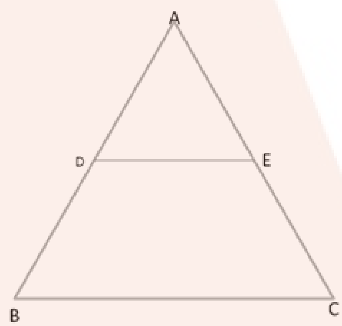
Book Name: Selina Concise

EXERCISE 15(A)

Solution 1:

(i) False
(ii) True
(iii) True
(iv) False
(v) True
(vi) True
(vii) True

Solution 2:



In Δ ADE and Δ ABC, DE is parallel to BC, so corresponding angles are equal.

$\angle ADE = \angle ABC$

$\angle AED = \angle ACB$

Hence, $\Delta ADE \sim \Delta ABC$ (By AA similarity criterion)

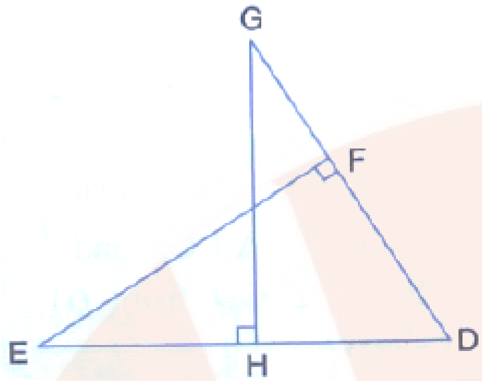
$$\therefore \frac{AD}{AB} = \frac{DE}{BC}$$

$$\frac{12}{12+24} = \frac{DE}{8}$$

$$DE = \frac{12}{36} \times 8 = \frac{8}{3} = 2\frac{2}{3}$$

Hence, $DE = 2\frac{2}{3}$ cm

Solution 3:



In ΔDHG and ΔDFE,

$\angle GHD = \angle DFE = 90^{\circ}$

$\angle D = \angle D$ (Common)

$\therefore \Delta DHG \sim \Delta DFE$

$$\Rightarrow \frac{DH}{DF} = \frac{DG}{DE}$$

$$\Rightarrow \frac{8}{12} = \frac{3x-1}{4x+2}$$

$\Rightarrow 32x + 16 = 36x - 12$

$\Rightarrow 28 = 4x$

$\Rightarrow x = 7$

$\therefore DG = 3 \times 7 - 1 = 20$

$DE = 4 \times 7 + 2 = 30$

Solution 4:

In ΔADC and ΔBAC,

$\angle ADC = \angle BAC$ (Given)

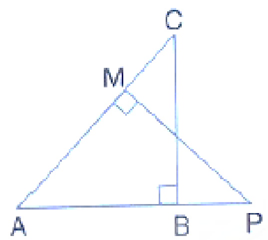
$\angle ACD = \angle ACB$ (Common)

$\therefore \Delta ADC \sim \Delta BAC$

$$\therefore \frac{CA}{CB} = \frac{CD}{CA}$$

Hence, $CA^2 = CB \times CD$

Solution 5:



(i) In Δ ABC and Δ AMP,

∠ BAC = ∠PAM [Common]

∠ ABC = ∠ PMA [Each = 90°]

Δ ABC ~ Δ AMP [AA Similarity]

(ii)

$AM = \sqrt{AP^2 - PM^2} = \sqrt{15^2 - 12^2} = 11$

Since Δ ABC – Δ AMP,

$$\frac{AB}{AM} = \frac{BC}{PM} = \frac{AC}{AP}$$

$$\Rightarrow \frac{AB}{AM} = \frac{BC}{PM} = \frac{AC}{AP}$$

$$\Rightarrow \frac{AB}{11} = \frac{BC}{12} = \frac{10}{15}$$

From this we can write,

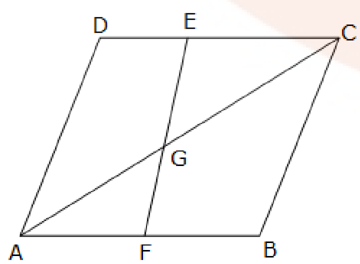
$$\frac{AB}{11} = \frac{10}{15}$$

$$\Rightarrow AB = \frac{10 \times 11}{15} = 7.33$$

$$\frac{BC}{12} = \frac{10}{15}$$

$$\Rightarrow BC = 8cm$$

Solution 6:



In ΔEGC and ΔFGA

$\angle ECG = \angle FAG$ (Alternate angles as AB ∥ CD)

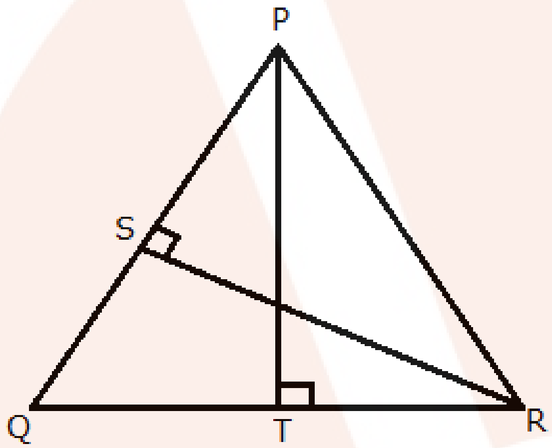
$\angle EGC = \angle FGA$ (Vertically opposite angles)

$\Delta EGC \sim \Delta FGA$ (By AA – similarity)

$\therefore \frac{EG}{FG} = \frac{CG}{AG}$

$AG \times EG = FG \times CG$

Solution 7:



(i)

In ΔPQT and ΔQRS,

$\angle PTQ = \angle RSQ = 90^\circ$ (Given)

$\angle PQT = \angle RQS$ (Common)

$\Delta PQT \sim \Delta RQS$ (By AA similarity)

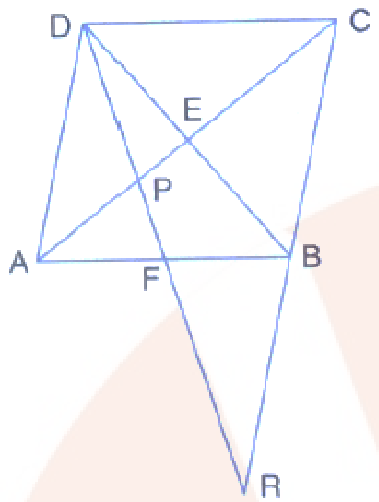
(ii)

Since, triangle PQT and RQS are similar

$\therefore \frac{PQ}{RQ} = \frac{QT}{QS}$

$\Rightarrow PQ \times QS = RQ \times QT$

Solution 8:



In ΔDPA and ΔRPC,

$\angle DPA = \angle RPC$ (Vertically opposite angles)

$\angle PAD = \angle PCR$ (Alternate angles)

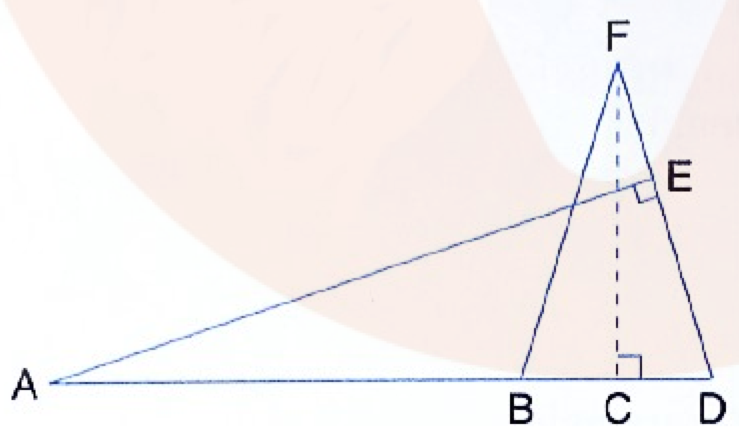
$\Delta DPA \sim \Delta RPC$

$\therefore \dfrac{DP}{PR} = \dfrac{AD}{CR}$

$\dfrac{DP}{PR} = \dfrac{DC}{CR}$ (AD = DC, as ABCD is rhombus)

Hence, $DP \times CR = DC \times PR$

Solution 9:

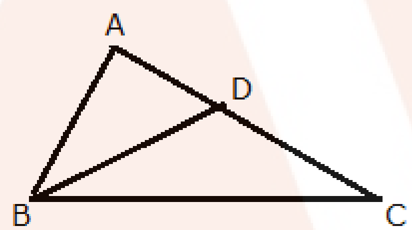


Given, FB = FD

$\therefore \angle FDB = \angle FBD$ ……….(1)

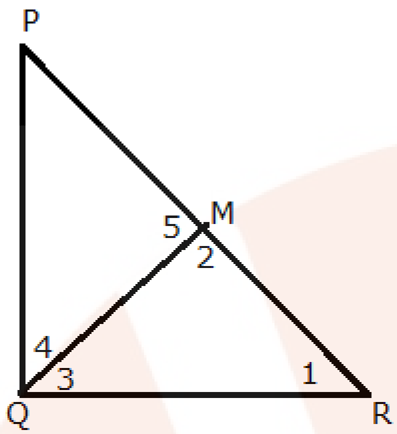
In $\Delta AED = \Delta FCB$,
$\angle AED = \angle FCB = 90^\circ$
$\angle ADE = \angle FBC$ [using (1)]
$\Delta AED \sim \Delta FCB$ [By AA similarity]
$\therefore \frac{AD}{FB} = \frac{ED}{BC}$
$\frac{FB}{AD} = \frac{BC}{ED}$

Solution 10:



(i) Since, BD is the bisector of angle B,
$\angle ABD = \angle DBC$
Also, given $\angle B = 2\angle C$
$\therefore \angle ABD = \angle DBC = \angle ACB$ .... (1)
In Δ ABC and Δ ABD,
$\angle BAC = \angle DAB$ (Common)
$\angle ACB = \angle ABD$ (Using (1))
$\therefore \Delta ABC \sim \Delta ADB$ (By AA similarity)
(ii) Since, triangles ABC and ADB are similar,
$\therefore \frac{BC}{BD} = \frac{AB}{AD}$
$\frac{BC}{AB} = \frac{BD}{AD}$
$\frac{BC}{AB} = \frac{DC}{AD}$ $(\angle DBC = \angle DCB \Rightarrow DC = BD)$
BC : AB = DC : AD

Solution 11:



(i) In ΔPQM and ΔPQR,

$\angle PMQ = \angle PQR = 90^\circ$

$\angle QPM = \angle RPQ$ (Common)

$\therefore \Delta PQM \sim \Delta PRQ$ (By AA Similarity)

$$\therefore \frac{PQ}{PR} = \frac{PM}{PQ}$$

$\Rightarrow PQ^2 = PM \times PR$

(ii) In ΔQMR and ΔPQR,

$\angle QMR = \angle PQR = 90^\circ$

$\angle QRM = \angle QRP$ (Common)

$\therefore \Delta QRM \sim \Delta PQR$ (By AA similarity)

$$\therefore \frac{QR}{PR} = \frac{MR}{QR}$$

$\Rightarrow QR^2 = PR \times MR$

(iii) Adding the relations obtained in (i) and (ii), we get,

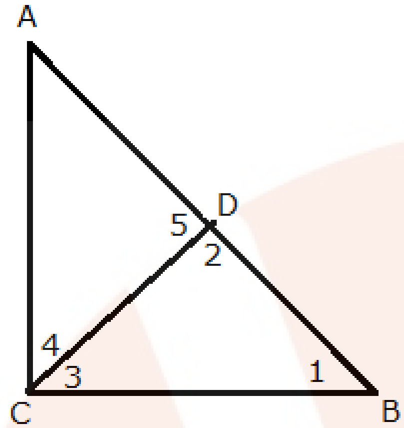
$PQ^2 + QR^2 = PM \times PR + PR \times MR$

$= PR(PM + MR)$

$= PR \times PR$

$= PR^2$

Solution 12:



In Δ CDB,

$\angle 1 + \angle 2 + \angle 3 = 180^{\circ}$

$\angle 1 + \angle 3 = 90^{\circ}$ ...... (1)(Since, $\angle 2 = 90^{\circ}$)

$\angle 3 + \angle 4 = 90^{\circ}$ ...... (2) (Since, $\angle ACB = 90^{\circ}$)

From (1) and (2),

$\angle 1 + \angle 3 = \angle 3 + \angle 4$

$\angle 1 = \angle 4$

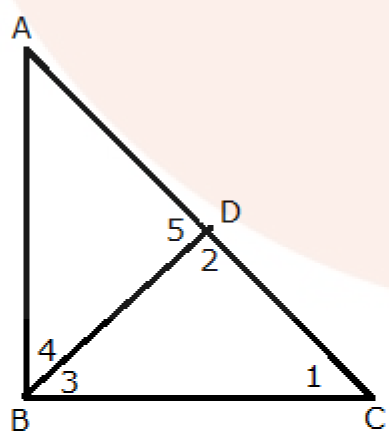
Also, $\angle 2 = \angle 5 = 90^{\circ}$

$\therefore \Delta BDC \sim \Delta CDA$ (By AA similarity)

$$\Rightarrow \frac{DB}{CD} = \frac{CD}{AD}$$

$$\Rightarrow CD^2 = AD \times DB$$

Solution 13:



(i) In Δ CDB,
$\angle 1 + \angle 2 + \angle 3 = 180^o$
$\angle 1 + \angle 3 = 90^o$ ..... (1)(Since, $\angle 2 = 90^o$)
$\angle 3 + \angle 4 = 90^o$ .....(2) (Since, $\angle ABC = 90^o$)
From (1) and (2),
$\angle 1 + \angle 3 = \angle 3 + \angle 4$
$\angle 1 = \angle 4$
Also, $\angle 2 = \angle 5 = 90^o$
$\therefore \Delta CDB \sim \Delta BDA$ (By AA similarity)

$$\Rightarrow \frac{CD}{BD} = \frac{BD}{AD}$$
$$\Rightarrow BD^2 = AD \times CD$$
$$\Rightarrow (8)^2 = AD \times 10$$
$$\Rightarrow AD = 6.4$$

Hence, AD = 6.4 cm
(ii) Also, by similarity, we have:

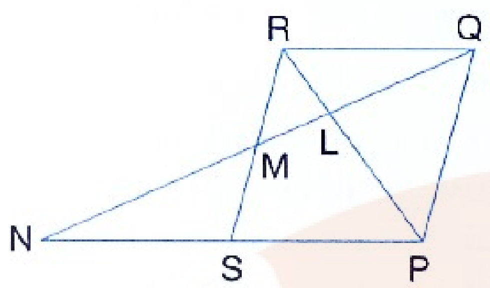
$$\frac{BD}{DA} = \frac{CD}{BD}$$
$$BD^2 = 6 \times (18 - 6)$$
$$BD^2 = 72$$
$$\text{Hence, } BD = 8.5 \text{ cm}$$

(iii)
Clearly, $\Delta ADB \sim \Delta ABC$

$$\therefore \frac{AD}{AB} = \frac{AB}{AC}$$
$$AD = \frac{7 \times 7}{9} = \frac{49}{9} = 5\frac{4}{9}$$
$$\text{Hence, } AD = 5\frac{4}{9} \text{ cm}$$

Solution 14:

In ΔRLQ and ΔPLN,
$\angle RLQ = \angle PLN$ (Vertically opposite angles)
$\angle LRQ = \angle LPN$ (Alternate angles)



$\Delta RLQ \sim \Delta PLN$ (AA Similarity)

$$\therefore \frac{RL}{LP} = \frac{RQ}{PN}$$

$$\frac{2}{3} = \frac{10}{PN}$$

PN = 15 cm

In ΔRLM and ΔPLQ

$\angle RLM = \angle PLQ$ (Vertically opposite angles)

$\angle LRM = \angle LPQ$ (Alternate angles)

$\Delta RLM \sim \Delta PLQ$ (AA Similarity)

$$\therefore \frac{RM}{PQ} = \frac{RL}{LP}$$

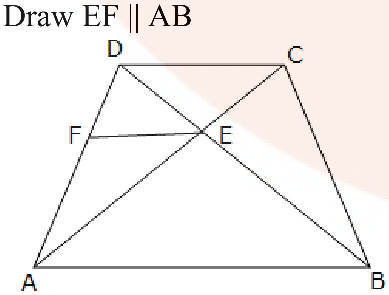
$$\frac{RM}{16} = \frac{2}{3}$$

$$RM = \frac{32}{3} = 10\frac{2}{3} \text{ cm}$$

Solution 15:

Given, AE : EC = BE : ED

Draw EF || AB



In ΔABD, EF || AB

Using Basic Proportionality theorem,

$$\frac{DF}{FA}=\frac{DE}{EB}$$

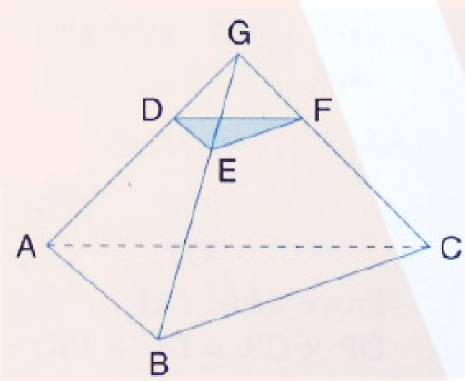
But, given $\frac{DE}{EB}=\frac{CE}{EA}$

$\therefore \frac{DF}{FA}=\frac{CE}{EA}$

Thus, in ΔDCA, E and F are points on CA and DA respectively such that $\frac{DF}{FA}=\frac{CE}{EA}$

Thus, by converse of Basic proportionality theorem, FE || DC.
But, FE || AB.
Hence, AB || DC.
Thus, ABCD is a trapezium.

Solution 16:



(i) In ΔAGB, DE || AB , by Basic proportionality theorem,

$$\frac{GD}{DA}=\frac{GE}{EB} \quad \ldots\ldots\ldots\ldots(1)$$

In ΔGBC, EF || BC, by Basic proportionality theorem,

$$\frac{GE}{EB}=\frac{GF}{FC} \ldots\ldots\ldots\ldots\ldots(2)$$

From (1) and (2), we get,

$$\frac{GD}{DA}=\frac{GF}{FC}$$

$$\frac{AD}{DG}=\frac{CF}{FG}$$

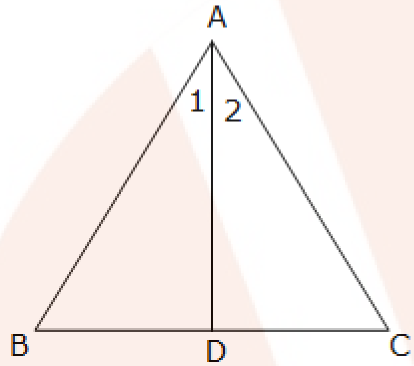
(ii)
From (i), we have:

$\frac{AD}{DG} = \frac{CF}{FG}$

$\angle DGF = \angle AGC$ (Common)

$\therefore \Delta DFG \sim \Delta ACG$ (SAS similarity)

Solution 17:



Given $AD^2 = BD \times DC$

$\frac{AD}{DC} = \frac{BD}{AD}$

$\angle ADB = \angle ADC = 90^\circ$

$\therefore \Delta DBA \sim \Delta DAC$ (SAS similarity)

So, these two triangles will be equiangular.

$\therefore \angle 1 = \angle C$ and $\angle 2 = \angle B$

$\angle 1 + \angle 2 = \angle B + \angle C$

$\angle A = \angle B + \angle C$

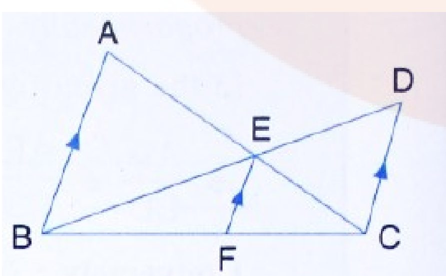
By angle sum property,

$\angle A + \angle B + \angle C = 180°$

$2\angle A = 180°$

$\angle A = \angle BAC = 90°$

Solution 18:



(i) The three pair of similar triangles are:
ΔBEF and ΔBDC
ΔCEF and ΔCAB
ΔABE and ΔCDE
(ii) Since, ΔABE and ΔCDE are similar,

$$\frac{AB}{CD}=\frac{AE}{CE}$$

$$\frac{67.5}{40.5}=\frac{52.5}{CE}$$

CE = 31.5 cm
Since, ΔCEF and ΔCAB are similar,

$$\frac{CE}{CA}=\frac{EF}{AB}$$

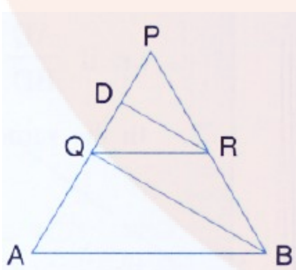
$$\frac{31.5}{52.5+31.5}=\frac{EF}{67.5}$$

$$\frac{31.5}{84}=\frac{EF}{67.5}$$

$$EF=\frac{2126.25}{84}$$

$$EF=\frac{405}{16}=25\frac{5}{16}\text{cm}$$

Solution 19:



Given, QR is parallel to AB. Using Basic proportionality theorem,

$$\Rightarrow \frac{PQ}{PA}=\frac{PR}{PB} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

Also, DR is parallel to QB. Using Basic proportionality theorem,

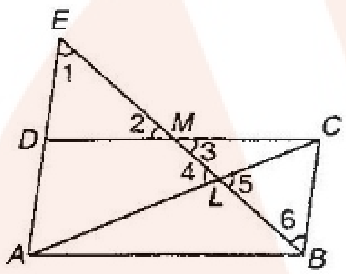
$\Rightarrow \frac{PD}{PQ} = \frac{PR}{PB}$ ……………..(2)

From (1) and (2), we get,

$\frac{PQ}{PA} = \frac{PD}{PQ}$

$PQ^2 = PD \times PA$

Solution 20:



$\angle 1 = \angle 6$ (Alternate interior angles)

$\angle 2 = \angle 3$ (Vertically opposite angles)

DM = MC (M is the mid-point of CD)

$\therefore \Delta DEM \cong \Delta CBM$ (AAS congruence criterion)

So, DE = BC (Corresponding parts of congruent triangles)

Also, AD = BC (Opposite sides of a parallelogram)

$\Rightarrow$ AE = AD + DE = 2BC

Now, $\angle 1 = \angle 6$ and $\angle 4 = \angle 5$

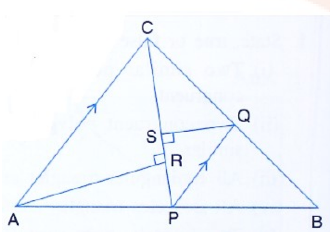
$\therefore \Delta ELA \sim \Delta BC$ (AA similarity)

$\Rightarrow \frac{EL}{BL} = \frac{EA}{BC}$

$\Rightarrow \frac{EL}{BL} = \frac{2BC}{BC} = 2$

$\Rightarrow$ E = 2BL

Solution 21:



(i) Given, AP : PB = 4 : 3.
Since, PQ || AC. Using Basic Proportionality theorem,

$$\frac{AP}{PB}=\frac{CQ}{QB}$$

$$\Rightarrow \frac{CQ}{QB}=\frac{4}{3}$$

$$\Rightarrow \frac{BQ}{BC}=\frac{3}{7} \quad \ldots\ldots\ldots\ldots(1)$$

Now, ∠PQB = ∠ACB (Corresponding angles)
∠QPB = ∠CAB (Corresponding angles)
∴ΔPBQ ~ ΔABC (AA similarity)

$$\Rightarrow \frac{PQ}{AC}=\frac{BQ}{BC}$$

$$\Rightarrow \frac{PQ}{AC}=\frac{3}{7} \quad \text{[using (1)]}$$

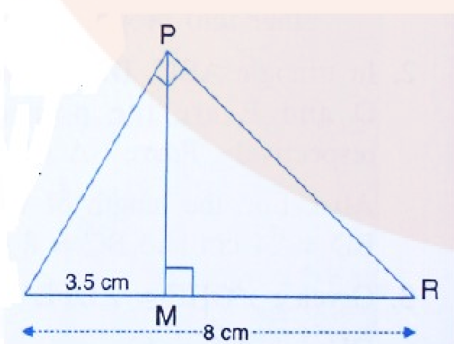
(ii) ∠ARC = ∠QSP = 90°
∠ACR = ∠SPQ (Alternate angles)
∴ΔARC ~ ΔQSP (AA similarity)

$$\Rightarrow \frac{AR}{QS}=\frac{AC}{PQ}$$

$$\Rightarrow \frac{AR}{QS}=\frac{7}{3}$$

$$\Rightarrow AR=\frac{7\times 6}{3}=14\text{cm}$$

Solution 22:



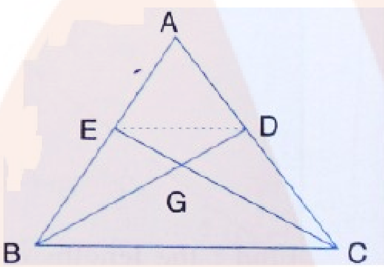
We have

$\angle QPR = \angle PMR = 90°$
$\angle PRQ = \angle PRM$ (common)
$\Delta PQR \sim \Delta MPR$ (AA similarity)

$$\therefore \frac{QR}{PR} = \frac{PR}{MR}$$

$PR^2 = 8 \times 4.5 = 36$
$PR = 6$ cm

Solution 23:



(i) Since, BD and CE are medians.
$AD = DC$
$AE = BE$
Hence, by converse of Basic Proportionality theorem,
$ED \parallel BC$
In ΔEGD and ΔCGB,
$\angle DEG = \angle GCB$ (alternate angles)
$\angle EGD = \angle BGC$ (Vertically opposite angles)
$\Delta EGD \sim \Delta CGB$ (AA similarity)
(ii) since, $\Delta EGD \sim \Delta CGB$

$$\frac{GD}{GB} = \frac{ED}{BC} \quad \ldots\ldots\ldots (1)$$

In ΔAED and ΔABC,
$\angle AED = \angle ABC$ (Corresponding angles)
$\angle EAD = \angle BAC$ (Common)
$\Delta EAD \sim \Delta BAC$ (AA similarity)

$$\therefore \frac{ED}{BC} = \frac{AE}{AB} = \frac{1}{2} \quad \text{(since, E is the mid – point of AB)}$$

$$\Rightarrow \frac{ED}{BC} = \frac{1}{2}$$

From (1),

$$\frac{GD}{GB} = \frac{1}{2}$$
$GB = 2GD$

EXERCISE. 15(B)

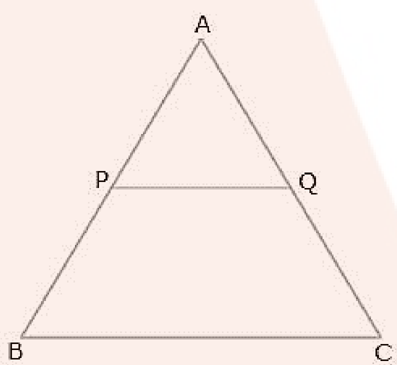
Solution 1:

We know that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

(i) Required ratio $= \frac{2^2}{5^2} = \frac{4}{25}$

(ii) Required ratio $= \sqrt{\frac{98}{128}} = \sqrt{\frac{49}{64}} = \frac{7}{8}$

Solution 2:



(i) $AP = \frac{1}{3} PB \Rightarrow \frac{AP}{PB} = \frac{1}{3}$

In ΔAPQ and ΔABC,
As PQ || BC, corresponding angles are equal
$\angle APQ = \angle ABC$
$\angle AQP = \angle ACB$
$\Delta APQ \sim \Delta ABC$
$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta APQ} = \frac{AB^2}{AP^2}$$
$$= \frac{4^2}{1^2} = 16 : 1$$

$\left(\frac{AP}{PB}=\frac{1}{3}\Rightarrow\frac{AB}{AP}=\frac{4}{1}\right)$

$$\frac{\text{Area of }\Delta APQ}{\text{Area of trapezium PBCQ}}$$

$$=\frac{\text{Area of }\Delta APQ}{\text{Area of }\Delta ABC\text{ -Area of }\Delta APQ}$$

$$=\frac{1}{16-1}=1:5$$

Solution 3:

Let $\Delta ABC \sim \Delta DEF$

Then, $\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}=\frac{AB+BC+AC}{DE+EF+DF}$

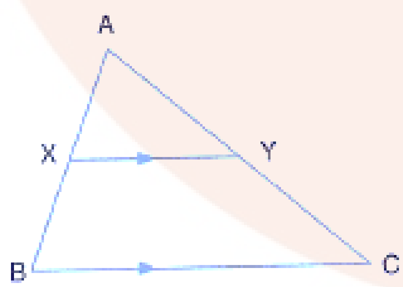
$$=\frac{\text{Perimeter of }\Delta ABC}{\text{Perimeter of }\Delta DEF}$$

$$\Rightarrow\frac{\text{Perimeter of }\Delta ABC}{\text{Perimeter of }\Delta DEF}=\frac{AB}{DE}$$

$$\Rightarrow\frac{30}{24}=\frac{12}{DE}$$

$\Rightarrow DE = 9.6$ cm

Solution 4:



Given, $\frac{AX}{XB}=\frac{3}{5}\Rightarrow\frac{AX}{AB}=\frac{3}{8}$ ……………..(1)

(i)

In ΔAXY and ΔABC,

As XY ∥ BC, Corresponding angles are equal

$\angle AXY = \angle ABC$

$\angle AYX = \angle ACB$

$\Delta AXY \sim \Delta ABC$

$$\Rightarrow \frac{AX}{AB} = \frac{XY}{BC}$$

$$\Rightarrow \frac{3}{8} = \frac{18}{BC}$$

$\Rightarrow BC = 48$ cm

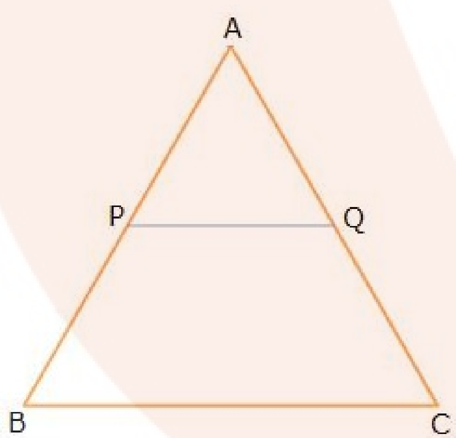
(ii)

$$\frac{\text{Area of } \Delta AXY}{\text{Area of } \Delta ABC} = \frac{AX^2}{AB^2} = \frac{9}{64}$$

$$\frac{\text{Area of } \Delta ABC \text{ -Area of } \Delta AXY}{\text{Area of } \Delta ABC} = \frac{64-9}{64} = \frac{55}{64}$$

$$\frac{\text{Area of trapezium XBCY}}{\text{Area of } \Delta ABC} = \frac{55}{64}$$

Solution 5:



From the given information, we have:

$$\text{ar}(\Delta APQ) = \frac{1}{2}\text{ar}(\Delta ABC)$$

$$\Rightarrow \frac{\text{ar}(\Delta APQ)}{\text{ar}(\Delta ABC)} = \frac{1}{2}$$

$$\Rightarrow \frac{AP^2}{AB^2} = \frac{1}{2}$$

$$\Rightarrow \frac{AP}{AB} = \frac{1}{\sqrt{2}}$$

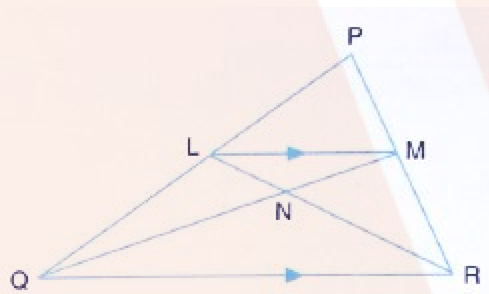
$$\Rightarrow \frac{AB - BP}{AB} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow 1 - \frac{BP}{AB} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{BP}{AB} = 1 - \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{BP}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}} = \frac{2 - \sqrt{2}}{2}$$

Solution 6:



(i)

In ΔPLM and ΔPQR,

As LM || QR, Corresponding angles are equal

$\angle PLM = \angle PQR$

$\angle PML = \angle PRQ$

$\Delta PLM \sim \Delta PQR$

$$\Rightarrow \frac{3}{7} = \frac{LM}{QR}\left(\because \frac{PM}{MR} = \frac{3}{4} \Rightarrow \frac{PM}{PR} = \frac{3}{7}\right)$$

Also, by using basic proportionality theorem, we have:

$$\frac{PL}{LQ} = \frac{PM}{MR} = \frac{3}{4}$$

$$\Rightarrow \frac{LQ}{PL} = \frac{4}{3}$$

$$\Rightarrow 1+\frac{LQ}{PL}=1+\frac{4}{3}$$

$$\Rightarrow \frac{PL+LQ}{PL}=\frac{3+4}{3}$$

$$\Rightarrow \frac{PQ}{PL}=\frac{7}{3}$$

$$\Rightarrow \frac{PL}{PQ}=\frac{3}{7}$$

(ii) Since ΔLMN and ΔMNR have common vertex at M and their bases LN and NR are along the same straight line

$$\therefore \frac{\text{Area of } \Delta LMN}{\text{Area of } \Delta MNR}=\frac{LN}{NR}$$

Now, in ΔLNM and ΔRNQ

$\angle NLM = \angle NRQ$ (Alternate angles)

$\angle LMN = \angle NQR$ (Alternate angles)

$\Delta LMN \sim \Delta RNQ$ (AA Similarity)

$$\therefore \frac{MN}{QN}=\frac{LN}{NR}=\frac{LM}{QR}=\frac{3}{7}$$

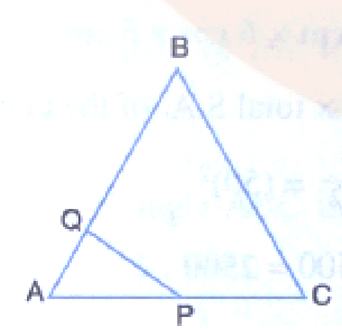
$$\therefore \frac{\text{Area of } \Delta LMN}{\text{Area of } \Delta MNR}=\frac{LN}{NR}=\frac{3}{7}$$

(iii) Since ΔLQM and ΔLQN have common vertex at L and their bases QM and QN are along the same straight line

$$\frac{\text{Area of } \Delta LQM}{\text{Area of } \Delta MNR}=\frac{QM}{QN}=\frac{10}{7}$$

$$\left(\because \frac{MN}{QN}=\frac{3}{7} \Rightarrow \frac{QM}{QN}=\frac{10}{7}\right)$$

Solution 7:



(i)
Given, $\Delta AQP \sim \Delta ACB$

$$\Rightarrow \frac{AQ}{AC} = \frac{AP}{AB}$$

$$\Rightarrow \frac{3}{4 + AP} = \frac{AP}{3 + 12}$$

$\Rightarrow AP^2 + 4AP - 45 = 0$
$\Rightarrow (AP + 9)(AP - 5) = 0$
$\Rightarrow AP = 5$ units (as length cannot be negative)

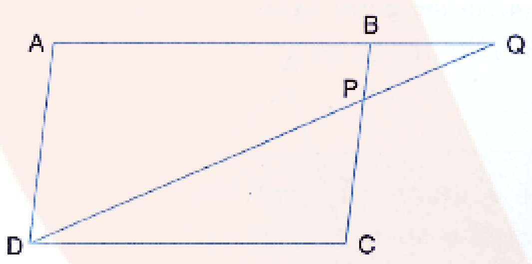
(ii)
Since, $\Delta AQP \sim \Delta ACB$

$$\therefore \frac{ar(\Delta APQ)}{ar(\Delta ACB)} = \frac{PQ^2}{BC^2}$$

$$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{BC^2} \quad (PQ = AP)$$

$$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \left(\frac{5}{15}\right)^2 = \frac{1}{9}$$

Solution 8:



(i) In ΔBPQ and ΔCPD
$\angle BPQ = \angle CPD$ (Vertically opposite angles)
$\angle BQP = \angle PDC$ (Alternate angles)
$\Delta BPQ \sim \Delta CPD$ (AA similarity)

$$\therefore \frac{BP}{PC} = \frac{PQ}{PD} = \frac{BQ}{CD} = \frac{1}{2} \quad \left(\because \frac{BP}{PC} = \frac{1}{2}\right)$$

$$\text{Also, } \frac{ar(\Delta BPQ)}{ar(\Delta CPD)} = \left(\frac{BP}{PC}\right)^2$$

$\Rightarrow \dfrac{10}{ar(\Delta CPD)} = \dfrac{1}{4}$ [ar(ΔBPQ) = $\dfrac{1}{2}$ × ar(ΔCPQ), ar(CPQ)=20]

$\Longrightarrow$ ar(ΔCPD) = 40 cm^2

(ii) In ΔBAP and ΔAQD

As BP ∥ AD, corresponding angles are equal

∠QBP = ∠QAD

∠BQP = ∠AQD (Common)

ΔBQP ~ ΔAQD (AA similarity)

$\therefore \dfrac{AQ}{BQ} = \dfrac{QD}{QP} = \dfrac{AD}{BP} = 3 \left(\because \dfrac{BP}{PC} = \dfrac{PQ}{PD} = \dfrac{1}{2} \Rightarrow \dfrac{PQ}{QD} = \dfrac{1}{3}\right)$

Also, $\dfrac{ar(\Delta AQD)}{ar(\Delta BQP)} = \left(\dfrac{AQ}{BQ}\right)^2$

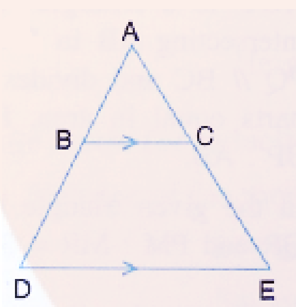
$\Rightarrow \dfrac{ar(\Delta AQD)}{10} = 9$

$\Longrightarrow$ ar(ΔAQD) = 90 cm^2

∴ ar(ADPB) = ar(ΔAQD) – ar(ΔBQP) = 90 cm^2 – 10 cm^2 = 80 cm^2

ar(ABCD) = ar(ΔCDP) + ar(ADPB) = 40 cm^2 + 80 cm^2 = 120 cm^2

Solution 9:



In ΔABC and Δ ADE,

As BC ∥ DE, corresponding angles are equal

∠ABC = ∠ADE

∠ACB = ∠AED

ΔABC ~ ΔADE

$\therefore \dfrac{ar(\Delta ABC)}{ar(\Delta ADE)} = \dfrac{BC^2}{DE^2}$

$\dfrac{25}{49} = \dfrac{BC^2}{14^2}$ (ar (ΔADE) = ar(ΔABC) + ar(trapezium BCED))

$BC^2 = 100$

$BC = 10$ cm

In trapezium BCED,

Area = $\frac{1}{2}$(Sum of parallel sides) × h

Given : Area of trapezium BCED = 24 cm^2, BC = 10 cm, DE = 14cm

$$\therefore 24 = \frac{1}{2}(10+14)\times h$$

$$\Rightarrow h = \frac{48}{(10+14)}$$

$$\Rightarrow h = \frac{48}{24}$$

$$\Rightarrow h = 2$$

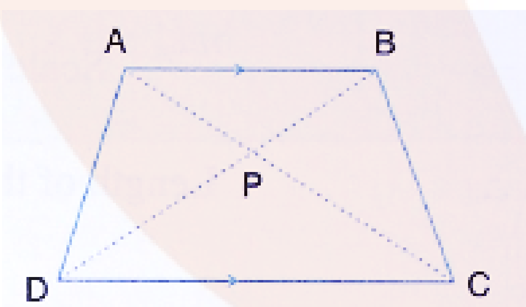
Area of ΔBCD = $\frac{1}{2}$ × base × height

$$= \frac{1}{2}\times BC\times h$$

$$= \frac{1}{2}\times 10\times 2$$

∴ Area of ΔBCD = 10 cm^2

Solution 10:



(i) Since ΔAPB and ΔCPB have common vertex at B and their bases AP and PC are along the same straight line

$$\therefore \frac{ar(\Delta APB)}{ar(\Delta CPB)} = \frac{AP}{PC} = \frac{3}{5}$$

(ii) Since ΔDPC and ΔBPA are similar

$$\therefore \frac{ar(\Delta DPC)}{ar(\Delta BPA)} = \left(\frac{PC}{AP}\right)^2 = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

(iii) Since ΔADP and ΔAPB have common vertex at A and their bases DP and PB are along the same straight line

$$\therefore \frac{ar(\Delta ADP)}{ar(\Delta APB)} = \frac{DP}{PB} = \frac{5}{3}$$

$$\left(\Delta APB \sim \Delta CPD \Rightarrow \frac{AP}{PC} = \frac{BP}{PD} = \frac{3}{5}\right)$$

(iv) Since ΔAPB and ΔADB have common vertex at A and their bases BP and BD are along the same straight line.

$$\therefore \frac{ar(\Delta APB)}{ar(\Delta ADB)} = \frac{PB}{BD} = \frac{3}{8}$$

$$\left(\Delta APB \sim \Delta CPD \Rightarrow \frac{AP}{PC} = \frac{BP}{PD} = \frac{3}{5} \Rightarrow \frac{BP}{BD} = \frac{3}{8}\right)$$

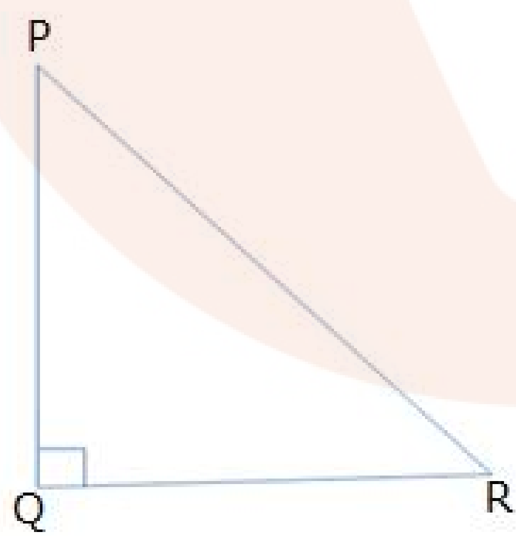
Solution 11:

Scale:-1 : 250000

∴ 1 cm represents 250000cm

$$= \frac{250000}{1000 \times 100} = 2.5 \text{ km}$$

∴ 1 cm represents 2.5 km



(i)
Actual length of PQ = 3 × 2.5 = 7.5 km
Actual length of QR = 4 × 2.5 = 10 km
Actual length of PR = $\sqrt{(7.5)^2 + (10)^2}\text{ km} = 12.5\text{km}$
(ii)
Area of Î”PQR = $\frac{1}{2} \times PQ \times QR = \frac{1}{2}(3)(4)\text{cm}^2 = 6\text{cm}^2$
1cm represents 2.5 km
1 cm^2 represents 2.5 × 2.5 km^2
The area of plot = 2.5 × 2.5 × 6 km^2 = 37.5 km^2

Solution 12:

Scale factor = k = $\frac{1}{200}$
(i) Length of model = k × actual length of the ship
⟹ Actual length of the ship = 4 × 200 = 800 m
(ii) Area of the deck of the model = k^2 × area of the deck of the ship

$$= \left(\frac{1}{200}\right)^2 \times 160000\text{ m}^2 = 4\text{ m}^2$$

(iii) Volume of the model = k^3 × volume of the ship
Volume of the ship

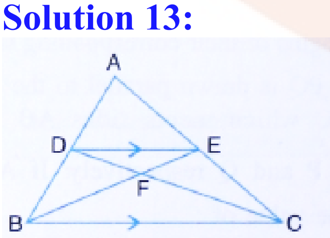
$$= \frac{1}{k^3} \times 200\text{ liters}$$

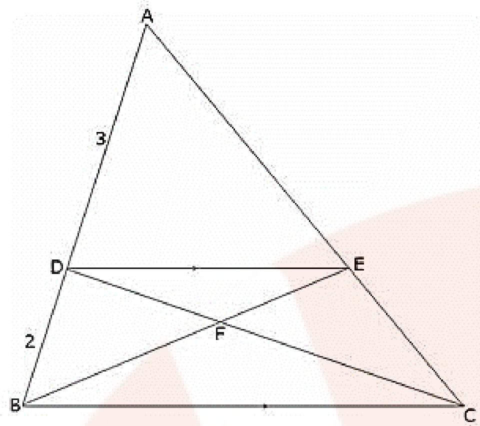
$$= (200)^3 \times 200\text{ liters}$$

$$= 1600000000\text{ liters}$$

$$= 1600000\text{ m}^3$$

Solution 13:





(i) Given, DE || BC and $\frac{AD}{DB} = \frac{3}{2}$

In ΔADE and ΔABC,

$\angle A = \angle A$(Corresponding Angles)

$\angle ADE = \angle ABC$(Corresponding Angles)

$\therefore \Delta ADE \sim \Delta ABC$ (By AA- similarity)

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Now $\frac{AD}{AB} = \frac{AD}{AD + DB} = \frac{3}{3+2} = \frac{3}{5}$

Using (1), we get $\frac{AD}{AE} = \frac{3}{5} = \frac{DE}{BC} \quad \ldots\ldots\ldots\ldots\ldots (2)$

(ii) ΔIn DEF and ΔCBF,

$\angle FDE = \angle$ FCB(Alternate Angle)

$\angle DFE = \angle BFC$(Vertically Opposite Angle)

$\therefore \Delta$ DEF $\sim \Delta$CBF(By AA- similarity)

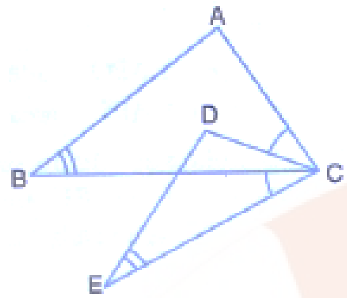
$\frac{EF}{FB} = \frac{DE}{BC} = \frac{3}{5}$ Using (2)

$\frac{EF}{FB} = \frac{3}{5}$

(iii) Since the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides, therefore.

$$\frac{\text{Area of } \Delta DFE}{\text{Area of } \Delta CBF} = \frac{EF^2}{FB^2} = \frac{3^2}{5^2} = \frac{9}{25}$$

Solution 14:



Given, $\angle ACD = \angle BCE$

$\angle ACD + \angle BCD = \angle BCE + \angle BCD$

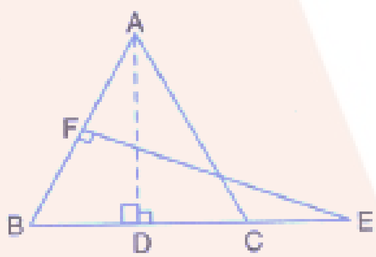
$\angle ACB = \angle DCE$

Also, given $\angle B = \angle E$

$\therefore \Delta ABC \sim \Delta DEC$

$$\frac{ar(\Delta ABC)}{ar(\Delta DEC)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{10.4}{7.8}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

Solution 15:



(i) $AB = AC$(Given)

$\therefore \angle FBE = \angle ACD$

$\angle BFE = \angle ADC$

$\Delta EFB \sim \Delta ADC$ (AA similarity)

$$\therefore \frac{ar(\Delta ADC)}{ar(\Delta EFB)} = \left(\frac{AC}{BE}\right)^2$$

$$= \left(\frac{AC}{BC + CE}\right)^2$$

$$= \left(\frac{13}{18}\right)^2 = \frac{169}{324} \quad \text{...............(1)}$$

(ii) Similarly, it can be proved that $\Delta ADB \sim \Delta EFB$

$$\therefore \frac{ar(\Delta ADB)}{ar(\Delta EFB)} = \left(\frac{AB}{BE}\right)^2$$

$$= \left(\frac{13}{18}\right)^2$$

$$= \frac{169}{324} \quad \ldots\ldots\ldots\ldots\ldots..(2)$$

From (1) and (2),

$$\frac{ar(\Delta ABC)}{ar(\Delta EFB)} = \frac{169+169}{324} = \frac{338}{324} = \frac{169}{162}$$

$\therefore$ ar(ΔEFB) : ar(ΔABC) = 162 : 169

Solution 16:

15cm represents = 30 m

1cm represents $\frac{30}{15} = 2\text{m}$

1 cm^2 represents 2m × 2m = 4 m^2

Surface area of the model = 150 cm^2

Actual surface area of aeroplane = 150 × 2 × 2 m^2= 600 m^2

50 m^2 is left out for windows

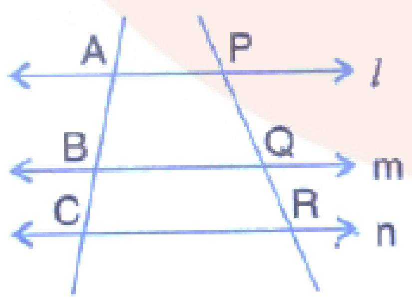
Area to be painted = 600 − 50 =50 m^2

Cost of painting per m^2 = Rs. 120

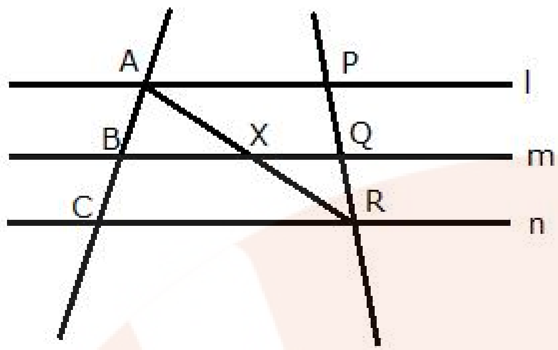
Cost of painting 550 m^2= 120 × 550 = Rs. 66000

EXERCISE. 15 (C)

Solution 1:



Join AR.



In ΔACR, BX || CR. By Basic Proportionality theorem,

$$\frac{AB}{BC} = \frac{AX}{XR} \quad \ldots\ldots\ldots\ldots..(1)$$

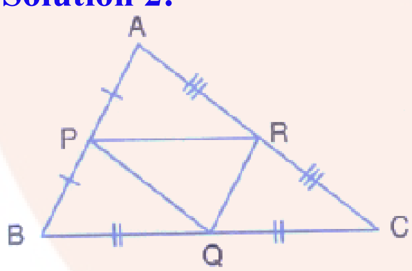
In ΔAPR, XQ || AP. By Basic Proportionality theorem,

$$\frac{PQ}{QR} = \frac{AX}{XR} \quad \ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

From (1) and (2), we get,

$$\frac{AB}{BC} = \frac{PQ}{QR}$$

Solution 2:



In ΔABC, PR || BC. By Basic proportionality theorem,

$$\frac{AP}{PB} = \frac{AR}{RC}$$

Also, in ΔPAR and ΔABC,

$\angle PAR = \angle BAC$ (common)

$\angle APR = \angle ABC$ (Corresponding angles)

$\Delta PAR \sim \Delta BAC$ (AA similarity)

$$\frac{PR}{BC} = \frac{AP}{AB}$$

$\frac{PR}{BC} = \frac{1}{2}$ (As P is the mid-point of AB)

$\frac{\text{PR}}{\text{BC}} = \frac{1}{2}\text{BC}$

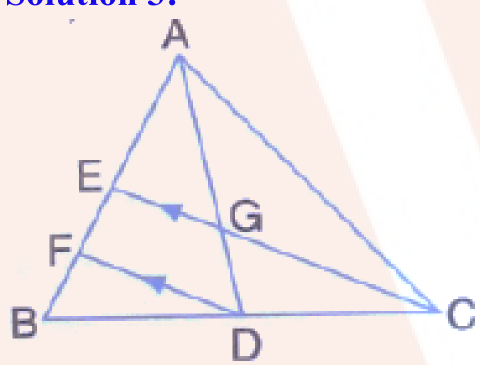
Similarity, $\text{PQ} = \frac{1}{2}\text{AC}$

$\text{RQ} = \frac{1}{2}\text{AB}$

Thus, $\frac{\text{PR}}{\text{BC}} = \frac{\text{PQ}}{\text{AC}} = \frac{\text{RQ}}{\text{AB}}$

$\Longrightarrow \Delta\text{QRP} \sim \Delta\text{ABC}$ (SSS similarity)

Solution 3:



(i)

In ΔBFD and ΔBEC,

$\angle\text{BFD} = \angle\text{BEC}$ (Corresponding angles)

$\angle\text{FBD} = \angle\text{EBC}$ (Common)

$\Delta\text{BFD} \sim \Delta\text{BEC}$ (AA Similarity)

$\therefore \frac{\text{BF}}{\text{BE}} = \frac{\text{BD}}{\text{BC}}$

$\frac{\text{BF}}{\text{BE}} = \frac{1}{2}$ (As D is the mid – point of BC)

BE = 2BF

BF = FE = 2BF

Hence, EF = FB

(ii) In ΔAFD, EG || FD. Using Basic Proportionality theorem,

$$\frac{AE}{EF} = \frac{AG}{GD} \quad \ldots\ldots\ldots (1)$$

Now, AE = EB (as E is the mid-point of AB)

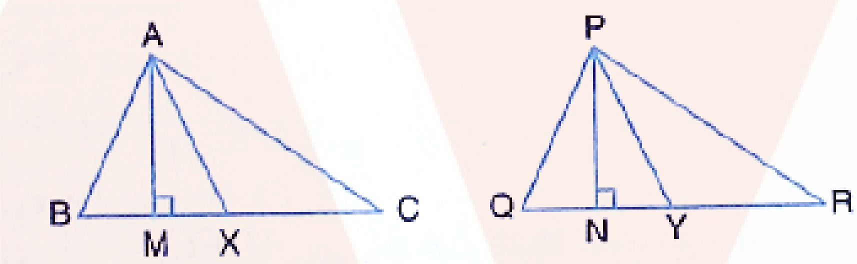
AE = 2EF (Since, EF = FB, by (i))

From (1),

$$\frac{AG}{GD} = \frac{2}{1}$$

Hence, AG : GD = 2 : 1

Solution 4:



Since $\Delta ABC \sim \Delta PQR$

So, their respective sides will be in proportion

Or, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$

Also, $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$

In ΔABM and ΔPQN,

$\angle ABM = \angle PQN$ (Since, ABC and PQR are similar)

$\angle AMB = \angle PNQ = 90°$

$\Delta ABM \sim \Delta PQN$ (AA similarity)

$$\therefore \frac{AM}{PN} = \frac{AB}{PQ} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Since, AX and PY are medians so they will divide their opposite sides.

Or, $BX = \frac{BC}{2}$ and $QY = \frac{QR}{2}$

Therefore, we have:

$$\frac{AB}{PQ} = \frac{BX}{QY}$$

$\angle B = \angle Q$

So, we had observed that two respective sides are in same proportion in both triangles and also angle included between them is respectively equal.

Hence, $\Delta ABX \sim \Delta PQY$ (by SAS similarity rule)

So, $\frac{AB}{PQ} = \frac{AX}{PY}$ ………………..(2)

From (1) and (2),

$\frac{AM}{PN} = \frac{AX}{PY}$

Solution 5:

Let us assume two similar triangles as $\Delta ABC \sim \Delta PQR$

Now $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

Since area (ΔABC) = area (ΔPQR)

Therefore, $AB = PQ$

$BC = QR$

$AC = PR$

So, respective sides of two similar triangles

Are also of same length

So, $\Delta ABC \cong \Delta PQR$ (by SSS rule)

Solution 6:

The ratio between the altitudes of two similar triangles is same as the ratio between their sides.

(i) The ratio between the medians of two similar triangles is same as the ratio between their sides.

∴ Required ratio = 3 : 5

(ii) The ratio between the perimeters of two similar triangles is same as the ratio between their sides.

∴ Required ratio = 3 : 5

(iii) The ratio between the areas of two similar triangles is same as the square of the ratio between their corresponding sides.

∴ Required ratio = (3)2 : (5)2 = 9 : 25

Solution 7:

The ratio between the areas of two similar triangles is same as the square of the ratio between their corresponding sides.

So, the ratio between the sides of the two triangles = 4 : 5

(i) The ratio between the perimeters of two similar triangles is same as the ratio between their sides.

∴ Required ratio = 4 : 5

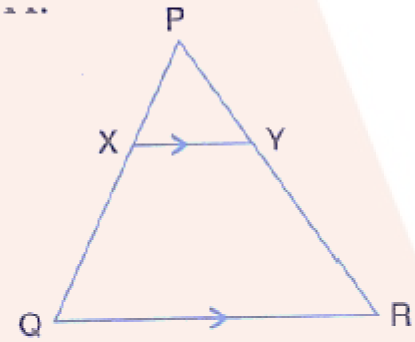
(ii) The ratio between the altitudes of two similar triangles is same as the ratio between their sides.

∴ Required ratio = 4 : 5

(iii) The ratio between the medians of two similar triangles is same as the ratio between their sides.

∴ Required ratio = 4 : 5

Solution 8:



In ΔPXY and ΔPQR, XY is parallel to QR, so corresponding angles are equal.

∠PXY = ∠PQR

∠PYX = ∠PRQ

Hence, ΔPXY ~ ΔPQR (By AA similarity criterion)

$$\frac{PX}{PQ} = \frac{XY}{QR}$$

$$\Rightarrow \frac{1}{4} = \frac{XY}{QR} \quad (PX : XQ = 1 : 3 \Rightarrow PX : PQ = 1 : 4)$$

$$\Rightarrow \frac{1}{4} = \frac{XY}{9}$$

⟹ XY = 2.25 cm

(i) We know that the ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

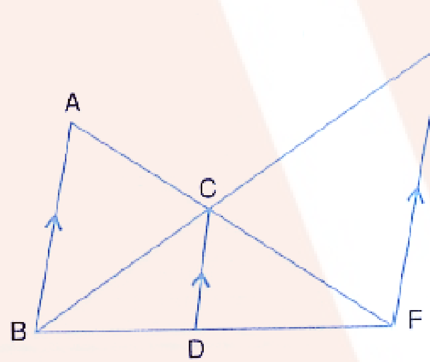
$$\frac{Ar(\Delta PXY)}{Ar(\Delta PQR)} = \left(\frac{PX}{PQ}\right)^2$$

$$\frac{x}{Ar(\Delta PQR)} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

$$Ar(\Delta PQR) = 16x \text{ cm}^2$$

(ii) Ar (trapezium XQRY) = Ar (ΔPQR) - Ar (ΔPXY)
= $(16x - x)$ cm^2
= $15x$ cm^2

Solution 9:



In ΔFDC and ΔFBA,
$\angle FDC = \angle FDA$ (Corresponding angles)
$\angle DFC = \angle BFA$ (Common)
$\Delta FDC \sim \Delta FBA$ (AA similarity)

$$\frac{CD}{AB} = \frac{FC}{FA}$$

$$\frac{Y}{6} = \frac{x}{x+4} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

In ΔFCE and ΔACB,
$\angle FCE = \angle ACB$ (vertically opposite angles)
$\angle CFE = \angle CAB$ (Alternate angles)
$\Delta FCE \sim \Delta ACB$ (AA similarity)

$$\frac{FC}{AC} = \frac{EF}{AB}$$

$$\frac{x}{4} = \frac{10}{6} \Rightarrow x = \frac{20}{3} = 6\frac{2}{3}\text{cm}$$

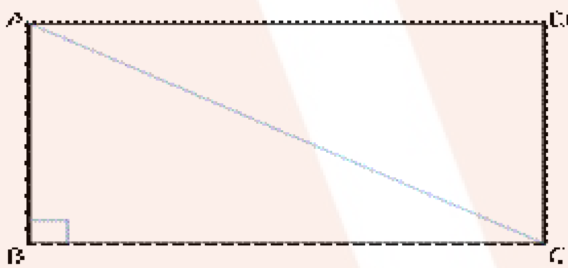
From (1):

$$y = \frac{6 \times \frac{20}{3}}{\frac{20}{3} + 4} = 3.75$$

Solution 10:

Scale :- 1 : 20000

1 cm represents 20000 cm = $\frac{20000}{1000 \times 100} = 0.2$ km



(i)

$AC^2 = AB^2 + BC^2$

$= 24^2 + 32^2$

$= 576 + 1024 = 1600$

AC = 40 cm

Actual length of diagonal = 40 × 0.2 km = 8 km

(ii)

1 cm represents 0.2 km

1 cm^2 represents 0.2 × 0.2 km^2

The area of the rectangle ABCD = AB × BC

= 24 × 32 = 768 cm^2

Actual area of the plot = 0.2 × 0.2 × 768 km^2 = 30.72 km^2

Solution 11:

The dimensions of the building are calculated as below.

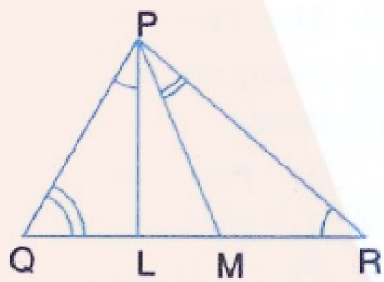
Length = 1 × 50 m = 50 m

Breadth = 0.60 × 50 m = 30 m

Height = 1.20 × 50 m = 60 m

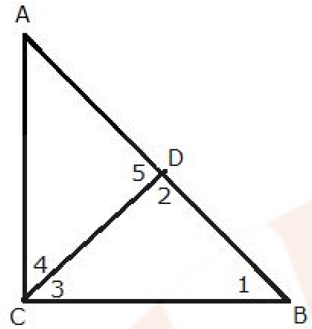
Thus, the actual dimensions of the building are 50 m × 30 m × 60 m.
(i)
Floor area of the room of the building =
$$50\times\left(\frac{50}{1}\right)^2=125000\text{ cm}^2=\frac{125000}{100\times100}=12.5\text{ m}^2$$
(ii)
Volume of the model of the building
$$=90\left(\frac{1}{50}\right)^3=90\times\left(\frac{1}{50}\right)\times\left(\frac{1}{50}\right)\times\left(\frac{1}{50}\right)=90\times\left(\frac{100\times100\times100}{50\times50\times50}\right)\text{cm}^3$$
$$=720\text{ cm}^3$$

Solution 12:



In ΔPQL and ΔRMP
$\angle LPQ=\angle QRP$ (Given)
$\angle RQP=\angle RPM$ (Given)
$\Delta PQL\sim\Delta RMP$ (AA similarity)
(ii)
As $\Delta PQL\sim\Delta RMP$ (proved above)
$$\frac{PQ}{RP}=\frac{QL}{PM}=\frac{PL}{RM}$$
$$\Rightarrow QL\times RM=PL\times PM$$
(iii)
$\angle LPQ=\angle QRP$ (Given)
$\angle Q=\angle Q$ (Common)
$\Delta PQL\sim\Delta RQP$ (AA similarity)
$$=\frac{PQ}{RQ}=\frac{QL}{QP}=\frac{PL}{PR}$$
$$\Rightarrow PQ^2=QR\times QL$$

Solution 13:



In ΔCDB,

$\angle 1 + \angle 2 + \angle 3 = 180^o$

$\angle 1 + \angle 3 = 90^o$ … (1) (Since, $\angle 2 = 90^0$)

$\angle 3 + \angle 4 = 90^o$ … (2) (Since, $\angle ACB = 90^0$)

From (1) and (2),

$\angle 1 + \angle 3 = \angle 3 + \angle 4$

$\angle 1 = \angle 4$

Also, $\angle ADC = \angle ACB = 90^o$

$\therefore \Delta ACD \sim \Delta ABC$ (AA similarity)

$$\therefore \frac{AC}{AB} = \frac{AD}{AC}$$

$AC^2 = AB \times AD$ ………….. (1)

Now $\angle BDC = \angle ACB = 90^0$

$\angle CBD = \angle ABC$ (common)

$\Delta BCD \sim \Delta BAC$ (AA similarity)

$$\therefore \frac{BC}{BA} = \frac{BD}{BC} \quad \text{…………. (2)}$$

$BC^2 = BA \times BD$

From (1)and (2), we get,

$$\frac{BC^2}{AC^2} = \frac{BA \times BD}{AB \times AD} = \frac{BD}{AD}$$

Solution 14:

Triangle ABC is enlarged to DEF. So, the two triangles will be similar.

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Longest side in $\Delta ABC = BC = 6$ cm

Corresponding longest side in ΔDEF = EF = 9 cm

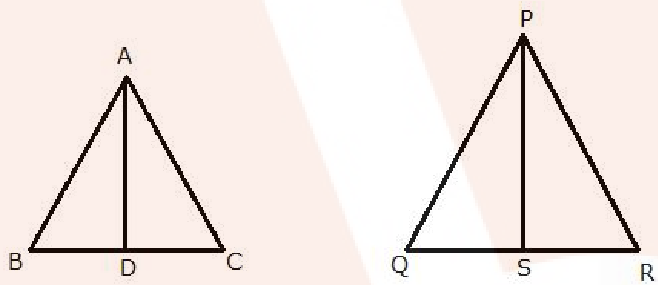
$$\text{Scale factor} = \frac{EF}{BC} = \frac{9}{6} = \frac{3}{2} = 1.5$$

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{2}{3}$$

$$DE = \frac{3}{2}AB = \frac{9}{2} = 4.5 \text{ cm}$$

$$DF = \frac{3}{2}AC = \frac{12}{2} = 6 \text{ cm}$$

Solution 15:



Let ABC and PQR be two isosceles triangles.

$$\text{Then, } \frac{AB}{AC} = \frac{1}{1} \text{ and } \frac{PQ}{PR} = \frac{1}{1}$$

Also, ∠A = ∠P (Given)

∴ ΔABC ~ ΔPQR (SAS similarity)

Let AD and PS be the altitude in the respective triangles.

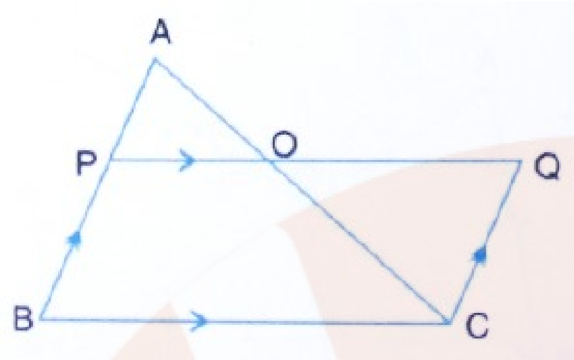
We know that the ratio of areas of two similar triangles is equal to the square of their corresponding altitudes.

$$\frac{Ar(\triangle}{Ar(\triangle} \qquad \left(\frac{AD}{PS}\right)^2$$

$$\frac{16}{25} = \left(\frac{AD}{PS}\right)^2$$

$$\frac{AD}{PS} = \frac{4}{5}$$

Solution 16:



In triangle ABC, PO || BC. Using Basic proportionality theorem,

$$\frac{AP}{PB}=\frac{AO}{OC}$$

$$\Rightarrow \frac{AO}{OC}=\frac{2}{3} \quad \ldots\ldots\ldots\ldots..(1)$$

(i) $\angle PAO = \angle BAC$ (common)

$\angle APO = \angle ABC$ (Corresponding angles)

$\Delta APO \sim \Delta ABC$ (AA similarity)

$$\therefore \frac{Ar(\Delta APO)}{Ar(\Delta ABC)}=\left(\frac{AO}{AC}\right)^2=\left(\frac{2}{2+3}\right)^2=\left(\frac{2}{5}\right)^2=\frac{4}{25}$$

(ii)

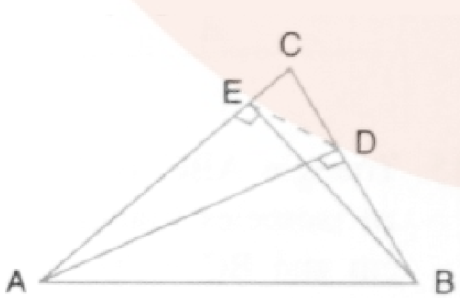
$\angle POA = \angle COQ$ (vertically opposite angles)

$\angle PAO = \angle QCO$ (alternate angles)

$\Delta AOP \sim \Delta COQ$ (AA similarity)

$$\therefore \frac{Ar(\Delta AOP)}{Ar(\Delta COQ)}=\left(\frac{AO}{CO}\right)^2=\left(\frac{2}{3}\right)^2=\frac{4}{9}$$

Solution 17:



$\angle ADC = \angle BEC = 90°$

$\angle ACD = \angle BCE$ (Common)

$\Delta ADC \sim \Delta BEC$ (AA similarity)

(ii) From part (i),

$$\frac{AC}{BC} = \frac{CD}{EC} \quad \dots\dots\dots (1)$$

$\Rightarrow CA \times CE = CB \times CD$

(iii) In ΔABC and ΔDEC,

From (1),

$$\frac{AC}{BC} = \frac{CD}{EC} \Rightarrow \frac{AC}{CD} = \frac{BC}{EC}$$

$\angle DCE = \angle BCA$ (Common)

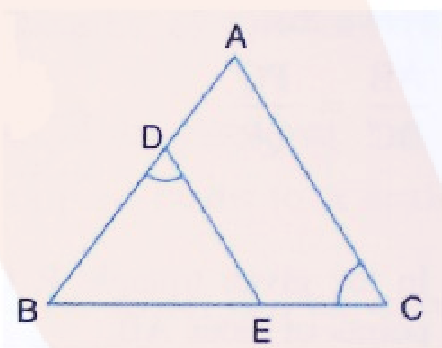
$\Delta ABC \sim \Delta DEC$ (SAS similarity)

(iv) From part (iii),

$$\frac{AC}{DC} = \frac{AB}{DE}$$

$\Rightarrow CD \times AB = CA \times DE$

Solution 18:



In ΔABC and ΔEBD,

$\angle ACB = \angle EDB$ (given)

$\angle ABC = \angle EBD$ (common)

$\Delta ABC \sim \Delta EBD$ (by AA – similarity)

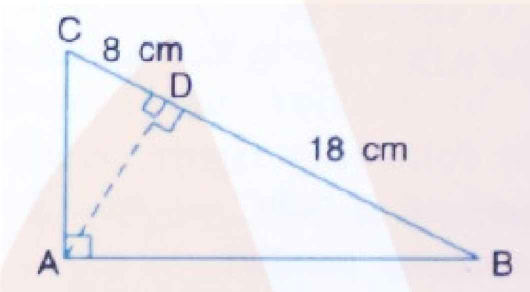
(i) we have, $\frac{AB}{BE} = \frac{BC}{BD} \Rightarrow AB = \frac{6 \times 10}{5} = 12$ cm

(ii) $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BED} = \left(\frac{AB}{BE}\right)^2$

$\Rightarrow$ Area of $\Delta ABC = \left(\frac{12}{6}\right)^2 \times 9\ cm^2$

$= 4 \times 9\ cm^2 = 36\ cm^2$

Solution 19:



(i) let $\angle CAD = x$

$\Rightarrow m\ \angle dab = 90^\circ - x$

$\Rightarrow m\ \angle DBA = 180° - (90° + 90° - x) = x$

$\Rightarrow \angle CDA = \angle DBA$ ……… (1)

In ΔADB and ΔCDA,

$\angle ADB = \angle CDA$ …… [each 90°]

$\angle ABD = \angle CAD$ …… [From (1)]

$\therefore \Delta ADB \sim \Delta CDA$ …….. [By A.A]

(ii) Since the corresponding sides of similar triangles are proportional, we have.

$$\frac{BD}{AD} = \frac{AD}{CD}$$

$$\Rightarrow \frac{18}{AD} = \frac{AD}{8}$$

$\Rightarrow AD^2 = 18 \times 8 = 144$

$\Rightarrow AD = 12$ cm

(iii) The ratio of the areas of two similar triangles is equal to the ratio of the square of their corresponding sides.

$$\Rightarrow \frac{Ar(\Delta ADB)}{Ar(\Delta CDA)} = \frac{AD^2}{CD^2} = \frac{12^2}{8^2} = \frac{144}{64} = \frac{9}{4} = 9:4$$