CHEME 5440 Final - Shu-Han Wang (sw2227)

1.

a) From Lecture notes:

$$\begin{cases} \dfrac{\partial N_1}{\partial t} = F(D_2) - \gamma_A N_1 & -(2) \\ \dfrac{\partial D_1}{\partial t} = G(N_1) - \gamma_D D_1 & -(3) \\ \dfrac{\partial N_2}{\partial t} = F(D_1) - \gamma_N N_2 & -(4) \\ \dfrac{\partial D_2}{\partial t} = G(N_2) - \gamma_D D_2 & -(5) \end{cases} \xrightarrow[\nu = \gamma_D/\gamma_N]{\tau = \gamma_N t} \begin{cases} \dfrac{\partial N_1}{\partial \tau} = f(D_2) - N_1 \\ \dfrac{\partial D_1}{\partial \tau} = (g(N_1) - D_1)\nu \\ \dfrac{\partial N_2}{\partial \tau} = f(D_1) - N_2 \\ \dfrac{\partial D_2}{\partial \tau} = (g(N_2) - D_2)\nu \end{cases}$$

$f(D')$ and $g(N)$ are from experiments (Sprinzale, Nature 2010)

$$f(D') = \frac{F(D')}{\gamma_N} = \frac{D'^2}{0.1 + D'^2} \quad -(6)$$

$$g(N) = \frac{G(N)}{\gamma_D} = \frac{1}{1 + 10N^2} \quad -(7)$$

Using $\boxed{\nu = \frac{\gamma_D}{\gamma_N} \ll 1}$, meaning $\gamma_N \gg \gamma_D$ and $\boxed{\tau = \gamma_D t}$ to make equations (2) ~ (5) dimensionless

$$\Rightarrow \begin{cases} \nu \dfrac{\partial N_1}{\partial \tau} = f(D_2) - N_1 \\ \dfrac{\partial D_1}{\partial \tau} = g(N_1) - D_1 \\ \nu \dfrac{\partial N_2}{\partial \tau} = f(D_1) - N_2 \\ \dfrac{\partial D_2}{\partial \tau} = g(N_2) - D_2 \end{cases} \text{and } \because \nu \ll 1 \text{, thus} \longrightarrow \begin{cases} \boxed{f(D_2) - N_1 \doteq 0} & -(2^*) \\ \dfrac{\partial D_1}{\partial \tau} = g(N_1) - D_1 & -(3^*) \\ \boxed{f(D_1) - N_2 \doteq 0} & -(4^*) \\ \dfrac{\partial D_2}{\partial \tau} = g(N_2) - D_2 & -(5^*) \end{cases}$$

From (2^*) and (4^*), one can observe that the Notch activity quickly settles into a steady state by approching a certain value.

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1. a) cont'd

Substituting (2*) and (4*) into (3*) and (5*), the equations of Delta can be written as

$$\Rightarrow \begin{cases} \dfrac{\partial D_1}{\partial \tau} = g(f(D_2)) - D_1 & \text{— } (3^{**}) \\ \dfrac{\partial D_2}{\partial \tau} = g(f(D_1)) - D_2 & \text{— } (5^{**}) \end{cases}$$

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1. b) From 1. a) equations (6) and (7) and equations (2*) and (4*) and equations (3**) and (5**),

$$\Rightarrow \frac{\partial D_1}{\partial \tau} = \frac{1}{1+10\left(\frac{D_2^2}{0.1+D_2^2}\right)^2} - D_1$$

$$\frac{\partial D_2}{\partial \tau} = \frac{1}{1+10\left(\frac{D_1}{0.1+D_1}\right)^2} - D_2$$

Phase portrait will be plotted using python and shown in other files in Github resp.