1.

(a) The specific material balance equations are given by:

$$\begin{cases} \dot{m}_i = r_{X,i} u_i - (\mu + \theta_{m,i}) m_i + \lambda_i & i = 1, 2, \dots N \quad ① \\ \dot{p}_i = r_{L,i} w_i - (\mu + \theta_{p,i}) p_i \end{cases}$$

where μ is the dilution term.

Dilution: (intracellular)

$$\dot{n}_{x,acc,j} = \dot{n}_{x,in,j}^{0} - \dot{n}_{x,out,j}^{0} + \dot{n}_{x,gen,j} \quad j = 1, 2, \dots M$$

$$\Rightarrow \dot{n}_{x,acc,j} = \dot{n}_{x,gen,j} \quad j = 1, 2, \dots, M$$

$$\Rightarrow \frac{d}{dt} \int_B x_j \, dB = \int_B (\dots) \, dB \quad j = 1, 2, 3, \dots M$$

$$\Rightarrow \dot{x}_j = (\dots) - x_j B^{-1} \dot{B} \quad j = 1, 2, \dots M$$

and $\dot{B} = \dot{x} V_R + \dot{V}_R x$

$$B^{-1}\dot{B} = \frac{\dot{x}}{x}\frac{V_R}{V_R} + \frac{\dot{V}_R}{x}\cdot\frac{x}{V_R} = \frac{\dot{x}}{x} + \frac{\dot{V}_R^{\,0}}{V_R}$$ not changing volume

$$\Rightarrow B^{-1}\dot{B} = \frac{\dot{x}}{x} = \mu$$

However, this is cell-free system, i.e. there's no dilution effect

$$\Rightarrow \mu = 0$$

$$\Rightarrow \begin{cases} \dot{m}_i = r_{X,i} u_i - \theta_{m,i} m_i + \lambda_i \\ \dot{p}_i = r_{L,i} w_i - \theta_{p,i} p_i \end{cases}$$

#