

Varieties of contextuality – emphasizing (non)embeddability

<http://tph.tuwien.ac.at/~svozil/publ/2021-QCQMB2021-pres.pdf>

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 - 2.2. empirical / counterfactual / stochastic based on gadget graphs with input/output terminals—aka pre-/postselection of pure quantum states: (Kochen-)Specker bug (1965, aka Hardy-type, cf Stigler's law of eponymy), Belinfante, Stairs, Cabello, ...;

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3. nonempirical / logical / algebraic / theoretical / counterfactual structure of observables with nonclassical interpretations:

- 3.1. **nonfaithful embedding into Boolean algebras associated with inseparability, nonunital value assignments, and other nonclassical properties;**
- 3.2. nonexistence of any classical interpretation aka two-valued (even partial) states: Gleason, Specker, Zierler-Schlessinger, Kamber, Kochen-Specker, Pitowsky, Hrushovski-Pitowsky, Cabello, Abbot-Calude-Svozil ...;

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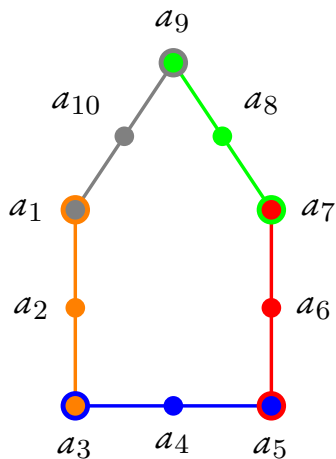
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Choice of the distribution depends on the physical / psychological *etc* realization of the *BOO*.

Anecdotal example: probabilities on a cyclic whose respective hypergraph is a pentagon aka pentagram aka house



1) **classical** probability distributions in terms of convex combinations of the 11 two-valued states thereon;

2) **quantum** probability distributions according to Born, Gleason, and Lovász;

3) **exotic** probability according to Gerelle & Greechie & Miller (1974) and Wright (1978)

4) — ... ?

So far we only spoke about comparing
different probability distributions on fixed collections of
(intertwined)observables ...

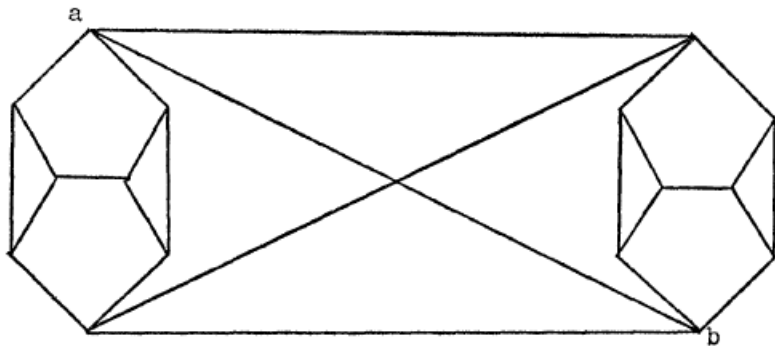
∞ *interlude* ∞

... now we shall be talking about
“weird” nonclassical collections of (intertwined)observables ...

Inseparability 101: Kochen & Specker's demarcation criterion

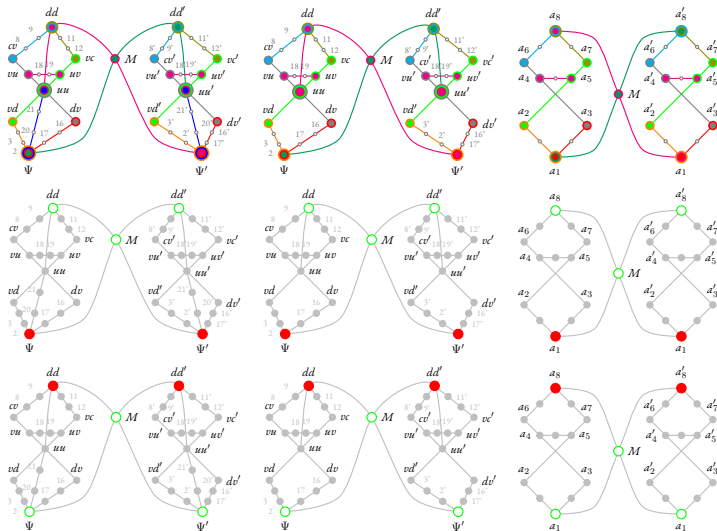
1967, Theorem 0 of DOI: 10.1512/iumj.1968.17.17004

Theorem 0. *Let \mathfrak{K} be a partial Boolean algebra. A necessary and sufficient condition that \mathfrak{K} is imbeddable in a Boolean algebra B is that for every pair of distinct elements a, b in \mathfrak{K} there is a homomorphism $h: \mathfrak{K} \rightarrow \mathbb{Z}_2$ such that $h(a) \neq h(b)$.*



Graph of Γ_3

Hypergraphs with nonseparable set of two-valued states (third column is Kochen & Specker (1967, Γ_3))



Hypergraph with nonunital set of 6 value assignments

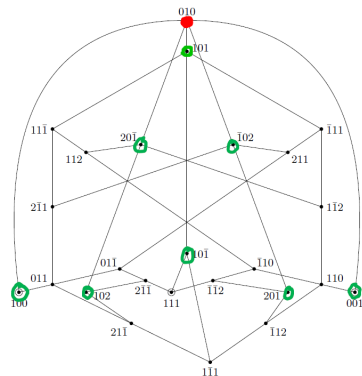


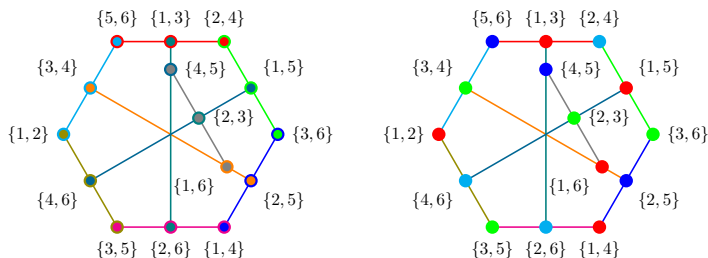
Fig. 2 'Almost' Greechie diagram of a suborthoposet of $L(\mathbb{R}^3)$ without a unital set of two-valued states [e.g., $12\bar{1} = \text{Sp}(1, 2, -1)$].

Josef Tkadlec, [DOI:10.1023/A:1026646229896](https://doi.org/10.1023/A:1026646229896) based on Erna Clavadetscher-Seeberger, Diss. ETH Zürich (Specker) [handle ETH: 20.500.11850/138142](https://handle.ethz.ch/20.500.11850/138142) based on Schütte's letters to Specker, April 22nd, 1965 & November 3rd, 1983 (communicated to KS by Specker).

Hypergraph with exotic contextuality derived from coloring

Hypergraph of biconnected intertwined contexts representing complete graphs with a separating set of 6 two-valued states which is non-partitionable: G_{32} , cf. Figure 6, p. 121 Greechie (1971)

DOI: 10.1016/0097-3165(71)90015-X



Mohammad H. Shekarriz & KS, vertex labeling by partitions of $\{1, 2, 3, 4, 5, 6\}$ with no faithful orthogonal representation
[arXiv:2105.08520](https://arxiv.org/abs/2105.08520).

Thank you for your attention!

