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MATHEMATICAL METHODS OF THEORETICAL PHYSICS

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For the sake of an example, consider again the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \tag{1}$$

and the associated Eigensystem

$$\begin{aligned} &\{\{\lambda_1, \lambda_2, \lambda_3\}, \{\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3\}\} \\ &= \left\{ \{0,1,2\}, \left\{ \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \right\} \right\}. \end{aligned} \tag{2}$$

The projectors associated with the eigenvalues, and, in particular, $\mathbf{E}_1$, can be obtained from the set of eigenvalues $\{0,1,2\}$ by

$$\begin{aligned} p_1(A) &= \left(\frac{A - \lambda_2 \mathbb{I}}{\lambda_1 - \lambda_2}\right)\left(\frac{A - \lambda_3 \mathbb{I}}{\lambda_1 - \lambda_3}\right) \\ &= \frac{\left(\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} - 1\cdot\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right)}{(0-1)} \cdot \frac{\left(\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} - 2\cdot\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right)}{(0-2)} \\ &= \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix} = \mathbf{E}_1. \end{aligned} \tag{3}$$

For the sake of another, degenerate example, consider again the matrix

$$B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} \tag{4}$$

Again, the projectors $\mathbf{E}_1, \mathbf{E}_2$ can be obtained from the set of eigenvalues $\{0,2\}$ by

$$\begin{aligned} p_1(A) &= \frac{A - \lambda_2 \mathbb{I}}{\lambda_1 - \lambda_2} = \frac{\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} - 2\cdot\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}{(0-2)} = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix} = \mathbf{E}_1, \\ p_2(A) &= \frac{A - \lambda_1 \mathbb{I}}{\lambda_2 - \lambda_1} = \frac{\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} - 0\cdot\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}{(2-0)} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \mathbf{E}_2. \end{aligned} \tag{5}$$

Note that, in accordance with the spectral theorem, $\mathbf{E}_1 + \mathbf{E}_2 = \mathbb{I}$ and $0\cdot\mathbf{E}_1 + 2\cdot\mathbf{E}_2 = B$.