Partial Fraction – Mathematica: `Apart[ 1/((4 n + 1) (4 n - 3))]`

$$\sum_{n=0}^{\infty} \frac{1}{(4n+1)(4n-3)}$$

$$= \frac{1}{4}\left[\sum_{n=0}^{\infty} \frac{1}{4n-3} - \sum_{n=0}^{\infty} \frac{1}{4n+1}\right]$$

index change in first sum: $4m+1 = 4n-3 \Rightarrow n = m+1, m = n-1$]

$$= \frac{1}{4}\left[\sum_{m=-1}^{\infty} \frac{1}{4m+1} - \sum_{n=0}^{\infty} \frac{1}{4n+1}\right]$$

index change in first sum: $m = n$]

$$= \frac{1}{4}\left[\sum_{n=-1}^{\infty} \frac{1}{4n+1} - \sum_{n=0}^{\infty} \frac{1}{4n+1}\right]$$

$$= \frac{1}{4}\left[\frac{1}{4(-1)+1} + \sum_{n=0}^{\infty} \frac{1}{4n+1} - \sum_{n=0}^{\infty} \frac{1}{4n+1}\right]$$

$$= \frac{1}{4}\left[\frac{1}{(-3)} + 0\right] = -\frac{1}{12}$$

$$\left\{ \text{ and, by the way, } = \ldots = \sum_{n=0}^{\infty} n \right\}$$