Exercise 12.1

Question 1:

Evaluate

(i) 3^{-2} (ii) $(-4)^{-2}$ (iii)

$\left(\dfrac{1}{2}\right)^{-5}$ Answer:

(i) $3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$ $\left(a^{-m} = \dfrac{1}{a^m}\right)$

(ii) $(-4)^{-2} = \dfrac{1}{(-4)^2} = \dfrac{1}{16}$ $\left(a^{-m} = \dfrac{1}{a^m}\right)$

(iii) $\left(\dfrac{1}{2}\right)^{-5} = \dfrac{1}{(2)^{-5}} = (2)^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

Question 2:

Simplify and express the result in power notation with positive exponent.

(i) $(-4)^5 \div (-4)^8$ (ii) $\left(\dfrac{1}{2^3}\right)^2$

(iii) $(-3)^4 \times \left(\dfrac{5}{3}\right)^4$ (iv) $\left(3^{-7} \div 3^{-10}\right) \times 3^{-5}$

(v) $2^{-3} \times (-7)^{-3}$

Answer:

(i) $(-4)^5 \div (-4)^8 = (-4)^{5-8}$ ($a^m \div a^n = a^{m-n}$)

$= (-4)^{-3}$

$= \dfrac{1}{(-4)^3}$ $\left(a^{-m} = \dfrac{1}{a^m}\right)$

(ii)
$$\left(\frac{1}{2^3}\right)^2 = \frac{1}{\left(2^3\right)^2} = \frac{1}{2^6} \qquad \left(\left(a^m\right)^n = a^{mn}\right)$$

(iii)
$$(-3)^4 \times \left(\frac{5}{3}\right)^4 = (-1 \times 3)^4 \times \frac{5^4}{3^4}$$

$$= (-1)^4 \times 3^4 \times \frac{5^4}{3^4} \qquad \left[(ab)^m = a^m \times b^m\right]$$

$$= (-1)^4 \times 5^4$$

$$= 5^4 \qquad \left[(-1)^4 = 1\right]$$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5} = (3^{-7-(-10)}) \times 3^{-5}$ $(a^m \div a^n = a^{m-n})$

$= 3^3 \times 3^{-5}$

$= 3^{3+(-5)}$ $(a^m \times a^n = a^{m+n})$

$= 3^{-2}$

$$= \frac{1}{3^2} \qquad \left(a^{-m} = \frac{1}{a^m}\right)$$

(v) $2^{-3} \times (-7)^{-3} = \dfrac{1}{2^3} \times \dfrac{1}{(-7)^3}$ $\qquad \left(a^{-m} = \frac{1}{a^m}\right)$

$$= \frac{1}{\left[2 \times (-7)\right]^3} \qquad \left[a^m \times b^m = (ab)^m\right]$$

$$= \frac{1}{(-14)^3}$$

Question 3:

Find the value of.

(i) $(3^0 + 4^{-1}) \times 2^2$ (ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$

(iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$ (iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$

(v) $\left\{\left(\dfrac{-2}{3}\right)^{-2}\right\}^{2}$

Answer:

(i) $(3^{0}+4^{-1})\times 2^{2} = \left(1+\dfrac{1}{4}\right)\times 2^{2}$ $\left(a^{0}=1 \text{ and } a^{-m}=\dfrac{1}{a^{m}}\right)$

$=\dfrac{5}{4}\times 4 = 5$

(ii) $(2^{-1}\times 4^{-1})\div 2^{-2} = [2^{-1}\times\{(2)^{2}\}^{-1}]\div 2^{-2}$

$=(2^{-1}\times 2^{-2})\div 2^{-2}$ $\left((a^{m})^{n}=a^{mn}\right)$

$= 2^{-1+(-2)}\div 2^{-2}$ $(a^{m}\times a^{n}=a^{m+n})$

$= 2^{-3}\div 2^{-2}$

$= 2^{-3-(-2)}$ $(a^{m}\div a^{n}=a^{m-n})$

$= 2^{-3+2} = 2^{-1}$

$=\dfrac{1}{2}$ $\left(a^{-m}=\dfrac{1}{a^{m}}\right)$

(iii) $\left(\dfrac{1}{2}\right)^{-2}+\left(\dfrac{1}{3}\right)^{-2}+\left(\dfrac{1}{4}\right)^{-2} = \left(\dfrac{2}{1}\right)^{2}+\left(\dfrac{3}{1}\right)^{2}+\left(\dfrac{4}{1}\right)^{2}$ $\left(\therefore a^{-m}=\dfrac{1}{a^{m}}\right)$

$= 2^{2}+3^{2}+4^{2} = 4+9+16 = 29$

(iv) $(3^{-1}+4^{-1}+5^{-1})^{0}$ $=\left(\dfrac{1}{3}+\dfrac{1}{4}+\dfrac{1}{5}\right)^{0}$ $\left(a^{-m}=\dfrac{1}{a^{m}}\right)$

$= 1$ $(a^{0}=1)$

(v) $\left\{\left(\dfrac{-2}{3}\right)^{-2}\right\}^{2} = \left\{\left(\dfrac{3}{-2}\right)^{2}\right\}^{2}$ $\left(a^{-m}=\dfrac{1}{a^{m}}\right)$

$$= \left\{ \frac{3^2}{(-2)^2} \right\}^2 \qquad\qquad \left[\left(\frac{a}{b} \right)^m = \frac{a^m}{b^m} \right]$$

$$= \left(\frac{9}{4} \right)^2 = \frac{81}{16}$$

Question 4:

Evaluate (i) $\dfrac{8^{-1} \times 5^3}{2^{-4}}$ (ii) $\left(5^{-1} \times 2^{-1} \right) \times 6^{-1}$

Answer:

(i) $\dfrac{8^{-1} \times 5^3}{2^{-4}} = \dfrac{2^4 \times 5^3}{8^1}$ $\qquad\qquad \left(a^{-m} = \dfrac{1}{a^m} \right)$

$= \dfrac{2^4 \times 5^3}{2^3} = 2^{4-3} \times 5^3$ $\qquad\qquad \left(a^m \div a^n = a^{m-n} \right)$

$= 2 \times 125 = 250$

(ii) $(5^{-1} \times 2^{-1}) \times 6^{-1} = \left(\dfrac{1}{5} \times \dfrac{1}{2} \right) \times \dfrac{1}{6}$ $\qquad \left(a^{-m} = \dfrac{1}{a^m} \right)$

$= \dfrac{1}{10} \times \dfrac{1}{6} = \dfrac{1}{60}$

Question 5:

Find the value of m for which $5^m \div 5^{-3} = 5^5$.

Answer:

$5^m \div 5^{-3} = 5^5$

$5^{m-(-3)} = 5^5 \ (a^m \div a^n = a^{m-n})$

$5^{m+3} = 5^5$

Since the powers have same bases on both sides, their respective exponents must be equal.

m + 3 = 5

m = 5 − 3

m = 2

Question 6:

Evaluate (i) $\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}$ (ii) $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}$

Answer:

(i) $\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}=\left\{\left(\frac{3}{1}\right)^{1}-\left(\frac{4}{1}\right)^{1}\right\}^{-1}$ $\left(a^{-m}=\frac{1}{a^{m}}\right)$

$=\{3-4\}^{-1}=(-1)^{-1}=\frac{1}{-1}=-1$

(ii) $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}=\frac{5^{-7}}{8^{-7}}\times\frac{8^{-4}}{5^{-4}}$ $\left[\left(\frac{a}{b}\right)^{m}=\frac{a^{m}}{b^{m}}\right]$

$=\frac{8^{7}}{5^{7}}\times\frac{5^{4}}{8^{4}}$ $\left(a^{-m}=\frac{1}{a^{m}}\right)$

$=\frac{8^{7-4}}{5^{7-4}}$ $\left(a^{m}\div a^{n}=a^{m-n}\right)$

$=\frac{8^{3}}{5^{3}}=\frac{512}{125}$

Question 7:

Simplify. (i) $\frac{25\times t^{-4}}{5^{-3}\times10\times t^{-8}}(t\neq0)$ (ii) $\frac{3^{-5}\times10^{-5}\times125}{5^{-7}\times6^{-5}}$

Answer:

(i) $\frac{25\times t^{-4}}{5^{-3}\times10\times t^{-8}}=\frac{5^{2}\times t^{-4}}{5^{-3}\times5\times2\times t^{-8}}$

$$= \frac{5^2 \times t^{-4}}{5^{-3+1} \times 2 \times t^{-8}} \qquad \left(a^m \times a^n = a^{m+n}\right)$$

$$= \frac{5^2 \times t^{-4}}{5^{-2} \times 2 \times t^{-8}}$$

$$= \frac{5^{2-(-2)} t^{-4-(-8)}}{2} \qquad \left(a^m \div a^n = a^{m-n}\right)$$

$$= \frac{5^4 t^4}{2} = \frac{625 \, t^4}{2}$$

(ii) $$\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}} = \frac{3^{-5} \times (2 \times 5)^{-5} \times 5^3}{5^{-7} \times (2 \times 3)^{-5}}$$

$$= \frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}} \qquad \left[(a \times b)^m = a^m \times b^m\right]$$

$$= 3^{-5-(-5)} \times 2^{-5-(-5)} \times 5^{-5+3-(-7)} \qquad \left(a^m \div a^n = a^{m-n}\right)$$

$$= 3^0 \times 2^0 \times 5^5 \qquad \left(a^0 = 1\right)$$

$$= 5^5$$

Exercise 12.2

Question 1:

Express the following numbers in standard form.

(i) 0.0000000000085 (ii) 0.00000000000942

(iii) 6020000000000000 (iv) 0.00000000837

(v) 31860000000

(i) $0.0000000000085 = 8.5 \times 10^{-12}$

(ii) $0.00000000000942 = 9.42 \times 10^{-12}$

(iii) $6020000000000000 = 6.02 \times 10^{15}$

(iv) $0.00000000837 = 8.37 \times 10^{-9}$

(v) $31860000000 = 3.186 \times 10^{10}$

(i) 3.02×10^{-6} (ii) 4.5×10^{4}

(iii) 3×10^{-8} (iv) 1.0001×10^{9}

(v) 5.8×10^{12} (vi) 3.61492×10^{6}

Answer:

(i) $3.02 \times 10^{-6} = 0.00000302$

(ii) $4.5 \times 10^{4} = 45000$

(iii) $3 \times 10^{-8} = 0.00000003$

(iv) $1.0001 \times 10^{9} = 1000100000$

(v) $5.8 \times 10^{12} = 5800000000000$

(vi) $3.61492 \times 10^{6} = 3614920$

Question 3:

Express the number appearing in the following statements in standard form.

(i) 1 micron is equal to $\dfrac{1}{1000000}$ m.

(ii) Charge of an electron is 0.000, 000, 000, 000, 000, 16 coulomb.

(iii) Size of a bacteria is 0.0000005 m

(iv) Size of a plant cell is 0.00001275 m

(v) Thickness of a thick paper is 0.07 mm

Answer:

(i) $\dfrac{1}{1000000}$ = 1 × 10^{-6}

(ii) 0.000, 000, 000, 000, 000, 000, 16 = 1.6 × 10^{-19}

(iii) 0.0000005 = 5 × 10^{-7}

(iv) 0.00001275 = 1.275 × 10^{-5}

(v) 0.07 = 7 × 10^{-2}

In a stack there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack?

Thickness of each book = 20 mm

Hence, thickness of 5 books = (5 × 20) mm = 100 mm

Thickness of each paper sheet = 0.016 mm

Hence, thickness of 5 paper sheets = (5 × 0.016) mm = 0.080 mm

Total thickness of the stack = Thickness of 5 books + Thickness of 5 paper sheets

= (100 + 0.080) mm

= 100.08 mm

= 1.0008 × 10^{2} mm