Relationship between PSC strengths and network parameters

Unitary conductance. A PSC from in response to a spike at time $t = 0$ is

$$g(t) = \frac{c_{\text{syn}}}{\tau_s} e^{-t/\tau_s}\, \Theta(t) \tag{1}$$

Assuming current-only interaction, $I_{\text{syn}} = g(t)\Delta V_{\text{syn}}$, where $\Delta V_{\text{syn}} = V_L - V_{\text{syn}}$. The PSP for $t > 0$ is governed by

$$C\frac{dV}{dt} = -g_L V - \frac{c_{\text{syn}}\, \Delta V_{\text{syn}}}{\tau_s}\, e^{-t/\tau_s} \tag{2}$$

$$\frac{dV}{dt} = -\frac{V}{\tau_0} - \frac{c_{\text{syn}}\, \Delta V_{\text{syn}}}{C\tau_s}\, e^{-t/\tau_s} \tag{3}$$

Assuming $V(0) = 0$,

$$V(t) = -\frac{c_{\text{syn}}\, \Delta V_{\text{syn}}}{C}\, \frac{\tau_0}{\tau_0 - \tau_s}\, \left(e^{-t/\tau_0} - e^{-t/\tau_s}\right) \tag{4}$$

Extremum:

$$t_p = \frac{\tau_0\, \tau_s}{\tau_0 - \tau_s} \log\left(\frac{\tau_0}{\tau_s}\right) \tag{5}$$

Extreme value

$$V_{\text{extr}} = -\frac{c_{\text{syn}}\, \Delta V_{\text{syn}}}{C}\, \frac{\tau_0}{\tau_0 - \tau_s} \left[\left(\frac{\tau_s}{\tau_0}\right)^{\frac{\tau_s}{\tau_0 - \tau_s}} - \left(\frac{\tau_s}{\tau_0}\right)^{\frac{\tau_0}{\tau_0 - \tau_s}}\right] \tag{6}$$

Defining

$$f(\tau_0, \tau_s) = \frac{\tau_0}{\tau_0 - \tau_s} \left[\left(\frac{\tau_s}{\tau_0}\right)^{\frac{\tau_s}{\tau_0 - \tau_s}} - \left(\frac{\tau_s}{\tau_0}\right)^{\frac{\tau_0}{\tau_0 - \tau_s}}\right] \tag{7}$$

we obtain

$$c_{\text{syn}} = -\frac{V_{\text{extr}}\, C}{\Delta V_{\text{syn}}\, f(\tau_0, \tau_s)} \tag{8}$$

Asynchronized, noisy thalamic input

There are K_T neurons outside of the network projecting to one neurons in the network. These neurons fire in a Poisson manner with a rate R_T and strength of unitary conductance $c_{\text{syn,b}}$. The time-average conductance a neuron receives is

$$g_T = K_T R_T \left\langle \frac{c_{\text{syn,b}}}{\tau_s}\, e^{-t/\tau_s}\, \Theta(t) \right\rangle_t = K_T\, R_T\, c_{\text{syn,b}} \tag{9}$$

The response of a neuron to a single PSC is given by an Equation similar to Eq. 8. Therefore,

$$g_T = -\frac{K_T\, R_T\, V_{\text{extr}}\, C}{\Delta V_{\text{syn}}\, f(\tau_0, \tau_s)} \tag{10}$$

Response to time-constant background noise and threshold crossing

$$\frac{dV}{dt} = -\frac{V}{\tau_0} - \frac{g_T\,\Delta V_{\rm syn}}{C} \tag{11}$$

At sub-threshold steady state,

$$V = -\frac{g_T\,\Delta V_{\rm syn}\tau_0}{C} \tag{12}$$

Using Eq. 10

$$V = \frac{K_T\,R_T\,V_{\rm extr}\,\tau_0}{f(\tau_0,\tau_s)} \tag{13}$$

The threshold condition is

$$K_T\,R_T = \frac{V_{\rm th} f(\tau_0,\tau_s)}{V_{\rm extr}\,\tau_0} \tag{14}$$

If $\tau_s = 3\,\mathrm{ms}$, $f(\tau_0,\tau_s)$ is 0.597 for $\tau_0 = 10\,\mathrm{ms}$ and 0.715 for $\tau_0 = 20\,\mathrm{ms}$.

In order to get the value of $K_T\,R_T$ in Hz, we multifpy the numerical value obtained in Eq. 14 by 1000. Assuming $V_{\rm th} = 0.5\,\mathrm{mV}$ and $V_{\rm extr} = 10\,\mathrm{mV}$, $K_T\,R_T = 1194\,\mathrm{Hz}$ for $\tau_0 = 10\,\mathrm{ms}$ and 715 Hz for $\tau_0 = 20\,\mathrm{ms}$.

Conductance-based model

Extended Wang-Buzsáki model (Hansel and van Vreeswijk 2012).

E neuron: $g_L = 0.05\,\mathrm{mS/cm^2}$ ($\tau_0 \approx 20\,\mathrm{ms}$), $g_{KZ} = 0.5\,\mathrm{mS/cm^2}$.

I neuron: $g_L = 0.1\,\mathrm{mS/cm^2}$ ($\tau_0 \approx 10\,\mathrm{ms}$), $g_{KZ} = 0$.

The average thalamic current is:

$$I_{\rm syn} = g_{\rm syn}\left[\rho\,(V - V_{\rm syn}) + (1-\rho)\,(V_L - V_{\rm syn})\right] \tag{15}$$

$\rho = 1$ means coupling via conductance, and $\rho = 0$ means coupling via current. We take $\rho = 0.5$. If $g_{\rm syn}$ is constant in time, we compute the bifurcation diagram with $g_{\rm syn}$ being the bifurcation parameter. The critical values $g_{\rm syn,c}$ (in $\mathrm{mS/cm^2}$) are 2.622×10^{-3} for an E neuron and 8.329×10^{-3} for an I neuron.

From Eq. 10, the critical value for $K_T\,R_T$ is

$$(K_T\,R_T)_c = -\frac{g_{\rm syn,c}\,\Delta V_{\rm syn}\,f(\tau_0,\tau_s)}{V_{\rm extr}\,C} \tag{16}$$

We get $K_T\,R_T = 244\,\mathrm{Hz}$ for E neurons and $K_T\,R_T = 646\,\mathrm{Hz}$ for I neurons.

Recurrent connections: scaling the coupling constants

We use the matrix notation. For example, "IE" means from E to I. For a single recurrent connection, the coupling coefficient is computed using Eq. 8

EE: $\tau_0 = 20\,\text{ms}$, $\tau_s = 3\,\text{ms}$, $V_{\text{extr}} = 0.5\ \text{mV}$, $\Delta V_{\text{syn}} = -65\,\text{mV}$, $c_{\text{syn}} = 0.0108\,\mu\text{F/cm}^2$.

IE: $\tau_0 = 10\,\text{ms}$, $\tau_s = 3\,\text{ms}$, $V_{\text{extr}} = 1\,\text{mV}$, $\Delta V_{\text{syn}} = -65\,\text{mV}$, $c_{\text{syn}} = 0.0258\,\mu\text{F/cm}^2$.

EI: $\tau_0 = 20\,\text{ms}$, $\tau_s = 3\,\text{ms}$, $V_{\text{extr}} = 1\ \text{mV}$, $\Delta V_{\text{syn}} = 20\,\text{mV}$, $c_{\text{syn}} = 0.0699\,\mu\text{F/cm}^2$.

IE: $\tau_0 = 10\,\text{ms}$, $\tau_s = 3\,\text{ms}$, $V_{\text{extr}} = 1\ \text{mV}$, $\Delta V_{\text{syn}} = 20\,\text{mV}$, $c_{\text{syn}} = 0.0838\,\mu\text{F/cm}^2$.

Weak synapses scenario: The coupling is $c_{\text{syn,AB}} = J_{AB}/K_{AB}$

EE: $K = 400$, $J_{AB} = 4.3\,\mu\text{F/cm}^2$.

IE: $K = 800$, $J_{AB} = 20.6\,\mu\text{F/cm}^2$.

EI: $K = 100$, $J_{AB} = 7\,\mu\text{F/cm}^2$.

II: $K = 100$, $J_{AB} = 8.38\,\mu\text{F/cm}^2$.

Strong synapses or balanced network scenario: The coupling is $c_{\text{syn,AB}} = J_{AB}/\sqrt{K_{AB}}$

EE: $K = 400$, $J_{AB} = 0.22\,\mu\text{F/cm}^2$.

IE: $K = 800$, $J_{AB} = 0.73\,\mu\text{F/cm}^2$.

EI: $K = 100$, $J_{AB} = 0.7\,\mu\text{F/cm}^2$.

II: $K = 100$, $J_{AB} = 0.84\,\mu\text{F/cm}^2$.