Spurious, Emergent Laws in Number Worlds

Cristian S. Calude

Department of Computer Science, University of Auckland,

Private Bag 92019, Auckland, New Zealand

email: cristian@cs.auckland.ac.nz, URL: http://www.cs.auckland.ac.nz/˜cristian

and

Karl Svozil

Institute for Theoretical Physics, Vienna University of Technology,

Wiedner Hauptstrasse 8-10/136, 1040 Vienna, Austria

email: svozil@tuwien.ac.at, URL: http://tph.tuwien.ac.at/˜svozil

November 7, 2018

Abstract

We study some aspects of the emergence of *lógos* from *xáos* on a simple model of the universe using methods and techniques from algorithmic information and Ramsey theories. Thereby an intrinsic and unusual mixture of meaningful and (spurious) emerging laws surfaces. The emergent laws outnumber the meaningful ones, a picture which is compatible with the lawfulness hypothesis. In accord with the ancient Greek theogony one could say that *lógos*, the Gods and the laws of the universe, originate from "the void," or from *xáos*.

1   Introduction

What if the universe, on the most fundamental layer, just consisted of numbers? This is a suspicion at least as old as the Pythagoreans: as Schrödinger notes [61, Chapter III], "The basic doctrine of the Pythagoreans, we are told, was that *things are numbers.*" More recently Tegmark's Mathematical Universe Hypothesis [67, 68] states that "the physical universe is not merely

described by mathematics, but is a mathematical structure". As a consequence, mathematical existence equals physical existence, and all structures that exist mathematically (even in a non-constructive way) exist physically as well. How could things be numbers? A world "spanned" by numbers can be represented by a single (binary) sequence, or, equivalently, a single real number. All entities encoded therein, including observers as well as measured objects, must be embedded in [69, 66]; that is, they must themselves be (formed out of) numbers or symbols [7]. Non-numeric properties associated with such a "world on a sequence" can arise by way of a structural, levelled hierarchy [3].

Epistemologically this can be perceived as a sort of emergence of reality, which is the inverse of reductionism to some more fundamental, basic levels, involving explanations in terms of ever "smaller" entities: laws – in particular, relational and probabilistic ones – emerge as effective patterns and structures "bottom-up" (rather than "top-down").

Such concepts were quite popular in the *fin de siécle* Viennese physical circles: stimulated by the apparent indeterminacy manifesting in Rutherford's decay law and its corroboration by Schweidler [63], Exner's 1908 inaugural lecture as *Rector Magnificus* included the suggestion that [29, p. 18] "we have to perceive all so-called exact laws as probabilistic which are not valid with absolute certainty; but the more individual processes are involved the higher their certainty". Also Schrödinger's inaugural lecture in Zürich entitled *"What is a natural law?* adopted and promoted Exner's ideas [59, 60], well in accord with Born's later inclinations [8].

In what follows we shall, in a "Humean spirit" [40], study emerging "laws" as a consequence of spurious correlations in data. Two guiding theories will be applied: one is algorithmic information theory, the other is Ramsey theory. The gist of these two ways of looking at data is twofold: "all very long, even irregular" data sequences contain "very large" (indeed, as long as you prefer) regular, computable and thus, in physical terms, *deterministic*, subsequences. Secondly, it is impossible and inevitable for any arbitrary data set *not* to contain a variety of spurious correlations; that is, relational properties which could physically be wrongly interpreted as laws "governing" that universe of data.

2 Natural law

The notion of law in natural sciences, or law of the universe [5, 23, 58, 48, 30] has a long ambivalent history. It might not be overstated to claim that the conjecture that there are laws of nature is the core to what science is and how it was and is performed. Of course, one can refute this view and this lawless hypothesis has been discussed by various authors, see [31, 4, 70, 16, 57, 17, 71, 47, 11]. Contemplating a lawful universe usually amounts to assuming that the laws of nature are objective, have always existed and will exist, and they are written in the language of mathematics. Taken this for granted is an assumption which raises many problems, some of which will be discussed later. In this tradition science can be done in one way, the Galileo-Newton one; but if there are no laws, we can be freed to pursue other methodological options, some of which are not entirely unproblematic. Continuing to enrich the fundamental Greek practice of scientific observation, thinking and debating on different theoretical interpretations of phenomena with other methods, like the experimental methods (since Galileo) and the mathematical models (since Descartes and Newton) is obviously desirable. A step in this direction is to incorporate robust data analytics as a scientific method, see [54, 65, 62, 55]. However, suggestions to narrow down the scientific methods to just a collection of "empirical evidences", to advance purely speculative theories (see [28] for physics) or to promote the "philosophy" according to which correlation supersedes causation and theorising (see [15]) are dangerous.[1]

The laws governing "physis" (nature) and those under which human societies are ruled have often been conflated and postulated to be of the same origin. At the dawn of western civilization Heraclitus held that *lógos*[2] permeates everything, an arrangement common to all things yet incomprehensible to man (DK 22B1, DK 22B2 [24, 22] and [43, 197, 198]). However, there are crucial differences between these laws. As Aristotle argued, a law is "by nature" if it is justified by appeal to something other than an agreement or a decision; in contrast, the laws human societies are ruled by are agreed upon in the Agora. While the former laws have been considered "absolute", the latter are clearly conventional. For example, the laws of movement are natural in contrast with the institutional structure of Greek democracy which is

---

[1]See the Appendix for a more formal discussion.

[2]*Lógos* is the apparent antithesis of *xáos* in Hesiod's *Theogony* [39].

the result of human consensus. In Rhetoric, I.13, Aristotle discusses also the compatibility between the natural and the conventional laws. a characteristic of human justice, in contrast to divine justice. Both these laws are different from the concept of "natural law" developed in the Greek (Aristotle) as well as the Roman (Cicero) philosophies. In this philosophical sense a "natural law" asserts certain rights inherent by virtue of *human nature.* Endowed by nature – by God or a transcendent source – such a law can be understood universally through human reason [21]. Two typical laws of Aristotelian "physis" are: (i) Nothing moves unless one pushes it (there must be a 'mover' in order to move it). (ii) Because motion does exist, the above law implies that there must be a self-moved mover, i.e., a 'Prime Mover'. Finally, according to the definition of "natural" found in the Nicomachean Ethics, V.7, God is both a lawgiver for humans and the governor of nature, a view which was inherited by Christianity.

3 Laws and limit constructions

The scientific revolution grounded the proposal of new laws of nature on observation and iterable experimentation; sometimes these types of laws were simply guessed or invented, but nonetheless on the grounds of a "meaningful" (physical, theoretical and practical) framework. For example, after several experiments, some of which were just imagined, Galileo and others [26] proposed the "law" of inertia. This law is a fundamental conservation principle, the conservation of momentum, and is a limit principle since no physical body actually moves at constant speed along an Euclidean line – a straight line with no thickness. Yet, by extrapolating from his observations made on the object of bodies as their friction was changed, Galileo was able to deduce the concept of inertia, and closely analyse what circumstances affect this asymptotic movement: friction and gravitation. Thus, by this scientific process of induction, deduction, extrapolation and abduction [52, 25, 42], an Aristotelian, God given, absolute, notion of a law of "physis" was radically modified. The advantage of this notion of physical law based on limit principles and symmetry is visible once Newton made the connection between falling apples and planets: there is no need to be anyone pulling nor pushing the planets to move them around. Indeed, Newton's law of gravitation gives the trajectories of any two bodies in inertial movement within a gravitational field, including apples and planets. On the one hand it became possible to de-

rive Kepler's trajectories and laws for one sun and one planet from Newton's law, without the need for a Prime Mover that is constantly pushing. On the other hand, Newton realised that, with two or more planets, reciprocal interactions destabilise the planets' trajectories (which later would be recognised as a result of chaotic non-linearity). He thus assumed the aid of occasional interventions of God in order to assure the stability of the planetary system *in secula seculorum*: God, through a few sapient touches, was the only guarantor of the long term stability of the Solar System [45]. Poincaré later confirmed mathematically this deep intuition of Newton on the asymptotic chaos within the Solar System (see below for more discussion of this). We should note, however, that this analysis only makes sense in the mathematical continua. Inertia is conceived as a limit property; moreover, its understanding as a conservation law (of momentum) alongside the conservation of energy, as a symmetry in the equations (as a result of Noether's theorems relating symmetries to conserved quantities [49]), is based on continuous symmetries: they are invariant with respect to continuous translations in space or time. A few years later, Galileo, Boyle and Mariotte proposed another limit law: they traced the isothermal hyperbolas of pressure and volume for perfect gases. Of course, actual gases, as a result of friction, gravitation, inter-particle interactions, etc., do not follow this peculiar conic section; yet its abstract, algebraic formulation and its geometric representation, allowed a uniform and general understanding of the earliest law of thermodynamics. Principles referring to inexistent ideal trajectories, at the external limit of phenomena, continued to rule knowledge constructions in physics. As another example, let us consider Boltzmann's ergodic principle: *In a perfect gas a particle stays in a region of a given space for an amount of time proportional to the volume of that region.* Once again this is an asymptotic principle, as it uniformly holds only at the infinite limit in time. On these grounds, Boltzmann's thermodynamic integral that allows the deduction of the second law of thermodynamics (regarding the increase in entropy) is also formulated as a limit construction (an integral): it holds only at the infinite limit of the number of particles in the volume of gas. Can one prove, or at least corroborate, these asymptotic principles? There is no way to put oneself or a measurement instrument at these limit conditions and check for Euclidean straight lines, hyperbolas or behaviour at the asymptotic limit in time. One may only try to falsify some consequences [53]; yet, even in such cases the derivation itself may be wrong, but not necessarily the principle. As has already been pointed out by many philosophers, among them Hume, Berkeley, Kant and Schopenhauer, all we

can produce – and this is a crucial point – is *scientific knowledge*: we understand a lot, but not everything, through these limit principles that unify all movements, all gases, etc., as specific instances of inexistent movements and gases. And, more importantly, as a result we can construct fantastic tools and machines that work reasonably well – but not perfectly well, of course – and have radically changed our lives. With these machines the westerners dominated the world after the scientific revolution, a non trivial consequence of their science and its "absolute" laws. We are typing, reading and exchanging data in networks of the latest of these inventions, an excellent, but not perfect, instance of a limit machine – the Turing machine. One of the limit principles of these machines is Turing's distinction between hardware and software and the identification between program with data that allows abstract, mathematical styles of programming all the while (almost) disregarding their material realisation.

Another important consequence was the discovery of limits of computing, specifically the incomputability of the halting problem, and more generally the development of theoretical computer science [36]. At the same time these limit principles obscured the role played by physics in computing: because of the separation between hardware and software, the role of hardware in computation was largely ignored in theoretical computer science, arguably delaying with a few decades the understanding and development of physics of computation, reversible computing and quantum computing, [44, 32, 46].

4   Order within disordered sequences

In intuitive terms, Ramsey theory states that there exists a certain degree of order in all sets/sequences, regardless of their composition. Heuristically speaking this is so because it is impossible for a collection of data not to have any (spurious) correlations, that is, relational properties among its constituents which are determined only by the size of the data. The simplest example of such (spurious) correlation is given by the *Dirichlet's pigeonhole principle* stating that n pigeons sitting in $m < n$ holes result in at least one hole being filled with at least two pigeons. Or in a party of any six people, some three of them are either mutually acquaintances, or complete strangers to each other [35, 9]. [3] This seemingly obvious statements can be used to

---

[3]Iin fact, there is a second trio who are either mutually acquainted or unacquainted [20].

demonstrate unexpected results; for example, the pigeonhole principle implies that there are two people in Paris who have the same number of hairs on their heads. The pigeonhole principle is true for at least two pigeons and one whole; the party result needs at least six people. A common drawback of both results is their non-effectivity: we know that two people in Paris have the same number of hairs on their heads, but we don't know who they are.

An important result in Ramsey theory is Van der Waerden theorem (see [34]) which states that *in every binary sequence at least one of the two symbols must occur in arithmetical progressions of every length.*[4] The theorem describes a set of arbitrary large strong correlations – in the sequence $x_1 x_2 \ldots x_n \ldots$ there exist arbitrary large k, N such that equidistant positions $k, k+t, k+2t, \ldots k + Nt$ contain the same element (0 or 1), that is, $x_k = x_{k+t} = x_{k+2t}, \cdots = x_{k+Nt}$.[5] Crucial here is the fact that the property holds true for *every* sequence, ordered or disordered. Can we say that these correlations are "spurious"? According to Oxford Dictionary, *spurious* means "Not being what it purports to be; false or fake. False, although seeming to be genuine. Based on false ideas or ways of thinking." The (dictionary) definition of the word "spurious" is semantic, that is, it depends on an assumed theory: one correlation can be spurious according to one theory, but meaningful with respect to another one.

Can we give a definition of "spurious correlation" which is independent of *any* theory? In [15] a *spurious correlation* is defined in a very restrictive way as follows: "a correlation is *spurious* if it appears in a randomly generated sequence". Indeed, in the above sense a spurious correlation is "meaningless" according to any reasonable interpretation because, by construction, its values are chosen at "random", as all data in the sequence. Of course, there are other reasons making a correlation spurious, even within a "non-random" sequence. Van der Waerden theorem proves that in every sequence there are spurious correlations in the above sense – they can be said to "emerge." Therefore, these spurious correlations can also be re-interpreted as "emergent laws." It is important to keep in mind that these "laws" are not properties of a particular sequence, – indeed, they are satisfied by *all* sequences as Van der Waerden theorem proves. How do the spurious correlations manifest themselves in a number world? The more bits of the sequence describing the

---

[4]If we interpret 0 and 1 as colours, then the theorem says that in every binary sequence there exist arbitrarily long monochromatic arithmetical progressions.

[5]Again, the proof is not constructive.

number world we can observe, the longer are the lengths of monochromatic arithmetical progressions. So, once there are (sufficiently many) data, regardless of their intrinsic structure, "laws from nowhere" (*ex nihilo*) emerge. The larger the data set, the greater is the number of emerging laws. How "large" is the set of spurious correlations, in the above sense? Using an argument based on algorithmic information theory, in [15] it is proved that the size of spurious correlations tends exponentially to 1 with the size of the data. Furthermore, the increase of some types of spurious correlations, i.e. emerging laws, can be quantified: Goodman's inequality [33, 64] yields lower bounds on how many spurious correlations are observed as a function of the size of data. Conversely, Pawliuk recently suggested [51] that Goodman's inequality can be utilised for testing the (null) hypothesis that a dataset is random: if the bounds are over-satisfied, the correlations might be not spurious, and thus the dataset might not be stochastic. Can we distinguish between meaningful laws and emerging laws? The answer seems to be negative at least from an computational point of view.

5 Emergence of Turing complete (universal) computation

In view of the "quantification" of information content [18, 13], how could complexity and structures such as universal computation, evolve even in principle? The answer to this question is in the algorithmic information content (complexity) of the number world.

The proof of Turing completeness[6] of the Game of Life provided by Conway in [6, Chapter 25, What Is Life?] is a useful method for exploring how complex behaviour like Turing completeness can emerge from very simple rules, in this case, the rules of cellular automata (see more in [56]). With a universal self-delimiting Turing machine[7] and all algorithmic random strings[8] one can generate *all* strings [13].

Is this phenomenon also possible for sequences, that is, for number worlds?

---

[6] A model of computation is Turing complete – sometimes called Turing universal – if it can simulate any Turing machine, or equivalently, if it can simulate a universal Turing machine.

[7] A machine with a prefix-free domain.

[8] A string is algorithmic random if it cannot be compressed by the self-delimiting universal Turing machine by more than a fixed number of bits.

The answer is affirmative. According to a theorem by Kučera-Gács-Hertlinger [13, p. 179], there effectively exists a process F – which is continuous computable operator – which generates all sequences from the set of Martin-Löf random sequences[9]: in other words, every sequence is the image from F of Martin-Löf random sequence.

6   Is the world number computable?

Of course, there exist infinitely (countable) computable world numbers.

Can we decide whether the sequence describing a given world number is computable? Answering this question is probably impossible both theoretically and empirically. However, we can answer a simpler variant of the question: What is the probability that a world number is computable? If we take as probability the Lebesgue measure [13], then the answer is zero.[10]

The above result shows that the probability that a world number can be generated by an algorithm is zero. If we weaken the above requirement and ask about the probability that there exists an algorithm which generates infinitely many bits of a world number, then the answer remains the same: this probability is nil. This result follows from a theorem in algorithmic information theory saying that the complement of the above set – the set of bi-immune sequences[11] – has probability one [13]. A consequence of this fact, corroborated by an extension of the Kochen-Specker theorem proving value indefiniteness of quantum observables relative to rather weak physical assumptions [2], is that with probability one a number world is produced by repeatedly measuring of such a value indefinite observable.

7   Non-uniform evolution

Two examples of world numbers are particularly interesting: Champernowne world number and Chaitin world number. A Champernowne world number

---

[9]A sequence is Martin-Löf random if there exists a fixed constant such that every finite prefix (string) of the sequence is algorithmically random [13].

[10]One should not think that this means that there are no computable world numbers, which we already stated is false! The result follows from the fact that the computable sequences form a countable set.

[11] A sequence is bi-immune if its corresponding set of natural numbers nor its complement contain an infinite computably enumerable subset.

in base two is given by the sequence

$$C_2 = 010001101100000101001110010111011110000\ldots$$

which consists in the concatenation of all binary strings enumerated in quasi-lexicographical order [19].[12] A Chaitin world number is given by a Chaitin Ω_U number (or halting probability), that is the probability that the universal self-delimiting Turing machine U halts [14]. Both world numbers are Borel normal in the sense that every binary string x appears in these sequences infinitely many times with the same frequency, namely $2^{-|x|}$, where $|x|$ is the length of x. In such a world every text – codified in binary – which was written and will be ever written appears infinitely many times and with the same frequency, which depends only on the length of the text. In particular, any correlation appears in such a world infinitely many times. However, these worlds are also very different: A Champernowne world number is computable, but a Chaitin world number is highly incomputable because it is Martin-Löf random. As a consequence, while both number worlds have all possible correlations repeated infinitely many times, the status of those correlations are different: in a Chaitin world number these correlations are spurious (because of its randomness), but in a Champernowne world number they are not (because its computability, hence highly non-randomness).

How an embedded observer would "feel" to live in such a world? This is a deep question which needs more study. Here we will make only a few simple remarks (see also [16]).

First, no observer or rational agent could decide in a finite time whether they live in a Champernowne or Chaitin world. Second, any observer or rational agent surviving, or at least recording experimental outcomes, a sufficiently long time will see many of the previously discovered accepted "laws" being refuted.

Third, suppose observers "surf" the number world by a long succession of bits, that is long prefixes of Champernowne and Chaitin infinite sequences. Because of the Borel normality of these sequences, the strings "surfed" by observers are Borel normal as finite objects, that is, are distributed uniformly up to finite corrections [12]. How would intrinsic observers experience such variations? In one scenario one may speculate that intrinsically such "interim" periods of monotony may not count at all; that is, these will not be

---

[12]In base 10, $C_{10} = 1234567891011121314151 6\ldots$.

operationally recognized as such: for an embedded observer [69, 66], the world number will remain "dormant" while the number world remains monotonous.

Another option, maybe even more speculative, is to assume that, as long as the world number allows for a sufficiently wide variety of substrings, the intrinsic phenomenology will, through emergent character of (self-)perception, "pick" its own segment or pieces (of numbers) from all the available ones. Indeed, it might not matter at all for intrinsic perception whether, for instance, the cycle time is altered (reduced, increased), or whether the lapse of cycles is arbitrarily exchanged or even inverted: as long as there are still "sufficient" patterns and number states emergence could "process" and "use," lawfulness and consciousness will always ensue [27].

8 Summary

According to Heidegger [37], the most profound and foundational metaphysical issue is to think the existent as the existent (*"das Seiende als das Seiende denken"*). Here the existent is metaphorically interpreted as a sequence of bits. Rather than answering the primary question [38] of why there is existence rather than nothingness, this paper has been mostly concerned with the formal consequences of existence under the least amount of extra assumptions [41]. As it turns out, existence implies an intrinsic and unusual mixture of meaningful and (spurious) emerging laws, in which the emergent laws outnumber the meaningful ones, a picture which is compatible with both lawfulness and lawless hypotheses. Furthermore, the axioms in mathematics find their correspondents in the laws of physics as a sort of *"lógos"* upon which the respective mathematical universe is "created by the formal system" and, by analogy, our own universe might be based upon.

As in biological living systems, the dynamics described above is not a matter of stable or unstable equilibrium, but of far from equilibrium processes which are "structurally stable". This "duality" is supported in physics by the hierarchical layers theory [3, 50]. The simultaneous structural stability and non-conservative behavior in biology, which is a blend of stability and instability is due to the coexistence of opposite properties such as order/disorder and integration/differentiation [10].

Such an active and mindful (some might say self-delusional and projective) approach to order in and purpose of the universe may be interpreted in accord with the ancient Greek theogony [39] by saying that *lógos*, the

Gods and the laws of the universe, originate from "the void," or, in a less certain interpretation, from *xáos*. Very similar concepts were developed in ancient China probably around the same time as Homerus and Hesiod: the *I Ching* utlizes relational properties of symbols from sophisticated stochastic procedures providing inspiration, meaning and advice on what has been understood as divine intent and the way the universe operates.

9   Acknowledgments

References

[1] Spurious correlations. http://www.tylervigen.com/spurious-correlations, Nov 2015.

[2] A. A. Abbott, C. S. Calude, and K. Svozil. Value indefiniteness is almost everywhere. *Physical Review A*, 89(3):032109–032116, 2014.

[3] P. W. Anderson. More is different. Broken symmetry and the nature of the hierarchical structure of science. *Science*, 177(4047):393–396, August 1972.

[4] D. M. Armstrong. *What is a Law of Nature?* Cambridge Studies in Philosophy. Cambridge University Press, Cambridge, 1983.

[5] H. Beebee. Hume and the problem of causation. In P. Russell, editor, *The Oxford Handbook of Hume*, Oxford Handbooks. Oxford University Press, Oxford, New York, 2014,2016.

[6] E. Berlekamp, J. H. Conway, and R. Guy. *Winning Ways*. Academic Press, New York, 1982.

[7] J. L. Borges. *La biblioteca de Babel – The Library of Babel*. Editorial Sur, Buenos Aires, 1941 (Spanish), 1962 (English). See also Borges' 1939 essay "The Total Library".

[8] M. Born. Zur Quantenmechanik der Stoßvorgänge. *Zeitschrift für Physik*, 37(12):863–867, Dec 1926.

[9] C. W. Bostwick, J. Rainwater, and J. D. Baum. E1321. *American Mathematical Monthly*, 66, 02 1959.

[10] M. Buiatti and G. Longo. Randomness and multilevel interactions in biology. *Theory Bioscience*, 132:139–158, 2013.

[11] A. Cabello. A simple explanation of Born's rule, Jan 2018.

[12] C. Calude. Borel normality and algorithmic randomness. In G. Rozenberg and A. Salomaa, editors, *Developments in Language Theory*, pages 113–129. World Scientific, Singapore, 1994.

[13] C. Calude. *Information and Randomness—An Algorithmic Perspective*. Springer, Berlin, second edition, 2002.

[14] C. S. Calude and G. J. Chaitin. What is … a halting probability? *Notices of the AMS*, 57(2):236–237, 2007.

[15] C. S. Calude and G. Longo. The deluge of spurious correlations in big data. *Foundations of Science*, pages 1–18, 2016.

[16] C. S. Calude and W. F. Meyerstein. Is the universe lawful? *Chaos Solitons & Fractals*, 10:1075–1084, Jun 1999.

[17] C. S. Calude, W. F. Meyerstein, and A. Salomaa. *Computable Universe: Understanding and Exploring Nature as Computation*, chapter The Universe is Lawless or "Panton chrematon metron anthropon einai", pages 525–537. World Scientific, Singapore, 2013.

[18] G. J. Chaitin. *Algorithmic Information Theory*. Cambridge Tracts in Theoretical Computer Science, Volume 1. Cambridge University Press, Cambridge, revised edition edition, 1987,2003.

[19] D. G. Champernowne. The construction of decimals normal in the scale of ten. *The Journal of London Mathematical Society*, 8:254–260, 1933.

[20] F. Cheney. E1321. *American Mathematical Monthly*, 66, 02 1959.

[21] R. J. Corbett. The question of natural law in aristotle. *History of Political Thought*, 30(2):229–250, 2009.

[22] P. Curd and R. D. McKirahan. *A Presocratics Reader (Second Edition). Selected Fragments and Testimonia*. Hackett Publishing Co., Indianapolis, Cambridge, second edition, 2011.

[23] G. De Pierris and M. Friedman. Kant and Hume on causality. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2013 edition, 2013.

[24] H. Diels and W. Kranz. *Die Fragmente der Vorsokratiker*. Weidmannsche Buchhandlung, Berlin, sixth edition, 1906,1952.

[25] I. Douven. Abduction. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, summer 2017 edition, 2017.

[26] S. Drake. Galileo and the law of inertia. *American Journal of Physics*, 32(8):601–608, 1964.

[27] G. Egan. *Permutation City*. 1994. accessed January 4, 2017.

[28] G. Ellis and J. Silk. Scientific method: Defend the integrity of physics. *Nature*, 516:321–323, 2014.

[29] F. S. Exner. *Über Gesetze in Naturwissenschaft und Humanistik: Inaugurationsrede gehalten am 15. Oktober 1908*. Hölder, Ebooks on Demand Universittsbibliothek Wien, Vienna, 1909, 2016. handle https://hdl.handle.net/11353/10.451413, o:451413, Uploaded: 30.08.2016.

[30] R. P. Feynman. *The Character of Physical Law*. MIT Press, Cambridge, MA, 1965.

[31] B. C. V. Fraassen. *Laws and Symmetry*. Clarendon Press, Oxford, 1989.

[32] M. Frank, C. Vieri, J. Ammer, N. Love, N. H. Margolus, and T. Knight. A scalable reversible computer in silicon. In C. S. Calude, J. Casti, and M. J. Dinneen, editors, *Unconventional Models of Computation*, pages 183–200, Singapore, 1998. Springer.

[33] A. W. Goodman. On sets of acquaintances and strangers at any party. *American Mathematical Monthly*, 66, 11 1959.

[34] R. Graham. Some of my favorite problems in Ramsey Theory. *INTEGERS, The Electronic Journal of Combinatorial Number Theory*, 7(2):#A2, 2007.

[35] R. E. Greenwood and A. M. Gleason. Combinatorial relations and chromatic graphs. *Canadian Journal of Mathematics*, 7:1–7, 1955.

[36] J. Gruska. *Foundations of Computing*. International Thompson Computer Press, London, April 1997.

[37] M. Heidegger. *Was ist Metaphysik?* Klostermann, Frankfurt, 1929,1943,1949.

[38] M. Heidegger. *Einführung in die Metaphysik (Freiburger Vorlesung Sommersemester 1935)*, volume 40 of *Martin Heidegger Gesamtausgabe*. Klostermann, Frankfurt, 1935,1953,1983.

[39] Hesiod. *Hesiod: Volume I, Theogony. Works and Days. Testimonia (Loeb Classical Library No. 57)*. Harvard University Press, Cambridge, MA and London, England, 2006.

[40] D. Hume. *A Treatise of Human Nature: Volume 1: Texts*, volume 1 of *The Clarendon Edition of the Works of David Hume*. Clarendon Press and Oxford University Press, 2007. edited by David Fate Norton and Mary J. Norton.

[41] E. T. Jaynes. *Probability Theory: The Logic Of Science*. Cambridge University Press, Cambridge, 2003.

[42] J. R. Josephson and S. G. Josephson. *Abductive Inference: Computation, Philosophy, Technology*. Cambridge University Press, Cambridge, New York, Melbourne, 1994.

[43] G. S. Kirk, J. E. Raven, and M. Schofield. *The Presocratic Philosophers: A Critical History with a Selcetion of Texts*. Cambridge University Press, Cambridge, 2 edition, 1957,1983.

[44] R. Landauer. Computation: A fundamental physical view. *Physica Scripta*, 35(1):88, 1987.

[45] J. Laskar. A numerical experiment on the chaotic behaviour of the solar system. *Nature*, 338, 1989.

[46] D. N. Mermin. *Quantum Computer Science*. Cambridge University Press, Cambridge, 2007.

[47] M. P. Mueller. Could the physical world be emergent instead of fundamental, and why should we ask? (short version), 2017.

[48] S. Mumford and R. L. Anjum. *Causation: A Very Short Introduction*. Very Short Introductions. Oxford University Press, 2014.

[49] E. Noether. Invariante variationsprobleme. *Gott. Nachr.*, (235–257), 1918.

[50] H. H. Pattee. *Postscript: Unsolved Problems and Potential Applications of Hierarchy Theory*, pages 111–124. Springer Netherlands, Dordrecht, 1973, 2012. Volume 7 of the series Biosemiotics.

[51] M. Pawliuk. Statistical Ramsey behavior in large datasets. Technical report, 2017.

[52] C. S. Peirce, C. Hartshorne, P. Weiss, and A. W. Burks. *Collected Papers of Charles Sanders Peirce*. Harvard University Press, Belknap Press, 1932.

[53] K. R. Popper. *The Logic of Scientific Discovery*. Hutchinson & Co and Routledge, New York, 1959, 1992.

[54] A. Rajaraman and J. D. Ullman. *Mining of Massive Datasets*. Cambridge University Press, Cambridge, UK, 2011.

[55] D. A. Reed and J. Dongarra. Exascale computing and big data. *Commun. ACM*, 58(7):56–68, June 2015.

[56] P. Rendell. *Turing Machine Universality of the Game of Life*. Springer International Publishing, Cham, Heidelberg, New York, Dordrecht, London, 2016.

[57] J. Rosen. *Lawless Universe: Science and the Hunt for Reality*. The John Hopkins University Press, Balrtimreo, Maryland, 2010.

[58] B. Russell. I. On the notion of cause. *Proceedings of the Aristotelian Society (Hardback))*, 13, 06 1913.

[59] E. Schrödinger. Was ist ein Naturgesetz? *Naturwissenschaften (The Science of Nature)*, 17, 01 1929.

[60] E. Schrödinger. *Science And The Human Temperament*. George Allen & Unwin, 1935.

[61] E. Schrödinger. *Nature and the Greeks*. Cambridge University Press, Cambridge, 1954, 2014.

[62] R. Schutt and C. O'Neil. *Doing Data Science*. O'Reilly Media, 2014.

[63] E. v. Schweidler. *Über Schwankungen der radioaktiven Umwandlung*, pages German part, 1–3. Paris, 1906.

[64] A. J. Schwenk. Acquaintance graph party problem. *American Mathematical Monthly*, 79, 12 1972.

[65] J. M. Stanton. *Introduction to Data Science*. Syracuse University, Syracuse, 2012.

[66] K. Svozil. Extrinsic-intrinsic concept and complementarity. In H. Atmanspacher and G. J. Dalenoort, editors, *Inside versus Outside*, volume 63 of *Springer Series in Synergetics*, pages 273–288. Springer, Berlin Heidelberg, 1994.

[67] M. Tegmark. The mathematical universe. *Foundations of Physics*, 38(2):101–150, 2007.

[68] M. Tegmark. *Our Mathematical Universe: My Quest for the Ultimate Nature of Reality*. Penguin Random House LLC., New York, 2014.

[69] T. Toffoli. The role of the observer in uniform systems. In G. J. Klir, editor, *Applied General Systems Research: Recent Developments and Trends*, pages 395–400. Plenum Press, Springer US, New York, London, and Boston, MA, 1978.

[70] B. C. van Fraassen. *Laws and Symmetry*. Oxford University Press, 1989, 2003.

[71] N. Yanofsky. Chaos makes the multiverse unnecessary. *Nautilus*, pages 1–16, June 22 2017.

Appendix
Causation and correlation: Two formal models

To understand better the difference between causation and correlation we present two simple models. In the first model we have two hypotheses, x and y which can true or false and we denote by $x \succ y$ the proposition "x is a cause for y" and by $C(x, y)$ the proposition "x and y are correlated". The logical representations of the new propositions are enumerated in the following table:

x	y	$x \succ y$	$C(x, y)$
0	0	0	1
0	1	0	0
1	0	0	0
1	1	1	1

Causation versus correlation: a logical model

Indeed, $x \succ y = 1$ if x is true, then y is true, that is, $x = y = 1$. Note that $x \succ y$ is a "more restrictive" operator than the logical implication which is true also when $0 \to y = 1$, for every $y \in \{0, 1\}$. We have $C(x, y = 1)$ if and only if both x and y are either true or false, that is, $x = y$. If follows that $x \succ y$ implies $C(x, y) = 1$, but the converse is false.

For the second model we assume that data is represented by two sets X and Y. If $f \colon X \to Y$ is a function from X to Y, then we denote the graph of f by $G_f = \{(x, f(x)) \in X \times Y \mid f(x) = y\}$. A relation R between X and Y is a set $R \subseteq X \times Y$. We say that $x \in X$ is an f-cause for $y \in Y$ if $f(x) = y$ and we write $x \succ_f y$. The elements x, y are correlated by the relation R, in writing, $C_R(x, y)$, if $(x, y) \in R$. Assume that $G_f \subset R$; if $x \succ_f y$, then $C_R(x, y)$ but the converse implication is not true.

Both models show that correlation is symmetric, but causation is not. However, the models above do not reflect a crucial difference between causation and correlation: the former contributes to the understanding, in an

imperfect way, of the phenomenon, but the second is just a syntactical observation. Causation invites testing, revision, even abandonment; correlation is static and without further analysis could be misleading, see [1].

References

[1] Spurious correlations. http://www.tylervigen.com/spurious-correlations, Nov 2015.

[2] A. A. Abbott, C. S. Calude, and K. Svozil. Value indefiniteness is almost everywhere. *Physical Review A*, 89(3):032109–032116, 2014.

[3] P. W. Anderson. More is different. Broken symmetry and the nature of the hierarchical structure of science. *Science*, 177(4047):393–396, August 1972.

[4] D. M. Armstrong. *What is a Law of Nature?* Cambridge Studies in Philosophy. Cambridge University Press, Cambridge, 1983.

[5] H. Beebee. Hume and the problem of causation. In P. Russell, editor, *The Oxford Handbook of Hume*, Oxford Handbooks. Oxford University Press, Oxford, New York, 2014, 2016.

[6] E. Berlekamp, J. H. Conway, and R. Guy. *Winning Ways*. Academic Press, New York, 1982.

[7] J. L. Borges. *La biblioteca de Babel – The Library of Babel*. Editorial Sur, Buenos Aires, 1941 (Spanish), 1962 (English). See also Borges' 1939 essay "The Total Library".

[8] M. Born. Zur Quantenmechanik der Stoßvorgänge. *Zeitschrift für Physik*, 37(12):863–867, Dec 1926.

[9] C. W. Bostwick, J. Rainwater, and J. D. Baum. E1321. *American Mathematical Monthly*, 66, 02 1959.

[10] M. Buiatti and G. Longo. Randomness and multilevel interactions in biology. *Theory Bioscience*, 132:139–158, 2013.

[11] A. Cabello. A simple explanation of Born's rule, Jan 2018, arXiv:1801.06347.

[12] C. Calude. Borel normality and algorithmic randomness. In G. Rozenberg and A. Salomaa, editors, *Developments in Language Theory*, pages 113–129. World Scientific, Singapore, 1994.

[13] C. Calude. *Information and Randomness—An Algorithmic Perspective*. Springer, Berlin, second edition, 2002.

[14] C. S. Calude and G. J. Chaitin. What is . . . a halting probability? *Notices of the AMS*, 57(2):236–237, 2007.

[15] C. S. Calude and G. Longo. The deluge of spurious correlations in big data. *Foundations of Science*, pages 1–18, 2016.

[16] C. S. Calude and W. F. Meyerstein. Is the universe lawful? *Chaos Solitons & Fractals*, 10:1075–1084, Jun 1999.

[17] C. S. Calude, W. F. Meyerstein, and A. Salomaa. *Computable Universe: Understanding and Exploring Nature as Computation*, chapter The Universe is Lawless or "Panton chrematon metron anthropon einai", pages 525–537. World Scientific, Singapore, 2013.

[18] G. J. Chaitin. *Algorithmic Information Theory*. Cambridge Tracts in Theoretical Computer Science, Volume 1. Cambridge University Press, Cambridge, revised edition edition, 1987, 2003.

[19] D. G. Champernowne. The construction of decimals normal in the scale of ten. *The Journal of London Mathematical Society*, 8:254–260, 1933.

[20] F. Cheney. E1321. *American Mathematical Monthly*, 66, 02: 141–142, 1959.

[21] R. J. Corbett. The question of natural law in Aristotle. *History of Political Thought*, 30(2):229–250, 2009.

[22] P. Curd and R. D. McKirahan. *A Presocratics Reader (Second Edition). Selected Fragments and Testimonia*. Hackett Publishing Co., Indianapolis, Cambridge, second edition, 2011.

[23] G. De Pierris and M. Friedman. Kant and Hume on causality. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Winter 2013 edition, 2013.

[24] H. Diels and W. Kranz. *Die Fragmente der Vorsokratiker*. Weidmannsche Buchhandlung, Berlin, sixth edition, 1906, 1952.

[25] I. Douven. Abduction. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Summer 2017 edition, 2017.

[26] S. Drake. Galileo and the law of inertia. *American Journal of Physics*, 32(8):601–608, 1964.

[27] G. Egan. *Permutation City*. 1994. accessed January 4, 2017.

[28] G. Ellis and J. Silk. Scientific method: Defend the integrity of physics. *Nature*, 516:321–323, 2014.

[29] F. S. Exner. *Über Gesetze in Naturwissenschaft und Humanistik: Inaugurationsrede gehalten am 15. Oktober 1908*. Hölder, Ebooks on Demand Universitätsbibliothek Wien, Vienna, 1909, 2016. handle https://hdl.handle.net/11353/10.451413, o:451413, Uploaded: 30.08.2016.

[30] R. P. Feynman. *The Character of Physical Law*. MIT Press, Cambridge, MA, 1965.

[31] B. C. V. Fraassen. *Laws and Symmetry*. Clarendon Press, Oxford, 1989.

[32] M. Frank, C. Vieri, J. Ammer, N. Love, N. H. Margolus, and T. Knight. A scalable reversible computer in silicon. In C. S. Calude, J. Casti, and M. J. Dinneen, editors, *Unconventional Models of Computation*, pages 183–200, Singapore, 1998. Springer.

[33] A. W. Goodman. On sets of acquaintances and strangers at any party. *American Mathematical Monthly*, 66, 11: 778-783, 1959.

[34] R. Graham. Some of my favorite problems in Ramsey Theory. *INTEGERS, The Electronic Journal of Combinatorial Number Theory*, 7(2):#A2, 2007.

[35] R. E. Greenwood and A. M. Gleason. Combinatorial relations and chromatic graphs. *Canadian Journal of Mathematics*, 7:1–7, 1955.

[36] J. Gruska. *Foundations of Computing*. International Thompson Computer Press, London, April 1997.

[37] M. Heidegger. *Was ist Metaphysik?* Klostermann, Frankfurt, 1929, 1943, 1949.

[38] M. Heidegger. *Einführung in die Metaphysik (Freiburger Vorlesung Sommersemester 1935)*, volume 40 of *Martin Heidegger Gesamtausgabe*. Klostermann, Frankfurt, 1935, 1953, 1983.

[39] Hesiod. *Hesiod: Volume I, Theogony. Works and Days. Testimonia (Loeb Classical Library No. 57)*. Harvard University Press, Cambridge, MA and London, England, 2006.

[40] D. Hume. *A Treatise of Human Nature: Volume 1: Texts*, volume 1 of *The Clarendon Edition of the Works of David Hume*. Clarendon Press and Oxford University Press, 2007. edited by David Fate Norton and Mary J. Norton.

[41] E. T. Jaynes. *Probability Theory: The Logic Of Science*. Cambridge University Press, Cambridge, 2003.

[42] J. R. Josephson and S. G. Josephson. *Abductive Inference: Computation, Philosophy, Technology*. Cambridge University Press, Cambridge, 1994.

[43] G. S. Kirk, J. E. Raven, and M. Schofield. *The Presocratic Philosophers: A Critical History with a Selcetion of Texts*. Cambridge University Press, Cambridge, 2 edition, 1957,1983.

[44] R. Landauer. Computation: A fundamental physical view. *Physica Scripta*, 35(1):88, 1987.

[45] J. Laskar. A numerical experiment on the chaotic behaviour of the solar system. *Nature*, 338, 1989.

[46] D. N. Mermin. *Quantum Computer Science*. Cambridge University Press, Cambridge, 2007.

[47] M. P. Mueller. Could the physical world be emergent instead of fundamental, and why should we ask? (Short version), arXiv:1712.01816, 2017.

[48] S. Mumford and R. L. Anjum. *Causation: A Very Short Introduction.* Very Short Introductions. Oxford University Press, Oxford, 2014.

[49] E. Noether. Invariante variationsprobleme. *Gott. Nachr.*, (235–257), 1918.

[50] H. H. Pattee. *Postscript: Unsolved Problems and Potential Applications of Hierarchy Theory*, pages 111–124. Springer Netherlands, Dordrecht, 1973, 2012. Volume 7 of the series Biosemiotics.

[51] M. Pawliuk. Statistical Ramsey behavior in large datasets. Technical report, 2017.

[52] C. S. Peirce, C. Hartshorne, P. Weiss, and A. W. Burks. *Collected Papers of Charles Sanders Peirce.* Harvard University Press, Belknap Press, 1932.

[53] K. R. Popper. *The Logic of Scientific Discovery.* Hutchinson & Co and Routledge, New York, 1959, 1992.

[54] A. Rajaraman and J. D. Ullman. *Mining of Massive Datasets.* Cambridge University Press, Cambridge, UK, 2011.

[55] D. A. Reed and J. Dongarra. Exascale computing and big data. *Comm. ACM*, 58(7):56–68, 2015.

[56] P. Rendell. *Turing Machine Universality of the Game of Life.* Springer International Publishing, Cham, Heidelberg, New York, Dordrecht, London, 2016.

[57] J. Rosen. *Lawless Universe: Science and the Hunt for Reality.* The John Hopkins University Press, Maryland, 2010.

[58] B. Russell. I. On the notion of cause. *Proceedings of the Aristotelian Society (Hardback))*, 13, 06 1913.

[59] E. Schrödinger. Was ist ein Naturgesetz? *Naturwissenschaften (The Science of Nature)*, 17, 01 1929.

[60] E. Schrödinger. *Science And The Human Temperament.* George Allen & Unwin, 1935.

[61] E. Schrödinger. *Nature and the Greeks*. Cambridge University Press, Cambridge, 1954, 2014.

[62] R. Schutt and C. O'Neil. *Doing Data Science*. O'Reilly Media, 2014.

[63] E. v. Schweidler. *Über Schwankungen der radioaktiven Umwandlung*, pages German part, 1–3. Paris, 1906.

[64] A. J. Schwenk. Acquaintance graph party problem. *American Mathematical Monthly*, 79, 12: 1113–1117, 1972.

[65] J. M. Stanton. *Introduction to Data Science*. Syracuse University, Syracuse, 2012.

[66] K. Svozil. Extrinsic-intrinsic concept and complementarity. In H. Atmanspacher and G. J. Dalenoort, editors, *Inside versus Outside*, volume 63 of *Springer Series in Synergetics*, pages 273–288. Springer, Berlin, 1994.

[67] M. Tegmark. The mathematical universe. *Foundations of Physics*, 38(2):101–150, 2007.

[68] M. Tegmark. *Our Mathematical Universe: My Quest for the Ultimate Nature of Reality*. Penguin Random House LLC., New York, 2014.

[69] T. Toffoli. The role of the observer in uniform systems. In G. J. Klir, editor, *Applied General Systems Research: Recent Developments and Trends*, pages 395–400. Plenum Press, Springer US, New York, London, and Boston, MA, 1978.

[70] B. C. van Fraassen. *Laws and Symmetry*. Oxford University Press, 1989, 2003.

[71] N. Yanofsky. Chaos makes the multiverse unnecessary. *Nautilus*, pages 1–16, 2017.