GOAL PROGRAMMING

Ordinary linear program models are not able to solve management cases that require certain goals to reach all objectives optimally at the same time. In the linear program model there is a Slack variable in the constraint function in the form of a delimiter and a surplus in the constraint function in the form of conditions. In solving the case of a linear program the two variables function to accommodate the advantages and disadvantages of the left hand side value of a constraint function so that it equals the value of the right hand side. However, both of these variables are completely uncontrollable in solving the case of linear programs, so the linear program model was developed by A. Charles and W. M. Cooper as a goal programming model.

If there are variables in a linear program that have characteristics similar to the Slack and Surplus variables, and are in a constraint equation, then the variable is controlled so that the left segment value of a constraint is equal to the value of the right segment. The goal programming model is able to solve cases of linear programs that have more than one goal to be achieved. The goal or target is a constant value on the right hand side of the constraint function

Basically the structure of goal programming and linear programming is the same. The concept of goal programming is to introduce additional auxiliary variables called deviations that act not as variable decision, but only as facilitators to formulate models. This deviation is the difference between the desired target value and the results obtained.

The goal programming model is an extension of the linear program model so that all assumptions, formation notations, mathematical models, formulation procedures, models and solutions are no different. The difference lies only in the deviation variable that appears in the constraint function and the objective function.

1. Deviation Variable

Deviation variables are variables that hold the deviation of results towards desired goals. Differentiated as follows:

a. Deviational variables to contain the deviation below desired target

Target is the value of the right segment in one of the goal constraints (b_i). In other words, this deviation variable serves to support negative deviation. We use d^- notation to use this type of deviational variable. Because the d^- deviational variable works for a negative deviation, so the equation can be written as follows:

$$\sum_{j=1}^{n} a_{ij}X_{ij} = b_i - d_i^-$$

or

$$\sum_{j=1}^{n} a_{ij}X_{ij} + d_i^- = b_i$$

b. Deviational variables to contain the deviation above desired target

Target is the value of the right segment in one of the goal constraints (b_i). In other words, this deviation variable serves to support positive deviation. We use d^+ notation to use this type of deviational variable. Because the d^+ deviational variable works for a positive deviation, so the equation can be written as follows:

$$\sum_{j=1}^{n} a_{ij}X_{ij} = b_i + d_i^+$$

or

$$\sum_{j=1}^{n} a_{ij}X_{ij} - d_i^+ = b_i$$

with :
a_{ij} : coefficient of decision variable
X_{ij} : decision variable

The value of deviation variables must be positive or can be written as follows :

$$d_i^- \geqslant 0 \; for \; i = 1, 2, \ldots, m$$
$$d_i^+ \geqslant 0 \; for \; i = 1, 2, \ldots, m$$

In the goal programming the deviation variable in the objective function must be minimized so that the value of the left hand side of an equation is as close as possible to the value of the right segment

2. General Form of Goal programming

The general form of mathematical goal programming models can be formulated as follows:
Objective Function :

$$Minimize \sum_{i=1}^{m}(d_i^+ + d_i^-)$$

Goal Constraints :

$$\sum_{j=1}^{n}(a_{ij}X_j) + d_i^- - d_i^+ = b_i$$
$$X_j, d_i^-, d_i^+ \geqslant 0$$

$\forall i = 1, 2, \ldots, m \; and \; j = 1, 2, \ldots, n$

with :
d_i^- : lower bond of the achievement of goal i
d_i^+ : upper bond of the achievement of goal i
a_{ij} : coefficient of decision variable
x_j : decision variable

b_i : goal or target

3. Goal Constraints

The use of deviational variables to achieve managerial goals can be distinguished as follows:

a. To achive a goal with certain value

The targets to be achieved are denoted in b_i parameters. For this target to be achieved, the deviations below and above (b_i value must be minimized. So that the function of goal constraints becomes:

$$\sum_{j=1}^{n} a_{ij}X_{ij} + d_i^- - d_i^+ = b_i$$

For d_i^- and d_i^+ to be minimum, the objective function equation becomes:

$$Minimize \sum_{i=1}^{m} d_i^- + d_i^+$$

b. To achive a goal below a certain value

The targets to be achieved are denoted in b_i parameters and may not be exceeded, therefore deviations above the b_i value must be minimized so that the results is less then or equal to the value of b_i. So that the function of goal constraints becomes:

$$\sum_{j=1}^{n} a_{ij}X_{ij} - d_i^+ = b_i$$

For d_i^+ to be minimum, the objective function equation becomes:

$$Minimize \sum_{i=1}^{m} d_i^+$$

c. To achive a goal above a certain value

Deviations below the b_i value must be minimized so that the results of the equation is greater or equal to the value of b_i. So that the function of goal constraints becomes:

$$\sum_{j=1}^{n} a_{ij}X_{ij} + d_i^- = b_i$$

For d_i^- to be minimum, the objective function equation becomes:

$$Minimize \sum_{i=1}^{m} d_i^-$$

d. To achive a goal in certain interval

If intervel is limited by a_i and b_i then the expected equal results will be between these intervals, or $c_i \leq \sum_{j=1}^{n} a_{ij} X_{ij} \leq b_i$, so the goal constraint is :

$$c_i - d_i^- \leq \sum_{j=1}^{n} a_{ij} X_{ij} \leq b_i + d_i^+$$

in this case it is equivalent to:

$$\sum_{j=1}^{n} a_{ij} X_{ij} + d_i^- \geq c_i$$

and

$$\sum_{j=1}^{n} a_{ij} X_{ij} - d_i^+ \leq b_i$$

For d_i^- and d_i^+ to be minimum, the objective function becomes:

$$\text{Minimize} \sum_{i=1}^{m} d_i^- + d_i^+$$