Ley probabilidad total

$$P(A), \Omega = U_i B_i,\ A_i \cap B_j = \phi, i \neq s$$

$$A = A \cap \Omega$$

$$= A \cap (U_i B)$$

$$= U(A \cap B_i)$$

$$= P(A) = P(U_i(A \cap B_i)) = \sum_i P(A \cap B) = \sum_i P(A, B_i)$$

donde:

$$\sum_i P(A, B_i) = P(A \mid B_i) P(B_i)$$

y finalmente tenemos que;

$$P(A) = \sum_i P(A \mid B_i) P(B_i) \rightarrow (Probabilidad\ total)$$

También vamos a usar la ley del valor esperado total

$$\mathbb{E}[Y] = \sum_j y_j\, P(Y = y) = \sum_j y_j \sum_i P(Y = y_j \mid X = x_i) P(X = x_i)$$

Escribimos el valor esperado en terminos de la probabilidad total

$$\mathbb{E}[Y] = \sum_i P(Y = y_j \mid X = x)\ P(X_i = x_i)$$

$$\mathbb{E}[y] = \sum_i P(X = x_i) \sum_j y_j\, P(Y = y_j \mid X = x_i)$$

Donde:

$$\sum_j y_j\, P(Y = y_j \mid X = x_i) = \mathbb{E}[y \mid X = x_i]$$

y queda igual:

$$\mathbb{E}[Y] = \sum_i \mathbb{E}[Y \mid X = x_i] P(X = x_i)$$

Primer problema

consiste el siguiente espacio de resultados:

	T	H
T	TT	TH
H	HT	HH

Y definiremos nuestros 3 eventos:
$T = \frac{1}{2}$
$HT = \frac{1}{4}$
$HH = \frac{1}{4}$

$$\mathbb{E}[Y] = \mathbb{E}[y \mid T]\ P(T) + \mathbb{E}[y \mid HT]\ P(HT) + \mathbb{E}[y \mid HH]\ P(HH)$$

$$= \mathbb{E}[y \mid T]\frac{1}{2} + \mathbb{E}[Y \mid HT]\frac{1}{4} + 2\left(\frac{1}{4}\right)$$

$$\mathbb{E}[Y \mid T] = 1 + \mathbb{E}[Y]$$

$$\mathbb{E}[y \mid HT] = 2 + \mathbb{E}[Y]$$

$$\mathbb{E}[y] = (1 + \mathbb{E}[y])\frac{1}{2} + (2 + \mathbb{E}[y])\frac{1}{4} + \frac{1}{2}$$

$$= \frac{1}{2} + \frac{1}{2}\mathbb{E}[y] + \frac{1}{2} + \frac{1}{4}\mathbb{E}[y] + \frac{1}{2}$$

$$= \frac{3}{2} + \left(\frac{1}{2} + \frac{1}{4}\right)\mathbb{E}[y]$$

$$= \frac{3}{2} + \frac{3}{4}\mathbb{E}[y]$$

$$\left(1 - \frac{3}{4}\right)\mathbb{E}[y] = \frac{3}{2}$$

$$\mathbb{E}[y] = \frac{3}{2} - \frac{1}{4}$$

$$\mathbb{E}[y] = 6$$

Segundo ejercicio

$$P(X_2 = 3 \mid X_0 = 1) = P_{ij}^2$$

$$P(X_2 = j\ \mid X_0 = 1) = \frac{P(X_2 = J, X_0 = i)}{P(X_0 = i)}$$

$$(X_2 = J, X_0 = i) = U_k(X_2 = J, X_1 = P_i, X_0 = 1)$$

$$P(X_2 = J, X_0 = i) = \sum_j P(X_2 = J, X_1 = k, X_o = i) = \sum_k P(X_1 = J, X_i = h)P(X_1 = k, X_u = 1)$$

no se pudo demostrar 💀

Prueba inducción matemática que $P(X_n = J \mid X_0 = J) = P_{ij}^n$ con base $n = 2$