Modulation

Last Class

- Propagation

Free Space Loss

- Free space loss accounting for gain of other antennas

$$\frac{P_t}{P_r} = \frac{(4\pi)^2 (d)^2}{G_r G_t \lambda^2} = \frac{(\lambda d)^2}{A_r A_t} = \frac{(cd)^2}{f^2 A_r A_t}$$

- G_t = gain of transmitting antenna
- G_r = gain of receiving antenna
- A_t = effective area of transmitting antenna
- A_r = effective area of receiving antenna

Expression E_b/N_0

- Ratio of signal energy per bit to noise power density per Hertz

$$\frac{E_b}{N_0} = \frac{S/R}{N_0} = \frac{S}{\mathrm{k}TR}$$

- The bit error rate for digital data is a function of E_b/N_0
  - Given a value for E_b/N_0 to achieve a desired error rate, parameters of this formula can be selected
  - As bit rate R increases, transmitted signal power must increase to maintain required E_b/N_0

Spectral Efficiency

Shannon's Capacity:

$$C = B\log_2\left(1+\mathrm{SNR}\right) = B\log_2\left(1+\mathrm{S/N}\right)$$

$$\frac{C}{B} = \log_2\left(1+\mathrm{SNR}\right) = \log_2\left(1+\mathrm{S/N}\right)$$

Spectral Efficiency

$$\frac{E_b}{N_0} = \frac{S/R}{N_0} = \frac{S}{N_0 R} = \frac{S}{N}\frac{B}{R}$$

Notice $N = N_0 B$

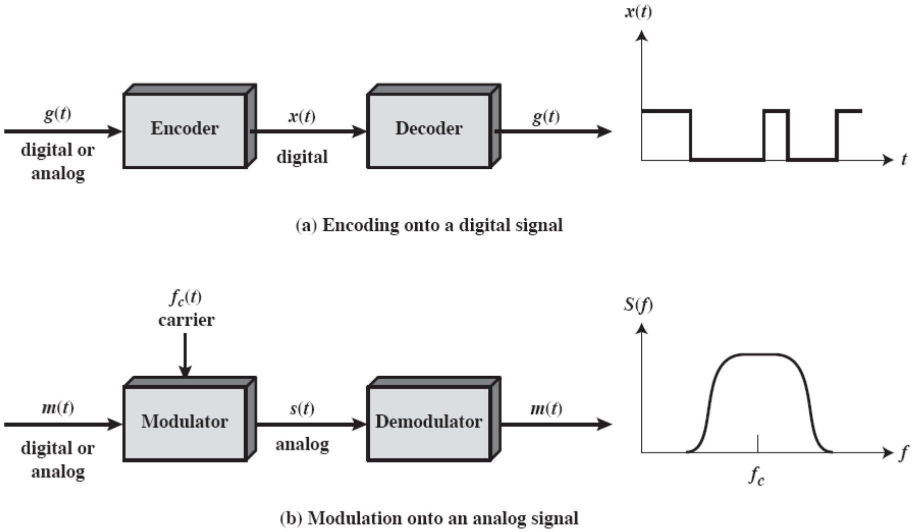
$$\frac{E_b}{N_0} = \frac{B}{C}(2^{C/B} - 1)$$

Today

- Modulation/Signal Encoding Technique
- Digitization

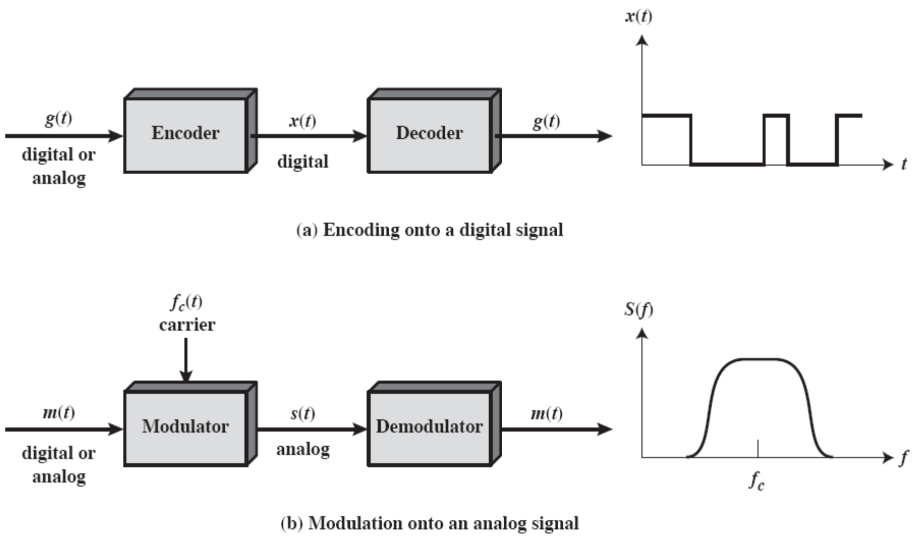
Reasons for Choosing Encoding Techniques

- Digital data, digital signal
  - Equipment less complex and expensive than digital-to-analog modulation equipment
- Analog data, digital signal
  - Permits use of modern digital transmission and switching equipment



Reasons for Choosing Encoding Techniques

- Digital data, analog signal
  - Some transmission media will only propagate analog signals
  - *E.g., optical fiber and unguided media*
- Analog data, analog signal
  - Analog data in electrical form can be transmitted easily and cheaply
  - Done with voice transmission over voice-grade lines



Signal Encoding Criteria

- What determines how successful a receiver will be in interpreting an incoming signal?
  - Signal-to-noise ratio
  - Data rate
  - Bandwidth
- An increase in data rate increases bit error rate
- An increase in SNR decreases bit error rate
- An increase in bandwidth allows an increase in data rate

Basic Modulation Techniques

- Digital data to analog signal
  - Amplitude-shift keying (ASK)
    - Amplitude difference of carrier frequency
  - Frequency-shift keying (FSK)
    - Frequency difference near carrier frequency
  - Phase-shift keying (PSK)
    - Phase of carrier signal shifted

Basic Encoding Techniques

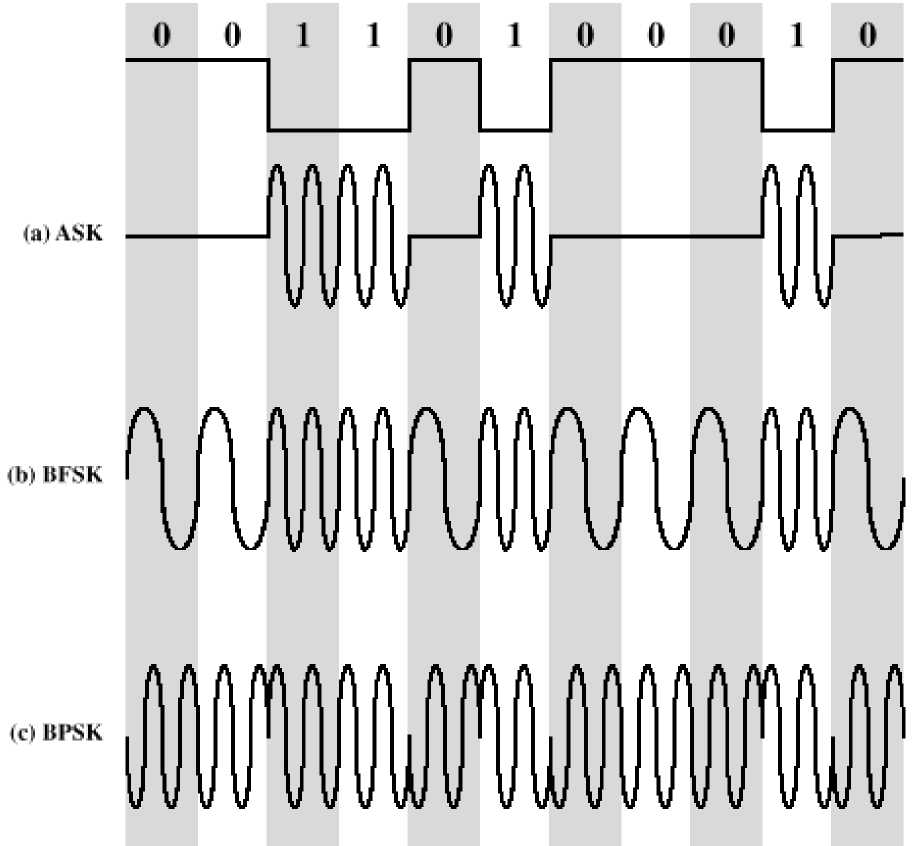


Figure 6.2 Modulation of Analog Signals for Digital Data

Amplitude-Shift Keying

- One binary digit represented by presence of carrier, at constant amplitude
- Other binary digit represented by absence of carrier

$$s(t) = \begin{cases} A\cos(2\pi f_c t) & \text{binary } 1 \\ 0 & \text{binary } 0 \end{cases}$$

- where the carrier signal is $A\cos(2\pi f_c t)$

Amplitude-Shift Keying

- Susceptible to sudden gain changes
- Inefficient modulation technique
- On voice-grade lines, used up to 1200 bps
- Used to transmit digital data over optical fiber

Binary Frequency-Shift Keying (BFSK)

- Two binary digits represented by two different frequencies near the carrier frequency

$$s(t) = \begin{cases} A\cos(2\rho f_1 t) & \text{binary } 1 \\ A\cos(2\rho f_2 t) & \text{binary } 0 \end{cases}$$

where f_1 and f_2 are offset from carrier frequency f_c by equal but opposite amounts

Binary Frequency-Shift Keying (BFSK)

- Less susceptible to error than ASK
- On voice-grade lines, used up to 1200bps
- Used for high-frequency (3 to 30 MHz) radio transmission
- Can be used at higher frequencies on LANs that use coaxial cable

Multiple Frequency-Shift Keying (MFSK)

- More than two frequencies are used
- More bandwidth efficient but more susceptible to error

$$s_i(t) = A\cos 2\pi f_i t \quad 1 \le i \le M$$

- $f_i = f_c + (2i - 1 - M)f_d$
- f_c = the carrier frequency
- f_d = the difference frequency
- M = number of different signal elements = 2^L
- L = number of bits per signal element

Multiple Frequency-Shift Keying (MFSK)

- To match data rate of input bit stream, each output *signal element* is held for:

  $$T_s = LT \text{ seconds}$$

  - where T is the bit period (data rate = $1/T$)

One signal element encodes L bits

(Signal element is also termed symbol)

Multiple Frequency-Shift Keying (MFSK)

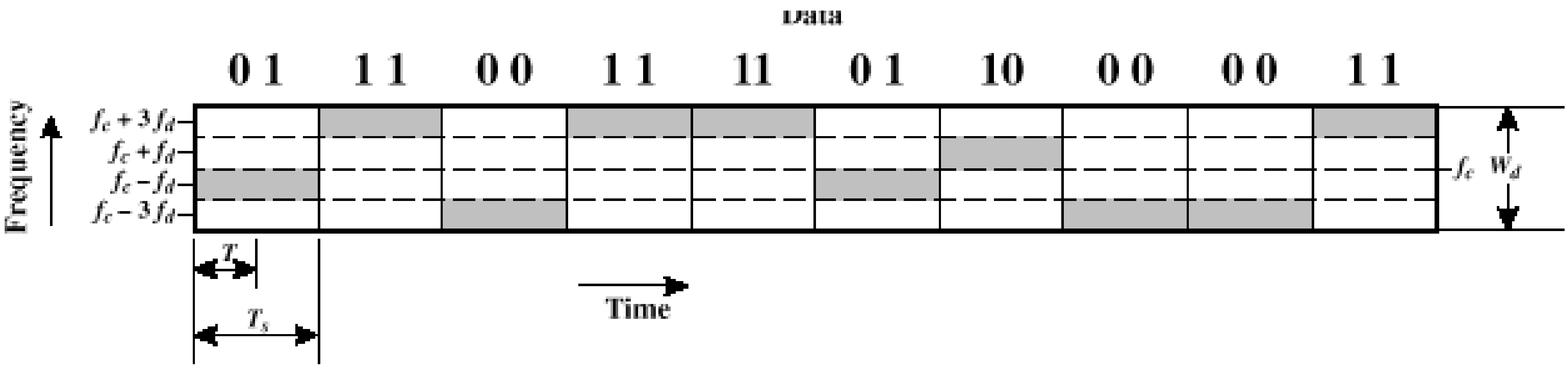


Figure 6.4 MFSK Frequency Use (M = 4)

Phase-Shift Keying (PSK)

- Two-level PSK (BPSK)
  - Uses two phases to represent binary digits

$$s(t) = \begin{cases} A\cos(2\pi f_c t) & \text{binary } 1 \\ A\cos(2\pi f_c t + \pi) & \text{binary } 0 \end{cases}$$

$$= \begin{cases} A\cos(2\pi f_c t) & \text{binary } 1 \\ -A\cos(2\pi f_c t) & \text{binary } 0 \end{cases}$$

Phase-Shift Keying (PSK)

- Four-level PSK (QPSK)
  - Each element represents more than one bit

$$
s(t) = \begin{cases}
A\cos\left(2\pi f_c t + \dfrac{\pi}{4}\right) & 11 \\[2ex]
A\cos\left(2\pi f_c t + \dfrac{3\pi}{4}\right) & 01 \\[2ex]
A\cos\left(2\pi f_c t - \dfrac{3\pi}{4}\right) & 00 \\[2ex]
A\cos\left(2\pi f_c t - \dfrac{\pi}{4}\right) & 10
\end{cases}
$$

Phase-Shift Keying (PSK)

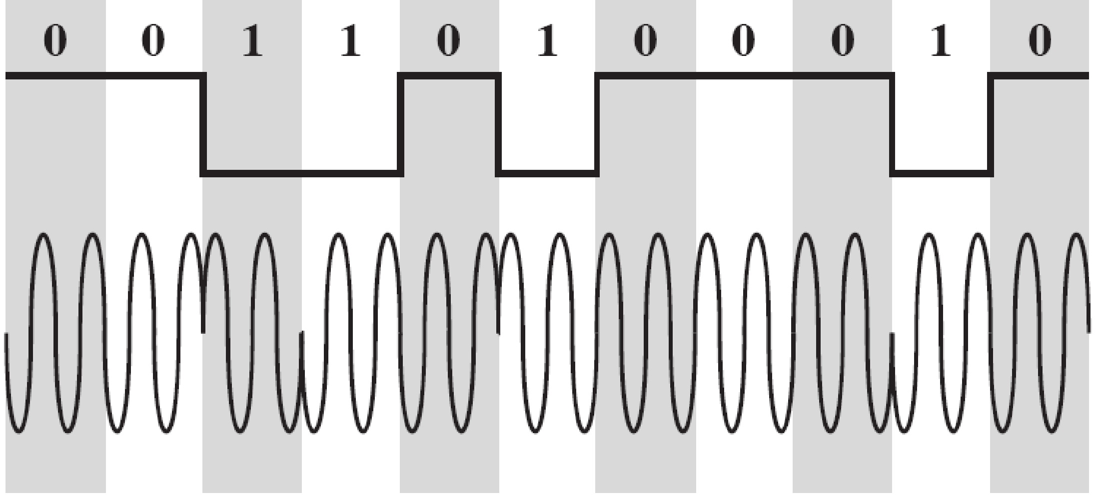
- Multilevel PSK
  - Using multiple phase angles with each angle having more than one amplitude, multiple signals elements can be achieved

$$D = \frac{R}{L} = \frac{R}{\log_2 M}$$

- D = modulation symbol rate, baud
- R = data rate, bps
- M = number of different signal elements = 2^L
- L = number of bits per signal element

Phase-Shift Keying (PSK)

- Differential PSK (DPSK)
  - Phase shift with reference to previous bit
    - Binary 0 – signal burst of same phase as previous signal burst
    - Binary 1 – signal burst of opposite phase to previous signal burst



Performance – Bandwidth Efficiency

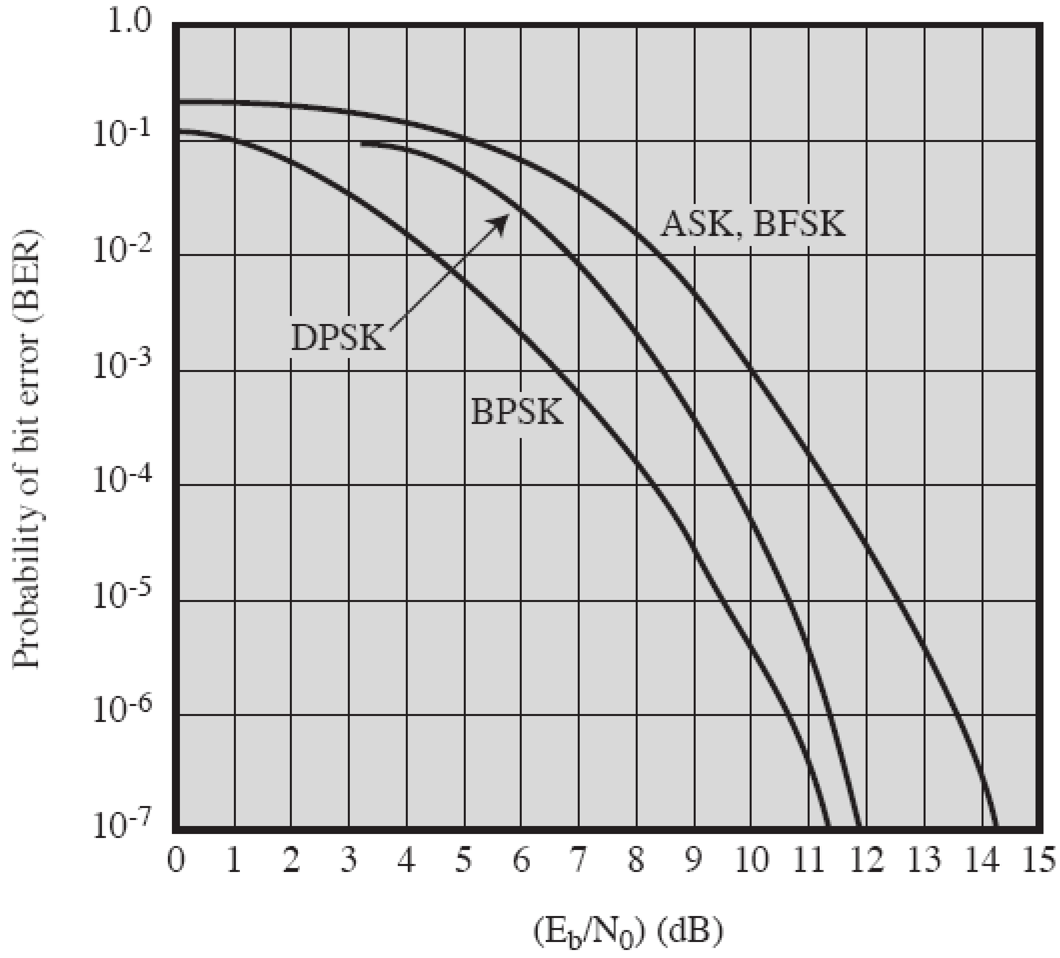
- Bandwidth of modulated signal (B_T)
  - ASK, PSK $B_T=(1+r)R$
  - FSK $B_T=2\Delta F+(1+r)R$

    - R = bit rate
    - 0 < r < 1; related to how signal is filtered
    - $\Delta F = f_2 - f_c = f_c - f_1$

Performance – Bandwidth Efficiency

- Bandwidth of modulated signal (B_T)
  - MPSK $$B_T = \left(\frac{1+r}{L}\right)R = \left(\frac{1+r}{\log_2 M}\right)R$$
  - MFSK $$B_T = \left(\frac{(1+r)M}{\log_2 M}\right)R$$
    - L = number of bits encoded per signal element
    - M = number of different signal elements

Performance – Error Rate



Quadrature Amplitude Modulation

- QAM is a combination of ASK and PSK
  - Two different signals sent simultaneously on the same carrier frequency

$$s(t) = d_1(t)\cos 2\pi f_c t + d_2(t)\sin 2\pi f_c t$$

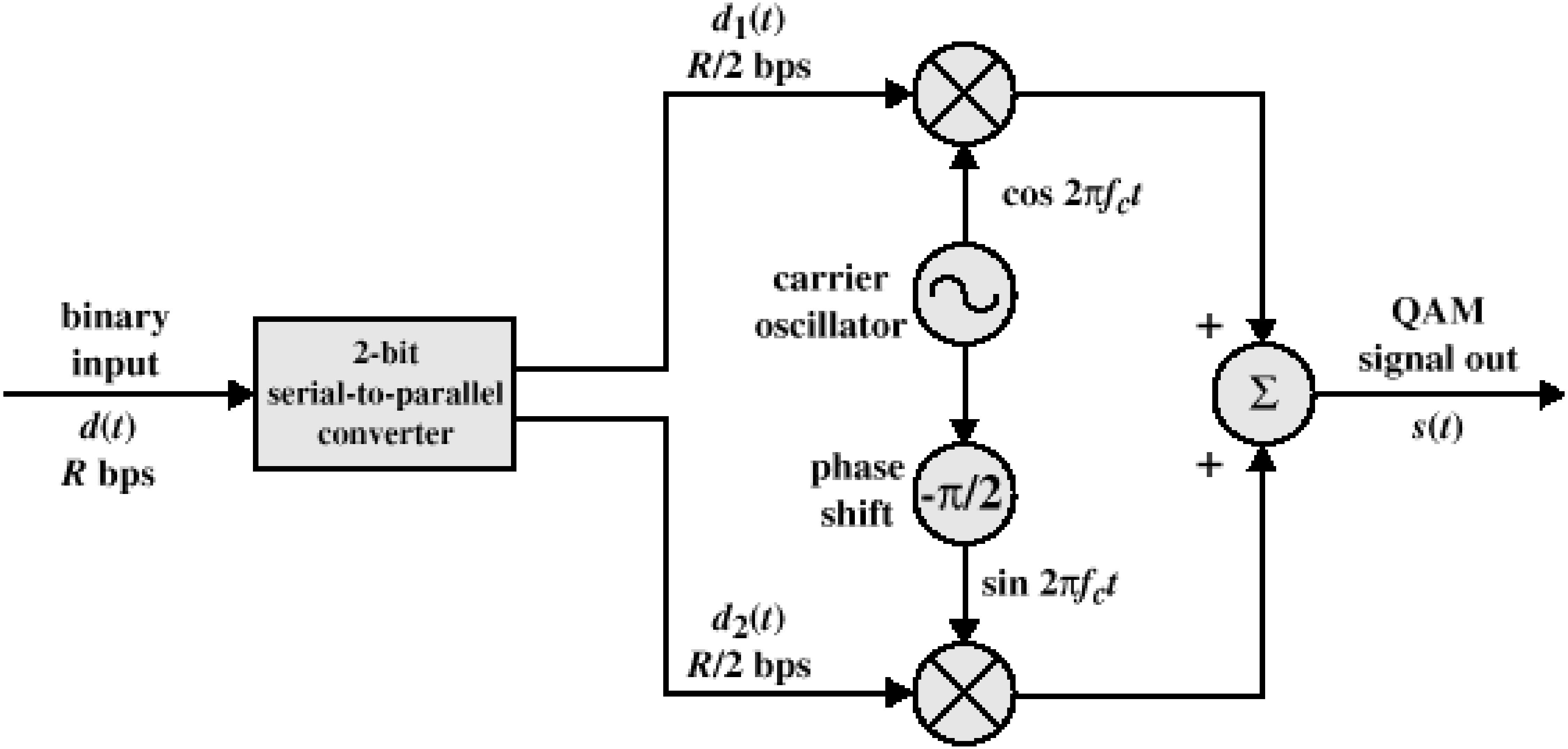
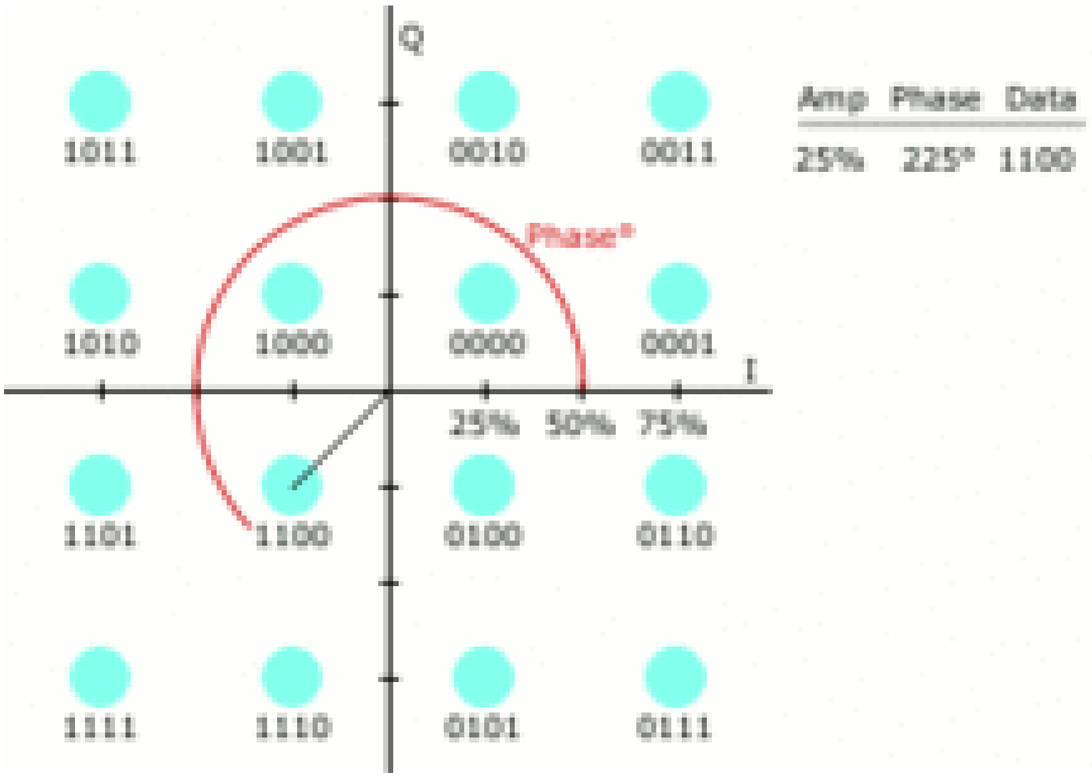


Figure 6.10 QAM Modulator

16 - QAM



Ref: https://en.wikipedia.org/wiki/Quadrature_amplitude_modulation

Summary

- Encoding—Digital->Analog
  - ASK
  - FSK
  - PSK
- Multiple Level Keying
  - One signal element denotes multiple bits
  - QAM

Next Part

- Digitization

Basic Encoding Techniques

- Analog data to digital signal
  - Pulse code modulation (PCM)
  - Delta modulation (DM)

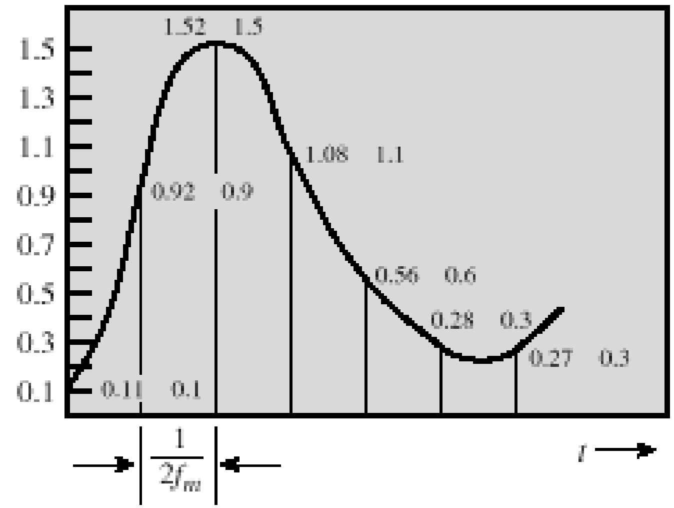
Analog Data to Digital Signal

- Once analog data have been converted to digital signals, the digital data:
  - can be transmitted using NRZ-L
  - can be encoded as a digital signal using a code other than NRZ-L
  - can be converted to an analog signal, using different filtering techniques

Pulse Code Modulation

- Based on the sampling theorem
- Each analog sample is assigned a binary code
  - Analog samples are referred to as pulse amplitude modulation (PAM) samples
- The digital signal consists of block of *n* bits, where each *n*-bit number is the amplitude of a PCM pulse

Pulse Code Modulation



(a)

Digit	Binary Equivalent	PCM waveform	Digit	Binary Equivalent	PCM waveform
0	0000		8	1000	
1	0001		9	1001	
2	0010		10	1010	
3	0011		11	1011	
4	0100		12	1100	
5	0101		13	1101	
6	0110		14	1110	
7	0111		15	1111	

Figure 6.15 Pulse-Code Modulation

Pulse Code Modulation

- By quantizing the PAM pulse, original signal is only approximated
- Leads to *quantizing noise*
- Signal-to-noise ratio for quantizing noise

$$\mathrm{SNR}_{\mathrm{dB}} = 20\log 2^{n} + 1.76\,\mathrm{dB} = 6.02n + 1.76\,\mathrm{dB}$$

- Thus, each additional bit increases SNR by 6 dB, or a factor of 4

Delta Modulation

- Analog input is approximated by staircase function
  - Moves up or down by one quantization level (δ) at each sampling interval
- The bit stream approximates derivative of analog signal (rather than amplitude)
  - 1 is generated if function goes up
  - 0 otherwise

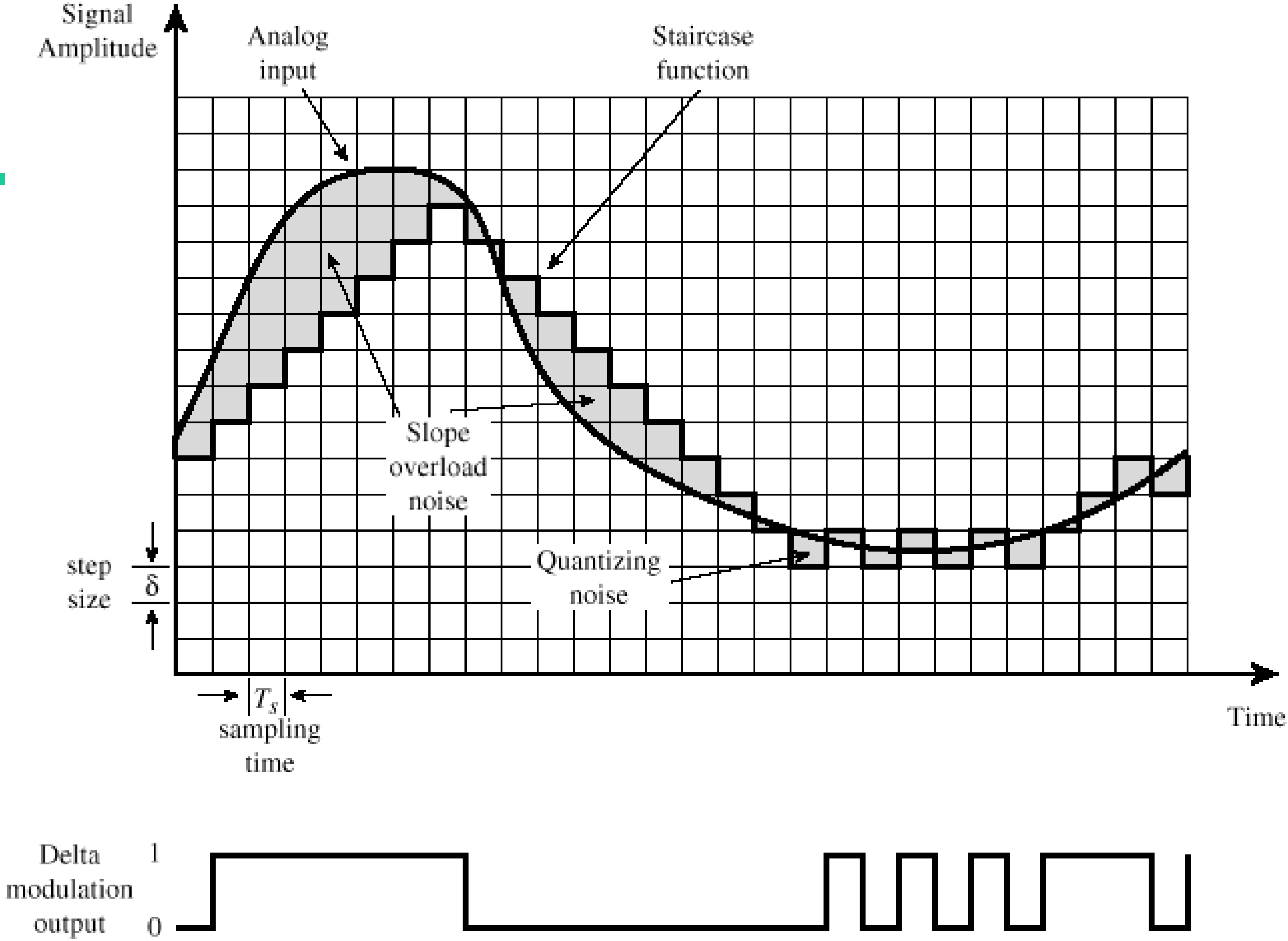


Figure 6.18 Example of Delta Modulation

Delta Modulation

- Two important parameters
  - Size of step assigned to each binary digit (δ)
  - Sampling rate
- Accuracy improved by increasing sampling rate
  - However, this increases the data rate
- Advantage of DM over PCM is the simplicity of its implementation