Shu-Han Wang

3.

a)

$$\dot{m}_i = r_{x,i}\,\bar{u}_i - (\mu + \theta_{m,i})\,m_i \qquad i = 1, 2, \ldots N$$

$$\dot{p}_i = r_{L,i}\,\omega_i - (\mu + \theta_{p,i})\,p_i$$

At steady state, $\dot{m}_i, \dot{p}_i = 0$

$$\Rightarrow \quad r_{x,i}\,\bar{u}_i = (\mu + \theta_{m,i})\,m_i^*$$

$$r_{L,i}\,\omega_i = (\mu + \theta_{p,i})\,p_i^*$$

$$m_i^* = \frac{r_{x,i}\,\bar{u}_i}{\mu + \theta_{m,i}}, \quad m_i^* = K_{x,i}\,\bar{u}_i \quad \therefore \quad K_{x,i} = \frac{r_{x,i}}{\mu + \theta_{m,i}} = 0.575 \ \left(\text{nmol/gDW}\right)$$

(From Prelim 1 Soln)

$$p_i^* = \frac{r_{L,i}\,\omega_i}{\mu + \theta_{p,i}} \qquad \text{---} \ (1)$$

From Lecture $\rightarrow$

$$r_{L,i} = \frac{m_i}{\tau_{L,i} K_{L,i} + (\tau_{L,i}+1)\,m_i}\left(K_{E,i}^{L}\,R_{L,T}\right)$$

take limit for $\tau_{L,i} K_{L,i} \gg (\tau_{L,i}+1)\,m_i$

$$\Rightarrow \quad r_{L,i} = \frac{m_i}{\tau_{L,i}\,K_{L,i}}\left(K_{E,i}^{L}\,R_{L,T}\right) \qquad \text{---} \ (2)$$

Put (2) in (1)

$$p_i^* = \frac{m_i^*\left(K_{E,i}^{L} R_{L,T}\right)\omega_i}{\tau_{L,i} K_{L,i}\,(\mu + \theta_{p,i})}$$

$$\Rightarrow \quad p_i^* = \frac{r_{x,i}\,\bar{u}_i\left(K_{E,i}^{L} R_{L,T}\right)\omega_i}{\tau_{L,i} K_{L,i}\,(\mu + \theta_{p,i})(\mu + \theta_{m,i})}$$

$$= \left(\frac{r_{x,i}}{\mu + \theta_{m,i}}\right)\cdot\left(\frac{K_{E,i}^{L} R_{L,T}}{\tau_{L,i} K_{L,i}(\mu + \theta_{p,i})}\right)\bar{u}_i \cdot \omega_i$$

$\downarrow$ $K_{x,i}$ $\qquad$ $\downarrow$ $K_{L,i}$

$$= K_{x,i}\,K_{L,i}\,\bar{u}_i\,\omega_i$$

#

Shu-Han Wang

3.

b)

$$P_i^* \simeq K_{x,i} K_{L,i} \bar{u}_i \omega_i = \left(\frac{\gamma_{x,i}}{\mu + \theta_{m,i}}\right)\left(\frac{k_{Ei}^L R_{L,T}}{\tau_{L,i} K_{L,i} (\mu + \theta_{p,i})}\right) \bar{u}_i \omega_i$$

	Parameter	Value (unit)	Source	Denote/Calculation
✓	$K_{x,i}$	$0.575 \left(\frac{nmol}{gDW}\right)$	Prelim 1 Soln	Transcription gain
✓	$k_{E,i}^L$	$3.3 \left(\frac{1}{min}\right)$	Calculated	$\frac{k_{L,i}}{L_p} = k_{E,i}^L$, Elongation rate
✓	$R_{L,T}$	$2.3 (\mu M)$	PS2 Soln	Ribosome concentration
✓	$\tau_{L,i}$	$1.5 (s)$	Given	Translation initiation time
	$k_{L,i}$	$16.5 \left(\frac{aa}{s}\right)$	PS2 Soln	Translation elongation rate
✓	μ	$0.01733 \left(\frac{1}{min}\right)$	Calculated	$\frac{\ln 2}{\tau_D} = \mu$, Growth rate
✓	$\theta_{p,i}$	$4.814 \times 10^{-4} \left(\frac{1}{min}\right)$	Calculated	$\theta_{p,i} = \frac{\ln 2}{\tau_{1/2}}$, Degrade rate
✓	ω_i	1 (-)	Given	Translational control function for protein i
	L_p	300 (aa)	Given	Protein length
	τ_D	40 (min)	Given	E. coli doubling time
	$\tau_{1/2}$	24 (hr)	Given	Half life of P_i
✓	$K_{L,i}$	$200 (\mu M)$	Given	Translation saturation coeff

$$\Rightarrow P_i^* = 0.575 \left(\frac{nmol}{gDW}\right) \times \frac{3.3 \left(\frac{1}{min}\right) \cdot 2.3 (\mu M) \cdot 1}{1.5 (s) \cdot 200 (\mu M)\left(0.01733 \left(\frac{1}{min}\right) + 4.814 \times 10^{-4} \left(\frac{1}{min}\right)\right)} \cdot \bar{u}_i$$

$$= 0.575 \left(\frac{nmol}{gDW}\right) \times 1.4204 \left(\frac{1}{s}\right) \bar{u}_i$$

⇒ Using data and file from Prelim 1, the plot will be shown by using Jupyter notebook (python) (uploaded on Github)