Probabilities on logics with lean sets of two-valued states

http://tph.tuwien.ac.at/~svozil/publ/2017-Svozil-Cordoba-pres.pdf

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Some questions one could ask, and answers one might expect

- Does a(n empirical) structure of propositions (logic) induce a probability? No!
- What kind of non-Boolean (non-classical) structure of propositions can one imagine?
  - Classical Boolean algebras
  - Wright's generalized urn model (partition logics)
  - Moore's finite automaton state identification problem (partition logics)
  - quantum logics (Hilbert lattices)
  - general logics constructed by the pasting of Boolean subalgebras (contexts, blocks)
- What criteria/axioms to assume for probabilities? Gleason-type frame functions: additivity of mutual exclusive events, totally (im)probable events have probability 0 and 1, respectively.

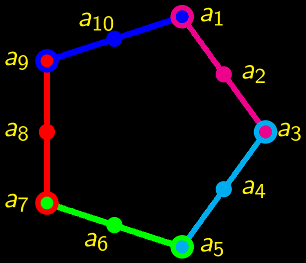
Geometric strategies to classical probabilities

- Froissart (1981), Pitowsky (1986), Tsirelson (1993): geometric interpretation of probability distributions as surface of a convex polytope "spanned" by vertices aka "mutually exclusive extreme cases."
  - The vertices are encoded by two-valued states on the logic.
  - The face (in)equalities indicating "inside-outside relations" are very similar to Boole's "conditions of possible experience" (1854,1862).
  - The *hull problem* of finding these faces is NP-complete in the number of vertices.

Geometric strategies to (quasi)classical probabilities

- I suggest to generalize these methods for (quasi)classical models to situations when there are "enough" [i.e., the set of two-valued states is separating (Kochen-Specker, 1967)] two-valued states on the logic.
- These can be used to finding (quasi)classical probability distributions on, say, partition logics (from generalized urn models or finite automaton state identification).

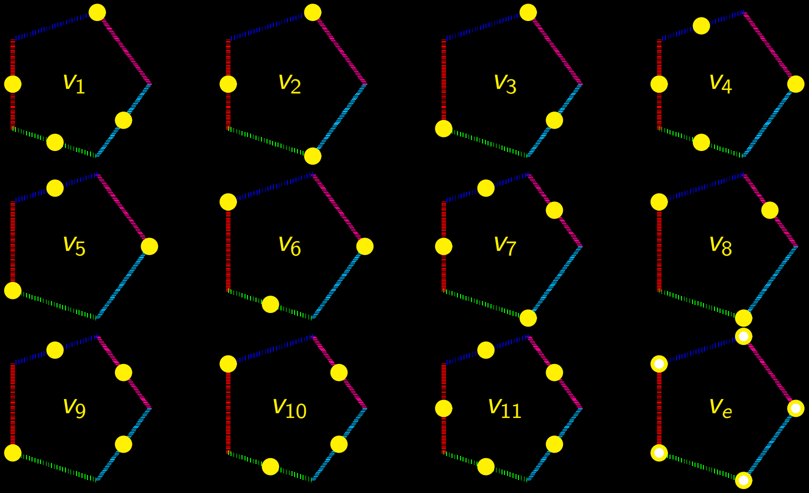
Example I: Pentagon logic



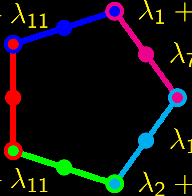
Example I: two-valued states on the pentagon logic (Wright, 1978)

#	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
v_1	1	0	0	1	0	1	0	1	0	0
v_2	1	0	0	0	1	0	0	1	0	0
v_3	1	0	0	1	0	0	1	0	0	0
v_4	0	0	1	0	0	1	0	1	0	1
v_5	0	0	1	0	0	0	1	0	0	1
v_6	0	0	1	0	0	1	0	0	1	0
v_7	0	1	0	0	1	0	0	1	0	1
v_8	0	1	0	0	1	0	0	0	1	0
v_9	0	1	0	1	0	0	1	0	0	1
v_{10}	0	1	0	1	0	1	0	0	1	0
v_{11}	0	1	0	1	0	1	0	1	0	1
v_e	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0

Example I: two-valued states on the pentagon logic (Wright, 1978)



Example I: Probabilities on partition logics from two-valued states on the pentagon logic – with $\lambda_i \geq 0$, $i = 1, \ldots 11$, $\sum_{i=1}^{11} \lambda_i = 1$



Example I: hull computation on the pentagon logic

The full hull computations for the probabilities $p_1, \ldots, p_{10}$ on all atoms $a_1, \ldots, a_{10}$ reduces to 16 inequalities, among them

$$\begin{aligned} p_4 + p_8 + p_9 &\geq +p_1 + p_2 + p_6, \\ 2p_1 + p_2 + p_6 + p_{10} &\geq 1 + p_4 + p_8. \end{aligned} \tag{1}$$

If one considers only the five probabilities on the intertwining atoms, then the Bub-Stairs) inequalitiy (Bub, 2009)

$$p_1 + p_3 + p_5 + p_7 + p_9 \leq 2 \tag{2}$$

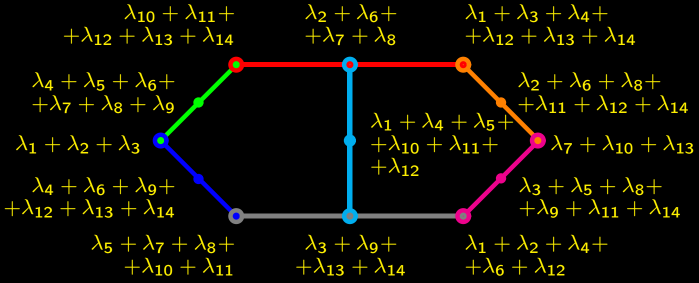
result. Concentration on the four non-intertwining atoms yields

$$p_2 + p_4 + p_6 + p_8 + p_{10} \geq 1. \tag{3}$$

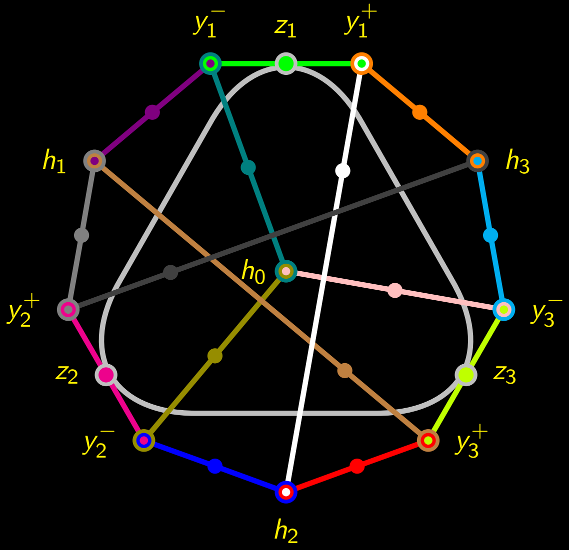
Limiting the hull computation to adjacent pair expectations of dichotomic ± 1 observables yields the Klyachko-Can-Biniciogolu-Shumovsky inequality (2008)

$$E_{13} + E_{35} + E_{57} + E_{79} + E_{91} \geq 3. \tag{4}$$

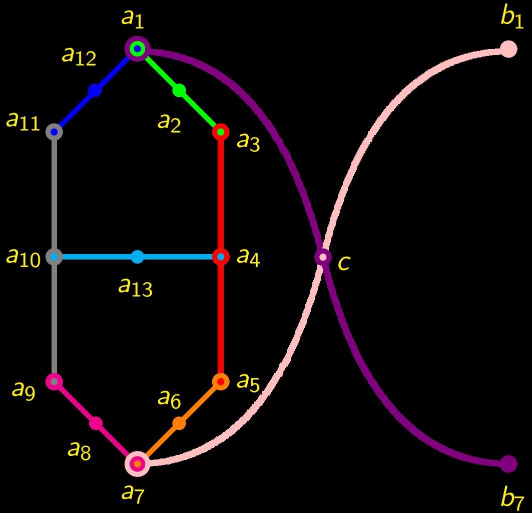
Example II: Specker's "Käfer" (bug) logic (Kochen&Specker, 1965, 67) - true (1) implies false (0) logic – with $\lambda_i \geq 0$, $i = 1, \ldots 14$, $\sum_{i=1}^{14} \lambda_i = 1$



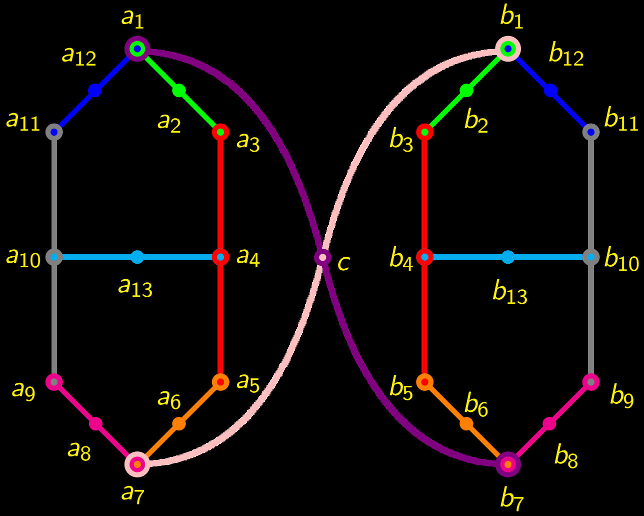
Example III: True (1) implies three times false (0) logic (Yu&Oh, 2012)



Example IV: True (1) implies true (1) logic (Kochen&Specker, 1967)



Example V: Combo of two linked Specker bug logics inducing a non-separating set of two-valued states



Quantum challenge & outlook: Hilbert logics without a two-valued states (Gleason 1957, Specker 1960)

- In such situations the classical strategy to build probabilities from two-valued states fails entirely.
- Gleason suggested a new strategy, which, for pure states (formalizable as normalized vector) can be based upon the "Pythagorean (theorem) view from orthonormal bases."
- In such situations, value indefiniteness rulez (Pitowsky 1998,2004; Abbott,Calude,Conder,KS 2012,2014, arXiv:1503.01985, doi 10.1063/1.49316582014 2015)
- more "exotic" phenomena – e.g. demanding Wright's "exotic" dispersionless state?

Thank you for your attention!