Natural computing

Assignment 2

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1 Exercise 1

You and a friend are in an Japanese restaurant, and the owner offers both of you an 8-pieces sushi for free under the following condition. Each of you must simultaneously announce how many pieces you would like; that is, each player $i1, 2$ names his desired amount of sushi, $0 \leq s_i \leq 8$.

If $s_1 + s_2 \leq 8$ then the players get their demands (and the owner eats any leftover pieces). If $s_1 + s_2 > 8$, then the players get nothing. Assume that you each care only about how much sushi you individually consume, and the more the better.

(a) *Write out or graph each players best-response correspondence.*

(b) *What outcomes can be supported as pure-strategy Nash equilibria?*

The first thing that can be done, is writing out the payoff matrix. The following table is the payoff matrix for this problem.

The best response choices to the opponent's (P_2) choice are bold and are summarized in the following table.

Suppose that (n, m) is a Nash equilibria. First notice that it must hold that $n = 8-m$, since that is the best possible choice after observing m. If $n > 8 - m$, then the reward is 0 for both players. If $n < 8 - m$, then $n = 8 - m$ is a higher reward. So $n = 8 - m$ is the highest possible reward. For the Nash equilibrium, it must hold that n is the best response to m and m is the best response to n. So this yields two equations: $n = 8 - m$ and $m = 8 - n$. All the bold states in the payoff matrix are therefore Nash equilibria.

		P_2								
		0	1	2	3	4	5	6	7	8
P_1	0	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	**0,8**
	1	1,0	1,1	1,2	1,3	1,4	1,5	1,6	**1,7**	0,0
	2	2,0	2,1	2,2	2,3	2,4	2,5	**2,6**	0,0	0,0
	3	3,0	3,1	3,2	3,3	3,4	**3,5**	0,0	0,0	0,0
	4	4,0	4,1	4,2	4,3	**4,4**	0,0	0,0	0,0	0,0
	5	5,0	5,1	5,2	**5,3**	0,0	0,0	0,0	0,0	0,0
	6	6,0	6,1	**6,2**	0,0	0,0	0,0	0,0	0,0	0,0
	7	7,0	**7,1**	0,0	0,0	0,0	0,0	0,0	0,0	0,0
	8	**8,0**	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0

Table 1: Payoff matrix.

Choice of P_2	**Best response of P_1**
0	8
1	7
2	6
3	5
4	4
5	3
6	2
7	1
8	0

Table 2: Best responses.

2

(Replicator Dynamics.) Consider a two-player game with strategies A and B and payoffs $\pi(A,A) = 3$, $\pi(A,B) = \pi(B,A) = 0$, $\pi(B,B) = 1$. *Denote by x the fraction of population with strategy A.*

(a) *Compute the expected payoff $\pi(A,x)$ and $\pi(B,x)$ of strategy A and B.*

(b) *Give the replicator dynamics equation for the game (use equation in slide 40).*

(c) *Compute the fixed points.*

(d) *Describe the evolution of the population (use example at the bottom of slide 41 as guideline).*

We will first calculate the expected payoff $\pi(A,x)$. Since x denotes the fraction of the population using strategy A, this is also the probability of encountering an individual with strategy A. And this also means that $(1-x)$ is the fraction of the population using strategy B and again also the probability of encountering an individual with strategy B.

And thus we get:

$\pi(A,x) = x * \pi(A,A) + (1-x) * \pi(A,B) = x * 3 + (1-x) * 0 = 3x$

And similarly for the expected payoff of playing strategy B:

$\pi(B,x) = x * \pi(B,A) + (1-x) * \pi(B,B) = x * 0 + (1-x) * 1 = 1-x$

We can then use these results to give the replicator dynamics of the population. The change of the fraction of the population using strategy A is given by:

$\frac{dx}{dt} = (\pi(A,x) - \bar{\pi}(x))x$

where $\bar{\pi}(x)$ is the average payoff given the fraction of the population using strategy A. The average payoff can be calculated by multiplying the fraction of the population using a strategy by the payoff of that strategy and adding those up:

$\bar{\pi}(x) = x*\pi(A,x)+(1-x)*\pi(B,x) = x*3x+(1-x)(1-x) = 3x^2+x^2-2x+1 = 4x^2-2x+1$

We can now fill in this result of the average payoff in the equation for the change in fraction of the population using strategy A:

$\frac{dx}{dt} = (\pi(A,x) - \bar{\pi}(x))x = (3x - 4x^2 + 2x - 1) * x = (5x - 4x^2 - 1) * x = 5x^2 - 4x^3 - x$

We can now equalize this to zero to find the fixed points:

$5x^2 - 4x^3 - x = 0$

We immediately find the first solution at $x = 0$. We can now divide the equation by x to search for the other possible solutions:

$5x - 4x^2 - 1 = 0$

$4x^2 - 5x = -1$

$x^2 - \frac{10}{8} * x = -\frac{1}{4}$

$(x - \frac{5}{8})^2 - (\frac{5}{8})^2 = -\frac{1}{4}$

$(x - \frac{5}{8})^2 = -\frac{1}{4} + \frac{25}{64} = -\frac{16}{64} + \frac{25}{64} = \frac{9}{64}$

$x - \frac{5}{8} = \frac{3}{8}$ or $x - \frac{5}{8} = -\frac{3}{8}$

$x = 1$ or $x = \frac{2}{8} = \frac{1}{4}$

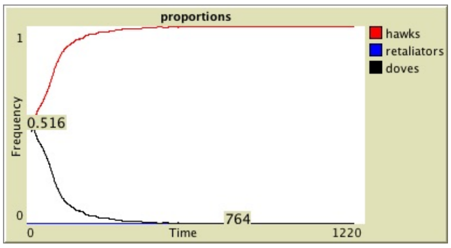
We have now found the three solutions for the fixed points. $x = 0$, $x = \frac{1}{4}$ and $x = 1$ are all fixed points and are all valid numbers for a fraction of the population, which is a number between zero and one. The fixed points $x = 0$ and $x = 1$ are both stable fixed points, while the fixed point at $x = \frac{1}{4}$ is an unstable one. That means that the population will evolve to $x = 0$, if the current population has a fraction of the population using strategy A of less than $\frac{1}{4}$. In other words, in this case individuals using strategy A will go extinct. On the other hand, if the current population has a fraction of population using strategy A that is higher than this fraction $\frac{1}{4}$, the population will evolve to $x = 1$. In other words, in this case the individuals using strategy B will go extinct.

3

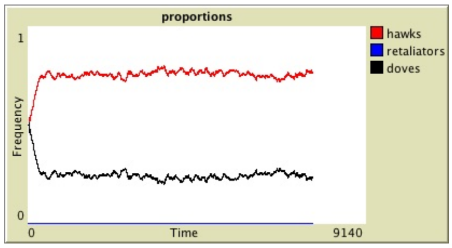
(Experimenting with existing software.) Consider the NetLogo model available at `http://ccl.northwestern.edu/netlogo/models/community/GameTheory`.

(a) Run the model following the steps described in the ##HOW TO USE IT section. Try to justify the results obtained using the different settings described in the section.

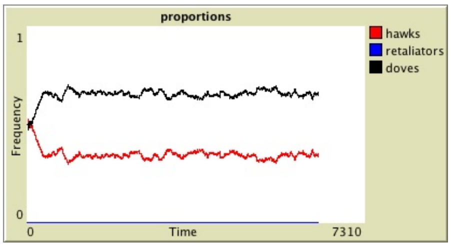
(b) Discuss briefly your experience with the use of such software (pro's and con's).



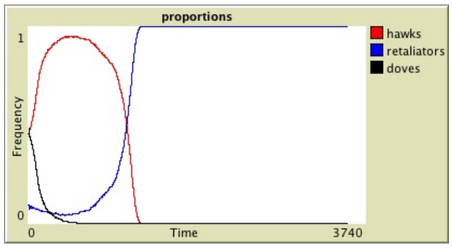
(a) v > 2c **v=5; c=2**



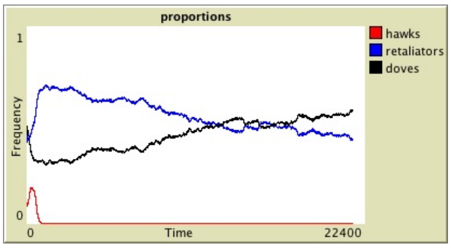
(b) 2c > v > c **v=3; c=2**



(c) c > v **v=2;c=3**



(d) v > 2c
v=5;c=2
$i_h = 50$;$i_d = 50$;$i_r = 10$



(e) v > 2c
v=5;c=2
$i_h = 10$;$i_d = 50$;$i_r = 50$

Figure 1: plots of different netlogo GameTheory runs with different cost value settings. Note that for a,b,c the initial populations was 50/50. For d and e i_h,i_d,i_r represent the initial population sizes of the hawks, doves and retaliators respectively.

3.1 a

Let f_h, f_d, f_r be the frequency of the hawks, doves and retaliators respectively. And let v_h, v_d, v_r be the average value of the hawks, doves and retaliators respectively. If no retaliators are present then $v_h = (.5v - c) * f_h + (1 - f_h) * v$ and the doves have an average of $v_d = .5v * (1 - f_h)$

(a) Let's assume that $v = 2c + \epsilon$ where $\epsilon > 0$ (and thus) $v > 2c$. Then $v_h = .5\epsilon * f_h + (1 - f_h) * v$ and $v_d = .5v * (1 - f_h)$. For every f_h it holds that: $.5v * (1 - f_h) < .5\epsilon * f_h + (1 - f_h) * v$. And thus the doves will always go extinct.

(b) Now $v = 2c - \epsilon v$ but $0 < \epsilon v < c$. Filling it in gives $v_h = -.5\epsilon v * f_h + (1 - f_h) * v$ and $v_d = .5v * (1 - f_h)$ So we have a stable population where:

$$
\begin{aligned}
v_h &= v_d \\
-.5\epsilon v * f_h + v * (1 - f_h) &= .5v * (1 - f_h) \\
.5\epsilon v * f_h &= .5v * (1 - f_h) \\
\epsilon f_h &= (1 - f_h) \\
f_h &= \frac{1}{\epsilon} - \frac{1}{\epsilon} f_h \\
f_h + \frac{1}{\epsilon} f_h &= \frac{1}{\epsilon} \\
f_h &= \frac{\frac{1}{\epsilon}}{1 + \frac{1}{\epsilon}}
\end{aligned}
$$

In general the hawks and doves population will go to a stable point such that the chance of meeting a dove and it's average advantage counterparts that of the average disadvantage of the average number of times it meets another hawk.

If $\epsilon = 1$ and thus $c = v$ then the hawks and doves will have an equal population. if $c > v$ doves will be in the majority and if $c < v$ hawks will be in the majority.
In the case of figure b $\epsilon = \frac{1}{3}$ and thus $f_h = \frac{3}{4}$

(c) See answer b, in the case of figure c $\epsilon = 2$ and thus $f_h = \frac{.5}{1.5} = \frac{1}{3}$

(d) In figure d and e we see two cases. (d) in which the hawks make the doves go extinct like they did in case (a) before the retaliators can make the hawks go extinct. This is due to a relative large population size of the hawks and small of the retaliators.

In case (e) we see that the retaliators make the hawks extinct before they can make the doves go extinct since there are relatively way less hawks now.