Chapter 8

Rotational Motion

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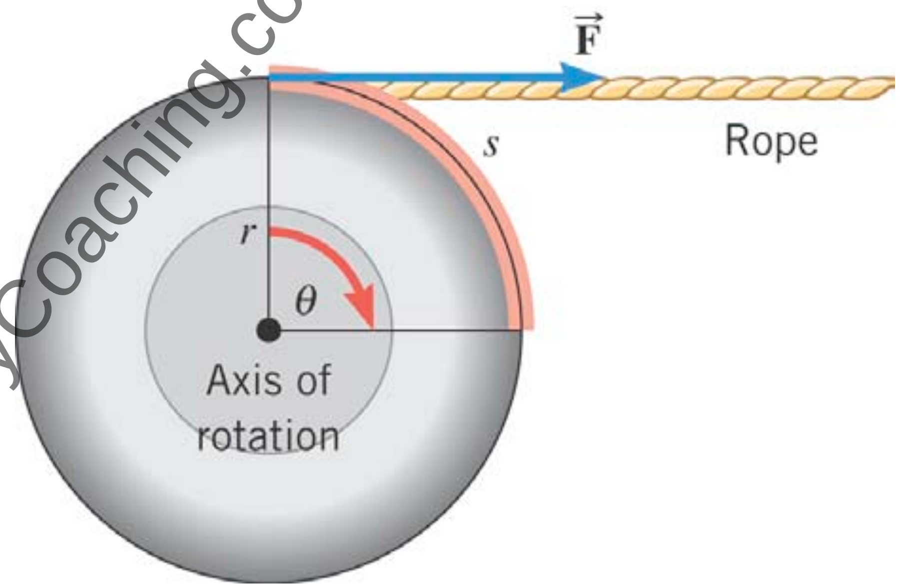
Rotational Work and Energy

$$s = r\theta$$

Consider the work done in rotating a wheel with a tangential force, F, by an angle θ.

$$W = Fs = Fr\theta$$



$$\tau = Fr$$

$$W = \tau\theta$$

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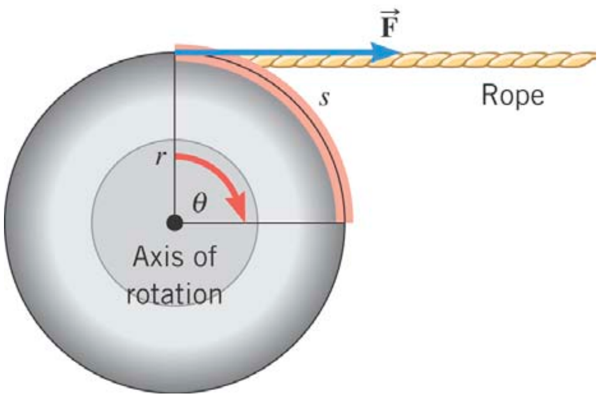
DEFINITION OF ROTATIONAL WORK

The rotational work done by a constant torque in turning an object through an angle is

$$W_R = \tau\theta$$



Requirement: The angle must be expressed in radians.

SI Unit of Rotational Work: joule (J)

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Rotational Work and Energy

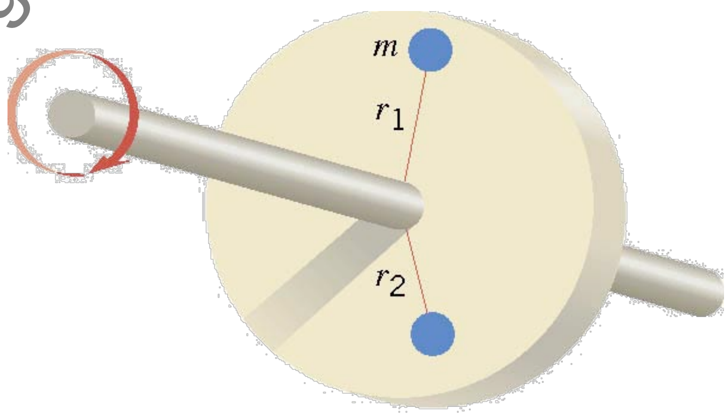
According to the Work-Energy theorem: $W = KE_f - KE_0$

So W_R should be able to produce rotational kinetic energy.

Calculate the kinetic energy of a mass m undergoing rotational motion at radius r and moving with tangential speed v_T

$$KE = \tfrac{1}{2} m v_T^2 = \tfrac{1}{2} m r^2 \omega^2$$

$$v_T = r\omega$$



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For a system of rotating masses, the total kinetic energy is the sum over the kinetic energies of the individual masses,

$$KE = \sum \left(\tfrac{1}{2} m r^2 \omega^2\right) = \tfrac{1}{2}\left(\sum m r^2\right)\omega^2 = \tfrac{1}{2} I \omega^2$$

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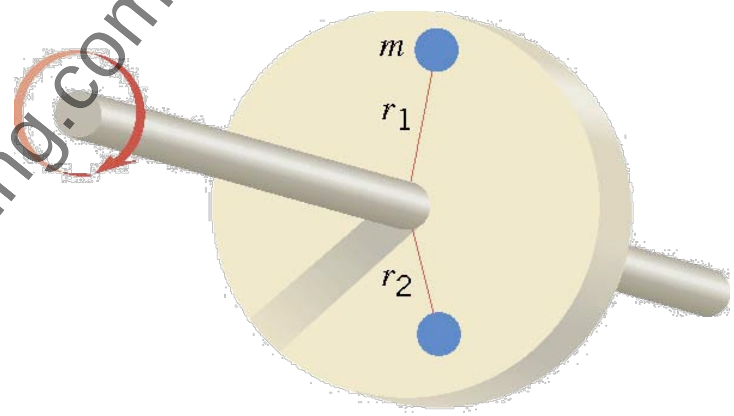
DEFINITION OF ROTATIONAL KINETIC ENERGY

The rotational kinetic energy of a rigid rotating object is

$$KE_R = \tfrac{1}{2} I\omega^2$$



Requirement: The angular speed must be expressed in rad/s.

SI Unit of Rotational Kinetic Energy: joule (J)

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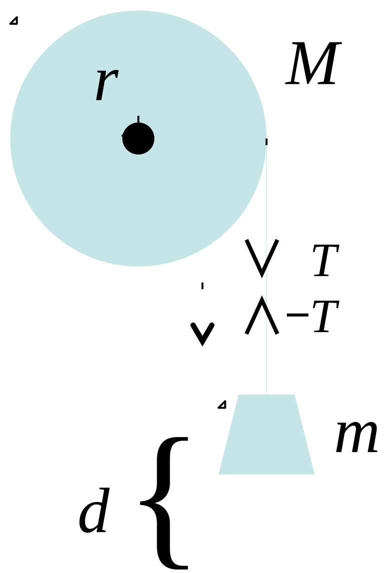
Thus, the **rotational version of the Work-Energy theorem** is:

$$W_R = KE_{Rf} - KE_{R0} \quad \text{where} \quad \begin{cases} W_R = \tau\theta \\ KE_R = \tfrac{1}{2} I\omega^2 \end{cases}$$

Example: **A hanging mass rotating a solid disk.** As seen in the figure, a 2.0 kg mass attached to a string is rotating a solid disk of mass 10.0 kg and radius 0.20 m pivoting around its center. If the system is initially at rest, what is the angular velocity of the disk after the mass falls 0.70 m?

$$m = 2.0 \text{ kg} \quad M = 10.0 \text{ kg} \quad r = 0.20 \text{ m} \quad d = 0.70 \text{ m}$$

$$\rightarrow \text{Find } \omega_f$$



$\Rightarrow$ Work done on the disk:

$$W_R = \Delta KE_R \Rightarrow \quad \tau\theta = Tr\theta = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_0^2$$

$$\text{Since: } r\theta = d,\ I_{disk} = \tfrac{1}{2}Mr^2,\ \omega_0 = 0 \Rightarrow \quad Td = \tfrac{1}{4}Mr^2\omega_f^2$$

$\Rightarrow$ Work done on the hanging mass:

$$W_{NC} = \Delta E \Rightarrow -Td = \left(\tfrac{1}{2}mv_f^2 + mgh_f\right) - \left(\tfrac{1}{2}mv_0^2 + mgh_0\right)$$

$$\text{Since: } v_0 = 0 \quad v_f = r\omega_f \quad h_f - h_0 = -d \Rightarrow \quad -Td = \tfrac{1}{2}mr^2\omega_f^2 - mgd$$

$$\text{Add disk eq. + hanging mass eq. } \Rightarrow 0 = \tfrac{1}{4}Mr^2\omega_f^2 + \tfrac{1}{2}mr^2\omega_f^2 - mgd$$

$$\therefore \omega_f = \frac{2}{r}\sqrt{\frac{mgd}{M+2m}} = \frac{2}{0.20}\sqrt{\frac{(2.0)(9.8)(0.70)}{10.0+2(2.0)}} = 9.9 \text{ rad/s}$$

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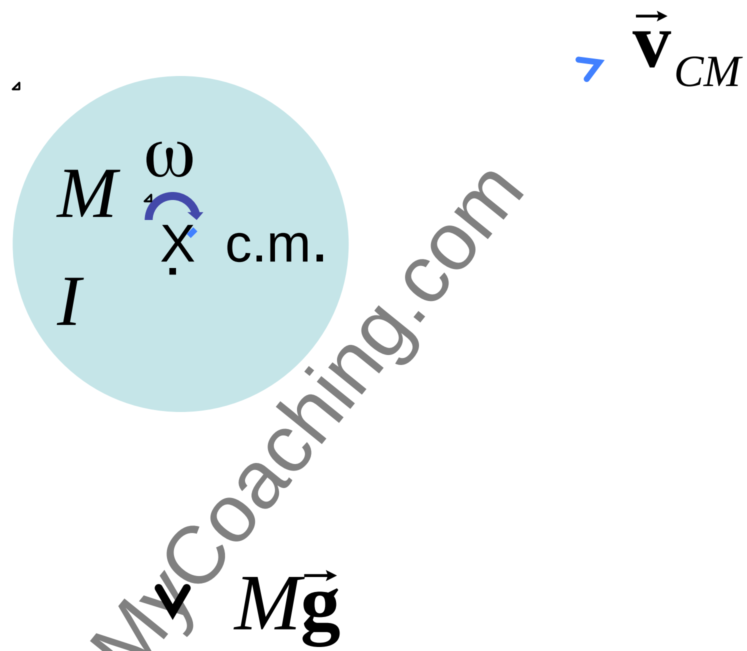
Total energy of a rotating and translating rigid body in a gravitational field



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$$\text{total energy} = E = E_{\substack{\text{rotation}\\\text{about CM}}} + E_{\substack{\text{translation}\\\text{of CM}}} = \tfrac{1}{2} I\omega^2 + \tfrac{1}{2} M v_{CM}^2 + Mgh_{CM}$$

$$\text{Since a gravitational field is a conservative force} \quad \Rightarrow \quad E_f = E_0$$

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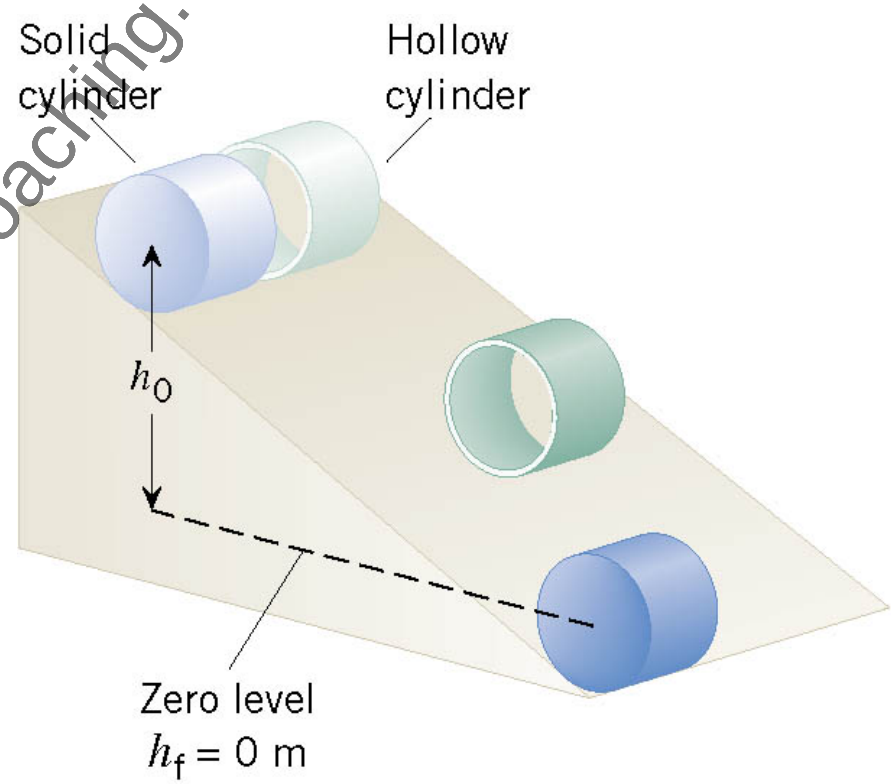
Example: **Rolling Cylinders**

A thin-walled hollow cylinder (mass = *m*, radius = *r*) and a solid cylinder (also, mass = *m*, radius = *r*) start from rest at the top of an incline.

Determine which cylinder has the greatest translational speed upon reaching the bottom.



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Rotational Work and Energy

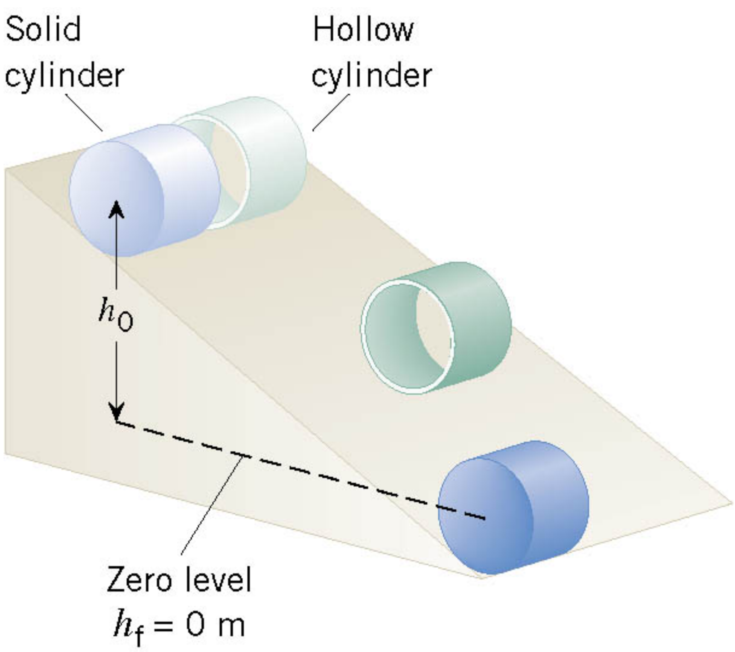
$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + mgh$$

ENERGY CONSERVATION

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgh_f = \tfrac{1}{2}mv_o^2 + \tfrac{1}{2}I\omega_o^2 + mgh_o$$

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 = mgh_o$$

$$\omega_f = v_f / r$$



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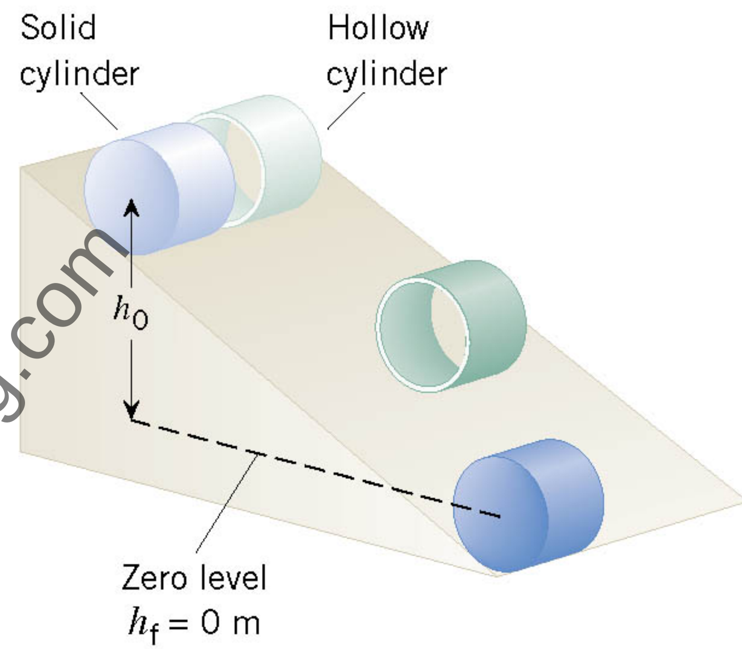
Rotational Work and Energy

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\,v_f^2/r^2 = mgh_o$$

$$v_f = \sqrt{\frac{2mgh_o}{m + I/r^2}}$$



The cylinder with the **smaller** moment of inertia will have a **greater** final translational speed.

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Since $I_{solid} = (1/2)mr^2$ and $I_{hollow} = mr^2$

Then, $I_{solid} < I_{hollow}$ ➔ $v_{f\,solid} > v_{f\,hollow}$

DEFINITION OF ANGULAR MOMENTUM

The angular momentum *L* of a body rotating about a fixed axis is the product of the body's moment of inertia and its angular velocity with respect to that axis:

$$L = I\omega$$

Requirement: The angular speed must be expressed in rad/s.

SI Unit of Angular Momentum: kg·m^2/s

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Consider the rotational version of Newton's 2nd Law:

$$\sum \bar{\tau}_{EXT} = I\bar{\alpha} = I\frac{\Delta\omega}{\Delta t} = \frac{\Delta\left(I\omega\right)}{\Delta t} = \frac{\Delta L}{\Delta t}$$

$$\therefore \left(\sum \bar{\tau}_{EXT}\right)\Delta t = \Delta L$$

$\Rightarrow$ "angular impulse-angular momentum theorm"

If $\left(\sum \bar{\tau}_{EXT}\right) = 0$

$$\Rightarrow \quad \Delta L = 0 \quad \Rightarrow \quad L_f = L_0$$

$\Rightarrow$ Conservation of angular momentum

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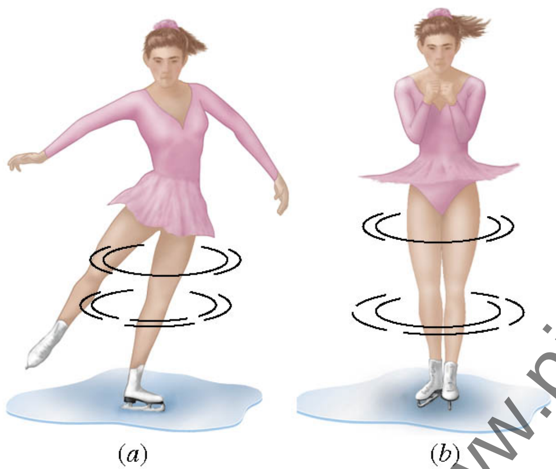
PRINCIPLE OF CONSERVATION OF ANGULAR MOMENTUM

The angular momentum of a system remains constant (is conserved) if the net external torque acting on the system is zero.



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$$L_f = L_0$$

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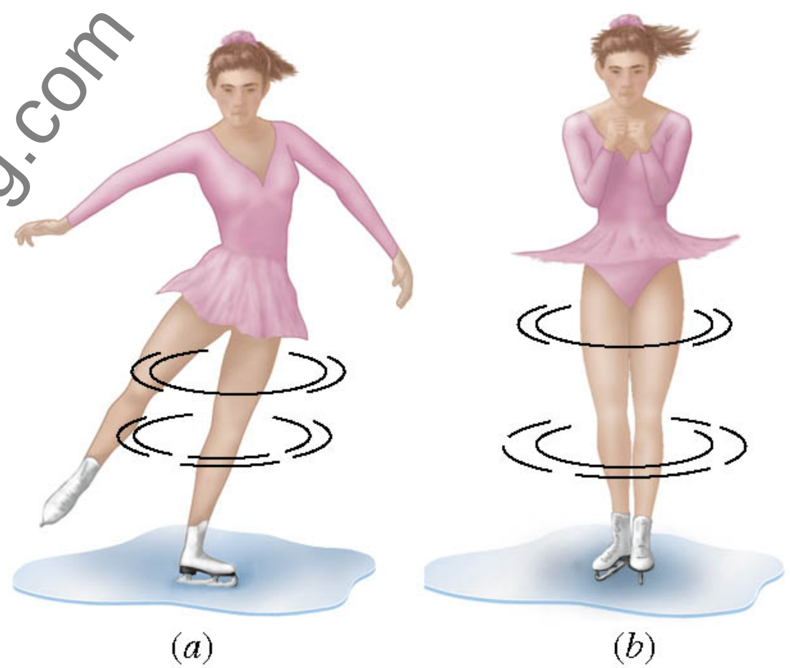
Conceptual Example: **A Spinning Skater**

An ice skater is spinning with both arms and a leg outstretched. She pulls her arms and leg inward and her spinning motion changes dramatically.

Use the principle of conservation of angular momentum to explain how and why her spinning motion changes.



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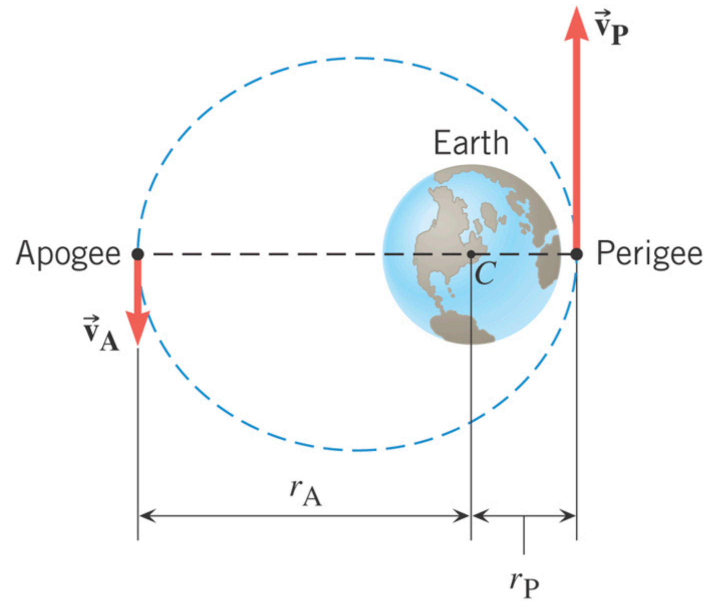
Example: A Satellite in an Elliptical Orbit

An artificial satellite is placed in an elliptical orbit about the earth. Its point of closest approach is 8.37×10^6 m from the center of the earth, and its point of greatest distance is 25.1×10^6 m from the center of the earth.

The speed of the satellite at the perigee is 8450 m/s. Find the speed at the apogee.



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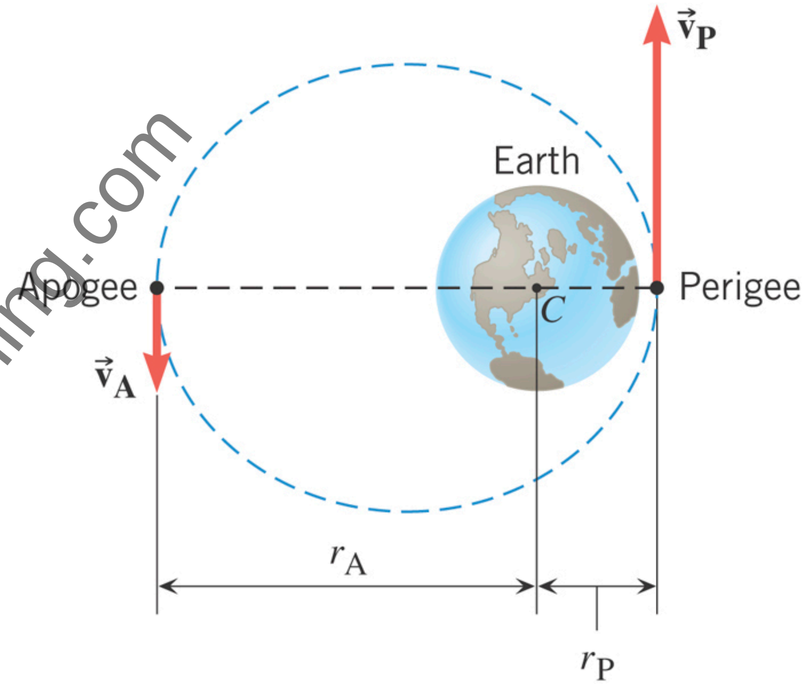
Angular Momentum

$$L = I\omega$$

Since no external torques are present in this case, we have angular momentum conservation



$$I_A\omega_A = I_P\omega_P$$

$$I = mr^2 \quad \omega = v/r$$

$$mr_A^2\frac{v_A}{r_A} = mr_P^2\frac{v_P}{r_P}$$

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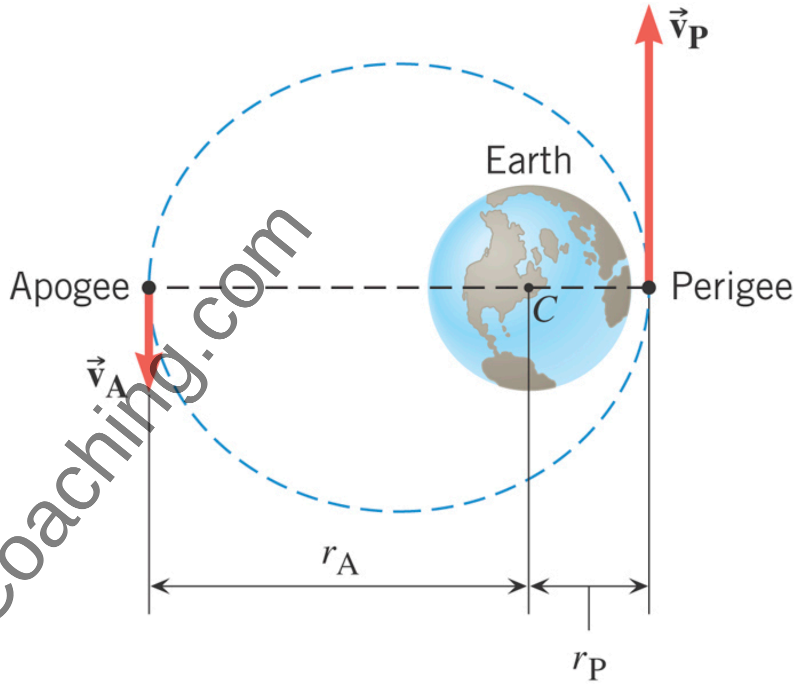
Angular Momentum

$$mr_A^2 \frac{v_A}{r_A} = mr_P^2 \frac{v_P}{r_P}$$



$$r_A v_A = r_P v_P$$

$$v_A = \frac{r_P v_P}{r_A} = \frac{(8.37\times10^6\ \text{m})(8450\,\text{m/s})}{25.1\times10^6\ \text{m}} = 2820\,\text{m/s}$$

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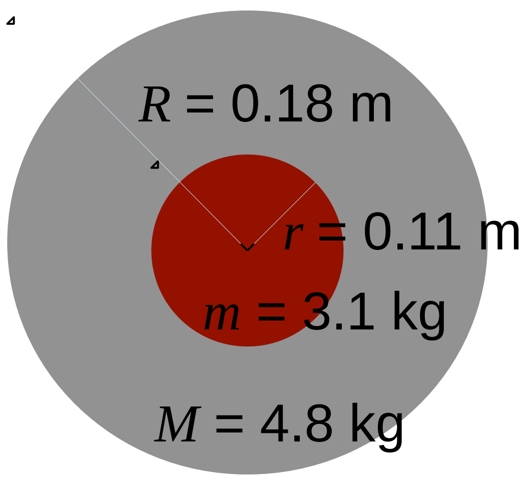
Example: A potter's wheel is rotating around a vertical axis through its center at a frequency of 2.0 rev/s. The wheel can be considered a uniform disk of mass 4.8 kg and diameter 0.36 m. The potter then throws a 3.1 kg chunk of clay, approximately shaped as a flat disk of radius 11 cm, onto the center of the rotating wheel. (a)What is the frequency of the wheel after the clay sticks to it? (b) What fraction of the original mechanical energy of the wheel is lost to friction after the collision with the clay?

ω_0 = 2.0 rev/s = 13 rad/s



a) $L_f = L_0 \Rightarrow I_f \omega_f = I_0 \omega_0$

$$\omega_f = \frac{I_0}{I_f}\omega_0 = \frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + \frac{1}{2}mr^2}\omega_0$$

$$= \frac{(4.8)(0.18)^2}{(4.8)(0.18)^2 + (3.1)(0.11)^2}(2.0)$$

$$= 1.6 \text{ rev/s} = 10 \text{ rad/s}$$

b) $$\frac{KE_0 - KE_f}{KE_0} = 1 - \frac{KE_f}{KE_0} = 1 - \frac{\frac{1}{2}I_f\omega_f^2}{\frac{1}{2}I_0\omega_0^2} = 1 - \frac{\omega_f}{\omega_0} = 1 - \frac{10}{13} = 0.23 \Rightarrow 23\% \text{ lost}$$

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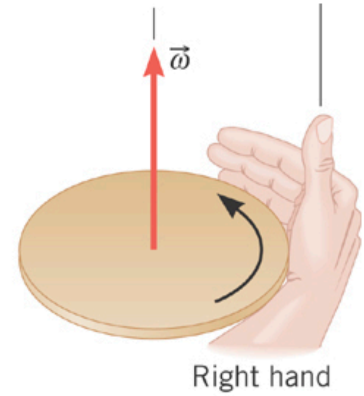
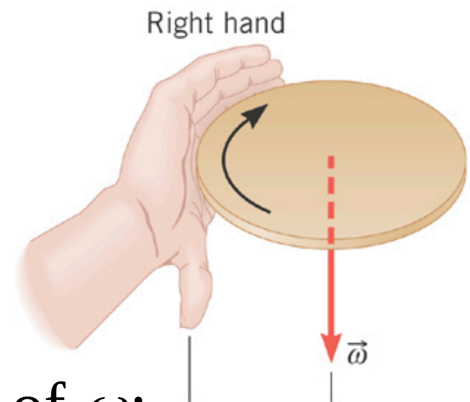
The Vector Nature of Angular Variables



Right-Hand Rule: Grasp the axis of rotation with your right hand, so that your fingers circle the axis in the same sense as the rotation.

Your extended thumb points along the axis in the direction of the angular velocity.



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$\therefore$ we can express L as a vector in the direction of ω:

$$\vec{L} = I\vec{\omega}$$

and write conservation of angular momentum in vector form:

$$\vec{L}_f = \vec{L}_0$$

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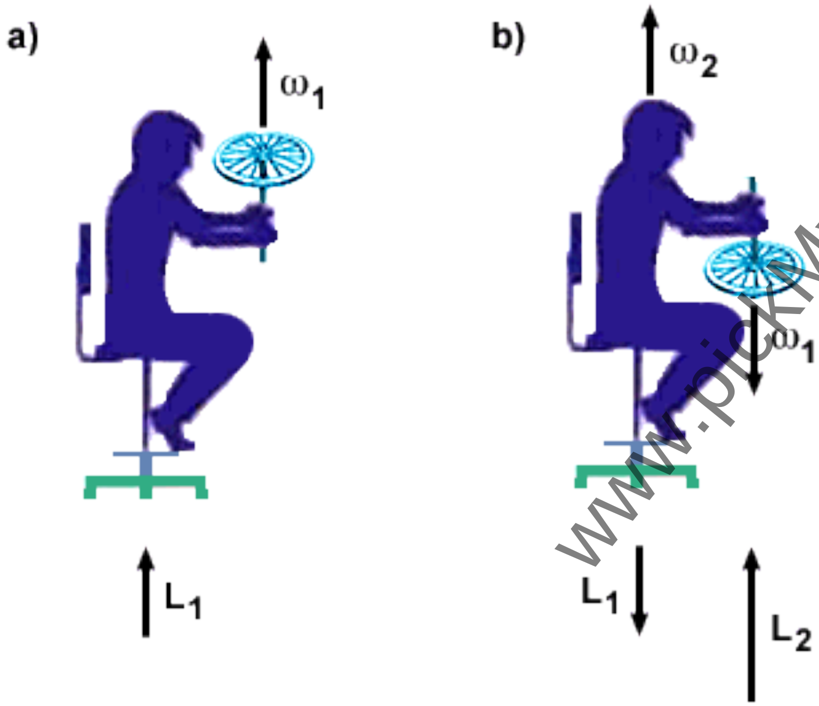
Example: A person sitting on a chair that can rotate is initially at rest and holding a bicycle wheel which is spinning with its angular momentum vector in the vertically up direction and with magnitude 20 rad/s. The mass and radius of the bicycle wheel are 5.0 kg and 0.30 m, respectively, approximated as a solid disk. The mass and average radius of the person through a vertical axis are 90 kg and 0.35 m, respectively, approximated as a solid cylinder. If the person now flips the spinning wheel so that the angular momentum vector is vertically down, what is the angular velocity of the person?



$$\vec{L}_f = \vec{L}_0 \Rightarrow \quad -\vec{L}_1 + \vec{L}_2 = \vec{L}_1$$

$$\therefore \vec{L}_2 = 2\vec{L}_1 \text{ , in upward direction}$$

$$I_2\omega_2 = 2I_1\omega_1 \Rightarrow \quad \omega_2 = \frac{2I_1}{I_2}\omega_1$$

$$\omega_2 = 2\frac{\frac{1}{2}(5.0)(0.30)^2}{\frac{1}{2}(90)(0.35)^2}(20) = 1.6 \text{ rad/s}$$

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