Physical aspects of Kelly James Clark's bowling *vs.* curling metaphor

Karl Svozil
Vienna University of Technology
Institute for Theoretical Physics
Wiedner Hauptstrasse 8-10/136
A-1040 Vienna, Austria, EU

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Abstract

This is a very brief speculative excursion, or rather an update to Frank's *"The Law of Causality and its Limits,"* on if and how transcendence could manifest itself in the physical world.

Inspired by Nicholas of Cusa (*aka* Nicolaus Cusanus, 1401 – 1464),
author of *On Learned Ignorance*
who inevitably[1] failed in constructing the *quadrature of the circle* (Böhlandt, 2009, Chapter 4).

1 The historic context

To spare the Reader a bloated introduction I just name-drop a few previous references pertinent to the subject: according to a familiar and influential narrative, the European Enlightenment[2] evolved as a courageous, thorough and highly successful[3] exorcism of transcendence; in particular, the rejection of law-defying *miracles* (Swinburne, 1970); moreover, the empirical sciences "established natural laws" of regular, reliable tempo-spacial coincidences which appear to be existentially trustworthy – think of air travel[4].

The denial of any direct breach or "rupture" of the laws of nature (Frank, 1932; Frank & R. S. Cohen (Editor), 1997, Sect. III, 10) has pushed the boundaries of conceivable transcendental real-time interventions, and, in particular, divine providence, to the fringe of "gaps" (Frank, 1932; Frank & R. S. Cohen (Editor), 1997, Sect. III, 12) in the laws of nature – indeterminate situations where applicable laws have not (yet?) been identified.

[1] *Squaring the circle* is an ancient geometric challenge to construct a square exactly equal in area to a given circle by finite means limited to the use of ruler and compass alone. Because π, the ratio of the circumference to the diameter of a circle, turned out to be a transcendental number, this task is provable impossible.

[2] For a subjective introduction to similar Greek periods see Schrödinger's *"Nature and the Greeks"* (Schrödinger, 2014).

[3] The criterion of success is taken relative to and in terms of full-spectrum dominance compared to alternative worldviews grounded in esoteric thought.

[4] I am unaware of any religious orthodoxy refusing to board an airplane or any other water or ground vessel based on the belief that the technology of this machinery is flawed because their respective deity disobeys such "natural laws." (This does not exclude the occasional prayer for a safe journey; in particular, in case of emergencies.) Indeed, one may argue that much of today's decline of competence of the religious orthodoxy originates in this and other examples of hypocrisy: they act opportunistically when it suits their purposes but look the other way when absorbing the respective consequences.

As effective as the formal (Wigner, 1962) and natural sciences are in terms of utility, they turn out to be as means and context relative[5] as any construct of thought: those imaginations of the human mind (Berkeley, 1710; Goldschmidt & Pearce, 2018) cannot deliver any "Archimedean point" or "ontological anchor" upon which an "objective reality" (whatever that is) can be based[6]. As Lakatos suggested (Lakatos, 1978) (and this Author concurs in the spirit of Berkeley (Berkeley, 1710)), all that we can ever hope for are successions of "scientific research programs" (*aka* narratives) without a recognizable coherent conceptual convergence. There is no "certainty from rationality," and most likely, there never will be, as all our findings are context and means relative with respect to the assumptions and ego investments made, and are, therefore, "suspended in free human thought"[7].

Consider metamathematics, for example: Cantor took a bold step in introducing informal "entities of thought" (Hilbert's paradise (Hilbert, 1926, 1984)) into the mathematical discourse (Cantor, 1895): *"By a 'set' we mean any summary M of certain well-differentiated objects m of our outlook or thinking (which are called the 'elements' of M) into a whole."* Soon this turned out to be inconsistent; and yet, any remedy – at least insofar it includes sufficiently[8] "strong" formalizations – is provable incomplete in the sense that not all "true" statements; and, in particular, its consistency, are derivable by intrinsic (within that respective formalization) means alone (Gödel, 1931; Turing, 1937; Chaitin, 2003).

Furthermore, any formalization of physical (in)determinism by (in)computability, as well as physical randomness as algorithmic incompressibility, as well as general induction (Gold, 1967; Blum & Blum, 1975; Angluin & Smith, 1983; Adleman & Blum, 1991; Li & Vitányi, 1992) would require transfinite means not available (Gandy, 1982) in this Universe (Svozil, 1993, 1996, 2011). This is because the associated formal proofs are blocked by the aforementioned Gödel-Turing-type incompleteness/incomputability results.

Therefore, one cannot expect that the formal and natural sciences offer absolute corroboration of any type of semantic statements. All they allow is a systematic exploitation of syntax and narratives which are true relative to and for all means and practical purposes.

In what follows we shall first discuss what general options of existence can be imagined; and then proceed with a discussion of their possible concrete physical *modi operandi.*

2 Bowler type scenario of a clockwork universe

The assumption of a "clockwork universe" – that is, "stuff" such as matter, energy and the like, together with its assorted evolution laws which are uniformly valid and unique (leaving no room for alternatives) – entails a "bowler"-type god[9]. Once this universe is created *ex nihilo* and put into motion god does not in any way interfere with it; as all necessary and sufficient conditions exist to determine its evolution uniquely and completely from a "previous" state into a "later" one[10].

How could physics facilitate and support such a view?

[5]Means relativity of an entity such as an idea is the dependence (eg., validity, existence) of this entity on the means or assumptions employed. Context relativity relates to whatever side and additional means and assumptions are employed. Different means may lead to very different situations: for instance, Gödel's (in)completeness theorems state that first order logic is complete (Gödel, 1930) whereas "stronger" second order logic is incomplete (Gödel, 1931).

[6]Anyone believing in the human capacity to grasp some "ontology" acts like the prisoners in Plato's *cave metaphor*; fools of which Goethe in *Elective Affinities* once remarked, *"no one is more a slave than the man who thinks himself free while he is not."*

[7]The desperation, if not nihilism, that results from the deconstruction of long-held believes and narratives has been very vividly decribed by Schopenhauer (Schopenhauer, 1912), as well as through Nietzsche's *Übermensch* (Nietzsche, 009 a,b) and Camus' *Sisyphe* (Camus, 1942).

[8]Sufficiency is understood in terms of the capacity to universally compute.

[9]Deity is understood as an entity creating existence; a sort of "programmer of the Universe."

[10]In such a scenario free will appears to be illusory and subjectively, as per assumption choices are merely fictitious and delusional[11]. For instance, addictions appear as the "inner perception" of the brain's dopamine system preferences; in particular, the accumulation of ΔFosB (Nestler et al., 2001).

- The description of a unique physical state as a *function* of some operational physical quantity such as time – indeed, the very notion of *total function* (as opposed to partiality (Kleene, 1936)), *Laplace's demon*, *causal (Norton, 2003) determinism* and the *principle of sufficient reason* are scientific tropes and schemes signifying clockwork universes. They were widely held in pre-statistical physics and quantum areas until around *fin de siècle*.

  In the context of ordinary differential equations of classical continuum mechanics and classical electrodynamics the semantic notion of "determinism" is formalized by the *uniqueness* of the solutions, which are guaranteed by a Lipschitz continuity condition[12] (Svozil, 2018, Chapter 17).

- The quantum state evolution is postulated to be unique and deterministic[13]. But in general – that is, in the case of coherent superposition or mixed states – the quantum state is not operationally accessible. Therefore this sort of quantum determinacy cannot be given any direct empirical meaning.

- *Deterministic chaos* is characterized by a unique initial value – a "seed" supposed to be taken from the mathematical continuum and thus incomputable and even random[14] with probability one – whose information or digits are "revealed" by some suitable deterministic temporal evolution. To be suitable a temporal evolution needs to be very sensitive to changes of initial seeds such that very small fluctuations may produce very large effects. This is similar to Maxwell's gap scenario discussed later.

  Like the quantum evolution, deterministic chaos might be considered both an argument for as well as against classical determinism: because the assumption of the continuum renders almost all seeds formally random (Martin-Löf, 1966), thereby passing all statistical tests of randomness; in particular "elementary" test such as Borel normality, certifying that all sequences of arbitrary length occur with the expected frequency [15], but also much stronger ones.

  In this respect classical machinery designed to utilize extreme sensitivities of the temporal evolution to the initial seed, such as the Athenian (Dow, 1939) $\kappa\lambda\eta\rho\omega\tau\eta\rho\iota\omega\nu$ (*kleroterion*), for all practical purposes is not inferior to a quantum oracle for randomness, such as *QUANTIS* (ID Quantique SA, 2010), based on the "evangelical" belief of irreducible quantum randomness (Zeilinger, 2005).

- In the context of system science or virtual physics, this modus could be referred to as *virtual reality, computational gaming environment* or *simulation* (*aka* simulacrum); but only if there is no interference from "the outside" (*aka* beyond): the respective universe is hermetic. No participation is possible; only passive (without interference) observation.

How does physics contradict such a view?

- Classical gaps are characterized by *instabilities* at *singular points*, such that very small fluctuations may produce very large effects. To quote Maxwell (Maxwell, 1999, pp. 211,212), *"for example, the*

[12] According to the Picard-Lindelöf theorem an *initial value problem* defined by a first order ordinary differential equation of the form $y'(t) = f(t, y(t))$ and the initial value $y(t_0) = y_0$ has a *unique* solution if f satisfies the Lipschitz condition and is continuous as a function of t. A mapping f satisfies (global/local) *Lipschitz continuity* (or, used synonymously, *Lipschitz condition*) with finite positive constant $0 < k < \infty$ if it increases the distance between any two points y_1 and y_2 (of its entire domain/some neighborhood) by a factor at most k: $|f(t, y_2) - f(t, y_1)| \leq k|y_2 - y_1|$. f may be nonlinear as long as it does not separate different points y_1 and y_2 "too much."

[13] Formally it is represented by a unitary transformation, that is, a generalized rotation mapping one orthonormal basis into another one. Such a state evolution is one-to-one and thus reversible and unique. However, if the preparation context differs from the measurement context, the quantum state does not identify outcomes uniquely, thereby allowing one particular kind of quantum indeterminacy.

[14] Randomness of an infinite string is taken to be algorithmically incompressible (Martin-Löf, 1966).

[15] Unfortunately Borel normality is no guarantee of randomness because very regular sequences, for instance, the Champernowne constant (Sloane, 2019) C_{10} in base 10 is just the sequence obtained by concatenating successive numbers (encoded in base 10), turn out to be normal.

rock loosed by frost and balanced on a singular point of the mountain-side, the little spark which kindles the great forest ... At these points, influences whose physical magnitude is too small to be taken account of by a finite being, may produce results of the greatest importance.

- In some physical situations the Lipschitz continuity is violated, yielding no unique solutions. The Norton dome (Norton, 2008; van Strien, 2014) is a contemporary example of such a situation.

- Spontaneous symmetry breaking, a physical (re)source of nonuniqueness, is a spontaneous process by which a physical system in a symmetric state ends up in an asymmetric state. This is facilitated by some appropriate "Mexican hat" potential, not dissimilar to Norton's dome or Maxwell's (Maxwell, 1999, pp. 211,212) *"rock loosed by frost and balanced on a singular point"* mentioned earlier.

  In particle physics the Higgs mechanism, the spontaneous symmetry breaking of gauge symmetries, plays an important role in the origin of particle masses in the standard model of particle physics. All of these ruptures or breaches of uniqueness depend on the assumptions and models involved.

- Quantum indeterminacy, in particular, complementarity, contextuality (*aka* value indefiniteness), and aspects (such as the exact decay time) of the occurrence of certain single events are postulated to signify indeterminism.

Because of both formal as well as empirical reasons these scenarios might no be interrelated and not separate: for instance, one might suspect that Maxwell's instabilities at singular points could be formalized by "Mexican hat" type potentials discussed in spontaneous symmetry breaking, or by ordinary differential equations yielding Norton dome-type configurations. One might even speculate that all violations of Lipschitz continuity amount to some kind of symmetry breakdown.

Empirically one might argue that, for all practical purposes (Bell, 1990), Maxwell's scenario as well as Norton dome-type configurations (related to violations of Lipschitz continuity) or spontaneous symmetry breaking, never "actually" happen. Because for all practical purposes a rock loosed by frost is never (with probability zero) totally symmetrically balanced at a singular point; rather the position of its center of gravity will fluctuate around the tip, thereby spoiling symmetry. Also one may argue that, due to fluctuations related to the character of matter, singular points make no operational sense whatsoever; they are (over)idealized concepts invented by the human mind for mere convenience. In particular, microscopic quantum zero point fluctuations, as well as thermal fluctuations (Smoluchowski, 1912) ultimately spoil symmetries. Therefore, all such exploitations of such singularities confuse epistemic convenience with an ontology that has no physical, operational grounds.

3 Scenario of a stochastic, disorganized universe

The "converse" of a Laplacean determinism governed by a unique state evolution "tied to" previous states, as mentioned in the previous section, is one in which any given state is independent[16] of the respective previous (and future) states. In such a most extreme scenario among many conceivable degrees of stochasticity the universe is "completely" stochastic and disorganized on the most fundamental level. For the embedded observer's intrinsic perspective, due to irreducible contingency and chance, it appears as if such a world is constantly created anew by throwing some sort of dice[17].

[16]Two events A and B are statistically independent if their joint probability $P(A \cap B)$ can be written as the product of their single probabilities $P(A)$ and $P(B)$; that is, $P(A \cap B) = P(A)P(B)$. It turns out that this results in a journey down a rabbit hole, as the concept of probability is a nontrivial one (Uffink, 2011).

[17]This may be considered an extreme form of *creatio continua.* However, *extrinsically* – that is, from an external, extrinsic, perspective – this may be considered *creatio ex nihilo* as no active, real-time participation is assumed. Indeed, one may speculate that if the temporal ordering of events (as well as causality) turns out to be epistemic – an intrinsically emerging concept/observable of (self-)cognition/observation – then any differentiation based on temporal creation – such as *creatio continua* versus *ex nihilo* – turns out to be a "red herring." Alas, without granting "time" some ontology, also differentiations between a "bowling" or "curling" god collapse.

Whether and how some sort of structural continuity of existence can emerge and be maintained under such circumstances is a fascinating question. As in such a scenario space and time, as much as notions of causality and the laws, are emergent concepts, continuity might emerge with them.

Indeed, one might speculate that "the laws" are some sort of expressions of chaos [18], the formation of matter and genes are expressions of these laws, the individuals carrying those genes are expressions thereof (Hamilton, 1963), and that the ideas about the world are expressions of these individuals. In that transitive way, the Universe contemplates itself through our ideas – ideas such as religion, mathematics, ethics, and so on.

Contemporary physics supports such a view in postulating that many elementary events – such as the spontaneous or stimulated emission of photons – occur acausally, irreducibly pure and simple (Born, 1926; Zeilinger, 2005). Indeed, both classical statistical physics at finite resolution[19], as well as quantum mechanics, support such a view.

Already Exner (Hiebert, 2000; Stöltzner, 1999), motivated by statistical physics and the radiation law (Schweidler, 1906), suggested that (Exner, 2016, p. 7,18) *"... laws do not exist in nature, those are only formulated by man, he makes use of it as a linguistic and computational aid and only wants to say that the processes in nature run as if matter, like a sentient being, would obey these laws. ... So we must understand all so-called exact laws only as average laws, which are not valid with absolute certainty, but with the higher probability the more individual processes they result from. All physical laws go back to molecular processes of random nature and from them follows the result according to the laws of probability calculus ... ."*

Even in totally "random" datasets, some sort of structure must necessarily emerge by the law of large numbers: for instance, if two dice are thrown sufficiently often, the number seven appears to be the most likely sum of their two faces. Modern arguments for the emergence of laws from chaos employ, among other methods (Armstrong, 1983; van Fraassen, 2003; Calude & Meyerstein, 1999; Rosen, 2010; Calude et al., 2013; Yanofsky, 2017; Mueller, 2017; Cabello, 2019), Ramsey theory for structure formation and structural continuity through spurious correlations (Calude & Svozil, 2019). Thereby it is irrelevant whether or not these events occur "absolutely randomly" – indeed, as has been pointed out earlier, on an individual level and with finitistic means, "absolute randomness" appears to be a vacuous concept.

4 The intermediate curler case

Intuitively the curler case discussed here (Clark, 2017) is one in which the natural laws – whatever their form and origin – predominate, but there are situations in which such laws do not exist, or if laws exist they are violated. The first "weak" case of indeterminism can be realized by gaps[20].

Theologically this could be perceived as a mild form of *creatio continua*[21]: god has created laws which are not violated, but god also left "some room" to communicate *via* gaps.

A "god of the gaps" has been rephrased in many ways. This concept is also quite popular since, on the one hand, the obvious regularities of experience and life express correlations or laws which appear evident: the daily cycle of the sun, the yearly cycle of the seasons, life, death; apples and other stuff falling down and not up, and so on. So denial of regularities appears futile; one way of integrating them into religion is to say that a "bowling god" made them so. On the other hand, humans experience fate and uncontrollable

[18] This is not dissimilar to the impossible choice not to communicate (Watzlawick et al., 1967).

[19] A Laplacian demon with unbounded resources might be able to determine future states from present ones with arbitrary precision.

[20] As mentioned earlier (Frank, 1932; Frank & R. S. Cohen (Editor), 1997, Sect. III, 10) "stronger" forms of curling involve a "rupture" of the laws of nature, as they are in direct violations of those laws as mentioned in Voltaire's Philosophical Dictionary (Voltaire, 1764, Chapter 330). Although nobody can *a priori*exclude such latter cases we shall henceforth stick with Hume's attitude towards miracles (Hume, 2007, Section X) and neglect them.

[21] *Cf.* my earlier remarks on *creatio continua* in footnote 17.

circumstances quite often. In a similar reaction, the primitive mind (re)interpreted such "evidence" as god's signal.

As more and more "fateful" behaviors became "understood" and even controllable[22] it is not unreasonable to speculate that, maybe, eventually, there will be no such gaps left – in which case one recovers the bowler, *ex nihilo,* scenario. Alternatively some "pure" gaps in the causal fabric of our universe might "turn out" – that is, relative to the assumptions and means employed – to be irreducible and final: those gaps cannot be eliminated and might remain forever. In secular terms, this could be suspected to signify irreducible indeterminism or randomness (Zeilinger, 2005). But there exist other, possibly transcendental, interpretations involving *intentionality* across gaps.

That these latter scenarios are not purely speculative can be demonstrated by an interactive gaming scenario: If one is considering an interactive virtual reality environment (Galouye, 1964; Egan, 1994) one usually assumes that the virtual reality is "sustained" or "supported" by a computational process "running" on some kind of computer whose physical characteristics are not directly related[23] to the simulacrum. To be interactive the two universes need to be intertwined and connected by some sort of (bidirectional) gap through which information flows in both "directions"[24]. For an intrinsic (Svozil, 1994) observer embedded (Toffoli, 1978) in the virtual environment and bound by its operational capacities *the capacity to send an arbitrary signal through the interface – from the simulating universe (aka "the beyond") to the simulacrum – can only be realized by a gap.* Because without a gap, the signal must remain immanent; that is, it reduces to either lawful or chaotic behavior.

Gaps potentially allow some "transcendental" exchange of signals but do not necessarily imply such a conversation or dialogue. Therefore, gaps are a necessary but not a sufficient condition for transcendence – just because gaps have been located does not imply the existence of "active" transcendental entities.

From a theological perspective, gaps can be employed to realize individual (human) soul/mind-body dualism (Eccles, 1990), and also divine providence (Frank, 1932; Frank & R. S. Cohen (Editor), 1997, Sect. III, 9-16).

How does physics support gaps? Or can physics rule them out? The following is an update and extension of Frank's discussion on physical gaps.

- As has been mentioned earlier, in the classical domain of ordinary differential equations some breach of the Lipschitz continuity condition (Svozil, 2018, Chapter 17) could result in nonunique solutions. Often such types of gaps are identified with instabilities at their singular points (Maxwell, 1999, pp. 211,212), (Frank, 1932; Frank & R. S. Cohen (Editor), 1997, Sect. III, 13).

- Quantum complementarity, and, as an extension thereof, quantum contextuality (*aka* value indefiniteness) can be interpreted as the impossibility to co-represent (Peres, 1993; Kochen & Specker, 1967; Abbott et al., 2015) certain (even finite) sets of – necessarily *counterfactual* because they are complementary – quantum observables, relative to the asssumptions[25]. In my opinion, this is problematic as the corresponding experimental protocols ("prepare a pure state and measure a different one")

[22]Think of medical treatments and also volcanic eruptions, floods or weather phenomena such as lightning and thunder.

[23]To be feasible and nonmonotonic it can be assumed without loss of generality that both the universe in which the simulation is implemented as well as the simulated universe are capable of universal computation in the sense of Chuch-Turing.

[24]This could result in a sort of *dialogue* between those realms. This could lead to a "backflow" from the simulacrum to the universe in which the simulation takes place, such that the former simulacrum performs "empirical studies" on the latter, thereby fully and actively participating in it. In this very speculative scenario, "transcendence becomes immanence." Think of evolving artificial intelligence in a computer simulation becoming aware of its situation and asking online players questions about its situation and the general situation. However, as symmetric as such an exchange through the interface may appear, it is asymmetric in one aspect: whereas the simulacrum cannot exist without the world in which the simulation takes place the latter can exist without the former.

[25]One assumption entering those proofs are the (context) independence of outcomes of measurements for "intertwine" observables occurring in more than one context. For reasons of being able to intertwine contexts formalized by orthonormal bases this can only happen in vector spaces of dimension higher than two.

seem to suggest that they "reveal" some pre-existing property – indicated by the (non)occurrence of a detector click. Alas this could be misleading, as the respective click might either be subject to debate and interpretation[26] or merely signify the capacity of the measurement apparatus to "translate an improper question;" thereby introducing stochastic noise (Svozil, 2004). This appears to be related to notorious inconsistencies in quantum physics proper (von Neumann, 1996, 1955; Everett III, 1957; Wigner, 1962; Everett III, 2012) due to the assumption of irreversible quantum measurements.

- Aspects of certain individual, single events in quantized systems such as the time of emission or absorption of single quanta of light, are postulated to be indeterministic.

5 The (un)known (un)knowns

The relativity of the considerations on the respective assumptions and means invested or taken for granted results in an echo-chamber of sorts: whatever one puts in one gets out. There does not seem to exist any "firm (meta)physical ground," no undisputable "Archimedean ontological anchor" upon which such speculations can be based. And the tendency of the mind to rationalize, project (Jaynes, 1990; Freud, 1999, 1958) and empathically embrace opinions which are favorable to one's ego-investments increases delusions about particular beliefs and corroborations thereof even further.

At this point, the Reader might get frustrated: a negative message (akin to a negative theology) has been delivered[27]. Alas, unfortunately, this is all that can be safely stated.

Therefore we should accept the sober fact that there is certainty only in our uncertainty. This has been expressed by many insightful individuals of many religions and at various times. Aurelius Augustinus, for instance, writes (Augustine of Hippo, 2019, Book XI, chapter 25.32), *"Do I perhaps not know how to express what I do know? Woe is me: I do not even know what it is I do not know!"*

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[26] A debate (Braunstein & Kimble, 1998; Bouwmeester et al., 1998) on the alleged "*a posteriori* teleportation" is an example for such a nonunique semantic perception of syntactically undisputed detector clicks.

[27] One positive side effect might be the abandoning of what the Vienna Circle (in a Humean tradition) called "meaningless pseudo-statements" (Hahn, 1930; Carnap, 1931, 1959) targeting a particular kind of hocus-pocus, abracadabra delusional (thought) rituals delivered by sophistic philosophers and an orthodox clergy. However, one has to be very careful not to "throw the baby out with the bathwater." Shortly after these seemingly bold rejections of metaphysical entities, it turned out that their program based on empirical evidence and formal logic proposed could not be carried ous as completely as desired (Gödel, 1931; Turing, 1937; Smullyan, 1992, 1993, 1994; Chaitin, 2003).

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