Instructions for Exercise 1:
In many real applications, the functions associated with an optimization problem can be nonlinear. Nonlinear optimization is usually more difficult and error-prone as compared to linear optimization.

Generally speaking, there are no practical methods to verify whether a given point is an optimal solution. Most solvers iteratively converge to a point that satisfies 'Karush-Kuhn-Tucker' (KKT) conditions. SNOPT is one such solver. If the problem is convex or if certain other conditions are satisfied, then the KKT point is also an optimal point. In this session, we will encounter convex functions in the first two exercises and nonconvex in the remaining two.

In this lab, we will model some problems that require nonlinear objective function or constraints. Just like we used Gurobi to solve linear problems, we can use the option

```
option solver snopt;
```

to ensure that AMPL calls SNOPT to solve the problem. One can write nonlinear functions of constraints and objective in the same way as linear functions. The nonlinear expressions in AMPL are written just like you would write them on paper. $x*y$ denotes product of two variables. Similarly $x/y, \ln(x), x\hat{}y, \cos(x), \sin(x)$ have the usual meanings. `sqrt(x)` is used to denote the square-root of x.

Exercise 1: Unconstrained Nonlinear Optimization. You are assigned the task of placing a reactor at an optimal position in a chemical plant. The reactor receives through pipes seven different raw materials from feeders distributed spatially in the plant. The coordinates of the seven feeders along the three spatial axis are

Feeder	A	B	C	D	E	F	G
Coordinates	(0.5,0,0)	(10,10,0)	(0,0,5)	(2,10,0)	(3,0,15)	(8,0,8)	(0,12,0)
Quantity	5	10	7	9	3	2	2

The last row denotes the quantity of material that needs to be moved from each feeder to the reactor for each production-run. Your goal is to find the location such that the total distance travelled by all the material is minimized.

1. **[R]** Formulate this problem as an unconstrained nonlinear optimization problem.
2. Model this problem in AMPL and solve using SNOPT.
3. **[R]** Report the optimal solution and the number of iterations the solver took to solve it.
4. **[R]** What is the minimum quantity that should flow from feeder-A so that the optimal location of the main reactor is within 2 units of feeder-A?

Exercise 2: Assignment Problem
Bindu constructions has to build $n \in \mathbb{N}$ different types of facilities, one at each of the n locations. The cost of constructing the i^{th} facility at the j^{th} location provided in the table below. Bindu wants to minimize the sum of the costs of assigning all the facilities to the locations.

1. **[R]** Write a mathematical model of the above assignment problem. Define all the variables and constraints clearly.
2. Write an AMPL model for this problem for a general n. You can assume that the cost matrix is given as data file.
3. Use the table below to make a data file for your model.
4. **[R]** Solve the problem and report which facility must be opened at each location.
5. **[R]** Now change the integer variables in your model to continuous variables, and re-solve the problem. Report the solution.
6. **[R]** Are the optimal costs for both problems same? If yes, explain why.
7. Will the solution to the continuous problem become fractional (non-integer) if the costs are changed to non-integer values? Try changing the costs and whether the solution to the LP becomes fractional.
8. **[R]** Now suppose that, due to some reason, faciliy 1 can not be assigned to location 2, facility 11 can not be assigned to location 11 and facility 8 can not be assigned to location 8. What changes in `.mod` or in `.dat` file will you make? Solve the integer problem and report the solution.

Facility	Location											
	1	2	3	4	5	6	7	8	9	10	11	12
1	22	12	18	19	22	21	17	20	16	15	21	24
2	19	22	19	21	22	24	18	17	21	19	22	23
3	18	23	20	20	21	22	19	18	20	23	19	19
4	23	21	20	18	17	19	24	16	18	16	20	24
5	18	17	16	19	24	21	23	21	20	21	22	21
6	22	20	17	16	20	23	22	25	24	19	17	20
7	23	18	17	15	22	24	23	20	22	19	23	20
8	21	22	21	23	18	17	16	19	24	21	20	23
9	24	20	17	18	16	24	19	17	18	20	21	23
10	20	22	21	24	20	23	19	18	23	24	25	20
11	19	24	22	20	23	19	18	16	22	24	21	24
12	22	23	24	20	21	20	20	19	17	19	20	22

Exercise 3: Max-Cut of a Graph

Let $G = (V, E)$ be an undirected graph, where V and E are, respectively, the set of vertices and edges. Let $w_{ij} \geq 0$ be the weight of the edge $(i, j) \in E$.

A cut in a weighted undirected graph is defined as a partition of the vertices into two sets S and $V \setminus S$ (say), and the weight W_S of the cut $(S, V \setminus S)$ is the summation of the weights of the edges who have one end point in S and the other in $V \setminus S$, i.e.,

$$W_S = \sum_{(i,j) \in E : i \in S, j \in V \setminus S} w_{ij}$$

The max-cut problem is defined as the problem to find a cut in the graph G with maximum weight, i.e.,

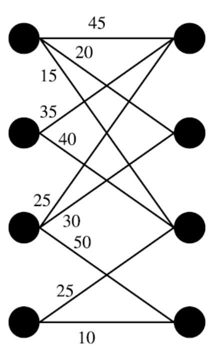
$$\max_{S \subsetneq V} W_S$$

1. [R] Write an integer programming formulation to find the max cut of a given graph. Define all the variables and constraints clearly.

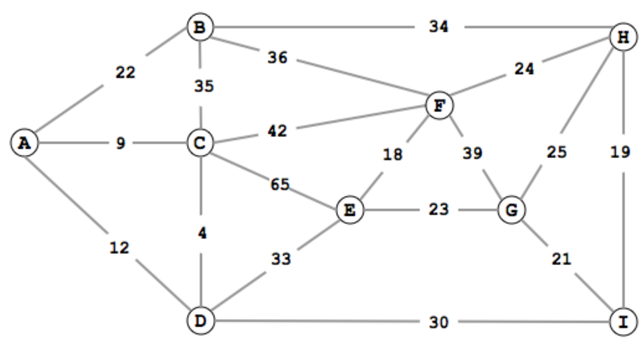
2. **[R]** Consider the first graph given below. Use your intuition to find the max-cut for the graph (a). Explain why your intuition is correct.

3. Write a proper model and data file for the graph (b) and solve the integer problem using CPLEX or Gurobi.

4. **[R]** Write down the solution and the value of max-cut for graph (b).



(a) Graph 1



(b) Graph 2