

**Table 1** Structure and operation of a three-tape reversible Turing machine. The computation proceeds in three stages using different sets of quadruples and control states, linkage occurring through states  $A_j$  and  $C_j$ . On the right the contents of the tapes are shown symbolically at the beginning and end of each stage. The underbar denotes the position of the head. The initial state is  $A_1$  and, for a terminating computation,  $C_1$  is the final state.

Stage	Quadruples	Contents of tape		
		Working tape	History tape	Output tape
Compute <sup>a</sup>	1) $\begin{cases} A_1[b/b] \rightarrow [b+b]A_1' \\ A_1'[b/b] \rightarrow [+10]A_2 \end{cases}$	<u>INPUT</u>	<u>          </u>	<u>          </u>
	$\vdots$			
	m) $\begin{cases} A_j[T/b] \rightarrow [T'+b]A_m' \\ A_m'[b/b] \rightarrow [\sigma m 0]A_k \end{cases}$			
Copy output <sup>b</sup>	$\vdots$	<u>OUTPUT</u>	<u>HISTORY</u>	<u>          </u>
	N) $\begin{cases} A_{j-1}[b/b] \rightarrow [b+b]A_N' \\ A_N'[b/b] \rightarrow [0 N 0]A_j \end{cases}$			
	$\begin{aligned} &A_j[b N b] \rightarrow [b N b]B_1' \\ &B_1'[b/b] \rightarrow [+0+]B_1 \\ x \neq b: &\{ B_1[x N b] \rightarrow [x N x]B_1' \} \\ &B_1[b N b] \rightarrow [b N b]B_2' \\ &B_2'[b/b] \rightarrow [-0-]B_2 \\ x \neq b: &\{ B_2[x N x] \rightarrow [x N x]B_2' \} \\ &B_2[b N b] \rightarrow [b N b]C_j \end{aligned}$			
Retrace	N) $\begin{cases} C_j[b N b] \rightarrow [0 b 0]C_N' \\ C_N'[b/b] \rightarrow [b-b]C_{j-1} \end{cases}$	<u>OUTPUT</u>	<u>HISTORY</u>	<u>OUTPUT</u>
	$\vdots$			
	m) $\begin{cases} C_k[b m b] \rightarrow [-\sigma b 0]C_m' \\ C_m'[T'/b] \rightarrow [T-b]C_j \end{cases}$			
	$\vdots$	<u>INPUT</u>	<u>          </u>	<u>OUTPUT</u>
	1) $\begin{cases} C_2[b/b] \rightarrow [-b 0]C_1' \\ C_1'[b/b] \rightarrow [b-b]C_1 \end{cases}$			

<sup>a</sup>The labels 1) . . . m) . . . N) are not part of the machine. They indicate correspondence to the quintuples of the original irreversible machine, which the reversible machine emulates.

<sup>b</sup>In the second stage the small braces indicate sets of quadruples, with one quadruple for each nonblank tape letter  $x$ .