CHEME 5440 - HW#4
Spring 2020
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1.(a)

From $\frac{d[x^*]}{dt}=0$, $\frac{d[y^*]}{dt}=0$ and $\frac{d[R^*]}{dt}=0$ for steady-state, we get:

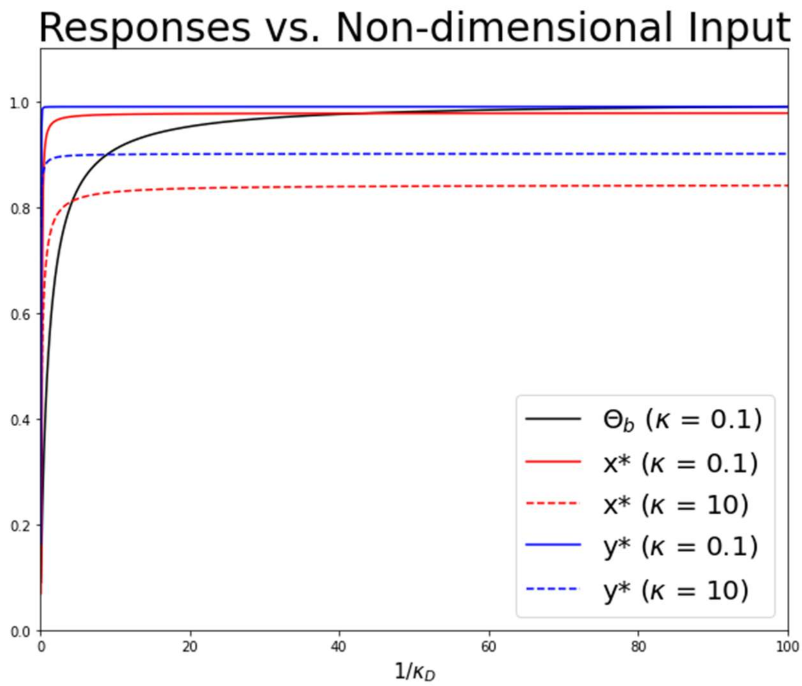
$$x^* = -\frac{1+\kappa_1+\frac{V_1}{V_2}\times(-1+\kappa_2)+\sqrt{\left(1+\kappa_1+\frac{V_1}{V_2}\times(-1+\kappa_2)\right)^2+4\times\left(-1+\frac{V_1}{V_2}\right)\times\frac{V_1}{V_2}\times\kappa_2}}{2\times\left(-1+\frac{V_1}{V_2}\right)}$$

$$y^* = -\frac{1+\frac{V_3}{V_4}-\kappa_3-\frac{V_3}{V_4}\kappa_4+\sqrt{\left(1+\kappa_3+\frac{V_3}{V_4}\times(-1+\kappa_4)\right)^2+4\times\left(-1+\frac{V_3}{V_4}\right)\times\frac{V_3}{V_4}\times\kappa_4}}{2\times\left(-1+\frac{V_3}{V_4}\right)}$$

$\theta_B = \frac{1-\theta_B}{\kappa_D}$, thus $\theta_B = \frac{1}{1+\kappa_D} = \frac{\frac{1}{\kappa_D}}{\frac{1}{\kappa_D}+1}$

1.(b)

For $\kappa_1 = \kappa_2 = \kappa_3 = \kappa_4 = \kappa$



*Code in github resp.

1.(c)

*Code in github resp.

- Did not success

1.(d)

Using numpy.interp(), we can obtain the y value when given a x value on a plot.

θ_B percentage change : 0.41666666666666666

x* (κ = 0.1) percentage change : 1.4664528944969721

x* (κ = 10) percentage change : 0.24254211403159665

y* (κ = 0.1) percentage change : 2.3916752044004874

y* (κ = 10)percentage change : 0.04332996901435626

*Code in github resp.

1.(e)

The consequences of zero-order sensitivity are that the signals have very sharp responses for lower values of κ; on the other hand, there are smaller differences between input and output when the κ value increases.

2.(a)

Using Python scipy, we could calculate steady-state [A], [B] and [C] without inhibitors to be approximately:

[A] = 1.11

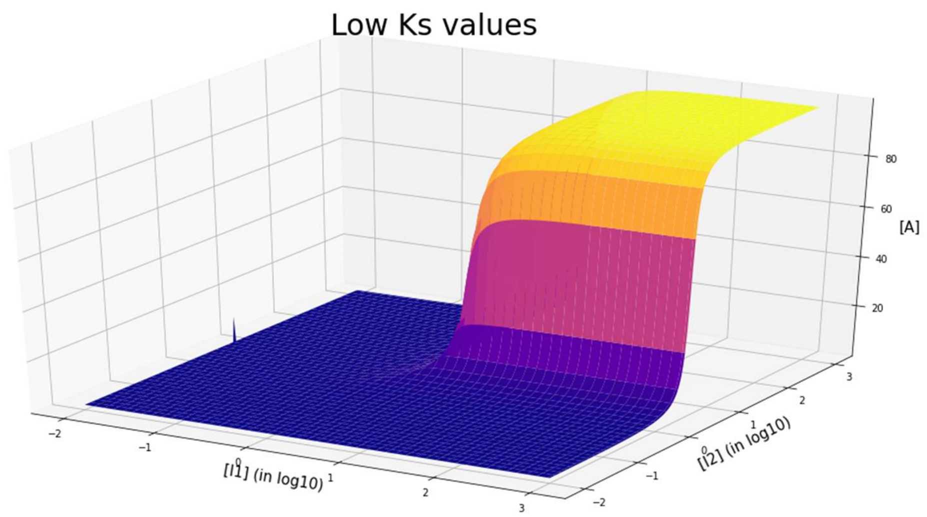
[B] = 49.45

[C] = 49.45

*Code in github resp.

2.(b)

Using Matplot, we can construct the 3D plot with a mesh of 190:



*Code in github resp.

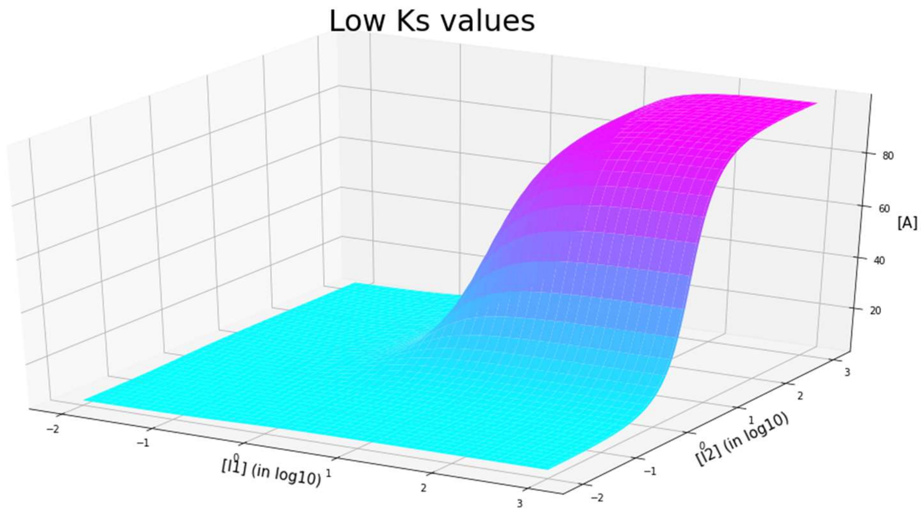
2.(c)

From part (b) plot, we can see as [I1] and [I2] increase, the concentration of A also increases, meaning that reactions A→B and A→C are inhibited. From part (a), we can also see that when there are no inhibitors, B and C concentrations are quite high and [A] is low. However, if there is only I1, the level of A does not increase. Both I1 and I2 concentrations should be high to result in a significant increase in [A].

Thus, the relationship should be an AND gate circuit, with [I1] and [I2] being the input and [A] being the output.

2.(d)

Similar to part (b), we can construct the 3D plot while $K_{S1}=K_{S2}=K_{S3}=K_{S4}=35.0$:



*Code in github resp.

From the above plot, we can observe that the slope for the change in [A] is much shallower compared to that of the plot in part (b), which could be that the gate is less sensitive and has more [A] while either [I1] or [I2] is not at all high. This can be considered a "fuzzy" gate since the gate output results are not as definite as part (b).

2.(e)

From Goldbeter and Goshland's (1981) cell signaling work, they demonstrated that zero-order ultrasensitivity is important for tuning sharp responses. They showed that the dominant kind will have a sharp response if the K value is low, and this response is very similar to a logical gate. However, it the K value is high, then this could lead to more "fuzzy" results.