9 = 36

(ii) Let the number of toys produced be x.

∴ Cost of production of each toy = Rs (55 − x)

It is given that, total production of the toys = Rs 750

$\therefore x(55-x)=750$

$\Rightarrow x^2-55x+750=0$

$\Rightarrow x^2-25x-30x+750=0$

$\Rightarrow x(x-25)-30(x-25)=0$

$\Rightarrow (x-25)(x-30)=0$

Either $x-25$ = 0 or x − 30 = 0

i.e., x = 25 or x = 30

Hence, the number of toys will be either 25 or 30.

Question 4:

Which of the following are APs? If they form an A.P. find the common difference *d* and write three more terms.

(i) 2, 4, 8, 16 …

(ii) $2, \frac{5}{2}, 3, \frac{7}{2} \ldots$

(iii) – 1.2, – 3.2, – 5.2, – 7.2 …

(iv) – 10, – 6, – 2, 2 …

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2} \ldots$

(vi) 0.2, 0.22, 0.222, 0.2222 ….

(vii) 0, – 4, – 8, – 12 …

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \ldots$

(ix) 1, 3, 9, 27 …

(x) a, $2a$, $3a$, $4a$ …

(xi) a, a^2, a^3, a^4 …

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \ldots$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12} \ldots$

(xiv) 1^2, 3^2, 5^2, 7^2 …

(xv) 1^2, 5^2, 7^2, 73 …

Answer:

(i) 2, 4, 8, 16 …

It can be observed that

$a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 8 - 4 = 4$

$a_4 - a_3 = 16 - 8 = 8$

i.e., $a_{k+1} - a_k$ is not the same every time. Therefore, the given numbers are not forming an A.P.

(ii) $2, \frac{5}{2}, 3, \frac{7}{2} \ldots$

It can be observed that

$$a_2 - a = \frac{5}{2} - 2 = \frac{1}{2}$$

$$a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2}$$

$$a_4 - a_3 = \frac{7}{2} - 3 = \frac{1}{2}$$

i.e., $a_{k+1} - a_k$ is same every time.

Therefore, $d = \frac{1}{2}$ and the given numbers are in A.P.

Three more terms are

$$a_5 = \frac{7}{2} + \frac{1}{2} = 4$$

$$a_6 = 4 + \frac{1}{2} = \frac{9}{2}$$

$$a_7 = \frac{9}{2} + \frac{1}{2} = 5$$

(iii) −1.2, −3.2, −5.2, −7.2 …

It can be observed that

$a_2 - a_1 = (-3.2) - (-1.2) = -2$

$a_3 - a_2 = (-5.2) - (-3.2) = -2$

$a_4 - a_3 = (-7.2) - (-5.2) = -2$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = -2$

The given numbers are in A.P.

Three more terms are

$a_5 = -7.2 - 2 = -9.2$

$a_6 = -9.2 - 2 = -11.2$

$a_7 = -11.2 - 2 = -13.2$

(iv) −10, −6, −2, 2 …

It can be observed that

$a_2 - a_1 = (-6) - (-10) = 4$

$a_3 - a_2 = (-2) - (-6) = 4$

$a_4 - a_3 = (2) - (-2) = 4$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = 4$

The given numbers are in A.P.

Three more terms are

$a_5 = 2 + 4 = 6$

$a_6 = 6 + 4 = 10$

$a_7 = 10 + 4 = 14$

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$

It can be observed that

$$a_2 - a_1 = 3+\sqrt{2} - 3 = \sqrt{2}$$
$$a_3 - a_2 = 3+2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$$
$$a_4 - a_3 = 3+3\sqrt{2} - 3 - 2\sqrt{2} = \sqrt{2}$$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = \sqrt{2}$

The given numbers are in A.P.

Three more terms are

$a_5 = 3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$

$a_6 = 3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$

$a_7 = 3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$

(vi) 0.2, 0.22, 0.222, 0.2222 ….

It can be observed that

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

$a_4 - a_3 = 0.2222 - 0.222 = 0.0002$

i.e., $a_{k+1} - a_k$ is not the same every time.

Therefore, the given numbers are not in A.P.

(vii) 0, −4, −8, −12 …

It can be observed that

$a_2 - a_1 = (-4) - 0 = -4$

$a_3 - a_2 = (-8) - (-4) = -4$

$a_4 - a_3 = (-12) - (-8) = -4$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = -4$

The given numbers are in A.P.

Three more terms are

$a_5 = -12 - 4 = -16$

$a_6 = -16 - 4 = -20$

$a_7 = -20 - 4 = -24$

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \ldots$

It can be observed that

$$a_2 - a_1 = \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = 0$$

$$a_3 - a_2 = \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = 0$$

$$a_4 - a_3 = \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = 0$$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = 0$

The given numbers are in A.P.

Three more terms are

$$a_5 = -\frac{1}{2} - 0 = -\frac{1}{2}$$

$$a_6 = -\frac{1}{2} - 0 = -\frac{1}{2}$$

$$a_7 = -\frac{1}{2} - 0 = -\frac{1}{2}$$

(ix) 1, 3, 9, 27 …

It can be observed that

$a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 9 - 3 = 6$

$a_4 - a_3 = 27 - 9 = 18$

i.e., $a_{k+1} - a_k$ is not the same every time.

Therefore, the given numbers are not in A.P.

(x) a, $2a$, $3a$, $4a$ …

It can be observed that

$a_2 - a_1 = 2a - a = a$

$a_3 - a_2 = 3a - 2a = a$

$a_4 - a_3 = 4a - 3a = a$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = a$

The given numbers are in A.P.

Three more terms are

$a_5 = 4a + a = 5a$

$a_6 = 5a + a = 6a$

$a_7 = 6a + a = 7a$

(xi) $a, a^2, a^3, a^4 \ldots$

It can be observed that

$a_2 - a_1 = a^2 - a = a(a-1)$

$a_3 - a_2 = a^3 - a^2 = a^2(a-1)$

$a_4 - a_3 = a^4 - a^3 = a^3(a-1)$

i.e., $a_{k+1} - a_k$ is not the same every time.

Therefore, the given numbers are not in A.P.

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \ldots$

It can be observed that

$a_2 - a_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$a_3 - a_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$a_4 - a_3 = \sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

i.e., $a_{k+1} - a_k$ is same every time.

Therefore, the given numbers are in A.P.

And, $d = \sqrt{2}$

Three more terms are

$a_5 = \sqrt{32} + \sqrt{2} = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$

$a_6 = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72}$

$a_7 = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}$ ...

It can be observed that

$a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3\times 2} - \sqrt{3} = \sqrt{3}\left(\sqrt{2} - 1\right)$

$a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6} = \sqrt{3}\left(\sqrt{3} - \sqrt{2}\right)$

$a_4 - a_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - \sqrt{3\times 3} = \sqrt{3}\left(2 - \sqrt{3}\right)$

i.e., $a_{k+1} - a_k$ is not the same every time.

Therefore, the given numbers are not in A.P.

(xiv) $1^2, 3^2, 5^2, 7^2$ …

Or, 1, 9, 25, 49 …..

It can be observed that

$a_2 - a_1 = 9 - 1 = 8$

$a_3 - a_2 = 25 - 9 = 16$

$a_4 - a_3 = 49 - 25 = 24$

i.e., $a_{k+1} - a_k$ is not the same every time.

Therefore, the given numbers are not in A.P.

(xv) $1^2, 5^2, 7^2$, 73 …

Or 1, 25, 49, 73 …

It can be observed that

$a_2 - a_1 = 25 - 1 = 24$

$a_3 - a_2 = 49 - 25 = 24$

$a_4 - a_3 = 73 - 49 = 24$

i.e., $a_{k+1} - a_k$ is same every time.

Therefore, the given numbers are in A.P.

And, $d = 24$

Three more terms are

$a_5 = 73 + 24 = 97$

$a_6 = 97 + 24 = 121$

$a_7 = 121 + 24 = 145$

Question 3:

Find two numbers whose sum is 27 and product is 182.

Answer:

Let the first number be x and the second number is $27 - x$.

Therefore, their product = $x(27 - x)$

It is given that the product of these numbers is 182.

Therefore, $x(27-x) = 182$

$\Rightarrow x^2 - 27x + 182 = 0$

$\Rightarrow x^2 - 13x - 14x + 182 = 0$

$\Rightarrow x(x-13) - 14(x-13) = 0$

$\Rightarrow (x-13)(x-14) = 0$

Either $x - 13 = 0$ or $x - 14 = 0$

i.e., $x = 13$ or $x = 14$

If first number = 13, then

Other number = 27 − 13 = 14

If first number = 14, then

Other number = 27 − 14 = 13

Therefore, the numbers are 13 and 14.

Question 4:

Find two consecutive positive integers, sum of whose squares is 365.

Answer:

Let the consecutive positive integers be x and $x + 1$.

$$\text{Given that } x^2+(x+1)^2=365$$
$$\Rightarrow x^2+x^2+1+2x=365$$
$$\Rightarrow 2x^2+2x-364=0$$
$$\Rightarrow x^2+x-182=0$$
$$\Rightarrow x^2+14x-13x-182=0$$
$$\Rightarrow x(x+14)-13(x+14)=0$$
$$\Rightarrow (x+14)(x-13)=0$$

Either $x + 14 = 0$ or $x - 13 = 0$, i.e., $x = -14$ or $x = 13$

Since the integers are positive, x can only be 13.

$\therefore\ x + 1 = 13 + 1 = 14$

Therefore, two consecutive positive integers will be 13 and 14.

Question 5:

The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Answer:

Let the base of the right triangle be x cm.

Its altitude = $(x - 7)$ cm

From pythagoras theorem,

$$\text{Base}^2 + \text{Altitude}^2 = \text{Hypotenuse}^2$$

$$\therefore x^2 + (x-7)^2 = 13^2$$

$$\Rightarrow x^2 + x^2 + 49 - 14x = 169$$

$$\Rightarrow 2x^2 - 14x - 120 = 0$$

$$\Rightarrow x^2 - 7x - 60 = 0$$

$$\Rightarrow x^2 - 12x + 5x - 60 = 0$$

$$\Rightarrow x(x-12) + 5(x-12) = 0$$

$$\Rightarrow (x-12)(x+5) = 0$$

Either $x - 12 = 0$ or $x + 5 = 0$, i.e., $x = 12$ or $x = -5$

Since sides are positive, x can only be 12.

Therefore, the base of the given triangle is 12 cm and the altitude of this triangle will be (12 − 7) cm = 5 cm.

Question 6:

A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

Answer:

Let the number of articles produced be x.

Therefore, cost of production of each article = Rs $(2x + 3)$

It is given that the total production is Rs 90.

$\therefore x(2x+3)=90$

$\Rightarrow 2x^2+3x-90=0$

$\Rightarrow 2x^2+15x-12x-90=0$

$\Rightarrow x(2x+15)-6(2x+15)=0$

$\Rightarrow (2x+15)(x-6)=0$

Either $2x + 15 = 0$ or $x - 6 = 0$, i.e., $x = \frac{-15}{2}$ or $x = 6$

As the number of articles produced can only be a positive integer, therefore, x can only be 6.

Hence, number of articles produced = 6

Cost of each article = 2 × 6 + 3 = Rs 15

Exercise 4.3

Question 1:

Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 - 7x + 3 = 0$ (ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3x} + 3 = 0$ (iv) $2x^2 + x + 4 = 0$

Answer:

(i) $2x^2 - 7x + 3 = 0$

$\Rightarrow 2x^2 - 7x = -3$

On dividing both sides of the equation by 2, we obtain

$\Rightarrow x^2 - \frac{7}{2}x = -\frac{3}{2}$

$\Rightarrow x^2 - 2 \times x \times \frac{7}{4} = -\frac{3}{2}$

On adding $\left(\frac{7}{4}\right)^2$ to both sides of equation, we obtain

$\Rightarrow (x)^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 = \left(\frac{7}{4}\right)^2 - \frac{3}{2}$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{49}{16} - \frac{3}{2}$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$

$\Rightarrow \left(x - \frac{7}{4}\right) = \pm\frac{5}{4}$

$\Rightarrow x = \frac{7}{4} \pm \frac{5}{4}$

$\Rightarrow x = \frac{7}{4} + \frac{5}{4}$ or $x = \frac{7}{4} - \frac{5}{4}$

$\Rightarrow x = \frac{12}{4}$ or $x = \frac{2}{4}$

$\Rightarrow x = 3$ or $\frac{1}{2}$

(ii) $2x^2 + x - 4 = 0$

$\Rightarrow 2x^2 + x = 4$

On dividing both sides of the equation by 2, we obtain

$$\Rightarrow x^2 + \frac{1}{2}x = 2$$

On adding $\left(\frac{1}{4}\right)^2$ to both sides of the equation, we obtain

$$\Rightarrow (x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = 2 + \left(\frac{1}{4}\right)^2$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$$

$$\Rightarrow x + \frac{1}{4} = \pm\frac{\sqrt{33}}{4}$$

$$\Rightarrow x = \pm\frac{\sqrt{33}}{4} - \frac{1}{4}$$

$$\Rightarrow x = \frac{\pm\sqrt{33} - 1}{4}$$

$$\Rightarrow x = \frac{\sqrt{33} - 1}{4} \text{ or } \frac{-\sqrt{33} - 1}{4}$$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

$$\Rightarrow (2x)^2 + 2 \times 2x \times \sqrt{3} + \left(\sqrt{3}\right)^2 = 0$$

$$\Rightarrow \left(2x + \sqrt{3}\right)^2 = 0$$

$$\Rightarrow \left(2x + \sqrt{3}\right) = 0 \text{ and } \left(2x + \sqrt{3}\right) = 0$$

$$\Rightarrow x = \frac{-\sqrt{3}}{2} \text{ and } x = \frac{-\sqrt{3}}{2}$$

(iv) $2x^2 + x + 4 = 0$

$\Rightarrow 2x^2 + x = -4$

On dividing both sides of the equation by 2, we obtain

$\Rightarrow x^2 + \frac{1}{2}x = -2$

$\Rightarrow x^2 + 2 \times x \times \frac{1}{4} = -2$

On adding $\left(\frac{1}{4}\right)^2$ to both sides of the equation, we obtain

$\Rightarrow (x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 - 2$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1}{16} - 2$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = -\frac{31}{16}$

However, the square of a number cannot be negative.

Therefore, there is no real root for the given equation.

Question 2:

Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

Answer:

(i) $2x^2 - 7x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = -7, c = 3$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{49 - 24}}{4}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{25}}{4}$$

$$\Rightarrow x = \frac{7 \pm 5}{4}$$

$$\Rightarrow x = \frac{7+5}{4} \text{ or } \frac{7-5}{4}$$

$$\Rightarrow x = \frac{12}{4} \text{ or } \frac{2}{4}$$

$$\therefore x = 3 \text{ or } \frac{1}{2}$$

(ii) $2x^2 + x - 4 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = 1, c = -4$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 + 32}}{4}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{33}}{4}$$

$$\therefore x = \frac{-1 + \sqrt{33}}{4} \text{ or } \frac{-1 - \sqrt{33}}{4}$$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 4, b = 4\sqrt{3}, c = 3$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm \sqrt{48 - 48}}{8}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm 0}{8}$$

$$\therefore\ x = \frac{-\sqrt{3}}{2} \text{ or } \frac{-\sqrt{3}}{2}$$

(iv) $2x^2 + x + 4 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = 1, c = 4$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 - 32}}{4}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{-31}}{4}$$

However, the square of a number cannot be negative.

Therefore, there is no real root for the given equation.

Question 3:

Find the roots of the following equations:

(i) $x - \frac{1}{x} = 3, x \neq 0$ (ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

Answer:

(i) $x-\frac{1}{x}=3 \Rightarrow x^2-3x-1=0$

On comparing this equation with $ax^2+bx+c=0$, we obtain

$a=1, b=-3, c=-1$

By using quadratic formula, we obtain

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{3\pm\sqrt{9+4}}{2}$$

$$\Rightarrow x=\frac{3\pm\sqrt{13}}{2}$$

Therefore, $x=\frac{3+\sqrt{13}}{2}$ or $\frac{3-\sqrt{13}}{2}$

(ii) $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}$

$$\Rightarrow \frac{x-7-x-4}{(x+4)(x-7)}=\frac{11}{30}$$

$$\Rightarrow \frac{-11}{(x+4)(x-7)}=\frac{11}{30}$$

$$\Rightarrow (x+4)(x-7)=-30$$

$$\Rightarrow x^2-3x-28=-30$$

$$\Rightarrow x^2-3x+2=0$$

$$\Rightarrow x^2-2x-x+2=0$$

$$\Rightarrow x(x-2)-1(x-2)=0$$

$$\Rightarrow (x-2)(x-1)=0$$

$$\Rightarrow x=1 \text{ or } 2$$

Question 4:

The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Answer:

Let the present age of Rehman be *x* years.

Three years ago, his age was (*x* − 3) years.

Five years hence, his age will be (*x* + 5) years.

It is given that the sum of the reciprocals of Rehman's ages 3 years ago and 5 years from now is $\frac{1}{3}$.

$$\therefore \frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$

$$\frac{x+5+x-3}{(x-3)(x+5)}=\frac{1}{3}$$

$$\frac{2x+2}{(x-3)(x+5)}=\frac{1}{3}$$

$$\Rightarrow 3(2x+2)=(x-3)(x+5)$$

$$\Rightarrow 6x+6=x^2+2x-15$$

$$\Rightarrow x^2-4x-21=0$$

$$\Rightarrow x^2-7x+3x-21=0$$

$$\Rightarrow x(x-7)+3(x-7)=0$$

$$\Rightarrow (x-7)(x+3)=0$$

$$\Rightarrow x=7,-3$$

However, age cannot be negative.

Therefore, Rehman's present age is 7 years.

Question 5:

In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Answer:

Let the marks in Maths be *x*.

Then, the marks in English will be 30 − *x*.

According to the given question,

$$(x+2)(30-x-3)=210$$
$$(x+2)(27-x)=210$$
$$\Rightarrow -x^2+25x+54=210$$
$$\Rightarrow x^2-25x+156=0$$
$$\Rightarrow x^2-12x-13x+156=0$$
$$\Rightarrow x(x-12)-13(x-12)=0$$
$$\Rightarrow (x-12)(x-13)=0$$
$$\Rightarrow x=12,13$$

If the marks in Maths are 12, then marks in English will be 30 − 12 = 18

If the marks in Maths are 13, then marks in English will be 30 − 13 = 17

Question 6:

The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Answer:

Let the shorter side of the rectangle be *x* m.

Then, larger side of the rectangle = (*x* + 30) m

Diagonal of the rectangle $= \sqrt{x^2 + (x+30)^2}$

It is given that the diagonal of the rectangle is 60 m more than the shorter side.

$$\therefore \sqrt{x^2 + (x+30)^2} = x + 60$$

$$\Rightarrow x^2 + (x+30)^2 = (x+60)^2$$

$$\Rightarrow x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$$

$$\Rightarrow x^2 - 60x - 2700 = 0$$

$$\Rightarrow x^2 - 90x + 30x - 2700 = 0$$

$$\Rightarrow x(x-90) + 30(x-90)$$

$$\Rightarrow (x-90)(x+30) = 0$$

$$\Rightarrow x = 90, -30$$

However, side cannot be negative. Therefore, the length of the shorter side will be

90 m.

Hence, length of the larger side will be (90 + 30) m = 120 m

Question 7:

The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Answer:

Let the larger and smaller number be *x* and *y* respectively.

According to the given question,

$$x^2 - y^2 = 180 \text{ and } y^2 = 8x$$
$$\Rightarrow x^2 - 8x = 180$$
$$\Rightarrow x^2 - 8x - 180 = 0$$
$$\Rightarrow x^2 - 18x + 10x - 180 = 0$$
$$\Rightarrow x(x-18) + 10(x-18) = 0$$
$$\Rightarrow (x-18)(x+10) = 0$$
$$\Rightarrow x = 18, -10$$

However, the larger number cannot be negative as 8 times of the larger number will be negative and hence, the square of the smaller number will be negative which is not possible.

Therefore, the larger number will be 18 only.

$$x = 18$$
$$\therefore\ y^2 = 8x = 8 \times 18 = 144$$
$$\Rightarrow\ y = \pm\sqrt{144} = \pm 12$$
$$\therefore\ \text{Smaller number} = \pm 12$$

Therefore, the numbers are 18 and 12 or 18 and −12.

Question 8:

A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Answer:

Let the speed of the train be *x* km/hr.

Time taken to cover 360 km $= \frac{360}{x}$ hr

According to the given question,

$$(x+5)\left(\frac{360}{x}-1\right)=360$$
$$\Rightarrow (x+5)\left(\frac{360}{x}-1\right)=360$$
$$\Rightarrow 360-x+\frac{1800}{x}-5=360$$
$$\Rightarrow x^2+5x-1800=0$$
$$\Rightarrow x^2+45x-40x-1800=0$$
$$\Rightarrow x(x+45)-40(x+45)=0$$
$$\Rightarrow (x+45)(x-40)=0$$
$$\Rightarrow x=40,-45$$

However, speed cannot be negative.

Therefore, the speed of train is 40 km/h

Question 9:

Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Answer:

Let the time taken by the smaller pipe to fill the tank be *x* hr.

Time taken by the larger pipe = (*x* − 10) hr

Part of tank filled by smaller pipe in 1 hour = $\frac{1}{x}$

Part of tank filled by larger pipe in 1 hour = $\frac{1}{x-10}$

It is given that the tank can be filled in $9\frac{3}{8}=\frac{75}{8}$ hours by both the pipes together. Therefore,

$$\frac{1}{x}+\frac{1}{x-10}=\frac{8}{75}$$
$$\frac{x-10+x}{x(x-10)}=\frac{8}{75}$$
$$\Rightarrow \frac{2x-10}{x(x-10)}=\frac{8}{75}$$
$$\Rightarrow 75(2x-10)=8x^2-80x$$
$$\Rightarrow 150x-750=8x^2-80x$$
$$\Rightarrow 8x^2-230x+750=0$$
$$\Rightarrow 8x^2-200x-30x+750=0$$
$$\Rightarrow 8x(x-25)-30(x-25)=0$$
$$\Rightarrow (x-25)(8x-30)=0$$
$$i.e., x=25,\frac{30}{8}$$

Time taken by the smaller pipe cannot be $\frac{30}{8}$ = 3.75 hours. As in this case, the time taken by the larger pipe will be negative, which is logically not possible.

Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and 25 − 10 =15 hours respectively.

Question 10:

An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speeds of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

Answer:

Let the average speed of passenger train be x km/h.

Average speed of express train = $(x + 11)$ km/h

It is given that the time taken by the express train to cover 132 km is 1 hour less than the passenger train to cover the same distance.

$$\therefore \frac{132}{x} - \frac{132}{x+11} = 1$$

$$\Rightarrow 132\left[\frac{x+11-x}{x(x+11)}\right] = 1$$

$$\Rightarrow \frac{132 \times 11}{x(x+11)} = 1$$

$$\Rightarrow 132 \times 11 = x(x+11)$$

$$\Rightarrow x^2 + 11x - 1452 = 0$$

$$\Rightarrow x^2 + 44x - 33x - 1452 = 0$$

$$\Rightarrow x(x+44) - 33(x+44) = 0$$

$$\Rightarrow (x+44)(x-33) = 0$$

$$\Rightarrow x = -44, 33$$

Speed cannot be negative.

Therefore, the speed of the passenger train will be 33 km/h and thus, the speed of the express train will be 33 + 11 = 44 km/h.

Question 11:

Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

Answer:

Let the sides of the two squares be x m and y m. Therefore, their perimeter will be $4x$ and $4y$ respectively and their areas will be x^2 and y^2 respectively.

It is given that

$4x - 4y = 24$

$x - y = 6$

$x = y + 6$

Also, $x^2 + y^2 = 468$

$\Rightarrow (6+y)^2 + y^2 = 468$

$\Rightarrow 36 + y^2 + 12y + y^2 = 468$

$\Rightarrow 2y^2 + 12y - 432 = 0$

$\Rightarrow y^2 + 6y - 216 = 0$

$\Rightarrow y^2 + 18y - 12y - 216 = 0$

$\Rightarrow y(y+18) - 12(y+18) = 0$

$\Rightarrow (y+18)(y-12) = 0$

$\Rightarrow y = -18 \text{ or } 12.$

However, side of a square cannot be negative.

Hence, the sides of the squares are 12 m and (12 + 6) m = 18 m

Exercise 4.4

Question 1:

Find the nature of the roots of the following quadratic equations.

If the real roots exist, find them;

(I) $2x^2 - 3x + 5 = 0$

(II) $3x^2 - 4\sqrt{3}x + 4 = 0$

(III) $2x^2 - 6x + 3 = 0$

Answer:

We know that for a quadratic equation $ax^2 + bx + c = 0$, discriminant is $b^2 - 4ac$.

(A) If $b^2 - 4ac > 0 \rightarrow$ two distinct real roots

(B) If $b^2 - 4ac = 0 \rightarrow$ two equal real roots

(C) If $b^2 - 4ac < 0 \rightarrow$ no real roots

(I) $2x^2 - 3x + 5 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = -3, c = 5$

Discriminant $= b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40$

$= -31$

As $b^2 - 4ac < 0$,

Therefore, no real root is possible for the given equation.

(II) $3x^2 - 4\sqrt{3}x + 4 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 3,\ b = -4\sqrt{3}, c = 4$

Discriminant $= b^2 - 4ac = \left(-4\sqrt{3}\right)^2 - 4(3)(4)$

= 48 − 48 = 0

As $b^2 - 4ac = 0$,

Therefore, real roots exist for the given equation and they are equal to each other.

And the roots will be $\frac{-b}{2a}$ and $\frac{-b}{2a}$.

$$\frac{-b}{2a} = \frac{-\left(-4\sqrt{3}\right)}{2\times 3} = \frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$$

Therefore, the roots are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$.

(III) $2x^2 - 6x + 3 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2$, $b = -6$, $c = 3$

Discriminant $= b^2 - 4ac = (-6)^2 - 4(2)(3)$

= 36 − 24 = 12

As $b^2 - 4ac > 0$,

Therefore, distinct real roots exist for this equation as follows.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4}$$

$$= \frac{3 \pm \sqrt{3}}{2}$$

Therefore, the roots are $\frac{3+\sqrt{3}}{2}$ or $\frac{3-\sqrt{3}}{2}$.

Question 2:

Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(I) $2x^2 + kx + 3 = 0$

(II) $kx(x - 2) + 6 = 0$

Answer:

We know that if an equation $ax^2 + bx + c = 0$ has two equal roots, its discriminant

$(b^2 - 4ac)$ will be 0.

(I) $2x^2 + kx + 3 = 0$

Comparing equation with $ax^2 + bx + c = 0$, we obtain

$a = 2$, $b = k$, $c = 3$

Discriminant $= b^2 - 4ac = (k)^2 - 4(2)(3)$

$= k^2 - 24$

For equal roots,

Discriminant = 0

$k^2 - 24 = 0$

$k^2 = 24$

$k = \pm\sqrt{24} = \pm 2\sqrt{6}$

(II) $kx(x - 2) + 6 = 0$

or $kx^2 - 2kx + 6 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = k$, $b = -2k$, $c = 6$

Discriminant = $b^2 - 4ac = (-2k)^2 - 4(k)(6)$

$= 4k^2 - 24k$

For equal roots,

$b^2 - 4ac = 0$

$4k^2 - 24k = 0$

$4k(k - 6) = 0$

Either $4k = 0$ or $k = 6 = 0$

$k = 0$ or $k = 6$

However, if $k = 0$, then the equation will not have the terms 'x^2' and 'x'.

Therefore, if this equation has two equal roots, k should be 6 only.

Question 3:

Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2?

If so, find its length and breadth.

Answer:

Let the breadth of mango grove be l.

Length of mango grove will be $2l$.

Area of mango grove = $(2l)(l)$

$= 2l^2$

$$2l^2 = 800$$

$$l^2 = \frac{800}{2} = 400$$

$$l^2 - 400 = 0$$

Comparing this equation with $al^2 + bl + c = 0$, we obtain

$a = 1$ $b = 0$, $c = 400$

Discriminant = $b^2 - 4ac = (0)^2 - 4 \times (1) \times (-400) = 1600$

Here, $b^2 - 4ac > 0$

Therefore, the equation will have real roots. And hence, the desired rectangular mango grove can be designed.

$$l = \pm 20$$

However, length cannot be negative.

Therefore, breadth of mango grove = 20 m

Length of mango grove = $2 \times 20 = 40$ m

Question 4:

Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Answer:

Let the age of one friend be x years.

Age of the other friend will be $(20 - x)$ years.

4 years ago, age of 1st friend = $(x - 4)$ years

And, age of 2nd friend = $(20 - x - 4)$

$= (16 - x)$ years

Given that,

$(x - 4)(16 - x) = 48$

$16x - 64 - x^2 + 4x = 48$

$-x^2 + 20x - 112 = 0$

$x^2 - 20x + 112 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 1, b = -20, c = 112$

Discriminant $= b^2 - 4ac = (-20)^2 - 4(1)(112)$

$= 400 - 448 = -48$

As $b^2 - 4ac < 0$,

Therefore, no real root is possible for this equation and hence, this situation is not possible.

Question 5:

Is it possible to design a rectangular park of perimeter 80 and area 400 m^2? If so find its length and breadth.

Answer:

Let the length and breadth of the park be l and b.

Perimeter $= 2(l + b) = 80$

$l + b = 40$

Or, $b = 40 - l$

Area $= l \times b = l(40 - l) = 40l - l^2$

$40l - l^2 = 400$

$l^2 - 40l + 400 = 0$

Comparing this equation with

$al^2 + bl + c = 0$, we obtain

$a = 1$, $b = -40$, $c = 400$

Discriminate = $b^2 - 4ac = (-40)^2 - 4(1)(400)$

$= 1600 - 1600 = 0$

As $b^2 - 4ac = 0$,

Therefore, this equation has equal real roots. And hence, this situation is possible.

Root of this equation,

$$l = -\frac{b}{2a}$$

$$l = -\frac{(-40)}{2(1)} = \frac{40}{2} = 20$$

Therefore, length of park, $l = 20$ m

And breadth of park, $b = 40 - l = 40 - 20 = 20$ m