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(sw2227)

2. (a)

$$\frac{d\tilde{X}}{d\tilde{t}} = \frac{\tilde{\alpha}_X + \tilde{\beta}_X S}{1 + S + \left(\frac{Z}{\tilde{Z}_X}\right)^{n_{ZX}}} - \tilde{\delta}_X \tilde{X}$$

$$\frac{d\tilde{Y}}{d\tilde{t}} = \frac{\tilde{\alpha}_Y + \tilde{\beta}_Y S}{1 + S + \left(\frac{\tilde{X}}{\tilde{X}_Y}\right)^{n_{XY}}} - \tilde{\delta}_Y \tilde{Y} \qquad \text{(Equation 2)}$$

$$\frac{d\tilde{Z}}{d\tilde{t}} = \frac{\tilde{\alpha}_Z}{1 + \left(\frac{\tilde{X}}{\tilde{X}_Z}\right)^{n_{XZ}} + \left(\frac{\tilde{Y}}{\tilde{Y}_Z}\right)^{n_{YZ}}} - \tilde{\delta}_Z \tilde{Z} \leftarrow$$

In the paper, the $\tilde{Z}$ is without a tilde, which should be an error

$\Rightarrow$ For Figure 1-B, only X and Z is considered, therefore,

$$\begin{cases} \dfrac{d\tilde{X}}{d\tilde{t}} = \dfrac{\tilde{\alpha}_X + \tilde{\beta}_X S}{1 + S + \left(\frac{\tilde{Z}}{\tilde{Z}_X}\right)^{n_{ZX}}} - \tilde{\delta}_X \tilde{X} & \text{——— (1)*} \\ \dfrac{d\tilde{Z}}{d\tilde{t}} = \underbrace{\dfrac{\tilde{\alpha}_Z}{1 + \left(\frac{\tilde{X}}{\tilde{X}_Z}\right)^{n_{XZ}}}}_{\text{production term}} - \underbrace{\tilde{\delta}_Z \tilde{Z}}_{\text{degradation term}} & \text{——— (2)*} \end{cases}$$

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Note that $\tilde{X}$ is suppressed by $\tilde{Z}$ and $\tilde{Z}$ is suppressed by $\tilde{X}$ by looking at the denominator.

* Figure 1-B is uploaded on Github resp.

2. (b)

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From the STAR methods,

Measuring time in units of the degradation rate of protein X, all temporal variables can be non-dimensionalized

as, $\begin{cases} \delta_z = \dfrac{\tilde{\delta}_z}{\tilde{\delta}_x} \\ \delta_x = \dfrac{\tilde{\delta}_x}{\tilde{\delta}_x} = 1 \\ t = \tilde{t}\,\delta_x \longleftarrow ! \end{cases}$ provided by the paper.

$\Rightarrow$ should be $t = \tilde{t}\,\tilde{\delta}_x$

However, for $t = \tilde{t}\,\delta_x$, since δ_x is already non-dimensionalized, it could not non-dimensionalize $\tilde{t}$, thus there is an error.

concentration and rates can be non-dimensionalized using the timescale of $\tilde{\delta}_x$ and the production rate $\tilde{\alpha}_z$

$$\alpha_x = \frac{\tilde{\alpha}_x}{\tilde{\alpha}_z}, \quad \beta_x = \frac{\tilde{\beta}_x}{\tilde{\alpha}_x}$$

$$\alpha_z = \frac{\tilde{\alpha}_z}{\tilde{\alpha}_z} = 1$$

$$X_z = \frac{\tilde{X}_z\,\tilde{\delta}_x}{\tilde{\alpha}_z}, \quad Z_x = \frac{\tilde{Z}_x\,\tilde{\delta}_x}{\tilde{\alpha}_z}$$

$$X = \frac{\tilde{X}\,\tilde{\delta}_x}{\tilde{\alpha}_z}, \quad Z = \frac{\tilde{Z}\,\tilde{\delta}_x}{\tilde{\alpha}_z}$$

using the above non-dimensional system equations and substitute into $(1)^*$ and $(2)^*$:

For $(1)^*$:

$$\frac{d\tilde{X}}{d\tilde{t}} = \frac{d\left(\frac{X\tilde{\alpha}_z}{\tilde{\delta}_x}\right)}{d\left(\frac{t}{\tilde{\delta}_x}\right)} = \frac{dX}{dt}\tilde{\alpha}_z = \frac{\tilde{\alpha}_z(\alpha_x) + S(\beta_x\tilde{\alpha}_x)}{1 + S + \left(\dfrac{\frac{Z\tilde{\alpha}_z}{\tilde{\delta}_x}}{\frac{Z_x\tilde{\alpha}_z}{\tilde{\delta}_x}}\right)^{n_{zx}}} - \cancel{\tilde{\delta}_x}\left(\frac{X\tilde{\alpha}_z}{\cancel{\tilde{\delta}_x}}\right)$$

$$\Rightarrow \frac{dX}{dt} = \frac{\alpha_x + S\beta_x}{1 + \left(\frac{Z}{Z_x}\right)^{n_{zx}}} - X \quad \text{——} \quad (1)^{**}$$

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For $(2)^*$:

$$\frac{d\tilde{z}}{d\hat{t}} = \frac{d\left(\frac{z\tilde{\alpha}_z}{\tilde{\delta}_x}\right)}{d\left(\frac{t}{\tilde{\delta}_x}\right)} = \frac{dz}{dt}\tilde{\alpha}_z = \frac{\alpha_z(\tilde{\alpha}_z)}{1+\left(\frac{\frac{x\tilde{\alpha}_z}{\tilde{\delta}_x}}{\frac{x_z\tilde{\alpha}_z}{\tilde{\delta}_x}}\right)^{n_{yz}}} - \tilde{\delta}_z\left(\frac{\tilde{\alpha}_z z}{\tilde{\delta}_x}\right)$$

$$\Rightarrow \frac{dz}{dt} = \frac{1}{1+\left(\frac{x}{x_z}\right)^{n_{yz}}} - \delta_z Z \quad \text{——} \quad (2)^{**}$$

#.

2. (c)

Under steady-state,

$$0 = \frac{\alpha_x + S\beta_x}{1+\left(\frac{z}{z_x}\right)^{n_{zx}}} - x$$

$$0 = \frac{\alpha_z}{1+\left(\frac{x}{x_z}\right)^{n_{xz}}} - \delta_z Z$$

The plot is on python, and it showed that the lines in Figure 1-B are reproducible.