

Exercise 7.1

Question 1:

Which of the following numbers are not perfect cubes?

(i) 216

(ii) 128

(iii) 1000

(iv) 100

(v) 46656

Answer:

(i) The prime factorisation of 216 is as follows.

2	216
2	108
2	54
3	27
3	9
3	3
	1

$$216 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} = 2^3 \times 3^3$$

Here, as each prime factor is appearing as many times as a perfect multiple of 3, therefore, 216 is a perfect cube.

(ii) The prime factorisation of 128 is as follows.

2	128
2	64

2	32
2	16
2	8
2	4
2	2
	1

$$128 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 2$$

Here, each prime factor is not appearing as many times as a perfect multiple of 3. One 2 is remaining after grouping the triplets of 2. Therefore, 128 is not a perfect cube.

(iii) The prime factorisation of 1000 is as follows

2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$1000 = \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$$

Here, as each prime factor is appearing as many times as a perfect multiple of 3, therefore, 1000 is a perfect cube.

(iv) The prime factorisation of 100 is as follows.

2	100
2	50
5	25
5	5
	1

$$100 = 2 \times 2 \times 5 \times 5$$

Here, each prime factor is not appearing as many times as a perfect multiple of 3. Two 2s and two 5s are remaining after grouping the triplets. Therefore, 100 is not a perfect cube.

(v) The prime factorisation of 46656 is as follows.

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81

3	27
3	9
3	3
	1

$$46656 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$$

Here, as each prime factor is appearing as many times as a perfect multiple of 3, therefore, 46656 is a perfect cube.

Question 2:

Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube.

(i) 243

(ii) 256

(iii) 72

(iv) 675

(v) 100

Answer:

$$(i) 243 = \underline{3 \times 3 \times 3} \times 3 \times 3$$

Here, two 3s are left which are not in a triplet. To make 243 a cube, one more 3 is required.

In that case, $243 \times 3 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} = 729$ is a perfect cube.

Hence, the smallest natural number by which 243 should be multiplied to make it a perfect cube is 3.

$$(ii) 256 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 2 \times 2$$

Here, two 2s are left which are not in a triplet. To make 256 a cube, one more 2 is required.

Then, we obtain

$$256 \times 2 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} = 512 \text{ is a perfect cube.}$$

Hence, the smallest natural number by which 256 should be multiplied to make it a perfect cube is 2.

(iii) $72 = \underline{2 \times 2 \times 2} \times 3 \times 3$

Here, two 3s are left which are not in a triplet. To make 72 a cube, one more 3 is required.

Then, we obtain

$72 \times 3 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} = 216$ is a perfect cube.

Hence, the smallest natural number by which 72 should be multiplied to make it a perfect cube is 3.

(iv) $675 = \underline{3 \times 3 \times 3} \times 5 \times 5$

Here, two 5s are left which are not in a triplet. To make 675 a cube, one more 5 is required.

Then, we obtain

$675 \times 5 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5} = 3375$ is a perfect cube.

Hence, the smallest natural number by which 675 should be multiplied to make it a perfect cube is 5.

(v) $100 = 2 \times 2 \times 5 \times 5$

Here, two 2s and two 5s are left which are not in a triplet. To make 100 a cube, we require one more 2 and one more 5.

Then, we obtain

$100 \times 2 \times 5 = \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5} = 1000$ is a perfect cube

Hence, the smallest natural number by which 100 should be multiplied to make it a perfect cube is $2 \times 5 = 10$.

Question 3:

Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.

(i) 81

(ii) 128

(iii) 135

(iv) 192

(v) 704

Answer:

(i) $81 = \underline{3 \times 3 \times 3} \times 3$

Here, one 3 is left which is not in a triplet.

If we divide 81 by 3, then it will become a perfect cube.

Thus, $81 \div 3 = 27 = \underline{3 \times 3 \times 3}$ is a perfect cube.

Hence, the smallest number by which 81 should be divided to make it a perfect cube is 3.

(ii) $128 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 2$ Here,
one 2 is left which is not in a triplet.

If we divide 128 by 2, then it will become a perfect cube. Thus,

$128 \div 2 = 64 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$ is a perfect cube.

Hence, the smallest number by which 128 should be divided to make it a perfect cube is 2.

(iii) $135 = \underline{3 \times 3 \times 3} \times 5$

Here, one 5 is left which is not in a triplet.

If we divide 135 by 5, then it will become a perfect cube.

Thus, $135 \div 5 = 27 = \underline{3 \times 3 \times 3}$ is a perfect cube.

Hence, the smallest number by which 135 should be divided to make it a perfect cube is 5.

(iv) $192 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 3$

Here, one 3 is left which is not in a triplet.

If we divide 192 by 3, then it will become a perfect cube. Thus,

$192 \div 3 = 64 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$ is a perfect cube.

Hence, the smallest number by which 192 should be divided to make it a perfect cube is 3.

(v) $704 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 11$

Here, one 11 is left which is not in a triplet.

If we divide 704 by 11, then it will become a perfect cube.

Thus, $704 \div 11 = 64 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$ is a perfect cube.

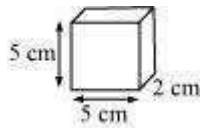
Hence, the smallest number by which 704 should be divided to make it a perfect cube is 11.

Question 4:

Parikshit makes a cuboid of plasticine of sides 5 cm, 2 cm, 5 cm. How many such cuboids will he need to form a cube?

Answer:

Here, some cuboids of size $5 \times 2 \times 5$ are given.



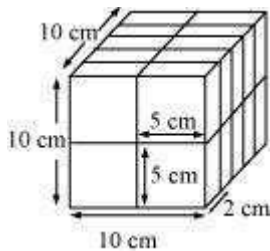
When these cuboids are arranged to form a cube, the side of this cube so formed will be a common multiple of the sides (i.e., 5, 2, and 5) of the given cuboid. LCM of 5, 2, and 5 = 10

Let us try to make a cube of 10 cm side.

For this arrangement, we have to put 2 cuboids along with its length, 5 along with its width, and 2 along with its height.

Total cuboids required according to this arrangement = $2 \times 5 \times 2 = 20$

With the help of 20 cuboids of such measures, a cube is formed as follows.



Alternatively

Volume of the cube of sides 5 cm, 2 cm, 5 cm

$$= 5 \text{ cm} \times 2 \text{ cm} \times 5 \text{ cm} = (5 \times 5 \times 2) \text{ cm}^3$$

Here, two 5s and one 2 are left which are not in a triplet.

If we multiply this expression by $2 \times 2 \times 5 = 20$, then it will become a perfect cube.

Thus, $(5 \times 5 \times 2 \times 2 \times 2 \times 5) = (5 \times 5 \times 5 \times 2 \times 2 \times 2) = 1000$ is a perfect cube.

Hence, 20 cuboids of 5 cm, 2 cm, 5 cm are required to form a cube.

Exercise 7.2

Question 1:

Find the cube root of each of the following numbers by prime factorisation method.

(i) 64

(ii) 512

(iii) 10648

(iv) 27000

(v) 15625

(vi) 13824

(vii) 110592

(viii) 46656

(ix) 175616

(x) 91125

Answer:

(i) Prime factorisation of $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$$\therefore \sqrt[3]{64} = 2 \times 2 = 4$$

(ii) Prime factorisation of $512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$$\therefore \sqrt[3]{512} = 2 \times 2 \times 2 = 8$$

(iii) Prime factorisation of $10648 = 2 \times 2 \times 2 \times 11 \times 11 \times 11$

$$\therefore \sqrt[3]{10648} = 2 \times 11 = 22$$

(iv) Prime factorisation of $27000 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$

$$\therefore \sqrt[3]{27000} = 2 \times 3 \times 5 = 30$$

(v) Prime factorisation of $15625 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$

$$\therefore \sqrt[3]{15625} = 5 \times 5 = 25$$

(vi) Prime factorisation of $13824 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$$\therefore \sqrt[3]{13824} = 2 \times 2 \times 2 \times 3 = 24$$

(vii) Prime factorisation of $110592 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

$$\therefore \sqrt[3]{110592} = 2 \times 2 \times 2 \times 3 = 48$$

(viii) Prime factorisation of $46656 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

$$\therefore \sqrt[3]{46656} = 2 \times 2 \times 3 \times 3 = 36$$

(ix) Prime factorisation of $175616 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7}$

$$\therefore \sqrt[3]{175616} = 2 \times 2 \times 2 \times 7 = 56$$

(x) Prime factorisation of $91125 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

$$\therefore \sqrt[3]{91125} = 3 \times 3 \times 5 = 45$$

Question 2:

State true or false.

- (i) Cube of any odd number is even.
- (ii) A perfect cube does not end with two zeroes.
- (iii) If square of a number ends with 5, then its cube ends with 25.
- (iv) There is no perfect cube which ends with 8.
- (v) The cube of a two digit number may be a three digit number.
- (vi) The cube of a two digit number may have seven or more digits.
- (vii) The cube of a single digit number may be a single digit number.

Answer:

For finding the cube of any number, the number is first multiplied with itself and this product is again multiplied with this number.

- (i) False. When we find out the cube of an odd number, we will find an odd number as the result because the unit place digit of an odd number is odd and we are multiplying three odd numbers. Therefore, the product will be again an odd number. For example, the cube of 3 (i.e., an odd number) is 27, which is again an odd number.

(ii) True. Perfect cube will end with a certain number of zeroes that are always a perfect multiple of 3.

For example, the cube of 10 is 1000 and there are 3 zeroes at the end of it. The cube of 100 is 1000000 and there are 6 zeroes at the end of it.

(iii) False. It is not always necessary that if the square of a number ends with 5, then its cube will end with 25.

For example, the square of 25 is 625 and 625 has its unit digit as 5. The cube of 25 is 15625. However, the square of 35 is 1225 and also has its unit place digit as 5 but the cube of 35 is 42875 which does not end with 25.

(iv) False. There are many cubes which will end with 8. The cubes of all the numbers having their unit place digit as 2 will end with 8.

The cube of 12 is 1728 and the cube of 22 is 10648.

(v) False. The smallest two-digit natural number is 10, and the cube of 10 is 1000 which has 4 digits in it.

(vi) False. The largest two-digit natural number is 99, and the cube of 99 is 970299 which has 6 digits in it. Therefore, the cube of any two-digit number cannot have 7 or more digits in it.

(vii) True, as the cube of 1 and 2 are 1 and 8 respectively.

Question 3:

You are told that 1331 is a perfect cube. Can you guess without factorization what is its cube root? Similarly, guess the cube roots of 4913, 12167, 32768 Answer:

Firstly, we will make groups of three digits starting from the rightmost digit of the number as $\overline{1\ 331}$.

There are 2 groups, 1 and 331, in it.

Considering 331,

The digit at its unit place is 1. We know that if the digit 1 is at the end of a perfect cube number, then its cube root will have its unit place digit as 1 only. Therefore, the unit place digit of the required cube root can be taken as 1. Taking the other group i.e., 1,

The cube of 1 exactly matches with the number of the second group. Therefore, the tens digit of our cube root will be taken as the unit place of the smaller number whose cube is near to the number of the second group i.e., 1 itself. 1 will be taken as tens place of the cube root of 1331.

Hence, $\sqrt[3]{1331} = 11$

The cube root of 4913 has to be calculated.

We will make groups of three digits starting from the rightmost digit of 4913,

as $\overline{4\ 913}$. The groups are 4 and 913.

Considering the group 913,

The number 913 ends with 3. We know that if the digit 3 is at the end of a perfect cube number, then its cube root will have its unit place digit as 7 only. Therefore, the unit place digit of the required cube root is taken as 7. Taking the other group i.e., 4,

We know that, $1^3 = 1$ and $2^3 = 8$

Also, $1 < 4 < 8$

Therefore, 1 will be taken at the tens place of the required cube root.

Thus, $\sqrt[3]{4913} = 17$

The cube root of 12167 has to be calculated.

We will make groups of three digits starting from the rightmost digit of the number

12167, as $\overline{12\ 167}$. The groups are 12 and 167.

Considering the group 167,

167 ends with 7. We know that if the digit 7 is at the end of a perfect cube number, then its cube root will have its unit place digit as 3 only. Therefore, the unit place digit of the required cube root can be taken as 3. Taking the other group i.e., 12,

We know that, $2^3 = 8$ and $3^3 = 27$

Also, $8 < 12 < 27$

2 is smaller between 2 and 3. Therefore, 2 will be taken at the tens place of the required cube root.

Thus, $\sqrt[3]{12167} = 23$

The cube root of 32768 has to be calculated.

We will make groups of three digits starting from the rightmost digit of the number

32768, as $\overline{32\,768}$.

Considering the group 768,

768 ends with 8. We know that if the digit 8 is at the end of a perfect cube number, then its cube root will have its unit place digit as 2 only. Therefore, the unit place digit of the required cube root will be taken as 2. Taking the other group i.e., 32,

We know that, $3^3 = 27$ and $4^3 = 64$

Also, $27 < 32 < 64$

3 is smaller between 3 and 4. Therefore, 3 will be taken at the tens place of the required cube root.

Thus, $\sqrt[3]{32768} = 32$