Incomplete Information

POSC 1020 – Introduction to International Relations

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Goal for Today

Discuss importance of incomplete information in game theory.

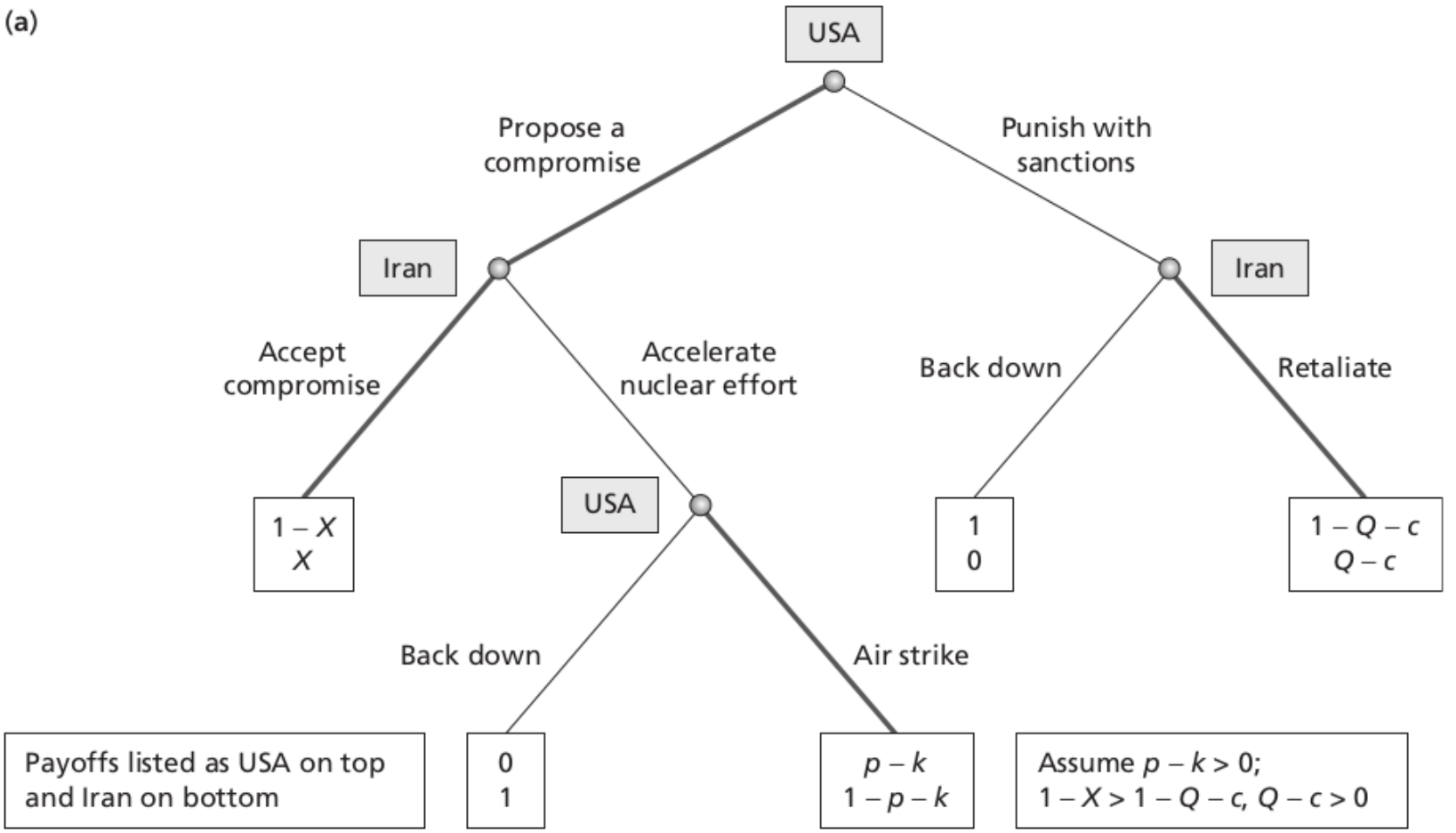
Recall our Previous Discussion

Iran-U.S. strategic interaction is more than an expected utility problem.

- The U.S. has the option to sanction Iran or offer a deal.
- Iran can accept a deal, if offered, or reject it.
- If Iran rejects, the U.S. may consider attacking Iran.

Iran-U.S. Strategic Interaction Game

FIGURE 3.1 **An Iranian-U.S. Strategic Interaction**



Solving the Game

Solving the game requires knowing if the U.S. has value in attacking Iran.

- When p - $k > 0$, the U.S. induces Iran to accept a deal.
- When p - $k < 0$, the U.S. sanctions Iran instead of offering a deal that gets rejected.

Both are subgame perfect Nash equilibria (under different assumptions) when the game is one of complete and perfect information.

Making the Problem More Interesting

Subgame perfect Nash equilibrium is fine only under conditions of complete and perfect information.

- i.e. Iran/U.S. know, with certainty, that p - $k > 0$ (or p - $k < 0$)

When one or the other is true, and everyone knows it, we can solve the game rather simply.

- The probability of an attack is effectively zero as well.

Incomplete Information

What if Iran doesn't know Obama's payoffs?

- That is: Khameini doesn't know if p - $k > 0$ or p - $k < 0$.

We already know Iran prefers a compromise if the former condition were true, but Iran will get (and accept the costs of) a sanctions regime if the latter were true.

The Uncertainty in Iran-U.S. Strategic Interaction

This suggests Obama may know his resolve on the issue, but Iran doesn't know how committed the U.S is.

The Uncertainty in Iran-U.S. Strategic Interaction

Why Iran has reason to believe p - $k > 0$.

- U.S. public opinion is fairly united on the Iran nuclear issue.
- USAF power *far* exceeds Iran's defenses.
- Iran has no (good) allies in the region, nor likely assistance from Russia and China.

Why Iran has reason to believe p - $k < 0$.

- Iran's nuclear facilities are well-concealed, protected, and diffuse.
- An air strike would torpedo U.S. standing in the region.
- The U.S. public may have war fatigue.

Obama may know if the U.S. is committed (i.e. p - $k > 0$), but Khameini has (credible) reasons to believe that Obama's threats are hollow.

Incomplete Information

In short, this strategic situation is one of **incomplete information**.

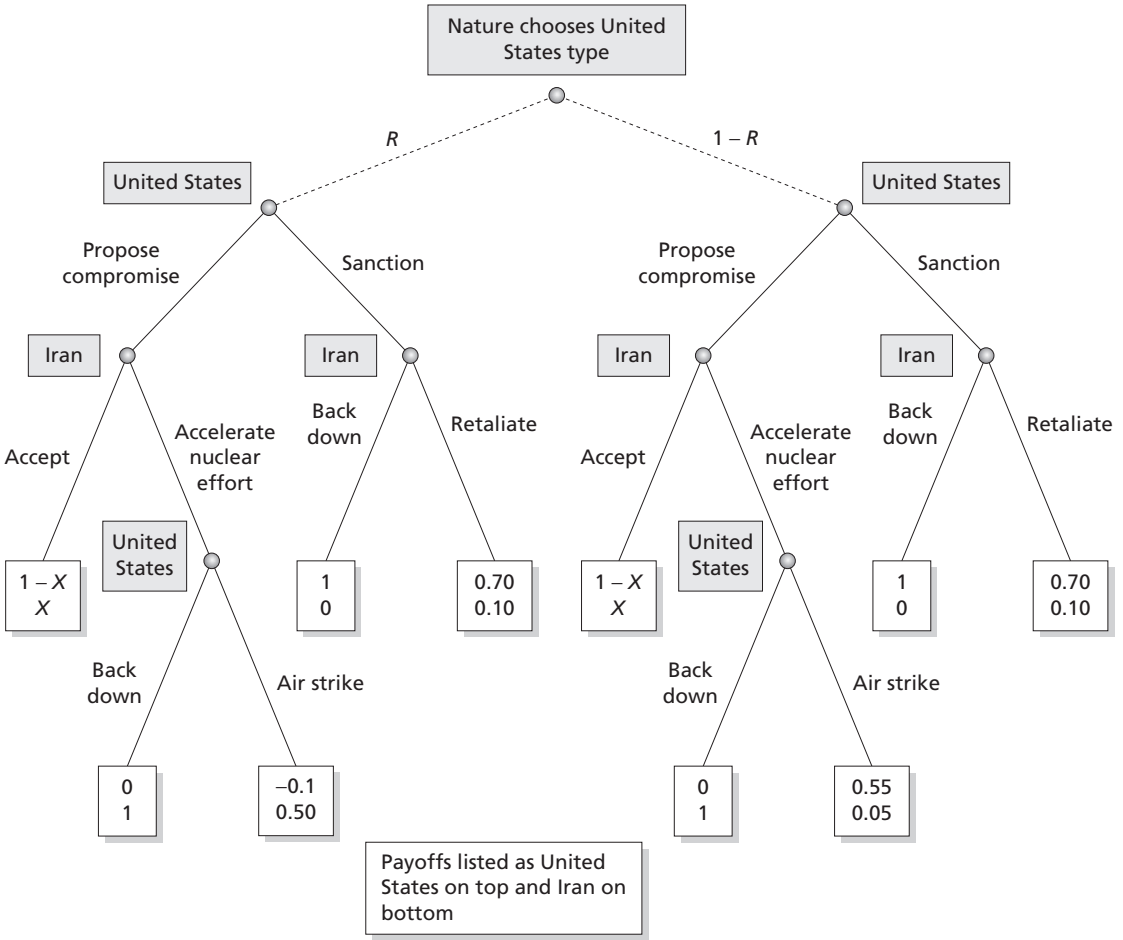
- *Definition*: situations in which the characteristics of a strategic game (e.g. player type, action, beliefs, preferences) are not common knowledge.

In our example, Khameini doesn't know for sure whether Obama is committed or not committed to this issue.

- With probability R, Obama is weak (i.e. p - $k < 0$).
- With probability 1 - R, Obama is strong (i.e. p - $k > 0$).

The New Game

FIGURE 3.2 **Choosing Actions under Uncertainty: Converting Incomplete Information into Imperfect Information**



The Game with Uncertainty

The new game is much more interesting.

- Nature assigns a type (weak or strong) to the United States.
- The U.S. knows its type, but Iran does not.
  - You can think of this in poker logic. It'd be illustrative.
- Khameini and Iran's belief of the U.S. type is expressed in probability (R, 1 - R).

Understanding the Game

We can illustrate how the game unfolds by plugging in some values.

- Assume, for simplicity: $p = .75$, $k = .20$.

The U.S. has a fairly high confidence that it can attack Iran, and cheaply.

- However, Iran does not know this!
- It can only assume with probability *1 - R* that this might be the case.

Understanding the Game

The U.S. knows its expected utility of attacking Iran is .55 (i.e. p - k = .75 - .20 $>$ 0).

- Iran's expected utility of being attacked (as the U.S. calculates it) is .05 (i.e. 1 - p - k = .05).

There's your bargaining space, by the way: .55 $\geq X \geq$ .05.

- The most the U.S. would offer is just short of research into weapons-grade fuel.
  - See: Figure 2.1.
- The least Iran would accept (as the U.S. calculates it) is X = .05.
  - Incidentally, this approximates the level to which both sides agreed in Vienna.

Ths U.S. will make the minimal offer (to maximize its utility), which it expects Iran to accept.

When the Minimum is No Longer Acceptable

However, Khameini would *reject* this offer in a world of uncertainty.

- Recall: Iran accepts the minimum offer only when it is 100% convinced an air strike would follow a rejected deal.
- i.e. Iran would accept $X = .05$ only when $R = 0$.

The moment $R > 0$ is the moment Khameini needs a better offer in order to accept a compromise.

Calculating Payoffs Under Uncertainty

How does this work?

- Recall: Khameini believes the U.S. is weak with probability R and the U.S. is strong with probability $1 - R$.
- If the U.S. is weak and Iran rejects the U.S. offer, Iran gets a 1 (i.e. the U.S. backs down).
- If the U.S. is strong and Iran rejects the U.S. offer, the U.S. attacks.
  - Iran's payoff would be $1 - p - k = 1 - .75 - .20 = .05$.

Iran receives a payoff of 1 with a probability of R and a payoff of .05 with probability $1 - R$.

Calculating Payoffs Under Uncertainty

What happens to this if the U.S. offers $X = .05$ while $R = 0$?

$$\begin{aligned} EU(\text{Iran}|\text{Reject Compromise}) &= R(1) + (1 - R)(.05) \\ &= 0(1) + (1 - 0)(.05) \\ &= .05 \end{aligned} \tag{1}$$

The offer is the minimum that Khameini would accept over risking an attack from the United States.

Calculating Payoffs Under Uncertainty

What happens to this if the U.S. offers $X = .05$ while $R = .01$?

$$\begin{aligned} EU(\text{Iran}|\text{Reject Compromise}) &= R(1) + (1 - R)(.05) \\ &= (.01)(1) + (1 - .01)(.05) \\ &= .0595 \end{aligned} \tag{2}$$

Khameini *rejects* what the United States otherwise believes is the minimum acceptable offer.

- Iran is not 100% convinced that p - $k > 0$.

Making a Better Deal

Under uncertainty, the U.S. offer must be more attractive for Iran.

- *But* it should still be more attractive for the U.S. than sanctions, per our assumptions ($1 - X \geq 1 - Q - c$).

Assume: $Q = .20$, $c = .10$.

- The U.S. values a compromise of X when $1 - X \geq 1 - .2 - .1$.

The minimum value of X that satisfies this is .3.

- The U.S. maximizes its utility with sanctions if $X > .3$.

Making a Better Deal

What if the U.S. offers $X = .3$ to Iran?

- It would be an improvement for Iran from the status quo of $X = .2$.

However, we do not know if Iran would actually accept this.

- After all, Iran is unconvinced that p - $k > 0$ for the U.S.

Thus, we need to know when $.30 \geq R(1) + (1 - R)(1 - .75 - .20)$.

Would Iran Accept $X = .30$?

$$
\begin{aligned}
.30 &\geq R(1) + (1 - R)(1 - .75 - .20) \\
.30 &\geq R + 1 - .75 - .20 - R + .75R + .20R \\
.30 &\geq 1 - .75 - .20 + .95R \\
.30 - 1 + .75 + .20 &\geq .95R \\
.25 &\geq .95R \\
\\
\frac{.25}{.95} &\geq R \qquad (3)
\end{aligned}
$$

Would Iran Accept $X = .30$?

What does this say in plain English?

- Khameini accepts the compromise offer of $X = .3$ (the most the United States would grant over sanctions) when Iran's probability of the United States *not* attacking Iran is, at the most, $\frac{.25}{.95}$, or $R = .263$.

If Iran believes $R = .50$, for example, then the U.S. should sanction.

- However, the threat of bargaining failure (i.e. war) looms large.

Conclusion

Differences of subjective beliefs about payoffs and observed actions complicate states' ability to locate a bargaining space.

- One way the U.S. can credibly parlay to Iran that it is strong (i.e. $p - k > 0$) is through sending a **costly signal** to Iran.
- A public threat on television from Obama could be credible if it informs Iran of U.S. resolve.
- Private threats via negotiation are often **cheap talk** that don't communicate meaningful information.

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