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Numerical Phase retrieval from beam intensity measurements in three planes

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ABSTRACT

A system and method have been developed at CEA to retrieve phase information from multiple intensity measurements along a laser beam. The device has been patented.

Commonly used devices for beam measurement provide phase and intensity information separately or with a rather poor resolution whereas the MIROMA method provides both at the same time, allowing direct use of the results in numerical models.

Usual phase retrieval algorithms use two intensity measurements, typically the image plane and the focal plane (Gerschberg-Saxton algorithm) related by a Fourier transform, or the image plane and a lightly defocus plane (D.L. Misell).

The principal drawback of such iterative algorithms is their inability to provide unambiguous convergence in all situations. The algorithms can stagnate on bad solutions and the error between measured and calculated intensities remains unacceptable.

If three planes rather than two are used, the data redundancy created confers to the method good convergence capability and noise immunity. It provides an excellent agreement between intensity determined from the retrieved phase data set in the image plane and intensity measurements in any diffraction plane.

The method employed for MIROMA is inspired from GS algorithm, replacing Fourier transforms by a beam-propagating kernel with gradient search accelerating techniques and special care for phase branch cuts. A fast one dimensional algorithm provides an initial guess for the iterative algorithm.

Applications of the algorithm on synthetic data find out the best reconstruction planes that have to be chosen. Robustness and sensibility are evaluated. Results on collimated and distorted laser beams are presented.

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1. INTRODUCTION

Commonly used methods for beam characterization provide phase and intensity information separately or with a poor resolution. This leads to difficulties in establishing the correspondence between phase and intensity profiles. The main interest of the proposed numerical method is that it provides both at the same time. The complex field is then obtained without any further treatment.

Usual phase retrieval algorithms use two intensity measurements, typically the image plane and the focal plane [1], or the image plane and a lightly defocus plane [2].

In the MIROMA method beam phase is calculated from measurements taken in at least three intensity planes. The beam intensity repartition in any diffraction plane is then calculated using Fresnel approximation and Fourier methods for beam propagation. This method is able to accurately model beam diffraction from near field to far field. Numerical simulations show that the problem of the uniqueness of the solution is not an obstacle for the method since simple measurement rules are verified [3]. A phase analyzer using a MIROMA algorithm has been designed. Experimental data show that this is a very efficient tool.

2. PRESENTATION

Gerschberg and Saxton already studied the problem of phase determination from intensity measurements. They proposed two reference planes for the phase determination, one being the image plane and the other one a focal plane. Nevertheless, experiments made at CEA since 1992 have shown that the use of laser beam images in two reference planes did not give acceptable results.

Usual methods for the measurement of the wavefront also use interferential techniques with a reference wave or a Hartmann-Shack mask. Interferometric methods combine the wavefront to be measured with a reference wave related in phase with the wavefront. Hartmann-Shack analyzers use a phase mask for the evaluation of the wave gradient at every point. The wavefront is then reconstructed. The first system is complex and costly. The second system provides data that are difficult to treat because of the strong inhomogeneity of the pupil energy and because of the lack of resolution due to mask sampling. Accurate diffracted intensity reconstruction is in fact impossible.

That was the reason why the MIROMA method was developed. This proposed method uses indeed intensity measurements in at least three planes for the beam wavefront determination. Excellent accordance has been found between measured intensity profiles and calculated profiles from the image field. This correspondence is much better than what can be obtained with a Hartmann-Shack or a ZYGO analyzer. Moreover, it is far less costly. This method has given rise to the patent [4] and publications [5 6].

3. CONTEXT

The problem of phase determination was already dealt with under the name of phase retrieval problem. Different solutions exist, for example in [5] and [8]. The well-known Gerschberg and Saxton algorithm is similar to our proposed method. It uses [1] an image plane and a diffraction plane at infinity. These two planes can also be close from one another [2]. Numerous studies focus on electronic microscopy and antenna emissions. Algorithmic improvements have already been proposed, but no proposal was made relative to the increasing of the number of planes. [11] for example proposes to change the image plane and the Fourier plane. Numerous studies were made to evaluate and compare these different methods (interferometric methods, Hartmann Shack systems and phase retrieval problems from intensity profiles). Experiments made at CEA since 1992 with laser beams have shown that accuracy obtained with our method is much better than with a Hartmann-Shack based system.

4. TECHNICAL DESCRIPTION

The proposed MIROMA method allows laser beam complex field determination in the reference pupil from intensity measurements made in several planes. The method forces the calculated profiles to be as close as possible to the measured profile in every measurement plane (at least three planes).
The complete laser beam characterization in a z_0 cut plane needs the determination of the intensity $I(x,y,z_0)$ and of the phase $\varphi(x,y,z_0)$ at z_0 so that $\sqrt{I(x,y,z_0)} \cdot e^{i\varphi(x,y,z_0)}$ is the complex field. $\varphi(x,y,z)$ is then determined for any $z \neq z0$ using scalar optic approximation and Fresnel hypothesis.

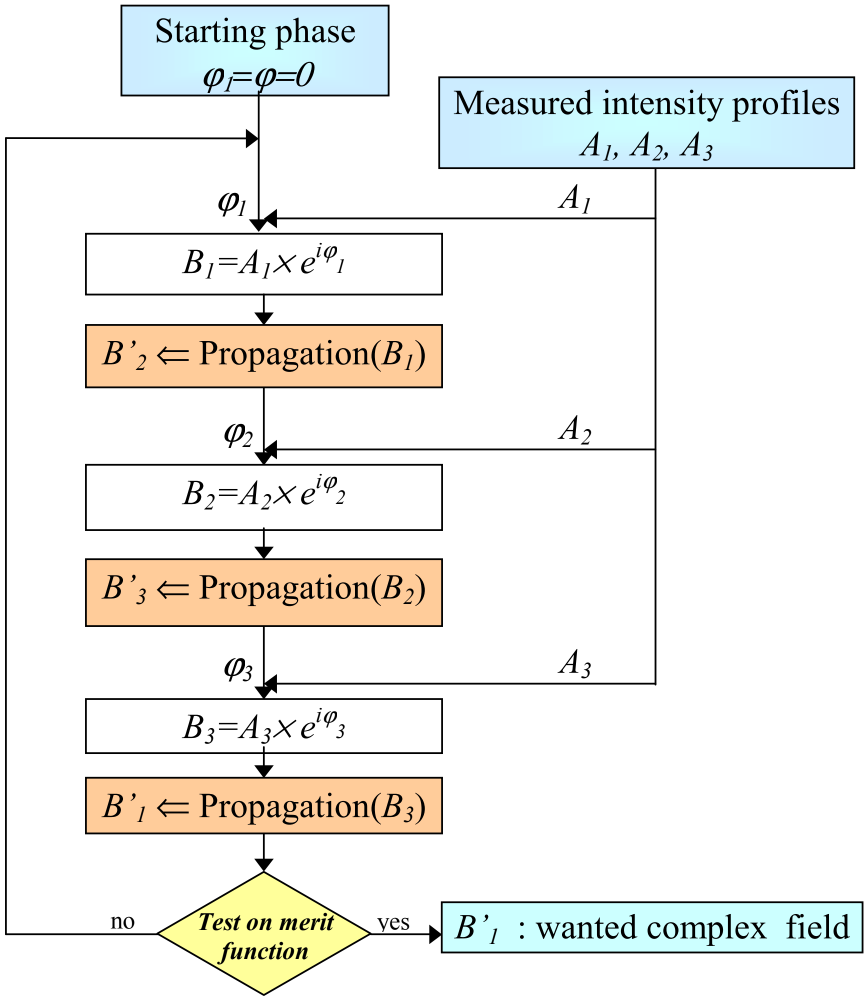


Fig. 1. Algorithm of MIROMA phase reconstruction

Figure 1 shows the general algorithm. The first plane (A1) is the plane where the phase is calculated. In the beginning of the process the phase in (A1) can be an estimation of $\varphi(x,y,z_0)$, generally zero. The beam is propagated to the second plane (A2), the calculated phase is then zero. In (A2) the calculated intensity is replaced by the measured intensity. The beam is further propagated to (A3) and so on. Then the process comes back to (A1).
The beam propagation is calculated using a linear FFT convolution of the field $B(x,y)$.

$$B'(x,y) = B(x,y) \otimes P(x,y) = \iint B(x',y').e^{\frac{2i\pi}{\lambda d}\left[(x'-x)^2+(y'-y)^2\right]} dx'dy'$$

Ten to a hundred iterations are necessary before convergence, depending on the phase shape. Propagation functions are calculated using the fast Fourier transform. 50.10^6 floating-point operations are needed every iteration for 256x256 pixel images. Each iteration needs less than one second on a 1 GHz computer.

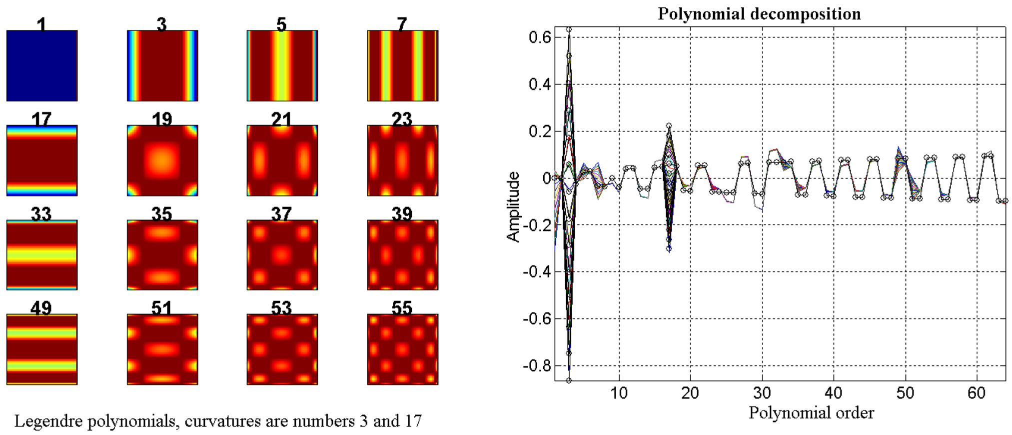
5. SIMULATION RESULTS

Numerical simulations have been made in order to test the accuracy of the method. The first simulation shows that the curvature parameters of a laser beam are retrieved even with a distorted phase shape and noise. The second simulation is given as a reminder of the difficulty of retrieving a phase shape from only two measured planes.

1. Curvature calculation

Beam curvature parameters are obtained from a Legendre or Zernicke polynomial decomposition of the wavefront (figure 2). The synthetic wavefront used is obtained from a weighted sum of Legendre polynomials (little circles on figure 2 graph). The simulations show that the curvature parameters are not correlated to other terms of the decomposition. They also show that the curvature parameters determination remain accurate to lambda/10 even in the case of a bad signal to noise ratio of 50 for a 64 terms Legendre decomposition. The reconstruction error is of lambda/40 in the no noise case.

The simulations show that a unique solution is generally obtained when the curvature radius is greater than the maximal distance between the different planes [3].



Legendre polynomials, curvatures are numbers 3 and 17

o = Legendre coefficients
plain curves = retrieved coefficients
Whatever the curvature, all the coefficients are restored

Fig. 2. Some Legendre polynomials and retrieved coefficients for curvature varying from –1 λ to 1 λ. The two curvatures are the polynomials numbered 3 and 17 for the decomposition on 64 terms (8x8).

2. Three planes interest

The nearest plane from the image plane takes the high frequencies of the wavefront into account. The most distant plane models low frequency fluctuations (including curvature). This information allows precise reconstitution of all polynomial coefficients. Figure 3 shows the results of a three-plane calculation with a 64x64-pixel image and an error reconstruction of lamda/7 peak to peak. Only low frequencies could be retrieved in the case where the two last planes are the same. This latter case corresponds to the Gerchberg and Saxton two planes method. The wavefront reconstitution is not exact in this case as figure 3 shows.

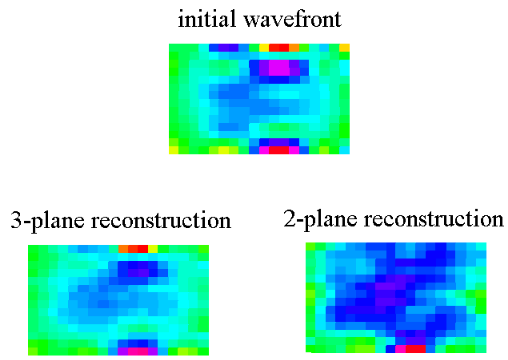


Fig. 3 Three plane reconstitution versus two-plane reconstitution. The correct shape of the wavefront is retrieved only with the use of three measurement planes.

6. CUT PLANE CHOICE

The algorithm convergence tends to reduce the merit functions calculated in the chosen planes. As a result planes must be chosen for their high sensibility to phase variations in the pupil. Merit function is defined as the normalized quadratic distance between the measured and calculated images; good convergence is achieved when its value is less than 0.2. As shown on figure 4, high sensitivity is obtained in two separates domains:

High spatial frequency domain corresponds to distances less than 25 cm, and low spatial frequency corresponds to distances beyond 100 cm approximately.

In term of Fresnel numbers ($N = r^2/(\lambda.d)$, where r is the beam radius, λ the wavelength and d the distance of propagation), the high frequency domain corresponds to $N > 15$ and the other one to $N < 4$.

These planes contain all information in term of spatial frequencies, with no gap. Curvature information is contained around 1/2 frequency on the graph, and the highest frequency attainable depends on the measurements accuracy.

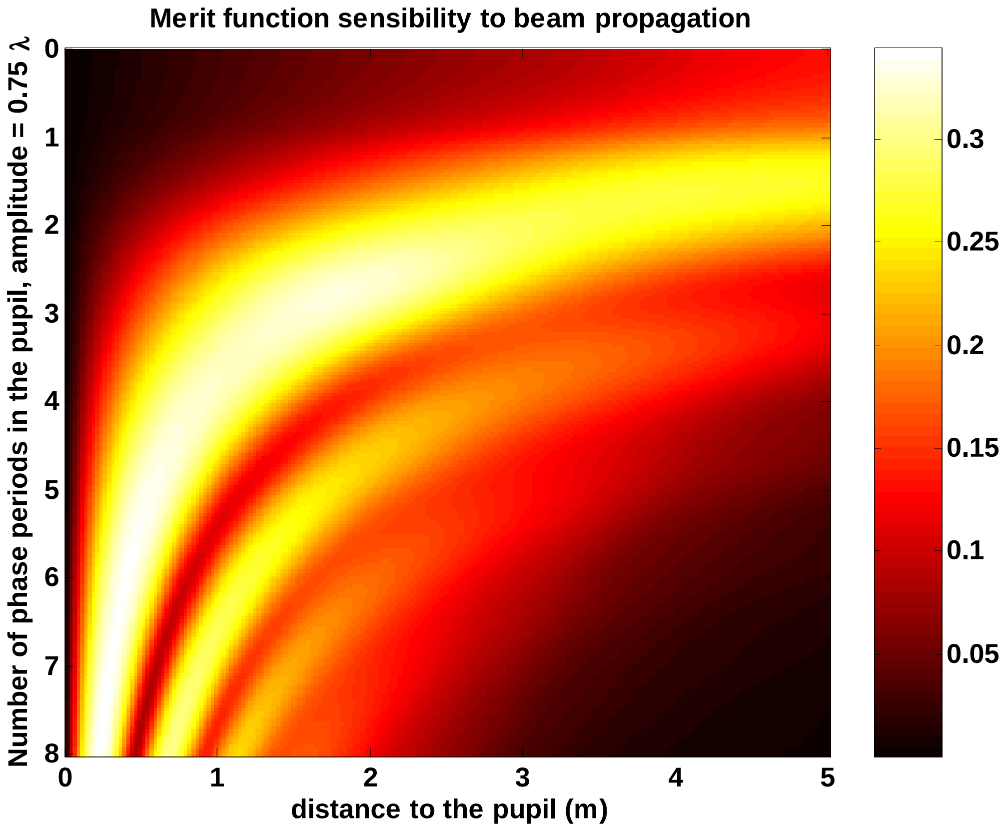


Fig. 4: Merit function sensibility of the number of phase periods in the pupil and function of the distance to the pupil for a 3 mm diameter 633 nm laser beam. This chart shows that high frequency components of the phase modify mainly the beam at short propagating distance and vice versa. It exemplifies the choice of measurement planes.

7. CONVERGENCE IMPROVEMENT

There is no mathematical guarantee of the uniqueness of the solution [10], and there is no guarantee that the phase solution describes a surface. During the iterative process phase jumps can appear in such a way unwrapping the phase profile is impossible. These phase jumps called residues can annihilate by pair as soon as they join together, forming a closed loop. The unwrapping process is then possible and the solution physically acceptable.

Convergence acceleration by a gradient method [11] is a good way to get rid of these problems. The iterative gradient is calculated every second iteration and is used to speed up the next iteration.

Figure 5 show the phase obtained with the basic algorithm, a branch cut appeared and will not disappear even after 600 iterations. The figure 6 gives the result obtained with convergence acceleration, the result is physically acceptable and good convergence is obtained here after 86 iterations.

Successful calculus of thousands of phase profiles on a deformable laser beam gives us confidence on this approach.

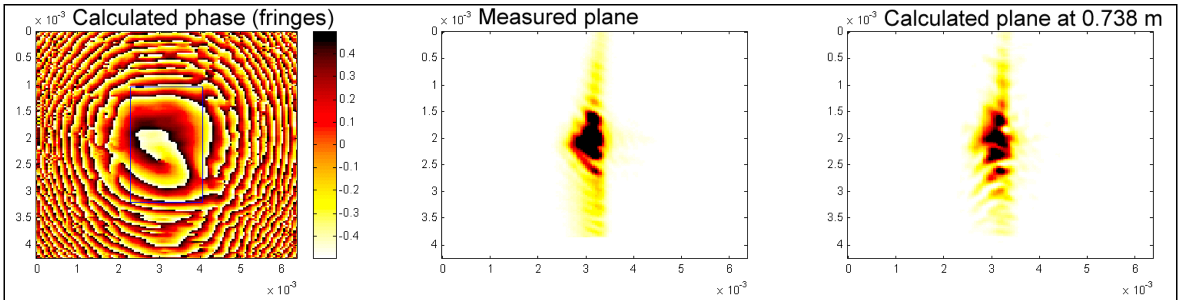


Fig. 5: Phase profile showing bad convergence process and phase residue (600 iterations)

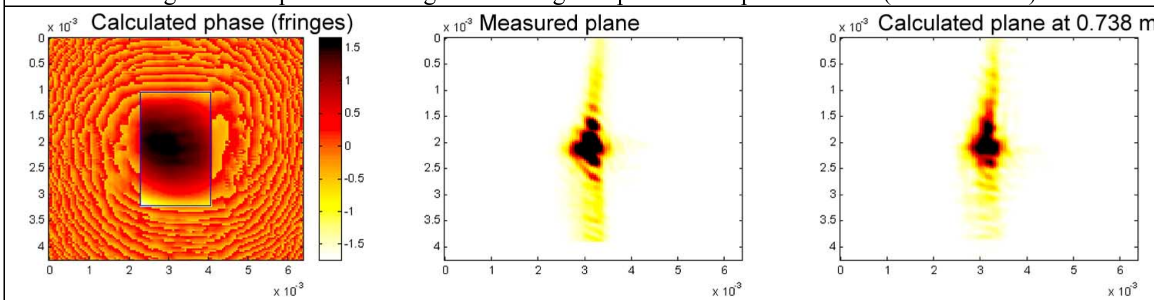


Fig. 6: Same data as Fig. 5, but good convergence process with convergence acceleration by a gradient method (86 iterations)

8. EXPERIMENTAL RESULTS

Validation was needed for the newly designed MIROMA system. The phase information delivered by the MIROMA analyzer was used to propagate a laser beam. Intensity measurements were then realized in several planes in order to verify the accordance with the experimental data. It is worth noting that this method provides an estimation of the phase information that permits an accurate determination of the intensity images from near field to far field.

Figure 7 shows the case of a krypton laser beam Ø2.8 mm; a non-polished glass thin plate has distorted this beam. Three planes were used for the phase reconstitution (0; 34 mm; 800 mm). Four more planes were used for validation (from –34 mm to 800 mm). Images measured in all the seven planes were in excellent accordance with images calculated from the three planes only.

The same calculation has been made with a dye laser beam with a rectangular pupil. Excellent results are also obtained on figure 8.

9. CONCLUSION

Precise characterization of laser beam propagation is often needed. That was the reason why a phase analyzer using a MIROMA algorithm was designed in order to determine the complex field of a laser beam in any diffraction plane. The algorithm is a generalization of the well known Gerschberg and Saxton algorithm. Wave reconstruction is based on an iterative method using multiple data acquisitions in at least three planes. The use of more planes is not necessary according to results on laser beam measurements. Beam diffraction is then directly calculated in any other plane. This method provides a phase estimation that allows an accurate determination of the intensity images from near field to far field.

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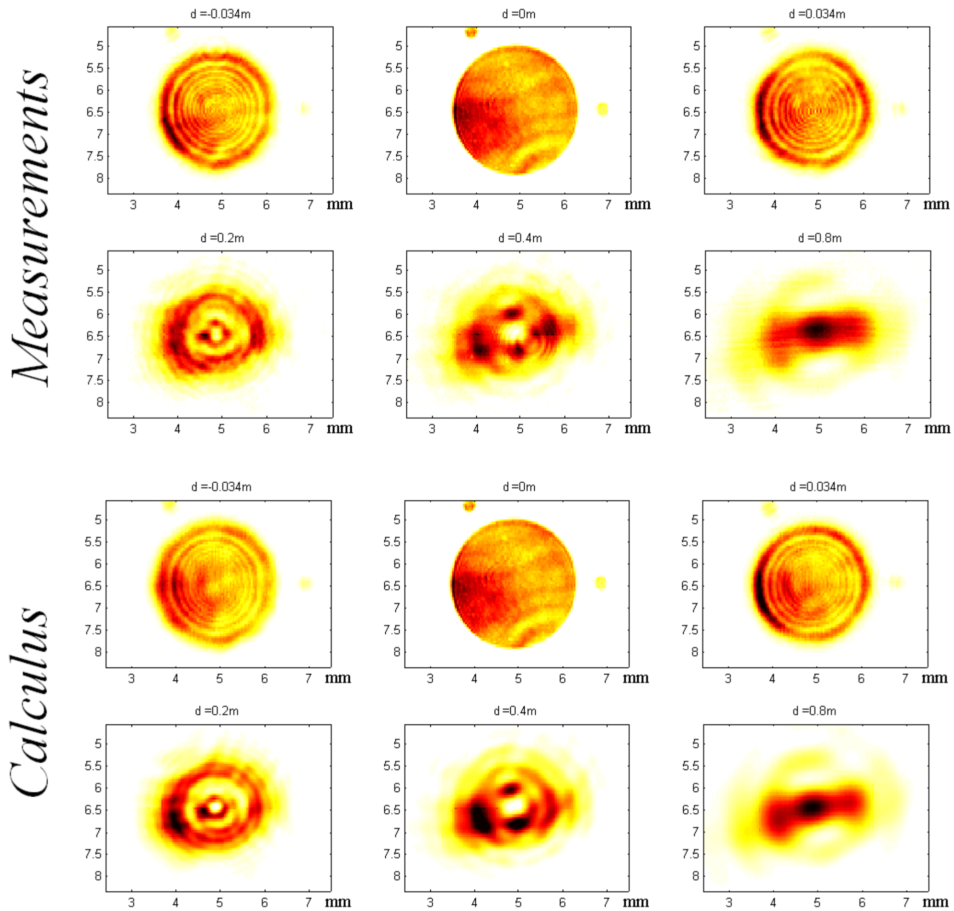

Fig. 7 : Krypton laser beam images in six different planes from –34mm to 800mm, measured images and calculated images are indistinguishable. The differences are in the magnitude order of the beam fluctuations.

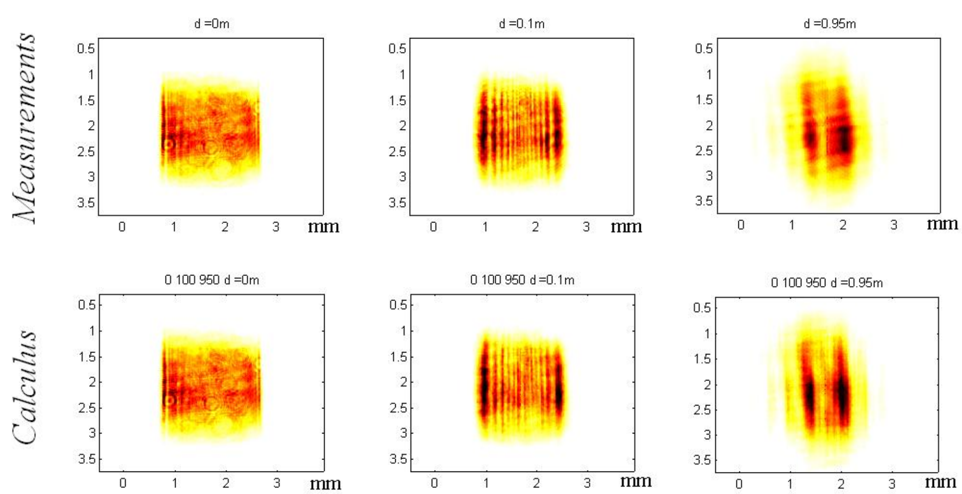

Fig. 8 : Comparison between measurement and calculation of the dye laser beam shapes in three planes (0; 100mm; 950mm)