Dynamics

Ex.

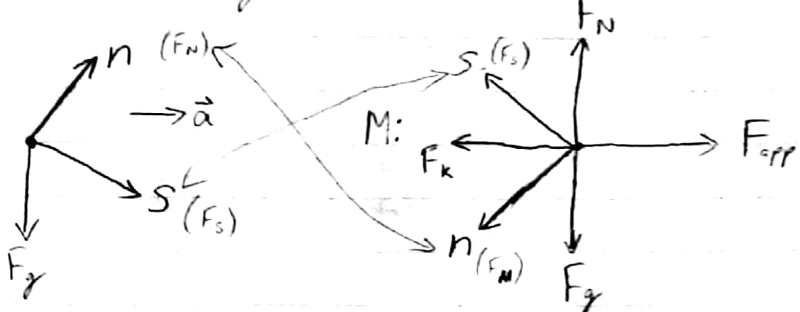
There is an F_{min} and F_{max} here such that the block m doesn't slide on the wedge.

$$\boxed{F_s \le F_s^{max} = \mu_s n}$$

Free body:

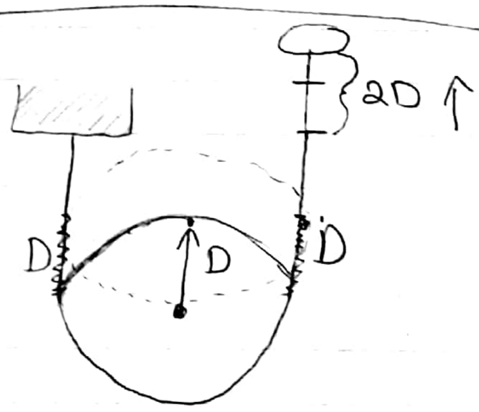
$F_K = \mu_K F_N$

m:



note the effect of Newton's third law on n (F_N) and s (F_s).

Note: you can use two different coordinate systems for two different free-body diagrams.



$$\Delta E_{tot} = 0$$
$$= \Delta U_g + \Delta U_s + \Delta K + \Delta E_{other}$$

$\Delta K \to \Delta E_{trans} + E_{rot}$

$W_{net} = \Delta K$

$\Delta \vec{p}_{tot} = 0$

$\Delta L_{tot} = 0$

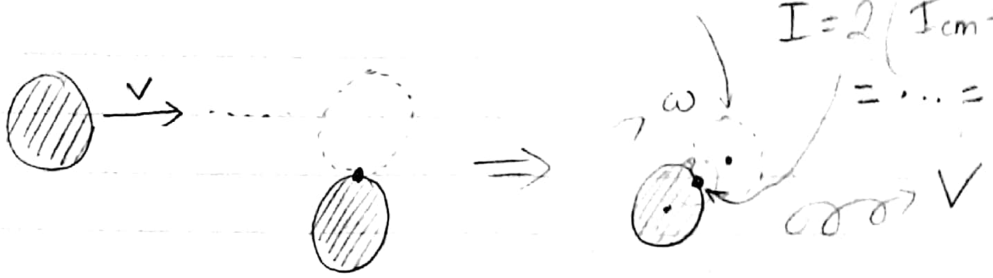
Scanned by CamScanner

for point mass:
$I = mr^2$

parallel axis theorem

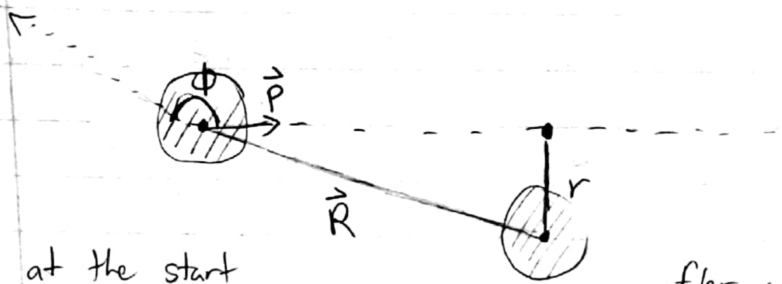
$I_{cm} = \frac{1}{2}mr^2$

$I = 2(I_{cm} + mr^2)$
$= \dots = 3mr^2$



radius r
mass m
$\Rightarrow I_{cm} = \frac{1}{2}mr^2$

angular momentum: $\vec{L} = \vec{r} \times \vec{p}$
$L = I\omega$



at the start
$L_i = Rp\sin(\phi)$
$= rp$
$L_i = rmv$
$L_f = I\omega$

apply conservation of angular momentum

after contact and starts to spin
$\omega = \frac{rmv}{I} = \frac{rm}{3mr^2}v$
$= \frac{v}{3r}$

apply conservation of linear momentum
$p_i = mv$
$p_f = 2mV$ $\Rightarrow V = \frac{1}{2}v$

$K_i = \frac{1}{2}mv^2$
$K_f = \frac{1}{2}(2m)V^2 + \frac{1}{2}I\omega^2 = \frac{5}{12}mv^2$

$\Delta K = K_f - K_i = \left(\frac{K_f}{K_i} - 1\right)K_i = \left(\frac{5}{6} - 1\right)K_i = \frac{-1}{6}K_i$

$\Delta E_{tot} = 0$
$= \Delta K + \Delta E_{other}$

Scanned by CamScanner

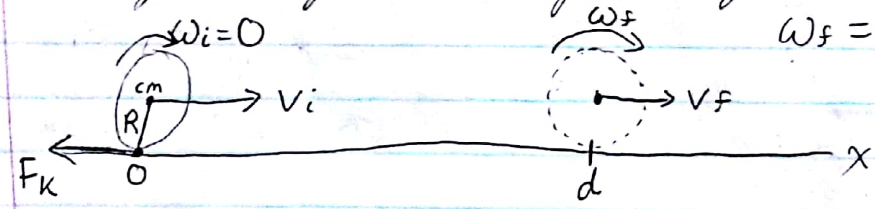
Bowling Ball Problem

Conservation approach

Initially sliding $\rightarrow$ finally rolling

F_K $\nearrow$ decreases v

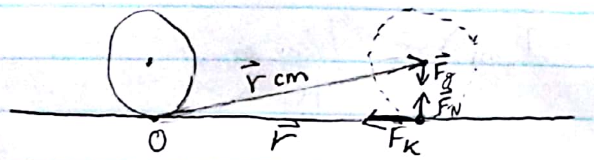
F_K $\searrow$ increases ω



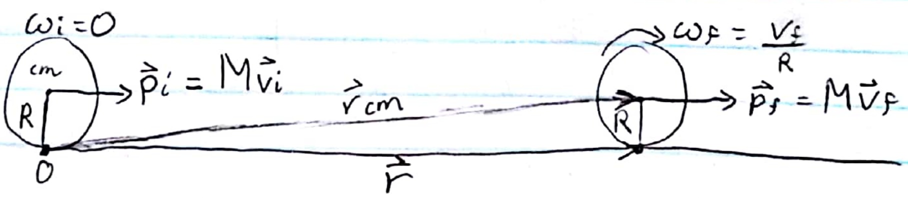
$\omega_f = \frac{v_f}{R}$ since it ends fully rolling

reaches d at time t

note: $\omega_i \neq \frac{v_i}{R}$ since it gradually starts rolling but starts off sliding



$$\sum \vec{\tau} = (\vec{r}_{cm} \times \vec{F}_g) + (\vec{r} \times \vec{F}_N) + (\vec{r} \times \vec{F}_k)$$
$$= \vec{r} \times (\vec{F}_g + \vec{F}_N) = 0$$
$$\vec{L} = \text{constant}$$



$$L_f = |\vec{r}_{cm} \times \vec{p}_f| + I_{cm}\omega_f$$
$$= RMv_f + I_{cm}\left(\frac{v_f}{R}\right)$$
$$\Rightarrow v_f = \frac{1}{1 + \frac{I_{cm}}{MR^2}} v_i$$
$$\Rightarrow v_f = \frac{5}{7} v_i$$

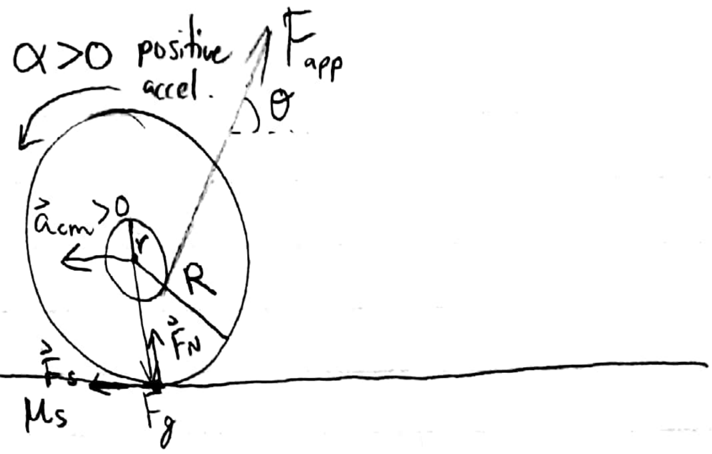
for a solid ball with $I_{cm} = \frac{2}{5}MR^2$

$$0 = \Delta E_{tot} = \Delta K + \Delta E_{other}$$

$$\Delta E_{other} = \frac{1}{1 + \frac{MR^2}{I_{cm}}} K_i$$
$$= \frac{2}{7} K_i$$

iematics approach:

Scanned by CamScanner



$$\sum F_x = F_s - F_{app}\cos\theta = M a_{cm}$$
$$\sum F_y = F_N - F_g + F_{app}\sin\theta = 0$$
} translational

$$\sum \tau_{ccw} = r F_{app} - R F_s = I_{cm}\alpha = \frac{I_{cm}}{R} a_{cm}$$

⊕ counterclockwise

$$\Rightarrow \quad a_{cm} = \frac{\left(\frac{r}{R} - \cos\theta\right)}{M + \frac{I_{cm}}{R^2}} F_{app}$$

Scanned by CamScanner