

# 7

## Individual Bank Runs and Systemic Risk

Bank panics were a recurrent phenomenon in the United States until 1934. According to Kemmerer (1910), the country experienced 21 bank panics between 1890 and 1908. Similarly, Friedman and Schwartz (1963) enumerate five bank panics between 1929 and 1933,<sup>1</sup> the most severe period in the financial history of the United States. Miron (1986) extensively documents this phenomenon, recalling its seasonal pattern prior to the founding of the Federal Reserve System (the Fed). Moreover, whereas the average yearly growth rate of real gross national product (GNP) was 3.75 during this period, Miron finds that if the years in which a bank panic occurred (or following a bank panic) are taken out of the sample,<sup>2</sup> the average growth rate becomes 6.82 percent. Similar phenomena affected England before the establishment of a Central Bank, as well as other European countries (Bordo 1990; Eichengreen and Portes 1987).

More recently, many countries have experienced banking crises, in part initiated by the general movement toward financial deregulation. For example, banking crises in East Asia, Mexico, and the Scandinavian countries started when the savings and loan crisis began to ebb away in the United States.

Therefore, it seems that without regulation, bank runs and bank panics are inherent to the nature of banking, and more specifically to the fractional reserve system. Indeed, bank deposit contracts usually allow depositors to withdraw the nominal value of their deposits on demand. As soon as a fraction of these deposits is used for financing illiquid and risky loans or investments, there is a possibility of a liquidity crisis. This chapter examines whether such deposit contracts are efficient and whether the fractional reserve system is justified despite the possibility of bank runs.

Most theoretical models have addressed this question in an aggregate framework, representing the whole banking industry by a unique entity. However, it is important to distinguish between *bank runs*, which affect an individual bank, and *bank panics*, which concern the whole banking industry,<sup>3</sup> the payment system, and the interbank market.

The conventional explanation for a bank run is that when depositors learn bad news about their bank, they fear bankruptcy and respond by withdrawing their own deposits. This bad news can be about the value of the bank's assets (fundamental bank run) or about large withdrawals (speculative bank run). Withdrawals in excess of the current expected demand for liquidity generate a negative externality for the bank experiencing the liquidity shortage because they imply an increase in the bank's probability of failure. But they can also generate an externality for the whole banking system if the agents view the failure as a symptom of difficulties occurring throughout the industry.

In such a case, a bank run may develop into a bank panic. Bagehot (1873) was one of the first to analyze how the Central Bank could prevent such contagion by playing the part of a lender of last resort (LLR). Section 7.7 is devoted to this question. The other sections are organized as follows. Section 7.1 recalls the model of liquidity insurance presented in section 2.2, and section 7.2 introduces a fractional reserve banking system and studies its stability. Sections 7.3 and 7.4 discuss bank runs. Section 7.5 is dedicated to interbank markets, and section 7.6 examines systemic risk and contagion.

## 7.1 Banking Deposits and Liquidity Insurance

This section recalls the simple model of liquidity insurance (Bryant 1980; 1981; Diamond and Dybvig 1983) introduced in chapter 2. It then discusses different institutional arrangements that can provide this liquidity insurance to individual economic agents.

### 7.1.1 A Model of Liquidity Insurance

Consider a one-good, three-dates economy in which a continuum of agents, each endowed with one unit of good at date  $t = 0$ , want to consume at dates  $t = 1$  and  $t = 2$ . These agents are ex ante identical, but they are subject to independently identically distributed (i.i.d.) liquidity shocks in the following sense: with some probability  $\pi_i$  ( $i = 1, 2$ , with  $\pi_1 + \pi_2 = 1$ ), they need to consume at date  $t = i$ . The utility of agents of type  $i = 1$  (impatient consumers) is  $u(C_1)$ , and that of agents of type  $i = 2$  (patient consumers) is  $u(C_2)$ . Ex ante all agents have the same utility:<sup>4</sup>

$$U = \pi_1 u(C_1) + \pi_2 u(C_2). \quad (7.1)$$

Assume that  $u$  is increasing and concave.

There is a storage technology that allows transfer of the good without cost from one date to the next. More important, there is also a long-term illiquid technology (with constant returns to scale): one unit invested at  $t = 0$  gives a return  $R > 1$  at

$t = 2$ . The term *illiquid* reflects the fact that investments in this long-term technology give a low return  $\ell \leq 1$  if they are liquidated prematurely at  $t = 1$ . This section determines the characteristics of the optimal (symmetric) allocation. It begins by studying two benchmarks: the autarkic situation and the allocation obtained when a financial market is opened.

### 7.1.2 Autarky

Autarky corresponds to the absence of trade between agents. Each of them independently chooses at  $t = 0$  the level  $I$  of his investment in the long-term technology and stores the rest  $(1 - I)$ . In the case of a liquidity shock at date  $t = 1$ , the investment is liquidated, yielding a consumption level

$$C_1 = \ell I + 1 - I. \quad (7.2)$$

If consumption occurs at date  $t = 2$ , the consumption level obtained is

$$C_2 = RI + 1 - I. \quad (7.3)$$

At date  $t = 0$ , consumers choose  $I$  so as to maximize  $U$  under constraints (7.2) and (7.3). Notice that since  $\ell < 1 < R$ , then  $C_1 \leq 1$  and  $C_2 \leq R$ , with at least one strict inequality. This comes from the fact that the investment decision is always ex post inefficient with a positive probability: if  $i = 1$ , the efficient decision is  $I = 0$ , whereas it is  $I = 1$  if  $i = 2$ . This inefficiency can be mitigated by opening a financial market.

### 7.1.3 The Allocation Obtained When a Financial Market Is Opened

Suppose that a bond market is opened at  $t = 1$ , whereby  $p$  units of good at  $t = 1$  are exchanged against the promise to receive one unit of good at  $t = 2$ . The consumption levels obtained by each consumer at dates 1 and 2 become

$$C_1 = pRI + 1 - I \quad (7.4)$$

and

$$C_2 = RI + \frac{1 - I}{p}. \quad (7.5)$$

In the first case, the impatient agent has sold  $RI$  bonds (instead of liquidating his long-term investment), whereas in the second case the patient agent has bought  $(1 - I)/p$  bonds at  $t = 1$  (instead of storing the good for another period). Notice that  $C_1 = pC_2$  and that the utility of the agent is increasing in  $I$  if  $pR > 1$ , and decreasing if  $pR < 1$ . Since agents choose at date  $t = 0$  the amount  $I$  they invest in the long-run technology, an interior maximum exists only when  $pR = 1$ . Therefore, the only (interior) equilibrium price of bonds is  $p = 1/R$ , and the allocation obtained

is  $C_1 = 1, C_2 = R$ , which Pareto-dominates the autarkic allocation. This is because the existence of a financial market ensures that the investment decisions are efficient. However, this market allocation is not Pareto-optimal in general, because liquidity risk is not properly allocated.

#### 7.1.4 The Optimal (Symmetric) Allocation

Agents being ex ante identical, it is legitimate to focus on the (unique) symmetric optimal allocation obtained by

$$\mathcal{P}_1 \left\{ \begin{array}{l} \max_{C_1, C_2, I} U = \pi_1 u(C_1) + \pi_2 u(C_2) \\ \text{under the constraints} \\ \pi_1 C_1 = 1 - I, \\ \pi_2 C_2 = RI. \end{array} \right. \quad (7.6)$$

$$(7.7)$$

Replacing  $C_1$  and  $C_2$  by their values given by (7.6) and (7.7),  $U$  becomes a function of the single variable  $I$ :

$$U(I) = \pi_1 u\left(\frac{1-I}{\pi_1}\right) + \pi_2 u\left(\frac{RI}{\pi_2}\right).$$

The solution  $(C_1^*, C_2^*, I^*)$  of  $\mathcal{P}_1$  is thus determined by the constraints (7.6) and (7.7) and the first-order condition:

$$-u'(C_1^*) + Ru'(C_2^*) = 0. \quad (7.8)$$

In general, the market allocation  $(C_1 = 1, C_2 = R, I = \pi_2)$  does not satisfy (7.8) except in the peculiar case in which  $u'(1) = Ru'(R)$ . An interesting situation arises when  $u'(1) > Ru'(R)$ .<sup>5</sup> In this case, impatient consumers get more in the optimal allocation than in the market equilibrium ( $C_1^* > 1$ ): they need to be insured against a liquidity shock at  $t = 1$ . The next section shows how a fractional reserve banking system can provide this liquidity insurance.

#### 7.1.5 A Fractional Reserve Banking System

The optimal allocation characterized in the previous section can be implemented by a fractional reserve banking system in which banks collect the endowments of consumers (deposits) and invest a fraction of them in long-term investments while offering depositors the possibility of withdrawal on demand. A deposit contract  $(C_1, C_2)$  specifies the amounts  $C_1$  and  $C_2$  that can be withdrawn, respectively, at dates  $t = 1, 2$  for a unit deposit at  $t = 0$ . Competition between banks leads them to offer the optimal feasible deposit contract  $(C_1^*, C_2^*)$  characterized earlier. A crucial question is whether this fractional reserve system is stable, that is, whether the banks will be

able to fulfill their contractual obligations. This depends very much on the behavior of patient consumers, which in turn depends on their anticipations about the safety of their bank.

Consider the case of a patient consumer who anticipates that the bank will be able to fulfill its obligations. The consumer has the choice between withdrawing  $C_2^*$  at date  $t = 2$  or withdrawing  $C_1^*$  at date  $t = 1$  and storing it until  $t = 2$ . Since  $R > 1$  and  $u'$  decreases, equation (7.8) shows that  $C_2^* > C_1^*$ .

This means that if the patient consumer trusts her bank, she will always prefer to withdraw at  $t = 2$ . By the law of large numbers, the proportion of withdrawals at  $t = 1$  will be exactly  $\pi_1$ . This determines the amount  $\pi_1 C_1^*$  of liquid reserves that the bank has to make in order to avoid premature liquidation. With these reserves, the bank will be solvent with probability 1, and the consumers' expectations will be fulfilled. Thus there is an equilibrium of the banking sector that implements the optimal allocation. However, another equilibrium also exists, which leads to an inefficient allocation.

Suppose the patient consumer anticipates that all other patient consumers want to withdraw at  $t = 1$ . The bank will be forced to liquidate its long-term investments, yielding a total value of assets  $\pi_1 C_1^* + (1 - \pi_1 C_1^*)\ell$ , which is less than 1 and thus less than the total value of its liabilities ( $C_1^*$ ). In the absence of other institutional arrangements, the bank will fail and nothing will be left at  $t = 2$ . Anticipating this, the optimal strategy for a patient consumer is to withdraw at  $t = 1$ . Thus the initial expectations of the consumer are self-fulfilling. In other words, there is a second Nash equilibrium<sup>6</sup> of the withdrawal game in which all consumers withdraw at  $t = 1$  and the bank is liquidated: this is what is called an inefficient bank run.

**Result 7.1** In a fractional reserve banking system in which investment returns are high enough ( $R > 1$ ), two possible situations may arise at equilibrium:

- An efficient allocation, when patient depositors trust the bank and withdraw only at  $t = 2$
- An inefficient bank run, when all depositors withdraw at  $t = 1$

Change in expectations was supposed to be the main channel of contagion in the nineteenth century and the beginning of the twentieth century. This channel seems also important today in countries that are vulnerable to currency crises. This type of crisis closely resembles bank runs because they occur as self-fulfilling prophecies. The bankruptcy of a first bank makes depositors update their beliefs concerning the other banks' solvency. Under the updated beliefs, depositors may prefer to run the second bank, which goes bankrupt. It does not matter whether the beliefs concern the second bank's solvency (its return on assets  $R$ , in the notation we have been using) or the proportion of agents that will run the bank. In both cases, patient depositors are

perfectly rational in running the bank. As a consequence, contagion through change in expectations is easily modeled using the Bryant-Diamond-Dybvig framework and does not require extended development here.

## 7.2 The Stability of the Fractional Reserve System and Alternative Institutional Arrangements

### 7.2.1 The Causes of Instability

As was just shown, the fractional reserve banking system leads to an optimal allocation only if patient consumers do not withdraw early. There are two reasons they might want to withdraw.

If the relative return of date 2 deposits with respect to date 1 deposits ( $C_2^*/C_1^* - 1$ ) is less than what patient consumers can obtain elsewhere, either by storage (as in the present model) or more generally by reinvesting in financial markets, as in Von Thadden (1996), they will prefer to withdraw early. If consumers' types were observable, this could be avoided by forbidding patient consumers to withdraw early. In practice, however, liquidity needs are not publicly observable, and incentive compatibility constraints must be introduced. In a continuous time extension of the Bryant-Diamond-Dybvig model, Von Thadden (1996) shows that these incentive compatibility constraints are *always* binding somewhere, which severely limits the provision of liquidity insurance that can be obtained through a fractional reserve banking system.

The literature has paid more attention to a second cause of instability, arising from the fact that the game between depositors has two equilibria, one efficient and one inefficient. The inefficient equilibrium arises only when there is a coordination failure among depositors, coming from a lack of confidence in their bank. In any case, theoreticians dislike multiple equilibria, and they have tried to offer selection devices. For instance, Anderlini (1989) suggests a recourse to exogenous uncertainty ("sunspots") to determine which equilibrium will prevail. This might explain sudden confidence crises in real-world banking systems. On the other hand, Postlewaite and Vives (1987) suggest that some agents may observe signals that give them some information about the likelihood of a bank run (see problem 7.8.2); these are "information-based" bank runs. The following sections discuss several institutional arrangements that have been proposed to solve the instability problem of the fractional reserve system.

### 7.2.2 A First Remedy for Instability: Narrow Banking

A natural way to prevent the instability of the banking system is to require that under any possible circumstance all banks can fulfill their contractual obligations.

This idea given rise to the term *narrow banking*, which refers to a set of regulatory constraints on banks' investment opportunities that would make them safe in any possible event. But this is open to different interpretations, leading to three alternative views of narrow banking: (1) a bank with enough liquidity to guarantee repayment to all depositors even in case of bank run, (2) a bank that obtains enough liquidity after liquidation of its long-run technology to face a bank run, and (3) a bank that obtains enough liquidity after the securitization of its long-run technology to cope with a bank run. The discussion shows why, in terms of risk and resource allocation, narrow banking leads to an inefficient allocation for each of the three interpretations.

In the first version of narrow banking, the bank is required to have a reserve ratio of 100 percent: liquid reserves  $(1 - I)$  at least equal to  $C_1$ , the maximum possible amount of withdrawals at date 1.<sup>7</sup> In practice, the maturity structure of banks' assets should be perfectly matched with that of their liabilities. In the present context, this means that the deposit contract  $(C_1, C_2)$  offered by the bank must satisfy  $C_1 \leq 1 - I$ , and  $C_2 \leq RI$ .

The best deposit contract  $(C_1, C_2)$  that can offer such a narrow bank is defined by

$$\mathcal{P}_2 \begin{cases} \max_{I, C_1, C_2} U = \pi_1 u(C_1) + \pi_2 u(C_2) \\ \text{under the constraints} \\ C_1 \leq 1 - I, \quad C_2 \leq RI. \end{cases}$$

As Wallace (1988; 1996) points out, the solution of  $\mathcal{P}_2$  is dominated by that of  $\mathcal{P}_1$ . In fact, it is even dominated by the autarkic situation, which will be obtained if the second, milder version of the narrow banking proposal is adopted, in which the banks are allowed to liquidate some of their assets in order to satisfy unexpected withdrawals. If the bank has offered the deposit contract  $(C_1, C_2)$ , the amount  $I$  invested in the long-term technology must now be such that

$$C_1 \leq \ell I + (1 - I)$$

(the liquidation value of the bank's assets at  $t = 1$  covers the maximum possible amount of withdrawals). Similarly, at  $t = 2$ ,

$$C_2 \leq RI + 1 - I,$$

which means a return to the autarkic situation.

Finally, the more modern, weaker version of narrow banking suggests replacing banks by money market funds that use the deposits they collect to buy (riskless) financial securities (Gorton and Pennacchi 1993). Alternatively, banks would be allowed to securitize their long-term assets in order to satisfy the withdrawals of depositors. It is easy to see that the best deposit contract that can be offered by such a money

market fund is the market equilibrium characterized in section 7.1.3. Therefore, even with this last version involving money market funds (or monetary service companies), the narrow banking proposal is antagonistic to the efficient provision of liquidity insurance. It remains to be seen whether this efficient provision can be obtained under other institutional arrangements that would guarantee the stability of the banking system.

### 7.2.3 Regulatory Responses: Suspension of Convertibility or Deposit Insurance

If liquidity shocks are perfectly diversifiable, and if the proportion<sup>8</sup>  $\pi_1$  of impatient consumers is known, one can get rid of the coordination problem that gives rise to inefficient bank runs. For instance, the bank could announce that it will not serve more than  $\pi_1 C_1^*$  withdrawals at date  $t = 1$ . After this threshold, convertibility is suspended. Patient consumers therefore know that the bank will be able to satisfy its engagements at date 2, and thus they have no interest in withdrawing at date 1. The threat of a bank run disappears.<sup>9</sup>

An equivalent way to get rid of inefficient bank runs is to insure depositors. In this case, even if the bank is not able to fulfill its obligations, depositors receive the full value of their deposits. The difference is paid by a new institution, the deposit insurance system, financed by insurance premiums paid *ex ante* by the bank (or by taxes, if the system is publicly run). In the present simple framework, the existence of deposit insurance is enough to get rid of bank failures.<sup>10</sup>

The equivalence between these two systems breaks down as soon as one allows for a variability of the proportion  $\pi_i$ . In that case, the equilibrium without bank runs is characterized by a random amount of date 1 withdrawals. But since the level of investment has been already chosen, two situations may arise. First, if the realized value of  $\pi_1$  is too high, the investment in the long-run technology will have to be liquidated at a loss, and the bank will not be able to meet its obligations at date 2. If  $\pi_1$  is too low, the level of investment is also too low, and time 2 depositors will again not obtain the promised return. More generally, any type of regulation that is intended to cope with random withdrawals must take into account the fact that time 2 returns are contingent on time 1 withdrawals. In such a case, it is inefficient to set a critical level of liquidity demand (type 1 deposits) that triggers the suspension of convertibility. If this level is  $\hat{f}$ , a realization of  $\pi_1$  with  $\pi_1 > \hat{f}$  implies that type 1 agents will be rationed. Conversely, a realization with  $\pi_1 < \hat{f}$  implies that a bank run may still develop because type 2 agents are actually too numerous in relation to the promised return, given the amount the bank has invested. Thus, even if it is true that the suspension of convertibility will eliminate bank runs, it will do so at a cost because the deposit contracts will then be less efficient in terms of risk sharing.

Deposit insurance, however, will allow for a contingent allocation. For instance, if the deposit insurance system is publicly run and financed by taxes, the government



can levy a tax based on the realization of  $\pi_1$ . If the tax rate is the same across agents, it can be interpreted as resulting from the adjustment of the time 1 price of the good (inflationary tax). Of course, as pointed out by Wallace (1988) this is only possible if the potential taxpayers have not already consumed the good.

### 7.2.4 Jacklin's Proposal: Equity versus Deposits

Since a demand deposit economy achieves better risk sharing than a market economy but is vulnerable to bank runs, it is interesting to investigate whether other contractual arrangements achieve the same allocations without being prone to bank runs. Jacklin (1987) has shown that sometimes equity can do as well as deposit contracts.

Instead of the mutual bank of the previous section, consider a bank entirely financed by equity, which announces it will distribute a dividend  $d$  at date 1. Accordingly, it keeps an amount of reserves equal to  $d$  and invests  $(1 - d)$  in long-run technology. The shares of the bank are traded during period 1, once agents know their types and after the dividend is paid. Each share gives a right to  $R(1 - d)$  consumption units at time 2. The equilibrium price  $p$  of the ex-dividend share in terms of time 1 consumption good depends on  $d$ . Therefore, changing the value of  $d$  affects the period 1 utilities of the agents, so this mechanism will also allow for some improvement of the ex ante expected utility with respect to the market economy.

For a dividend  $d$  and a price  $p$  for the ex-dividend share, the behavior of each type of shareholder of the bank can be determined. Type 1 agents (impatient consumers) receive their dividends and sell their shares,

$$C_1 = d + p, \quad (7.9)$$

whereas type 2 agents (patient consumers) use their dividends  $d$  to buy  $d/p$  new shares, which gives them at date  $t = 2$ ,

$$C_2 = \left(1 + \frac{d}{p}\right) R(1 - d). \quad (7.10)$$

The price  $p$  is determined by equality of supply and demand of shares:

$$\pi_1 = \pi_2 \frac{d}{p}, \quad (7.11)$$

which gives

$$p = \frac{\pi_2 d}{\pi_1}$$

and

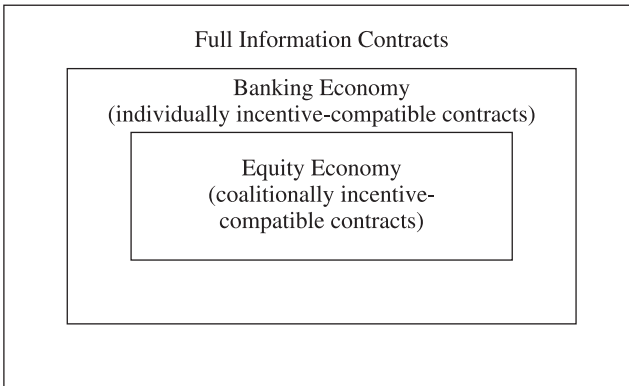
$$C_1 = \frac{d}{\pi_1}, \quad C_2 = \frac{R(1-d)}{\pi_2}. \quad (7.12)$$

Finally, the level of  $d$  is determined ex ante (at  $t = 0$ ) by stockholders,<sup>11</sup> who unanimously choose it to maximize  $U$  under the constraints (7.12). Eliminating  $d$  between these constraints yields the same budget constraint as for deposit contracts:

$$\pi_1 C_1 + \pi_2 \frac{C_2}{R} = 1.$$

Therefore, in the simple specification adopted here, the efficient allocation ( $C_1^*, C_2^*$ ) can also be obtained by participation contracts in which consumers are shareholders of the bank instead of depositors. The advantage of these participation contracts is that they are immune to bank runs. However, for more general specifications of agent utilities, Jacklin shows that equity contracts can be dominated by efficient deposit contracts, yielding a trade-off between stability and efficiency because deposit contracts can be destabilized by bank runs. Figure 7.1 illustrates this point. If the efficient allocation happens to be implementable in an equity economy, the three contractual frameworks are equivalent. This is the Diamond-Dybvig case. Still, if the efficient allocation is outside the banking economy area, the banking economy will typically perform better.

The reason for this domination is that equity contracts are necessarily *coalitionally incentive-compatible* in the sense that they are immune to early withdrawals (deviations) of coalitions of patient consumers, whereas deposit contracts are only *individually incentive-compatible*. If agents are allowed to trade their deposit contracts, these contracts become equivalent to equity contracts. This is the case because a time  $t = 2$



**Figure 7.1**  
Different sets of contracts.

deposit has then a market price; in equilibrium, agents should be indifferent between selling their time  $t = 2$  deposits in the market or cashing them at the bank.

### 7.3 Bank Runs and Renegotiation

Diamond and Rajan (2001) provide an alternative justification for banking based on the disciplinary role of bank runs. Their argument is based on the fact that contracting parties have the power to renegotiate the original agreed rules. The same asset may therefore be worth more to an entrepreneur developing a project than to its main financier, and the value of the asset to its main financier with experience in dealing with this kind of assets is higher than to other financiers. In this context, the mechanism of bank runs may limit the renegotiation power of a financier, since the threat to diminish the amount of repayment to depositors could trigger a bank run, whereas if the financier is dealing with a unique depositor the withdrawal of deposits is not a credible threat, and the financier can easily renegotiate a lower payment. In other words, the possibility of a bank run, which is usually characteristic of banks' fragility, here gives more bargaining power to depositors, and therefore may lead to a higher level of financing.

#### 7.3.1 A Simple Model

Consider an economy with an excess of savings, so that the opportunity cost of funds is equal to 1. Entrepreneurs have projects but no cash, so they will borrow from agents that we call financiers. Still, as in Hart and Moore (1994), entrepreneurs cannot commit to the project in the future because they cannot alienate their human capital. As a consequence, the entrepreneurs may threaten to quit and bargain a better deal on their loan contract.

Formally, assume an entrepreneur invests  $Iy$  ( $I < 1$ ) in the project. This allows it to obtain a riskless cash flow  $y$  at date  $t = 1$ . A financier is willing to provide funds in exchange for a repayment  $R$ . If the financier liquidates the project before time  $t = 1$ , it gets  $V_1$ .

A financier will be a specialized financial intermediary in the sense that it has experience in liquidating the type of firm it is lending to. If another external financier should replace the first one in the contract, it would only obtain a liquidation value for the assets equal to  $\alpha V_1$ , where  $0 < \alpha < 1$ . This means that the market is not ex post competitive, either because of a lack of specialized financiers, an interpretation that would make financiers akin to providers of venture capital, or because of the fact that there is a relationship with the borrower that is valuable to both parties (see section 3.6). Both interpretations, which are not incompatible, provide a rationale for financial intermediaries.

### 7.3.2 Pledgeable and Nonpledgeable Cash Flows

Assume that the borrower has all the bargaining power. As in Hart and Moore (1994), any contractual repayment higher than  $V_1$  will be reduced to that level, so renegotiation-proof contracts will be those satisfying  $R \leq V_1$ .

The implication is that even if the entrepreneur has a high future expected cash flow  $y$ , it will not be able to borrow against this cash flow but only against the liquidation value of its assets, and the maximum amount of funds the entrepreneur is able to borrow is  $V_1$ .

Assume only the uninformed lender has funds. Two possible financial arrangements are then possible: (1) The entrepreneur borrows directly from the uninformed lender, and (2) the entrepreneur borrows from the financial intermediary, which in turn borrows from the uninformed lender. (This can be interpreted as a loan sale.)

Assume the entrepreneur has all the bargaining power when negotiating with another party and that the financial intermediary has all the bargaining power when negotiating with the uninformed lender. Then the entrepreneur can only fund its project up to the level of  $\alpha V_1$ . A larger amount would not be renegotiation-proof.<sup>12</sup>

In case 1, where the entrepreneur borrows directly from the uninformed lender, the Hart and Moore argument applies, and the maximum value of the repayment is  $\alpha V_1$ .

In case 2 the entrepreneur could, in principle, borrow  $V_1$  because this is the amount it can extract from the entrepreneur. For this to be possible, the financial intermediary has to be able to raise  $V_1$  from the uninformed lender. Still, the same renegotiation problem that limits the funds available to the entrepreneur will now limit the funds to the financial intermediary. Indeed, any repayment to the uninformed lender larger than  $\alpha V_1$  will be renegotiated, and the uninformed lender's best option will be to accept the offer. In anticipation of this, the uninformed lender will only lend  $\alpha V_1$ , and therefore both cases lead to financing the project only to the level of  $\alpha V_1$ .

The existence of the financial intermediary, which could theoretically provide funds up to the amount  $V_1$ , is of no help because it is itself unable to raise (credibly) the required funds.

### 7.3.3 Bank Runs as a Discipline Device

Consider the case where the financial institution chooses to be funded not by a unique uninformed lender but by a demand deposit structure, thus committing to serve all deposits on a first-come, first-served basis. It is then possible for the bank to commit to a total repayment  $V_1$ .

Any attempt by the bank to threaten to withdraw its specific collection skills and to renegotiate down the total repayment, say, to  $\alpha V_1$ , will then trigger a bank run.

To see this, assume the financial intermediary raises an amount equal to  $V_1$  by offering a deposit contract to two external financiers. This contract gives depositors the right to withdraw at any time an amount

$$\frac{d}{2} = \frac{V_1}{2},$$

and has the property of sequential service, first-come, first-served.

In this case, any attempt to renegotiate the payment on deposit by decreasing it by  $\varepsilon$  will leave the two depositors facing the following game:

	Withdraw	Wait
Withdraw	$\left(\frac{\alpha V_1}{2}, \frac{\alpha V_1}{2}\right)$	$\left(\frac{d}{2}, \alpha V_1 - \frac{d}{2} - \varepsilon\right)$
Wait	$\left(\alpha V_1 - \frac{d}{2} - \varepsilon, \frac{d}{2}\right)$	$\left(\frac{d}{2} - \varepsilon, \frac{d}{2} - \varepsilon\right)$

The equilibrium outcome of the game is (withdraw, withdraw), that is, a bank run. The depositors will not gain anything by running the bank. Indeed, once they are in possession of the loan, the entrepreneur will make them a take-it-or-leave-it offer, and they will obtain  $V_1/2$ . The outcome is then the same as for the case of a loan sale: the entrepreneurs' rent increases. The loser will be the bank, which will go bankrupt.

Thus, a demand deposit structure allows the bank to commit to a larger nonrenegotiable repayment. This implies that the bank is able to act as a delegated monitor, committing in a credible way not to renegotiate the promised repayment to depositors. Thus the demand deposit structure of contracts gives the financial intermediary the ability to borrow additional funds, which in turn can be passed down to the entrepreneur. In other words, it is only because of its deposit structure that the services of the relationship lender can be used by the entrepreneur.

To summarize, in this model, the role of the bank is to "tie human capital to assets" (Diamond and Rajan 2000).

### 7.3.4 The Role of Capital

Under the previous assumptions, it is always optimal for a bank to hold zero capital. Still, when the liquidation value becomes random, the bank's structure of capital becomes a relevant issue. The randomness of  $V_1$  could lead to runs even in the absence of opportunistic behavior by the banker. This justifies a role for bank capital, characterized by Diamond and Rajan (2000) as a softer claim that can be renegotiated in bad times.

In order to summarize the main result of Diamond and Rajan (2000), we assume that when the bank managers renegotiate with the equity holders, they have all the bargaining power. Also, we assume that in the event of a bank run, depositors cannot employ the services of the bank managers for a fee but simply get the value of the liquidated asset,  $\alpha V$ .

Assume  $V$  can take two values  $V_H$  and  $V_L$  ( $V_H > V_L$ ) with probabilities  $p$  and  $1 - p$ , respectively. Assume also  $V_L < \alpha V_H$ .<sup>13</sup>

In the case  $d = V_L$ , if  $V_L$  occurs, depositors are paid, and neither the bank managers nor the equity holders obtain any payment. If  $V_H$  occurs, then the bank managers threaten not to extract from the entrepreneurs all the value  $V_H$  but only  $\alpha V_H$ . Since the bank managers have all the bargaining power, this leaves the equity holders with  $\alpha V_H - V_L$  and the bank managers with  $V_H(1 - \alpha)$ . The maximum amount of funding the project will obtain is therefore  $p(\alpha V_H - V_L) + V_L$ , which is strictly lower than  $E(V)$ . The difference between the two is due to the cost of renegotiation when  $H$  occurs. Equity holders will enter the project, but renegotiation implies that there is ex ante a cost of equity finance.

In the case  $d = V_H$ , with a probability  $1 - p$  that the bank is bankrupt, there is a bank run and there is no role for equity finance because equity holders always receive a zero return. Nevertheless, the choice of  $d = V_L$  or  $d = V_H$  is endogenous, so a high level of deposits  $d = V_H$  might be chosen whenever this allows raising a higher amount of funds. For  $d = V_H$ , the maximum amount raised is  $pV_H + (1 - p)\alpha V_L$ , and this could be larger than the amount obtained for  $d = V_L$ .

To summarize, as shown by comparing the cases for  $d = V_L$  and  $d = V_H$ , capital allows avoiding bankruptcy, that is, having to pay  $V_H$  when the value of assets are  $V_L$ . But this comes at a cost—the cost of bank managers extracting rents in case of success—a cost that investors will take into account ex ante.

## 7.4 Efficient Bank Runs

In the models examined up to now, the investment returns are certain. This implies that bank runs have a purely speculative origin. Yet it is reasonable to think that leakage of bad performance of bank loan portfolios should also trigger bank runs. Empirical evidence on bank runs seems to point in that direction. Thus bank runs can also have a fundamental origin, motivated by the expectation of poor performance of the banks.

There is no need to model explicitly the chain of events that trigger a fundamental bank run. When agents perceive a bad signal on the bank return during period 1, they may rationally decide to withdraw early. As Jacklin and Bhattacharya (1988) point out, the information on future returns modifies the relevant incentive-compatible constraint, and therefore agents of type 2 may prefer to withdraw at  $t = 1$ . Fun-

damental bank runs can thus provide an efficient mechanism for closing down inefficient banks. However, in practice it may be difficult to distinguish *ex ante* a fundamental bank run from a speculative one.

Gorton (1985) suggests a simple model in which agents obtain at  $t = 1$  some information about the expected return on the bank's assets at  $t = 2$ . If the expected return on deposits is lower than the expected return on currency, there is a fundamental bank run. This simple structure enables Gorton to provide a rationale for the suspension of convertibility when there is asymmetric information on the date 2 return on deposits. To do so, it suffices to assume that the banks are able, by paying a verification cost, to transmit the true value of the expected return to depositors. If there is a bank run on a solvent bank, the bank is able to suspend convertibility and pay the verification cost, which stops the run (see problem 7.8.3 for a simplified version of Gorton's model). This explains "a curious aspect of suspension . . . that despite its explicit illegality, neither banks, depositors nor the courts opposed it at any time" (Gorton 1985, 177).

Chari and Jagannathan (1988) consider a model close to the Diamond-Dybvig (1983) model, in which they introduce a random return on investment that may be observed by some of the type 2 agents. If the signal that the agents receive indicates a poor performance, this makes them prefer the type 1 deposit, which is less sensitive to the variations in period 2 returns. The agents observe the total amount of withdrawals and use this information to decide their own action (withdraw or wait). Since the proportion of type 1 agents is not observable, it is impossible for an uninformed type 2 depositor to distinguish if the origin of the large withdrawal he observes comes from informed type 2 agents or simply from a large proportion of type 1 agents. The rational expectations equilibrium that is obtained therefore combines fundamental bank runs (the ones justified by the poor performance of the bank) and speculative bank runs, which develop as in the Diamond-Dybvig model but are here triggered by the fear of poor performance, anticipated by informed depositors.<sup>14</sup> Notice that this model assumes that the management keeps the bank open even if this implies a decrease in its net wealth. If management has incentives to maximize the bank's total value, then fundamental bank runs will never occur, and speculative bank runs will not occur either. However, under limited liability, the bank management may have an incentive to keep the bank running if this increases shareholder value, even if the total value of the bank is decreased.<sup>15</sup> Provided that the bank is worth more dead than alive, bank runs are efficient because they correct, at least partly, the incentives of management to forbear. Closing down the bank will be efficient whenever, given a signal  $S$  on the future return for the long-run technology, the liquidation value is larger than the expected conditional return:

$$\ell > E(\tilde{R}|S).$$

The fact that deposits are demandable is therefore a key characteristic that would be desirable even if all consumers were of equal type ( $\pi_1 = 1$ ). Thus in the Chari and Jaggannathan model we have a disciplining role for demandable debt, a point subsequently developed and emphasized by Calomiris and Kahn (1991), Qi (1998), and Diamond and Rajan (2001).

Three other contributions extend the Diamond-Dybvig framework to a context of random returns on the bank's assets. They do not focus directly on bank runs but rather on the risk sharing provided by demand deposit contracts in this framework.

Jacklin and Bhattacharya (1988) address the question of the relative performance of equity versus demand deposit in banks' financing, in a context where some type 2 agents are informed on the bank's future return. In an equity economy, equilibrium prices are fully revealing; in a demand deposit equilibrium, with suspension of convertibility, there is rationing on type 1 deposits, so these deposits are shared between type 1 and informed type 2 agents. Comparison of the relative performances for specific values of the model's parameters shows that for a lower dispersion of returns, demand deposits perform better, whereas for a large dispersion, equity financing is preferred. Still, the demand deposit contract can be improved if it is required to remain incentive-compatible after type 2 agents become informed (Alonso 1991).

Gorton and Pennacchi (1990) also consider that some of the type 2 agents are informed,<sup>16</sup> but the agents' behavior is not competitive, so the prices of an equity economy are not fully revealing. Informed traders benefit from trading in the equity market. Demand deposit contracts emerge then because they are riskless, so their value cannot be affected by informed trading. Gorton and Pennacchi establish that in equilibrium uninformed traders invest in deposits and informed traders invest in equity of the financial intermediary, so there is no other market for equity. In that way, the authors elaborate on Jacklin's (1987) contention that equity could perform as well as demand deposit contracts.

Allen and Gale (1998) consider the efficiency properties of bank runs in the case where the long-run technology yields a random return. They argue that if liquidation of the long-run technology is impossible ( $\ell = 0$ ), then bank runs can be best information constrained efficient, thus allowing for optimal risk sharing between early and late withdrawing depositors. This is the case because, when returns on the long-run technology happen to be low, patient consumers will have an incentive to run the bank. By so doing, the remaining patient consumers withdrawing at time  $t = 2$  will have a larger consumption, and this will go on until the equality  $C_1 = C_2$  is reached and the incentives to run the bank disappear. Thus, as Allen and Gale argue, the problem is not bank runs per se but rather the cost of liquidating the long-run technology. When the cost of liquidation is taken into account, runs are not limited to a fraction of patient customers, and the best cannot be reached any longer.



### 7.5 Interbank Markets and the Management of Idiosyncratic Liquidity Shocks

Until now the discussion has maintained the convenient fiction that there is only one collective mutual bank. This section drops this assumption and focuses on the problems that arise precisely because of a multiplicity of banks.<sup>17</sup> These problems are again based on the coexistence of demand deposits on the liability side with non-marketable loans on the asset side.

#### 7.5.1 The Model of Bhattacharya and Gale

The Bhattacharya-Gale (1987) model is a variant of Diamond-Dybvig in which it is assumed that  $\ell = 0$  (liquidation is impossible) and the consumption good cannot be stored, so there are no bank runs. The novelty is that there are now several banks confronted with i.i.d. liquidity shocks in the sense that their proportion of patient consumers (who withdraw at date 1) can be  $\pi_L$  or  $\pi_H$  (with  $\pi_L < \pi_H$ ), with respective probabilities  $p_L$  and  $p_H$ . Assume there are a large number of banks, so liquidity shocks of banks are completely diversifiable; the proportion of banks with few ( $\pi_L$ ) early withdrawals is exactly  $p_L$ .

In an autarkic situation (absence of trade between the banks), each bank is completely restricted by its ex ante choice of investment  $I$ . The bank can offer only contingent deposit contracts

$$C_1(\pi) = \frac{1 - I}{\pi}, \quad C_2(\pi) = \frac{IR}{1 - \pi},$$

where  $\pi$  can be  $\pi_L$  or  $\pi_H$ . Therefore, depositors bear the liquidity risk of their bank. This risk can be eliminated by opening an interbank market, which can decentralize the optimal allocation, obtained by solving

$$\left\{ \begin{array}{l} \max_{I, C_1^k, C_2^k} \sum_{k=L, H} p_k [\pi_k u(C_1^k) + (1 - \pi_k) u(C_2^k)] \\ \sum_{k=L, H} p_k \pi_k C_1^k = 1 - I, \\ \sum_{k=L, H} p_k (1 - \pi_k) C_2^k = RI, \end{array} \right.$$

where  $(C_1^k, C_2^k)$  is the deposit contract offered by a bank of type  $k$ ,  $k = L, H$ .

The solution to this problem satisfies

$$C_1^k \equiv C_1^* = \frac{1 - I^*}{\pi_a}, \quad C_2^k \equiv C_2^* = \frac{RI^*}{1 - \pi_a} \quad (k = L, H), \quad (7.13)$$

where  $\pi_a = p_L\pi_L + p_H\pi_H$  is the average proportion of early withdrawals across all banks. Equations (7.13) show that consumers are now completely insured against the liquidity risk faced by their bank:  $C_1^*$  and  $C_2^*$  are independent of  $k$ .

### 7.5.2 The Role of the Interbank Market

The implementation of this allocation by the interbank market is realized as follows. Banks of type  $k = L$  face fewer early withdrawals than the average; therefore they have excess liquidity  $M_L = 1 - I^* - \pi_L C_1^*$ . On the contrary, banks of type  $k = H$  have liquidity needs  $M_H = \pi_H C_1^* - (1 - I^*)$ . Conditions (7.13) imply that on aggregate, supply and demand of liquidity are perfectly matched:

$$p_L M_L = p_H M_H.$$

At date 2, banks of type  $k = H$  will have excess liquidities, which they will use to repay the interbank loan they obtained at date 1. The interest rate  $r$  on the interbank market will thus be determined by equaling this repayment with  $(1 + r)M_H$ , where  $M_H$  is the amount of the loan obtained at date 1:

$$(1 + r)M_H = RI^* - (1 - \pi_H)C_2^*.$$

Computations yield the following:

$$1 + r = \left( \frac{\pi_a}{1 - \pi_a} \right) \left( \frac{I^*}{1 - I^*} \right) R. \quad (7.14)$$

### 7.5.3 The Case of Unobservable Liquidity Shocks

Bhattacharya and Gale (1987) also study the more difficult case in which the liquidity shock (the type  $k$  of the bank) and the investment of the bank in the illiquid technology are not publicly observed. In that case, the best allocation derived earlier will typically not be implementable. Suppose, for instance, that the equilibrium interest rate  $r$  on the interbank market (defined by (7.14)) is smaller than  $R - 1$ . Then all banks will have an interest in declaring that they are of type  $H$  because this will entitle them to an interbank loan that they will use for investing in the illiquid technology, obtaining a positive excess return  $R - (1 + r)$ . To prevent this, the second-best solution will involve imperfect insurance of depositors (in the sense that  $C_1^L < C_1^H$  and  $C_2^L > C_2^H$ ) and overinvestment with respect to the best solution  $I^*$ . This case occurs when liquidity shocks are small, which can be shown to imply that the interest rate  $r$  defined by (7.14) is smaller than  $R - 1$ .

Symmetrically, when liquidity shocks are large, then  $1 + r > R$ , and the second-best solution involves, on the contrary, underinvestment and reverse ordering of the consumption profiles ( $C_1^L > C_1^H$  and  $C_2^L < C_2^H$ ).<sup>18</sup>

Freixas and Holthausen (2005) extend the analysis of the interbank market to cross-country bank lending. They assume that the main barrier to building an integrated international interbank market is the presence of asymmetric information between different countries, which may prevail in spite of monetary integration or successful currency pegging. In order to address this issue, they consider a Bryant-Diamond-Dybvig model and find not only that an equilibrium with integrated markets need not always exist but also that when it does, the integrated equilibrium may coexist with one of segmentation of the interbank market.

## 7.6 Systemic Risk and Contagion

Systemic risk is usually defined as any risk that may affect the financial system as a whole (de Bandt and Hartmann 2002). It may originate either in the banking industry or in the financial markets. We focus here on the mechanisms that trigger a systemic banking crisis.

A systemic crisis may develop either as a result of a macroeconomic shock or as a result of contagion. The models we examined in section 7.2 allow us to understand why an aggregated liquidity shock, if sufficiently large, may trigger a systemic crisis. This will occur if all banks have to liquidate the long-run technology up to the point where residual depositors have incentives to join the run. In the same vein, efficient bank runs (section 7.4) may occur at the aggregated level when the returns on banks' assets are highly correlated. Patient depositors will prefer to withdraw at time  $t = 1$ , which may or may not be efficient depending on the information structure and the level of the liquidation cost.

Apart from liquidity and productivity shocks, a third macroeconomic shock, the one of the exchange rate, is particularly relevant to the study of systemic risk; the East Asian crises pointed out the close links between the currency crisis and banking crisis within a country.

Liquidity and productivity shocks are no longer considered the main sources of systemic risk because Central Banks have tools to accommodate such shocks.<sup>19</sup>

On the other hand, a systemic crisis may be the result of contagion. The failure of a bank may propagate to the whole banking industry. Clearly, the macroeconomic environment will be important in setting the conditions for this domino effect to occur, since a lower yield on loans, due to high loan losses, depletes banks capital and reduces the buffers each bank has to cope with risks.

Contagion may occur through four different (nonexclusive) channels:

- Change in expectations of investors
- Large-value payment systems

- Over-the-counter operations (mainly on derivatives)
- Interbank markets

### 7.6.1 Aggregate Liquidity and Banking Crises

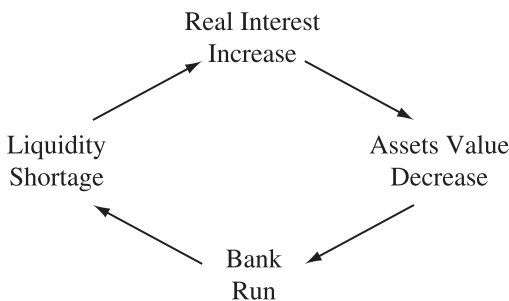
Diamond and Rajan (2005) study the effect of bank runs on the aggregate demand of liquidity and on interest rates, and find that the very existence of a demand deposit structure allowing the banker to commit not to renegotiate the deposit repayment (see section 7.3), might aggravate banking crises. This will occur because bank runs might destroy liquidity rather than create it. As mentioned, runs are triggered by the bank's inability to repay depositors. So, in contrast to Diamond-Dybvig (1983), Diamond and Rajan (2001; 2005) find that bank runs are triggered by the asset side of the bank's balance sheet.

The basic mechanism of Diamond and Rajan (2005) is somewhat related to Fisher's (1933) debt deflation mechanism, but whereas Fisher emphasized the effect a decrease in the value of assets would have on the supply of credit (because of the lower value of collateral), Diamond-Rajan find a decrease in the value of assets will trigger more bank runs (fig. 7.2).

It is clear that liquidity shortages increase interest rates and that this in turn decreases the value of assets promising future cash flows, so the key issue is to understand why bank runs may exacerbate liquidity shortages, that is, why the excess demand for liquidity may be increased by bank runs.

In order to understand how bank runs may absorb liquidity, it is useful to briefly describe some of the aspects of the Diamond-Rajan (2005) model, which inherits the main characteristics of their previous work.

There are three dates,  $t = 0, 1, 2$ , and three types of agents: investors, bankers, and entrepreneurs, all of them risk-neutral. Investors consume at time  $t = 1$ , and entrepreneurs and bankers consume at time  $t = 2$ .



**Figure 7.2**  
Debt deflation.

Entrepreneurs' projects are risky in the sense that the expected cash flow  $y$  may be obtained either at time  $t = 1$  (early project) or at time  $t = 2$  (late project).

As mentioned in section 7.3, banks are partly financed by deposits and any shortfall on repayment triggers a bank run. This depends on the proportion of early versus late projects the bank faces. But once a bank run occurs, inefficient liquidation will take place, so that both early and late projects will be liquidated. Assume that liquidation of a project produces an amount  $\alpha V_1$  at time  $t = 1$  and  $\alpha V_2$  at time  $t = 2$ .

On the other hand, early repayment as well as continuation of late projects leads to a repayment  $R$  to the bank and a profit  $y - R$  to the entrepreneur at the time when  $y$  is realized. Since  $\alpha(V_1 + V_2)$  is lower than  $y$ , project liquidation leads to a destruction of resources: the amount  $y - R$  lost by the entrepreneur plus the amount lost by the bank. If the equilibrium interest rate is  $\rho$ , the bank's loss equals

$$R - \alpha \left( V_1 + \frac{V_2}{1 + \rho} \right).$$

Absent a bank run, when the bank is confronted with a high proportion,  $\mu$ , of early repayments, its cash flow  $\mu R$  is sufficient to pay all the depositors. To understand why bank runs may deplete liquidity and therefore increase the real interest rate, consider a bank on the verge of insolvency (the value of its assets just equals its depositors' claims) that is confronted with a proportion  $\mu$  of early projects (and  $1 - \mu$  of late projects) and therefore has to liquidate all its late projects.

If no bank run occurs, the liquidity generated by the bank  $\mu R$  is absorbed by the depositors, but the bank will have to obtain additional liquidity from other banks in order to pay the remaining depositors  $d - \mu R$ . This will be possible by liquidating and selling the projects of the late entrepreneurs in proportion  $1 - \mu$ . Since the liquidated assets are assumed to be transparent, their price is the discount rate

$$\frac{1}{1 + \rho}.$$

Hence, the bank obtains  $\mu R$  from its projects and demands

$$(1 - \mu) \frac{\alpha V_2}{1 + \rho}$$

from other banks to satisfy its depositors. Consequently, the banks' depositors absorb

$$(1 - \mu) \frac{\alpha V_2}{1 + \rho}$$

liquid assets. Still, the bank generates liquid time  $t = 1$  cash flows for  $\mu(y - R)$  entrepreneurs.

By contrast if there is a bank run, depositors will repossess the bank's assets and thus destroy liquidity. Indeed, while the amount  $\alpha V_2$  will still be sold, keeping the demand for liquidity unchanged, the bank's liquidity is lower because  $\mu\alpha V_1 < \mu R$ , and so is the external liquidity because the amount  $\mu(y - R)$  that the early entrepreneurs received is destroyed.

Consequently, bank runs deplete liquidity. This leads to an increase in  $\rho$ , which in turn leads to additional (fundamental) bank runs.

Note that an increase in the interest rate  $\rho$  implies a higher demand for liquidity. This is indeed the case because in order to obtain a unit of liquidity, a bank has to liquidate a fraction  $\gamma$  of its late projects such that  $\gamma \cdot \alpha V_2 / (1 + \rho) = 1$ . So a higher  $\rho$  implies a higher value of  $\gamma$ .

Diamond and Rajan (2005) study the equilibrium and remark that multiple interest rates might be compatible with liquidity market clearing. So the coordination of beliefs will be the determinant for market allocation.

Finally, it is interesting to note that liquidity is defined here in terms of goods and that depositors' claims are also defined in real terms, so the introduction of money and prices, as in Allen and Gale (1998), may modify the model's implications.

### 7.6.2 Payment Systems and OTC Operations

In spite of their apparent differences, payment systems and over-the-counter (OTC) operations share some similarity because they both give rise to credit risk. We develop the analysis focusing on the payment system, as Freixas and Parigi (1998) do, and then we reinterpret the results in the context of OTC operations.

The motivation for the analysis of payment systems stems from the change that has taken place in this field. Banking authorities in developed countries have progressively introduced Real Time Gross System (RTGS), where each individual payment order becomes irrevocable in real time provided the sending bank has a sufficient balance on its account at the Central Bank. RTGS has completely replaced traditional net systems, where payment orders of each institution are netted against all the others, and a net credit or debit position is obtained for each of them. The issue that arises is therefore under what conditions this substitution of RTGS for net payment systems is efficient. Contagion is a key issue because in a net payment system banks are net creditors (debtors) of one another as a result of the operations their clients realize. The bankruptcy of a bank means that it will default on its payments to other banks, thus affecting their solvency.

Freixas and Parigi (1998) consider a Diamond-Dybvig economy with two locations where patient agents are uncertain about the location where they want to consume. As a consequence, there is a demand for a payment system, and the prop-

erties of net systems and RTGS could be compared in terms of expected future consumption.

In a Diamond-Dybvig framework the disadvantages of RTGS become apparent. Banks have to keep more reserves and are unable to invest as much as they would like in the more profitable long-run technology. As a consequence, when returns are certain, a net system always dominates. Still, when returns are uncertain, a net payment system causes the bankruptcy of one institution to propagate to others, whereas RTGS provides a perfect fire wall. The comparison of net system versus RTGS boils down to a trade-off between efficiency and safety.

The choice of a payment system, net or gross, depends on characteristics of the environment, in particular, how large the opportunity cost of holding reserves and how large the probability of a bank failure. RTGS is shown to be preferred when the probability of bank failures increases, when transaction volume increases, and when the opportunity cost of liquid reserves increases. These features may help to explain the recent move from net to gross systems.

An analysis of OTC versus market-based operations on derivatives could be pursued simply as a reinterpretation of the preceding results. Organized markets require traders to post collateral. In a Diamond-Dybvig framework this implies that banks invest less in the long-run technology in order to hold more liquidity. Hence OTC operations are equivalent to the net payment system, whereas organized market operations are the equivalent of RTGS.

A consequence of this analysis is that banks prefer net payment systems and OTC operations, whereas regulators prefer RTGS and organized (collateralized) markets. Holthausen and Rønde (2000) are concerned precisely with this issue.

### 7.6.3 Contagion through Interbank Claims

Contagion and systemic risk are explored by Allen and Gale (2000) and Freixas, Parigi, and Rochet (2000). Both contributions aim at explaining how contagion works, and they share some conclusions:

- The level of buffers each bank has, broadly defined as capital, subordinated debt, or the claim of patient depositors is a key determinant of contagion.
- The way in which the failure of a bank is resolved has an impact on the propagation of crises.
- The system of cross-holdings of assets and liabilities among banks, including those implicit in payment system arrangements (bilateral or multilateral credit lines), is essential in triggering systemic crises.
- The specific architecture of this system of cross-holdings matters. A system where each bank borrows only from one bank is more fragile than a system where the sources of funds are more diversified.

The focus of the two articles is different. In Allen-Gale the crisis is basically a liquidity crisis; in Freixas-Parigi-Rochet the crisis stems from a coordination problem. Although both articles conclude that a Central Bank is needed, they differ in the policy prescription. Allen-Gale advise preventing contagion by injecting liquidity globally (through repos or open market operations, as suggested by Goodfriend and King (1988)). Freixas-Parigi-Rochet require that liquidity be provided to a specific financial intermediary.

Both groups draw on the Diamond-Dybvig framework with a short-run storage technology and a long-run technology that is costly to liquidate. This framework is extended in order to consider  $N$  commercial banks ( $i = 1, \dots, N$ ).

### Contagion in the Allen-Gale Model

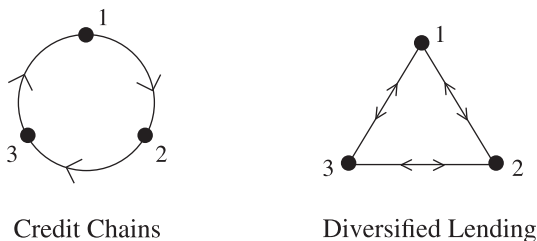
The model of Bhattacharya and Gale (1987) is the starting point of the Allen-Gale analysis. Absent aggregate liquidity shocks, (and abstracting from the issue of asymmetric information regarding liquidity shocks), efficient allocation corresponds to a consumption profile  $(C_1^*, C_2^*)$  that is independent of the banks' idiosyncratic liquidity shocks. To achieve this allocation, the banks with liquidity needs borrow from the ones with excess liquidity. The interbank market, credit lines, or cross-holdings of deposits among banks are possible mechanisms that implement efficient allocation by transferring liquidity among banks. It is therefore necessary that at least one of these mechanisms be in place for efficient allocation to be implemented.

Starting from this setup of aggregate certainty, Allen and Gale introduce the following perturbation: with probability zero, each bank faces the average demand for liquidity except one bank (say, bank 1), which faces the average demand plus  $\varepsilon$ . In this event, the aggregate liquidity shock is larger, and this implies that the allocation  $(C_1^*, C_2^*)$  is not feasible anymore. The shock, initially affecting bank 1, will propagate and affect all the other banks through the very same interbank links that were designed to channel liquidity efficiently and implement  $(C_1^*, C_2^*)$ .

The propagation mechanism is quite simple. Assume that  $N = 3$ , that all depositors are treated equally, and that each bank borrows from its neighbor, as in the credit chain of figure 7.3. An  $\varepsilon$  liquidity deficit forces bank 1 to borrow this amount from bank 2. Then bank 2 becomes illiquid and has to borrow from bank 3, which in turn will borrow from bank 1. But since bank 1 is already illiquid, it will be forced to liquidate its long-run assets at a cost, and this may trigger its bankruptcy. Then the value of bank 3's assets may become insufficient to cover its liquidity needs. Bank 3 is therefore forced to liquidate all its assets. The mechanism continues until all banks fail.

For bankruptcy and propagation to occur, banks have to have a high level of debt and a low level of buffers. Clearly, if the losses derived from the liquidation of assets at time  $t = 1$  could be borne by equity holders or by patient depositors (which would



**Figure 7.3**

Two examples of interbank borrowing architecture.

still reach a higher utility level by waiting than by forcing liquidation), the process would stop.

The extent of the contagion will also depend on the architecture of interbank borrowing. To see why, notice that although bank 1 does not have a sufficient buffer to cope with the liquidity shock, it could be the case that bank 1 and 3 together do. But the importance of the liquidity demand from one bank to another depends on the architecture of interbank borrowing claims. Comparing the two extreme cases of a credit chain and diversified lending, where each bank has deposited the same amount in every other bank (see fig. 7.3), an  $\varepsilon$  shock for bank 1 translates into the same shock for bank 3 in the first case and only to a shock of  $\varepsilon/2$  in the diversified lending case. As a consequence, the same buffer level is much more effective in avoiding a domino effect in the complete claims or diversified lending case than in the credit chain case.

### Contagion in the Freixas-Parigi-Rochet Model

Freixas, Parigi, and Rochet extend the Freixas and Parigi (1998) framework in order to explore contagion through interbank markets. The question they raise is whether the interbank market is a sufficient guarantee against a liquidity shock affecting one bank. Consider an economy where depositors travel and are uncertain about their consumption location. Depositors have two ways to transfer money at the right location: either they cash their deposit at time  $t = 1$  and travel to their consumption location with cash in their pockets, or they travel with a check payable at the location of destination (representing the use of the interbank market). In a Diamond-Dybvig framework, the first solution implies that too much cash is used. Therefore, as in Allen and Gale (2000), access to interbank credit reduces the cost of holding liquid assets.

Still, depositors have to make a strategic choice, whether to travel with cash or with a check. This depends on their assessment of the probability that the check will be repaid by the bank at the location where they travel. If they do not trust the bank at their destination, they will withdraw cash at their own bank, which generates externalities across banks. Consequently, two equilibria coexist: the efficient one,

where depositors travel with a check to their destination and long-run investment is preserved, and an inefficient one, called the gridlock equilibrium, without any interbank lending. In this equilibrium the expectations of each bank, that there will be no liquidity available, are self-fulfilled. Freixas-Parigi-Rochet analyze the possibilities of contagion when one of the banks is closed by supervisory authorities. As in Allen-Gale, they show that the likelihood of contagion depends on the architecture of interbank payments. A diversified lending situation is less fragile than a credit chain situation. They also show that some banks may be more likely than others to provoke contagion.

### 7.7 Lender of Last Resort: A Historical Perspective

Following the ideas of Bagehot (1873), the Central Banks of most countries have adopted a position of lender of last resort (LLR) in the sense that under certain conditions, commercial banks facing liquidity problems can turn to them for short-term loans. Although this definition is functionally correct, it amalgamates completely different interventions, which reflect different strategies on behalf of the Central Banks.

To begin with, the provision of liquidity to the market on a regular basis, through auctions or a liquidity facility is the responsibility of the LLR. It corresponds to lending (anonymously) to the market against good collateral and is one of the main tasks of a Central Bank.

Second, LLR lending could be directed at a specific group of solvent illiquid institutions. This type of facility could be arranged through a discount window or as an exceptional short-term loan. The term *emergency lending assistance* has been used to refer to this situation.

Third, LLR lending could be aimed at an insolvent institution, in which case the term *lending* barely conceals the fact that the LLR is injecting capital.

Notice that an institution could be ex ante solvent and ex post insolvent, or the other way around, so the decision of the LLR has to be taken with incomplete information. Also, since the rescue of a bank involves, directly or indirectly, taxpayer funds, a number of rescue operations are done under the pretense of granting a loan to an ex ante solvent institution so as to avoid political pressure.

Finally, the LLR could play a role as a crisis manager, involving no lending at all, but necessary so as to coordinate agents that would otherwise liquidate an otherwise solvent institution.

The discussion here examines various arguments justifying the role of the LLR. We start with a quick review of Bagehot's doctrine and of the criticisms that have addressed it. Then we examine the practice of LLR intervention as well as the historical evidence about the efficiency of the LLR system in preventing systemic risk.

### 7.7.1 Views on the LLR Role

The idea that market mechanisms cannot insure against liquidity shocks has to be based on arguments that sustain the existence of a market failure. The classical argument was forcefully put forward by Bagehot (1873), emphasizing the difficulty a bank will face if it must transmit credible information to the market during a crisis. In his words, “Every banker knows that if he has to prove that he is worthy of credit, however good may be his argument, in fact his credit is gone” (68). The classical price argument, which implies that an increase in interest rates would compensate lenders for the increased risk they take when lending to a bank facing a crisis, may in fact act as a signal of an unsound position and therefore discourage potential lenders. Market failure can thus be traced to asymmetric information on the banks’ solvency.

The idea of the Central Bank’s acting as the LLR is associated with the work of Bagehot. He argues that

- the LLR has a role in lending to illiquid, solvent financial institutions;
- these loans must be at a penalty rate, so that financial institutions cannot use the loans to fund their current lending operations;
- the lending must be open to solvent financial institutions provided they have good collateral (valued at prepanic prices);
- the LLR must make clear in advance its readiness to lend any amount to an institution that fulfills the conditions on solvency and collateral (credibility).

There have been some criticisms of Bagehot’s view:

- Goodhart (1987; 1995) writes that the clear-cut distinction between illiquidity and insolvency is a myth because the banks that require the assistance of the LLR are already under suspicion of being insolvent. The existence of contagion is the additional argument that may induce the systematic rescue of *any* bank.
- Goodfriend and King (1988) argue that LLR functions must be restricted to the use of open market operations. Humphrey (1986) claims that this would have been Bagehot’s viewpoint had he known of open market operations.
- Proponents of free banking do not challenge the existence of market failure but suggest that the market would still lead to a better allocation than a public LLR would.
- Repullo (2005) shows that when banks’ risk-taking decisions are explicitly taken into account, the existence of the LLR does not increase the incentives to take risk, whereas penalty rates do.

A second issue that evokes disagreement is the classical view that the rules governing LLR behavior should be clearly stated. Most of the time, this is opposed by Central Banks. In the United States, for instance, the Fed has always stressed that discounting is a privilege, not a right. The supporters of this view say that ambiguity in the policy will help to bring some market discipline (in contradiction to the view, also held by Central Banks, that disclosure could have a destabilizing effect on the payment system). In fact, the effect of ambiguity is a transfer of wealth from small to large banks, because there is no ambiguity that large institutions are “too big to fail.” Thus, ambiguity is to some extent illusory and is equivalent to repaying all large banks’ liabilities and rescuing only the solvent ones among the small banks (if they are able to prove that they are solvent).

It is clear that these positions should result from social welfare maximization, taking into account asymmetric (or costly) information and all the externalities that the behavior of the LLR may have: contagion, panics, and effects on securities markets, as well as the moral hazard problem. Therefore, at least theoretically, the differences among the views of the LLR’s role could be traced to differences in the appreciation of, say, the social cost of individual bank failure, bank panics, and contagion effects.<sup>20</sup>

### **7.7.2 Liquidity and Solvency: A Coordination Game**

Rochet and Vives (2004) revisit Bagehot’s assertion that the LLR should provide liquidity assistance to illiquid but solvent banks. This view had been criticized, in particular by Goodfriend and King (1988), on the grounds that in Bagehot’s time, interbank and money markets were underdeveloped and lacked the efficiency that now exists in most developed countries, where a solvent bank will be able to find liquidity assistance. So Central Bank lending is not needed anymore and should even be prohibited in the case where Central Banks are prone to forbearance, especially under political pressure.

Rochet and Vives (2004) are able to rejuvenate Bagehot’s doctrine by showing that even modern, sophisticated, interbank markets will not necessarily provide liquidity assistance to a solvent bank; in other words, a solvent bank can indeed be illiquid. The reason is a potential coordination problem between investors (typically other banks) who may have different opinions about the solvency of the bank requiring liquidity assistance. In this context, the decision of each individual investor to renew, say, a large certificate of deposit will be based not only on his own opinion about the bank’s solvency (fundamental risk) but also on his assessment about the decisions of other investors (strategic risk). The reason is that a large withdrawal by other investors (the modern form of a bank run, illustrated by what happened to Continental Illinois in 1985) might force the bank to liquidate some of its assets at a loss (“fire sales”) or simply borrow from other investors at a penalty rate or for an amount

strictly lower than the value of the assets that the bank can offer as collateral (“hair-cut”). As a result, liquidity problems might provoke the insolvency of an initially solvent bank. So, the optimal decision of each individual investor (renew the CD or not) depends on his expectation of what others will do. The higher the proportion of other investors that are ready to lend, the more likely each individual investor is to lend. Thus there is strategic complementarity between investors’ decisions. In the limit case of perfect information, where all investors know exactly the value of the bank’s assets, this strategic complementarity leads to the possibility of multiple equilibria. In fact, when the value of the bank’s assets is either very large or very low, there is still a unique equilibrium, characterized by lending in the former case and withdrawing in the latter, but in the intermediate region (where the bank is solvent but not “supersolvent”) there are two equilibria resembling bank run models à la Diamond-Dybvig.

By using global game techniques as in Morris and Shin (2000), Rochet and Vives (2004) show that in the imperfect information case, where investors have different opinions about the bank’s solvency, there is a unique equilibrium. In this equilibrium, the fraction of investors who withdraw continuously decreases (as the value of the bank’s assets increases) from 1 (when the bank is insolvent) to 0 (when the bank is “supersolvent”). There is a critical value of bank’s assets (between the solvency and supersolvency thresholds) such that whenever the value of the bank’s assets falls below this threshold (which is strictly higher than the solvency threshold) the proportion of investors who withdraw becomes so large that the bank will not find enough liquidity support from the interbank markets. Thus there is a possibility that a solvent bank might be illiquid.

Rochet and Vives (2004) then use their model to show how a Bagehot LLR might increase social welfare by avoiding inefficient closures of solvent banks. This is particularly clear if the Central Bank has access to regulatory information that allows it to assess properly the value of the bank’s assets.<sup>21</sup> Interestingly, it would not help in this case to instruct the supervisor (or the Central Bank) to disclose this information publicly, because this would lead to a multiplicity of equilibria (as in the perfect information case) and thus to some form of instability. Thus disclosure of supervisory information is not a good idea, implying a caveat on the promotion of a transparency policy.

Finally, Rochet and Vives (2004) discuss how other regulatory instruments like solvency and liquidity requirements can complement the LLR activities of the Central Bank so as to provide appropriate incentives for banks’ shareholders while avoiding too many bank closures.

An alternative link relating liquidity and bank crises is put forward by Acharya and Yorulmazer (2006). They consider a framework similar to that of Diamond and Rajan (2005) and analyze the effect of bank failures on the supply of assets available

for acquisition rather than on the interest rate. Since bank failures decrease available liquidity, a bank bankruptcy leads to a decrease in asset prices. So, bank failures are a cumulative phenomenon: they depress the price of banks' assets and this in turn increases the number of bank failures.

Confronted with this, the regulator has the option to intervene in an number of ways. Acharya and Yorulmazer show that when the number of bank failures is low, the optimal ex post policy is not to intervene, but when this number is sufficiently large, the regulator will optimally adopt a mixed strategy and choose randomly which banks to assist. This mixed strategy is justified by the fact that the regulator sets a liquidity target that may be lower than the amount required to bail out all banks. This liquidity target limits banks' assets sales and the fall of asset prices that could severely distort asset allocation (as inefficient users of the asset who are liquidity-long would then end up owing it all) and thus prevents additional bank bankruptcies.

Finally, Acharya and Yorulmazer remark that the policy of liquidity assistance to surviving banks in the purchase of failed banks is equivalent to an ex post bailout policy but provides better incentives ex ante.

### 7.7.3 The Practice of LLR Assistance

The practice of Central Banks in injecting money through regular repo auctions or other procedures falls outside the scope of this discussion. This section is concerned with the lending provided by the LLR (usually Central Banks) to individual institutions.

Casual empirical evidence seems to indicate that in general LLR assistance is directed to insolvent banks, or at least to banks that ex post are insolvent. In particular, "too big to fail" banks are always rescued, as illustrated by the Continental Illinois, Crédit Lyonnais, or Banesto events, even if the last two were insolvent as a result of fraudulent operations.

Although it is always difficult to distinguish between illiquid and insolvent banks, in the United States the discount window has been open to thrift institutions with the worst CAMELS ratings.<sup>22</sup> Thus, as Kaufman (1991) states, it has provided a disguised way to bail out insolvent banks.

The empirical analysis of bank resolutions confirms the idea that LLR lending is often directed to bail out banks. Goodhart and Schoenmaker (1995) support that view with evidence on the effective bailout policies of Central Banks all over the world. Out of a sample of 104 failing banks, 73 ended up being rescued and 31 liquidated. Since the Central Bank is in charge of an orderly liquidation, it is no surprise that absent institutional structures that would allow for an orderly closure of financial institutions, the Central Bank prefers to rescue them rather than risking a contagion.

This view is confirmed by Central Bank complementary measures when a bank is liquidated. Case studies of bank failures, for instance, the Herstatt bank in 1974 or Barings in 1995, show that the Central Bank was ready to lend to any bank that would have been hit by the bankruptcy, in order to limit a contagion effect.

The fact that LLR lending tends to bail out a bank or help it forbear is an important difference between nineteenth-century and modern practices, which may clarify the debates about the LLR. The existence of a closure/bailout policy is an important part of the overall regulation of the banking industry. The fact that it may be implemented by the LLR is, to say the least, confusing.

#### **7.7.4 The Effect of LLR and Other Partial Arrangements**

The evidence on the LLR mechanism points unambiguously to the conclusion that it has helped to avoid bank panics. Miron (1986), Bordo (1990), and Eichengreen and Portes (1987), among others, support this view.<sup>23</sup> Their results were obtained either by examining the effects of creating the LLR in a given country, thus assuming the *ceteris paribus* clause for changes in the banking system, or by comparing different countries and assuming that the *ceteris paribus* clause is satisfied for other factors affecting the frequency of financial panics. By monitoring the banks' solvency and payment system, the LLR mitigates the risk of contagion, the importance of which has been emphasized by Aharony and Swary (1983), Humphrey (1986), Guttentag and Herring (1987), Herring and Vankudre (1987), and Saunders (1987).

The evidence obtained by Miron (1986) on the effects of creating the Federal Reserve Board in the United States shows the importance it has had on limiting bank runs. Prior to the Fed's founding, autumn and spring were the stringent money quarters, during which panics tended to occur. The founding of the Fed provided the U.S. economy with an LLR, and the frequency of bank panics immediately decreased. The change in seasonal patterns for both interest rates and the loan-reserve ratio confirms the importance of the founding of the Fed as a way out of seasonal liquidity-triggered bank panics. Between 1915 and 1928 the banking system experienced no financial panics, although several recessions occurred during the periods 1918–1919, 1920–1921, 1923–1924, and 1926–1927. The 1920–1921 recession was quite severe.<sup>24</sup>

On the other hand, the panics observed during the 1929–1933 period may be considered as providing an argument against the effectiveness of the LLR policy. Still, it is clear that during that period the Fed did not conduct the open market operations necessary to provide banks with adequate reserves. According to Friedman and Schwartz (1963), the series of bank failures that produced an unprecedented decline in the money stock could have been prevented. Meltzer (1986) makes the same point: “The worst cases of financial panics arose because the Central Bank did not follow Bagehotian principles” (83).

Bordo (1990) examines the changes that occurred in the United States and the United Kingdom before and after the creation of an LLR system. Before 1866 the Bank of England tended to react by protecting its own gold reserves, which could even worsen panics. After that date, the Bank of England adopted Bagehot's policy and thus "prevented incipient crises in 1878, 1890, and 1914 from developing into full-blown panics, by timely announcements and action" (23). Bordo compares the two countries during the 1870–1913 periods and sees striking similarities in their business cycles: similar declines in output, price reversals, and declines in money growth. Still, the United States had four panics during this period while the United Kingdom had none. Evidence on Germany, Sweden, and Canada supports analogous views (Bordo 1986; Humphrey and Keleher 1984).

## 7.8 Problems

### 7.8.1 Bank Runs and Moral Hazard

Consider a Diamond-Dybvig economy with a unique good and three dates, where banks managers have a choice of the technology they implement. This choice is unobservable and consists in investing one unit in either project  $G$  or  $B$ , where project  $G$  yields  $G$  with probability  $p_G$  and zero otherwise, and project  $B$  yields  $B$  with probability  $p_B$  and zero otherwise, where  $G < B$ , and  $p_G G > p_B B$ .

A continuum of agents is endowed with one unit at time  $t = 0$ . Of these agents, a nonrandom proportion  $\pi_1$  will prefer to consume at time  $t = 1$ , and the complementary proportion  $\pi_2$  will prefer to consume at time  $t = 2$ .

The agents' utility function is

$$\begin{cases} U(C_1) & \text{for impatient consumers,} \\ \rho U(C_2) & \text{for patient consumers,} \end{cases}$$

so that the ex ante expected utility is  $\pi_1 U(C_1) + \pi_2 \rho U(C_2)$ .

If there are bank runs, they coincide with sunspots that occur with probability  $\alpha$ .

1. Assuming that the risk-neutral bank manager brings in equity, and the other agents have deposit contracts, compute under what conditions the  $G$  allocation is obtained. Interpret this condition in terms of regulation.
2. In what follows, we restrict our attention only to the particular case of risk-neutral depositors,  $U(C) = C$ . What is the optimal contract? What are the manager's incentives to implement  $G$ ? Do they depend upon  $\alpha$ ? Could we propose a better contract by defining an equity economy?



### 7.8.2 Bank Runs

Consider an economy with a unique good and three dates, with a storage technology that yields a zero net interest and a standard long-run technology that yields  $R$  units with certainty at time  $t = 2$  but yields only  $L$  ( $L < 1$ ) if prematurely liquidated at time  $t = 1$ . Both technologies are available to any agent.

A continuum of agents is endowed with one unit at time  $t = 0$ . Of these agents, a nonrandom proportion  $\pi_1$  will prefer to consume at time  $t = 1$ , and the complementary proportion  $\pi_2$  will prefer to consume at time  $t = 2$ .

The agents' utility function is

$$\begin{cases} \sqrt{C_1} & \text{for impatient consumers,} \\ \rho\sqrt{C_2} & \text{for patient consumers,} \end{cases}$$

so that the ex ante expected utility is  $\pi_1\sqrt{C_1} + \pi_2\rho\sqrt{C_2}$ .

Assume first that  $\rho R > 1$ .

1. Compute the first-order condition that fully characterizes the optimal allocation. Compare it with the market allocation that is characterized by  $C_1 = 1$  and  $C_2 = R$ .
2. Consider a banking contract where a depositor's type is private information. Are bank runs possible? If so, for what parameter values?
3. Is the optimal contract implementable within an equity economy, where each agent has a share of a firm that distributes dividends, and a market for ex-dividend shares opens at time  $t = 1$ , as suggested by Jacklin?

Assume now that  $\rho R < 1$ .

4. What would be the optimal banking contract? Are bank runs possible? if so, for what parameter values?
5. Is the optimal contract implementable within an equity economy à la Jacklin?

### 7.8.3 Information-Based Bank Runs

This problem is adapted from Postlewaite and Vives (1987). Consider a one-good, three-dates, two-agent economy in which the gross return is  $r_1$  ( $< 1$ ) for an investment during the first year ( $t = 0$  to  $t = 1$ ),  $r_2$  for an investment during the second year, and  $r_3$  for an investment during the third year. Assume  $2r_1 - 1 > 0$ , and  $2r_1r_2 - 1 > 0$ . The preferences can be of three types. If an agent is of type 1, her utility is  $U(x_1)$ ; of type 2,  $U(x_1 + x_2)$ ; and of type 3,  $U(x_1 + x_2 + x_3)$ . The probability that agent 1 is of type  $i$  and agent 2 is of type  $j$  is  $p_{ij}$ .

The (exogenous) banking contract allows each agent to withdraw the amount initially deposited without penalty at dates 1 and 2, but interest can be collected only if the agent waits until date 3.

1. Define  $a_t^i$  as the strategy that consists in withdrawing everything at time  $t$ . Write the matrix of payments when both agents initially deposit one unit.
2. Consider the restriction of the game to strategies  $a_1^i$  and  $a_2^i$ . What is the equilibrium if  $r_1 > (2r_1 - 1)r_2$ , and  $1 > r_1 r_2$ ? Is this an efficient allocation?
3. Returning to the initial matrix, assume that  $(2r_1 - 1)r_2 r_3 > 1$ . Describe the equilibrium by establishing the optimal strategy for each type. Will there be any bank runs?

#### 7.8.4 Banks' Suspension of Convertibility

This problem is adapted from Gorton (1985). Consider a three-dates economy ( $t = 0, 1, 2$ ) with a unique consumption good that cannot be stored but can be invested. There is a continuum of agents of total measure 1, each having one unit of the good as an initial endowment. Agents have identical risk-neutral preferences, represented by

$$U(C_1, C_2) = C_1 + \frac{1}{1 + \rho} C_2,$$

where  $C_t$  denotes consumption at time  $t$ .

The only available technology yields returns  $r_1$  at time 1, and  $r_2$  at time 2. A signal  $s$ , belonging to the interval  $[\underline{s}, \bar{s}]$ , characterizes the distribution of  $r_2$ . Assume that if  $s_1 > s_2$ , the distribution of  $r_2$  conditionally on  $s_1$  first-order dominates the distribution of  $r_2$  conditionally on  $s_2$ .

1. Show that the optimal consumption decision is given by

$$\begin{cases} C_1 = 1 + r_1 & \text{and } C_2 = 0 & \text{if } 1 + \rho > E[1 + \tilde{r}_2 | s], \\ C_1 = 0 & \text{and } C_2 = 1 + \tilde{r}_2 & \text{if } 1 + \rho < E[1 + \tilde{r}_2 | s], \\ \text{undetermined} & & \text{if } 1 + \rho = E[1 + \tilde{r}_2 | s]. \end{cases}$$

2. A mutual fund contract is defined as one in which an investment of  $I_0$  gives a right to  $I_0(1 + r_1)d_1$  at time 1, and an investment of  $I_1$  at time 1 gives a right to  $I_1(1 + \tilde{r}_2)(1 - d_1)$  at time 2, where  $d_1$  is the fraction that is withdrawn at period 1. Assume agents invest  $Q$  in the mutual fund equity, and that the fund is liquidated if and only if it has repurchased all the investor shares. Show that the optimal allocation is obtained.

3. A deposit contract is defined as the right to withdraw amounts  $d_1$  and  $d_2$  such that  $d_1 \leq D(1 + r_D)$  and  $d_2 = (D(1 + r_D) - d_1)(1 + r_D)$ , where  $D$  is the initial deposit and  $r_D$  is the promised rate on deposits, ( $r_D > \rho$ ). When the bank fails to pay the amount

due, its assets are distributed in proportion to the depositors' rights, so that if the bank fails during period 1,

$$d_1 = 1 + r_1, \quad 1 + r_1 < D(1 + r_D),$$

and if it fails during period 2,

$$d_2 = (1 + r_1 - d_1)(1 + r_2), \quad d_2 < D(1 + r_D - d_1)(1 + r_D).$$

Equity holders are period 2 residual claimants. The bank will choose to close only if this increases its expected net present value. Speculative bank runs are defined as ones that happen independently of  $s$ , and fundamental bank runs as ones that arise for low values of  $s$ . Also,  $\delta(\hat{d}_1, s)$  is defined as the expected period 2 return on deposits when the other agents withdraw  $\hat{d}_1$ .

3a. Show that  $\delta(\hat{d}_1, s)$  is increasing (resp. decreasing) in  $\hat{d}_1$  if  $1 + r_1 > D(1 + r_D)$  (resp.  $1 + r_1 < D(1 + r_D)$ ).

3b. Characterize the different Nash equilibria that obtain depending on the values of  $\rho(0, s)$ ,  $\rho(D(1 + r_D), s)$ , and  $1 + \rho$ , and show that for some of these values a speculative bank run obtains, while others result in a fundamental bank run.

3c. Show that this contract does not lead to the optimal allocation.

4. Assume now that  $r_2$  is observable by the bank's management, whereas depositors observe only  $s$ . Show that if the bank's equity holders find it profitable to pay an auditing cost  $c$  in order to make  $r_2$  publicly observable while suspending convertibility, it is Pareto-superior to do so.

### 7.8.5 Aggregated Liquidity Shocks

This problem, adapted from Hellwig (1994), studies the allocation of interest rate risk in an extension of the Bryant-Diamond-Dybvig model (see section 7.1). There are three possible technologies (all with constant returns to scale):

- A short-term investment at date 0 that yields a return  $r_1 = 1$  at date 1 (storage technology).
- A long-term investment at date 0 that yields a return  $R > 1$  as of date 2 but can also be liquidated at date 1 for a return  $L < 1$ .
- A short-term investment at date 1 that yields a random return  $\tilde{r}_2$  at date 2.  $\tilde{r}_2$  is observed only at date 1. It is assumed that  $1 \leq \tilde{r}_2 \leq R/L$ .

The consumption profile  $(C_1, C_2)$  may now depend on  $\tilde{r}_2$  (depositors may bear some of the interest rate risk). This is because when  $\tilde{r}_2$  is large, some quantity  $x(\tilde{r}_2)$  of the available consumption good can be invested in the short-term technology rather than used for immediate consumption by impatient consumers.

The optimal allocation is obtained by solving

$$\mathcal{P}_3 \begin{cases} \max E[\pi_1 u(C_1(\tilde{r}_2)) + \pi_2 u(C_2(\tilde{r}_2))] \\ \pi_1 C_1(\tilde{r}_2) + x(\tilde{r}_2) = 1 - I, \\ \pi_2 C_2(\tilde{r}_2) = RI + \tilde{r}_2 x(\tilde{r}_2), \\ x(\tilde{r}_2) \geq 0. \end{cases}$$

Let  $(C_1^*(r_2, M), C_2^*(r_2, M))$  denote the solution of

$$\begin{cases} \max \pi_1 u(C_1) + \pi_2 u(C_2), \\ \pi_1 C_1 + \frac{\pi_2 C_2}{r_2} = M. \end{cases}$$

1. Show that  $C_1^*$  is increasing in  $M$  and decreasing in  $r_2$ .

2a. Show that the solution of  $\mathcal{P}_3$  satisfies when some investment takes place at  $t = 1$  ( $x(\tilde{r}_2) > 0$ ):

$$\begin{cases} C_1(\tilde{r}_2) = C_1^*\left(\tilde{r}_2, 1 + \left(\frac{R}{\tilde{r}_2} - 1\right)I\right), \\ C_2(\tilde{r}_2) = C_2^*\left(\tilde{r}_2, 1 + \left(\frac{R}{\tilde{r}_2} - 1\right)I\right). \end{cases} \quad (7.15)$$

2b. Show that the solution of  $\mathcal{P}_3$  satisfies when no investment takes place at  $t = 1$ , and depositors bear no interest rate risk:

$$\begin{cases} C_1(\tilde{r}_2) = \frac{1 - I}{\pi_1}, \\ C_2(\tilde{r}_2) = \frac{RI}{\pi_2}. \end{cases} \quad (7.16)$$

3. Show that case 2a occurs when  $\tilde{r}_2$  is larger than some threshold  $r_2^*$ . In other words, when the investment opportunities are good enough at  $t = 1$  ( $\tilde{r}_2 \geq r_2^*$ ), it is optimal to let depositors bear some risk, even though a complete immunization would be possible, since the allocation defined by (7.16) is always feasible.

### 7.8.6 Charter Value

This problem is based on Calomiris and Kahn (1991). Consider a one-good, three-period economy with risk-neutral agents and zero (normalized) interest rates where banks invest one unit of good in order to obtain random cash flows  $\tilde{y}$ ,  $\tilde{y} \in \{\underline{y}, \bar{y}\}$  that are nonobservable, and denote by  $p$  the probability of success ( $\tilde{y} = \bar{y}$ ). The bank owner-manager brings in some capital  $K$ . Depositors invest  $D = 1 - K$  and

are promised a return  $R$  at time  $t = 2$  in a time deposit. The bank manager is able to “take the money and run,” but at a cost, so that he obtains only  $\alpha \bar{y}$ ,  $0 < \alpha < 1$ , and the bank’s assets are completely depleted.

1. For which values of  $K$  will financial intermediation exist depending on the probability  $p$ ? Compute the level of  $R$  for which the depositors break even as a function of  $K$ . Is the strong form of the Modigliani-Miller theorem (the value of a firm, sum of the market value of debt and equity is constant) satisfied? Why?
2. Assume the bank has a charter value, defined as the net present value of future profits. How will this change the incentive problem?

## 7.9 Solutions

### 7.9.1 Banks Runs and Moral Hazard

1. A contract gives the depositor the right to withdraw  $C_1$  at time 1 or wait and consume  $\bar{C}_2$  if the project is successful and  $\underline{C}_2$  if it fails. If  $I$  is invested in the long-run technology, the bank manager will implement the  $G$  technology provided that

$$p_G(IG - \pi_2 \bar{C}_2) \geq p_B(IB - \pi_2 \bar{C}_2).$$

In terms of regulation this implies that payments to the depositors cannot be too generous in case of success. The condition is satisfied if a sufficient amount of capital is brought in by the managers–equity holders.

2. Optimal contract:  $C_1 = \underline{C}_2 = 0$ ,  $I = 1$  provided  $p_G G > 1$ . The manager’s incentives are

$$p_G(G - \pi_2 \bar{C}_2) \geq p_B(B - \pi_2 \bar{C}_2).$$

There are no bank runs, so  $\alpha$  is irrelevant. An equity contract could replicate the deposit contract but cannot do better.

### 7.9.2 Bank Runs

1. The first-order condition is

$$C_2 = \rho^2 R^2 C_1,$$

and the efficient allocation is

$$C_1 = \frac{1}{\pi_1 + \rho^2 R \pi_2}, \quad C_2 = \frac{\rho^2 R^2}{\pi_1 + \rho^2 R \pi_2}.$$

2. Since  $C_2 > C_1$ , the only type of run we have to consider is the Nash equilibrium, where every agent is better off withdrawing when all other agents are doing the same. This will occur if  $C_1 > \pi_1 C_1 + (1 - \pi_1)L$ , that is, if

$$\frac{1}{\pi_1 + \rho^2 R \pi_2} > \dot{L}.$$

Thus, for instance, if  $\rho^2 R > 1$ , and  $L = 1$ , bank runs will never occur.

3. In a dividend economy with  $d$  being the cash dividend and  $p$  the price at time  $t = 1$  of the ex-dividend share,

$$C_1 = d + p \quad \text{and} \quad C_2 = (d + p) \frac{R(1 - d)}{p},$$

so

$$\frac{C_2}{C_1} = \rho^2 R^2 = \frac{R(1 - d)}{p}.$$

Equality of supply and demand of ex-dividend shares at  $t = 1$  implies

$$\pi_1 = \pi_2 \frac{d}{p}.$$

Then

$$\rho^2 R + \frac{\pi_1}{\pi_2} = \frac{1}{p},$$

that is,

$$p = \frac{\pi_2}{\pi_1 + \rho^2 R \pi_2} \quad \text{and} \quad d = \frac{\pi_1}{\pi_1 + \rho^2 R \pi_2}.$$

An equity economy will implement the best.

4. If  $\rho R < 1$ , the optimal allocation would imply  $C_2 < C_1$ , so type 2 agents prefer to withdraw and store. The optimal deposit allocation would imply  $C_2 = C_1$ , so it will not reach the best. Bank runs are always possible because  $C_2 = C_1 > 1$  and  $L < 1$ .

5. An equity economy will not reach the best because the demand for ex-dividend shares by impatient types is zero for

$$\frac{R(1 - d)}{p} < 1.$$

### 7.9.3 Information-Based Bank Runs

1. The matrix of payments is

		Agent 1		
		$a_1$	$a_2$	$a_3$
Agent 2	$a_1$	$(r_1, r_1)$	$(1, (2r_1 - 1)r_2)$	$(1, (2r_1 - 1)r_2r_3)$
	$a_2$	$((2r_1 - 1)r_2, 1)$	$(r_1r_2, r_1r_2)$	$(1, (2r_1r_2 - 1)r_3)$
	$a_3$	$((2r_1 - 1)r_2r_3, 1)$	$((2r_1r_2 - 1)r_3, 1)$	$(r_1r_2r_3, r_1r_2r_3)$

2. The restriction to  $(a_1, a_2)$  shows a game with the “prisoner’s dilemma” structure. Strategy  $a_1$  dominates strategy  $a_2$ . If  $r_2 > 1$ , the allocation is inefficient.

3. The optimal strategy will be

$$\begin{cases} a_1^i & \text{if } i\text{'s type is 1 or 2,} \\ a_3^i & \text{if } i\text{'s type is 3.} \end{cases}$$

Therefore bank runs occur when one of the agents is of type 2.

### 7.9.4 Banks’ Suspension of Convertibility

1. For a given realization of  $s$ , solve

$$\max_{0 \leq C_1 \leq 1+r_1} C_1 + \frac{1}{1+\rho} E[(1+r_1 - C_1)(1 + \tilde{r}_2)|s],$$

which gives the desired result.

2. Let  $d_1$  be the fraction of the mutual fund that is withdrawn at time 1. For a given realization of  $s$ , solve

$$\max_{0 \leq d_1 \leq 1} (1 - Q) \left\{ d_1(1 + r_1) + \frac{1}{1+\rho} E[(1 + r_1)(1 + \tilde{r}_2)(1 - d_1)] \right\},$$

so

$$d_1 = \begin{cases} 0 & \text{if } 1 + \rho > E[1 + \tilde{r}_2|s], \\ 1 & \text{if } 1 + \rho < E[1 + \tilde{r}_2|s], \\ \text{undetermined} & \text{if } 1 + \rho = E[1 + \tilde{r}_2|s]. \end{cases}$$

The rules on the closing of the mutual fund imply that agents withdrawing  $(1 - Q)(1 + r_1)$  also obtain their capital  $Q(1 + r_1)$  so that their consumption is optimal. If fees are introduced, the solution is unchanged if these fees are proportional to withdrawals or if they are redistributed to equity holders in the form of dividends.

3a. The equity holders have a call option on the time 2 value of the bank's assets. Therefore, if closing the bank generates a zero profit, which happens when  $1 + r_1 < D(1 + r_D)$ , the bank will never liquidate its investment, independently of the value of the signal  $s$ .

Consider the cases for which the bank does not close down during period 1.

Let

$$\delta(\hat{d}_1, s) = E \left[ \min \left[ \frac{(1 + r_1 - \hat{d}_1)(1 + \tilde{r}_2)}{D(1 + r_D) - \hat{d}_1}, 1 + r_D \right] \middle| s \right].$$

The depositor will choose  $d_1$  so as to solve

$$\begin{cases} \max_{d_1} d_1 + \frac{1}{1 + \rho} \delta(\hat{d}_1, s) [D(1 + r_D) - d_1] \\ 0 \leq d_1 \leq D(1 + r_D). \end{cases}$$

Since all agents are identical, a Nash equilibrium obtains for  $d_1 = \hat{d}_1$  when there is a unique solution, so it is a symmetrical equilibrium. Examine the symmetrical equilibria:

$$\delta(\hat{d}_1, s) = \frac{1 + r_1 - \hat{d}_1}{D(1 + r_D) - \hat{d}_1} \int_{-1}^{\hat{r}_2(\hat{d}_1)} (1 + r_2) \varphi(r_2) dr_2 + (1 + r_D) \int_{\hat{r}_2(\hat{d}_1)}^{\infty} \varphi(r_2) dr_2,$$

where  $\varphi(r_2)$  is the generalized density function of  $\tilde{r}_2$ , and  $\hat{r}_2(\hat{d}_1)$  is the value of  $r_2$  for which

$$\frac{(1 + r_1 - \hat{d}_1)(1 + r_2)}{D(1 + r_D) - \hat{d}_1} = 1 + r_D.$$

Canceling out the terms in  $\hat{r}_2(\hat{d}_1)$  yields

$$\frac{d\delta}{d\hat{d}_1} = \frac{1 + r_1 - D(1 + r_D)}{D(1 + r_D) - \hat{d}_1} \int_{-1}^{\hat{r}_2(\hat{d}_1)} (1 + r_2) \varphi(r_2) dr_2.$$

3b. If  $1 + r_1 > D(1 + r_D)$ ,  $\delta(D(1 + r_D), s) > \delta(0, s)$ , and

(1)  $d_1 = \hat{d}_1 = 0$  for  $1 + \rho < \delta(0, s)$ ,

(2)  $d_1 = \hat{d}_1 = d_1^*$  for  $1 + \rho < \delta(d_1^*, s)$ ,

(3)  $d_1 = (1 + r_D)D$  for  $1 + \rho < \delta(D(1 + r_D), s)$ ,

case (3) corresponds to a fundamental bank run. In case (2) the solution is undetermined, so (infinitely many) asymmetrical equilibria are obtained, provided that

$$\delta \left( \int_0^1 d_1(t) d\mu(t), s \right) = 1 + \rho.$$



If  $1 + r_1 < D(1 + r_D)$ ,  $\delta(0, s) > \delta(1 + r_1, s)$ , and

- (4)  $1 + \rho > \delta(0, s)$ , then  $d_1 = \hat{d}_1 = 1 + r_1$ , since  $1 + \rho > \delta(1 + r_1, s)$ ;
- (5)  $\delta(0, s) > 1 + \rho > \delta(1 + r_1, s)$ , there are three solutions,  $d_1 = \hat{d}_1 = 0$ ;  $\hat{d}_1$  with  $\delta(\hat{d}_1^*, s) = 1 + \rho$ ; and  $d_1 = \hat{d}_1 = 1 + r_1$ ;
- (6)  $1 + \rho < \delta(1 + r_1, s)$ , then  $d_1 = \hat{d}_1 = 0$ , since  $1 + \rho < \delta(0, s)$ .

Thus, in case (5) speculative bank runs occur.

3c. The optimal allocation clearly does not obtain in equilibrium.

4. The bank's management will decide to suspend convertibility only if time 2 profit is greater than the auditing cost. This implies that the suspension of convertibility will take place only when time 2 profits are strictly positive for the  $r_2$  that is observed. But this in turn implies that depositors obtain  $r_D$  with certainty, so they are better off under the suspension of convertibility.

### 7.9.5 Aggregated Liquidity Shocks

1.  $(C_1^*, C_2^*)$  is characterized by the first-order condition  $u'(C_1^*) = r_2 u'(C_2^*) = \lambda(r_2, M)$ , where  $\lambda(r_2, M)$ , the Lagrange multiplier associated to the budget constraint, is such that this budget constraint is satisfied. Namely,

$$\pi_1(u' - 1)(\lambda(r_2, M)) + \pi_2(u' - 1)\left(\frac{\lambda(r_2, M)}{r_2}\right) = M.$$

Denoting by  $\varphi(\lambda, r_2, M)$  the mapping

$$\varphi(\lambda, r_2, M) = \pi_1(u' - 1)(\lambda) + \pi_2(u' - 1)\left(\frac{\lambda}{r_2}\right) - M,$$

we see that  $\lambda(r_2, M)$  is defined implicitly by

$$\varphi(\lambda(r_2, M), r_2, M) = 0.$$

$u'$  being decreasing (since  $u$  is strictly concave), we see that  $\varphi$  is decreasing in  $\lambda$  and  $M$ , and increasing in  $r_2$ . Therefore,  $\lambda(r_2, M)$  is decreasing in  $r_2$  and increasing in  $M$ , as was to be established.

2a. If the third constraint ( $x \geq 0$ ) does not bind,  $x$  can be eliminated between the first and the second constraint, leading to a unique budget constraint:

$$\pi_1 C_1 + \pi_2 \frac{C_2}{r_2} = 1 + \left(\frac{R}{r_2} - 1\right)I.$$

Thus by definition of  $(C_1^*, C_2^*)$  the solution of  $\mathcal{P}_3$  satisfies

$$\begin{cases} C_1(r_2) = C_1^* \left( r_2, 1 + \left( \frac{R}{r_2} - 1 \right) I \right), \\ C_2(r_2) = C_2^* \left( r_2, 1 + \left( \frac{R}{r_2} - 1 \right) I \right). \end{cases}$$

2b. If  $x = 0$ , then the first and second constraints give directly

$$\begin{cases} C_1 = \frac{1 - I}{\pi_1}, \\ C_2 = \frac{RI}{\pi_2}. \end{cases}$$

3. In case 2a, we have  $x = 1 - I - \pi_1 C_1(r_2)$ . According to part 1,  $C_1$  is decreasing in  $r_2$ . Thus  $x(r_2) \geq 0 \Leftrightarrow r_2 \leq r_2^*$ , where  $r_2^*$  is defined by  $x(r_2^*) = 0$ .

### 7.9.6 Charter Value

1. The bank manager has incentives to abstain from absconding with the money, when  $y$  if:

$$y - R \geq \alpha y, \quad (7.17)$$

we consider two cases.

In case 1 (riskless case), expression (7.17) is satisfied for both  $\underline{y}$  and  $\bar{y}$ . (More precisely, it is satisfied for  $\underline{y}$  and this implies it will be satisfied for  $\bar{y}$ , because  $(1 - \alpha)\underline{y} \geq R$  implies  $(1 - \alpha)\bar{y} \geq R$ . This implies that there is no risk for depositors and thus  $R = 1 - K$ , which implies

$$(1 - \alpha)\underline{y} \geq 1 - K, \quad (7.18)$$

or equivalently,

$$K \geq 1 - (1 - \alpha)\underline{y}. \quad (7.19)$$

In case 2 (risky case), expression (7.17) is satisfied only for  $\underline{y}$ , and investors are only repaid with probability  $p$ , so  $R = (1 - K)/p$ :

$$(1 - \alpha)p\bar{y} \geq 1 - K > (1 - \alpha)p\underline{y} \quad (7.20)$$

Thus, if  $1 - K > (1 - \alpha)p\bar{y}$ , only case 1 occurs. If  $1 - K < (1 - \alpha)p\bar{y}$ , then either  $(1 - \alpha)\underline{y} < 1 - K$  and only case 2 occurs, or else  $(1 - \alpha)\underline{y} \geq 1 - K$  and the two equilibria for both cases are possible. This corresponds to a self-fulfilling prophesy. If depositors believe the equilibrium is risky, the repayment has to be set accordingly, and the manager has an incentive to abscond in case of low return. But if depositors

believe the equilibrium is riskless, the manager has the right incentives and does not take the money and run.

The Modigliani-Miller theorem will hold locally within regions. Still, it is clear that when we go from case 1 to case 2, an expected amount  $p(1 - \alpha)\underline{y}$  of the bank's assets are destroyed, so the structure of liabilities does affect the value of the firm.

2. The existence of a charter value is a substitute for  $K$ . It would decrease the value of  $K$  required for every type of equilibrium.

## Notes

1. The precise definition of a bank panic varies from one author to another. Kemmerer himself enumerates six major bank panics and fifteen minor ones during the period 1890–1908. Benston and Kaufman (1986), using another definition, found bank panics on three occasions: 1874, 1893, and 1908. Gorton (1985) also found panics in 1884, 1890, and 1896.

2. Miron uses this indicator to evaluate the real (as opposed to financial) effects of bank panics. However, the inverse causality cannot be dismissed; it could be argued that decreases in GNP tend to shrink the value of banks' assets, thus triggering bank panics.

3. Of course, bank runs can develop into bank panics; this is the contagion phenomenon.

4. For simplicity, we do not discount the utility of consumption at date 2.

5. This is true, for instance, if  $R \rightarrow Ru'(R)$  is decreasing, which corresponds to assuming that the elasticity of substitution between periods is smaller than 1.

6. There is also a mixed-strategy equilibrium that is not considered here.

7. See Kareken (1986) and Mussa (1986).

8. By the law of large numbers, the realized proportion equals the theoretical frequency. This changes for aggregate liquidity risk (see section 7.6).

9. Engineer (1989) has shown that suspension of convertibility may fail to prevent a bank run if the Diamond-Dybvig (1983) model is extended to a framework with four dates and three types of agents.

10. The situation is more complex if the return on banks' assets is uncertain, and if moral hazard can occur (see, for instance, Freeman 1988).

11. This is the key difference with respect to market equilibrium. In a market equilibrium agents are free to choose the level of investment  $I$ . Here this level is set so as to maximize expected utility.

12. This is only true insofar as the uninformed lender cannot make the entrepreneur and the financial intermediation compete. This is an important assumption in Diamond and Rajan (2001) that is embodied in the timing of offers and counteroffers.

13. The alternative assumption,  $V_L > \alpha V_H$ , is easier to deal with and allows for equity financing, but the cost of equity financing does not appear. In this case, if  $V$  takes the value  $V_H$ , the banker cannot renegotiate its payment to the equity holders without triggering a bank run. Thus there is no cost of using equity, and the maximum amount  $E(V)$  can be raised from the two types of claim holders: depositors and equity holders.

14. Temzelides (1997) studies a repeated version of the Diamond-Dybvig model and models equilibrium selection (between the efficient and the panic equilibria) by an evolutionary process. He shows that the probability of panic decreases with the size of banks, and he studies the possibility of contagion effects.

15. As reported in Benston et al. (1986), this "gambling for resurrection" behavior is frequently observed when a bank faces a crisis. Therefore, although it is unattractive from a theoretical viewpoint because the manager contract is not the optimal one, this assumption is consistent with casual empiricism.

16. See chapter 2.

17. Adao and Temzelides (1995) introduce Bertrand competition between banks in the Diamond-Dybvig model. They show that surprisingly Bertrand equilibria may imply positive profits.
18. Other interesting approaches to the role of the interbank market are provided by Aghion, Bolton, and Dewatripont (1988) and Bhattacharya and Fulghieri (1994).
19. Schwartz (1986) argues that the severe consequences of the Great Depression in the United States could have been considerably limited had the Fed properly conducted lender-of-last-resort operations.
20. See also Smith (1984) for a model of the role of the LLR in the presence of adverse selection.
21. In the case where the Central Bank has only imperfect information about the bank's asset value, there is a trade-off between lending to insolvent banks and refusing to lend to solvent banks.
22. CAMELS is a confidential supervisory rating given to each regulated financial institution (bank) as part of an examination process undertaken by federal and state banking agencies. This rating is based on financial statements of the bank and on-site examination. The components of a bank's condition that are assessed are Capital, Assets, Management, Earnings, Liquidity, and (since 1997) Sensitivity to market risk. The scale is from 1 (strongest) to 5 (weakest). These ratings are not released to the public but only to the top management of the banking company. This is to prevent a bank run on a bank with a bad CAMELS rating.
23. See also the references in these articles.
24. Miron (1986) makes a simple test using a Bernoulli distribution. He estimates that prior to the founding of the Fed the probability of having a panic during a given year was 0.316. This implies that the probability of having no bank panic during the fourteen years 1914–1928 was only 0.005. Miron rejects the hypothesis of no change in the frequency of panics at the 99 percent level of confidence.

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