GOAL PROGRAMMING

Goal progamming is the one of the mathematical model used to make solution for the problem having many target so it can be got optimal solution. Aran Puntosadewo (2013) said that Goal Programming is for decising the goal notated by numerical for each goal, making objective function for each function, and finding solution to minimize the deviation at the objective function. Goal Programming model try to minimize the deviation between several goal or target, so the value of the right side is as close as possible to same with left side of the equation.

Goal Programming model is the extention of Linier Programming develoved by A. Charles and W. M Cooper at 1956 so all of the asumption, notation, mathematical formula, procuder of model and the solution are not different. The different of this program is just in the deviation exist in objective function and constraint. Linier Pogrmmiing is the mathematical model used to find the optimal solution with minimizing and maximizing objective function based on one constraint. Beside that, Goal Programming have three part, that is decision variable, objective function, and constraint.

Having known that Goal Progamming have 3 part that are objective function, goal constraint, non negatif constraint.

1. Objective Function
   Objective Function in Goal Programming basically is the problem of minimization, because at the objective function, there are deviation variable that has to minimize. Objective Function in the Goal Programming is for minimizing all of the goal constraint that want to be reached.

2. Non Negatif Constraint
   Non Negatif constraint in Goal Progamming is all of the variable that has positive value or equal to zero. So the decision variable and deviation variable at the Goal Programming problem always has positive value or equal to zero. The notation of non-negative is $x_j, d_i^-, d_i^+ \geq 0$

3. Goal Contraint
   According to Rio Armindo (2006), at the Goal Programming, there are six kind of goal constraint different each other. The Goal from each constraint is decised by connection between objective function. this is the 6's kind of constraint.

NO	Kendala Tujuan	Variabel Deviasi dalam Fungsi Tujuan	Kemungkinan Simpangan	Penggunaan Nilai RHS yang Diingninkan
1	$C_{ij}x_{ij} + d_i^- = b_i$	d_i^-	Negatif	$= b_i$
2	$C_{ij}x_{ij} - d_i^+ = b_i$	d_i^-	Positif	$= b_i$
3	$C_{ij}x_{ij} + d_i^- - d_i^+ = b_i$	d_i^-	Negatif atau Positif	b_i atau lebih
4	$C_{ij}x_{ij} + d_i^- - d_i^+ = b_i$	d_i^-	Negatif atau Positif	b_i atau kurang
5	$C_{ij}x_{ij} + d_i^- - d_i^+ = b_i$	d_i^- dan d_i^+	Negatif atau Positif	$= b_i$
6	$C_{ij}x_{ij} - d_i^+ = b_i$	d_i^+(artifisial)	Tidak ada	$= b_i$

Common Model of Goal Programming

The first is the common model without priority factor in the structure :

$$\text{Minimize} \quad Z = \sum_{i=1}^{m} \left(d_i^+ + d_i^-\right)$$

with goal constraint :

$$\begin{aligned}
C_{11}x_1 + C_{12}x_2 + \cdots + C_{1n}x_n + d_1^- - d_1^+ &= b_1 \\
C_{21}x_1 + C_{22}x_2 + \cdots + C_{2n}x_n + d_2^- - d_2^+ &= b_2 \\
&\vdots \\
C_{m1}x_1 + C_{m2}x_2 + \cdots + C_{mn}x_n + d_m^- - d_m^+ &= b_m
\end{aligned}$$

non negative constraint : $x_j, d_i^-, d_i^+ \geq 0$, for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$

The Second is the problem at The Goal Programming with structur containing priority of weight :

$$\text{Minimize :} \qquad Z = P_1 d_i^- + \cdots + P_l d_i^- + P_{i+1} d_i^+ + \cdots + P_k d^+ i$$

with constraint :

$$\begin{aligned}
C_{11}x_1 + C_{12}x_2 + \cdots + C_{1n}x_n + d_1^- - d_1^+ &= b_1 \\
C_{21}x_1 + C_{22}x_2 + \cdots + C_{2n}x_n + d_2^- - d_2^+ &= b_2 \\
&\vdots \\
C_{m1}x_1 + C_{m2}x_2 + \cdots + C_{mn}x_n + d_m^- - d_m^+ &= b_m
\end{aligned}$$

Where P_k is the priority of the goal -k and

$$x_j, d_i^-, d_i^+ \geq 0$$

Based on model of Goal Programming, the reaching of the Goal or Target is done by minimizing deviation. There is two type of target, that is target with each of target have same priority and the target that is ordering the target based on level of the priority of the target. For the target ordering by the level of the priority has to be given weight factor. Weight factor is the numerical value that dont have dimention and being used to show the level of relative priority from any target. The value of the weight factor from each target is got from manipulation result by reasercher or people who make decision.

If the weight factor for priority of target function -i is called W_i, so it mathematically has characteristic by :

$$0 < W_i < 1$$

and

$$\sum_{i=1}^{k} W_i = 1$$

If there any statement that W_c is more than W_y it shows that target $-c$ is more important from target -y, and if $W_c = W_y$ so target -c and target $-y$ have same priority

Weighted Goal Programming

The first variant of a Goal Programming method presented in this section is called weighted Goal Programming. It can also be found in the literature as non-preemptive Goal Programming. The main idea of this variant is to attach penalty weights to the unwanted deviational variables in the objective function. These weights consist of two parts:

- The importance of the penalization at each deviational variable. We denote by ui the weight associated with the minimization of ni, while vi is the weight associated with the minimization of p_i, where $i = 1, 2, \ldots, m$. These weights show the relative importance of the minimization of a deviational variable. Note that deviational variables whose minimization is unimportant, e.g., the negative deviational variable of a goal for cost, are assigned a weight equal to 0.

- A normalization constant, ki, in order to assure a commensurability of the goals. These factors are necessary in order to scale all goals into the same units of measurement.

A weighted Goal Programming problem can be represented by the following formulation

$$\min\ z = \sum_{i=1}^{m} \left(\frac{u_i n_i}{k_i} + \frac{v_i p_i}{k_i} \right)$$

subject to

$$f_i(x) + n_i + p_i = b_i \quad , \quad x \in F$$
$$n_i, p_i \geq 0, i = 1, 2, \ldots, m$$

Lexicographic Goal Programming

The second variant of a Goal Programming method presented in this section is called lexicographic Goal Programming. It can also be found in the literature as preemptive Goal Programming. The main feature of this variant is the existence of a number of priority levels. This variant is used when the decision maker has a clear preference order for satisfying the goals. Each priority level consists of a number of unwanted deviations to be minimized. We define by L the number of priority levels with corresponding index $l = 1, 2, \ldots, L$. Each priority level is a function of a subset of unwanted deviational variables, $h_l(n, p)$. The consensus in the goalprogramming literature is that no more than five priority levels should be used in this variant.

A lexicographic Goal Programming problem can be represented by the following formulation

$$\min \quad z = h_1(n,p), h_2(n,p), \ldots, h_L(n,p)$$

$$\text{Subject to } f_i(x) + n_i + p_i = b_i \quad , \quad x \in F$$

$$n_i, p_i \geq 0, i = 1, 2, \ldots, m$$

Chebyshev Goal Programming

The third variant of a Goal Programming method presented in this section is called Chebyshev Goal Programming. This variant was introduced by Flavell [6] and it is known as Chebyshev Goal Programming, because it uses the Chebyshev distance or L∞ metric. It can also be found in the literature as Minmax Goal Programming. The main idea of this variant is to achieve a balance between the goals. Classical, weighted and lexicographic Goal Programming often find extreme solutions, i.e., points that lie in the intersection of goals, constraints, and axes. This can lead to an unbalanced solution since some goals are achieved and others are far from satisfactory. In Chebyshev Goal Programming, we introduce additional constraints in order to ensure balance between the goals. This is the only widely-used variant that can find optimal solutions that are not located at extreme points.

Let λ be the maximal deviation from amongst the set of goals, then a generic form of a Chebyshev Goal Programming problem is the following

$$
\begin{gathered}
\min \quad z = \lambda \\
\text{s.t. } \frac{u_i n_i}{k_i} + \frac{v_i p_i}{k_i} + \leq \lambda \\
f_i(x) + n_i + p_i = b_i \qquad , x \in F \\
n_i, p_i \geq 0, i = 1, 2, \ldots, m
\end{gathered}
$$