1 The General Model

1.1 Variables

x_1 = number of shirts,
x_2 = number of jackets

1.2 Constraints

$2x_1 + 4x_2 \leq 600$ Cotton Stock
$5x_1 + 3x_2 \leq 700$ Linen Stock
$100x_1 + 90x_2 \geq 18000$ Weekly Profit minimum
$2x_1 + 2x_2 \leq 380$ Man-hours available
$x_1 + x_2 \leq 200$ Machine Limitation
$x_1, x_2 \geq 60$ Minimum Production's Contract

2 Lexicographical Solution

Lexicographical Goal Programming use priority levels for each goals. The linear program will run separately for each priority level, running in sequence of order from the highest to lowest priority.

This will be the priority level for each of the goals :

• **Priority Level 1** Man-hours (Goal 4)

• **Priority Level 2** Weekly Profit (Goal 3)

• **Priority Level 3** Cotton and Linen stocks to buy (Goal 1 and Goal 2)

The first linear program is to solve Priority Level 1, which is to minimize man hour excess.

Priority Level 1 LP
Minimize d_4^+
Subject to

$$2x_1 + 4x_2 + d_1^- - d_1^+ = 600$$
$$5x_1 + 3x_2 + d_2^- - d_2^+ = 700$$
$$100x_1 + 90x_2 + d_3^- - d_3^+ = 18000$$
$$2x_1 + 2x_2 + d_4^- - d_4^+ = 600$$
$$x_1 + x_2 \leq 200$$
$$x_1, x_2 \geq 60$$
$$d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_4^+ \geq 0$$

This linear program can be solved using any method such as Simplex Method, Graphical Method or using any Solver. The results we get is $d_4^+ = 0$ which fully satisfy the goal.

The idea is to use the previous results for the next priority goal we want to accomplish. We will use the results from first Linear Program as a constraints for the next one. The second Linear Program is to achieve weekly profit.

Priority Level 2 LP
Minimize d_3^-
Subject to

$$2x_1 + 4x_2 + d_1^- - d_1^+ = 600$$
$$5x_1 + 3x_2 + d_2^- - d_2^+ = 700$$
$$100x_1 + 90x_2 + d_3^- - d_3^+ = 18000$$
$$2x_1 + 2x_2 + d_4^- - d_4^+ = 600$$
$$d_4^+ = 0$$
$$x_1 + x_2 \leq 200$$
$$x_1, x_2 \geq 60$$
$$d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_4^+ \geq 0$$

The results we get from the second Linear Program is $d_3^- = 0$ which fully satisfy the second level priority goal.

The third Linear Program will use the second Linear Program's constraints and its results.

Priority Level 3 LP
Minimize $d_1^+ + d_2^+$
Subject to

$$2x_1 + 4x_2 + d_1^- - d_1^+ = 600$$
$$5x_1 + 3x_2 + d_2^- - d_2^+ = 700$$
$$100x_1 + 90x_2 + d_3^- - d_3^+ = 18000$$
$$2x_1 + 2x_2 + d_4^- - d_4^+ = 600$$
$$d_4^+ = 0$$
$$d_3^- = 0$$
$$x_1 + x_2 \leq 200$$
$$x_1, x_2 \geq 60$$
$$d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_4^+ \geq 0$$

The result for this linear program is $d_1^+ = 0$, $d_1^- = 20$, $d_2^+ = 50$ and $d_2^- = 0$

Goal	Target	Achieved Value
1	600	580
2	700	750
3	18000	18000
4	380	380

This will be the final results :

$$x_1 = 90$$
$$x_2 = 100$$
$$d_1^+ = 0, \; d_1^- = 20$$
$$d_2^+ = 50, d_2^- = 0$$
$$d_3^+ = 0, \; d_3^- = 0$$
$$d_4^+ = 0, \; d_4^- = 0$$