Karl Svozil

Is the World a Machine?

Suppose the world is a machine. This is a long-held suspicion, at least as old as the Pythagoreans, which was revitalized by early modern natural sciences. Presently this intuition is formalized by both the computer sciences and constructive as well as discrete mathematics.

Of course, anybody claiming nowadays that the world is a machine is a heretic. This claim contradicts the canon of physics outright, at least at the moment. We are told that certain quantum mechanical events occur randomly and uncontrollably; and chaos theory claims that there is randomness even in classical continuum mechanics and electricity.

Otherwise regarded, the statement that the world is a machine is trivial; a self-fulfilling prophesy, if you like. Because anything that can be comprehended can automatically be called machine-like, causal, or constructive. Alternately, if there were no comprehension of the world, there would be no talk of its machine-like character. But then there would most probably be no talk at all.

Having said this as a preamble, let me explain more explicitly one particular consequence of the assumption that the world is a machine. There has been hardly any feature of quantum mechanics that has given rise to as many fruitless speculations as complementarity. Intuitively, complementarity states that it is impossible to (irreversibly) observe certain observables at the same time with arbitrary accuracy. The more precisely one of these observables is measured, the measurement of other, complementary observables is that much less precise. Typical examples of complementary observables are position/momentum (velocity), angular momentum in the x/y/z direction, and particle number/phase.[10, 18]

The intuition (if intuition makes any sense in the quantum domain) behind this feature is that the act of (irreversible) observation of a physical system gives rise to a loss of information by (irreversibly) interfering with the system. Thereby, the possibility of measuring other aspects of the system is destroyed.

Well, this is not the whole story. Indeed, there is reason to believe that — at least up to a certain amount of complexity — any measurement can be 'undone' by properly reconstructing the wave function. A necessary condition for this is that all information about the original measurement is lost. Schrödinger, the creator of wave mechanics, liked to think of the wave function as a sort of catalogue of predictions.[26] This prediction catalogue contains all potential information. Yet it can only be opened at a single particular page. The prediction catalogue may be closed before this page is read. Then it could be opened once more at another complementary page. In no way is it possible to open the prediction catalogue at one page, read and (irreversibly) memorize (measure) the page, close it, and then open it at another complementary page. (Two non-complementary pages, which correspond to two co-measurable observables, can be read simultaneously.)

This may sound a little bit like voodoo. It is tempting to speculate that complementarity can never be modeled by classical metaphors. Yet, classical examples abound. A trivial one is a dark room with a ball moving in it. Suppose that we want to measure its position and its velocity. We first try to measure the ball's position by touching it. This finite contact inevitably causes a finite change of the ball's motion. Therefore, we can no longer measure the initial velocity of the ball in arbitrary position.

There are a number of more faithful classical metaphors for quantum complementarity. Take, for instance, Cohen's "firefly-in-a-box" model,[5] Wright's urn model,[36] and Aerts's vessel model.[1] In what follows, we are going to explore a model of complementarity pioneered by Moore.16 It is based on extremely simple systems, probably the simplest systems you can think of — finite automata. The finite automata we will consider here are objects that have a finite number of internal states and a finite number of input and output symbols. Their time evolution is mechanistic and can be written down on tables in matrix form. There are no built-in infinities anywhere, no infinite tape or memory, no non-recursive bounds on the runtime, et cetera.

Let us develop computational complementarity, as it is often called, as a game between you, the reader, and me, the author. The rules of the game are as follows. First, I give you all you need to know about the intrinsic workings of the automaton. For example, I tell you, "if the automaton is in state 1 and you input the symbol 2, then the automaton will make a transition into state 2 and output the symbol 0," and so on. Then I show you a black box that contains a realization of the automaton. The black box has a keyboard, with which you input the input symbols. It has an output display, on which the output symbols appear. No other interfaces are allowed. Suppose that I can choose which initial state the automaton will be in at the beginning of the game. I do not tell you this state. Your goal is to find out by experiment which state I have chosen. You can simply guess or rely on your luck by throwing a dice. But you can also perform clever input-output experiments and analyze your data in order to find out. You win if you give the correct answer. I win if you guess incorrectly. (So I have to be mean and select worst-case examples).

Suppose that you try very hard. Is cleverness sufficient? Will you always be able to definitely determine the initial state of the automaton? The answer to that question is no. The reason is that there may be situations when the input causes an irreversible transition into a state that does not allow any further queries about the

Karl Svozil, born 1956 in Vienna. Studied physics in Vienna and in Heidelberg. Works at the Institute of Theoretical physics at the Technical University in Vienna. Research at the University of California, Berkeley, the Lawrence Berkeley Laboratory, at the Lomonosov University, Moscow and at the Lebedev Physical Institute. Lives in Vienna.

1. D. Aerts, "Example of a Macroscopic Classical Situation that Violates Bell Inequalities," Lettere al Nuovo Cimento 34: 4 (1982): 107-111.
2. C.H. Bennett, "The Thermodynamics of Computation — a Review," International Journal of Theoretical Physics 14, pp. 905-940; reprint [14], pp. 213-248.
3. R.J. Boskovich, *De spacio et tempore, ut a nobis cognoscuntur* (Vienna, 1755). English translation in [4].
4. R.J. Boskovich, "De spacio et tempore, ut a nobis cognoscuntur," *A Theory of Natural Philosophy*, J.M. Child, ed. (Open Court [1922] and Cambridge/ MA: MIT Press, 1966) pp. 203-205.
5. D.W. Cohen, *An Introduction to Hilbert Space and Quantum Logic* (New York: Springer, 1989).
6. M. Davis, *The Undecidable* (New York: Raven Press, 1965).
7. P. Frank, *Das Kausalgesetz und seine Grenzen* (Vienna: Springer, 1932).
8. K. Gödel, "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme," Monatshefte für Mathematik und Physik 38 (1931): 173-198. English translation in [9] and [6].
9. K. Gödel, *Collected Works. Publications 1929-1936*, vol. I, S. Feferman, J.W. Dawson, S.C. Kleene, G.H. Moore, R.M. Solovay, and J. van Heijenoort, eds. (Oxford: Oxford UP, 1986).
10. H. Havlicek and K. Svozil, "Density Conditions for Quantum Propositions," Journal of Mathematical Physics (1996).
11. G. Kalmbach, *Orthomodular Lattices* (New York: Academic Press, 1983).
12. S. Kochen and E.P. Specker, "The Problem of Hidden Variables in Quantum Mechanics," Journal of Mathematics and Mechanics 17: 1 (1967): pp. 59-87; reprint [27], pp. 235-263.
13. R. Landauer, "Information is Physical," Physics Today 44 (May 1991): 23-29.

initial state. This is the meaning of the term "self-interference." Any such irreversible loss of information about the initial value of the automaton can be traced back to many-to-one operations:[13] different states are mapped onto a single state with the same output. Many-to-one operations, such as "deletion of infomation," are the only source of entropy increase in mechanistic systems.[13, 2]

In the case of the automaton discussed above, one could, of course, restore reversibility and recover the automaton's initial state using Landauer's "Hänsel and Gretel" strategy. That is, one could introduce an additional marker at every many-to-one node, which would indicate the previous state before the transition. But then, since the combined automaton/marker system is reversible, going back to the initial state erases all previous knowledge. This is analogous to re-opening the pages of Schrödinger's prediction catalogue.

This might be a good moment to introduce a sufficiently simple example. Consider, therefore, an automaton, which could be in one of three states, denoted by 1, 2, and 3. This automaton accepts three input symbols, namely 1, 2, and 3. It outputs only two symbols, namely 0 and 1. The transition function of the automaton is as follows: on input 1, it makes a transition to (or remains in) state 1; on input 2, it makes a transition to (or remains in) state 2; on input 3, it makes a transition to (or remains in) state 3. This is a typical irreversible many-to-one operation, since a particular input steers the automaton into that state, no matter which one of the three possible states it was in previously. The output function is also many-to-one and rather simple: whenever both state and input coincide — that is, whenever the guess was correct — it outputs 1; otherwise it outputs 0. So, for example, if it was in state 2 or 3 and receives input 1, it outputs 0 and makes a transition to state 1. There it awaits another input. These automaton specifications can be conveniently represented by diagrams such as the one drawn in fig. 1(a)).

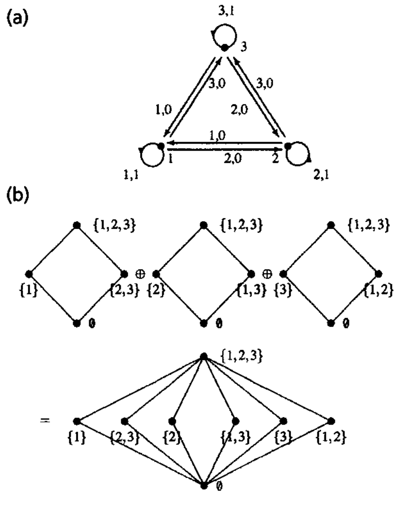

Fig. 1:
(a) Describes a complementary game using a diagram of the quantum-like (Mealy) automaton $M_3=(S, I, O, \delta, \lambda)$. The automaton has three states 1, 2, 3; three input symbols 1,2,3; and two output symbols 0,1.
Input and output symbols are separated by a comma. Arrows indicate transitions. Formulas could be

$$S = \{1, 2, 3\},$$
$$I = \{1, 2, 3\},$$
$$O = \{0, 1\}.$$

The transitional and output functions are ($\delta_{s,x}$ stands for the Kronecker delta function)

$$\delta(s, i) = i,$$
$$\lambda(s, i) = \delta_{s,i} \begin{cases} 1 & \text{if } s = i \\ 0 & \text{if } s \neq i \end{cases}$$

(b) Hasse diagram for the propositional structure. Lower elements imply higher ones, if they are connected by lines.

Computational complementarity manifests itself in the following way. If one does not know the automaton's initial state, one must choose among the input symbols 1, 2, or 3. This will correspond to the answer to the question of whether the automaton was initially in state 1, 2, or 3. If the output is 1, one knows that the answer is correct. If, on the other hand, the answer is output 0, then is might have been in 1 or 3 (corresponding to input 2), or in 1 or 2 8 (corresponding to input 3) In other words, whenever the automaton responds with a 0 (for failure), all one knows is that the automaton is not in the input state and must therefore be in one of the other two states. Otherwise, all the rest of the information about the automaton's initial state has been lost, since the transition function was chosen so that its final state would correspond to the input, completely independent of the initial state. The following propositions can be stated: on input 1, one obtains information that the automaton either was in state 1 (exclusive) or not in state 1, that is, in state 2 or 3. This is denoted by v(1)={{1}, {2, 3}}. On input 2, we obtain information that the automaton was either in state 2 (exclusive) or in state 1 or 3, denoted by v(2)={{2}, {1, 3}}. On input 3, we obtain information that the automaton either was in state 3 (exclusive) or in state 1 or 2, denoted by v(3)={{3}, {1, 2}}. In this way, we naturally arrive at the notion of partitioning automaton states according to the information obtained from input/output experiments. Every element of the partition stands for the proposition that the automaton is in (one of) the state(s) contained in that partition.

14. H.S. Leff and A.F. Rex, *Maxwell's Demon* (Princeton: Princeton UP, 1990).
15. N.D. Mermin, "Hidden Variables and the Two Theorems of John Bell," Reviews of Modern Physics 65 (1993): 803-815.
16. E.F. Moore, "Gedanken-experiments on sequential machines," Automata Studies, C.E.S. and J. McCarthy, eds. (Princeton: Princeton UP, 1956).
17. M. Navara and V. Rogalewicz, "The Pasting Constructions for Orthomodular Posets," Mathematische Nachrichten 154 (1991): 157-168.
18. A. Peres, *Quantum Theory: Concepts and Methods* (Dordrecht: Academic Publishers 1993).
19. R. Piziak, "Orthomodular Lattices and Quadratic Spaces: a Survey," Rocky Mountain Journal of Mathematics 21 (1991): 951.
20. P. Pták and S. Pulmannovà, *Orthomodular Structures as Quantum Logics* (Dordrecht: Kluwer Academic Publishers, 1991).
21. O.E. Rössler, "Endophysics," *Real Brains, Artificial Minds*, J.L. Casti and A. Karlquist, eds. (New York: North-Holland, 1987) p. 25.
22. O.E. Rössler, *Endophysics, Die Welt des inneren Beobachters*, with a foreword by Peter Weibel (Berlin: Merve Verlag, 1992).
23. M. Schaller and K. Svozil, "Partition Logics of Automata," Il Nuovo Cimento 109B (1994): 167-176.
24. M. Schaller and K. Svozil, "Automaton Partition Logic versus Quantum Logic," International Journal of Theoretical Physics 34: 8 (August 1995): 1741-1750.
25. M. Schaller and K. Svozil, "Automaton Logic," International Journal of Theoretical Physics 35: 5 (May 1996): 911-940.
26. E. Schrödinger, "Die gegenwärtige Situation in der Quantenmechanik," Naturwissenschaften 23 (1935): 807-812, 823-828, 844-849; English translation in [35], pp. 152-167.
27. E. Specker, Selecta (Basel: Birkhäuser, 1990).
28. K. Svozil, "On the Setting of Scales for Space and Time in Arbitrary Quantized Media," Lawrence Berkeley Laboratory preprint LBL-16097, May 1983.

From any partition we can construct the Boolean propositional calculus, which can be obtained if we identify its atoms with the elements of the partition. We then ‚paste' all Boolean propositional calculi (sometimes called subalgebras or blocks) together. This is a standard construction in quantum logics and the theory of orthomodular arranged masses.11, 20, 19, 17 In the above example, we arrive at a form of non-Boolean lattice, whose Hasse diagram MO3 is of the "Chinese lantern" type shown in fig.1 (b).

Let us go still a little bit further and ask which of the above automaton games people can play. The answer requires the systematic investigation of all possible non-isomorphic automaton propositional structures, or, equivalently, partition logics.30, 23, 24, 25 In fig. 2, the Hasse diagrams of all nonisomorphic four-state automaton propositional calculi are drawn.

New automata can be composed from old ones, by using parallel and serial compositions. In figs. 3 and 4, the Hasse diagrams for simple parallel compositions of two and three automata are drawn.

Recall that the method introduced here is not directly related to diagonalization and is a second, independent source of undecidability. It is already realizable at an elementary pre-diagonalization level, i.e., without the requirement of computational universality or its arithmetic equivalent. The corresponding machine model is the class of finite automata.

Since a universal computer can simulate any finite state automaton, complementarity is a feature of sufficiently complex, deterministic universes as well. To put it pointedly: if the physical universe is conceived as the product of a universal computation, then complementarity is an inevitable and necessary feature of the perception of intrinsic observers. It cannot be avoided.

Conversely, a sufficiently complex finite automaton can realize any computation. Therefore, the class of all complementary games is a unique one, encompassing all possible deterministic universes.

29. K. Svozil, "Connections between Deviations from Lorentz Transformation and Relativistic Energy-momentum Relation," Europhysics Letters 2 (1986): 83-85.
30. K. Svozil, *Randomness & Undecidability in Physics* (Singapore: World Scientific, 1993).
31. K. Svozil, "Extrinsic-intrinsic Concept and Complementarity," Inside versus Outside, H. Atmanspacher and G.J. Dalenoort, eds. (Heidelberg: Springer-Verlag, 1994) pp. 273-288.
32. K. Svozil and J. Tkadlec, "Greechie Diagrams, Nonexistence of Measures in Quantum Logics and Kochen-Specker Type Constructions," Journal of Mathematical Physics, 1996.
33. T. Toffoli and N. Margolus, *Cellular Automata Machines* (Cambridge/MA: MIT Press, 1987).
34. A.M. Turing, "On Computable Numbers, with an Application to the Entscheidungsproblem," Proceedings of the London Mathematical Society, Series 2: 42 and 43 (1936-7 and 1937): 230-265 and 544-546; reprint in [6].
35. J.A. Wheeler and W.H. Zurek, *Quantum Theory and Measurement* (Princeton: Princeton UP, 1983).
36. R. Wright, "Generalized Urn Models," Foundations of Physics 20 (1990): 881-903.

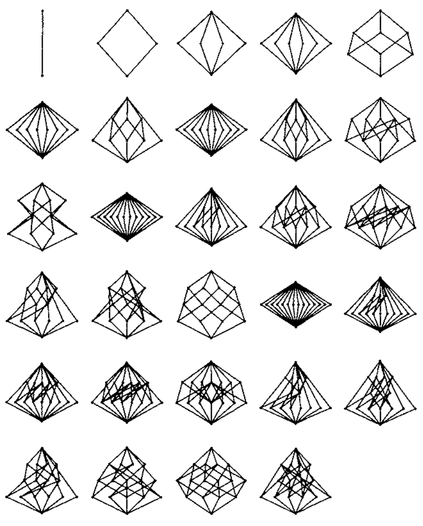

Fig. 2: Variations of the complementarity game for up to four automaton states.

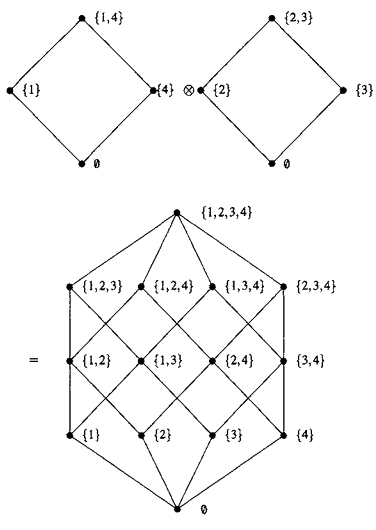

Fig. 3: Hasse diagram for a propositional calculus of two parallel automatons.

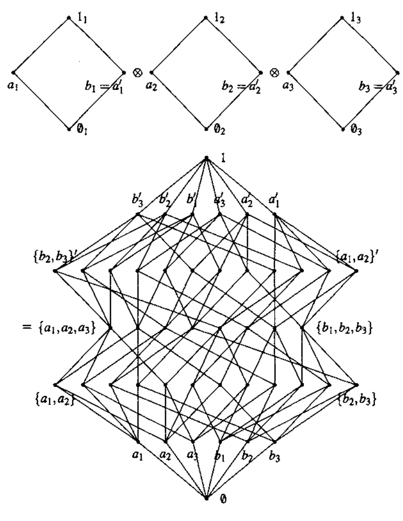

Fig. 4: Hasse diagram for a propositional calculus for three parallel automatons.