

Wondrous Möbius-Penrose-Escher type periodic diagrams and their quantum doubles

Mirko Navara^{*}

*Faculty of Electrical Engineering, Czech Technical University in Prague,
Technická 2, CZ-166 27 Prague 6, Czech Republic*

Karl Svozil[†]

*Institute for Theoretical Physics, TU Wien,
Wiedner Hauptstrasse 8-10/136, 1040 Vienna, Austria*

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Abstract

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I. INTRODUCTION

In the following we shall discuss a quantum analogue, in terms of orthogonality hypergraphs, of what Lionel Sharples Penrose and Roger Penrose, based on paradoxical drawings of Maurits Cornelis Escher [1], termed ‘impossible object’ [2]. Historically, the related Möbius strip has found many artistic representations in antiquity [3].

II. FAITHFUL ORTHOGONAL COORDINATIZATION OF THE MÖBIUS-ESCHER HYPERGRAPH

What will henceforth called the Möbius-Escher hypergraph was introduced in a previous publication [4, Fig. 3, Equ. (5)]

$$\alpha = 2 \cot^{-1} \left(\sqrt{\frac{11}{9} + \frac{1}{81} \sqrt[3]{2262816 - 69984\sqrt{69}} + \frac{2}{9} 2^{2/3} \sqrt[3]{97 + 3\sqrt{69}}} \right) \quad (1)$$

^{*} navara@fel.cvut.cz; <https://cmp.felk.cvut.cz/~navara>

[†] svozil@tuwien.ac.at; <http://tph.tuwien.ac.at/~svozil>

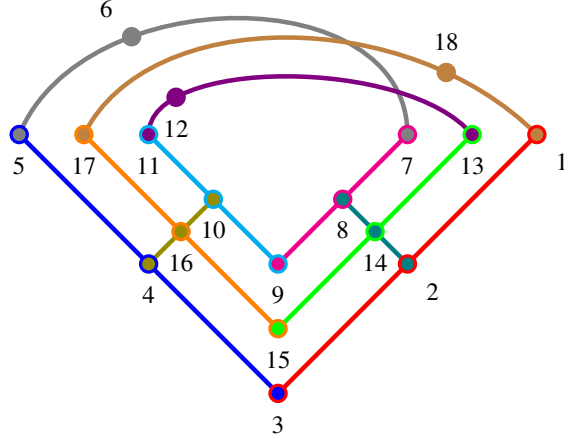


FIG. 1. Möbius-Penrose-Escher type periodic diagram.

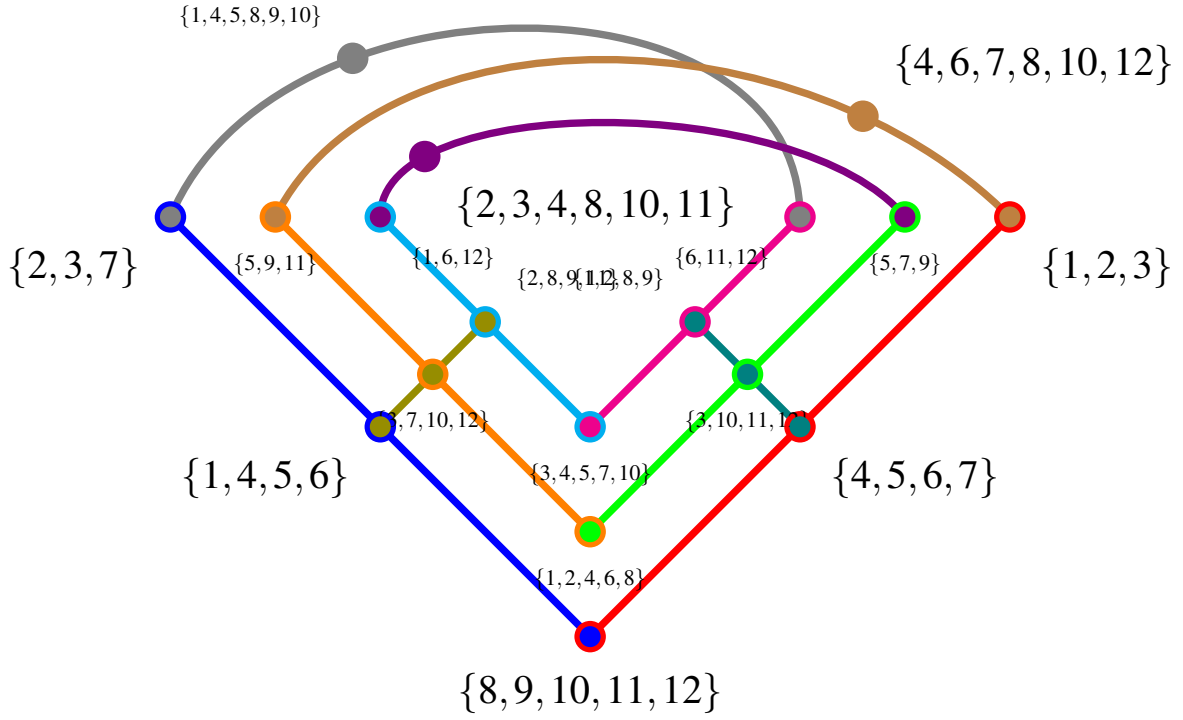


FIG. 2. Möbius-Penrose-Escher type periodic diagram.

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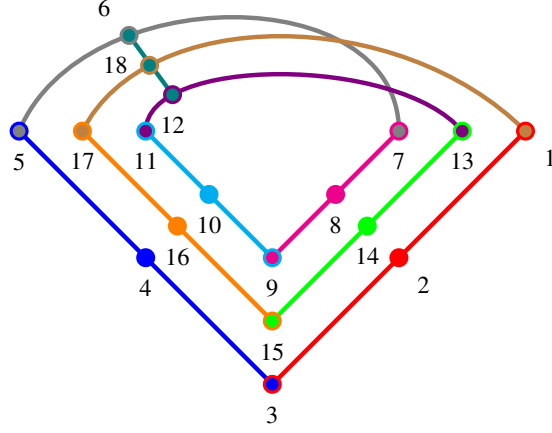


FIG. 3. Möbius-Penrose-Escher type periodic diagram.

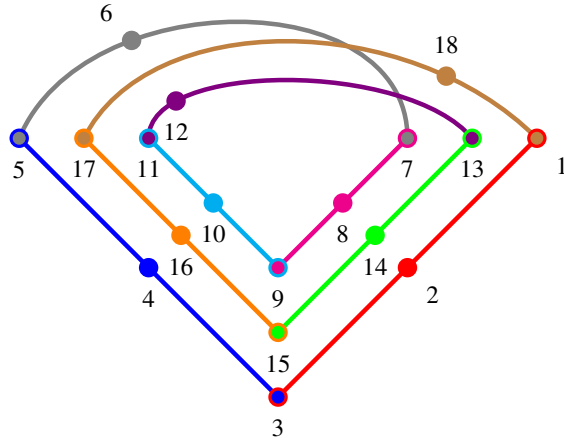


FIG. 4. Möbius-Penrose-Escher type periodic diagram.

Mladen Pavičić for providing a C++ program that heuristically computes the faithful orthogonal representations of hypergraphs written in MMP format, given possible vector components.

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[1] M. C. Escher, *M. C. Escher: Catalogus 118* (Stedelijk Museum, Amsterdam, Netherlands, 1954), catalogue for the M. C. Escher Exhibition at the Stedelijk Museum. On the Occasion of the International

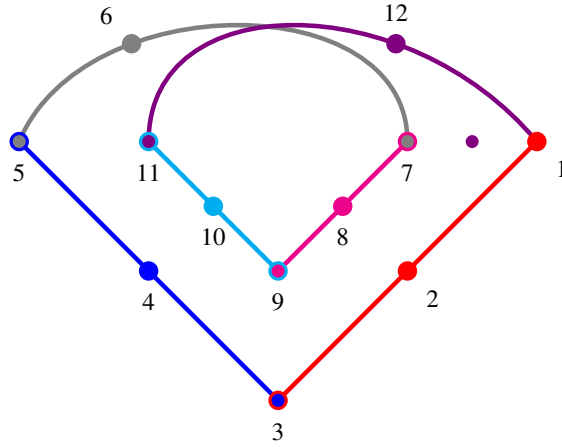


FIG. 5. Periodic diagram which is equivalent to a hexagon.

Congress of Mathematics in Amsterdam, Summer 1954. Book Design by Willem Sandberg.

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