CHAPTER – 2

POLYNOMIAL EXPRESSIONS

A polynomial expression S(x) in one variable x is an algebraic expression in x term as

$$S(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + ax + a_0$$

Where $a_n, a_{n-1}, ...a, a_0$ are constant and real numbers and a_n is not equal to zero.

Some Important points to Note:

S.no	Points
1	$a_n, a_{n-1}, a_{n-2}, ...a_1, a_0$ are called the coefficients for $x^n, x^{n-1}, ...x^1, x^0$
2	n is called the degree of the polynomial
3	when $a_n, a_{n-1}, a_{n-2}, ...a_1, a_0$ all are zero, it is called zero polynomial
4	A constant polynomial is the polynomial with zero degree, it is a constant value polynomial
5	A polynomial of one item is called monomial, two items binomial and three items as trinomial
6	A polynomial of one degree is called linear polynomial, two degree as quadratic polynomial and degree three as cubic polynomial

Important Concepts on Polynomial:

Concept	Description
Zero's or roots of the polynomial	It is a solution to the polynomial equation S(x)=0 i.e. a number "a" is said to be a zero of a polynomial if S(a) = 0. If we draw the graph of S(x) =0, the values where the curve cuts the X-axis are called Zeroes of the polynomial
Remainder Theorem's	If p(x) is an polynomial of degree greater than or equal to 1 and p(x) is divided by the expression (x-a),then the remainder will be p(a)
Factor's Theorem's	If x-a is a factor of polynomial p(x) then p(a)=0 or if p(a) =0,x-a is the factor the polynomial p(x)

Geometric Meaning of the Zeroes of the Polynomial:

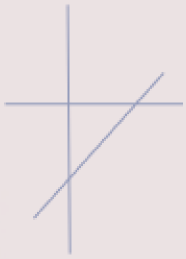
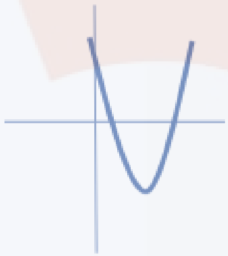
Let's us assume

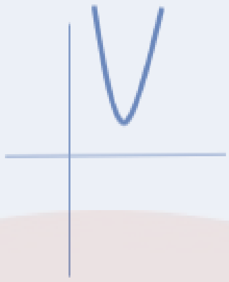
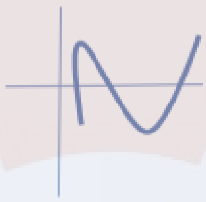
Y= p (x) where p(x) is the polynomial of any form.

Now we can plot the equation y=p(x) on the Cartesian plane by taking various values of x and y obtained by purring the values. The plot or graph obtained can be of any shapes.

The zeroes of the polynomial are the points where the graph meet x axis in the Cartesian plane. If the graph does not meet x axis, then the polynomial does not have any zero's.

Let us take some useful polynomial and shapes obtained on the Cartesian plane

S.no	y=p(x)	Graph obtained	Name of the graph	Name of the equation
1	y=ax+b where a and b can be any values ($a \neq 0$) Example y=2x+3		Straight line. It intersect the x-axis at (-b/a ,0) Example (-3/2,0)	Linear polynomial
2	$y=ax^2+bx+c$ where $b^2-4ac > 0$ and $a \neq 0$ and $a > 0$ Example $y=x^2-7x+12$		Parabola It intersect the x-axis at two points Example (3,0) and (4,0)	Quadratic polynomial
3	$y=ax^2+bx+c$ where $b^2-4ac > 0$ and $a \neq 0$ and $a < 0$ Example $y=-x^2+2x+8$		Parabola It intersect the x-axis at two points Example (-2,0) and (4,0)	Quadratic polynomial
4	$y=ax^2+bx+c$ where $b^2-4ac = 0$ and $a \neq 0$ $a > 0$ Example $y=(x-2)^2$		Parabola It intersect the x-axis at one points	Quadratic polynomial

5	$y=ax^2+bx+c$ where $b^2-4ac < 0$ and $a\neq0$ $a > 0$ Example $y=x^2-2x+6$		Parabola It does not intersect the x-axis It has no zero's	Quadratic polynomial
6	$y=ax^2+bx+c$ where $b^2-4ac < 0$ and $a\neq0$ $a < 0$ Example $y=-x^2-2x-6$		Parabola It does not intersect the x-axis It has no zero's	Quadratic polynomial
7	$y=ax^3+bx^2+cx+d$ where $a\neq0$	It can be of any shape 	It will cut the x-axis at the most 3 times	Cubic Polynomial
8	$a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots \ldots \ldots \ldots + ax + a_0$ Where $a_n \neq 0$	It can be of any shape 	It will cut the x-axis at the most n times	Polynomial of n degree

Relation between coefficient and zeros of the polynomial:

S.no	Type of Polynomial	General form	Zero's	Relationship between Zero's and coefficients
1	Linear polynomial	ax+b , a≠0	1	$k = \frac{-constant\ term}{Coefficent\ of\ x}$
2	Quadratic	ax^2+bx+c, a≠0	2	$k_1 + k_2 = -\frac{Coefficent\ of\ x}{Coefficent\ of\ x^2}$ $k_1 k_2 = \frac{Contant\ term}{Coefficent\ of\ x^2}$
3	Cubic	ax^3+bx^2+cx+d, a≠0	3	$k_1 + k_2 + k_3 = -\frac{Coefficent\ of\ x^2}{Coefficent\ of\ x^3}$
				$k_1 k_2 k_3 = -\frac{Contant\ term}{Coefficent\ of\ x^{32}}$ $k_1 k_2 + k_2 k_3 + k_1 k_3 = \frac{Coefficent\ of\ x}{Coefficent\ of\ x^2}$

Formation of polynomial when the zeroes are given:

Type of polynomial	Zero's	Polynomial Formed
Linear	k=a	(x-a)
Quadratic	k_1=a and k_2=b	(x-a)(x-b) Or x^2-(a+b)x +ab Or x^2-(Sum of the zero's)x +product of the zero's
Cubic	k_1=a ,k_2=b and k_3=c	(x-a)(x-b)(x-c)

Division algorithm for polynomial:

Let's p(x) and q(x) are any two polynomial with $q(x)\neq 0$, then we can find polynomial s(x) and r(x) such that

$P(x) = s(x)\, q(x) + r(x)$

Where r(x) can be zero or degree of r(x) < degree of g(x)

$\boxed{\textbf{Dividend = Quotient} \times \textbf{Divisor + Remainder}}$