(FAPP) Infinity (FAPP) Does It

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Mathematical Aspects in Non Equilibrium Systems — From
Micro to Macro, Erice, Sicily, Italy

Erice, 1981: Dirac, Teller, Wigner, Zichichi, (Andreotti, with wife), . . ., KS

"SEMINARIO INTERNAZIONALE SULLE IMPLICAZIONI MONDIALI DI UN CONFLITTO NUCLEARE Erice – 14/19 Agosto 1981" — The World-wide Implications of a Nuclear War. 1st International Seminar on Nuclear War, Erice, Italy, 14 – 19 August 1981

Eg, Paul Dirac, "The Futility of War", etc, "Hidden" Proceedings: DOI 10.1142/9789814536608

The Quantum Measurement Problem: Two types of quantum processes

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The Quantum Measurement Problem: Processes of type 2 can never "produce" any process of type 1. Or, can they?

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Von Neumann-Landau scheme of measurement

The 'object' is prepared in an initial state $|\psi\rangle$ which, with respect to a 'mismatching' context (relative to that preparation)—or, equivalently, orthonormal basis or maximal operator, is in a coherent superposition (linear combination) $|\psi\rangle = \sum_{i=1}^{n} a_i |\psi_i\rangle$ of (basis) elements $|\psi_i\rangle$ of that context.

The 'measurement ancilla'—along with synonymous terms such as 'provision', 'component', or 'arrangement'—should be represented by another state, denoted as $|\varphi\rangle$. This state, in relation to a suitable basis $|\varphi_1\rangle, \ldots, |\varphi_n\rangle$, can also be expressed as a coherent superposition: $|\varphi\rangle = \sum_{j=1}^{n} b_j |\varphi_j\rangle$. When an interaction occurs between the 'object' and the 'measurement ancilla', the combined state

$$|\Psi\rangle = \sum_{i,j=1}^{n} c_{ij} |\psi_i\rangle \otimes |\varphi_j\rangle = \sum_{i,j=1}^{n} c_{ij} |\psi_i \varphi_j\rangle \tag{1}$$

becomes a non-factorizable tensor product, meaning that the coefficients c_{ij} cannot be written as products $a_i b_j$.

Infinite chaining of Von Neumann-Landau scheme of measurement

The finite chaining of the von Neumann-Landau scheme of measurement is straightforward, infinite chaining requires infinite tensor products which are highly nontrivial (von Neumann 1939, with Murray 1936-43).

(i) We start with elementary tensors $\bigotimes_{n=1}^{\infty} |k_n\rangle$.

(ii) We define the inner product on elementary tensors by the product of the individual inner products:

$$\left\langle \bigotimes_{n=1}^{\infty} |k_n\rangle \middle| \bigotimes_{n=1}^{\infty} |j_n\rangle \right\rangle = \begin{cases} \prod_{n=1}^{\infty} \langle k_n | j_n \rangle_{\mathcal{H}_n} & \text{converging,} \\ 0 & \text{otherwise.} \end{cases}$$

(iii) We then consider finite linear combinations of these elementary tensors:

$$\sum_i c_i \left(\bigotimes_{n=1}^{\infty} |k_n^{(i)}\rangle \right),$$

where c_i are complex coefficients and $|k_n^{(i)}\rangle$ are basis vectors in the elementary tensor product labeled by a countable (enumerable) index i.

(iv) We finally obtain the Hilbert space $\bigotimes_{n=1}^{\infty} \mathcal{H}_n$ by taking the completion of the space of finite linear combinations of elementary tensors.

Problems with infinite products: cardinality

However, this construction does not directly address the uncountable infinity of elementary products. To illustrate this, we can draw an analogy with the representation of real numbers as expansions in an n-ary system, where they are encoded using a (finite) set of basis elements. Just as Cantor's diagonal argument shows that the reals cannot be enumerated by any countable set of indices, so too can we not enumerate the uncountable infinity of elementary products in the infinite tensor product. In von Neumann's own words (1939, http://eudml.org/doc/88704):

"generalisations of the direct product lead to higher set-theoretical powers (G. Cantor's "Alephs")."

Problems with infinite products: inner product and orthogonality

When we extend to an infinite tensor product, say $|\Psi\rangle = \bigotimes_{i=1}^{\infty} |\psi_i\rangle$ and $|\Phi\rangle = \bigotimes_{i=1}^{\infty} |\phi_i\rangle$, the inner product would apparently be $\langle\Psi|\Phi\rangle = \prod_{i=1}^{\infty}\langle\psi_i|\phi_i\rangle$.

Suppose, for the sake of demonstration, that each $\langle\psi_i|\phi_i\rangle$ is very slightly less than 1. As a consequence, the infinite product can converge to zero, and thus those vectors which are only 'slightly apart' appear orthogonal. Formally, suppose that $\langle\psi_i|\phi_i\rangle = 1 - \epsilon_i = \delta_i$, or $\epsilon_i = 1 - \langle\psi_i|\phi_i\rangle$, where $0 < \epsilon_i \ll 1$. For a large number of factors, the infinite product behaves approximately as

$$\langle\Psi|\Phi\rangle = \prod_{i=1}^{\infty}(1 - \epsilon_i) \approx \exp\left(-\sum_{i=1}^{\infty}\epsilon_i\right). \tag{2}$$

If the series $\sum_{i=1}^{\infty}\epsilon_i$ diverges (even if slowly), this product will converge to zero, that is, $\prod_{i=1}^{\infty}(1 - \epsilon_i) \to 0$.

Furthermore, the inner product would also become zero for states $|\Psi\rangle$ and $|\Phi\rangle$ that differ in only a single or a finite number of the infinitely many subfactor components, where $\langle\psi_i|\phi_i\rangle = 0$, with all the rest being identical.

Take-home message: "Problems with/loss of orthogonality (and other issues) indicate breakdown of unitary equivalence."

Sectorization

Already von Neumann 1939, http://eudml.org/doc/88704 suggested a 'solution' by partitioning the product space into equivalence classes 'sectors' containing only those infinite tensor products that are located 'close to' each other, such that their deviations from each other are 'small'. Two vectors $|\Psi\rangle$ and $|\Phi\rangle$ are in the same sector if they are equivalent, denoted by $|\Psi\rangle \sim |\Phi\rangle$, when all but finitely many subfactors are either equal or unitary equivalent and to or 'close to' one another (only a unitary transformation apart); that is,

$$\sum_{i=1}^{\infty} (1 - \langle \psi_i | \phi_i \rangle) = \sum_{i=1}^{\infty} \epsilon_i \leq \sum_{i=1}^{\infty} |1 - \langle \psi_i | \phi_i \rangle| < \infty. \tag{3}$$

Grangier and Van Den Bossche 2023, DOI 10.1007/s10701-023-00678-x, 10.3390/e25121600, 10.1088/1742-6596/2533/1/012008 that those 'sectors' (of infinite-dimensional equivalent states) correspond to 'classical' counter readings.

Previous similar proposals by Hepp 1972, DOI 10.5169/seals-114381 have been criticized by Bell 1975, DOI 10.5169/seals-114661 on the basis of the requirement of transfinite means. (Cf. also Bridgman 1934 "A Physicist's Second Reaction to Mengenlehre".)

Further problems with unitary equivalence from factorization

Another issue is related to entanglement of infinite components, resulting in different 'factors' of type I, II, III that are not unitary equivalent:

"Problems with factorization indicate breakdown of unitary equivalence."

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- Specker sequences of computable numbers converge to an uncomputable limit. One example is Chaitin's constant, the halting probability of prefix-free program codes on a universal computer. whose rate of convergence is tied to the halting time, and therefore, 'grows faster' than any computable function.

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- ► If you are comfortable with the second law of thermodynamics, you might consider that the quantum measurement problem can be resolved by means relativity.
- ► If you still have issues with the irreversibility of quantum measurements, you might consider rethinking the classical *Umkehreinwand* and consider it unresolved.

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Thank you for your attention!