2.

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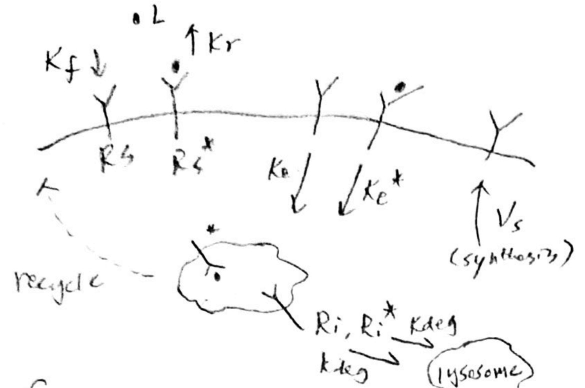
a)

From lecture notes:



$$\begin{cases} \text{Consume: } K_f R_s L_c(z) \\ \text{Generate: } K_r R_s^* + q \\ \text{Transport: } \underset{\text{(Given)}}{k_m(z)[L_b - L_c(z)]} \Rightarrow \frac{1}{n_c} k_m(z)[L_b - L_c(z)] \end{cases}$$

From accumulation = generation − consumption + transfer

$$\Rightarrow 0 = K_r R_s^* + q - k_f R_s L_c(z) + \frac{1}{n_c} k_m(z)[L_b - L_c(z)]$$

$$\frac{k_m(z)}{n_c} L_c(z) + k_f R_s L_c(z) = k_r R_s^* + q + \frac{k_m(z)}{n_c} L_b$$

$$L_c(z) = \frac{k_r R_s^* + q + \frac{k_m(z)}{n_c} L_b}{k_f R_s + \frac{k_m(z)}{n_c}} \quad (\#/m^3)$$

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Shu-Han Wang

2.

b)

$$L_c(z) = \frac{k_r R_s^* + q + \frac{k_m(z)}{n_c} L_b}{k_f R_s + \frac{k_m(z)}{n_c}} \quad (\text{from } 2.\text{a}))$$

1° For very small k_m :

$$\Rightarrow L_c(z) = \frac{k_r R_s^* + q}{k_f R_s} \quad — ①$$

As can seen from eq. ①, the transfer part is neglected when k_m is very small, meaning the $L_c(z)$ changes mainly with the binding of the receptors (including capture and release).

2° For very large k_m :

$$L_c(z) = \frac{\frac{k_m(z)}{n_c} L_b}{\frac{k_m(z)}{n_c}} = L_b \quad — ②$$

From eq. ②, one can observe that now the transfer part dominants and thus $L_c(z)$ mainly equals to L_b, which is the bulk concentration of ligands and that the bindings of receptors do not matter so much.

Shu-Han Wang

2. c)

From lecture notes:

Mass balances:

$$\frac{\partial R_s}{\partial t} = -k_f L R_s + k_r R_s^* - k_e R_s + V_s \quad \text{— ①}$$

$$\frac{\partial R_s^*}{\partial t} = k_f L R_s - k_r R_s^* - k_e^* R_s^* \quad \text{— ②}$$

$$\frac{\partial R_i^T}{\partial t} = k_e R_s + k_e^* R_s^* - k_{deg} R_i^T \quad (R_i^T = R_i^s + R_i^*)$$

$$\frac{\partial R_i^*}{\partial t} = k_e^* R_s^* - k_{deg} R_i^*$$

@ Steady state: $R_s^* = \frac{K_{ss} L}{1+K_{ss}L}\left(\frac{V_s}{K_e^*}\right)$ — ③, $R_i^* = \frac{k_e^*}{k_{deg}} R_s^*$ — ④

$R^*_{Total} = R_s^* + R_i^* = \left(\frac{1}{k_e^*} + \frac{1}{k_{deg}}\right)\left(\frac{K_{ss}L}{1+K_{ss}L}\right) V_s$; $K_{ss} = \frac{k_e^* k_f}{k_e(k_r + k_e^*)}$ (effective binding)

1° With $L_b = 0$, $L_c(z) = \dfrac{k_r R_s^* + q}{k_f R_s + \frac{k_m(z)}{n_c}}$ — ⑤

2° With $L_c K_{ss} \ll 1$, $R_s^* = K_{ss} L_c \left(\frac{V_s}{K_e^*}\right)$ — ③'

3° From ②, at s.s., $0 = k_f L_c R_s - (k_r + k_e^*) R_s^*$

$$\Rightarrow R_s = \frac{(k_r + k_e^*)}{k_f L_c} R_s^* \quad \text{— ⑥}$$

4° put ⑥ in ⑤ $\Rightarrow$ $L_c(z) = \dfrac{k_r R_s^* + q}{\frac{(k_r+k_e^*)}{L_c} R_s^* + \frac{k_m(z)}{n_c}}$

$$(k_r + k_e^*) R_s^* + \frac{k_m(z)}{n_c} L_c(z) = k_r R_s^* + q$$

$$\Rightarrow L_c(z) = (q - k_e^* R_s^*) \frac{n_c}{k_m(z)} \quad \text{— ⑦}$$

5° Put ⑦ in ③'

$$R_s^* = K_{ss}(q - k_e^* R_s^*) \frac{n_c}{k_m(z)} \left(\frac{V_s}{k_e^*}\right) \quad \text{— ⑧}$$

2. c) cont'd

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Rearranging (8),

$$\left(K_{ss}\cancel{k_e^*}\frac{n_c V_s}{k_{m(2)}}\cdot\frac{1}{\cancel{k_e^*}} + 1 \right) R_s^* = K_{ss}\, q\, \frac{n_c}{k_{m(2)}}\frac{V_s}{k_e^*}$$

$$\Rightarrow R_s^* = \frac{K_{ss}\, q\, \frac{n_c V_s}{k_{m(2)} k_e^*}}{K_{ss}\frac{n_c V_s}{k_{m(2)}} + 1} \quad \text{— (9)}$$

6° put (9) in (4)

$$R_i^* = \frac{\cancel{k_e^*}}{k_{deg}}\cdot K_{ss}\, q\, \frac{n_c V_s}{k_{m(2)}\cancel{k_e^*}}\left(\frac{1}{K_{ss}\frac{n_c V_s}{k_{m(2)}} + 1}\right)$$

$$= \frac{K_{ss}\, q\, n_c V_s}{k_{deg} k_{m(2)} \left(K_{ss}\frac{n_c V_s}{k_{m(2)}} + 1\right)} \quad \text{— (10)}$$

7° $R^*_{Total} = R_s^* + R_i^*$ (From Lecture notes)

Thus, (9) + (10) $= R^*_{Total}$

$$R^*_{Total} = \frac{K_{ss}\, q\, \frac{n_c V_s}{k_{m(2)} k_e^*}}{\left(K_{ss}\frac{n_c V_s}{k_{m(2)}} + 1\right)}\left(\frac{k_{deg} k_{m(2)}}{k_{deg} k_{m(2)}}\right) + \frac{K_{ss}\, q\, n_c V_s}{k_{deg} k_{m(2)}\left(K_{ss}\frac{n_c V_s}{k_{m(2)}} + 1\right)}$$

$$= \frac{K_{ss}\, q\, n_c V_s \left(\frac{1}{k_e^*} + \frac{1}{k_{deg}}\right)\cdot \cancel{k_{deg}}}{\cancel{k_{deg}}\, k_{m(2)} \left(K_{ss}\frac{n_c V_s}{k_{m(2)}} + 1\right)}$$

$$= \frac{K_{ss}\, q\, \frac{n_c V_s}{k_{m(2)}}}{K_{ss}\frac{n_c V_s}{k_{m(2)}} + 1}\left(\frac{1}{k_e^*} + \frac{1}{k_{deg}}\right) \quad (\#/\text{cell})$$

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2. d)

Sherwood number:

$$Sh_z = \frac{k_m(z)}{D_L/z} = \left(\frac{\dot{\gamma} z^2}{D_L}\right)^{\frac{1}{3}} \quad \text{with } \dot{\gamma} = 10^2,\ D_L = 10^{-10}$$

$$k_m(z) = \frac{D_L}{z}\left(\frac{\dot{\gamma} z^2}{D_L}\right)^{\frac{1}{3}} = D_L^{\frac{2}{3}} \cdot \dot{\gamma}^{\frac{1}{3}} \cdot z^{-\frac{1}{3}} \quad \text{—— (11)}$$

$$\text{Combining } R_{Total} = \frac{K_{ss} q \frac{n_c V_s}{k_m(z)}}{K_{ss} \frac{n_c V_s}{k_m(z)} + 1} \left(\frac{1}{k_e^*} + \frac{1}{k_{deg}}\right)$$

The plot of the predicted mitotic activity will be shown in other files in Github resp.