Probabilistic Aspects of Quantum Contextuality

We present an explicit instance of a quantum logic---defined by a collection of contexts realizable in an n-dimensional Hilbert space---that is not n-colorable. Consequently, it does not correspond to any "completable" non-contextual set of coexisting n-ary observables. This result serves as the chromatic analogue of the Kochen-Specker theorem. Furthermore, it introduces a novel and restrictive perspective on classical truth values—specifically, those two-valued measures that can be extended to an n-ary coloring and, through convex combinations, generate classical probability distributions. As a result, we derive new bounds for the house, pentagon, and pentagram hypergraphs, extending previous constraints.