Book Name: Selina Concise

EXERCISE- 7 (A)

Solution 1:

$a:b=5:3$

$\Rightarrow \frac{a}{b}=\frac{5}{3}$

$\frac{5a-3b}{5a+3b}=\frac{5\left(\frac{a}{b}\right)-30}{5\left(\frac{a}{b}\right)+3}$ (dividing each term by b)

$=\frac{5\left(\frac{5}{3}\right)-3}{5\left(\frac{5}{3}\right)+3}$

$=\frac{\frac{25}{3}-3}{\frac{25}{3}+3}$

$=\frac{25-9}{25+9}$

$=\frac{16}{34}=\frac{8}{17}$

Solution 2:

$x:y=4:7$

$\Rightarrow \frac{x}{y}=\frac{4}{7}$

$\frac{3x+2y}{5x+y}=$

$\frac{3\left(\frac{x}{y}\right)+2}{5\left(\frac{x}{y}\right)+1}$ (Dividing each term by y)

$$= \frac{3\left(\frac{4}{7}\right)+2}{5\left(\frac{4}{7}\right)+1}$$

$$= \frac{\frac{12}{7}+2}{\frac{20}{7}+1}$$

$$= \frac{12+14}{20+7}$$

$$= \frac{26}{27}$$

Solution 3:

$a:b=3:8$

$$\Rightarrow \frac{a}{b}=\frac{3}{8}$$

$$\frac{4a+3b}{6a-b}=\frac{4\left(\frac{a}{b}\right)+3}{6\left(\frac{a}{b}\right)-1} \quad (\text{Divinding each term by b})$$

$$= \frac{4\left(\frac{3}{8}\right)+3}{6\left(\frac{3}{8}\right)-1}$$

$$= \frac{\frac{3}{2}+3}{\frac{9}{4}-1}$$

$$= \frac{\frac{9}{2}}{\frac{5}{4}}$$

$$= \frac{18}{5}$$

Solution 4:

$$\frac{a-b}{a+b}=\frac{1}{11}$$

$$11a-11b=a+b$$

$$10a=12b$$

So, let $a=6k$ and $b=5k$

$$\frac{5a+4b+15}{5a-4b+3}=\frac{5(6k)+4(5k)+15}{5(6k)-4(5k)+3}$$

$$=\frac{30k+20k+15}{30k-20k+3}$$

$$=\frac{50k+15}{10k+3}$$

$$=\frac{5(10k+3)}{10k+3}$$

$$=5$$

Hence, $(5a+4b+15):(5a-4b+3)=5:1$

Solution 5:

$$\frac{y-x}{x}=\frac{3}{8}$$

$$\Rightarrow\frac{\frac{y}{x}-\frac{x}{x}}{\frac{x}{x}}=\frac{3}{8}$$

$$\Rightarrow\frac{\frac{y}{x}-1}{1}=\frac{3}{8}$$

$$\Rightarrow\frac{y}{x}=\frac{3}{8}+1=\frac{11}{8}$$

Solution 6:

$$\frac{m+n}{m+3n}=\frac{2}{3}$$
$$\Rightarrow 3m+3n=2m+6n$$
$$\Rightarrow m=3n$$
$$\Rightarrow \frac{m}{n}=\frac{3}{1}$$
$$\frac{2n^2}{3m^2+mn}=\frac{2}{3\left(\frac{m}{n}\right)^2+\left(\frac{m}{n}\right)}\ \left(\text{Dividing each term by } n^2\right)$$
$$=\frac{2}{3\left(\frac{3}{1}\right)^2+\left(\frac{3}{1}\right)}$$
$$=\frac{2}{27+3}=\frac{1}{15}$$

Solution 7:

$$x^2+6y^2=5xy$$
dividing both sides by y^2, we get,
$$\frac{x^2}{y^2}+\frac{6y^2}{y^2}=\frac{5xy}{y^2}$$
$$\left(\frac{x}{y}\right)^2+6=5\left(\frac{x}{y}\right)$$
$$\left(\frac{x}{y}\right)^2-5\left(\frac{x}{y}\right)+6=0$$
Let $\frac{x}{y}=a$
$$\therefore a^2-5a+6=0$$
$$\Rightarrow (a-2)(a-3)=0$$
$$\Rightarrow a=2,3$$
Hence, $\frac{x}{y}=2,3$

Solution 8:

$$\frac{2x-y}{x+2y}=\frac{8}{11}$$

$$22x-11y=8x+16y$$

$$14x=27y$$

Given, $\frac{x}{y}=\frac{27}{14}$

$$\therefore \frac{7x}{9y}=\frac{7\times 27}{9\times 14}=\frac{3}{2}$$

Solution 9:

Let the two numbers be 2x and 3x.

According to the given information,

$$\frac{2x+5}{3x+5}=\frac{5}{7}$$

$$14x+35=15x+25$$

$$x=10$$

Thus, the numbers are 2 × 10 = 20 and 3 × 10 = 30.

Solution 10:

Let the two numbers be 3x and 5x.

According to the given information

$(5x)^2 - (3x)^2 = 400$

$25x^2 - 9x^2 = 400$

$16x^2 = 400$

$x^2 = 25$

$x = 5$

Thus, the numbers are 3 × 5 = 15 and 5 × 5 = 25.

Solution 11:

Let x be subtracted from each term of the ratio 9: 17.

$$\frac{9-x}{17-x}=\frac{1}{3}$$

$$27-3x=17-x$$

$10 = 2x$
$x = 5$
Thus, the required number which should be subtracted is 5.

Solution 12:

Given that the pocket money of Ravi and Sanjeev
Are in the ratio 5:7
Thus, the pocket money of ravi is 5k and that of
Sanjeev is 7k
Also given that the expenditure of ravi and Sanjeev
Are in the ratio 3:5
Thus, the expenditure of ravi is 3m and that of
Sanjeev is 5m
And each of them saves Rs. 80
$\Longrightarrow 5k - 3m = 80$ ….. (1)
$7k - 5m = 80$ …….. (2)
Solving equations (1) and (2), We have,
K = 40, m = 40
Hence the monthly pocket money of Ravi is Rs. 200
And that of Sanjeev is Rs. 280

Solution 13:

Assuming that all the men do the same amount of work in one day and one day work of each man = 1 units, we have,
Amount of work done by $(x - 2)$ men in $(4x + 1)$ days
= Amount of work done by $(x - 2)(4x + 1)$ men in one day
$= (x - 2)(4x + 1)$ units of work
Similarly,
Amount of work done by $(4x + 1)$ men in $(2x - 3)$ days
$= (4x + 1)(2x - 3)$ units of work
According to the given information,

$$\frac{(x-2)(4x+1)}{(4x+1)(2x-3)} = \frac{3}{8}$$

$$\frac{x-2}{2x-3} = \frac{3}{8}$$

$8x - 16 = 6x - 9$

$2x = 7$

$x = \frac{7}{2} = 3.5$

Solution 14:

According to the given information,

Increased (new) bus fare = $\frac{9}{7}$ × original bus fare

(i) We have:

Increased (new) bus fare = $\frac{9}{7}$ × Rs. 245 = Rs. 315

∴ Increase in fare = Rs. 315 – Rs. 245 = Rs. 70

(ii) We have:

Rs 207 = $\frac{9}{7}$ × original bus fare

Original bus fare =Rs.207 × $\frac{7}{9}$ = Rs. 161

∴ Increase in fare = Rs. 207 – Rs. 161 = Rs. 46

Solution 15:

Let the cost of the entry ticket initially and at present be 10 x and 13x respectively.

Let the number of visitors initially and at present be 6y and 5y respectively.

Initially, total collection = 10x × 6y = 60 xy

At present, total collection = 13x × 5y = 65 xy

Ratio of total collection = 60 xy : 65 xy = 12 : 13

Thus, the total collection has increased in the ratio 12 : 13.

Solution 16:

Let the original number of oranges and apples be 7x and 13x.

According to the given information,

$\frac{7x - 8}{13x - 11} = \frac{1}{2}$

$14x - 16 = 13x - 11$

$x = 5$

Thus, the original number of oranges and apples are 7 × 5 = 35 and 13 × 5 = 65 respectively.

Solution 17:

Let the number of boys and girls in the class be 4x and 3x respectively.
According to the given information,

$$\frac{4x+20}{3x-12}=\frac{2}{1}$$
$$4x+20=6x-24$$
$$44=2x$$
$$x=22$$

Therefore,
Number of boys = 4 × 22 = 88
Number of girls = 3 × 22 = 66
∴ Number of students = 88 + 66 = 154

Solution 18:

(A)
(i)

$$\frac{A}{B}=\frac{3}{4}=\frac{3}{4}\times\frac{3}{3}=\frac{9}{12}$$
$$\frac{B}{C}=\frac{6}{7}=\frac{6}{7}\times\frac{2}{2}=\frac{12}{14}$$
$$A:B:C=9:12:14$$

(ii)

$$\frac{A}{B}=\frac{3}{4}$$
$$\frac{B}{C}=\frac{6}{7}$$
$$\therefore \frac{A}{C}=\frac{\frac{A}{C}}{\frac{C}{B}}=\frac{\frac{3}{4}}{\frac{7}{6}}=\frac{3}{4}\times\frac{6}{7}=\frac{9}{14}$$
$$\therefore A:C=9:14$$

(B) (i) To compare 3 ratios, the consequent of the first

Ratio and the antecedent of the 2^{nd} ratio must
Be made equal.
Given that A : B = 2 : 5 and A : C = 3 : 4
Interchanging the first ratio, we have,
B : A = 5 : 2 and A : C = 3 : 4
L.C.M of 2 and 3 is 6
$\Rightarrow$ B : A= 5 × 3 : 2 × 3 and A : C = 3 × 2 : 4 × 2
$\Rightarrow$ B : A = 15 : 6 and A : C = 6 : 8
$\Rightarrow$ B : A : C = 15 : 6 :8
$\Rightarrow$ A : B : C = 6 : 15 : 8

Solution 19:

3A = 4B = 6C

$3A = 4B \Rightarrow \frac{A}{B} = \frac{4}{3}$

$4B = 6C \Rightarrow \frac{B}{C} = \frac{6}{4} = \frac{3}{2}$

Hence, A: B: C = 4: 3: 2

Solution 20:

(i) Required compound ratio = 3 × 8: 5 × 15

$$= \frac{3 \times 8}{5 \times 15}$$

$$= \frac{8}{25} = 8 : 25$$

(ii) Required compound ratio = 2 × 9 × 14: 3 × 14 × 27

$$= \frac{2 \times 9 \times 14}{3 \times 14 \times 27}$$

$$= \frac{2}{9} = 2 : 9$$

(iii) Required compound ratio = 2a × mn × x: 3b × x^2 × n

$$= \frac{2a \times mn \times x}{3b \times x^2 \times n}$$

$$\frac{2am}{3bx} = 2am : 3bx$$

(iv) Required compound ratio = $\sqrt{2} \times 3 \times \sqrt{20} : 1 \times \sqrt{5} \times 9$

$$= \frac{\sqrt{2}\times 3\times \sqrt{20}}{1\times \sqrt{5}\times 9}$$

$$= \frac{\sqrt{2}\times \sqrt{4}}{3}$$

$$= \frac{2\sqrt{2}}{3} = 2\sqrt{2} : 3$$

Solution 21:

(i) Duplicate ratio of 3: 4 = 3^2: 4^2 = 9: 16

(ii) Duplicate ratio of $3\sqrt{3} : 2\sqrt{5} = \left(3\sqrt{3}\right)^2 : \left(2\sqrt{5}\right)^2 = 27 : 20$

Solution 22:

(i) Triplicate ratio of 1: 3 = 1^3: 3^3 = 1: 27

(ii) Triplicate ratio of

$$\frac{m}{2} : \frac{n}{3}$$

$$= \left(\frac{m}{2}\right)^3 : \left(\frac{n}{3}\right)^3 = \frac{m^3}{8} : \frac{n^3}{27} = \frac{\frac{m^3}{8}}{\frac{n^3}{27}} = 27m^3 : 8n^3$$

Solution 23:

(i) Sub-duplicate ratio of 9 : 16 $= \sqrt{9} : \sqrt{16} = 3 : 4$

(ii) Sub-duplicate ratio of $(x - y)^4$: $(x + y)^6$

$$= \sqrt{(x-y)^4} : \sqrt{(x+y)^6} = (x-y)^2 : (x+y)^3$$

Solution 24:

(i) Sub-triplicate ratio of 64 : 27 = $\sqrt[3]{64}:\sqrt[3]{27}=4:3$

(ii) Sub-triplicate ratio of x^3 : $125y^3$ = $\sqrt[3]{x^3}:\sqrt[3]{125y^3}=x:5y$

Solution 25:

(i) Reciprocal ratio of 5: 8 = $\frac{1}{5}:\frac{1}{8}=8:5$

(ii) Reciprocal ratio of $\frac{x}{3}:\frac{y}{7}=\frac{1}{\frac{x}{3}}:\frac{1}{\frac{y}{7}}=\frac{3}{x}:\frac{7}{y}=\frac{\frac{3}{x}}{\frac{7}{y}}=\frac{3y}{7x}=3y:7x$

Solution 26:

$$\frac{3x+4}{x+5}=\frac{(8)^2}{(15)^2}$$

$$\Rightarrow\frac{3x+4}{x+5}=\frac{64}{225}$$

$$\Rightarrow 675x+900=64x+320$$

$$\Rightarrow 611x=-580$$

$$\Rightarrow x=-\frac{580}{611}$$

Solution 27:

$$\frac{m}{n}=\frac{(m+x)^2}{(n+x)^2}$$

$$\frac{m}{n}=\frac{m^2+x^2+2mx}{n^2+x^2+2nx}$$

$$mn^2+mx^2+2mnx=m^2n+nx^2+2mnx$$

$$x^2(m-n)=mn(m-n)$$

$$x^2=mn$$

Solution 28:

$$\frac{4x+4}{9x-10}=\frac{(4)^3}{(5)^3}$$

$$\frac{4x+4}{9x-10}=\frac{64}{125}$$

$$500x+500=576x-640$$

$$576x-500x=500+640$$

$$76x=1140$$

$$x=\frac{1140}{76}=15$$

Solution 29:

Reciprocal ratio of 15 : 28 = 28 : 15

Sub-duplicate ratio of 36 : 49 = $\sqrt{36}:\sqrt{49}=6:7$

Triplicate ratio of 5: 4 = $5^3 : 4^3$ = 125 : 64

Required compounded ratio

$$=\frac{28\times6\times125}{15\times7\times64}=\frac{25}{8}=25:8$$

Solution 30:

$$\frac{a+b}{am+bn}=\frac{b+c}{mb+nc}=\frac{c+a}{mc+na}=\frac{\text{sum of antecedents}}{\text{sum of consequents}}$$

$$=\frac{a+b+b+c+c+a}{am+bn+mb+nc+mc+na}$$

$$=\frac{2(a+b+c)}{m(a+b+c)+n(a+b+c)}$$

$$=\frac{2}{m+n}$$

EXERCISE 7 (B)

Solution 1:

(i) Let the fourth proportional to 1.5, 4.5 and 3.5 be x.

$\Rightarrow 1.5 : 4.5 = 3.5 : x$

$\Rightarrow 1.5 \times \ x = 3.5 \times 4.5$

$\Rightarrow x = 10.5$

(i) Let the fourth proportional to 3a, $6a^2$ and $2ab^2$ be x.

$\Rightarrow 3a : 6a^2 = 2ab^2 : x$

$\Rightarrow 3a \times x = 2ab^2 \times 6a^2$

$\Rightarrow 3a \times x = 12a^3b^2$

$\Rightarrow x = 4a^2b^2$

Solution 2:

(i) Let the third proportional to $2\frac{2}{3}$ and 4 be x.

$\Rightarrow 2\frac{2}{3}$, 4, x are in continued proportion.

$\Rightarrow 2\frac{2}{3} : 4 = 4 : x$

$$\Rightarrow \frac{\frac{8}{3}}{4} = \frac{4}{x}$$

$$\Rightarrow x = 16 \times \frac{3}{8} = 6$$

(ii) Let the third proportional to a − b and $a^2 - b^2$ be x.

$\Rightarrow a - b$, $a^2 - b^2$, x are in continued proportion.

$\Rightarrow a - b : a^2 - b^2 = a^2 - b^2 : x$

$$\Rightarrow \frac{a-b}{a^2-b^2} = \frac{a^2-b^2}{x}$$

$$\Rightarrow x = \frac{\left(a^2-b^2\right)^2}{a-b}$$

$$\Rightarrow x = \frac{(a+b)(a-b)\left(a^2-b^2\right)}{a-b}$$

$$\Rightarrow x = (a+b)\left(a^2-b^2\right)$$

Solution 3:

(i) Let the mean proportional between 17.5 and 0.007 be x.
$\Rightarrow$ 17.5, x and 0.007 are in continued proportion.
$\Rightarrow 17.5 : x = x : 0.007$
$\Rightarrow x \times x = 17.5 \times 0.007$
$\Rightarrow x^2 = 0.1225$
$\Rightarrow x = 0.35$
(ii) Let the mean proportional between $6 + 3\sqrt{3}$ and $8 - 4\sqrt{3}$ be x.
$6 + 3\sqrt{3}$, x and $8 - 4\sqrt{3}$ are in continued proportion.
$6 + 3\sqrt{3} : x = x : 8 - 4\sqrt{3}$
$\Rightarrow x \times x = (6 + 3\sqrt{3})(8 - 4\sqrt{3})$
$\Rightarrow x^2 = 48 + 24\sqrt{3} - 24\sqrt{3} - 36$
$\Rightarrow x^2 = 12$
$\Rightarrow x = 2\sqrt{3}$
(iii) Let the mean proportional between $a - b$ and $a^3 - a^2b$ be x.
$\Rightarrow a - b, x, a^3 - a^2b$ are in continued proportion.
$\Rightarrow a - b : x = x : a^3 - a^2b$
$\Rightarrow x \quad x = (a - b)(a^3 - a^2b)$
$\Rightarrow x^2 = (a - b)a^2(a - b) = [a(a - b)]^2$
$\Rightarrow x = a(a - b)$

Solution 4:

Given, x + 5 is the mean proportional between x + 2 and x + 9.
$\Rightarrow (x + 2), (x + 5)$ and $(x + 9)$ are in continued proportion.
$\Rightarrow (x + 2) : (x + 5) = (x + 5) : (x + 9)$
$\Rightarrow (x + 5)^2 = (x + 2)(x + 9)$
$\Rightarrow x^2 + 25 + 10x = x^2 + 2x + 9x + 18$
$\Rightarrow 25 - 18 = 11x - 10x$
$\Rightarrow x = 7$

Solution 5:

Let the number added be x.
$\therefore (16 + x) : (7 + x) :: (79 + x) (43 + x)$

$$\frac{16+x}{7+x}=\frac{79+x}{43+x}$$
$$(16+x)(43+x)=(79+x)(7+x)$$
$$688+16x+43x+x^2=553+79x+7x+x^2$$
$$688-553=86x-59x$$
$$135=27x$$
$$x=5$$
Thus, the required number which must be added is 5.

Solution 6:

Let the number added be x.

∴ (6 + x) : (15 + x) :: (20 + x) (43 + x)

$$\frac{6+x}{15+x}=\frac{20+x}{43+x}$$
$$(6+x)(43+x)=(20+x)(15+x)$$
$$258+6x+43x+x^2=300+20x+15x+x^2$$
$$49x-35x=300-258$$
$$14x=42$$
$$x=3$$
Thus, the required number which should be added is 3.

Solution 7:

Let the number added be x.

∴ (16 + x) : (26 + x) :: (26 + x) (40 + x)

$$\frac{16+x}{26+x}=\frac{26+x}{40+x}$$
$$(162+x)(40+x)=\left(26+x^2\right)$$
$$640+16x+40x+x^2=676+52x+x^2$$
$$56x-52x=676-640$$
$$4x=36$$
$$x=9$$
Thus, the required number which should be added is 9.

Solution 8:

Let the number subtracted be x.

$\therefore (7 - x) : (17 - x) :: (17 - x) (47 - x)$

$$\frac{7-x}{17-x} = \frac{17-x}{47-x}$$

$$(7-x)(46-x) = (17-x)^2$$

$$329 - 47x - 7x + x^2 = 289 - 34x + x^2$$

$$329 - 289 = -34x + 54x$$

$$20x = 40$$

$$x = 2$$

Thus, the required number which should be subtracted is 2.

Solution 9:

Since y is the mean proportion between x and z

Therefore, $y^2 = xz$

Now, we have to prove that xy + yz is the mean proportional between $x^2 + y^2$ and $y^2 + z^2$, i.e.,

$$(xy + yz)^2 = (x^2 + y^2)(y^2 + z^2)$$

$$\text{LHS} = (xy + yz)^2$$

$$= [y(x+z)]^2$$

$$= y^2 (x+z)^2$$

$$= xz(x+z)^2$$

$$\text{RHS} = (x^2 + y^2)(y^2 + z^2)$$

$$= (x^2 + xz)(xz + z^2)$$

$$= x(x+z)z(x+z)$$

$$= xz(x+z)^2$$

LHS = RHS

Hence, proved.

Solution 10:

Given, q is the mean proportional between p and r.

$\Rightarrow q^2 = pr$

$$\text{L..H.S} = pqr(p+q+r)^3$$
$$= qq^2(p+q+r)^3$$
$$= q^3(p+q+r)^3 \quad \left[\because q^2 \quad pr\right]$$
$$= \left[q(p+q+r)\right]^3$$
$$= (pq+q^2+qr)^3$$
$$= (pq+pr+qr)^3 \quad \left[\because q^2 \quad pr\right]$$
$$= \text{R.H.S}$$

Solution 11:

Let x, y and z be the three quantities which are in continued proportion.
Then, $x : y :: y : z \Longrightarrow y^2 = xz$ .... (1)
Now, we have to prove that
$x : z = x^2 : y^2$
That is we need to prove that
$xy^2 = x^2z$
LHS = $xy^2 = x(xz) = x^2z$ = RHS [Using (1)]
Hence, proved.

Solution 12:

Given, y is the mean proportional between x and z.
$\Longrightarrow y^2 = xz$

$$\text{LHS} = \frac{x^2 - y^2 + z^2}{x^{-2} - y^{-2} + z^{-2}}$$
$$= \frac{x^2 - y^2 + z^2}{\frac{1}{x^2} - \frac{1}{y^2} + \frac{1}{z^2}}$$
$$= \frac{x^2 - xz + z^2}{\frac{1}{x^2} - \frac{1}{xz} + \frac{1}{z^2}}$$

$$=\frac{x^2-xz+z^2}{\frac{z^2-xz+x^2}{x^2z^2}}$$
$$=x^2z^2$$
$$=(xz)^2$$
$$=(y^2)^2 \qquad (\because Y^2 \quad XZ)$$
$$=y^4$$
$$=\text{RHS}$$

Solution 13:

$$\text{Let}\,\frac{a}{b}=\frac{c}{d}=k$$
$$\Rightarrow a=bk\,\text{and}\,c=dk$$
$$\text{LHS}=\frac{(a-c)b^2}{(b-d)cd}$$
$$=\frac{(bk-dk)b^2}{(b-d)dkd}$$
$$=\frac{k(b-d)b^2}{(b-d)d^2k}$$
$$=\frac{b^2}{d^2}$$
$$\text{RHS}=\frac{(a^2-b^2-ab)}{(c^2-d^2-cd)}$$
$$=\frac{(b^2k^2-b^2-bkb)}{(d^2k^2-d^2-dkd)}$$
$$=\frac{b^2(k^2-1-k)}{d^2(k^2-1-k)}$$
$$=\frac{b^2}{d^2}$$
$$\Rightarrow \text{LHS}=\text{RHS}$$

Hence proved.

Solution 14:

Let a and b be the two numbers, whose mean proportional is 12.

$\therefore ab = 12^2 \Rightarrow ab = 144 \Rightarrow b = \dfrac{144}{a}$.................(i)

Now, third proportional is 96

$\therefore a : b :: b : 96$

$\Rightarrow b^2 = 96a$

$\Rightarrow \left(\dfrac{144}{a}\right)^2 = 96a$

$\Rightarrow \dfrac{(144)^2}{a^2} = 96a$

$\Rightarrow a^3 = \dfrac{144 \times 144}{96}$

$\Rightarrow a^3 = 216$

$\Rightarrow a = 6$

$b = \dfrac{144}{6} = 24$

Therefore, the numbers are 6 and 24.

Solution 15:

Let the required third proportional be p.

$\Rightarrow \dfrac{x}{y} + \dfrac{y}{x}, \sqrt{x^2 + y^2}$, p are in continued proportion.

$\Rightarrow \dfrac{x}{y} + \dfrac{y}{x} : \sqrt{x^2 + y^2} = \sqrt{x^2 + y^2} : p$

$\Rightarrow p\left(\dfrac{x}{y} + \dfrac{y}{x}\right) = \left(\sqrt{x^2 + y^2}\right)^2$

$\Rightarrow p\left(\dfrac{x^2 + y^2}{xy}\right) = x^2 + y^2$

$\Rightarrow p = xy$

Solution 16:

$$\frac{p}{q}=\frac{r}{s}$$

$$\Rightarrow \frac{mp}{q}=\frac{mr}{s}$$

$$\Rightarrow \frac{mp}{q}+n=\frac{mr}{s}+n$$

$$\Rightarrow \frac{mp+nq}{q}=\frac{mr+ns}{s}$$

Hence, mp + nq : q = mr + ns : s.

Solution 17:

$$\frac{1}{q}+\frac{1}{s}=\frac{m}{r}$$

$$\frac{s+q}{qs}=\frac{m}{r}$$

$$\frac{s+q}{s}=\frac{mq}{r}$$

$$\frac{s+q}{s}=\frac{p+r}{r} \quad \left(\quad p+r=mq \right)$$

$$1+\frac{q}{s}=\frac{p}{r}+1$$

$$\frac{q}{s}=\frac{p}{r}$$

$$\frac{p}{q}=\frac{r}{s}$$

Hence, proved

Solution 18:

$\text{Let}\frac{a}{b}=\frac{c}{d}=k$

Then, $a=bk$ and $c=dk$

(i) $\frac{5a+4c}{5b+4d}=\frac{5(bk)+4(dk)}{5b+4b}=\frac{k(5b+4d)}{5b+4d}=k=\text{each given ratio}$

(ii) $\frac{13a-8c}{13b-8d}=\frac{13(bk)-8(dk)}{13b-8d}=\frac{k(13b-8d)}{13b-8d}=k=\text{each given ratio}$

(iii) $\sqrt{\frac{3a^2-10c^2}{3b^2-10d^2}}=\sqrt{\frac{3(bk)^2-10(dk)^2}{3b^2-10d^2}}=\sqrt{\frac{k^2(3b^2-10d^2)}{3b^2-10d^2}}=k=\text{each given ratio}$

(iv) $\left(\frac{8a^3+15c^3}{8b^3+15d^3}\right)^{\frac{1}{3}}=\left[\frac{8(bk)^3+15(dk)^3}{8b^3+15d^3}\right]^{\frac{1}{3}}=\left[\frac{k^3(8b)^3+15d^3}{8b^3+15d^3}\right]^{\frac{1}{3}}=k=\text{each given ratio}$

Solution 19:

$\text{Let}\frac{a}{b}=\frac{c}{d}=k(\text{say})$

Then, $a=bk$ and $c=dk$

(i) $\text{L..H.S}=\frac{13a+17b}{13c+17d}=\frac{13(bk)+17b}{13(bk)+17b}=\frac{b(13k+17)}{b(13k+17)}=\frac{b}{d}$

$\text{R.H.S}=\sqrt{\frac{2ma^2-3nb^2}{2mc^2-3nd^2}}=\sqrt{\frac{2m(bk)^2-3nb^2}{2m(dk)^2-3nd^2}}=\sqrt{\frac{b^2(2mk^2-3n)}{d^2(2mk^2-3n)}}=\frac{b}{d}$

Hence, L.H.S = R.H.S

(ii)L.H.S $= \sqrt{\dfrac{4a^2+9b^2}{4c^2+9d^2}} = \sqrt{\dfrac{4(bk)^2+9b^2}{4(dk)^2+9d^2}} = \sqrt{\dfrac{b^2(4k^2+9)}{d^2(4k^2+9)}} = \dfrac{b}{d}$

R.H.S $= \left(\dfrac{xa^3-5yb^3}{xc^3-5yd^3}\right)^{\frac{1}{3}} = \left[\dfrac{x(bk)^3-5yb^3}{x(dk)^3-5yd^3}\right]^{\frac{1}{3}}$

$= \left[\dfrac{b^3(xk^3-5y)}{d^3(xk^3-5y)}\right]^{\frac{1}{3}}$

$= \left[\dfrac{b^3}{d^3}\right]^{\frac{1}{3}} = \dfrac{b}{d}$

Hence, L.H.S = R.H.S

Solution 20:

Let $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c} = k$

Then, x = ak, y = bk and z = ck

L..H.S $= \dfrac{2x^3-3y^3+4z^3}{2a^3-3b^3+4c^3}$

$= \dfrac{2(ak)^3-3(bk)^3+4(ck)^3}{2a^3-3b^3+4c^3}$

$= \dfrac{2a^3k^3-3b^3k^3+4c^3k^3}{2a^3-3b^3+4c^3}$

$= \dfrac{k^3(2a^3-3b^3+4c^3)}{2a^3-3b^3+4c^3}$

$= k^3$

R.H.S $= \left(\dfrac{2x-3y+4z}{2a-3b+4c}\right)^3$

$= \left(\dfrac{2ak-3bk+4ck}{2a-3b+4c}\right)^3$

$= \left[\dfrac{k(2a-3b+4c)}{2a-3b+4c}\right]^3$

$= K^3$

Hence, L.H.S = R.H.S

EXERCISE .7 (c)

Solution 1:

(i) Given, $\frac{a}{b}=\frac{c}{d}$

$\Rightarrow \frac{5a}{7b}=\frac{5c}{7d} \quad \left(\text{Multiplying each side by } \frac{5}{7}\right)$

$\Rightarrow \frac{5a+7b}{5a-7b}=\frac{5c+7d}{5c-7d} \quad (\text{By componend and dividendo})$

(ii) Given, $\frac{a}{b}=\frac{c}{d}$

$\Rightarrow \frac{9a}{13b}=\frac{9c}{13d} \quad \left(\text{Multiplying each side by } \frac{9}{13}\right)$

$\Rightarrow \frac{9a+13b}{13a-13b}=\frac{9c+13d}{9c-13d} \quad (\text{By componend and dividendo})$

$\Rightarrow (9a+13b)(9c-13d)=(9c+13d)(9a-13b)$

(iii) Given, $\frac{a}{b}=\frac{c}{d}$

$\Rightarrow \frac{xa}{yb}=\frac{xc}{yd} \quad \left(\text{Multiplying each side by } \frac{x}{y}\right)$

$\Rightarrow \frac{xa+yb}{yb}=\frac{xc+yd}{yd} \quad (\text{By componend})$

$\Rightarrow \frac{xa+yb}{xc+yd}=\frac{yb}{yd}$

$\Rightarrow \frac{xa+yb}{xc+yd}=\frac{b}{d}$

Solution 2:

Given, $\frac{a}{b}=\frac{c}{d}$

$\Rightarrow \frac{6a}{7b}=\frac{6c}{7d} \quad \left(\text{Multiplying each side by } \frac{6}{7}\right)$

$\Rightarrow \frac{6a+7b}{7b}=\frac{6c+7d}{7d} \quad (\text{By componend})$

$\Rightarrow \frac{6a+7b}{6c+7d}=\frac{7b}{7d}=\frac{b}{d}$

$\Rightarrow$ Also, $\frac{a}{b}=\frac{c}{d}$

$\Rightarrow \frac{3a}{4b}=\frac{3c}{4d} \quad \left(\text{Multiplying each side by } \frac{3}{4}\right)$

$\Rightarrow \frac{3a-4b}{4b}=\frac{3c-4d}{4d} \quad (\text{By dividendo})$

$\Rightarrow \frac{3a-4b}{3c-4d}=\frac{4b}{4d}=\frac{b}{d} \quad \text{.........} (2)$

From (1) and $=(2)$

$\frac{6a+7b}{6c+7d}=\frac{3a-4b}{3c-4d}$

$(6a+7d)(3c-4d)=(6c+7d)(3a-4b)$

Solution 3:

$\Rightarrow \frac{a}{b}=\frac{c}{d}$

$\Rightarrow \frac{3a}{5b}=\frac{3c}{5d}\left(\text{Multiplying each side by } \frac{3}{5}\right)$

$\Rightarrow \frac{3a+5b}{3a-5b}=\frac{3c+5d}{3c-5d} \quad (\text{By componendo and dividendo})$

$\Rightarrow \frac{3a-5b}{3a+5b}=\frac{3c-5d}{3c+5d} \quad (\text{By alternendo})$

Solution 4:

$$\frac{5x-6y}{5u-6v}=\frac{5x-6y}{5u-6v} \quad (\text{By alternendo})$$

$$\frac{5x+6y}{5x-6y}=\frac{5u+6v}{5u-6v}$$

$$\frac{5x+6y+5x-6y}{5x+6y-5x+6y}=\frac{5u+6v+5u-6v}{5u+6v-5u+6v} \quad (\text{By componendo and dividendo})$$

$$\frac{10x}{12y}=\frac{10u}{12v}$$

$$\frac{x}{y}=\frac{u}{v}$$

Solution 5:

Given, $\frac{7a+8b}{7a-8b}=\frac{7c+8d}{7c-8d}$

Applying componendo and dividendo,

$$\frac{7a+8b+7a-8b}{7a+8b-7a+8b}=\frac{7c+8d+7c-8d}{7c+8d-7c+8d}$$

$$\Rightarrow \frac{14a}{16b}=\frac{14c}{16d}$$

$$\Rightarrow \frac{a}{b}=\frac{c}{d}$$

Hence, a : b = c : d.

Solution 6:

(i) $x=\frac{6ab}{a+b}$

$$\Rightarrow \frac{x}{3a}=\frac{2b}{a+b}$$

Applying componendo and dividendo,

$$\frac{x+3a}{x-3a}=\frac{2b+a+b}{2b-a-b}$$
$$\frac{x+3a}{x-3a}=\frac{3b+a}{b-a}$$
$$\text{Again, } x=\frac{6ab}{a+b}$$
$$\Rightarrow \frac{x}{3b}=\frac{2a}{a+b}$$

Applying componendo and dividendo,

$$\frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}=\frac{3b+a}{b-a}+\frac{3a+b}{a-b}$$
$$\frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}=\frac{-3b-a+3a+b}{a-b}$$
$$\frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}=\frac{2a-2b}{a-b}=2$$
$$\text{(ii)} a=\frac{4\sqrt{6}}{\sqrt{2}+\sqrt{3}}$$
$$\frac{a}{2\sqrt{2}}=\frac{2\sqrt{3}}{\sqrt{2}+\sqrt{3}}$$

Applying componendo and dividendo,

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}}=\frac{2\sqrt{3}+\sqrt{2}+\sqrt{3}}{2\sqrt{3}-\sqrt{2}+\sqrt{3}}$$
$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}}=\frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \quad ....(1)$$

Applying componendo and dividendo,

$$\frac{a+2\sqrt{3}}{a-2\sqrt{3}}=\frac{2\sqrt{2}+\sqrt{2}+\sqrt{3}}{2\sqrt{2}-\sqrt{2}-\sqrt{3}}$$
$$\frac{a+2\sqrt{3}}{a-2\sqrt{3}}=\frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} \quad ....(2)$$

From (1) and (2),

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}}+\frac{a+2\sqrt{3}}{a-2\sqrt{3}}=\frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}+\frac{3\sqrt{3}+\sqrt{2}}{\sqrt{2}-\sqrt{3}}$$
$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}}+\frac{a+2\sqrt{3}}{a-2\sqrt{3}}=\frac{3\sqrt{2}+\sqrt{3}-3\sqrt{3}-\sqrt{2}}{\sqrt{2}-\sqrt{3}}$$
$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}}+\frac{a+2\sqrt{3}}{a-2\sqrt{3}}=\frac{2\sqrt{2}-2\sqrt{3}}{\sqrt{2}-\sqrt{3}}=2$$

Solution 7:

Given, $\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$

Applying componendo and dividendo,

$$\frac{(a+b+c+d)+(a+b-c-d)}{(a+b+c+d)-(a+b-c-d)}=\frac{(a-b+c-d)+(a-b-c+d)}{(a-b+c-d)-(a-b-c+d)}$$

$$\frac{2(a+b)}{2(c+d)}=\frac{2(a-b)}{2(c-d)}$$

$$\frac{a+b}{c+d}=\frac{a-b}{c-d}$$

$$\frac{a+b}{a-d}=\frac{c+d}{c-d}$$

Applying componendo and dividendo,

$$\frac{a+b+a-b}{a+b-a+b}=\frac{c+d+c-d}{c+d-c+d}$$

$$\frac{2a}{2b}=\frac{2c}{2d}$$

$$\frac{a}{b}=\frac{c}{d}$$

Solution 8:

$$\frac{a-2b-3c+4d}{a+2b-3c-4d}=\frac{a-2b+3c-4d}{a+2b+3c+4d}$$

Applying componendo and dividendo,

$$\frac{(a-2b-3c+4d)+(a+2b-3c-4d)}{(a-2b-3c+4d)-(a+2b-3c-4d)}$$

$$=\frac{(a-2b+3c-4d)+(a+2b+3c+4d)}{(a-2b+3c-4d)-(a+2b+3c+4d)}$$

$$\frac{2(a-3c)}{2(-2b+4d)}=\frac{2(a+3c)}{2(-2b-4d)}$$

$$\frac{a-3c}{a+3c}=\frac{-2b+4d}{-2b-4d}$$

Applying componendo and dividendo,

$$\frac{a-3c+a+3c}{a-3c-a-3c}=\frac{-2b+4d-2b-4d}{-2b+4d+2b+4d}$$
$$\frac{2a}{-6c}=\frac{-4b}{8d}$$
$$\frac{a}{-3c}=\frac{-b}{2d}$$
$$2ad=3bc$$

Solution 9:

Given, $\left(a^2+b^2\right)\left(x^2+y^2\right)=\left(ax+by\right)^2$

$$a^2x^2+a^2y^2+b^2x^2+b^2y^2=a^2x^2+b^2y^2+2abxy$$
$$a^2y^2+b^2x^2-2abxy=0$$
$$\left(ay-bx\right)^2=0$$
$$ay-bx=0$$
$$ay=bx$$
$$\frac{a}{x}=\frac{b}{y}$$

Solution 10:

Given, a, b and c are in continued proportion.

$$\Rightarrow \frac{a}{b}=\frac{b}{c}=k\left(say\right)$$
$$\Rightarrow a=bk, b=ck$$
$$\Rightarrow a=\left(ck\right)k=ck^2, b=ck$$
$$(i) L..H.S=\frac{a^2+ab+b^2}{b^2+bc+c^2}$$
$$\frac{\left(ck^2\right)^2+\left(ck^2\right)\left(ck\right)+\left(ck\right)^2}{\left(ck\right)^2+\left(ck\right)c+c^2}$$

$$=\frac{c^2k^4+c^2k^3+c^2k^2}{c^2k^2+c^2k+c^2}$$

$$=\frac{c^2k^2\left(k^2+k+1\right)}{c^2\left(k^2+k+1\right)}$$

$$=k^2$$

$$\text{R.H.S}=\frac{a}{c}=\frac{ck^2}{c}=k^2$$

$$\therefore \text{L.H.S}=\text{R.H.S}$$

$$\text{(ii) L..H.S}=\frac{a^2+b^2+c^2}{\left(a+b+c\right)^2}$$

$$=\frac{\left(ck^2\right)^2+\left(ck^2\right)+c^2}{\left(ck^2+ck+c\right)^2}$$

$$=\frac{c^2k^4+c^2k^2+c^2}{c^2\left(k^2+k+1\right)^2}$$

$$=\frac{c^2\left(k^4+k^2+1\right)}{c^2\left(k^2+k+1\right)^2}$$

$$=\frac{k^4+k^2+1}{\left(k^2+k+1\right)^2}$$

$$\text{R.H.S}=\frac{a-b+c}{a+b+c}$$

$$=\frac{ck^2-ck+c}{ck^2+ck+c}$$

$$=\frac{k^2-k+1}{k^2+k+1}$$

$$=\frac{\left(k^2-k+1\right)\left(k^2+k+1\right)}{\left(k^2+k+1\right)^2}$$

$$=\frac{k^4+k^3+k^2-k^3-k^2-k+k^2+k+1}{\left(k^2+k+1\right)^2}$$

$$=\frac{k^4+k^2+1}{\left(k^2+k+1\right)^2}$$

$$\therefore \text{L..H.S}=\text{R.H.S}$$

Solution 11:

(i) $\frac{\sqrt{x+5}+\sqrt{x-16}}{\sqrt{x+5}-\sqrt{x-16}}=\frac{7}{3}$

Applying componendo and dividendo,

$$\frac{\sqrt{x+5}+\sqrt{x-16}+\sqrt{x+5}-\sqrt{x-16}}{\sqrt{x+5}+\sqrt{x-16}-\sqrt{x+5}+\sqrt{x-16}}=\frac{7+3}{7-3}$$

$$\frac{2\sqrt{x+5}}{2\sqrt{x-16}}=\frac{10}{4}$$

$$\frac{\sqrt{x+5}}{\sqrt{x-16}}=\frac{5}{2}$$

Squaring both sides,

$$\frac{x+5}{x-16}=\frac{25}{4}$$

$$4x+20=25x-400$$

$$21x=420$$

$$x=\frac{420}{21}=20$$

(ii) $\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}=\frac{4x-1}{2}$

Applying componendo and dividendo,

$$\frac{\sqrt{x+1}+\sqrt{x-1}+\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}-\sqrt{x+1}+\sqrt{x-1}}=\frac{4x-1+2}{4x-1-2}$$

$$\frac{2\sqrt{x+1}}{2\sqrt{x-1}}=\frac{4x+1}{4x-3}$$

Squaring both sides,

$$\frac{x+1}{x-1}=\frac{16x^2+1+8x}{16x^2+9-24x}$$

Applying componendo and dividendo,

$$\frac{x+1+x-1}{x+1-x+1}=\frac{16x^2+1+8x+16x^2+9-24x}{16x^2+1+8x-16x^2-9+24x}$$

$$\frac{2x}{2}=\frac{32x^2+10-16x}{32x-8}$$

$$16x^2-4x=16x^2+5-8x$$

$$4x=5$$

$$x=\frac{5}{4}$$

(iii) $\frac{3x+\sqrt{9x^2-5}}{3x-\sqrt{9x^2-5}}=5$

Applying componendo and dividendo,

$$\frac{3x+\sqrt{9x^2-5}+3x-\sqrt{9x^2-5}}{3x+\sqrt{9x^2-5}-3x+\sqrt{9x^2-5}}=\frac{5+1}{5-1}$$

$$\frac{6x}{2\sqrt{9x^2-5}}=\frac{6}{4}$$

$$\frac{x}{\sqrt{9x^2-5}}=\frac{1}{2}$$

Squaring both sides,

$$\frac{x^2}{9x^2-5}=\frac{1}{4}$$

$$4x^2=9x^2-5$$

$$5x^2=5$$

$$x^2=1$$

$$x=1$$

Solution 12:

Since, $\frac{x}{1}=\frac{\sqrt{a+3b}+\sqrt{a-3b}}{\sqrt{a+3b}-\sqrt{a-3b}}$

Applying componendo and dividendo, we get,

$$\frac{x+1}{x-1}=\frac{\sqrt{a+3b}+\sqrt{a-3b}+\sqrt{a+3b}-\sqrt{a-3b}}{\sqrt{a+3b}+\sqrt{a-3b-\sqrt{a+3b}+\sqrt{a-3b}}}$$

$$\frac{x+1}{x-1}=\frac{2\sqrt{a-3b}}{-2\sqrt{a-3b}}$$

Squaring both sides,

$$\frac{x^2+2x+1}{x^2-2x+1}=\frac{a+3b}{a-3b}$$

Again applying componendo and dividendo,

$$\frac{x^2+2x+1+x^2-2x+1}{x^2+2x+1-x^2+2x-1}=\frac{a+3b+a-3b}{a+3b-a+3b}$$

$$\frac{2\left(x^2+1\right)}{2(2x)}=\frac{2(a)}{2(3b)}$$

$$3b\left(x^2+1\right)=2ax$$

$3bx^2 + 3b = 2ax$

$3bx^2 - 2ax + 3b = 0.$

Solution 13:

$$\frac{x}{y}=\frac{\sqrt{a+b}+\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}}$$

Again applying componendo and dividendo,

$$\frac{x+y}{x-y}=\frac{\sqrt{a+b}+\sqrt{a-b}+\sqrt{a+b}-\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}-\sqrt{a+b}+\sqrt{a-b}}$$

$$\frac{x+y}{x-y}=\frac{2\sqrt{a+b}}{2\sqrt{a-b}}$$

$$\frac{x+y}{x-y}=\frac{\sqrt{a+b}}{\sqrt{a-b}}$$

Squaring both sides,

$$\frac{x^2+y^2+2xy}{x^2+y^2-2xy}=\frac{a+b}{a-b}$$

Again applying componendo and dividendo,

$$\frac{x^2+y^2+2xy+x^2+y^2-2xy}{x^2+y^2+2xy-x^2-y^2+2xy}=\frac{a+b+a-b}{a+b-a+b}$$

$$\frac{2\left(x^2+y^2\right)}{4xy}=\frac{2a}{2b}$$

$$\frac{x^2+y^2}{2xy}=\frac{a}{b}$$

$$bx^2+by^2=2axy$$

$$bx^2-2axy+by^2=0$$

Solution 14:

$$x = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$$

applying componendo and dividendo,

$$\frac{x+1}{x-1} = \frac{\sqrt{m+n}+\sqrt{m-n}+\sqrt{m+n}-\sqrt{m-n}}{c-\sqrt{m-n-\sqrt{m+n}+\sqrt{m-n}}}$$

$$\frac{x+1}{x-1} = \frac{2\sqrt{m+n}}{2\sqrt{m-n}}$$

$$\frac{x+1}{x-1} = \frac{\sqrt{m+n}}{\sqrt{m-n}}$$

Squaring both sides,

$$\frac{x^2+2x+1}{x^2-2x+1} = \frac{m+n}{m-n}$$

applying componendo and dividendo,

$$\frac{x^2+2x+1+x^2-2x+1}{x^2+2x+1-x^2+2x-1} = \frac{m+n+m-n}{m+n-m+n}$$

$$\frac{2x^2+2}{4x} = \frac{2m}{2n}$$

$$\frac{x^2+1}{2x} = \frac{m}{n}$$

$$\frac{x^2+1}{2mx} = \frac{1}{n}$$

$$n = \frac{2mx}{x^2+1}$$

Solution 15:

$$\frac{x^3+3xy^2}{3x^2y+y^3} = \frac{m^3+3mn^2}{3m^2n+n^3}$$

applying componendo and dividendo,

$$\frac{x^3+3xy^2}{3x^2y+y^3}=\frac{m^3+3mn^2}{3m^2n+n^3}$$

$$\frac{x^3+3xy^2+3x^2y+y^3}{x^3+3xy^2-3x^2y-y^3}=\frac{m^3+3mn^2+3m^2n+n^3}{m^3+3mn^2-3m^2n-n^3}$$

$$\frac{(x+y)^3}{(x-y)^3}=\frac{(m+n)^3}{(m-n)^3}$$

$$\frac{x+y}{x-y}=\frac{m+n}{m-n}$$

applying componendo and dividendo,

$$\frac{x+y+x-y}{x+y-x+y}=\frac{m+n+m-n}{m+n-m+n}$$

$$\frac{2x}{2y}=\frac{2m}{2n}$$

$$\frac{x}{y}=\frac{m}{n}$$

$$nx=my$$

EXERCISE.7 (D)

Solution 1:

Given, $\frac{a}{b}=\frac{3}{5}$

$$\frac{10a+3b}{5a+2b}$$

$$=\frac{10\left(\frac{a}{b}\right)+3}{5\left(\frac{a}{b}\right)+2}b$$

$$=\frac{10\left(\frac{3}{5}\right)+3}{5\left(\frac{3}{5}\right)+2}$$
$$=\frac{6+3}{3+2}$$
$$=\frac{9}{5}$$

Solution 2:

$$\frac{5x+6y}{8x+5y}=\frac{8}{9}$$
$$45x+54y=64x+40y$$
$$64x-45x=54y-40y$$
$$19x=14y$$
$$\frac{x}{y}=\frac{14}{19}$$

Solution 3:

$(3x - 4y): (2x - 3y) = (5x - 6y): (4x - 5y)$

$$\frac{3x-4y}{2x-3y}=\frac{5x-6y}{4x-5y}$$

applying componendo and dividendo,

$$\frac{3x-4y+2x-3y}{3x-4y-2x+3y}=\frac{5x-6y+4x-5y}{5x-6y-4x+5y}$$
$$\frac{5x-7y}{x-y}=\frac{9x-11y}{x-y}$$
$$5x-7y=9x-11y$$
$$11y-7y=9x-5x$$
$$4y=4x$$
$$\frac{x}{y}=\frac{1}{1}$$
$$x:1=1:1$$

Solution 4:

(i) Duplicate ratio of $2\sqrt{2}:3\sqrt{5}=\left(2\sqrt{2}\right)^2:\left(3\sqrt{5}\right)^2=8:45$

(ii) Triplicate ratio of 2a: 3b = $(2a)^3 : (3b)^3 = 8a^3 : 27b^3$

(iii) Sub-duplicate ratio of $9x^2a^4 : 25y^6b^2 = \sqrt{9x^2a^4} : \sqrt{25y^6b^2} = 3xa^2 : 5y^3b$

(iv) Sub-triplicate ratio of 216: 343 = $\sqrt[3]{216}:\sqrt[3]{343}=6:7$

(v) Reciprocal ratio of 3: 5 = 5: 3

(vi) Duplicate ratio of 5: 6 = 25: 36

Reciprocal ratio of 25: 42 = 42: 25

Sub-duplicate ratio of 36: 49 = 6: 7

Required compound ratio = $\dfrac{25\times42\times6}{36\times25\times7}=1:1$

Solution 5:

(i) (2x + 3): (5x - 38) is the duplicate ratio of $\sqrt{5}:\sqrt{6}$

Duplicate ratio of $\sqrt{5}:\sqrt{6}=5:6$

$$\frac{2x+3}{5x-38}=\frac{5}{6}$$

$$12x+18=25x-190$$

$$25x-12x=190+18$$

$$13x=208$$

$$x=\frac{208}{13}=16$$

(ii) (2x + 1): (3x + 13) is the sub-duplicate ratio of 9: 25

Sub-duplicate ratio of 9: 25 = 3: 5

$$\frac{2x+1}{3x+13}=\frac{3}{5}$$

$$10x+5=9x+39$$

$$10x-9x=39-5$$

$$x=34$$

(iii) (3x - 7): (4x + 3) is the sub-triplicate ratio of 8: 27

Sub-triplicate ratio of 8: 27 = 2: 3

$$\frac{3x-7}{4x+3}=\frac{2}{3}$$
$$9x-21=8x+6$$
$$9x-8x=6+21$$
$$x=27$$

Solution 6:

Let the required quantity which is to be added be p.
Then, we have:

$$\frac{x+p}{y+p}=\frac{c}{d}$$
$$dx+pd=cy+cp$$
$$pd-cp=cy-dx$$
$$p(d-c)=cy-dx$$
$$p=\frac{cy-dx}{d-c}$$

Solution 7:

Let the two numbers be 5x and 7x.
From the given information,

$$\frac{5x-3}{7x-3}=\frac{2}{3}$$
$$15x-9=14x-6$$
$$15x-14x=9-6$$
$$x=3$$

Thus, the numbers are 5x = 15 and 7x = 21.

Solution 8:

$$15\left(2x^2-y^2\right)=7xy$$
$$\frac{2x^2-y^2}{xy}=\frac{7}{15}$$

$$\frac{2x}{y}-\frac{y}{x}=\frac{7}{15}$$

Let $\frac{x}{y}=a$

$$\therefore 2a-\frac{1}{a}=\frac{7}{15}$$

$$\frac{2a^2-1}{a}=\frac{7}{15}$$

$$30a^2-15=7a$$

$$30a^2-7a-15=0$$

$$30a^2-25a+18a-15=0$$

$$5a(6a-5)+3(6a-5)=0$$

$$(6a-5)(5a+3)=0$$

$$a=\frac{5}{6},-\frac{3}{5}$$

But, a cannot be negative

$$\therefore a=\frac{5}{6}$$

$$\Rightarrow \frac{x}{y}=\frac{5}{6}$$

$$\Rightarrow x:y=5:6$$

Solution 9:

(i) Let the fourth proportional to $2xy$, x^2 and y^2 be n.

$\Rightarrow 2xy : x^2 = y^2 : n$

$\Rightarrow 2xy \times n = x^2 \times y^2$

$$\Rightarrow n=\frac{x^2y^2}{2xy}=\frac{xy}{2}$$

(ii) Let the third proportional to a^2-b^2 and $a+b$ be n.

$\Rightarrow a^2-b^2$, $a+b$ and n are in continued proportion.

$\Rightarrow a^2-b^2 : a+b = a+b : n$

$$\Rightarrow n=\frac{(a+b)^2}{a^2-b^2}=\frac{(a+b)^2}{(a+b)(a-b)}=\frac{a+b}{a-b}$$

(iii) Let the mean proportional to $(x-y)$ and (x^3-x^2y) be n.

$\Rightarrow$ (x − y), n, $(x^3 - x^2y)$ are in continued proportion

$\Rightarrow$ (x − y) : n = n : $(x^3 - x^2y)$

$$\Rightarrow n^2 = (x-y)(x^3 - x^2y)$$

$$\Rightarrow n^2 = x^2(x-y)(x-y)$$

$$\Rightarrow n^2 = x^2(x-y)^2$$

$$\Rightarrow n = x(x-y)$$

Solution 10:

Let the required numbers be a and b.

Given, 14 is the mean proportional between a and b.

$\Rightarrow$ a: 14 = 14: b

$\Rightarrow$ ab = 196

$$\Rightarrow a = \frac{196}{b} \text{........ (1)}$$

Also, given, third proportional to a and b is 112.

$\Rightarrow$ a: b = b: 112

$$\Rightarrow b^2 = 112a \text{..........(2)}$$

Using (1), we have:

$$b^2 = 112 \times \frac{196}{b}$$

$$b^3 = (14)^3(2)^3$$

$$b = 28$$

From (1),

$$a = \frac{196}{28} = 7$$

Thus, the two numbers are 7 and 28

Solution 11:

$$\text{Given, } \frac{x}{y} = \frac{(x+z)^2}{(y+z)^2}$$

$$x(y^2 + z^2 + 2yz) = y(x^2 + z^2 + 2xz)$$

$$xy^2 + xz^2 + 2xyz = x^2y + yz^2 + 2xyz$$

$$xy^2 + xz^2 = x^2y + yz^2$$
$$xy^2 - x^2y = yz^2 - xz^2$$
$$xy(y-x) = z^2(y-x)$$
$$xy = z^2$$

Hence, z is mean proportional between x and y.

Solution 12:

Since, q is the mean proportional between p and r,

$q^2 = pr$

$$\text{L..H.S} = \frac{p^3 + q^3 + r^3}{p^2q^2r^2}$$
$$= \frac{p^3 + (pr)q + r^3}{p^2(pr)r^2}$$
$$= \frac{p^3 + prq + r^3}{p^3r^3}$$
$$= \frac{1}{r^3} + \frac{q}{p^2r^2} + \frac{1}{p^3}$$
$$= \frac{1}{r^3} + \frac{q}{(q^2)^2} + \frac{1}{p^3}$$
$$= \frac{1}{r^3} + \frac{1}{q^3} + \frac{1}{p^3}$$
$$= \text{R.H.S}$$

Solution 13:

Given, a, b and c are in continued proportion.

$\Rightarrow$ a: b = b: c

$$\text{Let} \frac{a}{b} = \frac{b}{c} = k(\text{say})$$
$$\Rightarrow a = bk,\ b = ck$$
$$\Rightarrow a = ck^2,\ b = ck$$

$$\text{Now, L.H.S} = \frac{a}{c} = \frac{ck^2}{c} = k^2$$

$$\text{R.H.S} = \frac{a^2+b^2}{b^2+c^2}$$

$$= \frac{\left(ck^2\right)^2 + (ck)^2}{(ck)^2 + c^2}$$

$$= \frac{c^2k^2 + c^2k^2}{c^2k^2 + c^2}$$

$$= \frac{c^2k^2\left(k^2+1\right)}{c^2\left(k^2+1\right)}$$

$$= k^2$$

$$\therefore \text{ L.H.S} = \text{R.H.S}$$

Solution 14:

$$x = \frac{2ab}{a+b}$$

$$\frac{x}{a} = \frac{2ab}{a+b}$$

applying componendo and dividendo,

$$\frac{x+a}{x-a} = \frac{2b+a+b}{2b-a-b}$$

$$\frac{x+a}{x-a} = \frac{3b+a}{b-a} \qquad ....(1)$$

$$\text{Also, } x = \frac{2ab}{a+b}$$

applying componendo and dividendo,

$$\frac{x+b}{x-b}=\frac{2a+a+b}{2a-a-b}$$
$$\frac{x+b}{x-b}=\frac{3a+a}{a-b} \qquad ....(2)$$
From (1) and (2)
$$\frac{x+a}{x-a}+\frac{x+b}{x-b}=\frac{3b+a}{b-a}+\frac{3a+b}{a-b}$$
$$\frac{x+a}{x-a}+\frac{x+b}{x-b}=\frac{-3b-a+3a+b}{a-b}$$
$$\frac{x+a}{x-a}+\frac{x+b}{x-b}=\frac{2a-2b}{a-b}=2$$

Solution 15:

Given, $\frac{4a+9b}{4a-9b}=\frac{4c+9d}{4c-9d}$

applying componendo and dividendo,
$$\frac{4a+9b+4a-9d}{4a+9b-4a+9b}=\frac{4c+9d+4c-9d}{4c+9d-4c+9d}$$
$$\frac{a}{b}=\frac{c}{d}$$
$$\frac{8a}{18b}=\frac{8c}{18d}$$

Solution 16:

Let $\frac{a}{b}=\frac{c}{d}=k(\text{say})$

$\Rightarrow a=bk, c=dk$

$\text{L.H.S}=\frac{a+b}{c+d}$

$$= \frac{bk + b}{dk + d}$$
$$= \frac{b(k+1)}{d(k+1)}$$
$$= \frac{b}{d}$$
$$\text{R.H.S} = \frac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}}$$
$$= \frac{\sqrt{(bk)^2 + b^2}}{\sqrt{(dk)^2 + b^2}}$$
$$= \frac{\sqrt{b^2(k^2+1)}}{\sqrt{d^2(k^2+1)}}$$
$$= \frac{\sqrt{b^2}}{\sqrt{d^2}}$$
$$= \frac{b}{d}$$
$$\therefore \text{L..H.S} = \text{R.H.S}$$

Solution 17:

$$\text{Let } \frac{x}{a} = \frac{y}{b} = \frac{z}{c} = k(\text{say})$$
$$\Rightarrow x = ak, y = bk, z = ck$$
L.H.S
$$= \frac{ax - by}{(a+b)(x-y)} + \frac{by - cz}{(b+c)(y-z)} + \frac{cz - ax}{(c+a)(z-x)}$$

$$= \frac{a(ak)-b(bk)}{(a+b)(ak-bk)} + \frac{b(bk)-c(ck)}{(b+c)(bk-ck)} + \frac{c(ck)-a(ak)}{(c+a)(ck-ak)}$$

$$= \frac{k(a^2-b^2)}{k(a+b)(a-b)} + \frac{k(b^2-c^2)}{k(b+c)(b-c)} + \frac{k(c^2-a^2)}{k(c+a)(c-a)}$$

$$= \frac{k(a^2-b^2)}{k(a^2-b^2)} + \frac{k(b^2-c^2)}{k(b^2-c^2)} + \frac{k(c^2-a^2)}{k(c^2-a^2)}$$

$$= 1+1+1 = 3 = \text{R.H.S}$$

Solution 18:

Ratio of number of boys to the number of girls = 3: 1
Let the number of boys be 3x and number of girls be x.
∴ 3x + x = 36
4x = 36
x = 9∴ Number of boys = 27
Number of girls = 9
Le n number of girls be added to the council.
From given information, we have:

$$\frac{27}{9+n} = \frac{9}{5}$$

$$135 = 81 + 9n$$

$$9n = 54$$

$$n = 6$$

Thus, 6 girls are added to the council.

Solution 19:

Given, $\frac{x}{b-c} = \frac{y}{c-a} = \frac{z}{a-b} = k(\text{say})$

$$x = k(b-c), y = k(c-a), z = k(a-b)$$

$$ax + by + cz$$

$$= ak(b-c) + bk(c-a) + ck(a-b)$$

$$= abk - ack + bck - abk + ack - bck$$

$$= 0$$

Solution 20:

$7x - 15y = 4x + y$
$7x - 4x = y + 15y$
$3x = 16y$

$$\frac{x}{y} = \frac{16}{3}$$

$$\text{(i)}\ \frac{x}{y} = \frac{16}{3}$$

$$\Rightarrow \frac{9x}{5y} = \frac{144}{15} \qquad \left(\text{Multiplying both sides by } \frac{9}{5}\right)$$

$$\Rightarrow \frac{9x+5y}{9x-5y} = \frac{144+15}{144-15} \qquad (\text{applying componendo and dividendo})$$

$$\Rightarrow \frac{9x+5y}{9x-5y} = \frac{159}{129} = \frac{53}{43}$$

$$\text{(ii)}\ \frac{x}{y} = \frac{16}{3}$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{259}{9}$$

$$\Rightarrow \frac{3x^2}{2y^2} = \frac{786}{18} = \frac{128}{3} \qquad \left(\text{Multiplying both sides by } \frac{3}{2}\right)$$

$$\Rightarrow \frac{3x^2+2y^2}{3x^2-2y^2} = \frac{128+3}{128-3} \qquad (\text{applying componendo and dividendo})$$

$$\Rightarrow \frac{3x^2+2y^2}{3x^2-2y^2} = \frac{131}{125}$$

Solution 21:

$$\text{(i) Given, } \frac{4m+3n}{4m-3n} = \frac{7}{4}$$

applying componendo and dividendo,

$$\frac{4m+3n+4m-3n}{4m+3n-4m+3n} = \frac{7+4}{7-4}$$

$$\frac{8m}{6n} = \frac{11}{3}$$

$$\frac{m}{n}=\frac{11}{4}$$

(ii) $\frac{m}{n}=\frac{11}{4}$

$$\frac{m^2}{n^2}=\frac{121}{16}$$

$$\frac{2m^2}{11n^2}=\frac{2\times121}{11\times16}\quad\left(\text{Multiplying both sides by }\frac{2}{11}\right)$$

$$\frac{2m^2}{11n^2}=\frac{11}{8}$$

$$\frac{2m^2+11n^2}{2m^2-11n^2}=\frac{11+8}{11-8}\quad(\text{Applying componendo and dividendo})$$

$$\frac{2m^2+11n^2}{2m^2-11n^2}=\frac{19}{3}$$

$$\frac{2m^2-11n^2}{2m^2+11n^2}=\frac{3}{19}\quad(\text{Applying invertendo})$$

Solution 22:

∵ x, y, z are in continued proportion,

$$\therefore \frac{x}{y}=\frac{y}{z}\Rightarrow y^2=zx\ldots\ldots(1)$$

Therefore,

$$\frac{x+y}{y}=\frac{y+z}{z}\quad(\text{By componendo})$$

$$\Rightarrow\frac{x+y}{y+z}=\frac{y}{z}\quad(\text{By alternendo})$$

$$\Rightarrow\frac{(x+y)^2}{(y+z)^2}=\frac{y^2}{z^2}\quad(\text{Squaring both sides})$$

$$\Rightarrow\frac{(x+y)^2}{(y+z)^2}=\frac{zx}{z^2}\quad[\text{from}(1)]$$

$$\Rightarrow\frac{(x+y)^2}{(y+z)^2}=\frac{x}{z}$$

Hence Proved.

Solution 23:

$$x=\frac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}-\sqrt{a^2-b^2}}$$

By componendo and dividendo,

$$\frac{x+1}{x-1}=\frac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}+\sqrt{a^2+b^2}-\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}-\sqrt{a^2-b^2}-\sqrt{a^2+b^2}+\sqrt{a^2-b^2}}$$

$$\frac{x+1}{x-1}=\frac{2\sqrt{a^2+b^2}}{2\sqrt{a^2-b^2}}$$

Squaring both sides,

$$\frac{x^2+2x+1}{x^2-2x+1}=\frac{a^2+b^2}{a^2-b^2}$$

By componendo and dividendo,

$$\frac{\left(x^2+2x+1\right)+\left(x^2-2x+1\right)}{\left(x^2+2x+1\right)-\left(x^2-2x+1\right)}=\frac{\left(a^2+b^2\right)+\left(a^2-b^2\right)}{\left(a^2+b^2\right)-\left(a^2-b^2\right)}$$

$$\Rightarrow \frac{2\left(x^2+1\right)}{4x}=\frac{2a^2}{2b^2}$$

$$\Rightarrow \frac{x^2+1}{2x}=\frac{a^2}{b^2}$$

$$\Rightarrow b^2=\frac{2a^2x}{x^2+1}$$

Hence Proved

Solution 24:

$$\text{(i)Given, } \frac{x^2+y^2}{x^2-y^2}=2\frac{1}{8}$$

$$\frac{x^2+y^2}{x^2-y^2}=\frac{17}{8}$$

Applying componendo and dividendo,

$$\frac{x^2+y^2+x^2-y^2}{x^2+y^2-x^2+y^2}=\frac{17+8}{17-8}$$

$$\frac{2x^2}{2y^2}=\frac{25}{9}$$

$$\frac{x^2}{y^2}=\frac{25}{9}$$

$$\frac{x}{y}=\frac{5}{3}=1\frac{2}{3}$$

(ii) $\dfrac{x^3+y^3}{x^3-y^3}$

$$=\frac{\left(\frac{x}{y}\right)^3+1}{\left(\frac{x}{y}\right)^3-1}$$

$$=\frac{\left(\frac{5}{3}\right)^3+1}{\left(\frac{5}{3}\right)^3-1}$$

$$=\frac{\frac{125}{27}+1}{\frac{125}{27}-1}$$

$$=\frac{\frac{125+27}{27}}{\frac{125-27}{27}}$$

$$=\frac{125+27}{125-27}$$

$$=\frac{76}{49}=1\frac{27}{49}$$

Solution 25:

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rations such that $\frac{a}{b}=\frac{c}{d}$,

Then by componendo-dividendo,

We have $\dfrac{a+b}{a-b}=\dfrac{c+d}{c-d}$

Given that

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}}=9$$

$$\Rightarrow \frac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}}=\frac{9}{1}$$

$$\Rightarrow \frac{\left(\sqrt{3x+4}+\sqrt{3x-5}+\sqrt{3x+4}-\sqrt{3x-5}\right)}{\left(\sqrt{3x+4}+\sqrt{3x-5}-\sqrt{3x+4}-\sqrt{3x-5}\right)}=\frac{9+1}{9-1}\ \left[\text{Applying componendo – Dividendo}\right]$$

$$\Rightarrow \frac{2\sqrt{3x+4}}{2\sqrt{3x-5}}=\frac{10}{8}$$

$$\Rightarrow \frac{\sqrt{3x+4}}{\sqrt{3x-5}}=\frac{5}{4}$$

$$\Rightarrow 4\sqrt{3x+4}=5\sqrt{3x-5}$$

Squaring both the sides of the above equation, we have,

$$16(3x+4)=25(3x-5)$$

$$\Rightarrow 16(3x+4)=25(3x-5)$$

$$\Rightarrow 48x+64=75x-125$$

$$\Rightarrow 27x=189$$

$$\Rightarrow x=\frac{189}{27}$$

$$\Rightarrow x=7$$

Solution 26:

given that, $x=\frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$

By Applying componendo – dividendo,

$$\frac{x+1}{x-1}=\frac{\left(\sqrt{a+1}+\sqrt{a-1}\right)+\left(\sqrt{a+1}+\sqrt{a-1}\right)}{\left(\sqrt{a+1}+\sqrt{a-1}\right)-\left(\sqrt{a+1}-\sqrt{a-1}\right)}$$

$$\Rightarrow \frac{x+1}{x-1}=\frac{2\sqrt{a+1}}{2\sqrt{a-1}}$$

$$\Rightarrow \frac{x+1}{x-1}=\frac{\sqrt{a+1}}{\sqrt{a-1}}$$

Squaring both the sides if the equation, we have,

$$\Rightarrow \left(\frac{x+1}{x-1}\right)^2=\frac{a+1}{a-1}$$

$\Rightarrow (x+1)^2 (a-1) = (x-1)^2 (a+1)$

$\Rightarrow (x^2 + 2x + 1)(a-1) = (x^2 - 2x + 1)(a+1)$

$\Rightarrow a(x^2 + 2x + 1) - (x^2 + 2x + 1) = a(x^2 - 2x + 1) + (x^2 - 2x + 1)$

$\Rightarrow 4ax = 2x^2 + 2$

$\Rightarrow 2ax = x^2 + 1$

$\Rightarrow x^2 - 2ax + 1 = 0$