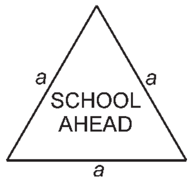
12 HERON'S FORMULA

EXERCISE 12.1

Q.1. *A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is* 180 *cm, what will be the area of the signal board?*



Sol. Each side of the triangle = a

Perimeter of the triangle = $3a$

$\therefore\ s = \dfrac{3a}{2}$

$\therefore$ Area of the signal board (triangle) = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{s(s-a)(s-a)(s-a)} \qquad [\because\ a = b = c]$$

$$= (s-a)\sqrt{s(s-a)} = \left(\frac{3a}{2} - a\right)\sqrt{\frac{3a}{2}\left(\frac{3a}{2} - a\right)}$$

$$= \frac{a}{2} \cdot \sqrt{\frac{3a^2}{4}} = \frac{a}{2} \cdot \frac{a}{2}\sqrt{3} = \frac{a^2}{4}\sqrt{3}$$

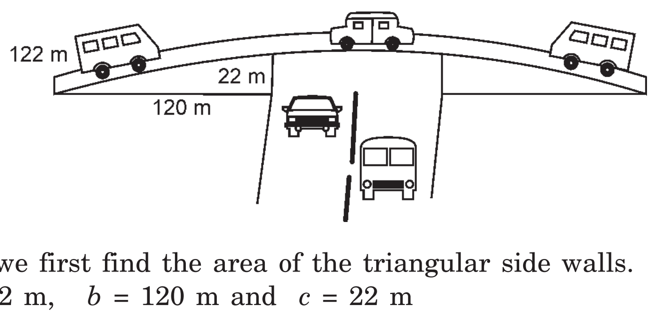
Hence, area of the signal board = $\mathbf{\dfrac{a^2}{4}\sqrt{3}}$ **sq units Ans.**

Now, perimeter = 180 cm

Each side of the triangle = $\dfrac{180}{3}$ cm = 60 cm

Area of the triangle = $\dfrac{(60)^2}{4} \times \sqrt{3}$ cm^2 = $\mathbf{900\sqrt{3}}$ **cm^2 Ans.**

Q.2. *The triangular side walls of a flyover have been used for advertisements. The sides of the walls are* 122 *m,* 22 *m and* 120 *m (see Fig.). The advertisements yield an earning of Rs* 5000 *per m^2 per year. A company hired one of its walls for* 3 *months. How much rent did it pay?*



Sol. Here, we first find the area of the triangular side walls.

a = 122 m, b = 120 m and c = 22 m

$\therefore\ s = \dfrac{122 + 120 + 22}{2}$ m = 132 m.

Area of the triangular side wall = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{132\,(132-122)\,(132-120)\,(132-22)}\ \text{m}^2$$

$$= \sqrt{132 \times 10 \times 12 \times 110}\ \text{m}^2 = 1320\ \text{m}^2$$

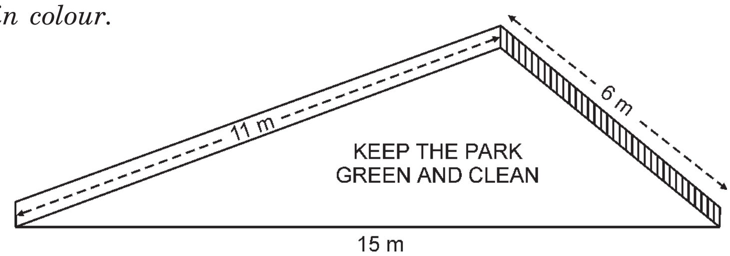
Rent of 1 m^2 of the wall for 1 year = Rs 5000

$\therefore$ Rent of 1 m^2 of the wall for 1 month = Rs $\dfrac{5000}{12}$

$\therefore$ Rent of the complete wall (1320 m^2) for 3 months

$$= \text{Rs } \frac{5000}{12} \times 1320 \times 3 = \textbf{Rs 16,50,000 Ans.}$$

Q.3. *There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN" (see Fig.). If the sides of the wall are* 15 *m,* 11 *m and* 6 *m, find the area painted in colour.*



Sol. Here a = 15 m, b = 11 m, c = 6 m

$$\therefore\ s = \frac{a+b+c}{2} = \frac{15+11+6}{2}\ \text{m} = 16\ \text{m}$$

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{16\,(16-15)\,(16-11)\,(16-6)}\ \ \text{m}^2$$

$$= \sqrt{16 \times 1 \times 5 \times 10}\ \ \text{m}^2 = 20\sqrt{2}\ \ \text{m}^2$$

Hence, the area painted in colour = $\mathbf{20\sqrt{2}\ m^2}$ **Ans.**

Q.4. *Find the area of a triangle two sides of which are* 18 *cm and* 10 *cm and the perimeter is* 42 *cm.*

Sol. Here a = 18 cm, b = 10 cm, c = ?

Perimeter of the triangle = 42 cm

$\Rightarrow a + b + c = 42$

$\Rightarrow 18 + 10 + c = 42$

$\Rightarrow c = 42 - 28 = 14$

Now, $s = \dfrac{a+b+c}{2} = \dfrac{42}{2}$ cm = 21 cm

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{21(21-18)\,(21-10)\,(21-14)}\ \ \text{cm}^2$$

$$= \sqrt{21 \times 3 \times 11 \times 7}\ \ \text{cm}^2 = \sqrt{7 \times 3 \times 3 \times 11 \times 7}\ \ \text{cm}^2$$

$$= 7 \times 3\ \sqrt{11}\ \text{cm}^2 = \mathbf{21\sqrt{11}\ cm^2}\ \ \textbf{Ans.}$$

Q.5. *Sides of a triangle are in the ratio of* 12 : 17 : 25 *and its perimeter is* 540 *cm. Find its area.*

Sol. Let the sides of the triangle be $12x$ cm $17x$ cm and $25x$ cm.

Perimeter of the triangle = 540 cm

$\therefore\ 12x + 17x + 25x = 540$

$\Rightarrow 54\ x = 540$

$$\Rightarrow \quad x = \frac{540}{54} = 10$$

$\therefore$ Sides of the triangle are (12 × 10) cm, (17 × 10) cm and (25 × 10) cm i.e., 120 cm, 170 cm and 250 cm.

Now, suppose a = 120 cm, b = 170 cm, c = 250 cm,

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{540}{2} \text{ cm} = 270 \text{ cm}$$

$$\text{Area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{270\,(270-120)\,(270-170)\,(270-250)} \text{ cm}^2$$

$$= \sqrt{270 \times 150 \times 100 \times 20} \text{ cm}^2 = \mathbf{9000\ cm^2\ Ans.}$$

Q.6. *An isosceles triangle has perimeter* 30 *cm and each of the equal sides is* 12 *cm. Find the area of the tirangle.*

Sol. Here, $a = b = 12$ cm,

Also, $a + b + c = 30 \quad \Rightarrow 12 + 12 + c = 30 \quad \Rightarrow c = 30 - 24 = 6$

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{30}{2} \text{ cm} = 15 \text{ cm}$$

$$\therefore \quad \text{Area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

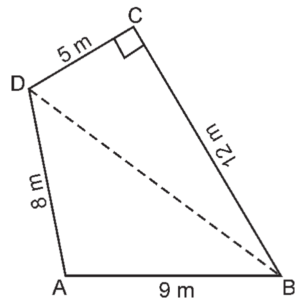
$$= \sqrt{15\,(15-12)\,(15-12)\,(15-6)} \text{ cm}^2$$

$$= \sqrt{15 \times 3 \times 3 \times 9} \text{ cm}^2 = \mathbf{9\sqrt{15}\ cm^2\ Ans.}$$

EXERCISE 12.2

Q.1. *A park, in the shape of a quadrilateral ABCD, has* $\angle C = 90°$, *AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?*

Sol. ABCD is the park as shown in the figure.



Join BD.

In ΔDBC, we have

$DB^2 = BC^2 + CD^2$ [Pythagoras theorem]

$\Rightarrow \quad DB^2 = (12)^2 + 5^2$

$\Rightarrow \quad DB = \sqrt{144 + 25} = \sqrt{169}$

$\Rightarrow \quad DB = 13$ m.

$$\text{Area of } \Delta\text{DBC} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 12 \times 5 \text{ m}^2 = 30 \text{ m}^2$$

In ΔABD, a = 9 m, b = 8 m, c = 13 m

$\therefore \quad s = \frac{a+b+c}{2} = \frac{9+8+13}{2}$ m = 15 m

$\therefore \quad$ Area of ΔABD = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15\,(15-9)\,(15-8)\,(15-13)}$ m^2

$= \sqrt{15 \times 6 \times 7 \times 2}$ m^2

$= \sqrt{1260}$ m^2 = 35.5 m^2 (approx.)

$\therefore$ Area of the park = area of ΔDBC + area of ΔABD

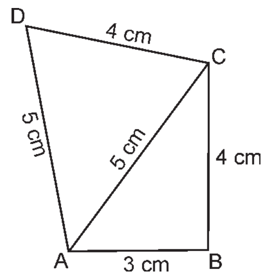
= (30 + 35.5) m^2 **= 65.5 m^2 Ans.**

Q.2. *Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.*

Sol. In ΔABC, we have

AB^2 + BC^2 = 9 + 16 = 25

= AC^2



Hence, ABC is a right triangle, right angled at B

[By converse of Pythagoras theorem]

$\therefore$ Area of ΔABC = $\frac{1}{2}$ × base × height

= $\frac{1}{2}$ × 3 × 4 cm^2 = 6 cm^2.

In ΔACD, a = 5 cm, b = 4 cm, c = 5 cm.

$\therefore \quad s = \frac{a+b+c}{2} = \frac{5+4+5}{2}$ cm = 7 cm

$\therefore$ Area of ΔACD = $\sqrt{s(s-a)(s-b)(s-c)}$

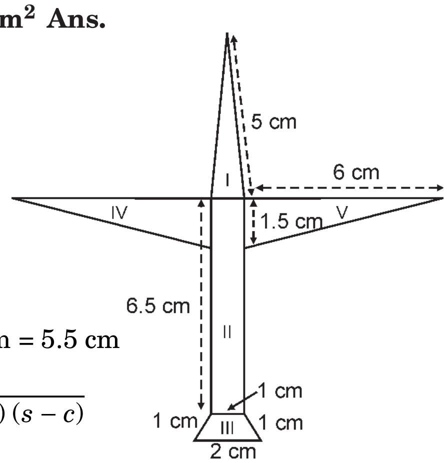
$= \sqrt{7 \times (7-5)\,(7-4)\,(7-5)}$ cm^2 = $\sqrt{7 \times 2 \times 3 \times 2}$ cm^2

= $\sqrt{84}$ cm^2 = 9.2 cm^2 (approx.)

$\therefore$ Area of the quadrilateral = area of ΔABC + area of ΔACD

= (6 + 9.2) cm^2 **= 15.2 cm^2 Ans.**

Q.3. *Radha made a picture of an aeroplane with coloured paper as shown in the figure. Find the total area of the paper used.*



Sol. For the triangle marked I :

a = 5 cm, b = 5 cm, c = 1 cm

$\therefore \quad s = \frac{a+b+c}{2} = \frac{5+5+1}{2}$ cm = $\frac{11}{2}$ cm = 5.5 cm

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{5.5\,(5.5-5)\,(5.5-5)\,(5.5-1)}\ \text{cm}^2$

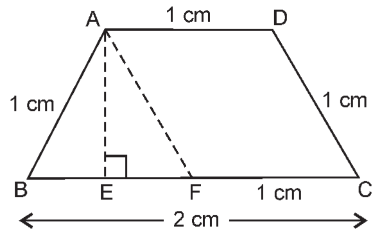
$= \sqrt{5.5\times0.5\times0.5\times4.5}\ \text{cm}^2 = \sqrt{6.1875}\ \text{cm}^2 = 2.5\ \text{cm}^2$

For the rectangle marked II :

Length = 6.5 cm, Breadth = 1 cm

Area of the rectangle = $6.5 \times 1\ \text{cm}^2 = 6.5\ \text{cm}^2$

For the trapezium marked III :



Draw AF || DC and AE ⊥ BC.

AD = FC = 1 cm, DC = AF = 1 cm

∴ BF = BC – FC = (2 – 1) cm = 1 cm

Hence, ΔABF is equilateral.

Also, E is the mid-point of BF.

∴ BE = $\frac{1}{2}$ cm = 0.5 cm

Also, $AB^2 = AE^2 + BE^2$ [Pythagoras theorem]

$\Rightarrow AE^2 = 1^2 - (0.5)^2 = 0.75$

$\Rightarrow$ AE = 0.9 cm (approx.)

Area of the trapezium = $\frac{1}{2}$ (sum of the parallel sides) × distance between them.

$= \frac{1}{2} \times (BC + AD) \times AE = \frac{1}{2} \times (2 + 1) \times 0.9\ \text{cm}^2 = 1.4\ \text{cm}^2.$

For the triangle marked IV :

It is a right-triangle

∴ Area of the triangle = $\frac{1}{2}$ × base × height

$= \frac{1}{2} \times 6 \times 1.5$ cm $\text{cm}^2 = 4.5\ \text{cm}^2.$

For the triangle marked V :

This triangle is congruent to the triangle marked IV.

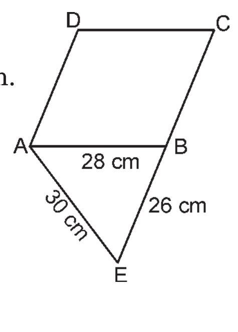
Hence, area of the triangle = $4.5\ \text{cm}^2$

Total area of the paper used = $(2.5 + 6.5 + 1.4 + 4.5 + 4.5)\ \text{cm}^2$

= 19.4 cm² Ans.

Q.4. *A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are* 26 *cm,* 28 *cm and* 30 *cm and the parallelogram stands on the base* 28 *cm, find the height of the parallelogram.*

Sol. In the figure, ABCD is a parallelogram and ABE is the triangle which stands on the base AB



For the triangle ABE, a = 30 cm, b = 28 cm, c = 26 cm.

$\therefore\ s = \frac{a+b+c}{2} = \frac{30+28+26}{2}$ cm = 42 cm

∴ Area of the ΔABE = $\sqrt{s(s-a)\,(s-b)\,(s-c)}$

$= \sqrt{42\,(42-30)\,(42-28)\,(42-26)}\ \text{cm}^2$

$= \sqrt{42 \times 12 \times 14 \times 16}\ \text{cm}^2 = \sqrt{112896}\ \text{cm}^2$

$= 336\ \text{cm}^2$

Now, area of the parallelogram = base × height

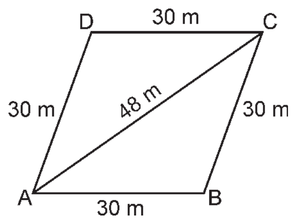
$\Rightarrow$ 336 = 28 × height [Given, area of the triangle = area of the parallelogram]

$\Rightarrow$ Height of the parallelogram = $\frac{336}{28}$ cm = **12 cm Ans.**

Q.5. *A rhombus shaped field has green grass for* 18 *cows to graze. If each side of the rhombus is* 30 *m and its longer diagonal is* 48 *m, how much area of grass field will each cow be getting?*

Sol. Clearly, the diagonal AC of the rhombus divides it into two congruent triangles.

For triangle ABC, $a = b = 30$ m, $c = 48$ m.



$\therefore\ s = \frac{a+b+c}{2} = \frac{30+30+48}{2}$ m = 54 m

$\therefore$ Area of the triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{54\,(54-30)\,(54-30)\,(54-48)}\ \text{m}^2$

$= \sqrt{54 \times 24 \times 24 \times 6}\ \text{m}^2 = 432\ \text{m}^2$

$\therefore$ Area of the rhombus = 2 × 432 m^2 = 864 m^2

Number of cows = 18

Hence, area of the grass field which each cow gets

$= \frac{864}{18}$ m^2 = **48 m^2 Ans.**

Q.6. *An umbrella is made by stitching* 10 *triangular pieces of cloth of two different colours (see Fig.), each piece measuring* 20 *cm,* 50 *cm, and* 50 *cm. How much cloth of each colour is required for the umbrella?*

Sol. First we find the area of one triangular piece.

Here, $a = b = 50$ cm, $c = 20$ cm

$\therefore\ s = \frac{a+b+c}{2} = \frac{50+50+20}{2}$ cm = 60 m

$\therefore$ Area of one triangular piece = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60\,(60-50)\,(60-50)\,(60-20)}\ \text{cm}^2$

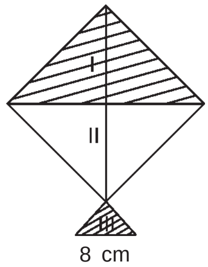
$= \sqrt{60 \times 10 \times 10 \times 40}\ \text{cm}^2 = 200\sqrt{6}\ \text{cm}^2$

$\therefore$ Area of 10 such triangular pieces = 10 × $200\sqrt{6}$ cm^2

$= 2000\sqrt{6}$ cm^2

Hence, cloth required for each colour = $\frac{2000\sqrt{6}}{2}$ cm^2 = **$1000\sqrt{6}$ cm^2 Ans.**

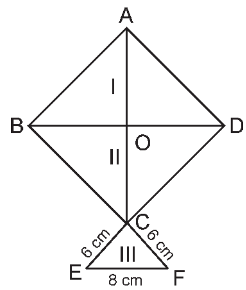
Q.7. *A kite in the shape of a square with a diagonal* 32 *cm and an isosceles triangle of base* 8 *cm and sides* 6 *cm each is to be made of three different shades as shown in figure. How much paper of each shade has been used in it?*



Sol. ABCD is a square.

So, AO = OC = OB = OD

and $\angle AOB = 90°$ [Diagonals of a square bisect each other at right angles]



BD = 32 cm (Given) $\Rightarrow$ OA = $\frac{32}{2}$ cm = 16 cm.

ΔABD is a right triangle.

So, area of $\Delta ABD = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 32 \times 16 \text{ cm}^2 = 256 \text{ cm}^2$

Thus, area of $\Delta BCD = 256 \text{ cm}^2$

For triangle CEF, $a = b = 6$ cm, $c = 8$ cm.

$\therefore\ s = \frac{a+b+c}{2} = \frac{6+6+8}{2}$ cm = 10 cm

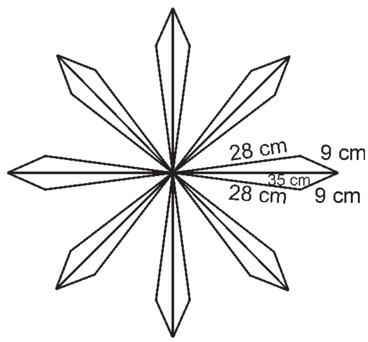
$\therefore$ Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{10(10-6)(10-6)(10-8)} \text{ cm}^2$

$= \sqrt{10 \times 4 \times 4 \times 2} \text{ cm}^2 = \sqrt{320} \text{ cm}^2 = 17.92 \text{ cm}^2$

Hence, paper needed for shade I = **256 cm^2, for shade II = 256 cm^2 and for shade III = 17.92 cm^2 Ans.**

Q.8. *A floral design on a floor is made up of* 16 *tiles which are triangular, the sides of the triangle being* 9 *cm,* 28 *cm and* 35 *cm (see figure). Find the cost of polishing the tiles at the rate of* 50 *p per cm^2.*



Sol. We have lengths of the sides of 1 triangular tile are $a = 35$ cm, $b = 28$ cm, $c = 9$ cm.

$\therefore\ s = \frac{a+b+c}{2} = \frac{35+28+9}{2}$ cm = 36 cm

$\therefore$ Area of 1 triangular tile $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{36\,(36-35)\,(36-28)\,(36-9)} \text{ cm}^2$

$= \sqrt{36 \times 1 \times 8 \times 27} \text{ cm}^2 = \sqrt{7776} \text{ cm}^2 = 88.2 \text{ cm}^2$

$\therefore$ Area of 16 such tiles = $16 \times 88.2 \text{ cm}^2$

Cost of polishing 1 cm^2 = 50 p = Re 0.50

$\therefore$ Total cost of polishing the floral design = Rs 16 × 88.2 × 0.50

= Rs 705.60 Ans.

Q.9. *A field is in the shape of a trapezium whose parallel sides are* 25 *m and* 10 *m. The non-parallel sides are* 14 *m and* 13 *m. Find the area of the field.*

Sol. In the figure ABCD is the field. Draw CF || DA and CG ⊥ AB.

DC = AF = 10 m, AD = FC = 13 m

For ΔBCF, a = 15 m, b = 14 m, c = 13 m

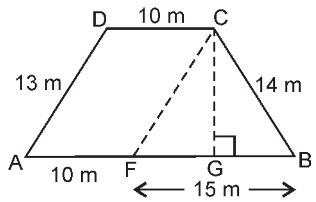
$$\therefore s = \frac{a+b+c}{2} = \frac{15+14+13}{2} \text{ m} = 21 \text{ m}$$

$$\therefore \text{Area of } \Delta BCF = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{21(21-15)(21-14)(21-13)} \text{ m}^2$$

$$= \sqrt{21 \times 6 \times 7 \times 8} \text{ m}^2$$

$$= \sqrt{7056} \text{ cm}^2 = 84 \text{ m}^2$$



Also, area of ΔBCF = $\frac{1}{2}$ × base × height

$$= \frac{1}{2} \times BF \times CG$$

$$\Rightarrow \quad 84 = \frac{1}{2} \times 15 \times CG$$

$$\Rightarrow \quad CG = \frac{84 \times 2}{15} \text{ m} = 11.2 \text{ m}$$

$\therefore$ Area of the trapezium = $\frac{1}{2}$ × sum of the parallel sides × distance between them.

$$= \frac{1}{2} \times (25 + 10) \times 11.2 \text{ m}^2$$

$$= 196 \text{ m}^2$$

Hence, area of the field **= 196 m^2 Ans.**

13 SURFACE AREAS AND VOLUMES

EXERCISE 13.1

Q.1. *A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is to be open at the top. Ignoring the thickness of the plastic sheet, determine :*
(i) The area of the sheet required for making the box.
(ii) The cost of sheet for it, if a sheet measuring 1 m^2 costs Rs 20.

Sol. Here, $l = 1.5$ m, $b = 1.25$ m, $h = 65$ cm $= 0.65$ m.
Since the box is open at the top, it has only five faces.
(i) So, surface area of the box $= lb + 2(bh + hl)$
$= 1.5 \times 1.25 \text{ m}^2 + 2\ (1.25 \times 0.65 + 0.65 \times 1.5) \text{ m}^2$
$= 1.875 + 2\ (1.7875) \text{ m}^2$
$= (1.875 + 3.575) \text{ m}^2 = 5.45 \text{ m}^2$
Hence, 5.45 m^2 of sheet is required **Ans.**
(ii) Cost of 1 m^2 of the sheet = Rs 20
$\therefore$ cost of 5.45 m^2 of the sheet = Rs 20 × 5.45 m^2 **= Rs 109 Ans.**

Q.2. *The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of Rs 7.50 per m^2.*

Sol. Here, $l = 5$ m, $b = 4$ m, $h = 3$ m
Surface area of the walls of the room and the ceiling
$= 2h\ (l + b) + lb$
$= [2 \times 3\ (5 + 4) + 5 \times 4] \text{ m}^2$
$= (6 \times 9 + 20) \text{ m}^2 = 74 \text{ m}^2$
Cost of white washing = Rs 7.50 per m^2
$\therefore$ total cost of white washing the walls and the ceiling of the room
= Rs 74 × 7.50 **= Rs 555 Ans.**

Q.3. *The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs 10 per m^2 is Rs 15000, find the height of the hall.*

Sol. Let length, breadth and height of the hall be l, b and h respectively.
Perimeter of the floor of the hall $= 2\ (l + b) = 250$ m.
Area of the four walls of the hall $= 2h\ (l + b)$ ... (i)
Also, area of the four walls of the hall $= \dfrac{15000}{10} \text{ m}^2$
$= 1500 \text{ m}^2$ ... (ii)
From (i) and (ii), we have
$2h\ (l + b) = 1500$
$\Rightarrow h \times 250 = 1500$ $\quad [\because 2(l + b) = 250]$
$\Rightarrow \quad h = \dfrac{1500}{250} = 6$
Hence, height of the hall is **6 m Ans.**

Q.4. *The paint in a certain container is sufficient to paint an area equal to* 9.375 *m^2. How many bricks of dimensions* 22.5 *cm* × 10 *cm* × 7.5 *cm can be painted out of this container?*

Sol. Here, l = 22.5 cm, b = 10 cm, h = 7.5 cm.

Total surface area of 1 brick = 2 ($lb + bh + hl$)

$= 2(22.5 \times 10 + 10 \times 7.5 + 7.5 \times 22.5)\ \text{cm}^2$

$= 2(225 + 75 + 168.75)\ \text{cm}^2 = 937.5\ \text{cm}^2$

$= \dfrac{937.5}{100 \times 100}\ \text{m}^2 = 0.09375\ \text{m}^2.$

$\therefore$ required number of bricks $= \dfrac{9.375}{0.09375}$ **= 100 Ans.**

Q.5. *A cubical box has each edge* 10 *cm and another cuboidal box is* 12.5 *cm long,* 10 *cm wide and* 8 *cm high.*

(i) Which box has the greater lateral surface area and by how much?

(ii) Which box has the smaller total surface area and by how much?

Sol. Here, a = 10 cm, l = 12.5 cm, b = 10 cm, h = 8 cm

(i) Lateral surface area of the cubical box = $4a^2$

$= 4 \times (10)^2\ \text{cm}^2 = 400\ \text{cm}^2$

Lateral surface area of the cuboidal box = $2h\ (l + b)$

$= 2 \times 8\ (12.5 + 10)\ \text{cm}^2$

$= 16 \times 22.5\ \text{cm}^2 = 360\ \text{cm}^2$

Difference in the lateral surface areas of the two boxes

$= (400 - 360)\ \text{cm}^2 = 40\ \text{cm}^2.$

Hence, the cubical box has greater lateral surface area by 40 cm^2. **Ans.**

(ii) Total surface area of the cubical box = $6a^2$

$= 6 \times (10)^2\ \text{cm}^2 = 600\ \text{cm}^2$

Total surface area of the cuboidal box = $2(lb + bh + hl)$

$= 2(12.5 \times 10 + 10 \times 8 + 8 \times 12.5)\ \text{cm}^2$

$= 2(125 + 80 + 100)\ \text{cm}^2$

$= 2 \times 305\ \text{cm}^2 = 610\ \text{cm}^2$

Difference in the total surface areas of the two boxes = $(610 - 600)\ \text{cm}^2$

$= 10\ \text{cm}^2$

Hence, the cubical box has smaller total surface area by **10 cm^2 Ans.**

Q.6. *A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tape. It is* 30 *cm long,* 25 *cm wide and* 25 *cm high.*

(i) What is the area of the glass?

(ii) How much of tape is needed for all the 12 *edges?*

Sol. Here, l = 30 cm, b = 25 cm, h = 25 cm.

(i) Total surface area of the herbarium = $2(lb + bh + hl)$

$= 2(30 \times 25 + 25 \times 25 + 25 \times 30)\ \text{cm}^2$

$= 2(750 + 625 + 750)\ \text{cm}^2$

$= 2 \times 2125\ \text{cm}^2 = 4250\ \text{cm}^2$

Hence, area of the glass **= 4250 cm^2 Ans.**

(ii) A cuboid has 12 edges. These consist of 4 lengths, 4 breadths and 4 heights.

$\therefore$ length of the tape required = $4l + 4b + 4h$
= $(4 \times 30 + 4 \times 25 + 4 \times 25)$ cm
= $(120 + 100 + 100)$ cm **= 320 cm Ans.**

Q.7. *Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm × 20 cm × 5 cm and the smaller of dimesnsions 15 cm × 12 cm × 5 cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is Rs 4 for 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each kind.*

Sol. **For bigger boxes :**

$l = 25$ cm, $b = 20$ cm, $h = 5$ cm

Total surface area of 1 bigger box = $2(lb + bh + hl)$
= $2(25 \times 20 + 20 \times 5 + 5 \times 25)$ cm^2
= $2(500 + 100 + 125)$ cm^2 = 1450 cm^2

Area of cardboard required for overlaps

$$= 5\% \text{ of } 1450 \text{ cm}^2 = \frac{1450 \times 5}{100} \text{ cm}^2 = 72.5 \text{ cm}^2.$$

Total area of cardboard needed for 1 bigger box
= $(1450 + 72.5)$ cm^2 = 1522.5 cm^2

Total area of cardboard needed for 250 bigger boxes = 1522.5×250 cm^2
= 380625 cm^2.

For smaller boxes :

$l = 15$ cm, $b = 12$ cm, $h = 5$ cm

Total surface area of 1 smaller box = $2(lb + bh + hl)$
= $2(15 \times 12 + 12 \times 5 + 5 \times 15)$ cm^2
= $2(180 + 60 + 75)$ cm^2 = 630 cm^2

Area of cardboard required for overlaps

$$= 5\% \text{ of } 630 \text{ cm}^2 = \frac{630 \times 5}{100} \text{ cm}^2 = 31.5 \text{ cm}^2$$

Total area of cardboard needed for 1 smaller box = $(630 + 31.5)$ cm^2
= 661.5 cm^2

Total area of cardboard needed for 250 smaller boxes
= 661.5×250 cm^2 = 165375 cm^2

Now, total area of cardboard needed for 500 boxes (250 bigger and 250 smaller boxes) = $(380625 + 165375)$ cm^2 = 546000 cm^2

Cost of 1000 cm^2 of cardboard = Rs 4

$\therefore$ Cost of 546000 cm^2 of cardboard = Rs $\frac{4}{1000} \times 546000$ **= Rs 2184 Ans.**

Q.8. *Parveen wanted to make a temporary shelter for her car, by making a box-like structuer with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m, with base dimesions 4 m × 3 m?*

Sol. Here, $l = 4$ m, $b = 3$ m, $h = 2.5$ m

The tarpaulin is needed to cover 5 faces only (excluding the floor)

Surface area of the shelter = $lb + 2\,(bh + hl)$

$= 4 \times 3 \text{ m}^2 + 2(3 \times 2.5 + 2.5 \times 4) \text{ m}^2$

$= 12 \text{ m}^2 + 2(7.5 + 10) \text{ m}^2$

$= (12 + 35) \text{ m}^2 = 47 \text{ m}^2$

Hence, 47 m^2 of tarpaulin is required to make the shelter **Ans.**

EXERCISE 13.2

Q.1. *The curved surface area of a right circular cylinder of height* 14 *cm is* 88 *cm*^{2}. *Find the diameter of the base of the cylinder.*

Sol. Here, h = 14 cm, curved, surface area = 88 cm^2, r = ?

Curved surface area of the cylinder = $2\pi rh$

$\Rightarrow\ 88 = 2 \times \frac{22}{7} \times r \times 14$

$\Rightarrow\ 88 = 44 \times 2 \times r$

$\Rightarrow\ r = \frac{88}{44 \times 2} = 1$

Hence, base diameter of the cylinder = 1 × 2cm = 2 cm **Ans.**

Q.2. *It is required to make a closed cylindrical tank of height* 1 *m and base diameter* 140 *cm from a metal sheet. How many square metres of the sheet are required for the same?.*

Sol. Here, h = 1 m, $r = \frac{140}{2}$ cm = 70 cm = 0.7 m

Total surface area of the cylinder = $2\pi r\,(h + r)$

$= 2 \times \frac{22}{7} \times 0.7\,(1 + 0.7) \text{ m}^2$

$= 44 \times 0.1 \times 1.7 \text{ m}^2 = 7.48 \text{ m}^2$

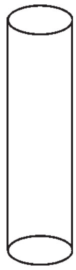
Hence, 7.48 m^2 of sheet is required **Ans.**

Q.3. *A metal pipe is* 77 *cm long. The inner diameter of a cross section is* 4 *cm, the outer diameter being* 4.4 *cm (see figure). Find its.*

(i) inner curved surface area,

(ii) outer curved surface area,

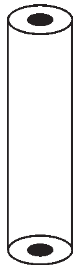
(iii) total surface area.



Sol. Here, h = 77 cm,

Outer radius (R) = $\frac{4.4}{2}$ cm = 2.2 cm,

Inner radius (r) = $\frac{4}{2}$ cm = 2 cm



(i) Inner curved surface area of the pipe

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 2 \times 77 \text{ cm}^2$

$= 2 \times 22 \times 22 \text{ cm}^2$ **= 968 cm^2 Ans.**

(ii) Outer curved surface area of the pipe = $2\pi Rh$

$$= 2 \times \frac{22}{7} \times 2.2 \times 77 \text{ cm}^2 = 44 \times 24.2 \text{ cm}^2$$

$$\mathbf{= 1064.80\ cm^2\ Ans.}$$

(iii) Total surface area of the pipe = inner curved surface area + outer curved surface area + areas of the two base rings.

$= 2\pi rh + 2\pi Rh + 2\pi (R^2 - r^2)$

$= 968 \text{ cm}^2 + 1064.80 \text{ cm}^2 + 2 \times \frac{22}{7} [(2.2)^2 - 2^2] \text{ cm}^2$

$= 2032.80 \text{ cm}^2 + 5.28 \text{ cm}^2$ $\mathbf{= 2038.08\ cm^2}$ **Ans.**

Q.4. *The diameter of a roller is* 84 *cm and its length is* 120 *cm. It takes* 500 *complete revolutions to move once over to level a playground. Find the area of the playground in* m^2.

Sol. Radius of the roller (r) = $\frac{84}{2}$ cm = 42 cm

Length of the roller (h) = 120 cm

Curved surface area of the roller = $2\pi rh$

$= 2 \times \frac{22}{7} \times 42 \times 120 \text{ cm}^2 = 44 \times 720 \text{ cm}^2 = 31680 \text{ cm}^2$

$\therefore$ area covered by the roller in 1 revolution = 31680 cm^2

$\therefore$ area covered by the roller in 500 revolutions = $31680 \times 500 \text{ cm}^2$

$= 15840000 \text{ cm}^2$

Hence, area of the playground = $\frac{15840000}{100 \times 100}$ m^2 $\mathbf{= 1584\ m^2}$ **Ans.**

Q.5. *A cylindrical pillar is* 50 *cm in diameter and* 3.5 *m in height. Find the cost of painting the curved surface of the pillar at the rate of Rs* 12.50 *per* m^2.

Sol. Here, $r = \frac{50}{2}$ cm = 25 cm = 0.25 m, h = 3.5 m

Curved surface area of the pillar = $2\pi rh$

$= 2 \times \frac{22}{7} \times 0.25 \times 3.5 \text{ m}^2 = 5.5 \text{ m}^2$

Cost of painting 1 m^2 = Rs 12.50

$\therefore$ Total cost of painting the curved surface of the pillar

= Rs 12.50 × 5.5 **= Rs 68.75 Ans.**

Q.6. *Curved surface area of a right circular cylinder is* 4.4 m^2. *If the radius of the base of the cylinder is* 0.7 *m, find its height.*

Sol. Curved surface area of the cylinder = 4.4 m^2, r = 0.7 m, h = ?

Curved surface area of the cylinder = $2\pi rh$.

$\Rightarrow \; 4.4 = 2 \times \frac{22}{7} \times 0.7 \times h$

$\Rightarrow \; h = \frac{4.4}{4.4} = 1$

Hence, height of the cylinder is 1 m **Ans.**

Q.7. *The inner diameter of a circular well is* 3.5 *m. It is* 10 *m deep. Find*
(i) its inner curved surface area,
(ii) the cost of plastering this curved surface at the rate of Rs 40 *per* m^2.

Sol. Here, $r = \frac{3.5 \text{ m}}{2}$, $h = 10$ m

(i) Inner curved surface area of the well

$= 2\pi rh = 2 \times \frac{22}{7} \times \frac{3.5}{2} \times 10\text{m}^2$

$= 22 \times 5 \text{ m}^2$ **= 110 m² Ans.**

(ii) Cost of plastering 1 m^2 = Rs 40

$\therefore$ Cost of plastering the curved surface area of the well

= Rs 110 × 40 **= Rs 4400 Ans.**

Q.8. *In a hot water heating system. there is a cylindrical pipe of length* 28 *m and diameter* 5 *cm. Find the total radiating surface in the system.*

Sol. Here, $r = \frac{5}{2}$ cm = 2.5 cm = 0.025 m, h = 28 m.

Total radiating surface in the system = total surface area of the cylinder

$= 2\pi\, r(h + r)$

$= 2 \times \frac{22}{7} \times 0.025\ (28 + 0.025)\ \text{m}^2$

$= \frac{44 \times 0.025 \times 28.025}{7}\ \text{m}^2$ **= 4.4 m² (approx) Ans.**

Q.9. *Find*
(i) the lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 *m in diameter and* 4.5 *m high.*
(ii) how much steel was actually used, if $\frac{1}{12}$ *of the steel actually used was wasted in making the tank.*

Sol. Here, $r = \frac{4.2}{2}$ m = 2.1 m, h = 4.5 m

(i) Curved surface area of the storage tank = $2\pi rh$

$= 2 \times \frac{22}{7} \times 2.1 \times 4.5\ \text{m}^2$ **= 59.4 m² Ans.**

(ii) Total surface area of the tank = $2\pi r\ (h + r)$

$= 2 \times \frac{22}{7} \times 2.1\ (4.5 + 2.1)\ \text{m}^2$

$= 44 \times 0.3 \times 6.6\ \text{m}^2 = 87.12\ \text{m}^2$

Let the actual area of steel used be x m^2.

Area of steel wasted $= \frac{1}{12}$ of x $\text{m}^2 = \frac{x}{12}$ m^2. ... (i)

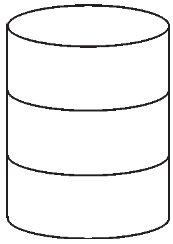
$\therefore$ area of the steel used in the tank $= \left(x - \frac{x}{12}\right)\text{m}^2 = \frac{11}{12}\ x\ \text{m}^2$

$\Rightarrow 87.12 = \frac{11}{12}\ x$

$\Rightarrow \quad x = \frac{87.12 \times 12}{11} = 95.04\ \text{m}^2$

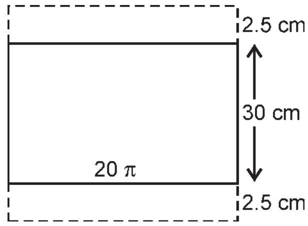
Hence, 95.04 m^2 of steel was actually used **Ans.**

Q.10. *In the figure, you see the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of* 20 *cm and height of* 30 *cm. A margin of* 2.5 *cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade.*



Sol. Here, $r = \frac{20}{2}$ cm = 10 cm

Height = 30 cm



Circumference of the base of the frame = $2\pi r$

$= 2\pi \times 10$ cm $= 20\pi$ cm

Height of the frame = 30 cm

Height of the cloth needed for covering the frame (including the margin) = (30 + 2.5 + 2.5) cm = 35 cm

Also, breadth of the cloth = circumference of the base of the frame.

∴ Area of the cloth required for covering the lampshade = length × breadth

$= 35 \times 20\pi \text{ cm}^2 = 35 \times 20 \times \frac{22}{7} \text{ cm}^2$ **= 2200 cm² Ans.**

Q.11. *The students of a Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base, using cardboard. Each penholder was to be of radius* 3 *cm and height* 10.5 *cm. The Vidyalaya was to supply the competitors with cardboard. If there were* 35 *competitors, how much cardboard was required to be bought for the competition?*

Sol. Here, $r = 3$ cm, $h = 10.5$ cm

The penholders have only one base i.e., these are open at one end.

Total surface area of 1 penholder

$= 2\pi rh + \pi r^2 = \pi r\,(2h + r)$

$= \frac{22}{7} \times 3\,(2 \times 10.5 + 3) \text{ cm}^2$

$= \frac{22}{7} \times 3 \times 24 \text{ cm}^2$

Total surface area of 35 penholders $= \frac{22}{7} \times 3 \times 24 \times 35 \text{ cm}^2 = 7920 \text{ cm}^2$

Hence, 7920 cm² of cardboard is needed **Ans.**

EXERCISE 13.3

Q.1. *Diameter of the base of a cone is* 10.5 *cm and its slant height is* 10 *cm. find its curved surface area.*

Sol. Here, $r = \frac{10.5}{2}$ cm = 5.25 cm, $l = 10$ cm.

Curved surface area of the cone $= \pi rl$

$= \frac{22}{7} \times 5.25 \times 10 \text{ cm}^2$ **= 165 cm² Ans.**

Q.2. *Find the total surface area of a cone, if its slant height is* 21 *m and diameter of its base is* 24 *m.*

Sol. Here, $l = 21$ m, $r = \frac{24}{2}$ m $= 12$ m

Total surface area of the cone $= \pi r(l + r)$

$$= \frac{22}{7} \times 12\ (21 + 12)\ \text{m}^2$$

$$= \frac{22}{7} \times 12 \times 33\ \text{m}^2 = \mathbf{1244.57\ m^2\ Ans.}$$

Q.3. *Curved surface area of a cone is* 308 *cm*^{2} *and its slant height is* 14 *cm. Find (i) radius of the base and (ii) total surface area of the cone.*

Sol. Here, $l = 14$ cm, curved surface area $= 308$ cm^2, $r = ?$

(i) Curved surface area of the cone $= \pi rl$

$$\Rightarrow 308 = \frac{22}{7} \times r \times 14$$

$$\Rightarrow r = \frac{308}{22 \times 2} = 7$$

Hence, base radius of the cone **= 7 cm.**

(ii) Total surface area of the cone $= \pi r\ (l + r)$

$$= \frac{22}{7} \times 7\ (14 + 7)\ \text{cm}^2 = 22 \times 21\ \text{cm}^2 = \mathbf{462\ cm^2\ Ans.}$$

Q.4. *A conical tent is* 10 *m high and the radius of its base is* 24 *m. Find*

(i) slant height of the tent.

(ii) cost of the canvas required to make the tent, if the cost of 1 *m*^{2} *canvas is Rs* 70.

Sol. Here, $h = 10$ m, $r = 24$ m

(i) We have, $l^2 = h^2 + r^2$

$$= (10)^2 + (24)^2$$

$$= 100 + 576 = 676$$

$$\Rightarrow \quad l = \sqrt{676} = \mathbf{26\ m\ Ans.}$$

(ii) Curved surface area of the tent $= \pi rl$

$$= \frac{22}{7} \times 24 \times 26\ \text{m}^2$$

Cost of 1 m^2 canvas = Rs 70

$\therefore$ Cost of $\frac{22}{7} \times 24 \times 26$ m^2 of canvas = Rs $70 \times \frac{22}{7} \times 24 \times 26$

= Rs 137280 Ans.

Q.5. *What length of tarpaulin* 3 *m wide will be required to make conical tent of height* 8 *m and base radius* 6 *m? Assume that the extra length of material that will be required for*

Stitching margins and wastage in cutting is approximately 29 *cm (use* π = 3.14)

Sol. Here $h = 6$ m, $r = 8$ m

We have, $l^2 = \sqrt{r^2 + h^2}$

$= \sqrt{36+64} = \sqrt{100} = 10$ m

Curved surface area of the tent $= \pi rl$

$= 3.14 \times 6 \times 10$ m^2

$\therefore$ required length of tarpaulin $= \dfrac{3.14 \times 6 \times 10}{3}$ m + 20 cm

$= 62.8$ m + 0.2 m **= 63 m Ans.**

Q.6. *The slant height and base diameter of a conical tomb are* 25 *m and* 14 *m respectively. Find the cost of white washing its curved surface at the rate of Rs* 210 *per* 100 *m^2.*

Sol. Here, $l = 25$ m, $r = \dfrac{14}{2}$ m = 7 m

Curved surface area of the tomb $= \pi rl$

$= \dfrac{22}{7} \times 7 \times 25$ m^2 = 550 m^2

Cost of white washing 100 m^2 = Rs 210

$\therefore$ Cost of white washing 550 m^2 = Rs $\dfrac{210}{100} \times 550$ **= Rs 1155 Ans.**

Q.7. *A joker's cap is in the form of a right circular cone of base radius* 7 *cm and height* 24 *cm. Find the area of the sheet required to make* 10 *such caps.*

Sol. Here, $r = 7$ cm, $h = 24$ cm

We have, $l = \sqrt{h^2 + r^2} = \sqrt{(24)^2 + 7^2}$

$= \sqrt{576 + 49} = \sqrt{625} = 25$ cm

Total curved surface area of 1 cap $= \pi rl$

$= \dfrac{22}{7} \times 7 \times 25$ cm^2 = 550 cm^2

Area of sheet required to make 10 such caps = 10 × 550 cm^2 **= 5500 cm^2 Ans.**

Q.8. *A bus stop is barricaded from the remaining part of the road, by using* 50 *hollow cones made of recycled cardboard. Each cone has a base diameter of* 40 *cm and height* 1 *m. If the outer side of each of the cones is to be painted and the cost of painting is Rs* 12 *per m^2, what will be the cost of painting all these cones? (Use* $\pi = 3.14$ *and take* $\sqrt{1.04} = 1.02$)

Sol. Here, $r = \dfrac{40}{2}$ cm = 20 cm = 0.20 m, $h = 1$ m

$l = \sqrt{h^2 + r^2} = \sqrt{1^2 + (0.2)^2} = \sqrt{1.04} = 1.02$ m

Curved surface area of 1 cone $= \pi rl$

Curved surface area of 50 cones = 50 × 3.14 × 0.2 × 1.02 m^2

= 32.028 m^2

Cost of painting an area of 1 m^2 = Rs 12

$\therefore$ Cost of painting an area of 32.028 m^2 = Rs 12 × 32.028

= Rs 384.34 (approx) Ans.

EXERCISE 13.4

Q.1. *Find the surface area of a sphere of radius :*

(i) 10.5 *cm* *(ii)* 5.6 *cm* *(iii)* 14 *cm*

Sol. (i) r = 10.5 cm

Surface area of the sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times (10.5)^2$ cm^2

$= 4 \times \frac{22}{7} \times 10.5 \times 10.5$ cm^2 **= 1386 cm^2 Ans.**

(ii) r = 5.6 cm

Surface area of the sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times (5.6)^2$ cm^2

$= 4 \times \frac{22}{7} \times 5.6 \times 5.6$ cm^2 **= 394.24 cm^2 Ans.**

(iii) r = 14 cm

Surface area of the sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 14 \times 14$ cm^2

$= 88 \times 28$ cm^2 **= 2464 cm^2 Ans.**

Q.2. *Find the surface area of sphere of a diameter :*

(i) 14 *cm* *(ii)* 21 *cm* *(iii)* 3.5 *m*

Sol. (i) $r = \frac{14}{2}$ cm = 7 cm

Surface area of the sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 7^2$ cm^2

$= 4 \times \frac{22}{7} \times 7 \times 7$ cm^2

88×7 cm^2 **= 616 cm^2 Ans.**

(ii) $r = \frac{21}{2}$ cm = 10.5 cm

Surface area of the sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times (10.5)^2$ cm^2

$= 4 \times \frac{22}{7} \times 10.5 \times 10.5$ cm^2 **= 1386 cm^2 Ans.**

(iii) $r = \frac{3.5}{2}$ m = 1.75 m

Surface area of the sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times (1.75)^2$ m^2

$= 4 \times \frac{22}{7} \times 1.75 \times 1.75$ m^2 **= 38.5 m^2 Ans.**

Q.3. *Find the total surface area of a hemisphere of radius* 10 *cm. (Use* $\pi = 3.14$*)*

Sol. $r = 10$ cm

Total surface area of the hemisphere $= 3\pi r^2$

$= 3 \times 3.14 \times (10)^2 \text{ cm}^2$

$= 3 \times 3.14 \times 100 \text{ cm}^2$ **= 942 cm^2 Ans.**

Q.4. *The radius of a spherical balloon increases from* 7 *cm to* 14 *cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.*

Sol. When r = 7 cm

Surface area of the balloon $= 4\pi r^2$

$= 4 \times \pi \times 7 \times 7 \text{ cm}^2$

When R = 14 cm :

Surface area of the balloon $= 4\pi r^2$

$= 4 \times \pi \times 14 \times 14 \text{ cm}^2$

Required ratio of the surface areas of the balloon

$$= \frac{4 \times \pi \times 7 \times 7}{4 \times \pi \times 14 \times 14} = \frac{1}{4} \quad \mathbf{= 1 : 4 \text{ Ans.}}$$

Q.5. *A hemispherical bowl made of brass has inner diameter* 10.5 *cm. Find the cost of tin-plating it on the inside at the rate of Rs* 16 *per* 100 *cm*^{2}.

Sol. Here $r = \frac{10.5}{2}$ cm = 5.25 cm

Inner surface area of the bowl $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times (5.25)^2 \text{ cm}^2$

$= 44 \times 0.75 \times 5.25 \text{ cm}^2 = 173.25 \text{ cm}^2$

Cost of tin plating 100 cm^2 = Rs 16

Cost of tin plating 173.25 cm^2 = Rs $\frac{16}{100} \times 173.25$ **= Rs 27.72 Ans.**

Q.6. *Find the radius of a sphere whose surface area is* 154 *cm*^{2}.

Sol. Surface area of the sphere $= 4\pi r^2$

$\Rightarrow 154 = 4 \times \frac{22}{7} \times r^2$

$\Rightarrow \quad r^2 = \frac{154 \times 7}{4 \times 22} = \frac{7 \times 7}{4}$

$\Rightarrow \quad r = \frac{7}{2} = 3.5$

Hence, radius of the sphere **= 3.5 cm Ans.**

Q.7. *The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.*

Sol. Let diameter of the earth $= 2r$

Then radius of the earth $= r$

$\therefore$ Diameter of the moon $= \frac{2r}{4} = \frac{r}{2}$

$\therefore$ Radius of the moon = $\frac{r}{4}$

Now, surface area of the moon = $4\pi \left(\frac{r}{4}\right)^2$

$= \frac{\pi r^2}{4}$ ... (i)

Surface area of the earth $= 4\pi r^2$ ... (ii)

$\therefore$ Required ratio = $\frac{\frac{\pi r^2}{4}}{4\pi r^2} = \frac{\pi r^2}{4 \times 4\pi r^2} = \frac{1}{16}$ **= 1 : 16 Ans.**

Q.8. *A hemispherical bowl is made of steel,* 0.25 *cm thick. The inner radius of the bowl is* 5 *cm. Find the outer curved surface area of the bowl.*

Sol. Inner radius of the bowl (r) = 5 cm

Thickness of the steel = 0.25 cm

$\therefore$ Outer radius of the bowl (R) = (5 + 0.25) cm = 5.25 cm

Outer curved surface area of the bowl

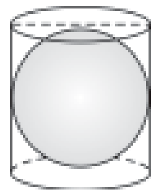
$= 2\pi R^2 = 2 \times \frac{22}{7} \times (5.25)^2$ cm^2 **= 173.25 cm^2 Ans.**

Q.9. *A right circular cylinder just encloses a sphere of radius r (see figure). Find*

(i) surface area of the sphere,

(ii) curved surface area of the cylinder,

(iii) ratio of the areas obtained in (i) and (ii).



Sol. Here, radius of the sphere = r

Radius of the cylinder = r

And, height of the cylinder = $2r$

(i) Surface area of the sphere **= $4\pi r^2$ Ans.**

(ii) Curved surface area of the cylinder = $2\pi rh$

$2\pi \times r \times 2r$ **= $4\pi r^2$ Ans.**

(iii) Required ratio = $\frac{4\pi r^2}{4\pi r^2} = \frac{1}{1}$ **= 1 : 1 Ans.**

EXERCISE 13.5

Q.1. *A matchbox measures* 4 *cm* × 2.5 *cm* × 1.5 *cm. What will be the volume of a packet containing* 12 *such boxes.*

Sol. Here, l = 4 cm, b = 2.5 cm, h = 1.5 cm

Volume of 1 matchbox = lbh

= 4 × 2.5 × 1.5 cm^3 = 15 cm^3

Volume of 12 matchboxes = 15 × 12 cm^2 **= 180 cm^3 Ans.**

Q.2. *A cuboidal water tank is* 6 *m long,* 5 *m wide and* 4.5 *m deep. How many litres of water can it hold?* (1 m^3 = 1000 l)

Sol. Here, l = 6 m, b = 5 m, h = 4.5 m

Volume of the tank = lbh

= 6 × 5 × 4.5 m^3 = 135 m^3

= 135 × 1000 litres = 1,35,000 litres.

Hence, the tank can hold 1,35,000 litres of water. **Ans.**

Q.3. *A cuboidal vessel is* 10 *m long and* 8 *m wide. How high must it be made to hold* 380 *cubic metres of a liquid?*

Sol. Here, l = 10 m, b = 8 m, h = ?

Volume of the vessel = lbh

$\Rightarrow$ 380 = 10 × 8 × h

$$\Rightarrow h = \frac{380}{10 \times 8} = 4.75$$

Hence, the tank must be made **4.75 m high Ans.**

Q.4. *Find the cost of digging a cuboidal pit* 8 *m long,* 6 *m broad and* 3 *m deep at the rate of Rs* 30 *per* m^3.

Sol. l = 8 m, b = 6 m, h = 3 m

Volume of the pit = lbh

= 8 × 6 × 3 m^3 = 144 m^3

Cost of digging 1m^3 = Rs 30

$\therefore$ Cost of digging 144 m^3 = Rs 30 × 144 **= Rs 4320 Ans.**

Q.5. *The capacity of a cuboidal tank is* 50000 *litres of water. Find the breadth of the tank, if its length and depth are respectively* 2.5 *m and* 10 *m.*

Sol. Here, l = 2.5 m, h = 10 m, b = ?

Capacity of the tank = 50000 lites = $\frac{50000}{1000}$ m^3 = 50 m^3

Also, capacity of the tank = lbh

$\Rightarrow$ 50 = 2.5 × b × 10 $\qquad \Rightarrow b = \frac{50}{25} = 2$

Hence, breadth of the tank **= 2 m Ans.**

Q.6. *A village, having a population of* 4000, *requires* 150 *litres of water per head per day. It has a tank measuring* 20 *m* × 15 *m* × 6 *m. For how many days will the water of this tank last?*

Sol. Here, l = 20 m, b = 15 m, h = 6 m

Population of the village = 4000

Water consumed by 1 person in 1 day = 150 litres

$\therefore$ Water consumed by 4000 persons in 1 day = 4000 × 150 litres

$$= \frac{4000 \times 150}{1000} \text{ m}^3 = 600 \text{ m}^3$$

Also, capacity of the tank = lbh

= 20 × 15 × 6 m^3

$$\therefore \text{Required number of days} = \frac{\text{Volume of the tank}}{\text{Water consumed in 1 day}}$$

$$= \frac{20 \times 15 \times 6}{600} = 3$$

Hence, the water of this tank will last for 3 days. **Ans.**

Q.7. *A godown measures* 40 *m* × 25 *m* × 10 *m. Find the maximum number of wooden crates each measuring* 1.5 *m* × 1.25 *m* × 0.5 *m that can be stored in the godown.*

Sol. Volume of the godown = 40 × 25 × 10 m^3

Volume 1 wooden crate = 1.5 × 1.25 × 0.5 m^3

$$\therefore \text{Required number of crates} = \frac{\text{Volume of the godown}}{\text{Volume of 1 crate}}$$

$$= \frac{40 \times 25 \times 10}{1.5 \times 1.25 \times 0.5} = 10666.67$$

Hence, the maximum number of wooden crates that can be stored in the godown **= 10666 Ans.**

Q.8. *A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.*

Sol. Here, a = 12 cm

Volume of the cube = $a^3 = (12)^3 \text{ cm}^3 = 1728 \text{ cm}^3$

Now, volume of 1 smaller cube = $\frac{1728}{8} \text{ cm}^3 = 216 \text{ cm}^3$

Let side of the new cube be A.

Then $A^3 = 216$

$\Rightarrow \quad A = \sqrt[3]{216} = 6$

Hence, side of the new cube **= 6 cm Ans.**

Total surface area of the bigger cube = $6a^2$

$= 6 \times (12)^2 \text{ cm}^2 = 6 \times 12 \times 12 \text{ cm}^2$

Total surface area of 1 smaller cube = $6A^2$

$= 6 \times 6^2 \text{ cm}^2 = 6 \times 6 \times 6 \text{ cm}^2$

Hence, required ratio = $\frac{6 \times 12 \times 12}{6 \times 6 \times 6} = \frac{4}{1}$ = 4 : 1 **Ans.**

Q.9. *A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute ?*

Sol. Here, b = 40 m, h = 3 m, l = 2 km = 2000 m

Volume of water flowing through the river in 1 hour

$= lbh = 2000 \times 40 \times 3 \text{ m}^3$

$\therefore$ Volume of water flowing through the river in 1 minute

$= \frac{2000 \times 40 \times 3}{60} \text{ m}^3$ **= 4000 m³ Ans.**

EXERCISE 13.6

Q.1. *The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold?* (1000 cm^3 = 1l)

Sol. Here, h = 25 cm, $2\pi r$ = 132 cm.

$2\pi r = 132$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 132$

$\Rightarrow \quad r = \frac{132 \times 7}{2 \times 22} \text{ cm} = 21 \text{ cm}$

Volume of the cylinder = $\pi r^2 h = \frac{22}{7} \times 21 \times 21 \times 25 \text{ cm}^3$

$= 34650 \text{ cm}^3$

$= \frac{34650}{1000}$ litres **= 34.65 litres Ans.**

Q.2. *The inner diameter of a cylindrical wooden pipe is* 24 *cm and its outer diameter is* 28 *cm. The length of the pipe is* 35 *cm. Find the mass of the pipe, if* 1 *cm*^{3} *of wood has a mass of* 0.6 *g.*

Sol. Here, inner radius (r) = $\frac{24}{2}$ cm = 12 cm

Outer radius (R) = $\frac{28}{2}$ cm = 14 cm, h = 35 cm

Volume of the wood used in the pipe = $\pi(R^2 - r^2)\, h$

$$= \frac{22}{7}\ [(14)^4 - (12)^2] \times 35 \text{ cm}^3$$

$$= \frac{22}{7} \times 26 \times 2 \times 35 \text{ cm}^3 = 5720 \text{ cm}^3$$

Mass of 1 cm^3 of wood = 0.6 g

$\therefore$ Mass of 5720 cm^3 of wood = 0.6 × 5720 g = 3432 g **= 3.432 kg Ans.**

Q.3. *A soft drink is available in two packs — (i) a tin can with a rectangular base of length* 5 *cm and width* 4 *cm, having a height of* 15 *cm and (ii) a plastic cylinder with circular base of diameter* 7 *cm and height* 10 *cm. Which container has greater capacity and by how much?*

Sol. **For tin can with rectangular 6 base.**

l = 5 cm, b = 4 cm, h = 15 cm

Volume of the tin can = lbh = 5 × 4 × 15 cm^3 = 300 cm^3

For plastic cylinder with circular base.

$$r = \frac{7}{2} \text{ cm} = 3.5 \text{ cm}, \quad h = 10 \text{ cm}$$

Volume of the plastic cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times 10 \text{ cm}^3 = 385 \text{ cm}^3$$

Difference in the capacities of the two containers

= (385 – 300) cm^3 = 85 cm^3

Hence, the plastic cylinder with circular base has greater capacity by 85 cm^3 **Ans.**

Q.4. *If the lateral surface of a cylinder is* 94.2 *cm*^{2} *and its height is* 5 *cm, then find (i) radius of its base (ii) its volume (Use* π = 3.14)

Sol. Here, h = 5 cm, $2\pi rh$ = 94.2 cm^2.

(i) $2\pi rh = 94.2$

$\Rightarrow 2 \times 3.14 \times r \times 5 = 94.2$

$$\Rightarrow r = \frac{94.2}{2 \times 3.14 \times 5} = 3$$

Hence, base radius of the cylinder = 3 cm **Ans.**

(ii) Volume of the cylinder = $\pi r^2 h$

= 3.14 × 3 × 3 × 5 cm^3 = 141.3 cm^3 **Ans.**

Q.5. *It costs Rs* 2200 *to paint the inner curved surface of a cylindrical vessel* 10 *m deep. If the cost of painting is at the rate of Rs* 20 *per m*^{2}*, find*

(i) Inner curved surface area of the vessel,

(ii) radius of the base,

(iii) capacity of the vessel.

Sol. Here, h = 10 m

(i) Inner curved surface area $= \dfrac{\text{Total cost}}{\text{Cost of painting per m}^2}$

$= \dfrac{2200}{20}\ \text{m}^2$ **= 110 m² Ans.**

(ii) We have, $2\pi rh = 110$

$\Rightarrow 2 \times \dfrac{22}{7} \times r \times 10 = 110$

$\Rightarrow r = \dfrac{110 \times 7}{2 \times 22 \times 10}$ **= 1.75 m Ans.**

(iii) Capacity of the vessel $= \pi r^2 h$

$= \dfrac{22}{7} \times 1.75 \times 1.75 \times 10\ \text{m}^3 = 96.25\ \text{m}^3$

= 96.25 kl Ans. [1 m³ = 1 *kl*]

Q.6. *The capacity of a closed cylindrical vessel of height* 1 *m is* 15.4 *litres. How many square metres of metal sheet would be needed to make it?*

Sol. Here, h = 1 m, volume = 15.4 litres

$= \dfrac{15.4}{1000}\ \text{m}^3 = 0.0154\ \text{m}^3$

Also, volume of the cylinderical vessel $= \pi r^2 h$

$\Rightarrow 0.0154 = \dfrac{22}{7} \times r^2 \times 1$

$\Rightarrow r^2 = \dfrac{0.0154 \times 7}{22} = 0.0049$

$\Rightarrow r = 0.07$ m

$\therefore$ Total surface area of the cylinder $= 2\pi r\ (h + r)$

$= 2 \times \dfrac{22}{7} \times 0.07\ (1 + 0.07)\ \text{m}^2$

$= 44 \times 0.01 \times 1.07\ \text{m}^2 = 0.4708\ \text{m}^2$

Hence, 0.4708 m² of metal sheet would be needed **Ans.**

Q.7. *A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is* 7 *mm and the dimeter of the graphite is* 1 *mm. If the length of the pencil is* 14 *cm, find the volume of the wood and that of the graphite.*

Sol. Here, h = 14 cm.

Radius of the pencil (R) $= \dfrac{7}{2}$ mm = 0.35 cm.

Radius of the graphite (r) $= \dfrac{1}{2}$ mm = 0.05 cm.

Volume of the the graphite $= \pi r^2 h$

$= \dfrac{22}{7} \times 0.05 \times 0.05 \times 14\ \text{cm}^3 = 0.11\ \text{cm}^3$

Volume of the the wood = π $(R^2 - r^2)h$

$$= \frac{22}{7} \times [(0.35)^2 - (0.05)^2] \times 14 \text{ cm}^3$$

$$= \frac{22}{7} \times 0.4 \times 0.3 \times 14 \text{ cm}^3 = 5.28 \text{ cm}^3$$

Hence, volume of the wood **= 5.28 cm^3** and volume of the graphite **= 0.11 cm^3 Ans.**

Q.8. *A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients?*

Sol. Here, $r = \frac{7}{2}$ cm = 3.5 cm, h = 4 cm

Capacity of 1 cylindrical bowl = $\pi r^2 h$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times 4 \text{ cm}^3 = 154 \text{ cm}^3$$

Hence, soup consumed by 250 patients per day

= 250 × 154 cm^3 **= 38500 cm^3 Ans.**

EXERCISE 13.7

Q.1. *Find the volume of the right circular cone with*

(i) radius 6 cm, height 7 cm *(ii) radius 3.5 cm, height 12 cm*

Sol. (i) Here, r = 6 cm, h = 7 cm

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 7 \text{ cm}^3 = \mathbf{264 \text{ cm}^3 \text{ Ans.}}$$

(ii) Here, r = 3.5 cm, h = 12 cm

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 12 \text{ cm}^3 = \mathbf{154 \text{ cm}^3 \text{ Ans.}}$$

Q.2. *Find the capacity in litres of a conical vessel with*

(i) radius 7 cm, slant height 25 cm

(ii) height 12 cm, slant height 13 cm

Sol. (i) Here, r = 7 cm, l = 25 cm

$$\therefore r = \sqrt{l^2 - r^2} = \sqrt{625 - 49} = \sqrt{576} = 24 \text{ cm.}$$

Volume of the conical vessel = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24 \text{ cm}^3 = 1232 \text{ cm}^3$$

$$= \frac{1232}{1000} \text{ litres} = \mathbf{1.232 \text{ litres Ans.}}$$

(ii) Here, h = 12 cm, l = 13 cm

$$\therefore h = \sqrt{l^2 - h^2} = \sqrt{13^2 - 12^2} = \sqrt{169 - 144} = \sqrt{25} = 5 \text{ cm}$$

Volume of the conical vessel = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 12 \text{ cm}^3 = \frac{22 \times 5 \times 5 \times 4}{7} \text{ cm}^3$

$= \frac{22 \times 5 \times 5 \times 4}{7 \times 1000}$ litres $= \mathbf{\frac{11}{35}}$ **litres Ans.**

Q.3. *The height of a cone is* 15 *cm. If its volume is* 1570 *cm*^{3}*, find the radius of the base.* (*Use* $\pi = 3.14$)

Sol. (i) Here, $h = 15$ cm, volume = 1570 cm^3

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$\Rightarrow 1570 = \frac{1}{3} \times 3.14 \times r^2 \times 15$

$\Rightarrow \quad r^2 = \frac{1570 \times 3}{3.14 \times 15} = 100$

$\Rightarrow \quad r = 10$

Hence, radius of the base **= 10 cm Ans.**

Q.4. *If the volume of a right circular cone of height* 9 *cm is* 48 π *cm*^{3}*, find the diameter of its base.*

Sol. Here, $h = 9$ cm, volume = 48π cm^3

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$\Rightarrow 48\pi = \frac{1}{3}\pi \times r^2 \times 9$

$\Rightarrow \quad r^2 = \frac{48\pi \times 3}{\pi \times 9} = 16$

$\Rightarrow \quad r = 4$

Hence, base diameter of the cone = 2 × 4 cm **= 8 cm Ans.**

Q.5. *A conical pit of top diameter* 3.5 *m is* 12 *m deep. What is its capacity in kilolitres?*

Sol. Here, $r = \frac{3.5}{2}$ m = 1.75 m, $h = 12$ m

Capacity of the pit = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 1.75 \times 1.75 \times 12 \text{ m}^3$

= 38.5 m^3 **= 38.5 *kl* Ans.**

Q.6. *The volume of a right circular cone is* 9856 *cm*^{3}*. If the diameter of the base is* 28 *cm, find*

(i) height of the cone

(ii) slant height of the cone

(iii) curved surface area of the cone.

Sol. Here, $r = \frac{28}{2}$ cm = 14 cm, volume = 9856 cm^3

(i) Volume of the cone = $\frac{1}{3}\pi r^2 h$

$\Rightarrow 9856 = \frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times h$

$\Rightarrow \quad h = \frac{9856 \times 3 \times 7}{22 \times 14 \times 14} = 48$

Hence, height of the cone = 48 cm **Ans.**

(ii) Slant height $l = \sqrt{h^2 + r^2} = \sqrt{(48)^2 + (14)^2}$

$= \sqrt{2304 + 196} = \sqrt{2500} = 50$

Hence, slant height of the cone **= 50 cm Ans.**

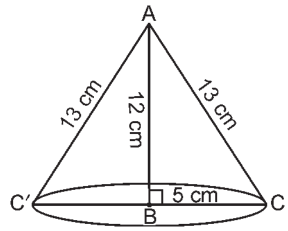
(iii) Curved surface area of the cone = πrl

$= \frac{22}{7} \times 14 \times 50$ cm^2 **= 2200 cm^2 Ans.**

Q.7. *A right triangle ABC with sides* 5 *cm,* 12 *cm and* 13 *cm is revolved about the side* 12 *cm. Find the volume of the solid so obtained.*

Sol. The solid formed is a cone, whose height

h = 12 cm, base radius r = 5 cm.

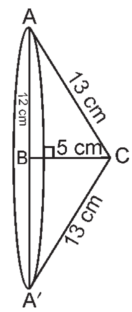


$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 5 \times 5 \times 12$ cm^3 **= 100 π cm^3 Ans.**

Q.8. *If the triangle ABC in the Question* 7 *above is revolved about the side* 5 *cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in questions* 7 *and* 8.

Sol. Here radius r of the cone = 12 cm and height h of the cone = 5 cm.



$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 12 \times 12 \times 5$ **= 240 πcm^3 Ans.**

Hence, required ratio $= \frac{100\pi}{240\pi} = \frac{5}{12}$ **= 5 : 12 Ans.**

Q.9. *A heap of wheat is in the form of a cone whose diameter is* 10.5 *m and height is* 3 *m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.*

Sol. Here, radius $r = \frac{10.5}{2}$ m = 5.25 m, h = 3 m

Volume of the heap = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 3 \text{ m}^3$ **= 86.625 m^3 Ans.**

Now, $l = \sqrt{h^2 + r^2} = \sqrt{3^2 + (5.25)^2}$

$= \sqrt{9 + 27.5625} = \sqrt{36.5625} = 6.05$ m (approx)

Curved surface area of the cone = πrl

$= \frac{22}{7} \times 5.25 \times 6.05 \text{ m}^2 = 99.825 \text{ m}^2$

Hence, 99.825 m^2 of canvas is needed. **Ans.**

EXERCISE 13.8

Q.1. *Find the volume of a sphere whose radius is*

(i) 7 *cm* *(ii)* 0.63 *m*

Sol. (i) Here, r = 7 cm

Volume of the sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 \text{ cm}^3 = \mathbf{1437\frac{1}{3} \text{ cm}^3}$ **Ans.**

(ii) Here, r = 0.63 m

Volume of the sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (0.63)^3 \text{ m}^3$ **= 1.05 m^3 (approx) Ans.**

Q.2. *Find the amount of water displaced by a solid spherical ball of diameter*

(i) 28 *cm* *(ii)* 0.21 *m*

Sol. (i) Here, $r = \frac{28}{2}$ cm = 14 cm

Volume of water displaced by the spherical ball

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14 \text{ cm}^3$

$\mathbf{= 11498\frac{2}{3} \text{ cm}^3}$ **Ans.**

(ii) Here, $r = \frac{0.21}{2}$ m = 0.105 m

Volume of the water displaced by the spherical ball

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 0.105 \times 0.105 \times 0.105 \text{ m}^3$

= 0.004851 m^3 Ans.

Q.3. *The diameter of a metallic ball is* 4.2 *cm. What is the mass of the ball, if the density of the metal is* 8.9 *g per cm*^{3}?

Sol. Here, $r = \frac{4.2}{2}$ cm = 2.1 cm

Volume of the ball $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1 \text{ cm}^3$$

$= 38.808 \text{ cm}^3$

Density of the metal = 8.9 g/cm^3

$\therefore$ Mass of the ball = 8.9 × 38.808 g

= 345.39 g (approx) Ans.

Q.4. *The diameter of the moon is approximately one fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?*

Sol. Let diameter of the earth be $2r$.

Then radius of the earth = r

So, diameter of the moon $= \frac{2r}{4} = \frac{r}{2}$

$\Rightarrow$ Radius of the moon $= \frac{r}{4}$

Volume of the earth $= \frac{4}{3}\pi r^3$ ... (i)

Volume of the moon $= \frac{4}{3}\pi\left(\frac{r}{4}\right)^3$ ... (ii)

$$\frac{\text{Volume of the earth}}{\text{Volume of the moon}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi\left(\frac{r}{4}\right)^3} \quad \text{[From (i) and (ii)]}$$

$$\frac{r^3}{\frac{r^3}{64}} = \frac{64}{1} = 64 \textbf{ Ans.}$$

$\Rightarrow$ Volume of the moon $= \frac{1}{64}$ × volume of the earth

Hence, volume of the moon is $\frac{1}{64}$ of volume of the earth. **Ans.**

Q.5. *How many litres of milk can a hemispherical bowl of diameter* 10.5 *cm hold?*

Sol. Here, $r = \frac{10.5}{2}$ cm = 5.25 cm

Volume of the hemispherical bowl $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 5.25 \text{ cm}^3 = 303 \text{ cm}^3 \text{ (approx)}$$

Hence, the hemispherical bowl can hold $\frac{303}{1000}$ litres **= 0.303 liters of milk. Ans.**

Q.6. *A hemispherical tank is made up of an iron sheet* 1 *cm thick. If the inner radius is* 1 *m, then find the volume of the iron used to make the tank.*

Sol. Here, inner radius of the tank (r) = 1 m

Thickness of the iron sheet = 1 cm = 0.01 m

$\therefore$ External radius of the tank (R) = (1 + 0.01) m = 1.01 m

Volume of the iron used to make the tank

$$= \frac{2}{3}\pi(R^3 - r^3)$$

$$= \frac{2}{3} \times \frac{22}{7} \times [(1.01)^3 - 1^3] \text{ m}^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 0.030301 \text{ m}^3$$

$$= \mathbf{0.06348 \text{ m}^3 \text{ Ans.}}$$

Q.7. *Find the volume of a sphere whose surface area is* 154 cm^2.

Sol. Here, $4\pi r^2 = 154$

$$\Rightarrow r^2 = 4 \times \frac{22}{7} \times r^2 = 154$$

$$\Rightarrow r^2 = \frac{154 \times 7}{4 \times 22} = \frac{49}{4}$$

$$\Rightarrow r = \frac{7}{2} = 3.5 \text{ cm}$$

$$\therefore \text{ Volume of the sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5 \text{ cm}^3 = \mathbf{179\frac{2}{3} \text{ cm}^3 \text{ Ans.}}$$

Q.8. *A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of Rs* 498.96. *If the cost of white-washing is Rs* 2.00 *per square metre, find the*

(i) inside surface area of the dome,

(ii) volume of the air inside the dome.

Sol. (i) Inner surface of the dome

$$= \frac{\text{Total cost}}{\text{Cost of white washing per m}^2}$$

$$= \frac{498.96}{2} \text{ m}^2 = \mathbf{249.48 \text{ m}^2 \text{ Ans.}}$$

(ii) Let radius of the dome be r m.

Then $2\pi r^2 = 249.48$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r^2 = 249.48$$

$$\Rightarrow \quad r^2 = \frac{249.48 \times 7}{2 \times 22} = 39.69$$

$$\Rightarrow \quad r = 6.3 \text{ cm}$$

$\therefore$ Volume of the air inside the dome = $\frac{2}{3}\pi r^3$

$$= \frac{2}{3}\times\frac{22}{7} \times 6.3 \times 6.3 \times 6.3 \text{ m}^3 = \mathbf{523.9 \text{ m}^3} \quad \textbf{Ans.}$$

Q.9. *Twenty seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S′. Find the*
(i) radius r′ of the new sphere.
(ii) ratio of S and S′.

Sol. (i) Volume of a sphere of radius $r = \frac{4}{3}\pi r^3$

$\therefore$ Volume of 27 such spheres = $27 \times \frac{4}{3}\pi r^3 = 36\pi r^3$

Volume of the sphere with radius $r' = \frac{4}{3}\pi r'^3$

$\therefore\ 36\pi r^3 = \frac{4}{3}\pi r'^3$

$\Rightarrow 27r^3 = r'^3$

$\Rightarrow r' = \sqrt[3]{27\,r^3}$

$\Rightarrow$ $\boldsymbol{r' = 3r}$ **Ans.**

(ii) Surface area (S) of the sphere with radius $r = 4\pi r^2$

Surface area (S′) of the sphere with radius $r' = 4\pi r'^2$

$$= 4\pi\,(3r)^2 = 36\pi r^2$$

$$\therefore \qquad \frac{S}{S'} = \frac{4\pi r^2}{36\pi r^2} = \frac{1}{9} \quad \mathbf{= 1 : 9} \ \textbf{Ans.}$$

Q.10. *A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?*

Sol. Here, $r = \frac{3.5}{2}$ mm = 1.75 mm

Volume of the capsule = $\frac{4}{3}\pi r^3$

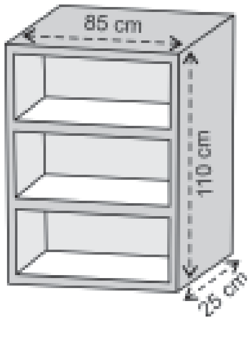
$$= \frac{4}{3} \times \frac{22}{7} \times 1.75 \times 1.75 \times 1.75 \text{ mm}^3$$

$$= 22.46 \text{ mm}^3 \text{ (approx)}$$

Hence, 22.46 mm^3 of medicine is needed to fill the capsule **Ans.**

EXERCISE 13.9 (Optional)

Q.1. *A wooden bookshelf has external dimensions as follows : Height = 110 cm, Depth = 25 cm, Breadth = 85 cm (see figure). The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2, find the total expenses required for polishing and painting the surface of the bookshelf.*



Sol. Here, external dimensions of the bookshelf are :

L = 110 cm, B = 85 cm, H = 25 cm

Thickness of the plank = 5 cm

Internal dimensions of the bookshelf are :

l = (110 – 5 – 5) cm = 100 cm,

b = (85 – 5 – 5) cm = 75 cm,

h = (25 – 5) cm = 20 cm

External surface area of the bookshelf

$= LB + 2\,(BH + HL)$

$= 110 \times 85 \text{ cm}^2 + 2(85 \times 25 + 25 \times 110) \text{ cm}^2$

$= (9350 + 9750) \text{ cm}^2 = 19100 \text{ cm}^2$

Surface are of the border

$= (4 \times 75 \times 5 + 110 \times 5 \times 2) \text{ cm}^2$

$= (1500 \times 1100) \text{ cm}^2 = 2600 \text{ cm}^2$

$\therefore$ Total surface area to be polished $= (19100 + 2600) \text{ cm}^2$

$= 21700 \text{ cm}^2$

$\therefore$ Cost of polishing the outer surface = Rs $\dfrac{21700 \times 20}{100}$ = Rs 4340 ... (i)

Inner surface are of the bookshelf $= lb + 2(bh + hl)$

$= 100 \times 75 \text{ cm}^2 + 2\,(75 \times 20 + 100) \text{ cm}^2$

$= 7500 \text{ cm}^2 + 2\,(1500 + 2000) \text{ cm}^2$

$= (7500 + 7000) \text{ cm}^2 = 14500 \text{ cm}^2$

Surface area of the two racks $= 4 \times 75 \times 20 \text{ cm}^2 = 6000 \text{ cm}^2$

Inner surface are covered by the racks $= (75 \times 5 \times 2 + 20 \times 5 \times 4) \text{ cm}^2$

$= (750 + 400) \text{ cm}^2 = 1150 \text{ cm}^2$

$\therefore$ Total surface are to be painted $= (14500 + 6000 - 1150) \text{ cm}^2$

$= 19350 \text{ cm}^2$

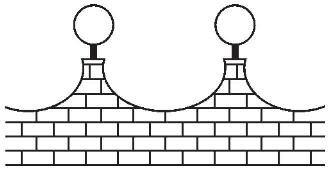
$\therefore$ Cost of painting the inner surface = Rs $\dfrac{19350 \times 10}{100}$ = Rs 1935 ... (ii)

From (i), and (ii), we have,

Total expenses required for polishing and painting the surface of the bookshelf.

= Rs (4340 + 1935) = Rs 6275 **Ans.**

Q.2. *The front compound wall of a house is decorated by wooden spheres of diameter* 21 *cm, placed on small supports as shown in the figure. Eight such spheres are used for this purpose, and are to be painted silver. Each support is a cylinder of radius* 1.5 *cm and height* 7 *cm and is to be painted black. Find the cost of paint required if silver paint costs* 25 *paise per cm*^{2} *and black paint costs* 5 *paise per cm*^{2}.



Sol. Radius of a sphere = $\dfrac{21}{2}$ cm = 10.5 cm.

Surface area of a sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times 10.5 \times 10.5 \text{ cm}^2 = 1386 \text{ cm}^2$$

Area of the base of the cylinder (support) = πR^2

$$= \pi \times (1.5)^2 = \frac{22}{7} \times 1.5 \times 1.5 \text{ cm}^2$$

$$= 7.07 \text{ cm}^2$$

Area of a sphere to painted silver

$$= (1386 - 7.07) \text{ cm}^2$$

$$= 1378.93 \text{ cm}^2$$

Area of spheres to be painted silver = 8×1378.93 cm^2

$\therefore$ cost of painting the spheres $= \text{Rs } \frac{8 \times 1378.93 \times 25}{100}$

$= \text{Rs } 2757.86$

Curved surface area of a cylinder (support)

$$= 2 \times \frac{22}{7} \times 1.5 \times 7 \text{ cm}^2$$

Curved surface area of 8 supports $= 8 \times 2 \times \frac{22}{7} \times 1.5 \times 7 \text{ cm}^2$

Cost of painting the supports $= \text{Rs } 8 \times 2 \times \frac{22}{7} \times 1.5 \times 7 \times \frac{5}{100}$

$= \text{Rs } 26.40$

Total cost requiredof paint = Rs (2757.86 + 26.40) **= Rs 2784.26 Ans.**

Q.3. *The diameter of a sphere is decreased by 25%. By what per cent does its curved surface area decrease?*

Sol. Let originally the diameter of the sphere be $2r$.

Then, radius of the sphere = r

Surface area of the sphere = $4\pi r^2$ ... (i)

New diameter of the sphere $= 2r - 2r \times \frac{25}{100} = \frac{3r}{2}$

$\therefore$ New radius of the sphere $= \frac{3r}{4}$

Surface area of the new sphere $= 4\pi\left(\frac{3r}{4}\right)^2 = \frac{9\pi r^2}{4}$

Decrease in surface area $= 4\pi r^2 - \frac{9\pi r^2}{4} = \frac{7\pi r^2}{4}$

Per cent decrease $= \dfrac{\frac{7\pi r^2}{4} \times 100}{4\pi r^2} = \frac{7}{16} \times 100 = \frac{175}{4} = 43.75$

Hence, the surface area decreases by **43.75% Ans.**