# Extending OLS: Fixed Effects, Controls, and Interactions

EH6105 – Quantitative Methods

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# Goal for Today

*Add some wrinkles to the OLS regression framework.*

# Introduction

By this point, I think you could be doing your own research.

- You know what variables are.
- You know how to describe them.
- You know how to propose an explanation for variations in them.
- You know how to set up a research design to test an argument.
- You even know how to interpret a regression coefficient!

# Limitations in Bivariate Regression

However, simple bivariate OLS is never enough.

- Variables of interest in political science are rarely interval.
- Bivariate regression does not control for confounders.

This lecture will deal with those topics accordingly.

# Dummy Variables as Predictors

Dummy variables are everywhere in applied social science.

- They play an important role in “fixed effects” regression.
- Sometimes we’re just interested in the effect of “one thing”.

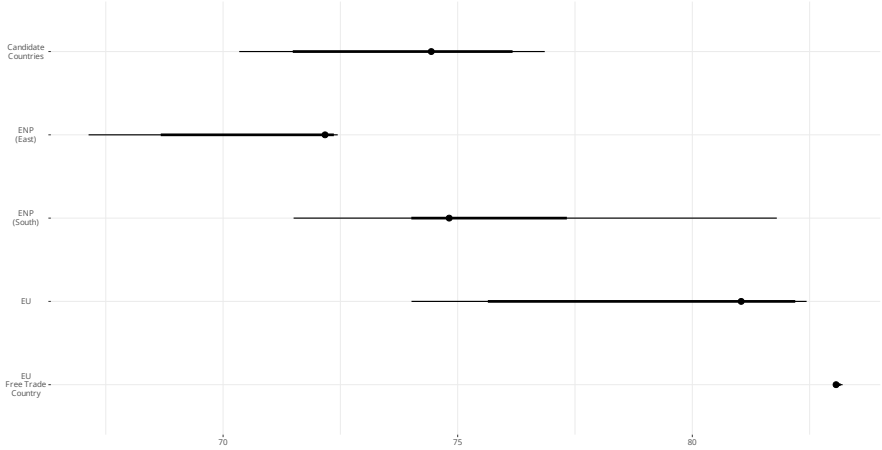
# Eurostat Categories and Life Expectancy

Return to our life expectancy example: what if we're just interested in categorical difference, by Eurostat category?

- Categories: EU (e.g. Sweden), EFTA (e.g. Norway), UK (i.e. those guys), EUCC (e.g. BiH), PC (i.e. Kosovo, Georgia), ENP-E (i.e. AM, BY, AZ), ENP-S (e.g. Algeria), OEC (i.e. Russia)
- Let's make this somewhat honest and drop the UK and Russia and combine the PC and EUCC countries.

## The Distribution of Life Expectancy in 2020, by Eurostat Category

The largest categorical differences seem to focus on the ENP-E countries as well as the free trade countries.




*Data: ?wbd_example in {stevedata} by way of World Bank*

# Eurostat Categories and Life Expectancy

Let's look at two things here:

1. A comparison of the ENP-E to the rest of the data.
2. A full comparison among categories.

**Table 1:** The Correlates of Life Expectancy for Eurostat Category States, 2020

| | **Model 1** |
|---|---|
| ENP (East) | -7.597** |
| | (2.315) |
| Intercept | 78.097*** |
| | (0.585) |
| Num.Obs. | 47 |
| R2 Adj. | 0.175 |

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

# Life Expectancy and the ENP-East

- The estimated life expectancy in other Eurostat category states is 78.1
- The estimated life expectancy in ENP-East states is 70.5
- The "ENP-East effect" is an estimated -7.6 (s.e.: 2.31).
- *t*-statistic: -7.6/2.31 = -3.28

We can rule out, with high confidence, an argument that being an ENP-East state has no effect on life expectancy.

- Our findings suggest a precise negative effect.

# What About Other Variation?

Obviously, this last regression isn't that informative.

- The baseline category is quite heterogeneous.
- It's impressive to pick up 17% of the variation with alone, though.

We can specify other categories as "fixed effects".

- These treat predictors as a series of dummy variables for each value of $x$.
- One predictor (or group) is left out as "baseline category".
    - Otherwise, we'd have no $y$-intercept.

**Table 2:** The Correlates of Life Expectancy for Eurostat Category States, 2020

| | Model 1 | Model 2 |
|---|---|---|
| ENP (East) | -7.597** | -9.107*** |
| | (2.315) | (1.793) |
| ENP (South) | | -4.019** |
| | | (1.192) |
| EU Free Trade | | 3.485+ |
| | | (1.793) |
| Candidate Country | | -5.589*** |
| | | (1.192) |
| Intercept | 78.097*** | 79.606*** |
| | (0.585) | (0.587) |
| Num.Obs. | 47 | 47 |
| R2 Adj. | 0.175 | 0.528 |

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

# Categorical Fixed Effects and Life Expectancy

How to interpret this regression:

- All coefficients communicate the effect of that category versus the baseline category.
  - I forced this to be the EU for ease of comparison, but default is whatever comes first.
  - Just be mindful: *everything* is benchmarked to the baseline.
- Estimated life expectancy in the EU is 79.61.
- Life expectancy in the candidate countries is discernibly lower than the EU ($t$ = -4.69).
- Life expectancy in the FTA countries is discernibly higher than the EU ($t$ = 1.94).
- Life expectancy in the ENP-East is discernibly lower than the EU ($t$ = -5.08).
- Life expectancy in the ENP-South is discernibly lower than the EU ($t$ = -5.08).

# Multiple Regression

Your previous example is basically an applied **multiple regression**.

- However, it lacks control variables.

Multiple regression produces **partial regression coefficients**.

# Multiple Regression

Let's return to what we did last time with human capital, but do more. Let:

- $x_1$: human capital score [0:1]
- $x_2$: real GDP per capita (2015 USD)
- $x_3$: categorical fixed effects

Important: we do this to "control" for potential confounders.

# The Rationale

Assume you are proposing a novel argument that human capital explains life expectancy. I might argue for omitted variable bias on these grounds:

- You've misspecified "capital"; it's more material than "human".
- You've missed that some regions "are just different".

In other words, I contend your argument linking human capital ($x$) to life expectancy ($y$) is spurious to these other factors ($z$).

- That's why you "control." Not to soak up variation but to test for effect of potential confounders.

**Table 3:** The Correlates of Life Expectancy for Eurostat Category States, 2020

| | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Human Capital | | | 26.617*** |
| | | | (5.815) |
| Real GDP per Capita | | | 0.000** |
| | | | (0.000) |
| ENP (East) | -7.597** | -9.107*** | -4.369** |
| | (2.315) | (1.793) | (1.360) |
| ENP (South) | | -4.019** | 2.258+ |
| | | (1.192) | (1.236) |
| EU Free Trade | | 3.485+ | 0.453 |
| | | (1.793) | (1.359) |
| Candidate Country | | -5.589*** | -0.537 |
| | | (1.192) | (1.056) |
| Intercept | 78.097*** | 79.606*** | 58.071*** |
| | (0.585) | (0.587) | (4.068) |
| Num.Obs. | 47 | 47 | 47 |
| R2 Adj. | 0.175 | 0.528 | 0.790 |

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

# Multiple Regression

Estimated life expectancy for EU state with no human capital and no money: 58.07

- This parameter is effectively useless, given how you modeled the data.
- (It's not a problem, though there are advanced tools available to make use of this).

Other takeaways:

- Partial [min-max] effect of human capital: 26.62
- Partial effect of GDP per capita is positive and significant.
  - *Don't* read much into coefficient size, only direction and significance.
  - e.g. a 45,000 USD increase in GDP per capita increases life expectancy by an estimated 2.8 years.
- Partialing out human capital and real GDP per capita, only the ENP differences remain.

# Interactive Effects

Multiple regression is linear and additive.

- However, some effects (say: $x_1$) may depend on the value of some other variable (say: $x_2$).

In regression, we call this an **interactive effect**.

# A Real World Example

Consider this example: we want to measure political trust in Sweden by ideology.

- *But*, a la Converse (1954) and Zaller (1992), political opinions are filtered through the politically aware.
- i.e. there's no standalone effect of ideology independent from political engagement.

Let's use 2019/2020 SOM data to evaluate whether there's something to this.

# Our Data

**IVs**: ideology, political interest

- Ideology: (0 = “clearly to the left”, 4 = “clearly to the right”)
- Political interest: (0 = “not at all interested” or “not particularly interested”, 1 = “very/rather interested”)

# Our Data

**DV**: latent political trust based on various items. Including:

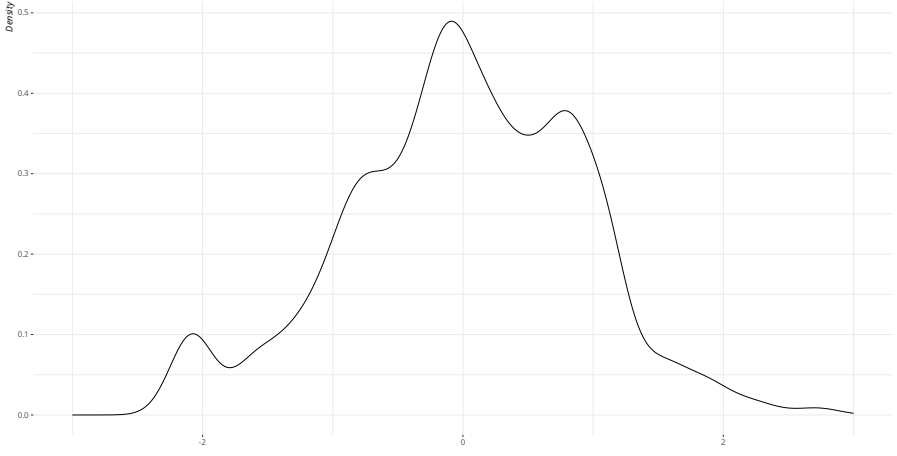
- government
- parliament
- political parties
- Swedish politicians

Emerging estimate has a mean of zero and standard deviation of one.

- Higher values = more political trust

## Density Plot of Latent Political Trust in Sweden, 2019-2020

The data were generated from a graded response model to have an approximate mean of 0 and standard deviation of 1.




*Data: SOM (2019-2020). Data available in {simqi}.*

## Interactive Effects

Our regression formula would look like this:

$$\hat{y} = \hat{a} + \hat{b_1}(x_1) + \hat{b_2}(x_2) + \hat{b_3}(x_1 * x_2)$$

where:

- $\hat{y}$ = estimated political trust score.
- $x_1$ = ideology (0 = "clearly to the left").
- $x_2$ = political interest (0 = "not at all/not particularly interested").
- $x_1 * x_2$ = product of the two variables.

# A Caution About Constituent Terms

*Be careful with interpreting regression coefficients for constituent terms of an interaction.*

- The regression coefficient for ideology is effect of increasing ideology when the interest variable = 0 (i.e. low/no-interest).
- The political interest variable is effect of interest when ideology = 0 (i.e. among the furthest Left).

**Table 4:** A Simple Interaction Between Ideology and Political Interest on Political Trust (SOM, 2019-2020)

| | **Model 1** |
|---|---|
| Political Interest | 0.374*** |
| | (0.075) |
| Ideology (L to R) | -0.172*** |
| | (0.028) |
| Political Interest*Ideology | -0.071* |
| | (0.032) |
| Intercept | 0.204** |
| | (0.066) |
| R2 Adj. | 0.104 |
| Num.Obs. | 2841 |

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

## Interactive Effects

How to interpret this table:

- Our estimate of political trust is 0.204 for the no-interest maximally Left
- $\hat{b_1}$, $\hat{b_2}$, and $\hat{b_3}$ are all statistically significant.
- When $x_1$ and $x_2 = 1$, subtract -0.071 from $\hat{y}$.

# Interactive Effects

Here's what this does for the maximally Left:

- $\hat{y}$ for low/no-interest Left: 0.204.
- $\hat{y}$ for high-interest Left: 0.578.

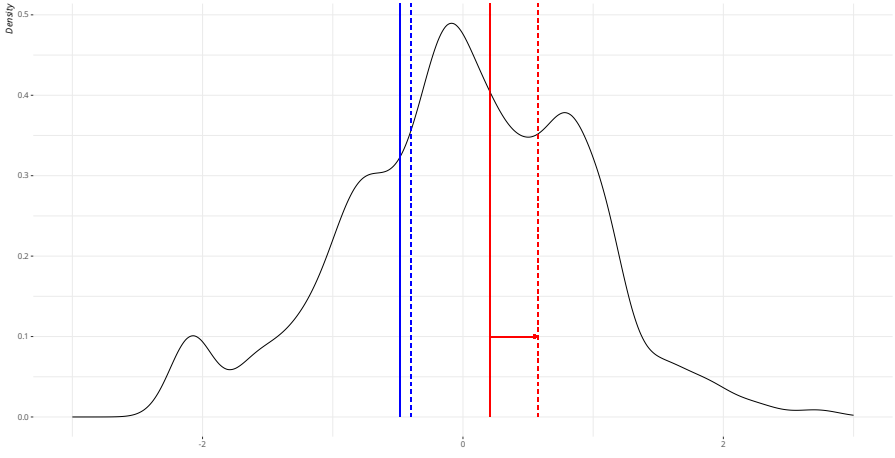
What this does for the maximally Right.

- $\hat{y}$ for low/no-interest Right: -0.486.
- $\hat{y}$ for high-interest Right: -0.398.

You see a huge effect of political interest on the Left, but a much smaller one on the right.

## Density Plot of Latent Political Trust in Sweden, 2019-2020

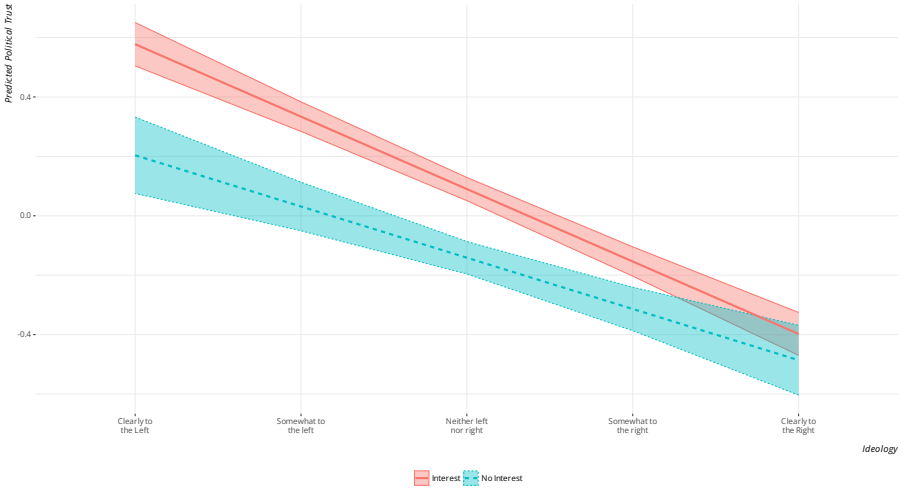
Notice the effect of political interest is much stronger for the left than right.




*Data: SOM (2019-2020). Data available in {simqi}.*

## Predicted Political Trust, by Political Interest and Ideology

Increasing ideology has a stronger effect on trust among those who are politically interested



# Conclusion

- Moving from bivariate OLS to multiple regression isn't really a big to-do.
  - It just means there are more parameters on the right-hand side of the equation.
  - What comes back are "partial" associations or regression coefficients.
  - This is where "ceteris paribus" language emerges.
- "Fixed effects" as you may encounter them = categorical dummy variables.
  - Something has to be a baseline, and that's what you're comparing against.
- Interactions = two (or more) things get multiplied together.
  - Constituent terms of x1 (x2): effect of x1 (x2) when x2 (x1) is 0.
  - Be mindful an "insignificant" interactive term may hide something.
  - Both things really have to have a 0 for the regression coefficients to communicate something.

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