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Title

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Abstract...

1 Introduction

1.1 Interaction representation

Given the Hamiltonian $H = H_0 + V$ with H_0 and V as noninteracting and interacting part respectively, evolution of states and operators are described by following equations

$$i\frac{\partial}{\partial t}|\psi_I(t)\rangle = V_I(t)|\psi_I(t)\rangle$$

$$-i\frac{\partial O_I(t)}{\partial t} = [H_0, O_I(t)]$$

with states and operators defined as

$$|\psi_I\rangle = e^{iH_0t}|\psi_S(t)\rangle$$

$$O_I(t) = e^{iH_0t}O_Se^{-iH_0t}$$

The evolution of the wavefunction is thus

$$|\psi_I(t)\rangle = U(t)|\psi_I(0)\rangle$$

so that

$$i\frac{\partial U(t)}{\partial t} = V(t)U(t)$$

using T-operator we obtain time-ordered exponential

$$U(t) = T\left[exp\left\{-i\int_0^t V(t')dt'\right\}\right]$$

1.2 Finite temperature

Switching from t to $i\tau$ with $\tau \in \{0, \beta = \frac{1}{k_B T}\}$ we obtain

$$U(\tau) = T\left[exp\left\{-\int_0^\tau V(\tau')d\tau'\right\}\right]$$

We can relate the partition function to the evolution operator as follows

$$Z = Tr\left[e^{-\beta H}\right] = Z_0 \left\langle U(\beta)\right\rangle_0$$

where

$$Z_0 = Tr\left[e^{-\beta H_0}\right]$$

$$\langle A\rangle_0 = \frac{Tr\left[e^{-\beta H_0}A\right]}{Z_0}$$

Now we can write ratio of the interacting to the non-interacting partition function

$$\frac{Z}{Z_0} = \langle T\ exp\left[-\int_0^\beta V(\tau)d\tau\right]\rangle$$

1.3 Perturbation expansion

In most general form interaction is

$$V(t) = \tilde{z}(t)c + c^\dagger z(t)$$

where $z(t)$ and $\tilde{z}(t)$ are the anticommuting forces which “create” and “annihilate” particles respectively.

Expanding the time-ordered exponential

$$\frac{Z}{Z_0} = \sum_{n=0}^{\inf} \frac{(-1)^n}{n!}\int_0^\beta ... \int_0^\beta d\tau_1 ... d\tau_n TV(\tau_1)...V(\tau_n)\rangle$$