Supplemental Material: Classical versus quantum probabilities & correlations

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(Dated: July 29, 2017)

A. The cddlib package

Fukuda's *cddlib package cddlib-094h* can be obtained from the package homepage [1]. Installation on Unix-type operating systems is with *gcc*; the free library for arbitrary precision arithmetic *GMP* (currently 6.1.2) [2], must be installed first.

In its elementary form of the *V-representation*, takes in the k vertices $|\mathbf{v}_1\rangle, \ldots, |\mathbf{v}_k\rangle$ of a convex polytope in an m-dimensional vector space as follows (note that all rows of vector components start with "1"):

```
V-representation
begin
k   m+1   numbertype
1  v_11  ...  v_1m
...............
1  v_k1  ...  v_km
end
```

cddlib responds with the faces, as encoded by n inequalities $\mathbf{A}|\mathbf{x}\rangle \leq |\mathbf{b}\rangle$ in the *H-representation* as follows:

```
H-representation
begin
n        m+1    numbertype
b       -A
end
```

Comments appear after an asterisk.

B. Trivial examples

1. One observable

The case of a single variable has two extreme cases: false$\equiv$ 0 and true$\equiv 1$, resulting in $0 \leq p_1 \leq 1$:

```
* one  variable
*
V-representation
begin
2    2    integer
1    0
1    1
end

~~~~~~  cddlib  response

H-representation
begin
 2  2  real
  1  -1
  0   1
end
```

2. Two observables

The case of two variables p_1 and p_2, and a joint variable p_{12}, result in

$$p_1 + p_2 - p_{12} \leq 1, \tag{1}$$
$$-p_1 + p_{12} \leq 0, \tag{2}$$
$$-p_2 + p_{12} \leq 0, \tag{3}$$
$$-p_{12} \leq 0, \tag{4}$$

and thus $0 \leq p_{12} \leq p_1, p_2$.

```
* two  variables:  p1,  p2,  p12=p1*p2
*
V-representation
begin
4    4    integer
1    0    0    0
1    0    1    0
1    1    0    0
1    1    1    1
end

~~~~~~  cddlib  response

H-representation
begin
 4  4  real
  1  -1  -1   1
  0   1   0  -1
  0   0   1  -1
  0   0   0   1
end
```

For dichotomic expectation values ± 1,

```
* two  expectation  values:  E1,  E2,  E12=E1*E2
*
V-representation
begin
4    4    integer
1    -1    -1    1
```


* svozil@tuwien.ac.at; http://tph.tuwien.ac.at/~svozil

```
1    -1    1    -1
1    1    -1    -1
1    1    1    1
end

~~~~~~ cddlib response

H-representation
begin
 4 4 real
  1 -1 -1  1
  1  1 -1 -1
  1 -1  1 -1
  1  1  1  1
end
```

3. Bounds on the (joint) probabilities and expectations of three observables

```
* four joint expectations:
* p1, p2, p3,
* p12=p1*p2, p13=p1*p3, p23=p2*p3,
* p123=p1*p2*p3
V-representation
begin
8    8    integer
1    0    0    0    0    0    0    0
1    0    0    1    0    0    0    0
1    0    1    0    0    0    0    0
1    0    1    1    0    0    1    0
1    1    0    0    0    0    0    0
1    1    0    1    0    1    0    0
1    1    1    0    1    0    0    0
1    1    1    1    1    1    1    1
end

~~~~~~ cddlib response

H-representation
begin
 8 8 real
  1 -1 -1 -1  1  1  1 -1
  0  1  0  0 -1 -1  0  1
  0  0  1  0 -1  0 -1  1
  0  0  0  1  0 -1 -1  1
  0  0  0  0  1  0  0 -1
  0  0  0  0  0  1  0 -1
  0  0  0  0  0  0  1 -1
  0  0  0  0  0  0  0  1
end
```

If single observable expectations are set to zero by assumption (axiom) and are not-enumerated, the table of expectation values may be redundand.

The case of three expectation value observables E_1, E_2 and E_3 (which are not explicitly enumerated), as well as all joint expectations E_{12}, E_{13}, E_{23}, and E_{123}, result in

$$-E_{12} - E_{13} - E_{23} \leq 1 \tag{5}$$
$$-E_{123} \leq 1, \tag{6}$$
$$E_{123} \leq 1, \tag{7}$$
$$-E_{12} + E_{13} + E_{23} \leq 1, \tag{8}$$
$$E_{12} - E_{13} + E_{23} \leq 1, \tag{9}$$
$$E_{12} + E_{13} - E_{23} \leq 1. \tag{10}$$

```
* four joint expectations:
* [E1, E2, E3, not explicitly enumerated]
* E12=E1*E2, E13=E1*E3, E23=E2*E3,
* E123=E1*E2*E3
V-representation
begin
8    5    integer
1    1    1    1    1
1    1   -1   -1   -1
1   -1    1   -1   -1
1   -1   -1    1    1
1   -1   -1    1   -1
1   -1    1   -1    1
1    1   -1   -1    1
1    1    1    1   -1
end

~~~~~~ cddlib response

H-representation
begin
 6 5 real
  1  1  1  1  0
  1  0  0  0  1
  1  0  0  0 -1
  1  1 -1 -1  0
  1 -1  1 -1  0
  1 -1 -1  1  0
end
```

C. 2 observers, 2 measurement configurations per observer

From a quantum physical standpoint the first relevant case is that of 2 observers and 2 measurement configurations per observer.

1. Bell-Wigner-Fine case: probabilities for 2 observers, 2 measurement configurations per observer

The case of four probabilities p_1, p_2, p_3 and p_4, as well as four joint probabilities p_{13}, p_{14}, p_{23}, and p_{24} result in

$$
\begin{aligned}
-p_{14} &\leq 0 && (11)\\
-p_{24} &\leq 0 && (12)\\
+p_1+p_4-p_{13}-p_{14}+p_{23}-p_{24} &\leq 1 && (13)\\
+p_2+p_4+p_{13}-p_{14}-p_{23}-p_{24} &\leq 1 && (14)\\
+p_2+p_3-p_{13}+p_{14}-p_{23}-p_{24} &\leq 1 && (15)\\
+p_1+p_3-p_{13}-p_{14}-p_{23}+p_{24} &\leq 1 && (16)\\
-p_{13} &\leq 0 && (17)\\
-p_{23} &\leq 0 && (18)\\
-p_1-p_4+p_{13}+p_{14}-p_{23}+p_{24} &\leq 0 && (19)\\
-p_2-p_4-p_{13}+p_{14}+p_{23}+p_{24} &\leq 0 && (20)\\
-p_2-p_3+p_{13}-p_{14}+p_{23}+p_{24} &\leq 0 && (21)\\
-p_1-p_3+p_{13}+p_{14}+p_{23}-p_{24} &\leq 0 && (22)\\
-p_1+p_{14} &\leq 0 && (23)\\
-p_2+p_{24} &\leq 0 && (24)\\
-p_3+p_{23} &\leq 0 && (25)\\
-p_3+p_{13} &\leq 0 && (26)\\
-p_1+p_{13} &\leq 0 && (27)\\
-p_2+p_{23} &\leq 0 && (28)\\
-p_4+p_{24} &\leq 0 && (29)\\
-p_4+p_{14} &\leq 0 && (30)\\
+p_2+p_4-p_{24} &\leq 1 && (31)\\
+p_1+p_4-p_{14} &\leq 1 && (32)\\
+p_2+p_3-p_{23} &\leq 1 && (33)\\
+p_1+p_3-p_{13} &\leq 1. && (34)
\end{aligned}
$$

```
* eight variables: p1, p2, p3, p4,
* p13, p14, p23, p24
*
V-representation
begin
16   9   integer
1   0   0   0   0   0   0   0   0
1   0   0   0   1   0   0   0   0
1   0   0   1   0   0   0   0   0
1   0   0   1   1   0   0   0   0
1   0   1   0   0   0   0   0   0
1   0   1   0   1   0   0   0   1
1   0   1   1   0   0   0   1   0
1   0   1   1   1   0   0   1   1
1   1   0   0   0   0   0   0   0
1   1   0   0   1   0   1   0   0
1   1   0   1   0   1   0   0   0
1   1   0   1   1   1   1   0   0
1   1   1   0   0   0   0   0   0
1   1   1   0   1   0   1   0   1
1   1   1   1   0   1   0   1   0
1   1   1   1   1   1   1   1   1
end

~~~~~~ cddlib response

H-representation
begin
 24 9 real
  0  0  0  0  0  0  1  0  0
  0  0  0  0  0  0  0  0  1
  1 -1  0  0 -1  1  1 -1  1
  1  0 -1  0 -1 -1  1  1  1
  1  0 -1 -1  0  1 -1  1  1
  1 -1  0 -1  0  1  1  1 -1
  0  0  0  0  0  1  0  0  0
  0  0  0  0  0  0  0  1  0
  0  1  0  0  1 -1 -1  1 -1
  0  0  1  0  1  1 -1 -1 -1
  0  0  1  1  0 -1  1 -1 -1
  0  1  0  1  0 -1 -1 -1  1
  0  1  0  0  0  0 -1  0  0
  0  0  1  0  0  0  0  0 -1
  0  0  0  1  0  0  0 -1  0
  0  0  0  1  0 -1  0  0  0
  0  1  0  0  0 -1  0  0  0
  0  0  1  0  0  0  0 -1  0
  0  0  0  0  1  0  0  0 -1
  0  0  0  0  1  0 -1  0  0
  1  0 -1  0 -1  0  0  0  1
  1 -1  0  0 -1  0  1  0  0
  1  0 -1 -1  0  0  0  1  0
  1 -1  0 -1  0  1  0  0  0
end
```

2. Clauser-Horne-Shimony-Holt case: expectation values for 2 observers, 2 measurement configurations per observer

The case of four expectation values E_1, E_2, E_3 and E_4 (which are not explicitly enumerated), as well as all joint expectations E_{13}, E_{14}, E_{23}, and E_{24} result in

$$
\begin{aligned}
+E_{13}-E_{14}-E_{23}-E_{24} &\leq 2 && (35)\\
-E_{24} &\leq 1 && (36)\\
-E_{23} &\leq 1 && (37)\\
-E_{13}+E_{14}-E_{23}-E_{24} &\leq 2 && (38)\\
-E_{14} &\leq 1 && (39)\\
-E_{13}-E_{14}+E_{23}-E_{24} &\leq 2 && (40)\\
-E_{13}-E_{14}-E_{23}+E_{24} &\leq 2 && (41)\\
-E_{13} &\leq 1 && (42)\\
-E_{13}+E_{14}+E_{23}+E_{24} &\leq 2 && (43)\\
+E_{24} &\leq 1 && (44)\\
+E_{23} &\leq 1 && (45)\\
+E_{13}-E_{14}+E_{23}+E_{24} &\leq 2 && (46)\\
+E_{14} &\leq 1 && (47)\\
+E_{13}+E_{14}-E_{23}+E_{24} &\leq 2 && (48)\\
+E_{13}+E_{14}+E_{23}-E_{24} &\leq 2 && (49)\\
+E_{13} &\leq 1. && (50)
\end{aligned}
$$

```
* four joint expectations:
* E13, E14, E23, E24
*
V-representation
begin
16  5  integer
1   1   1   1   1
1   1  -1   1  -1
1  -1   1  -1   1
1  -1  -1  -1  -1
1   1   1  -1  -1
1   1  -1  -1   1
1  -1   1   1  -1
1  -1  -1   1   1
1  -1  -1   1   1
1  -1   1   1  -1
1   1  -1  -1   1
1   1   1  -1  -1
1  -1  -1  -1  -1
1  -1   1  -1   1
1   1  -1   1  -1
1   1   1   1   1
end

~~~~~~ cddlib response

H-representation
begin
 16 5 real
  2 -1  1  1  1
  1  0  0  0  1
  1  0  0  1  0
  2  1 -1  1  1
  1  0  1  0  0
  2  1  1 -1  1
  2  1  1  1 -1
  1  1  0  0  0
  2  1 -1 -1 -1
  1  0  0  0 -1
  1  0  0 -1  0
  2 -1  1 -1 -1
  1  0 -1  0  0
  2 -1 -1  1 -1
  2 -1 -1 -1  1
  1 -1  0  0  0
end
```

3. Beyond the Clauser-Horne-Shimony-Holt case: 2 observers, 3 measurement configurations per observer

```
* 6 expectations:
* E1, ... , E6
* 9 joint expectations:
* E14, E15, E16, E24, E25, E26, E34, E35, E36
* 1,2,3 on one side
* 4,5,6 on other side
*
V-representation
begin
64    16    integer
1    1   1   1   1   1   1   1   1
     1   1   1   1   1   1   1
1    1   1   1   1   1  -1   1   1
    -1   1   1  -1   1   1  -1
1    1   1   1   1  -1   1   1  -1
     1   1  -1   1   1  -1   1
1    1   1   1   1  -1  -1   1  -1
    -1   1  -1  -1   1  -1  -1
1    1   1   1  -1   1   1  -1   1
     1  -1   1   1  -1   1   1
1    1   1   1  -1   1  -1  -1   1
    -1  -1   1  -1  -1   1  -1
1    1   1   1  -1  -1   1  -1  -1
     1  -1  -1   1  -1  -1   1
1    1   1   1  -1  -1  -1  -1  -1
    -1  -1  -1  -1  -1  -1  -1
1    1   1  -1   1   1   1   1   1
     1   1   1   1  -1  -1  -1
1    1   1  -1   1   1  -1   1   1
    -1   1   1  -1  -1  -1   1
1    1   1  -1   1  -1   1   1  -1
     1   1  -1   1  -1   1  -1
1    1   1  -1   1  -1  -1   1  -1
    -1   1  -1  -1  -1   1   1
1    1   1  -1  -1   1   1  -1   1
     1  -1   1   1   1  -1  -1
1    1   1  -1  -1   1  -1  -1   1
    -1  -1   1  -1   1  -1   1
1    1   1  -1  -1  -1   1  -1  -1
     1  -1  -1   1   1   1  -1
1    1   1  -1  -1  -1  -1  -1  -1
    -1  -1  -1  -1   1   1   1
1    1  -1   1   1   1   1   1   1
     1  -1  -1  -1   1   1   1
1    1  -1   1   1   1  -1   1   1
    -1  -1  -1   1   1   1  -1
1    1  -1   1   1  -1   1   1  -1
     1  -1   1  -1   1  -1   1
1    1  -1   1   1  -1  -1   1  -1
    -1  -1   1   1   1  -1  -1
1    1  -1   1  -1   1   1  -1   1
     1   1  -1  -1  -1   1   1
1    1  -1   1  -1   1  -1  -1   1
    -1   1  -1   1  -1   1  -1
1    1  -1   1  -1  -1   1  -1  -1
     1   1   1  -1  -1  -1   1
1    1  -1   1  -1  -1  -1  -1  -1
    -1   1   1   1  -1  -1  -1
1    1  -1  -1   1   1   1   1   1
     1  -1  -1  -1  -1  -1  -1
1    1  -1  -1   1   1  -1   1   1
    -1  -1  -1   1  -1  -1   1
1    1  -1  -1   1  -1   1   1  -1
     1  -1   1  -1  -1   1  -1
1    1  -1  -1   1  -1  -1   1  -1
    -1  -1   1   1  -1   1   1
1    1  -1  -1  -1   1   1  -1   1
     1   1  -1  -1   1  -1  -1
1    1  -1  -1  -1   1  -1  -1   1
    -1   1  -1   1   1  -1   1
1    1  -1  -1  -1  -1   1  -1  -1
     1   1   1  -1   1   1  -1
```

```
1     1    -1    -1    -1    -1    -1    -1    -1
     -1     1     1     1     1     1     1
1    -1     1     1     1     1     1    -1    -1
     -1     1     1     1     1     1     1
1    -1     1     1     1     1    -1    -1    -1
      1     1     1    -1     1     1    -1
1    -1     1     1     1    -1     1    -1     1
     -1     1    -1     1     1    -1     1
1    -1     1     1     1    -1    -1    -1     1
      1     1    -1    -1     1    -1    -1
1    -1     1     1    -1     1     1     1    -1
     -1    -1     1     1    -1     1     1
1    -1     1     1    -1     1    -1     1    -1
      1    -1     1    -1    -1     1    -1
1    -1     1     1    -1    -1     1     1     1
     -1    -1    -1     1    -1    -1     1
1    -1     1     1    -1    -1    -1     1     1
      1    -1    -1    -1    -1    -1    -1
1    -1     1    -1     1     1     1    -1    -1
     -1     1     1     1    -1    -1    -1
1    -1     1    -1     1     1    -1    -1    -1
      1     1     1    -1    -1    -1     1
1    -1     1    -1     1    -1     1    -1     1
     -1     1    -1     1    -1     1    -1
1    -1     1    -1     1    -1    -1    -1     1
      1     1    -1    -1    -1     1     1
1    -1     1    -1    -1     1     1     1    -1
     -1    -1     1     1     1    -1    -1
1    -1     1    -1    -1     1    -1     1    -1
      1    -1     1    -1     1    -1     1
1    -1     1    -1    -1    -1     1     1     1
     -1    -1    -1     1     1     1    -1
1    -1     1    -1    -1    -1    -1     1     1
      1    -1    -1    -1     1     1     1
1    -1    -1     1     1     1     1    -1    -1
     -1    -1    -1    -1     1     1     1
1    -1    -1     1     1     1    -1    -1    -1
      1    -1    -1     1     1     1    -1
1    -1    -1     1     1    -1     1    -1     1
     -1    -1     1    -1     1    -1     1
1    -1    -1     1     1    -1    -1    -1     1
      1    -1     1     1     1    -1    -1
1    -1    -1     1    -1     1     1     1    -1
     -1     1    -1    -1    -1     1     1
1    -1    -1     1    -1     1    -1     1    -1
      1     1    -1     1    -1     1    -1
1    -1    -1     1    -1    -1     1     1     1
     -1     1     1    -1    -1    -1     1
1    -1    -1     1    -1    -1    -1     1     1
      1     1     1     1    -1    -1    -1
1    -1    -1    -1     1     1     1    -1    -1
     -1    -1    -1    -1    -1    -1    -1
1    -1    -1    -1     1     1    -1    -1    -1
      1    -1    -1     1    -1    -1     1
1    -1    -1    -1     1    -1     1    -1     1
     -1    -1     1    -1    -1     1    -1
1    -1    -1    -1     1    -1    -1    -1     1
      1    -1     1     1    -1     1     1
1    -1    -1    -1    -1     1     1     1    -1
     -1     1    -1    -1     1    -1    -1
1    -1    -1    -1    -1     1    -1     1    -1
      1     1    -1     1     1    -1     1
1    -1    -1    -1    -1    -1     1     1     1
     -1     1     1    -1     1     1    -1
1    -1    -1    -1    -1    -1    -1     1     1
      1     1     1     1     1     1     1
end

~~~~~~ cddlib response

H-representation
begin
 684 16 real
  4  0 -1  1 -1 -1  0  1 -1  0  1  1  1 -1 -1
       1
  [...]
  4  1  1  0  1  1  0  1  1  1  1  1 -1  1 -1
       0
  [...]
end
```

D. Pentagon logic

E. Probabilities but no joint probabilities

Here is a computation which includes all probabilities but no joint probabilities:

```
* ten probabilities:
* p1 ... p10
*
begin
11  11  integer
1    1    0    0    1    0    1    0    1
     0    0
1    1    0    0    0    1    0    0    1
     0    0
1    1    0    0    1    0    0    1    0
     0    0
1    0    0    1    0    0    1    0    1
     0    1
1    0    0    1    0    0    0    1    0
     0    1
1    0    0    1    0    0    1    0    0
     1    0
1    0    1    0    0    1    0    0    1
     0    1
1    0    1    0    0    1    0    0    0
     1    0
1    0    1    0    1    0    0    1    0
     0    1
1    0    1    0    1    0    1    0    0
     1    0
1    0    1    0    1    0    1    0    1
     0    1
end

~~~~~~ cddlib response

H-representation
linearity 5  12 13 14 15 16
begin
 16 11 real
  0  0  0  0  0  0  1  0  0  0  0
```

```
  0  0  0  0  0  0  0  0  1  0  0
  0 -1  0  0  1  0  0  0  1  0  0
  0  0  0  0  1  0  0  0  0  0  0
  0  1  0  0  0  0  0  0  0  0  0
  1 -1 -1  0  1  0 -1  0  0  0  0
  0  0  1  0  0  0  0  0  0  0  0
  1 -2 -1  0  1  0 -1  0  1  0  0
  0  1  1  0 -1  0  0  0  0  0  0
  0  1  1  0 -1  0  1  0 -1  0  0
  1 -1 -1  0  0  0  0  0  0  0  0
 -1  1  1  1  0  0  0  0  0  0  0
  0 -1 -1  0  1  1  0  0  0  0  0
 -1  1  1  0 -1  0  1  1  0  0  0
  0 -1 -1  0  1  0 -1  0  1  1  0
 -1  2  1  0 -1  0  1  0 -1  0  1
end
```

$$+p_6 \geq 0 \tag{51}$$
$$+p_8 \geq 0 \tag{52}$$
$$-p_1+p_4+p_8 \geq 0 \tag{53}$$
$$+p_4 \geq 0 \tag{54}$$
$$+p_1 \geq 0 \tag{55}$$
$$-p_1-p_2+p_4-p_6 \geq -1 \tag{56}$$
$$+p_2 \geq 0 \tag{57}$$
$$-2p_1-p_2+p_4-p_6+p_8 \geq -1 \tag{58}$$
$$+p_1+p_2-p_4 \geq 0 \tag{59}$$
$$+p_1+p_2-p_4+p_6-p_8 \geq 0 \tag{60}$$
$$-p_1-p_2 \geq -1 \tag{61}$$
$$+p_1+p_2+p_3 \geq 1 \tag{62}$$
$$-p_1-p_2+p_4+p_5 \geq 0 \tag{63}$$
$$+p_1+p_2-p_4+p_6+p_7 \geq 1 \tag{64}$$
$$-p_1-p_2+p_4-p_6+p_8+p_9 \geq 0 \tag{65}$$
$$2p_1+p_2-p_4+p_6-p_8+p_{10} \geq 1. \tag{66}$$

F. Joint Expectations on all atoms

This is a full hull computation taking all joint expectations into account:

```
* 45 pair expectations:
* E12 ... E910
*
V-representation
begin
11   46   real
1 -1 -1 1 -1 1 -1 1 -1 -1 1 -1 1 -1 1 -1 1 1
    -1 1 -1 1 -1 1 1 -1 1 -1 1 -1 -1 -1 1 -1
    1 1 -1 1 -1 -1 -1 1 1 -1 -1 1
1 -1 -1 -1 1 -1 -1 1 -1 -1 1 1 -1 1 1 -1 1 1
    1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1 -1 1 -1
    -1 1 -1 1 1 -1 1 1 -1 -1 1
1 -1 -1 1 -1 -1 1 -1 -1 -1 1 -1 1 1 -1 1 1 1
    -1 1 1 -1 1 1 1 -1 -1 1 -1 -1 -1 1 -1 1 1
     1 -1 1 1 1 -1 -1 -1 1 1 1
1 1 -1 1 1 -1 1 -1 1 -1 -1 1 1 -1 1 -1 1 -1
    -1 -1 1 -1 1 -1 1 1 -1 1 -1 1 -1 -1 1 -1
    1 -1 -1 1 -1 1 -1 1 -1 -1 1 -1
1 1 -1 1 1 1 -1 1 1 -1 -1 1 1 1 -1 1 1 -1 -1
    -1 -1 1 -1 -1 1 1 1 -1 1 1 -1 1 -1 1 1 -1
     -1 1 1 -1 -1 -1 1 1 -1 -1
1 1 -1 1 1 -1 1 1 -1 1 -1 1 1 -1 1 1 -1 1 -1
    -1 1 -1 -1 1 -1 1 -1 1 1 -1 1 -1 1 1 -1 1
     -1 -1 1 -1 1 -1 1 -1 1 -1
1 -1 1 1 -1 1 1 -1 1 -1 -1 -1 1 -1 -1 1 -1 1
    1 -1 1 1 -1 1 -1 -1 1 1 -1 1 -1 -1 -1 1
    -1 1 1 -1 1 -1 -1 1 -1 -1 1 -1
1 -1 1 1 -1 1 1 1 -1 1 -1 -1 1 -1 -1 -1 1 -1
    1 -1 1 1 1 -1 1 -1 1 1 1 -1 1 -1 -1 -1 1
    -1 1 1 -1 1 1 -1 1 -1 1 -1
1 -1 1 -1 1 1 -1 1 1 -1 -1 1 -1 -1 1 -1 -1 1
    -1 1 1 -1 1 1 -1 -1 -1 1 -1 -1 1 1 -1 1 1
     -1 -1 1 1 -1 -1 -1 1 1 -1 -1
1 -1 1 -1 1 -1 1 1 -1 1 -1 1 -1 1 -1 -1 1 -1
    -1 1 -1 1 1 -1 1 -1 1 -1 -1 1 -1 -1 1 1
    -1 1 -1 -1 1 -1 1 -1 1 -1 1 -1
1 -1 1 -1 1 -1 1 -1 1 -1 -1 1 -1 1 -1 1 -1 1
    -1 1 -1 1 -1 1 -1 -1 1 -1 1 -1 1 -1 1 -1
    1 -1 -1 1 -1 1 -1 1 -1 -1 1 -1
end

~~~~~~ cddlib response

H-representation
linearity 35  12 13 14 15 16 17 18 19 20 21
   22 23 24 25 26 27 28 29 30 31 32 33 34 35
    36 37 38 39 40 41 42 43 44 45 46
begin
 46 46 real
  1 0 -1 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0
  1 1 0 0 0 0 0 -1 0 0 0 0 0 0 -1 0 0 0 0 0
     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0
  1 0 0 -1 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0
     0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0
 -1 -1 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 -1 0 1
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0
  0 -1 -1 0 0 0 0 0 0 0 0 0 0 -1 0 0 0 0 0 0
    -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0
  0 0 1 1 0 -1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 -1
     0 0 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0
     0 0 0 0 0 0
  1 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0
  0 0 0 0 0 -1 0 1 0 0 0 0 0 -1 0 1 0 0 0 0 0
     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0
  1 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 1 0 0 1 0 -1
     0 0 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0
     0 0 0 0 0 0
  0 0 0 -1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 1
     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0
```

```
  0  0  1  1  0  0  0  0  0  0  0  0  0  1  0  −1  0  0  0  0  1
       0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0
         0  0  0  0  0
−1  −1  0  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
     0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
     0  0  0  0  0
  0  0  −1  −1  0  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
         0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
       0  0  0  0  0  0
−1  −1  0  1  0  −1  0  1  1  0  0  0  0  0  0  0  0  0  0  0  0
       0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
       0  0  0  0  0
  1  0  −1  −1  0  1  0  −1  0  1  0  0  0  0  0  0  0  0  0  0
       0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
         0  0  0  0  0  0
  1  1  1  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0
         0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
       0  0  0  0  0
−1  −1  0  1  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0
     0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
     0  0  0  0  0
  0  0  0  −1  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0
       0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
         0  0  0  0  0
  0  0  1  1  0  0  0  0  0  0  0  0  0  1  1  0  0  0  0  0  0  0
         0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
       0  0  0  0  0
  0  0  0  −1  0  0  0  0  0  0  0  0  0  −1  0  1  1  0  0  0  0
         0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
       0  0  0  0  0  0
  0  1  1  1  0  0  0  0  0  0  0  0  0  1  0  −1  0  1  0  0  0
       0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
         0  0  0  0  0
  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  1  0  0
         0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
       0  0  0  0  0
−1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  −1  0  1
     1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
     0  0  0  0  0
  1  0  −1  −1  0  0  0  0  0  0  0  0  0  −1  0  1  0  0  1  0
       −1  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
       0  0  0  0  0  0  0
  0  1  1  1  0  0  0  0  0  0  0  0  0  1  0  −1  0  0  0  0  0
       0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
         0  0  0  0  0
  0  0  0  −1  0  0  0  0  0  0  0  0  0  −1  0  1  0  0  0  0  0
         0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
       0  0  0  0  0  0
  0  −1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
       0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
         0  0  0  0  0
−1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  −1  0  1
     0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
     0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  −1
       0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0
         0  0  0  0  0
−1  −1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  −1  0  1
       0  0  0  0  0  0  0  1  1  0  0  0  0  0  0  0  0  0  0  0
       0  0  0  0  0
  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  −1
       0  0  0  0  0  0  0  −1  0  1  0  0  0  0  0  0  0  0  0
       0  0  0  0  0  0
  1  0  0  0  0  −1  0  0  0  0  0  0  0  −1  0  0  0  0  1  0
       −1  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0
       0  0  0  0  0  0  0
  0  0  0  0  0  1  0  0  0  0  0  0  0  1  0  0  0  0  0  0  1  0
         0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0
       0  0  0  0  0
  0  0  0  0  0  0  0  −1  0  0  0  0  0  0  0  −1  0  0  0  0  0
         0  0  0  0  0  0  0  1  0  0  0  0  1  0  0  0  0  0  0
       0  0  0  0  0  0
  0  0  0  0  0  −1  0  1  0  0  0  0  0  −1  0  1  0  0  1  0
       −1  0  0  0  0  0  0  0  −1  0  0  0  0  0  1  0  0  0  0
         0  0  0  0  0  0  0
  1  1  0  −1  0  1  0  −1  0  0  0  0  0  1  0  −1  0  0  0  0
       1  0  0  0  0  0  0  0  1  0  0  0  0  0  0  1  0  0  0  0
         0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  −1  0  0
       0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0
         0  0  0  0  0
  0  0  0  0  0  −1  0  1  0  0  0  0  0  −1  0  1  0  0  1  0
       −1  0  0  0  0  0  0  0  −1  0  0  0  0  0  0  0  0  1  0
         0  0  0  0  0  0  0
  0  0  0  0  0  0  0  −1  0  0  0  0  0  0  0  −1  0  0  0  0  0
         0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  1  0
       0  0  0  0  0  0
  0  0  0  0  0  0  0  1  0  0  0  0  0  −1  0  1  0  0  0  0  −1
         0  0  0  0  0  0  0  −1  0  0  0  0  0  0  0  0  0  0  1
         0  0  0  0  0  0
  1  0  −1  −1  0  1  0  −1  0  0  0  0  0  0  0  0  0  0  0  0
       0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
         1  0  0  0  0  0
−1  0  1  1  0  0  0  1  0  0  0  0  0  1  0  0  0  0  −1  0  1
     0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
     1  0  0  0  0
  0  0  0  0  0  0  0  −1  0  0  0  0  0  0  0  0  0  0  0  0  0
       0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
         0  1  0  0  0
  1  0  0  0  0  −1  0  0  0  0  0  0  0  −1  0  0  0  0  1  0
       −1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
       0  0  0  0  1  0  0
  0  0  −1  −1  0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
         0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
       0  0  0  0  1  0
  1  1  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
         0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
       0  0  0  0  1
end
```

$$E_{13}+E_{14}-E_{34} \leq 1, \tag{67}$$
$$-E_{12}+E_{18}+E_{28} \leq 1, \tag{68}$$
$$E_{14}+E_{18}-E_{48} \leq 1, \tag{69}$$
$$E_{12}-E_{14}-E_{26}+E_{34}-E_{36} \leq -1, \tag{70}$$
$$E_{12}+E_{13}+E_{26}+E_{36} \leq 0, \tag{71}$$
$$-E_{13}-E_{14}+E_{16}-E_{18}+E_{36}+E_{48} \leq 0, \tag{72}$$
$$-E_{12}-E_{16}-E_{26} \leq 1, \tag{73}$$
$$E_{16}-E_{18}+E_{26}-E_{28} \leq 0, \tag{74}$$
$$E_{26}-E_{28}-E_{34}+E_{36}+E_{48} \leq 1, \tag{75}$$
$$E_{14}-E_{16}+E_{34}-E_{36} \leq 0, \tag{76}$$
$$-E_{13}-E_{14}-E_{26}+E_{28}-E_{36}-E_{48} \leq 0, \tag{77}$$
$$E_{12}-E_{14}-E_{15} \leq -1, \tag{78}$$
$$E_{13}+E_{14}-E_{16}-E_{17} \leq 0, \tag{79}$$
$$E_{12}-E_{14}+E_{16}-E_{18}-E_{19} \leq -1, \tag{80}$$
$$-E_{1,10}+E_{13}+E_{14}-E_{16}+E_{18} \leq 1, \tag{81}$$
$$-E_{12}-E_{13}-E_{23} \leq 1, \tag{82}$$
$$E_{12}-E_{14}-E_{24} \leq -1, \tag{83}$$
$$E_{14}-E_{25} \leq 0, \tag{84}$$
$$-E_{13}-E_{14}-E_{26}-E_{27} \leq 0, \tag{85}$$
$$E_{14}+E_{26}-E_{28}-E_{29} \leq 0, \tag{86}$$
$$-E_{12}-E_{13}-E_{14}-E_{2,10}-E_{26}+E_{28} \leq 0, \tag{87}$$
$$-E_{12}-E_{34}-E_{35} \leq 1, \tag{88}$$
$$E_{34}-E_{36}-E_{37} \leq -1, \tag{89}$$
$$E_{13}+E_{14}+E_{26}-E_{28}-E_{34}+E_{36}-E_{38} \leq 1, \tag{90}$$
$$-E_{12}-E_{13}-E_{14}-E_{26}+E_{28}-E_{39} \leq 0, \tag{91}$$
$$E_{14}+E_{26}-E_{28}-E_{3,10} \leq 0, \tag{92}$$
$$E_{12}-E_{45} \leq 0, \tag{93}$$
$$E_{34}-E_{36}-E_{46} \leq -1, \tag{94}$$
$$E_{36}-E_{47} \leq 0, \tag{95}$$
$$E_{12}+E_{34}-E_{36}-E_{48}-E_{49} \leq -1, \tag{96}$$
$$-E_{14}+E_{36}-E_{4,10}+E_{48} \leq 0, \tag{97}$$
$$E_{16}+E_{26}-E_{34}+E_{36}-E_{56} \leq 1, \tag{98}$$
$$-E_{16}-E_{26}-E_{36}-E_{57} \leq 0, \tag{99}$$
$$E_{18}+E_{28}-E_{48}-E_{58} \leq 0, \tag{100}$$
$$E_{16}-E_{18}+E_{26}-E_{28}-E_{34}+E_{36}+E_{48}-E_{59} \leq 0, \tag{101}$$
$$-E_{12}+E_{14}-E_{16}+E_{18}-E_{26}+E_{28}-E_{36}-E_{48}-E_{5,10} \leq 1, \tag{102}$$
$$E_{34}-E_{67} \leq 0, \tag{103}$$
$$E_{16}-E_{18}+E_{26}-E_{28}-E_{34}+E_{36}+E_{48}-E_{68} \leq 0, \tag{104}$$
$$E_{18}+E_{28}-E_{48}-E_{69} \leq 0, \tag{105}$$
$$-E_{18}+E_{26}-E_{28}+E_{36}+E_{48}-E_{6,10} \leq 0, \tag{106}$$
$$E_{13}+E_{14}-E_{16}+E_{18}-E_{78} \leq 1, \tag{107}$$
$$-E_{13}-E_{14}-E_{18}-E_{26}+E_{34}-E_{36}-E_{79} \leq -1, \tag{108}$$
$$E_{18}-E_{7,10} \leq 0, \tag{109}$$
$$E_{16}+E_{26}-E_{34}+E_{36}-E_{89} \leq 1, \tag{110}$$
$$E_{13}+E_{14}-E_{16}-E_{8,10} \leq 0, \tag{111}$$
$$-E_{12}-E_{13}-E_{9,10} \leq 1, \tag{112}$$

G. Bub-Stairs inequality

If one considers only the five probabilities on the intertwining atoms, then the following Bub-Stairs inequality $p_1+p_3+p_5+p_7+p_9 \leq 2$, among others, results:

```
* five probabilities on intertwining contexts
* p1, p3, p5, p7, p9
*
V-representation
begin
11  6   integer
1       1     0     0     0     0
1       1     0     1     0     0
1       1     0     0     1     0
1       0     1     0     0     0
1       0     1     0     1     0
1       0     1     0     0     1
1       0     0     1     0     0
1       0     0     1     0     1
1       0     0     0     1     0
1       0     0     0     0     1
1       0     0     0     0     0
end

~~~~~~ cddlib response

H-representation
begin
 11 6 real
  0   0   0   1   0   0
  1   0   0   0  -1  -1
  0   1   0   0   0   0
  1   0  -1  -1   0   0
  2  -1  -1  -1  -1  -1
  1  -1  -1   0   0   0
  0   0   0   0   1   0
  1  -1   0   0   0  -1
  1   0   0  -1  -1   0
  0   0   1   0   0   0
  0   0   0   0   0   1
end
```

One could also consider probabilities on the non-intertwining atoms yielding; in particular, $p_2+p_4+p_6+p_8+p_{10} \geq 1$.

```
* five probabilities
* on non-intertwining atoms
* p2, p4, p6, p8, p10
*
V-representation
begin
11  6   integer
1       0     1     1     1     0
1       0     0     0     1     0
1       0     1     0     0     0
1       0     0     1     1     1
1       0     0     0     0     1
1       0     0     1     0     0
1       1     0     0     1     1
1       1     0     0     0     0
```

```
1           1      1      0      0      1
1           1      1      1      0      0
1           1      1      1      1      1
end

~~~~~~ cddlib response

H-representation
begin
 11 6 real
  0  0  0  0  1  0
  0  0  0  0  0  1
  0  0  1  0  0  0
 -1  1  1  1  1  1
  0  1  0  0  0  0
  0  0  0  1  0  0
  1  1 -1  1 -1 -1
  1 -1  1 -1 -1  1
  1  1 -1 -1  1 -1
  1 -1  1 -1  1 -1
  1 -1 -1  1 -1  1
end
```

1. Klyachko-Can-Biniciogolu-Shumovsky inequalities

The following hull computation is limited to adjacent pair expectations; it yields the Klyachko-Can-Biniciogolu-Shumovsky inequality $E_{13}+E_{35}+E_{57}+E_{79}+E_{91} \geq 3$:

```
* five joint Expectations:
* E13 E35 E57 E79 E91
*
V-representation
begin
11   6    real
1        -1      1      1      1     -1
1        -1     -1     -1      1     -1
1        -1      1     -1     -1     -1
1        -1     -1      1      1      1
1        -1     -1     -1     -1      1
1        -1     -1      1     -1     -1
1         1     -1     -1      1      1
1         1     -1     -1     -1     -1
1         1      1     -1     -1      1
1         1      1      1     -1     -1
1         1      1      1      1      1
end

~~~~~~ cddlib response

H-representation
begin
 11 6 real
  1  0  0  0  1  0
  1  0  0  0  0  1
  1  0  1  0  0  0
  3  1  1  1  1  1
  1  1  0  0  0  0
  1  0  0  1  0  0
  1  1 -1  1 -1 -1
  1 -1  1 -1 -1  1
  1  1 -1 -1  1 -1
  1 -1  1 -1  1 -1
  1 -1 -1  1 -1  1
end
```

$$-E_{79} \leq 1 \tag{113}$$
$$-E_{91} \leq 1 \tag{114}$$
$$-E_{35} \leq 1 \tag{115}$$
$$-E_{13}-E_{35}-E_{57}-E_{79}-E_{91} \leq 3 \tag{116}$$
$$-E_{13} \leq 1 \tag{117}$$
$$-E_{57} \leq 1 \tag{118}$$
$$-E_{13}+E_{35}-E_{57}+E_{79}+E_{91} \leq 1 \tag{119}$$
$$+E_{13}-E_{35}+E_{57}+E_{79}-E_{91} \leq 1 \tag{120}$$
$$-E_{13}+E_{35}+E_{57}-E_{79}+E_{91} \leq 1 \tag{121}$$
$$+E_{13}-E_{35}+E_{57}-E_{79}+E_{91} \leq 1 \tag{122}$$
$$+E_{13}+E_{35}-E_{57}+E_{79}-E_{91} \leq 1. \tag{123}$$

H. Two intertwined pentagon logics forming a Specker Käfer (bug) or cat's cradle logic

1. Probabilities on the Specker bug logic

A *Mathematica* [3] code to reduce probabilities on the Specker bug logic:

```
Reduce[
p1 + p2 + p3 == 1
&& p3 + p4 + p5 == 1
&& p5 + p6 + p7 == 1
&&    p7 + p8 + p9 == 1
&& p9 + p10 + p11 == 1
&& p11 + p12 + p1 == 1
&&   p4 + p10 + p13 == 1,
{p3, p11, p5, p9, p4, p10}, Reals]

~~~~~~ Mathematica response

p1 == 3/2 - p12/2 - p13/2 - p2/2 - p6/2 - p7
    - p8/2 &&
 p3 == -(1/2) + p12/2 + p13/2 - p2/2 + p6/2 +
      p7 + p8/2 &&
 p11 == -(1/2) - p12/2 + p13/2 + p2/2 + p6/2
     + p7 + p8/2 &&
 p5 == 1 - p6 - p7 && p9 == 1 - p7 - p8 &&
 p4 == 1/2 - p12/2 - p13/2 + p2/2 + p6/2 - p8
    /2 &&
 p10 == 1/2 + p12/2 - p13/2 - p2/2 - p6/2 +
    p8/2
```

Computation of all the two-valued states thereon:

```
Reduce[p1 + p2 + p3 == 1 && p3 + p4 + p5 == 1
     && p5 + p6 + p7 == 1 &&
  p7 + p8 + p9 == 1 && p9 + p10 + p11 == 1 &&
       p11 + p12 + p1 == 1 &&
```

```
  p4 + p10 + p13 == 1 && p1^2 == p1 && p2^2
       == p2 && p3^2 == p3 &&
  p4^2 == p4 && p5^2 == p5 && p6^2 == p6 &&
       p7^2 == p7 && p8^2 == p8 &&
   p9^2 == p9 && p10^2 == p10 && p11^2 == p11
         && p12^2 == p12 &&
  p13^2 == p13]

~~~~~~ Mathematica response

(p9 == 0 && p8 == 0 && p7 == 1 && p6 == 0 &&
     p5 == 0 && p4 == 0 &&
   p3 == 1 && p2 == 0 && p13 == 0 && p12 == 1
         && p11 == 0 &&
   p10 == 1 && p1 == 0) || (p9 == 0 && p8 ==
        0 && p7 == 1 && p6 == 0 &&
    p5 == 0 && p4 == 0 && p3 == 1 && p2 == 0
         && p13 == 1 && p12 == 0 &&
    p11 == 1 && p10 == 0 && p1 == 0) || (p9
         == 0 && p8 == 0 &&
   p7 == 1 && p6 == 0 && p5 == 0 && p4 == 1
        && p3 == 0 && p2 == 1 &&
   p13 == 0 && p12 == 0 && p11 == 1 && p10 ==
         0 &&
   p1 == 0) || (p9 == 0 && p8 == 1 && p7 == 0
         && p6 == 0 && p5 == 1 &&
    p4 == 0 && p3 == 0 && p2 == 0 && p13 == 0
          && p12 == 0 &&
   p11 == 0 && p10 == 1 && p1 == 1) || (p9 ==
         0 && p8 == 1 &&
   p7 == 0 && p6 == 0 && p5 == 1 && p4 == 0
        && p3 == 0 && p2 == 1 &&
   p13 == 0 && p12 == 1 && p11 == 0 && p10 ==
         1 &&
   p1 == 0) || (p9 == 0 && p8 == 1 && p7 == 0
         && p6 == 0 && p5 == 1 &&
    p4 == 0 && p3 == 0 && p2 == 1 && p13 == 1
          && p12 == 0 &&
   p11 == 1 && p10 == 0 && p1 == 0) || (p9 ==
         0 && p8 == 1 &&
   p7 == 0 && p6 == 1 && p5 == 0 && p4 == 0
        && p3 == 1 && p2 == 0 &&
   p13 == 0 && p12 == 1 && p11 == 0 && p10 ==
         1 &&
   p1 == 0) || (p9 == 0 && p8 == 1 && p7 == 0
         && p6 == 1 && p5 == 0 &&
    p4 == 0 && p3 == 1 && p2 == 0 && p13 == 1
          && p12 == 0 &&
   p11 == 1 && p10 == 0 && p1 == 0) || (p9 ==
         0 && p8 == 1 &&
   p7 == 0 && p6 == 1 && p5 == 0 && p4 == 1
        && p3 == 0 && p2 == 1 &&
   p13 == 0 && p12 == 0 && p11 == 1 && p10 ==
         0 &&
   p1 == 0) || (p9 == 1 && p8 == 0 && p7 == 0
         && p6 == 0 && p5 == 1 &&
    p4 == 0 && p3 == 0 && p2 == 0 && p13 == 1
          && p12 == 0 &&
   p11 == 0 && p10 == 0 && p1 == 1) || (p9 ==
         1 && p8 == 0 &&
   p7 == 0 && p6 == 0 && p5 == 1 && p4 == 0
        && p3 == 0 && p2 == 1 &&
   p13 == 1 && p12 == 1 && p11 == 0 && p10 ==
         0 &&
   p1 == 0) || (p9 == 1 && p8 == 0 && p7 == 0
         && p6 == 1 && p5 == 0 &&
    p4 == 0 && p3 == 1 && p2 == 0 && p13 == 1
          && p12 == 1 &&
   p11 == 0 && p10 == 0 && p1 == 0) || (p9 ==
         1 && p8 == 0 &&
   p7 == 0 && p6 == 1 && p5 == 0 && p4 == 1
        && p3 == 0 && p2 == 0 &&
   p13 == 0 && p12 == 0 && p11 == 0 && p10 ==
         0 &&
   p1 == 1) || (p9 == 1 && p8 == 0 && p7 == 0
         && p6 == 1 && p5 == 0 &&
    p4 == 1 && p3 == 0 && p2 == 1 && p13 == 0
          && p12 == 1 &&
   p11 == 0 && p10 == 0 && p1 == 0)
```

2. *Hull calculation for the probabilities on the Specker bug logic*

```
* 13 probabilities on atoms a1...a13:
* p1 ... p13
*
V-representation
begin
14  14  real
1 1 0 0 0 1 0 0 0 1 0 0 0 1
1 1 0 0 1 0 1 0 0 1 0 0 0 0
1 1 0 0 0 1 0 0 1 0 1 0 0 0
1 0 1 0 0 1 0 0 0 1 0 0 1 1
1 0 1 0 0 1 0 0 1 0 0 1 0 1
1 0 1 0 1 0 1 0 0 1 0 0 1 0
1 0 1 0 1 0 0 1 0 0 0 1 0 0
1 0 1 0 1 0 1 0 1 0 0 1 0 0
1 0 1 0 0 1 0 0 1 0 1 0 1 0
1 0 0 1 0 0 0 1 0 0 0 1 0 1
1 0 0 1 0 0 1 0 1 0 0 1 0 1
1 0 0 1 0 0 1 0 0 1 0 0 1 1
1 0 0 1 0 0 0 1 0 0 1 0 1 0
1 0 0 1 0 0 1 0 1 0 1 0 1 0
end

~~~~~~ cddlib response


H-representation
linearity 7  17 18 19 20 21 22 23
begin
 23 14 real
  0  0  0  0  1  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  1  0  0  0  0  0  0  0
  0  1  1  0 -1  0  1  0 -1  0  0  0  0  0
  0  1  0  0  0  0  0  0  0  0  0  0  0  0
  0  1  1  0 -1  0  0  0  0  0  0  0  0  0
  0  1  2  0 -2  0  1  0 -1  0  0  0  0  0
  0  0  1  0 -1  0  1  0  0  0  0  0  0  0
  0  0  1  0  0  0  0  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0  0  0  1  0  0  0
  0  0  0  0  0  0  0  0  1  0  0  0  0  0
  0  0  1  0 -1  0  1  0 -1  0  1  0  0  0
  1  0  0  0 -1  0  0  0  0  0 -1  0  0  0
  1 -1 -1  0  1  0 -1  0  1  0 -1  0  0  0
```

```
  1 −1 −1  0  0  0  0  0  1  0 −1  0  0  0
  1 −1 −1  0  0  0  0  0  0  0  0  0  0  0
  1 −1 −1  0  1  0 −1  0  0  0  0  0  0  0
 −1  1  1  1  0  0  0  0  0  0  0  0  0  0
  0 −1 −1  0  1  1  0  0  0  0  0  0  0  0
 −1  1  1  0 −1  0  1  1  0  0  0  0  0  0
  0 −1 −1  0  1  0 −1  0  1  1  0  0  0  0
 −1  1  1  0 −1  0  1  0 −1  0  1  1  0  0
  0  0 −1  0  1  0 −1  0  1  0 −1  0  1  0
 −1  0  0  0  1  0  0  0  0  0  1  0  0  1
end
```

The resulting face inequalities are

$$-p_4 \leq 0, \tag{124}$$
$$-p_6 \leq 0, \tag{125}$$
$$-p_1 - p_2 + p_4 - p_6 + p_8 \leq 0, \tag{126}$$
$$-p_1 \leq 0, \tag{127}$$
$$-p_1 - p_2 + p_4 \leq 0, \tag{128}$$
$$-p_1 - 2p_2 + 2p_4 - p_6 + p_8 \leq 0, \tag{129}$$
$$-p_2 + p_4 - p_6 \leq 0, \tag{130}$$
$$-p_2 \leq 0, \tag{131}$$
$$-p_{10} \leq 0, \tag{132}$$
$$-p_8 \leq 0, \tag{133}$$
$$-p_2 + p_4 - p_6 + p_8 - p_{10} \leq 0, \tag{134}$$
$$+p_4 + p_{10} \leq +1, \tag{135}$$
$$+p_1 + p_2 - p_4 + p_6 - p_8 + p_{10} \leq +1, \tag{136}$$
$$+p_1 + p_2 - p_8 + p_{10} \leq +1, \tag{137}$$
$$+p_1 + p_2 \leq +1, \tag{138}$$
$$+p_1 + p_2 - p_4 + p_6 \leq +1, \tag{139}$$
$$-p_1 - p_2 - p_3 \leq -1, \tag{140}$$
$$+p_1 + p_2 - p_4 - p_5 \leq 0, \tag{141}$$
$$-p_1 - p_2 + p_4 - p_6 - p_7 \leq -1, \tag{142}$$
$$+p_1 + p_2 - p_4 + p_6 - p_8 - p_9 \leq 0, \tag{143}$$
$$-p_1 - p_2 + p_4 - p_6 + p_8 - p_{10} - p_{11} \leq -1, \tag{144}$$
$$+p_2 - p_4 + p_6 - p_8 + p_{10} - p_{12} \leq 0, \tag{145}$$
$$-p_4 - p_{10} - p_{13} \leq -1. \tag{146}$$

3. *Hull calculation for the expectations on the Specker bug logic*

```
* (13 expectations on atoms a1...a13:
* E1 ... E13 not enumerated)
* 6 joint expectations  E1*E3, E3*E5, ... ,
   E11*E1
*
V-representation
begin
14  7  integer
1   −1  −1  −1  −1  −1  −1
1   −1   1   1  −1  −1  −1
1   −1  −1  −1   1   1  −1
1    1  −1  −1  −1  −1   1
1    1  −1  −1   1  −1  −1
1    1   1   1  −1  −1   1
1    1   1  −1  −1  −1  −1
1    1   1   1   1  −1  −1
1    1  −1  −1   1   1   1
1   −1  −1  −1  −1  −1  −1
1   −1  −1   1   1  −1  −1
1   −1  −1   1  −1  −1   1
1   −1  −1  −1  −1   1   1
1   −1  −1   1   1   1   1
end

~~~~~~ cddlib response


H-representation
linearity 1  18
begin
 18 7 real
  1  0  0  0  1  0  0
  1 −1  0  0  1 −1  0
  1 −1  1 −1  1 −1  0
  1  0  0 −1  1 −1  0
  1  0  1  0  0  0  0
  1  1  0  0  0  0  0
  1  1 −1  1  0  0  0
  1  0  0  1  0  0  0
  1  1 −1  0 −1  0  0
  1  0  0  0 −1  0  0
  1  0 −1  1 −1  0  0
  1  1 −1  1 −1  1  0
  1  0  0 −1  0  0  0
  1 −1  1 −1  0  0  0
  1 −1  0  0  0  0  0
  1  0  0  0  0  1  0
  0  0 −1  0  0 −1  0
  0 −1  1 −1  1 −1  1
end
```

4. *Extended Specker bug logic*

Here is the *Mathematica* [3] code to reduce probabilities on the extended (by two contexts) Specker bug logics:

```
Reduce[
p1 + p2 + p3 == 1
&& p3 + p4 + p5 == 1
&& p5 + p6 + p7 == 1
&&   p7 + p8 + p9 == 1
&& p9 + p10 + p11 == 1
&& p11 + p12 + p1 == 1
&&   p4 + p10 + p13 == 1
&& p1 + pc + q7 ==1
&& p7 + pc + q1 ==1,
{p3, p11, p5, p9, p4, p10, q3, q11, q5, q9,
   q4, q10, p13, q13, pc}]

~~~~~~ Mathematica response

p1 == p7 + q1 − q7 && p3 == 1 − p2 − p7 − q1
   + q7 &&
```

```
p11 == 1 − p12 − p7 − q1 + q7 && p5 == 1 −
     p6 − p7 &&
p9 == 1 − p7 − p8 && p4 == −1 + p2 + p6 + 2
     p7 + q1 − q7 &&
p10 == −1 + p12 + 2 p7 + p8 + q1 − q7 &&
p13 == 3 − p12 − p2 − p6 − 4 p7 − p8 − 2 q1
     + 2 q7 &&
pc == 1 − p7 − q1
```

Computation of all the 112 two-valued states thereon:

```
Reduce[p1 + p2 + p3 == 1 && p3 + p4 + p5 == 1
     && p5 + p6 + p7 == 1 &&
  p7 + p8 + p9 == 1 && p9 + p10 + p11 == 1 &&
      p11 + p12 + p1 == 1 &&
  p4 + p10 + p13 == 1 && p1^2 == p1 && p2^2
     == p2 && p3^2 == p3 &&
  p4^2 == p4 && p5^2 == p5 && p6^2 == p6 &&
     p7^2 == p7 && p8^2 == p8 &&
   p9^2 == p9 && p10^2 == p10 && p11^2 == p11
      && p12^2 == p12 &&
  p13^2 == p13 &&  q1^2 == q1 &&  q7^2 == q7
     &&  pc^2 == pc]

~~~~~~ Mathematica response

q7 == 0 && q1 == 0 && pc == 0 && p9 == 0 &&
    p8 == 0 && p7 == 1 &&
   p6 == 0 && p5 == 0 && p4 == 0 && p3 == 1
      && p2 == 0 && p13 == 0 &&
   p12 == 1 && p11 == 0 && p10 == 1 && p1 ==
       0) || (q7 == 0 &&
   q1 == 0 && pc == 0 && p9 == 0 && p8 == 0
      && p7 == 1 && p6 == 0 &&
   p5 == 0 && p4 == 0 && p3 == 1 && p2 == 0
      && p13 == 1 && p12 == 0 &&
     p11 == 1 && p10 == 0 && p1 == 0) ||
     [...]
  || (q7 == 1 && q1 == 1 && pc == 1 && p9 ==
       1 && p8 == 0 &&
     p7 == 0 && p6 == 1 && p5 == 0 && p4 == 1
        && p3 == 0 && p2 == 1 &&
   p13 == 0 && p12 == 1 && p11 == 0 && p10 ==
        0 && p1 == 0)
```

I. Two intertwined Specker bug logics

Here is the *Mathematica* [3] code to reduce probabilities on two intertwined Specker bug logics:

```
Reduce[
p1 + p2 + p3 == 1
&& p3 + p4 + p5 == 1
&& p5 + p6 + p7 == 1
&&    p7 + p8 + p9 == 1
&& p9 + p10 + p11 == 1
&& p11 + p12 + p1 == 1
&&   p4 + p10 + p13 == 1
&& q1 + q2 + q3 == 1
&& q3 + q4 + q5 == 1
&& q5 + q6 + q7 == 1
&&   q7 + q8 + q9 == 1
&& q9 + q10 + q11 == 1
&& q11 + q12 + q1 == 1
&&  q4 + q10 + q13 == 1
&& p1 + pc + q7 ==1
&& p7 + pc + q1 ==1,
{p3, p11, p5, p9, p4, p10, q3, q11, q5, q9,
    q4, q10, p13, q13, pc}]

~~~~~~ Mathematica response

p1 == p7 + q1 − q7 && p3 == 1 − p2 − p7 − q1
     + q7 &&
 p11 == 1 − p12 − p7 − q1 + q7 && p5 == 1 −
     p6 − p7 &&
 p9 == 1 − p7 − p8 && p4 == −1 + p2 + p6 + 2
     p7 + q1 − q7 &&
 p10 == −1 + p12 + 2 p7 + p8 + q1 − q7 && q3
     == 1 − q1 − q2 &&
 q11 == 1 − q1 − q12 && q5 == 1 − q6 − q7 &&
     q9 == 1 − q7 − q8 &&
 q4 == −1 + q1 + q2 + q6 + q7 && q10 == −1 +
     q1 + q12 + q7 + q8 &&
 p13 == 3 − p12 − p2 − p6 − 4 p7 − p8 − 2 q1
     + 2 q7 &&
 q13 == 3 − 2 q1 − q12 − q2 − q6 − 2 q7 − q8
     && pc == 1 − p7 − q1
```

1. Hull calculation for the contexual inequalities corresponding to the Cabello, Estebaranz and García-Alcaine logic

```
* (13 expectations on atoms A1...A18:
*  not enumerated)
*  9 4th order expectations  A1A2A3A4
    A4A5A6A7 ... A2A9A11A18
*
V-representation
begin
262144   10    real
1   1    1    1    1    1    1    1    1    1
1   1    1    1    1    1   −1   −1    1    1
1   1    1    1    1    1   −1    1    1   −1
[[...]]
1   1    1    1    1    1   −1    1    1   −1
1   1    1    1    1    1   −1   −1    1    1
1   1    1    1    1    1    1    1    1    1
end

~~~~~~ cddlib response

H-representation
begin
 274  10  real
  1   0   0   0   0   0   0   0   0   1
  1   0   0   0   0   0   0   0   1   0
  7  −1  −1  −1  −1  −1  −1   1   1   1
  7  −1  −1  −1  −1  −1   1  −1   1   1
  7  −1  −1  −1  −1   1  −1  −1   1   1
  7  −1  −1  −1   1  −1  −1  −1   1   1
```

7	−1	−1	1	−1	−1	−1	−1	1	1
7	−1	1	−1	−1	−1	−1	−1	1	1
7	1	−1	−1	−1	−1	−1	−1	1	1
1	0	0	0	0	0	0	1	0	0
7	−1	−1	−1	−1	−1	1	1	−1	1
7	−1	−1	−1	−1	1	−1	1	−1	1
7	−1	−1	−1	1	−1	−1	1	−1	1
7	−1	−1	1	−1	−1	−1	1	−1	1
7	−1	1	−1	−1	−1	−1	1	−1	1
7	1	−1	−1	−1	−1	−1	1	−1	1
7	−1	−1	−1	−1	−1	1	1	1	−1
7	−1	−1	−1	−1	1	−1	1	1	−1
7	−1	−1	−1	1	−1	−1	1	1	−1
7	−1	−1	1	−1	−1	−1	1	1	−1
7	−1	1	−1	−1	−1	−1	1	1	−1
7	1	−1	−1	−1	−1	−1	1	1	−1
1	0	0	0	0	0	1	0	0	0
7	−1	−1	−1	−1	1	1	−1	−1	1
7	−1	−1	−1	1	−1	1	−1	−1	1
7	−1	−1	1	−1	−1	1	−1	−1	1
7	−1	1	−1	−1	−1	1	−1	−1	1
7	1	−1	−1	−1	−1	1	−1	−1	1
7	−1	−1	−1	−1	1	1	−1	1	−1
7	−1	−1	−1	1	−1	1	−1	1	−1
7	−1	−1	1	−1	−1	1	−1	1	−1
7	−1	1	−1	−1	−1	1	−1	1	−1
7	1	−1	−1	−1	−1	1	−1	1	−1
7	−1	−1	−1	−1	1	1	1	−1	−1
7	−1	−1	−1	1	−1	1	1	−1	−1
7	−1	−1	1	−1	−1	1	1	−1	−1
7	−1	1	−1	−1	−1	1	1	−1	−1
7	1	−1	−1	−1	−1	1	1	−1	−1
7	−1	−1	−1	−1	1	1	1	1	1
7	−1	−1	−1	1	−1	1	1	1	1
7	−1	−1	1	−1	−1	1	1	1	1
7	−1	1	−1	−1	−1	1	1	1	1
7	1	−1	−1	−1	−1	1	1	1	1
1	0	0	0	0	1	0	0	0	0
7	−1	−1	−1	1	1	−1	−1	−1	1
7	−1	−1	1	−1	1	−1	−1	−1	1
7	−1	1	−1	−1	1	−1	−1	−1	1
7	1	−1	−1	−1	1	−1	−1	−1	1
7	−1	−1	−1	1	1	−1	−1	1	−1
7	−1	−1	1	−1	1	−1	−1	1	−1
7	−1	1	−1	−1	1	−1	−1	1	−1
7	1	−1	−1	−1	1	−1	−1	1	−1
7	−1	−1	−1	1	1	−1	1	−1	−1
7	−1	−1	1	−1	1	−1	1	−1	−1
7	−1	1	−1	−1	1	−1	1	−1	−1
7	1	−1	−1	−1	1	−1	1	−1	−1
7	−1	−1	−1	1	1	−1	1	1	1
7	−1	−1	1	−1	1	−1	1	1	1
7	−1	1	−1	−1	1	−1	1	1	1
7	1	−1	−1	−1	1	−1	1	1	1
7	−1	−1	−1	1	1	1	−1	−1	−1
7	−1	−1	1	−1	1	1	−1	−1	−1
7	−1	1	−1	−1	1	1	−1	−1	−1
7	1	−1	−1	−1	1	1	−1	−1	−1
7	−1	−1	−1	1	1	1	−1	1	1
7	−1	−1	1	−1	1	1	−1	1	1
7	−1	1	−1	−1	1	1	−1	1	1
7	1	−1	−1	−1	1	1	−1	1	1
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7	−1	−1	−1	−1	−1	1	−1	−1	−1

```
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  7 -1 -1 -1 -1 -1 -1 -1  1 -1
  7 -1 -1 -1 -1 -1 -1 -1 -1  1
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end

~~~~~~ cddlib reverse vertex computation


V-representation
begin
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1	1	1	1	1	−1	1	−1	−1	−1
1	1	1	1	1	−1	−1	1	−1	−1
1	1	1	1	1	−1	−1	−1	−1	1
1	1	1	1	−1	1	1	1	−1	1
1	1	1	1	−1	1	1	−1	−1	−1
1	1	1	1	−1	1	−1	1	−1	−1
1	1	1	1	−1	1	−1	−1	−1	1
1	1	1	1	−1	−1	1	1	−1	−1
1	1	1	1	−1	−1	1	−1	−1	1
1	1	1	1	−1	−1	−1	1	−1	1
1	1	1	1	−1	−1	−1	−1	−1	−1
1	1	1	−1	1	1	1	1	−1	1
1	1	1	−1	1	1	1	−1	−1	−1
1	1	1	−1	1	1	−1	1	−1	−1
1	1	1	−1	1	1	−1	−1	−1	1
1	1	1	−1	1	−1	1	1	−1	−1
1	1	1	−1	1	−1	1	−1	−1	1
1	1	1	−1	1	−1	−1	1	−1	1
1	1	1	−1	1	−1	−1	−1	−1	−1
1	1	1	−1	−1	1	1	1	−1	−1
1	1	1	−1	−1	1	1	−1	−1	1
1	1	1	−1	−1	1	−1	1	−1	1
1	1	1	−1	−1	1	−1	−1	−1	−1
1	1	1	−1	−1	−1	1	1	−1	1
1	1	1	−1	−1	−1	1	−1	−1	−1
1	1	1	−1	−1	−1	−1	1	−1	−1
1	1	1	−1	−1	−1	−1	−1	−1	1
1	−1	1	1	1	1	1	1	−1	1
1	1	−1	1	1	1	1	1	−1	1
1	−1	1	1	1	1	1	−1	−1	−1
1	1	−1	1	1	1	1	−1	−1	−1
1	−1	1	1	1	1	−1	1	−1	−1
1	1	−1	1	1	1	−1	1	−1	−1
1	−1	1	1	1	1	−1	−1	−1	1
1	1	−1	1	1	1	−1	−1	−1	1
1	−1	1	1	1	−1	1	1	−1	−1
1	1	−1	1	1	−1	1	1	−1	−1
1	−1	1	1	1	−1	1	−1	−1	1
1	1	−1	1	1	−1	1	−1	−1	1
1	−1	1	1	1	−1	−1	1	−1	1

1	1	−1	1	1	−1	−1	1	−1	1
1	−1	1	1	1	−1	−1	−1	−1	−1
1	1	−1	1	1	−1	−1	−1	−1	−1
1	−1	1	1	−1	1	1	1	−1	−1
1	1	−1	1	−1	1	1	1	−1	−1
1	−1	1	1	−1	1	1	−1	−1	1
1	1	−1	1	−1	1	1	−1	−1	1
1	−1	1	1	−1	1	−1	1	−1	1
1	1	−1	1	−1	1	−1	1	−1	1
1	−1	1	1	−1	1	−1	−1	−1	−1
1	1	−1	1	−1	1	−1	−1	−1	−1
1	−1	1	1	−1	−1	1	1	−1	1
1	1	−1	1	−1	−1	1	1	−1	1
1	−1	1	1	−1	−1	1	−1	−1	−1
1	1	−1	1	−1	−1	1	−1	−1	−1
1	−1	1	1	−1	−1	−1	1	−1	−1
1	1	−1	1	−1	−1	−1	1	−1	−1
1	−1	1	1	−1	−1	−1	−1	−1	1
1	1	−1	1	−1	−1	−1	−1	−1	1
1	−1	−1	1	1	1	1	1	−1	−1
1	−1	1	−1	1	1	1	1	−1	−1
1	1	−1	−1	1	1	1	1	−1	−1
1	−1	−1	1	1	1	1	−1	−1	1
1	−1	1	−1	1	1	1	−1	−1	1
1	1	−1	−1	1	1	1	−1	−1	1
1	−1	−1	1	1	1	−1	1	−1	1
1	−1	1	−1	1	1	−1	1	−1	1
1	1	−1	−1	1	1	−1	1	−1	1
1	−1	−1	1	1	1	−1	−1	−1	−1
1	−1	1	−1	1	1	−1	−1	−1	−1
1	1	−1	−1	1	1	−1	−1	−1	−1
1	−1	−1	1	1	−1	1	1	−1	1
1	−1	1	−1	1	−1	1	1	−1	1
1	1	−1	−1	1	−1	1	1	−1	1
1	−1	−1	1	1	−1	1	−1	−1	−1
1	−1	1	−1	1	−1	1	−1	−1	−1
1	1	−1	−1	1	−1	1	−1	−1	−1
1	−1	−1	1	1	−1	−1	1	−1	−1
1	−1	1	−1	1	−1	−1	1	−1	−1
1	1	−1	−1	1	−1	−1	1	−1	−1
1	−1	−1	1	1	−1	−1	−1	−1	1
1	−1	1	−1	1	−1	−1	−1	−1	1
1	1	−1	−1	1	−1	−1	−1	−1	1
1	−1	−1	−1	1	1	1	1	−1	1
1	−1	−1	1	−1	1	1	1	−1	1
1	−1	1	−1	−1	1	1	1	−1	1
1	1	−1	−1	−1	1	1	1	−1	1
1	−1	−1	−1	1	1	1	−1	−1	−1
1	−1	−1	1	−1	1	1	−1	−1	−1
1	−1	1	−1	−1	1	1	−1	−1	−1
1	1	−1	−1	−1	1	1	−1	−1	−1
1	−1	−1	−1	1	1	−1	1	−1	−1
1	−1	−1	1	−1	1	−1	1	−1	−1
1	−1	1	−1	−1	1	−1	1	−1	−1
1	1	−1	−1	−1	1	−1	1	−1	−1
1	−1	−1	−1	1	1	−1	−1	−1	1
1	−1	−1	1	−1	1	−1	−1	−1	1
1	−1	1	−1	−1	1	−1	−1	−1	1
1	1	−1	−1	−1	1	−1	−1	−1	1
1	−1	−1	−1	−1	1	1	1	−1	−1
1	−1	−1	−1	1	−1	1	1	−1	−1
1	−1	−1	1	−1	−1	1	1	−1	−1
1	−1	1	−1	−1	−1	1	1	−1	−1
1	1	−1	−1	−1	−1	1	1	−1	−1

```
   1 -1 -1 -1 -1  1  1 -1 -1  1
   1 -1 -1 -1  1 -1  1 -1 -1  1
   1 -1 -1  1 -1 -1  1 -1 -1  1
   1 -1  1 -1 -1 -1  1 -1 -1  1
   1  1 -1 -1 -1 -1  1 -1 -1  1
   1 -1 -1 -1 -1 -1  1  1 -1  1
   1 -1 -1 -1 -1  1 -1  1 -1  1
   1 -1 -1 -1  1 -1 -1  1 -1  1
   1 -1 -1  1 -1 -1 -1  1 -1  1
   1 -1  1 -1 -1 -1 -1  1 -1  1
   1  1 -1 -1 -1 -1 -1  1 -1  1
   1 -1 -1 -1 -1 -1 -1  1 -1 -1
   1 -1 -1 -1 -1 -1  1 -1 -1 -1
   1 -1 -1 -1 -1  1 -1 -1 -1 -1
   1 -1 -1 -1  1 -1 -1 -1 -1 -1
   1 -1 -1  1 -1 -1 -1 -1 -1 -1
   1 -1  1 -1 -1 -1 -1 -1 -1 -1
   1  1 -1 -1 -1 -1 -1 -1 -1 -1
   1 -1 -1 -1 -1 -1 -1 -1  1 -1
   1 -1 -1 -1 -1 -1 -1 -1 -1  1
end
```

2. Hull calculation for the contexual inequalities corresponding to the pentagon logic

```
* (10 expectations on atoms A1...A10:
*  not enumerated)
*  5 3th order expectations  A1A2A3 A3A4A5
     ... A9A10A1
*  obtained through reverse Hull computation
V-representation
begin
 32 6 real
   1  1 -1 -1 -1 -1
   1  1 -1 -1 -1  1
   1  1 -1 -1  1 -1
   1  1 -1 -1  1  1
   1  1 -1  1 -1 -1
   1  1 -1  1 -1  1
   1  1 -1  1  1 -1
   1  1 -1  1  1  1
   1  1  1  1 -1 -1
   1  1  1  1 -1  1
   1  1  1  1  1  1
   1  1  1  1  1 -1
   1  1  1 -1  1  1
   1  1  1 -1  1 -1
   1  1  1 -1 -1  1
   1  1  1 -1 -1 -1
   1 -1  1  1  1  1
   1 -1  1  1  1 -1
   1 -1  1  1 -1  1
   1 -1  1  1 -1 -1
   1 -1  1 -1  1  1
   1 -1  1 -1  1 -1
   1 -1  1 -1 -1  1
   1 -1  1 -1 -1 -1
   1 -1 -1  1  1  1
   1 -1 -1  1  1 -1
   1 -1 -1  1 -1  1
   1 -1 -1  1 -1 -1
   1 -1 -1 -1  1  1
   1 -1 -1 -1  1 -1
   1 -1 -1 -1 -1  1
   1 -1 -1 -1 -1 -1
end

~~~~~~ cddlib response

H-representation
begin
 10 6 real
   1  0  0  0  0  1
   1  0  0  0  1  0
   1  0  0  1  0  0
   1  0  1  0  0  0
   1  1  0  0  0  0
   1  0  0  0  0 -1
   1  0  0  0 -1  0
   1  0  0 -1  0  0
   1  0 -1  0  0  0
   1 -1  0  0  0  0
end
```

3. Hull calculation for the contexual inequalities corresponding to Specker bug logics

```
* (13 expectations on atoms A1...A13:
*  not enumerated)
*  7 3th order expectations  A1A2A3 A3A4A5
     ... A11A12A1 A4A13A10
*  obtained through reverse Hull computation
V-representation
begin
 128 8 real
   1  1 -1 -1 -1 -1 -1 -1
   1  1 -1 -1 -1 -1 -1  1
   1  1 -1 -1 -1 -1  1 -1
   1  1 -1 -1 -1 -1  1  1
   1  1 -1 -1 -1  1 -1 -1
   1  1 -1 -1 -1  1 -1  1
   1  1 -1 -1 -1  1  1 -1
   1  1 -1 -1 -1  1  1  1
   1  1 -1 -1  1 -1 -1 -1
   1  1 -1 -1  1 -1 -1  1
   1  1 -1 -1  1 -1  1 -1
   1  1 -1 -1  1 -1  1  1
   1  1 -1 -1  1  1 -1 -1
   1  1 -1 -1  1  1 -1  1
   1  1 -1 -1  1  1  1 -1
   1  1 -1 -1  1  1  1  1
   1  1 -1  1 -1 -1 -1 -1
   1  1 -1  1 -1 -1 -1  1
   1  1 -1  1 -1 -1  1 -1
   1  1 -1  1 -1 -1  1  1
   1  1 -1  1 -1  1 -1 -1
   1  1 -1  1 -1  1 -1  1
   1  1 -1  1 -1  1  1 -1
   1  1 -1  1 -1  1  1  1
   1  1 -1  1  1 -1 -1 -1
```

```
  1  1 -1  1  1 -1 -1  1
  1  1 -1  1  1 -1  1 -1
  1  1 -1  1  1 -1  1  1
  1  1 -1  1  1  1 -1 -1
  1  1 -1  1  1  1 -1  1
  1  1 -1  1  1  1  1 -1
  1  1 -1  1  1  1  1  1
  1  1  1  1 -1 -1 -1 -1
  1  1  1  1 -1 -1 -1  1
  1  1  1  1 -1 -1  1 -1
  1  1  1  1 -1 -1  1  1
  1  1  1  1 -1  1 -1 -1
  1  1  1  1 -1  1 -1  1
  1  1  1  1 -1  1  1 -1
  1  1  1  1 -1  1  1  1
  1  1  1  1  1  1 -1 -1
  1  1  1  1  1  1 -1  1
  1  1  1  1  1  1  1  1
  1  1  1  1  1  1  1 -1
  1  1  1  1  1 -1  1  1
  1  1  1  1  1 -1  1 -1
  1  1  1  1  1 -1 -1  1
  1  1  1  1  1 -1 -1 -1
  1  1  1 -1  1  1  1  1
  1  1  1 -1  1  1  1 -1
  1  1  1 -1  1  1 -1  1
  1  1  1 -1  1  1 -1 -1
  1  1  1 -1  1 -1  1  1
  1  1  1 -1  1 -1  1 -1
  1  1  1 -1  1 -1 -1  1
  1  1  1 -1  1 -1 -1 -1
  1  1  1 -1 -1  1  1  1
  1  1  1 -1 -1  1  1 -1
  1  1  1 -1 -1  1 -1  1
  1  1  1 -1 -1  1 -1 -1
  1  1  1 -1 -1 -1  1  1
  1  1  1 -1 -1 -1  1 -1
  1  1  1 -1 -1 -1 -1  1
  1  1  1 -1 -1 -1 -1 -1
  1 -1  1  1  1  1  1  1
  1 -1  1  1  1  1  1 -1
  1 -1  1  1  1  1 -1  1
  1 -1  1  1  1  1 -1 -1
  1 -1  1  1  1 -1  1  1
  1 -1  1  1  1 -1  1 -1
  1 -1  1  1  1 -1 -1  1
  1 -1  1  1  1 -1 -1 -1
  1 -1  1  1 -1  1  1  1
  1 -1  1  1 -1  1  1 -1
  1 -1  1  1 -1  1 -1  1
  1 -1  1  1 -1  1 -1 -1
  1 -1  1  1 -1 -1  1  1
  1 -1  1  1 -1 -1  1 -1
  1 -1  1  1 -1 -1 -1  1
  1 -1  1  1 -1 -1 -1 -1
  1 -1  1 -1  1  1  1  1
  1 -1  1 -1  1  1  1 -1
  1 -1  1 -1  1  1 -1  1
  1 -1  1 -1  1  1 -1 -1
  1 -1  1 -1  1 -1  1  1
  1 -1  1 -1  1 -1  1 -1
  1 -1  1 -1  1 -1 -1  1
  1 -1  1 -1  1 -1 -1 -1
  1 -1  1 -1 -1  1  1  1
  1 -1  1 -1 -1  1  1 -1
  1 -1  1 -1 -1  1 -1  1
  1 -1  1 -1 -1  1 -1 -1
  1 -1  1 -1 -1 -1  1  1
  1 -1  1 -1 -1 -1  1 -1
  1 -1  1 -1 -1 -1 -1  1
  1 -1  1 -1 -1 -1 -1 -1
  1 -1 -1  1  1  1  1  1
  1 -1 -1  1  1  1  1 -1
  1 -1 -1  1  1  1 -1  1
  1 -1 -1  1  1  1 -1 -1
  1 -1 -1  1  1 -1  1  1
  1 -1 -1  1  1 -1  1 -1
  1 -1 -1  1  1 -1 -1  1
  1 -1 -1  1  1 -1 -1 -1
  1 -1 -1  1 -1  1  1  1
  1 -1 -1  1 -1  1  1 -1
  1 -1 -1  1 -1  1 -1  1
  1 -1 -1  1 -1  1 -1 -1
  1 -1 -1  1 -1 -1  1  1
  1 -1 -1  1 -1 -1  1 -1
  1 -1 -1  1 -1 -1 -1  1
  1 -1 -1  1 -1 -1 -1 -1
  1 -1 -1 -1  1  1  1  1
  1 -1 -1 -1  1  1  1 -1
  1 -1 -1 -1  1  1 -1  1
  1 -1 -1 -1  1  1 -1 -1
  1 -1 -1 -1  1 -1  1  1
  1 -1 -1 -1  1 -1  1 -1
  1 -1 -1 -1  1 -1 -1  1
  1 -1 -1 -1  1 -1 -1 -1
  1 -1 -1 -1 -1  1  1  1
  1 -1 -1 -1 -1  1  1 -1
  1 -1 -1 -1 -1  1 -1  1
  1 -1 -1 -1 -1  1 -1 -1
  1 -1 -1 -1 -1 -1  1  1
  1 -1 -1 -1 -1 -1  1 -1
  1 -1 -1 -1 -1 -1 -1  1
  1 -1 -1 -1 -1 -1 -1 -1
end

~~~~~~ cddlib response

H-representation
begin
 14 8 real
  1  0  0  0  0  0  0  1
  1  0  0  0  0  0  1  0
  1  0  0  0  0  1  0  0
  1  0  0  0  1  0  0  0
  1  0  0  1  0  0  0  0
  1  0  1  0  0  0  0  0
  1  1  0  0  0  0  0  0
  1  0  0  0  0  0  0 -1
  1  0  0  0  0  0 -1  0
  1  0  0  0  0 -1  0  0
  1  0  0  0 -1  0  0  0
  1  0  0 -1  0  0  0  0
  1  0 -1  0  0  0  0  0
  1 -1  0  0  0  0  0  0
end
```

4. Min-max calculation for the quantum bounds of two-two-state particles

```
(*
   ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
    *)
(* ~~~~~~~~~~~~~~~~~~~Start Mathematica Code
   ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~ *)
(*
   ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
    *)

(* old stuff

<<Algebra`ReIm`

Normalize[z_]:= z/Sqrt[z.Conjugate[z]];     *)

(*Definition of "my" Tensor Product*)
(*a,b are nxn and mxm-matrices*)

MyTensorProduct[a_, b_] :=
  Table[
   a[[Ceiling[s/Length[b]], Ceiling[t/Length[
       b]]]]*
    b[[s - Floor[(s - 1)/Length[b]]*Length[b
        ],
      t - Floor[(t - 1)/Length[b]]*Length[b
          ]]], {s, 1,
    Length[a]*Length[b]}, {t, 1, Length[a]*
        Length[b]}];


(*Definition of the Tensor Product between
    two vectors*)

TensorProductVec[x_, y_] :=
  Flatten[Table[
    x[[i]] y[[j]], {i, 1, Length[x]}, {j, 1,
        Length[y]}]];


(*Definition of the Dyadic Product*)

DyadicProductVec[x_] :=
  Table[x[[i]] Conjugate[x[[j]]], {i, 1,
      Length[x]}, {j, 1,
    Length[x]}];

(*Definition of the sigma matrices*)


vecsig[r_, tt_, p_] :=
 r*{{Cos[tt], Sin[tt] Exp[-I p]}, {Sin[tt]
     Exp[I p], -Cos[tt]}}

(*Definition of some vectors*)

BellBasis = (1/Sqrt[2]) {{1, 0, 0, 1}, {0, 1,
     1, 0}, {0, 1, -1,
     0}, {1, 0, 0, -1}};

Basis = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0,
    1, 0}, {0, 0, 0, 1}};



(*~~~~~~~~~~~~~~~~~~~~~~~~~~ 2 PARTICLES
   ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~*)

(*~~~~~~~~~~~~~~~~~~~~~~~~~~ 2 State System
   ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

% ~~~~~~~~~~~~~~~   2 x 2
% ~~~~~~~~~~~~~~~   2 x 2
% ~~~~~~~~~~~~~~~   2 x 2
% ~~~~~~~~~~~~~~~   2 x 2
% ~~~~~~~~~~~~~~~   2 x 2
% ~~~~~~~~~~~~~~~   2 x 2

*)


(*Definition of singlet state*)
vp = {1,0};
vm = {0,1};
psi2s = (1/Sqrt[2])*(TensorProductVec[vp, vm]
     -
     TensorProductVec[vm, vp])

DyadicProductVec[psi2s]

(*Definition of operators*)

(* Definition of one-particle operator *)

M2X = (1/2) {{0, 1}, {1, 0}};
M2Y = (1/2) {{0, -I}, {I, 0}};
M2Z = (1/2) {{1, 0}, {0, -1}};


Eigenvectors[M2X]
Eigenvectors[M2Y]
Eigenvectors[M2Z]

S2[t_, p_] := FullSimplify[M2X *Sin[t] Cos[p]
     + M2Y *Sin[t] Sin[p] + M2Z *Cos[t]]

FullSimplify[S2[\[Theta], \[Phi]]] //
    MatrixForm

FullSimplify[ComplexExpand[S2[Pi/2, 0]]] //
    MatrixForm
FullSimplify[ComplexExpand[S2[Pi/2, Pi/2]]]
    // MatrixForm
FullSimplify[ComplexExpand[S2[0, 0]]] //
    MatrixForm

Assuming[{0 <= \[Theta] <= Pi, 0 <= \[Phi] <=
     2 Pi}, FullSimplify[Eigensystem[S2[\[
    Theta], \[Phi]]], {Element[\[Theta],
    Reals],
  Element[\[Phi], Reals]}]]
```

```
FullSimplify[
 Normalize[
  Eigenvectors[S2[\[Theta], \[Phi]]][[1]]], {
      Element[\[Theta], Reals],
   Element[\[Phi], Reals]}]

ES2M[\[Theta]_,\[Phi]_] := {-E^(-I \[Phi])
    Tan[\[Theta]/2], 1}*Cos[\[Theta]/2]*E^(I
    \[Phi]/2)
ES2P[\[Theta]_,\[Phi]_] := {E^(-I \[Phi]) Cot
    [\[Theta]/2], 1}*Sin[\[Theta]/2]*E^(I \[
    Phi]/2)

FullSimplify[ES2M[\[Theta],\[Phi]] .Conjugate
    [ES2M [\[Theta],\[Phi]]], {Element[\[
    Theta], Reals],
  Element[\[Phi], Reals]}]
FullSimplify[ES2P[\[Theta],\[Phi]] .Conjugate
    [ES2P [\[Theta],\[Phi]]], {Element[\[
    Theta], Reals],
  Element[\[Phi], Reals]}]
FullSimplify[ES2P[\[Theta],\[Phi]] .Conjugate
    [ES2M[\[Theta],\[Phi]]], {Element[\[Theta
    ], Reals],
  Element[\[Phi], Reals]}]


ProjectorES2M [\[Theta]_,\[Phi]_] :=
    FullSimplify[DyadicProductVec[ES2M[\[
    Theta],\[Phi]]], {Element[\[Theta], Reals
    ],
  Element[\[Phi], Reals]}]
ProjectorES2P [\[Theta]_,\[Phi]_] :=
    FullSimplify[DyadicProductVec[ES2P[\[
    Theta],\[Phi]]], {Element[\[Theta], Reals
    ],
  Element[\[Phi], Reals]}]

 ProjectorES2M [\[Theta],\[Phi]] //MatrixForm
 ProjectorES2P [\[Theta],\[Phi]] //MatrixForm


(* verification of spectral form *)

FullSimplify[(-1/2)ProjectorES2M [\[Theta],\[
    Phi]] + (1/2)ProjectorES2P [\[Theta],\[Phi
    ]], {Element[\[Theta], Reals],
  Element[\[Phi], Reals]}]


SingleParticleSpinOneHalfeObservable[x_, p_]
    :=   FullSimplify[(1/2) (IdentityMatrix
    [2] + vecsig[1, x, p])] ;

SingleParticleSpinOneHalfeObservable[\[Theta
    ], \[Phi]] // MatrixForm

Eigensystem[FullSimplify[
    SingleParticleSpinOneHalfeObservable[x, p
    ]]]

(*Definition of single operators for
    occurrence of spin up*)

SingleParticleProjector2first[x_, p_, pm_] :=
       MyTensorProduct[1/2 (IdentityMatrix[2]
     + pm*vecsig[1, x, p]),  IdentityMatrix
    [2]]

SingleParticleProjector2second[x_, p_, pm_]
    :=  MyTensorProduct[IdentityMatrix[2],
    1/2 (IdentityMatrix[2] + pm*vecsig[1, x,
    p])]



(*Definition of two-particle joint operator
    for occurrence of spin up \
and down*)

JointProjector2[x1_, x2_, p1_, p2_, pm1_,
    pm2_] :=  MyTensorProduct[1/2 (
    IdentityMatrix[2] + pm1*vecsig[1, x1, p1
    ]),  1/2 (IdentityMatrix[2] + pm2*vecsig
    [1, x2, p2])]


(*Definition of probabilities*)


(*Probability of concurrence of two equal
    events for two-particle \
probability in singlet Bell state for
    occurrence of spin up*)

JointProb2s[x1_, x2_, p1_, p2_, pm1_, pm2_]
    :=
 FullSimplify[
  Tr[DyadicProductVec[psi2s].JointProjector2[
      x1, x2, p1, p2, pm1,
     pm2]]]

JointProb2s[x1, x2, p1, p2, pm1, pm2]

JointProb2s[x1, x2, p1, p2, pm1, pm2] //
    TeXForm

(*sum of joint probabilities add up to one*)

FullSimplify[
 Sum[JointProb2s[x1, x2, p1, p2, pm1, pm2], {
     pm1, -1, 1, 2}, {pm2, -1,
     1, 2}]]

(*Probability of concurrence of two equal
    events*)

P2Es[x1_, x2_, p1_, p2_] =
  FullSimplify[
   Sum[UnitStep[pm1*pm2]*
      JointProb2s[x1, x2, p1, p2, pm1, pm2], {
          pm1, -1, 1, 2}, {pm2, -1,
       1, 2}]];

P2Es[x1, x2, p1, p2]
```

```
(*Probability of concurrence of two non−equal
     events*)

P2NEs[x1_, x2_, p1_, p2_] =
  FullSimplify[
   Sum[UnitStep[−pm1*pm2]*
     JointProb2s[x1, x2, p1, p2, pm1, pm2], {
        pm1, −1, 1, 2}, {pm2, −1,
       1, 2}]];

P2NEs[x1, x2, p1, p2]

(*Expectation function*)

Expectation2s[x1_, x2_, p1_, p2_] =
 FullSimplify[P2Es[x1, x2, p1, p2] − P2NEs[x1
     , x2, p1, p2]]


(* ~~~~~~~~~~~~~~~~~~~~~~~ Min−Max
    calculation of the quantum correlation
    function ~~~~~~~~~~~~~~~~~~~~~~~ *)

JointExpectation2[t1_, t2_, p1_, p2_] :=
    MyTensorProduct[ 2 * S2[t1, p1] , 2 * S2[
    t2, p2] ]


FullSimplify[
 Eigensystem[
 JointExpectation2[t1 , t2 , p1 , p2 ]  ]]
        // MatrixForm

FullSimplify[
 Eigensystem[
 DyadicProductVec[psi2s]. JointExpectation2[
     t1, t2, p1 , p2 ] . DyadicProductVec[
     psi2s]  ]]    // MatrixForm

FullSimplify[
 Eigensystem[
 JointExpectation2[Pi/2 , Pi/2 , p1 , p2 ]
     ]]          // MatrixForm

FullSimplify[
  Eigensystem[
   DyadicProductVec[psi2s].JointExpectation2[
       Pi/2, Pi/2, p1,  p2 ].DyadicProductVec
       [psi2s]]] // MatrixForm


psi2mp = (1/Sqrt[2])*(TensorProductVec[vp, vm
    ] +
    TensorProductVec[vm, vp])

psi2mm = (1/Sqrt[2])*(TensorProductVec[vp, vp
    ] −
    TensorProductVec[vm, vm])

psi2pp = (1/Sqrt[2])*(TensorProductVec[vp, vp
    ] +
    TensorProductVec[vm, vm])
```

```
FullSimplify[
  Eigensystem[
   DyadicProductVec[psi2mp].JointExpectation2
       [Pi/2, Pi/2, p1,
     p2].DyadicProductVec[psi2mp]]] //
         MatrixForm

FullSimplify[
  Eigensystem[
   DyadicProductVec[psi2mm].JointExpectation2
       [Pi/2, Pi/2, p1,
     p2].DyadicProductVec[psi2mm]]] //
         MatrixForm

FullSimplify[
  Eigensystem[
   DyadicProductVec[psi2pp].JointExpectation2
       [Pi/2, Pi/2, p1,
     p2].DyadicProductVec[psi2pp]]] //
         MatrixForm

(* ~~~~~~~~~~~~~~~~~~~~~~~ Min−Max
    calculation of the Tsirelson bound
    ~~~~~~~~~~~~~~~~~~~~~~~ *)

JointProjector2Red[ p1_, p2_, pm1_, pm2_] :=
     JointProjector2[ Pi/2 , Pi/2 , p1, p2,
    pm1, pm2]

FullSimplify[ JointProjector2Red[ p1 , p2 ,
    pm1 , pm2 ]]

(* ~~~~~~~~~~~~~~~~~~~~~~~~ plausibility
    check *)

JointProb2sRed[p1_, p2_, pm1_, pm2_] :=
 FullSimplify[
  Tr[DyadicProductVec[psi2s].
      JointProjector2Red[p1, p2, pm1, pm2]]]

JointProb2sRed[p1, p2, pm1, pm2]

FullSimplify[
 JointProb2sRed[p1, p2, 1, 1] +
     JointProb2sRed[p1, p2, −1, −1] −
  JointProb2sRed[p1, p2, −1, 1] −
      JointProb2sRed[p1, p2, 1, −1]]

(* ~~~~~~~~~~~~~~~~~~~~~~~~ end plausibility
     check *)


TwoParticleExpectationsRed[ p1_, p2_] :=
    JointProjector2Red[ p1, p2, 1, 1]  +
    JointProjector2Red[ p1, p2, −1, −1] −
                                              JointProjector2
                                                 [
                                                 p1
                                                 ,
                                                 p2
                                                 ,
```

```
−1,
1]
−
Join
[
p1
,
p2
,
1,
−1]

(*  ~~~~~~~~~~~~~~~~~~~~~~~~  plausibility
    check  *)

FullSimplify[  Tr[DyadicProductVec[psi2s].
    TwoParticleExpectationsRed[A1, B1]]  ]

(*  ~~~~~~~~~~~~~~~~~~~~~~~~  end  plausibility
     check  *)

TwoParticleExpectationsRed[A1, B1]  //
    MatrixForm
TwoParticleExpectationsRed[A1, B1]  //  TeXForm

Eigenvalues[
 ComplexExpand[
  TwoParticleExpectationsRed[A1, B1] +
   TwoParticleExpectationsRed[A2, B1] +
   TwoParticleExpectationsRed[A1, B2] −
   TwoParticleExpectationsRed[A2, B2]  ]]

FullSimplify[
 Eigenvalues[
  ComplexExpand[
   TwoParticleExpectationsRed[A1, B1] +
    TwoParticleExpectationsRed[A2, B1] +
    TwoParticleExpectationsRed[A1, B2] −
    TwoParticleExpectationsRed[A2, B2]  ]]]


FullSimplify[
   TwoParticleExpectationsRed[A1, B1] +
    TwoParticleExpectationsRed[A2, B1] +
    TwoParticleExpectationsRed[A1, B2] −
    TwoParticleExpectationsRed[A2, B2]  ]

(*  observables  along  psi_singlet  *)


Eigenvalues[
 ComplexExpand[
  DyadicProductVec[
    psi2s].(TwoParticleExpectationsRed[A1, B1
        ] +
     TwoParticleExpectationsRed[A2, B1] +
     TwoParticleExpectationsRed[A1, B2] −
     TwoParticleExpectationsRed[A2, B2]).
        DyadicProductVec[psi2s]]]

FullSimplify[
 TrigExpand[
  Eigenvalues[
   ComplexExpand[
    DyadicProductVec[
      psi2s].(TwoParticleExpectationsRed[0,
          Pi/4] +
       TwoParticleExpectationsRed[Pi/2, Pi/4]
           +
       TwoParticleExpectationsRed[0, −Pi/4] −
       TwoParticleExpectationsRed[Pi/2, −Pi
           /4]).DyadicProductVec[
      psi2s]]]]]

(*  observables  along  psi_+  *)


Eigenvalues[
 ComplexExpand[
  DyadicProductVec[
    psi2mp].(TwoParticleExpectationsRed[A1,
        B1] +
     TwoParticleExpectationsRed[A2, B1] +
     TwoParticleExpectationsRed[A1, B2] −
     TwoParticleExpectationsRed[A2, B2]).
        DyadicProductVec[psi2mp]]]

FullSimplify[
 TrigExpand[
  Eigenvalues[
   ComplexExpand[
    DyadicProductVec[
      psi2mp].(TwoParticleExpectationsRed[0,
          Pi/4] +
       TwoParticleExpectationsRed[Pi/2, Pi/4]
           +
       TwoParticleExpectationsRed[0, −Pi/4] −
       TwoParticleExpectationsRed[Pi/2, −Pi
           /4]).DyadicProductVec[
      psi2mp]]]]]

(***  observables  along  phi_+  ***)


Eigenvalues[
 ComplexExpand[
  DyadicProductVec[
    psi2mm].(TwoParticleExpectationsRed[A1,
        B1] +
     TwoParticleExpectationsRed[A2, B1] +
     TwoParticleExpectationsRed[A1, B2] −
     TwoParticleExpectationsRed[A2, B2]).
        DyadicProductVec[psi2mm]]]

FullSimplify[
 TrigExpand[
  Eigenvalues[
```

```
    ComplexExpand[
     DyadicProductVec[
       psi2mm].(TwoParticleExpectationsRed[0,
           -Pi/4] +
         TwoParticleExpectationsRed[Pi/2, -Pi
             /4] +
         TwoParticleExpectationsRed[0, Pi/4] -
         TwoParticleExpectationsRed[Pi/2, Pi
             /4]).DyadicProductVec[
       psi2mm]]]]]


(*** observables along phi_+ ***)


Eigenvalues[
 ComplexExpand[
  DyadicProductVec[
    psi2pp].(TwoParticleExpectationsRed[A1,
        B1] +
      TwoParticleExpectationsRed[A2, B1] +
      TwoParticleExpectationsRed[A1, B2] -
      TwoParticleExpectationsRed[A2, B2]).
          DyadicProductVec[psi2pp]]]

FullSimplify[
 TrigExpand[
  Eigenvalues[
   ComplexExpand[
    DyadicProductVec[
      psi2pp].(TwoParticleExpectationsRed[0,
          -Pi/4] +
        TwoParticleExpectationsRed[Pi/2, -Pi
            /4] +
        TwoParticleExpectationsRed[0, Pi/4] -
        TwoParticleExpectationsRed[Pi/2, Pi
            /4]).DyadicProductVec[
      psi2pp]]]]]
```

5. Min-max calculation for the quantum bounds of two three-state particles

```
(*
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
     *)
(* ~~~~~~~~~~~~~~~~~~Start Mathematica Code
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~ *)
(*
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
     *)

(* old stuff

<<Algebra`ReIm`

Normalize[z_]:= z/Sqrt[z.Conjugate[z]];     *)

(*Definition of "my" Tensor Product*)
(*a,b are nxn and mxm-matrices*)

MyTensorProduct[a_, b_] :=
  Table[
   a[[Ceiling[s/Length[b]], Ceiling[t/Length[
       b]]]]*
    b[[s - Floor[(s - 1)/Length[b]]*Length[b
        ],
      t - Floor[(t - 1)/Length[b]]*Length[b
          ]]], {s, 1,
    Length[a]*Length[b]}, {t, 1, Length[a]*
        Length[b]}];


(*Definition of the Tensor Product between
    two vectors*)

TensorProductVec[x_, y_] :=
  Flatten[Table[
    x[[i]] y[[j]], {i, 1, Length[x]}, {j, 1,
        Length[y]}]];



(*Definition of the Dyadic Product*)

DyadicProductVec[x_] :=
  Table[x[[i]] Conjugate[x[[j]]], {i, 1,
      Length[x]}, {j, 1,
    Length[x]}];

(*Definition of the sigma matrices*)


vecsig[r_, tt_, p_] :=
 r*{{Cos[tt], Sin[tt] Exp[-I p]}, {Sin[tt]
     Exp[I p], -Cos[tt]}}

(*Definition of some vectors*)

BellBasis = (1/Sqrt[2]) {{1, 0, 0, 1}, {0, 1,
     1, 0}, {0, 1, -1,
     0}, {1, 0, 0, -1}};

Basis = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0,
    1, 0}, {0, 0, 0, 1}};


(*~~~~~~~~~~~~~~~~~~~~~~~~~  3  State System
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

% ~~~~~~~~~~~~~~~   2 x 3
% ~~~~~~~~~~~~~~~   2 x 3
% ~~~~~~~~~~~~~~~   2 x 3
% ~~~~~~~~~~~~~~~   2 x 3
% ~~~~~~~~~~~~~~~   2 x 3
% ~~~~~~~~~~~~~~~   2 x 3
% ~~~~~~~~~~~~~~~   2 x 3
% ~~~~~~~~~~~~~~~   2 x 3

*)


(*Definition of operators*)

(* Definition of one-particle operator *)
```

```
M3X = (1/Sqrt[2]) {{0, 1, 0}, {1, 0, 1},{0,
     1, 0}};
M3Y = (1/Sqrt[2]) {{0, -I, 0}, {I, 0, -I},
     {0, I, 0}};
M3Z =  {{1, 0, 0}, {0, 0, 0},{0, 0, -1}};


Eigenvectors[M3X]
Eigenvectors[M3Y]
Eigenvectors[M3Z]

S3[t_, p_] := M3X *Sin[t] Cos[p] + M3Y *Sin[t
     ] Sin[p] + M3Z *Cos[t]

FullSimplify[S3[\[Theta], \[Phi]]] //
     MatrixForm

FullSimplify[ComplexExpand[S3[Pi/2, 0]]] //
     MatrixForm
FullSimplify[ComplexExpand[S3[Pi/2, Pi/2]]]
     // MatrixForm
FullSimplify[ComplexExpand[S3[0, 0]]] //
     MatrixForm

Assuming[{0 <= \[Theta] <= Pi, 0 <= \[Phi] <=
      2 Pi}, FullSimplify[Eigensystem[S3[\[
     Theta], \[Phi]]], {Element[\[Theta],
     Reals],
   Element[\[Phi], Reals]}]]



FullSimplify[ComplexExpand[
 Normalize[
  Eigenvectors[S3[\[Theta], \[Phi]]][[1]]], {
      Element[\[Theta], Reals],
    Element[\[Phi], Reals]}]]

ES3M[\[Theta]_,\[Phi]_] := FullSimplify[
     ComplexExpand[
 Normalize[
  Eigenvectors[S3[\[Theta], \[Phi]]][[1]]]*E
      ^(I \[Phi])   , {Element[\[Theta], Reals
      ], Element[\[Phi], Reals]}]]

ES3M[\[Theta],\[Phi]]


ES3P[\[Theta]_,\[Phi]_] := FullSimplify[
     ComplexExpand[
 Normalize[
  Eigenvectors[S3[\[Theta], \[Phi]]][[2]]]*E
      ^(I \[Phi])   , {Element[\[Theta], Reals
      ], Element[\[Phi], Reals]}]]

ES3P[\[Theta],\[Phi]]

ES30[\[Theta]_,\[Phi]_] := FullSimplify[
     ComplexExpand[
 Normalize[
  Eigenvectors[S3[\[Theta], \[Phi]]][[3]]]*E
      ^(I \[Phi])   , {Element[\[Theta], Reals
      ], Element[\[Phi], Reals]}]]

ES30[\[Theta],\[Phi]]

FullSimplify[ES3M[\[Theta],\[Phi]] .Conjugate
     [ES3M [\[Theta],\[Phi]]], {Element[\[
     Theta], Reals],
   Element[\[Phi], Reals]}]
FullSimplify[ES3P[\[Theta],\[Phi]] .Conjugate
     [ES3P [\[Theta],\[Phi]]], {Element[\[
     Theta], Reals],
   Element[\[Phi], Reals]}]
FullSimplify[ES30[\[Theta],\[Phi]] .Conjugate
     [ES30 [\[Theta],\[Phi]]], {Element[\[
     Theta], Reals],
   Element[\[Phi], Reals]}]
FullSimplify[ES3P[\[Theta],\[Phi]] .Conjugate
     [ES3M[\[Theta],\[Phi]]], {Element[\[Theta
     ], Reals],
   Element[\[Phi], Reals]}]
FullSimplify[ES3P[\[Theta],\[Phi]] .Conjugate
     [ES30[\[Theta],\[Phi]]], {Element[\[Theta
     ], Reals],
   Element[\[Phi], Reals]}]
FullSimplify[ES30[\[Theta],\[Phi]] .Conjugate
     [ES3M[\[Theta],\[Phi]]], {Element[\[Theta
     ], Reals],
   Element[\[Phi], Reals]}]


ProjectorES30 [\[Theta]_,\[Phi]_] :=
     FullSimplify[ComplexExpand[
     DyadicProductVec[ES30[\[Theta],\[Phi]]],
     {Element[\[Theta], Reals],
   Element[\[Phi], Reals]}]]
ProjectorES3M [\[Theta]_,\[Phi]_] :=
     FullSimplify[ComplexExpand[
     DyadicProductVec[ES3M[\[Theta],\[Phi]]],
     {Element[\[Theta], Reals],
   Element[\[Phi], Reals]}]]
ProjectorES3P [\[Theta]_,\[Phi]_] :=
     FullSimplify[ComplexExpand[
     DyadicProductVec[ES3P[\[Theta],\[Phi]]],
     {Element[\[Theta], Reals],
   Element[\[Phi], Reals]}]]

 ProjectorES30 [\[Theta],\[Phi]] //MatrixForm
 ProjectorES3M [\[Theta],\[Phi]] //MatrixForm
 ProjectorES3P [\[Theta],\[Phi]] //MatrixForm

ProjectorES30 [\[Theta], \[Phi]] // MatrixForm
      // TeXForm
ProjectorES3M [\[Theta], \[Phi]] // MatrixForm
      // TeXForm
ProjectorES3P [\[Theta], \[Phi]] // MatrixForm
      // TeXForm

(* verification of spectral form *)

FullSimplify[0 * ProjectorES30 [\[Theta],\[Phi
     ]] +  (-1) * ProjectorES3M [\[Theta],\[Phi
     ]] + (+1) * ProjectorES3P [\[Theta],\[Phi
     ]], {Element[\[Theta], Reals],
   Element[\[Phi], Reals]}] //MatrixForm
```

```
(*   ~~~~~~~~~~~~~~~~~~~ general operator
     ~~~~~~~~~~~~~~~~~~~~~~~   *)

Operator3GEN[\[Theta]_,\[Phi]_] :=
    FullSimplify[LM * ProjectorES3M[\[Theta
    ],\[Phi]] + L0 * ProjectorES30[\[Theta
    ],\[Phi]] + LP * ProjectorES3P[\[Theta
    ],\[Phi]], {Element[\[Theta], Reals],
    Element[\[Phi], Reals]}];

Operator3GEN[\[Theta],\[Phi]]

JointProjector3GEN[x1_, x2_, p1_, p2_] :=
    MyTensorProduct[Operator3GEN[x1,p1],
    Operator3GEN[x2,p2]];

v3p = {1,0,0};
v30 = {0,1,0};
v3m = {0,0,1};

psi3s = (1/Sqrt[3])*(-TensorProductVec[v30,
    v30] + TensorProductVec[v3m, v3p] +
    TensorProductVec[v3p, v3m])


Expectation3sGEN[x1_, x2_, p1_, p2_] :=
    FullSimplify[ Tr[DyadicProductVec[psi3s].
    JointProjector3GEN[x1, x2, p1, p2]]];

Expectation3sGEN[x1, x2, p1, p2]


Ex3[LM_,L0_,LP_,x1_,x2_,p1_,p2_]:=
    FullSimplify[1/192 (24 L0^2 + 40 L0 (LM +
     LP) + 22 (LM + LP)^2 -
   32 (LM - LP)^2 Cos[x1] Cos[x2] +
   2 (-2 L0 + LM + LP)^2 Cos[
     2 x2] ((3 + Cos[2 (p1 - p2)]) Cos[2 x1]
         + 2 Sin[p1 - p2]^2) +
   2 (-2 L0 + LM + LP)^2 (Cos[2 (p1 - p2)] +
      2 Cos[2 x1] Sin[p1 - p2]^2) -
   32 (LM - LP)^2 Cos[p1 - p2] Sin[x1] Sin[x2
       ] +
   8 (-2 L0 + LM + LP)^2 Cos[p1 - p2] Sin[2
       x1] Sin[2 x2])];

Ex3[-1,0,1,x1,x2,p1,p2]



(* ~~~~~~~~~~~ natural spin observables
     ~~~~~~~~~~~~~~~~~~~~~ *)



JointProjector3NAT[x1_, x2_, p1_, p2_] :=
    MyTensorProduct[S3[x1,p1],S3[x2,p2]];

Expectation3sNAT[x1_, x2_, p1_, p2_] :=
    FullSimplify[ Tr[DyadicProductVec[psi3s].
    JointProjector3NAT[x1, x2, p1, p2]]];

Expectation3sNAT[x1, x2, p1, p2]
```

```
(* ~~~~~~~~~~~ Kochen-Specker observables
    ~~~~~~~~~~~~~~~~~~~~~ *)


(*
S3[t_, p_] := M3X *Sin[t] Cos[p] + M3Y *Sin[t
    ] Sin[p] + M3Z *Cos[t]

MM3X[ \[Alpha]_ ] := FullSimplify[S3[Pi/2, \[
    Alpha]]];
MM3Y[ \[Alpha]_ ] := FullSimplify[S3[Pi/2, \[
    Alpha]+Pi/2]];
MM3Z[ \[Alpha]_ ] := FullSimplify[S3[0, 0]];

SKS[ \[Alpha]_ ] := FullSimplify[ MM3X[\[
    Alpha]].MM3X[\[Alpha]] + MM3Y[\[Alpha]].
    MM3Y[\[Alpha]] + MM3Z[\[Alpha]].MM3Z[\[
    Alpha]] ];

FullSimplify[SKS[ \[Alpha] ]] // MatrixForm

FullSimplify[ComplexExpand[SKS[ 0]]] //
    MatrixForm
FullSimplify[ComplexExpand[SKS[ Pi/2]]] //
    MatrixForm

Assuming[{0 <= \[Theta] <= Pi, 0 <= \[Phi] <=
     2 Pi}, FullSimplify[Eigensystem[SKS[ \[
    Alpha] ]], {Element[\[Alpha], Reals]}]]

*)

Ex3[1, 0, 1, \[Theta]1, \[Theta]2, \[CurlyPhi
    ]1, \[CurlyPhi]2]

Ex3[0, 1, 0, \[Theta]1, \[Theta]2, \[CurlyPhi
    ]1, \[CurlyPhi]2]

Ex3[1, 0, 1, Pi/2, Pi/2, \[CurlyPhi]1, \[
    CurlyPhi]2]

Ex3[0, 1, 0, Pi/2, Pi/2, \[CurlyPhi]1, \[
    CurlyPhi]2]

Ex3[1, 0, 1, \[Theta]1, \[Theta]2, 0, 0]

Ex3[0, 1, 0, \[Theta]1, \[Theta]2, 0, 0]



(* min-max computation *)

(* define dichotomic operator based on spin-1
      expectation value , take \[Phi] = Pi/2
    *)

(* old, invalid parameterization
A[ \[Theta]1_ , \[Theta]2_  ] :=
    MyTensorProduct[ S3[\[Theta]1, Pi/2] ,
    S3[\[Theta]2, Pi/2] ]
```

```
(* Form the Klyachko-Can-Biniciogolu-
    Shumovsky operator  *)

T[\[Theta]1_, \[Theta]3_, \[Theta]5_, \[Theta
    ]7_, \[Theta]9_] :=
 A[\[Theta]1,\[Theta]3] + A[\[Theta]3,\[Theta
      ]5] +
  A[\[Theta]5,\[Theta]7] + A[\[Theta]7,\[
      Theta]9] +
  A[\[Theta]9,\[Theta]1]


FullSimplify[
 Eigenvalues[
  FullSimplify[
  T[\[Theta]1, \[Theta]3, \[Theta]5, \[Theta
      ]7, \[Theta]9]]]]

FullSimplify[
 Eigenvalues[
  T[2 Pi/5 , 4 Pi/5, 6 Pi/5, 8 Pi/5, 2 Pi]]]

 *)

A[ \[Theta]1_ , \[Theta]2_ ,\[CurlyPhi]1_, \[
    CurlyPhi]2_ ] :=   MyTensorProduct[ S3
    [\[Theta]1, \[CurlyPhi]1] , S3[\[Theta
    ]2, \[CurlyPhi]2] ]

(* Form the Klyachko-Can-Biniciogolu-
    Shumovsky operator  *)

T[\[Theta]1_, \[Theta]3_, \[Theta]5_, \[Theta
    ]7_, \[Theta]9_,\[CurlyPhi]1_, \[CurlyPhi
    ]3_,\[CurlyPhi]5_, \[CurlyPhi]7_,\[
    CurlyPhi]9_] :=
 A[\[Theta]1,\[Theta]3, \[CurlyPhi]1 ,\[
     CurlyPhi]3] + A[\[Theta]3,\[Theta]5,\[
     CurlyPhi]3,\[CurlyPhi]5] +
  A[\[Theta]5,\[Theta]7,\[CurlyPhi]5,\[
      CurlyPhi]7] + A[\[Theta]7,\[Theta]9,\[
      CurlyPhi]7,\[CurlyPhi]9] +
  A[\[Theta]9,\[Theta]1,\[CurlyPhi]9,\[
      CurlyPhi]1]


A1   =  CoordinateTransformData[ "Cartesian"
    -> "Spherical", "Mapping", {1,0,0  }] ;
A2   =  CoordinateTransformData[ "Cartesian"
    -> "Spherical", "Mapping", {0,1,0  }] ;
A3   =  (* CoordinateTransformData[ "
    Cartesian" -> "Spherical", "Mapping",
    {0,0,1  }] *)  {1,0,Pi/2} ;
A4   =  CoordinateTransformData[ "Cartesian"
    -> "Spherical", "Mapping", {1,-1,0 }] ;
A5   =  CoordinateTransformData[ "Cartesian"
    -> "Spherical", "Mapping", {1,1,0  }] ;
A6   =  CoordinateTransformData[ "Cartesian"
    -> "Spherical", "Mapping", {1,-1,2 }] ;
A7   =  CoordinateTransformData[ "Cartesian"
    -> "Spherical", "Mapping", {-1,1,1 }] ;
A8   =  CoordinateTransformData[ "Cartesian"
    -> "Spherical", "Mapping", {2,1,1  }] ;
A9   =  CoordinateTransformData[ "Cartesian"
    -> "Spherical", "Mapping", {0,1,-1 }] ;
A10  =  CoordinateTransformData[ "Cartesian"
    -> "Spherical", "Mapping", {0,1,1  }] ;



FullSimplify[
 Eigenvalues[
  FullSimplify[
  T[ A1[[2]],  A3[[2]],  A5[[2]],  A7[[2]],
      A9[[2]] , A1[[3]],  A3[[3]],  A5[[3]],
       A7[[3]],  A9[[3]]]]]]



{A1,
A2 ,
A3 ,
A4 ,
A5 ,
A6 ,
A7 ,
A8 ,
A9 ,
A10} //TexForm
```

6. Min-max calculation for two four-state particles

```
(*
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
     *)
(* ~~~~~~~~~~~~~~~~~ Start Mathematica Code
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~  *)
(*
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
     *)

(* old stuff

<<Algebra`ReIm`

Normalize[z_]:= z/Sqrt[z.Conjugate[z]];     *)

(*Definition of "my" Tensor Product*)
(*a,b are nxn and mxm-matrices*)

MyTensorProduct[a_, b_] :=
  Table[
   a[[Ceiling[s/Length[b]], Ceiling[t/Length[
       b]]]]*
    b[[s - Floor[(s - 1)/Length[b]]*Length[b
        ],
      t - Floor[(t - 1)/Length[b]]*Length[b
          ]]], {s, 1,
    Length[a]*Length[b]}, {t, 1, Length[a]*
        Length[b]}];
```

```
(*Definition of the Tensor Product between
    two vectors*)

TensorProductVec[x_, y_] :=
  Flatten[Table[
    x[[i]] y[[j]], {i, 1, Length[x]}, {j, 1,
        Length[y]}]];


(*Definition of the Dyadic Product*)

DyadicProductVec[x_] :=
  Table[x[[i]] Conjugate[x[[j]]], {i, 1,
      Length[x]}, {j, 1,
    Length[x]}];

(*Definition of the sigma matrices*)


vecsig[r_, tt_, p_] :=
 r*{{Cos[tt], Sin[tt] Exp[-I p]}, {Sin[tt]
     Exp[I p], -Cos[tt]}}

(*Definition of some vectors*)

BellBasis = (1/Sqrt[2]) {{1, 0, 0, 1}, {0, 1,
     1, 0}, {0, 1, -1,
     0}, {1, 0, 0, -1}};

Basis = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0,
    1, 0}, {0, 0, 0, 1}};


(*~~~~~~~~~~~~~~~~~~~~~~~~  4  State System
    ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

% ~~~~~~~~~~~~~~~   2 x 4
% ~~~~~~~~~~~~~~~   2 x 4
% ~~~~~~~~~~~~~~~   2 x 4
% ~~~~~~~~~~~~~~~   2 x 4
% ~~~~~~~~~~~~~~~   2 x 4
% ~~~~~~~~~~~~~~~   2 x 4
% ~~~~~~~~~~~~~~~   2 x 4
% ~~~~~~~~~~~~~~~   2 x 4

*)




(*Definition of operators*)

(* Definition of one-particle operator *)

M4X = (1/2) {{0,Sqrt[3],0,0 },{Sqrt[3],0,2,0
    },{0,2,0,Sqrt[3] },{0,0,Sqrt[3],0 }};
M4Y = (1/2) {{0,-Sqrt[3]I,0,0 },{Sqrt[3]I
    ,0,-2I,0},{0,2I,0,-Sqrt[3]I},{0,0,Sqrt[3]
    I,0  } };
M4Z = (1/2) {{3,0,0,0 },{0,1,0,0
    },{0,0,-1,0},{0,0,0,-3}};

Eigenvectors[M4X]
```

```
Eigenvectors[M4Y]
Eigenvectors[M4Z]

S4[t_, p_] :=  FullSimplify[M4X *Sin[t] Cos[p
    ] + M4Y *Sin[t] Sin[p] + M4Z *Cos[t]];



(*  ~~~~~~~~~~~~~~~~~~~ general operator
    ~~~~~~~~~~~~~~~~~~~~~~~   *)

LM32 =-3/2;
LM12 =-1/2;
LP32 =3/2;
LP12 =1/2;

ES4M32[\[Theta]_, \[Phi]_] :=   FullSimplify[
       Assuming[{0 < \[Theta] < Pi, 0 <= \[
    Phi] <= 2 Pi},     Normalize[
    Eigenvectors[S4[\[Theta], \[Phi
    ]]][[1]]]], {Element[\[Theta],      Reals
    ], Element[\[Phi], Reals]}];
ES4P32[\[Theta]_, \[Phi]_] :=  FullSimplify[
      Assuming[{0 < \[Theta] < Pi, 0 <= \[Phi
    ] <= 2 Pi},     Normalize[
    Eigenvectors[S4[\[Theta], \[Phi
    ]]][[2]]]], {Element[\[Theta],      Reals
    ], Element[\[Phi], Reals]}];
ES4M12[\[Theta]_, \[Phi]_] :=  FullSimplify[
      Assuming[{0 < \[Theta] < Pi, 0 <= \[Phi
    ] <= 2 Pi},     Normalize[
    Eigenvectors[S4[\[Theta], \[Phi
    ]]][[3]]]], {Element[\[Theta],      Reals
    ], Element[\[Phi], Reals]}];
ES4P12[\[Theta]_, \[Phi]_] :=  FullSimplify[
      Assuming[{0 < \[Theta] < Pi, 0 <= \[Phi
    ] <= 2 Pi},     Normalize[
    Eigenvectors[S4[\[Theta], \[Phi
    ]]][[4]]]], {Element[\[Theta],      Reals
    ], Element[\[Phi], Reals]}];



JointProjector4GEN[x1_, x2_, p1_, p2_] :=
    TensorProduct[S4[x1,p1],S4[x2,p2]];

v4P32 = ES4P32[0,0]
v4P12 = ES4P12[0,0]
v4M12 = ES4M12[0,0]
v4M32 = ES4M32[0,0]



psi4s = (1/2)*(TensorProductVec[v4P32, v4M32
    ]-TensorProductVec[v4M32, v4P32] -
    TensorProductVec[v4P12  , v4M12  ] +
    TensorProductVec[v4M12  , v4P12  ])


Expectation4sGEN[x1_, x2_, p1_, p2_] := Tr[
    DyadicProductVec[psi4s].
    JointProjector4GEN[x1, x2, p1, p2]];
```

```
FullSimplify[Expectation4sGEN[x1, x2, p1, p2
    ]]


(* ~~~~~~~~ general case ~~~~~~~~~ *)

EPPMM1[L4M32_ , L4M12_ , L4P12_ , L4P32_ ,
    \[Theta]_, \[Phi]_] :=   Assuming[{0 < \[
    Theta] < Pi, 0 <= \[Phi] <= 2 Pi},
    FullSimplify[
L4M32 * Assuming[{0 < \[Theta] < Pi, 0 <= \[
    Phi] <= 2 Pi},
 FullSimplify[
  DyadicProductVec[
   ES4M32[\[Theta], \[Phi]]], {Element[\[
        Theta], Reals],
   Element[\[Phi], Reals]}] ]   + L4M12 *
        Assuming[{0 < \[Theta] < Pi, 0 <= \[
        Phi] <= 2 Pi},
 FullSimplify[
  DyadicProductVec[
   ES4M12[\[Theta], \[Phi]]], {Element[\[
        Theta], Reals],
   Element[\[Phi], Reals]}] ]+
L4P32 * Assuming[{0 < \[Theta] < Pi, 0 <= \[
    Phi] <= 2 Pi},
 FullSimplify[
  DyadicProductVec[
   ES4P32[\[Theta], \[Phi]]], {Element[\[
        Theta], Reals],
   Element[\[Phi], Reals]}] ]+
L4P12 * Assuming[{0 < \[Theta] < Pi, 0 <= \[
    Phi] <= 2 Pi},
 FullSimplify[
  DyadicProductVec[
   ES4P12[\[Theta], \[Phi]]], {Element[\[
        Theta], Reals],
   Element[\[Phi], Reals]}] ]
]]


EPPMM1[-1,-1,1,1,\[Theta], \[Phi]] //
    MatrixForm

JointProjector4PPMM1[L4M32_ , L4M12_ , L4P12_
     , L4P32_ , x1_, x2_, p1_, p2_] :=
    Assuming[{0 < \[Theta] < Pi, 0 <= \[Phi]
    <= 2 Pi},
 FullSimplify[TensorProduct[EPPMM1[L4M32 ,
     L4M12 , L4P12 , L4P32 , x1,p1],EPPMM1[
     L4M32 , L4M12 , L4P12 , L4P32 ,x2,p2]],
     {Element[\[Theta], Reals],
   Element[\[Phi], Reals]}] ];

Expectation4PPMM1[L4M32_ , L4M12_ , L4P12_ ,
    L4P32_ , x1_, x2_, p1_, p2_] := Tr[
    DyadicProductVec[psi4s].
    JointProjector4PPMM1[L4M32 , L4M12 ,
    L4P12 , L4P32 ,x1, x2, p1, p2]];

FullSimplify[Expectation4PPMM1[-1,-1,1,1,x1,
    x2, p1, p2]]

Emmpp[x1_ ]= FullSimplify[Expectation4PPMM1
    [-1, -1, 1, 1, x1, 0, 0, 0]];
Emppm[x1_ ]= FullSimplify[Expectation4PPMM1
    [-1, 1, 1, -1, x1, 0, 0, 0]];
Empmp[x1_ ]= FullSimplify[Expectation4PPMM1
    [-1, 1, -1, 1, x1, 0, 0, 0]];


(*********** minmax calculation
    *************)

v12  = Normalize [ { 1,0,0,0     } ]  ;
v18  = Normalize [ { 0,1,0,0     } ]  ;
v17  = Normalize [ { 0,0,1,1     } ] ;
v16  = Normalize [ { 0,0,1,-1   } ]  ;
v67  = Normalize [ { 1,-1,0,0   } ]  ;
v69  = Normalize [ { 1,1,-1,-1  } ]  ;
v56  = Normalize [ { 1,1,1,1    } ]  ;
v59  = Normalize [ { 1,-1,1,-1  } ]  ;
v58  = Normalize [ { 1,0,-1,0   } ]  ;
v45  = Normalize [ { 0,1,0,-1   } ]  ;
v48  = Normalize [ { 1,0,1,0    } ]  ;
v47  = Normalize [ { 1,1,-1,1   } ]  ;
v34  = Normalize [ { -1,1,1,1   } ]  ;
v37  = Normalize [ { 1,1,1,-1   } ]  ;
v39  = Normalize [ { 1,0,0,1    } ]  ;
v23  = Normalize [ { 0,1,-1,0   } ]  ;
v29  = Normalize [ { 0,1,1,0    } ]  ;
v28  = Normalize [ { 0,0,0,1     } ]  ;

A12  = 2 * DyadicProductVec[ v12 ] -
    IdentityMatrix[4];
A18  = 2 * DyadicProductVec[ v18 ] -
    IdentityMatrix[4];
A17  = 2 * DyadicProductVec[ v17 ] -
    IdentityMatrix[4];
A16  = 2 * DyadicProductVec[ v16 ] -
    IdentityMatrix[4];
A67  = 2 * DyadicProductVec[ v67 ] -
    IdentityMatrix[4];
A69  = 2 * DyadicProductVec[ v69 ] -
    IdentityMatrix[4];
A56  = 2 * DyadicProductVec[ v56 ] -
    IdentityMatrix[4];
A59  = 2 * DyadicProductVec[ v59 ] -
    IdentityMatrix[4];
A58  = 2 * DyadicProductVec[ v58 ] -
    IdentityMatrix[4];
A45  = 2 * DyadicProductVec[ v45 ] -
    IdentityMatrix[4];
A48  = 2 * DyadicProductVec[ v48 ] -
    IdentityMatrix[4];
A47  = 2 * DyadicProductVec[ v47 ] -
    IdentityMatrix[4];
A34  = 2 * DyadicProductVec[ v34 ] -
    IdentityMatrix[4];
A37  = 2 * DyadicProductVec[ v37 ] -
    IdentityMatrix[4];
A39  = 2 * DyadicProductVec[ v39 ] -
    IdentityMatrix[4];
A23  = 2 * DyadicProductVec[ v23 ] -
    IdentityMatrix[4];
A29  = 2 * DyadicProductVec[ v29 ] -
    IdentityMatrix[4];
```

```
A28    = 2 * DyadicProductVec[ v28 ] -
     IdentityMatrix[4];


T=- MyTensorProduct[ A12, MyTensorProduct[
     A16, MyTensorProduct[ A17,  A18]]]-
     MyTensorProduct[ A34, MyTensorProduct[
         A45, MyTensorProduct[ A47,  A48]]]-
     MyTensorProduct[ A17, MyTensorProduct[
         A37, MyTensorProduct[ A47,  A67]]]-
     MyTensorProduct[ A12, MyTensorProduct[
         A23, MyTensorProduct[ A28,  A29]]]-
     MyTensorProduct[ A45, MyTensorProduct[
         A56, MyTensorProduct[ A58,  A59]]]-
     MyTensorProduct[ A18, MyTensorProduct[
         A28, MyTensorProduct[ A48,  A58]]]-
     MyTensorProduct[ A23, MyTensorProduct[
         A34, MyTensorProduct[ A37,  A39]]]-
     MyTensorProduct[ A16, MyTensorProduct[
         A56, MyTensorProduct[ A67,  A69]]]-
     MyTensorProduct[ A29, MyTensorProduct[
         A39, MyTensorProduct[ A59,  A69]]];


Sort[N[ Eigenvalues[FullSimplify[T]] ]]

~~~~~~~~~~~~~~~ Mathematica responds with

-6.94177, -6.67604, -6.33701, -6.28615,
    -6.23127, -6.16054, -6.03163, \
-5.96035, -5.93383, -5.84682, -5.73132,
    -5.69364, -5.56816, -5.51187, \
-5.41033, -5.37887, -5.30655, -5.19379,
    -5.16625, -5.14571, -5.10303, \
-5.05058, -4.94995, -4.88683, -4.81198,
    -4.76875, -4.64477, -4.59783, \
-4.51564, -4.46342, -4.44793, -4.36655,
    -4.33535, -4.26487, -4.24242, \
-4.18346, -4.11958, -4.05858, -4.00766,
    -3.94818, -3.91915, -3.86835, \
-3.83409, -3.77134, -3.7264, -3.68635,
    -3.63589, -3.59371, -3.54261, \
-3.48718, -3.47436, -3.4259, -3.35916,
    -3.35162, -3.29849, -3.24756, \
-3.23809, -3.18265, -3.14344, -3.09402,
    -3.07889, -3.03559, -3.02288, \
-2.98647, -2.88163, -2.84532, -2.80141,
    -2.76377, -2.72709, -2.67779, \
-2.65641, -2.64092, -2.5736, -2.53695,
    -2.48594, -2.46943, -2.42826, \
-2.40909, -2.3199, -2.27146, -2.26781,
    -2.23017, -2.19853, -2.14537, \
-2.1276, -2.1156, -2.08393, -2.02886,
    -2.01068, -1.95272, -1.90585, \
-1.8751, -1.81924, -1.80788, -1.77317,
    -1.71073, -1.67061, -1.61881, \
-1.58689, -1.56025, -1.52167, -1.47029,
    -1.43804, -1.41839, -1.39628, \
-1.33188, -1.2978, -1.26275, -1.24332,
    -1.17988, -1.16121, -1.12508, \
-1.06344, -1.04392, -0.981618, -0.9452,
    -0.93099, -0.902773, \
-0.866424, -0.847618, -0.797269, -0.749678,
    -0.718776, -0.667079, \
-0.655403, -0.621519, -0.563475, -0.535886,
    -0.505914, -0.488961, \
-0.477695, -0.438752, -0.413149, -0.385094,
    -0.329761, -0.313382, \
-0.267465, -0.251247, -0.186771, -0.162663,
    -0.135313, -0.115949, \
-0.0388241, -0.0285473, 0.0336107, 0.0472502,
    0.0664514, 0.0818923, \
0.137393, 0.170784, 0.18296, 0.254586,
    0.311604, 0.337846, 0.347853, \
0.351775, 0.395505, 0.422414, 0.481815,
    0.515078, 0.57488, 0.600515, \
0.655748, 0.703362, 0.727865, 0.763394,
    0.782482, 0.81889, 0.844406, \
0.888659, 0.920904, 1.00356, 1.02312,
    1.03976, 1.08469, 1.1021, \
1.11609, 1.14654, 1.20192, 1.22992, 1.28624,
    1.29287, 1.32196, \
1.36147, 1.43187, 1.52158, 1.5859, 1.61094,
    1.62377, 1.66645, \
1.68222, 1.77266, 1.8082, 1.86793, 1.92219,
    1.94603, 1.98741, \
2.04197, 2.06058, 2.12728, 2.16917, 2.20299,
    2.20934, 2.2568, \
2.34362, 2.38008, 2.38999, 2.44382, 2.47456,
    2.49679, 2.57822, \
2.62572, 2.63375, 2.67809, 2.73929, 2.81403,
    2.82569, 2.87209, \
2.94084, 2.94773, 2.99356, 3.03768, 3.0484,
    3.09975, 3.2194, 3.26743, \
3.2782, 3.30107, 3.41633, 3.43565, 3.49832,
    3.62058, 3.6639, 3.7087, \
3.78394, 3.83644, 3.94999, 3.98744, 4.01948,
    4.12536, 4.33452, \
4.37928, 4.42565, 4.47313, 4.53695, 4.71925,
    4.84841, 4.90328, \
4.95742, 5.0169, 5.17123, 5.28471, 5.39555,
    5.68376, 5.78503, 6.023}
```

---

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