**POLYNOMIALS AND FACTORIZATION**

**Basic Key Concepts:**

* A symbol having a fixed numerical value is called a ‘***Constant***’.

Ex: 4, 1/3. √5, 8.76….

* A symbol which takes various numerical values is known as a ‘***Variable***’.

Ex: x, y, z, m, n…

* A constant or a variable or product of constants and variables is called a ‘***Term***”.

Ex: 4x2y, 6xy3

* A term or combination of terms obtained by any binary operation is called “***Expression***”.

Ex:1) 4x3 + 5x2 – x + 1

2) 5√x + 3

3) x + 1/x

* The sum of powers of variables of a term is called ‘*Degree of the term*’.

Ex: Degree of 7x2y is 3

* The degree of any constant is ‘0’ except zero.
* Degree of zero is not define.

**Polynomial:**

* An algebraic expression in which the variables involved have only non-negative integral powers is called ‘***Polynomial***’.

Ex: P(x) = 3x2 + 5x -2

P(t) = 3t + 7

* All polynomials are algebraic expressions. But all algebraic expressions are not polynomials.
* The degree of a polynomial is the highest degree of its variable terms.
* The general form of a nth degree polynomial is

an xn + an-1 xn-1 + an-2 xn-2 + ……… + a0

* If a1 = a2 = a3 = …… = an = 0, then the polynomial is called ‘*Zero Polynomial*’.

**Types of polynomial according to number of terms:**

|  |  |  |
| --- | --- | --- |
| **Number of non zero terms** | **Name of the polynomial** | **Example** |
| 1 | Monomial | 6x3 |
| 2 | Binomial | 7x4 + 6 |
| 3 | Trinomial | 6x2 + 5x – 2 |
| More than 3 | Multinomial | 3x4 + 4x3 + x + 1 |

* Every polynomial is multinomial
* But every multinomial need not be polynomial.
* A polynomial can have any finite number of terms.

**Types of polynomials according to degree:**

|  |  |  |  |
| --- | --- | --- | --- |
| **Degree of the polynomial** | **Name of the polynomial** | **General form (a ≠ 0)** | **Maximum No.of terms** |
| Not define | Zero polynomial | 0 | 1 |
| 0 | Constant polynomial | Any constant | 1 |
| 1 | Linear polynomial | ax + b | 2 |
| 2 | Quadratic polynomial | ax2 + bx + c | 3 |
| 3 | Cubic polynomial | ax3 + bx2 + cx + d | 4 |
| 4 | Bi quadratic polynomial | ax4 + bx3 + cx2 + dx + e | 5 |

* A linear polynomial with one variable may be monomial or binomial.
* A quadratic polynomial with one variable may be monomial or binomial or trinomial.
* A cubic polynomial with one variable can have 4 terms. Means it may be monomial, binomial, trinomial or multinomial.
* Usually, a polynomial of degree ‘n’ is called ‘**nth degree polynomial**’.

**Zero of a polynomial:**

* The ***zero of a polynomial*** p(x) is the value of ‘x’ for which p(x) = 0.
* Zero of polynomial is also called as *a root of the polynomial*.

Ex: p(x) = x2 + 4x + 4

P (-2) = (-2)2 + 4(-2) + 4

= 4 – 8 + 4

= 0.

P (-2) = 0.So -2 is zero of the polynomial.

* If the degree of a polynomial is ‘n’ then the polynomial **may** have maximum ‘n’ zeroes.
* A constant polynomial does not have a zero.
* Zero polynomial have infinitely many zeroes.
* A linear polynomial may have maximum one zero.
* A quadratic polynomial may have at most two zeroes.
* A quadratic polynomial may have only one zero. (means both zeroes are equal)

Ex: zeroes of the polynomial x² + 4x + 4 are -2, -2.

* A quadratic polynomial may have no zeroes. It means zeroes are not real numbers.

Ex: P(x) = x² + 4 has no real zeroes.

* If p + is zero of ax² + bx + c, then p - is another zero o
* Zeroes of a polynomial are related to its coefficients of the terms including the constant term.

**Division Algorithm for Polynomials**

* The division algorithm is

Dividend = Divisor × Quotient + Remainder.

* If p(x) and g(x) are two polynomials with g(x) ≠ 0 then we can find two polynomials q(x) and r(x) such that

P(x) = g(x) × q(x) + r(x)

Where either r(x) = 0 or degree of r(x) < degree of g(x).

* If degree of p(x) is ‘m’ and degree of g(x) is ‘n’ then degree of q(x) is ‘m – n’
* If g(x) is linear polynomial then degree of r(x) is zero mean r(x) is constant.
* Degree of p(x) = Degree of g(x) + Degree of r(x)

**Remainder theorem:**

* Let P(x) be any polynomial of degree greater than or equal to one and let ‘a’ be any real number. If P(x) is divided by the linear polynomial (x – a), then the remainder is P(a).
* If the dividend is P(x), then according to remainder theorem,

|  |  |
| --- | --- |
| **Divisor** | **Remainder** |
| x – a | P(a) |
| x + a | P(-a) |
| ax - b | P(b/a) |
| ax + b | P(-b/a) |

**Factor theorem:**

* If P(x) is a polynomial of degree n ≥ 1 and ‘a’ is any real number, then (x – a) is factor of P(x), if P(a) = 0. This is called ‘Factor Theorem’.
* If (x – a) is a factor of P(x), then P(a) = 0.
* If (x + a) is a factor of P(x), then P(-a) = 0
* If (ax + b) is a factor of P(x), then P(-b/a) = 0
* If (ax – b) is a factor of P(x), then P(b/a) = 0. And their converse are also true.
* If ‘k’ is a zero of the polynomial, then (x – k) is a factor of P(x). Its converse also true.
* If (x – 1) is a factor of P(x) = ax4 + bx3 + cx² + dx + e, then a + b + c + d + e = 0.
* If (x + 1) is a factor of P(x) = ax4 + bx3 + cx² + dx + e then a + c + e = b + d.
* For any value of ‘n’ the polynomial xn - yn is divisible by (x – Y).
* If ‘n’ is even number, then the polynomial xn – yn is divisible by (x + y).
* If ‘n’ is odd number, then the polynomial xn + yn is divisible by (x + y).
* For no value of ‘n’, the polynomial xn + yn is divisible by (x – y).
* When the polynomial P(x) is divided by g(x) and the remainder r(x) = o then the divisor g(x) and quotient r(x) are factors of P(x).

**Algebraic Identities:**

* If the given equation is satisfied (becomes L.H.S = R.H.S) by any value of the variable occurring in it then the equation is called ‘Algebraic Identities’.
* These identities are useful in factorization of algebraic equations.
* **Some identities:**
* (a + b)² = a² + 2ab + b²
* (a – b)² = a² - 2ab + b²
* (a + b) (a – b) = a² - b²
* (a + b)² + (a – b)² = 2(a² + b²)
* (a + b)² - (a – b)² = 4ab
* a² + b² = (a + b)² - 2ab

= (a – b)² + 2ab

* (a + b + c)² = a² + b² + c² + 2ab + 2bc + 2ca
* (a + b)3 = a3 + 3a²b + 3ab² + b3

= a3 + b3 + 3ab (a + b).

* (a – b)3 = a3 - 3a²b + 3ab² - b3

= a3 - b3 - 3ab (a - b).

* a3 + b3  = (a + b)3 – 3ab(a + b)

= (a + b) (a² - ab + b²).

* a3 - b3  = (a - b)3 + 3ab(a - b)

= (a - b) (a² + ab + b²).

* a3 + b3 + c3 – 3abc = (a + b + c) (a² + b² + c² -ab –bc –ca)
* If a + b + c = 0 then a3 + b3 + c3 = 3abc
* = a + b
* = a – b
* = a² + ab + b²
* = a² - ab + b².
* (x + a) (x + b) = x² + x(a + b) + ab.
* (ax + b) (cx + d) = acx² + x(ad + bc) + bd.