What are the Most Important Statistical Ideas of the Past 50 Years?

Andrew Gelman and Aki Vehtari

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Introduction

- Overview of 8 significant statistical ideas from 1970 to 2021.
- Authors: Andrew Gelman and Aki Vehtari.
- Purpose: To provoke thought and discussion about modern statistical innovations and their impact on data science.

Authors' Background

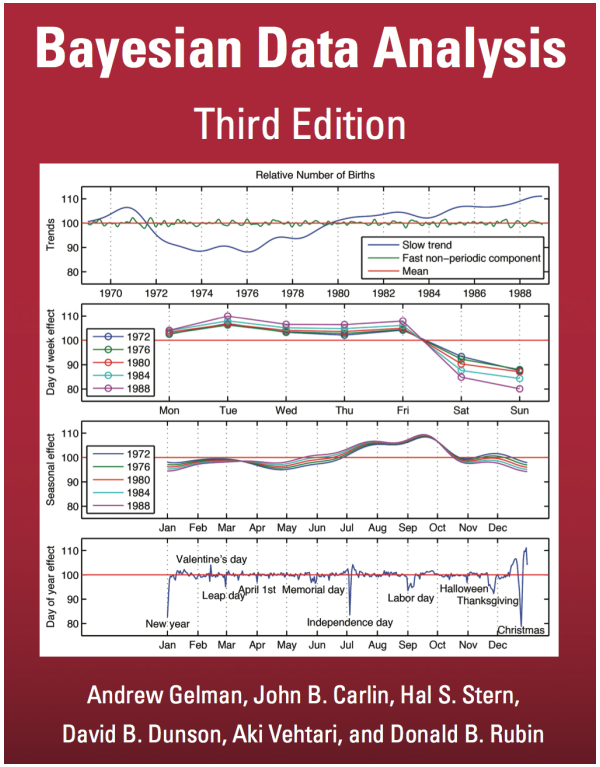
Andrew Gelman

- **Andrew Gelman**:
  - Professor of Statistics and Political Science, Columbia University.
  - Renowned for Bayesian statistics and multilevel modeling.

Aki Vehtari

- **Aki Vehtari**:
  - Professor of Computational Probabilistic Modeling, Aalto University.
  - Focused on Bayesian computation and model assessment.

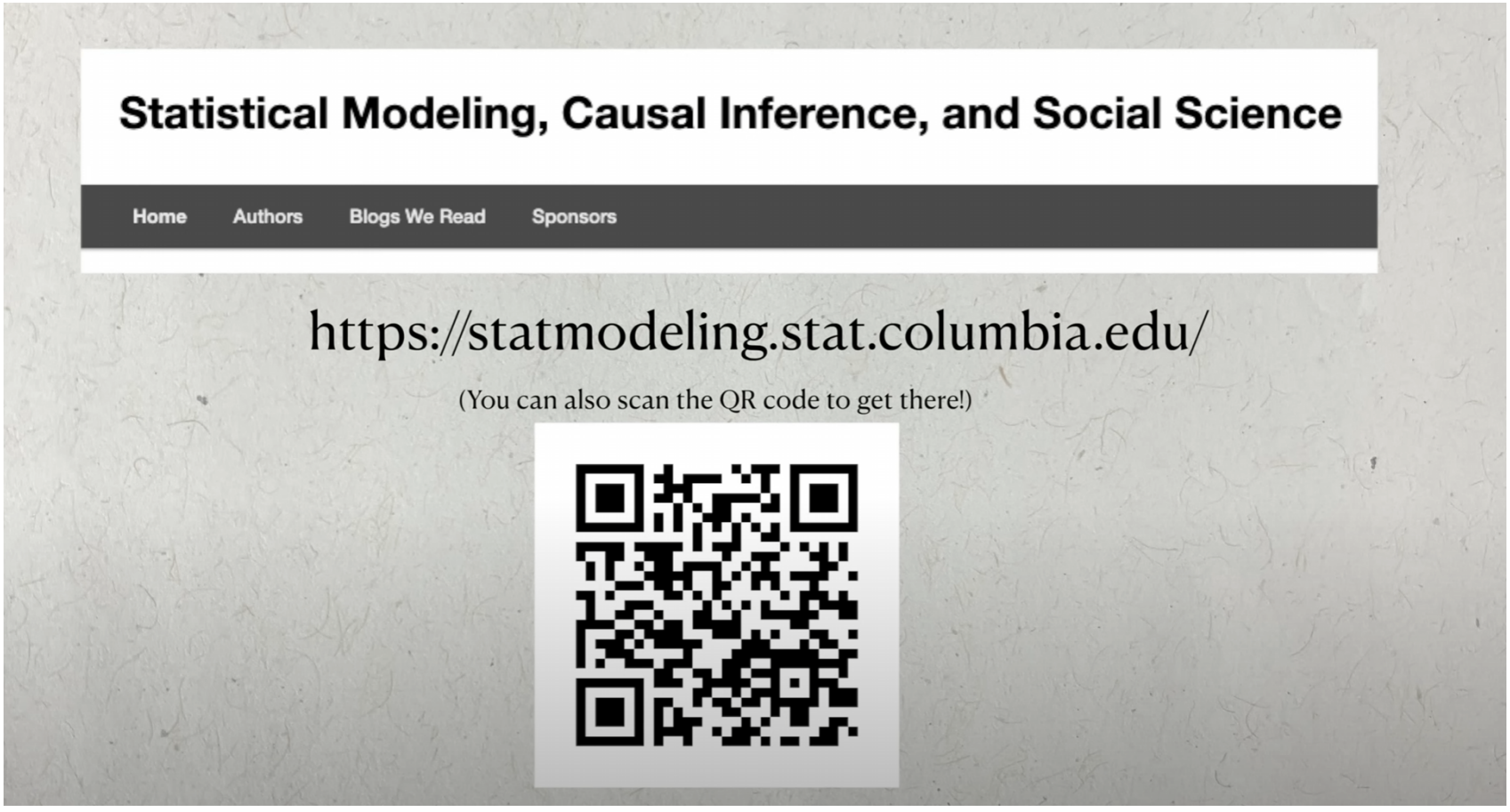
Authors' Background (cont.)



Bayesian Data Analysis

- **The Book: Bayesian Data Analysis**
  - Written by Andrew Gelman, Aki Vehtari, and others.
  - Widely regarded as the foundational text ("the bible") for Bayesian practitioners.
  - Covers theory, computation, and applied Bayesian methods.
- **Their Authority on the Topic**
  - Through this book, Gelman and Vehtari have shaped the modern understanding of Bayesian statistics.
  - Their extensive research and contributions give them unique insights to answer: *What are the most important statistical ideas of the past 50 years?*

Gelman's Blog



Overview of the Paper

- Timeframe: 1970 to 2021, focusing on the development of modern statistics.
- 8 statistical ideas selected based on their influence on statistical theory, computation, and applications.
- Published to the *Journal of the American Statistical Association* as an **essay**, not a research manuscript.
- Acknowledges that no definitive list can encompass all significant ideas.

Key Statistical Concepts

The 8 Statistical Ideas Covered in the Paper:

1. Counterfactual Causal Inference
2. Bootstrapping and Simulation-Based Inference
3. Overparameterized Models and Regularization
4. Bayesian Multilevel Models
5. Generic Computation Algorithms
6. Adaptive Decision Analysis
7. Robust Inference
8. Exploratory Data Analysis (EDA)

Purpose: These concepts reflect significant innovations in statistical theory, computation, and application from the last 50 years.

The Challenge of Observational Data

- **In an ideal world: Experimental Data**
  - Researchers control interventions.
  - Participants are assigned to "treatment" and "control" groups.
  - Enables *causal claims* about the effect of an intervention.
- **In the real world: Observational Data**
  - Researchers cannot control who receives the intervention.
  - Only allows for *correlational claims*.
  - Several fields of study are prone to having more observational data i.e. statistics, econometrics, psychometrics, epidemiology, and computer science.

Challenge: How can we infer causality in the absence of experimental data?

Introducing Counterfactuals

- **Model Setup:**
  - Let X represent attending class:
    - $x = 0$: Did not attend class.
    - $x = 1$: Did attend class.
  - In reality, I choose to attend class and receive a score Y_1 on my paper.

- **Key Question:** Did attending class improve my score?
  - To answer this, we need to consider an alternative reality:
  - What score (Y_0) would I have received if I *did not* attend class?

Key Idea: Counterfactuals

- Y_0: The score in the unobserved reality (if I did not attend class).
- This unobserved reality is called the *counterfactual*.

01: Counterfactual Causal Inference

- Allows causal inference using observational data.
- Framework based on "potential outcomes" or "counterfactuals."

$$\text{Causal Effect: } Y(1) - Y(0)$$

- $Y(1)$: Outcome if treated.
- $Y(0)$: Outcome if untreated.
- Challenge: Only one outcome is observed.

02: Bootstrapping and Simulation-Based Inference

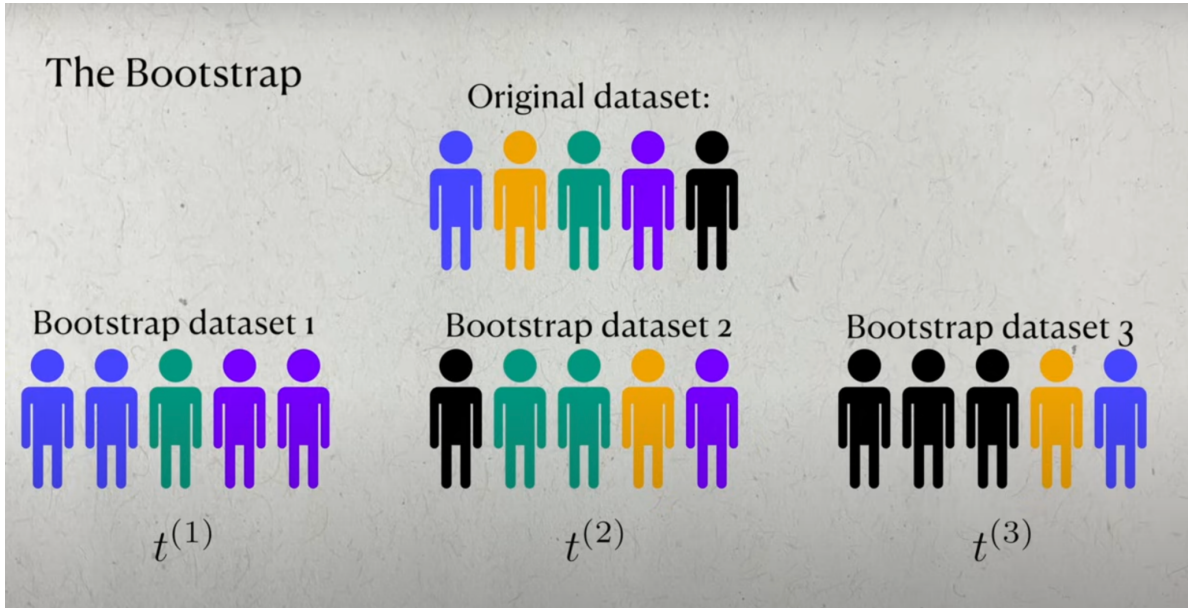
- Introduced by Bradley Efron (1979).
- Resampling technique to estimate sampling distributions without assumptions about data distribution.

Algorithm:

1. Resample the dataset with replacement.
2. Compute the statistic of interest (e.g., mean).
3. Repeat n times to estimate variability.

02: Bootstrapping and Simulation-Based Inference (cont.)

- **The Bootstrap Algorithm:**



- $\hat{F}(t)$ Goal: Estimate the CDF of some statistic.
- **Intuition:** Data sampled from the original dataset resembles a new dataset.
- For each bootstrap dataset, calculate a statistic of interest and derive its distribution:

$$t^{(1)}, t^{(2)}, t^{(3)} \rightarrow \hat{F}(t)$$

Simulation-Based Inference (cont.)

- **Example using Bayes' Rule:**

$$P(\theta \mid X) = \frac{P(X \mid \theta)\, P(\theta)}{P(X)}$$

- **Bayesian Approach:**
  - Bayesian encodes knowledge in the form of prior distributions.
  - Conduct **prior predictive checks:**

$$P(\Theta) \rightarrow \Theta \rightarrow X$$

    Checks if the data generated from the prior distribution aligns with realistic or expected values.
  - Conduct **posterior predictive checks:**

$$P(\Theta \mid X) \rightarrow \Theta \rightarrow X^*$$

    Evaluates how well the posterior distribution predicts new or observed data points.
- **Rise of Computational Power:**
  - Advances in computational power have made simulations easier and more widely applicable.

03: Overparameterized Models and Regularization

- Adding parameters to a model allows for more flexibility and complexity:

  Model 1: Simple Linear Regression

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

  - Explains the relationship between an outcome (Y) and a predictor (X).
  - Limitation: Cannot account for time-varying effects or individual differences.

  Model 2: Incorporating More Parameters

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 \text{time} + \beta_3 (X_i \cdot \text{time}) + \epsilon_i$$

  - Adds parameters for time and its interaction with treatment, increasing flexibility.
  - Still assumes uniform responses across individuals.

03: Overparameterized Models and Regularization (cont.)

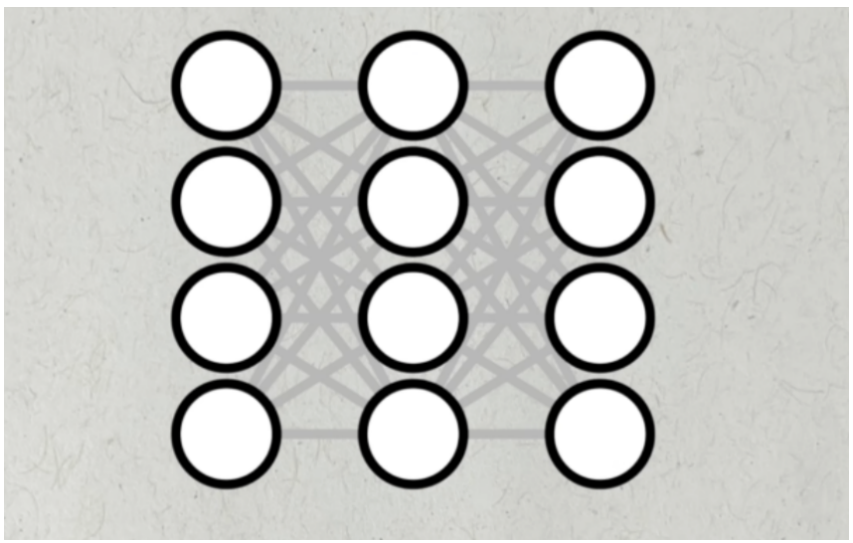
Model 3: Mixed Effects Model (More Parameters, More Flexibility)

$$Y_i = \beta_0 + b_{0i} + \beta_1 X_i + b_{1i} X_i + \beta_2 \text{time} + \beta_3 (X_i \cdot \text{time}) + \epsilon_i$$

- Includes subject-specific random effects (b_{0i}, b_{1i}), allowing for individual-level flexibility, increases model complexity.

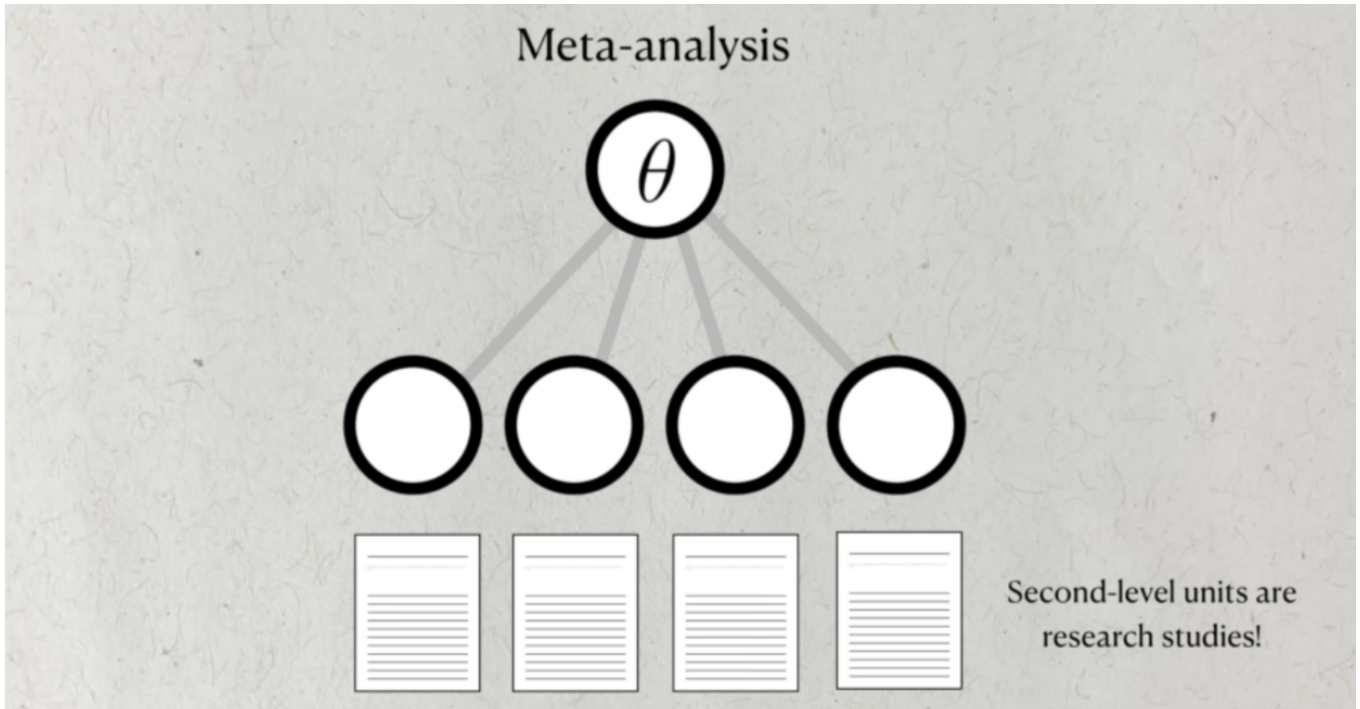
Example: Neural Networks

- Overparameterized models take these models to the extreme, with hundreds or thousands of parameters.
- In neural networks, each edge in the network is associated with a parameter or weight. By making these networks very large, they can approximate a wide variety of functions (**Universal Approximation Theorem**).



04: Bayesian Multilevel Models

- Multilevel (or hierarchical) models assume a structure over parameters.
- These models allow for varying effects at different levels of hierarchy:
  - **First-level units:** Generate treatment effects.
  - **Second-level units:** Generate observed data and can vary depending on the research question.
- Example: Aggregating data across different groups in meta-analysis.



04: Bayesian Multilevel Models (cont.)

- The structure of the multilevel model can be expressed as:

$$y_{ij} = \beta_0 + \beta_1 X_{ij} + u_j + \epsilon_{ij}$$

- Definitions:
  - u_j: Random effect for group j.
  - ϵ_{ij}: Error term for observation i in group j.

Advantage

- Combines individual-level and group-level variability for improved estimates.
- The Bayesian framework offers more flexibility in the modeling approach, as the choice of prior distributions plays a major role in capturing uncertainties and incorporating domain knowledge.

05: Generic Computation Algorithms

- **Importance of Computers in Statistics:**
  - Advances in computational power have been crucial for enabling and implementing complex statistical models.
  - Algorithms like EM and Metropolis-Hastings make it possible to analyze data that would otherwise be computationally infeasible.

The Expectation-Maximization (EM) Model

- **Purpose:** Solves estimation problems when data is incomplete or latent.
- **Process:** Uses available data to construct educated guesses for parameter values in a model ($\hat{\theta} \rightarrow \theta$).
- **Example:** Mixture models with latent classes, where group labels are unknown. The EM algorithm estimates parameters even without direct class information.

Metropolis-Hastings (M-H)

- **Purpose:** Allows sampling from complex distributions that are difficult to handle mathematically.
- **Bayesian Context:**
  - In Bayes' rule, the posterior distribution $P(\theta \mid X)$ can be so complicated that we cannot derive a closed-form formula for it.
  - M-H generates data samples directly from this posterior, enabling us to:
    - Recover key quantities like means, quantiles, and credible intervals.
- **Importance:** Demonstrates the power of computation in Bayesian statistics, as it bypasses analytical challenges.

06: Adaptive Decision Analysis

- **Framework for Decision-Making:**
  - Interim decision points: Collect and analyze data before the experiment concludes.
- **Application:** Stopping clinical stem cell trials early:
  - If evidence suggests treatment is futile, the trial can be stopped early.
  - If treatment shows early promise, the trial may also be stopped to proceed with broader application.
  - A well-designed trial allows us to control the consequences of our decision, such as type I error.
- **Frequentist Considerations:**
  - Stopping early may hurt power and complicate the interpretation of the p-value.

07: Robust Inference

- Statisticians often make assumptions about their models. When assumptions are reasonable, results can generally be presumed trustworthy.
- **However, assumptions are not always right.**
- **Definition:** Robust statistics provide trustworthy analyses even when some assumptions are incorrect.

Example: Outliers

- Outliers violate assumptions about the distribution of data (e.g., normality).
- Median-based estimators are less sensitive to outliers compared to mean-based estimators.

Key Insight

Robust inference allows valid results even when data deviates from assumptions.

Propensity Score Matching

- **Definition:** A method to match people in a treatment group to people in a control group who are very similar to them.
- Propensity Score Matching requires two models:
  - **Model 1: Exposure-Outcome Model**

$$Y_i = \beta_0 + \beta_1 T_i + E_i$$

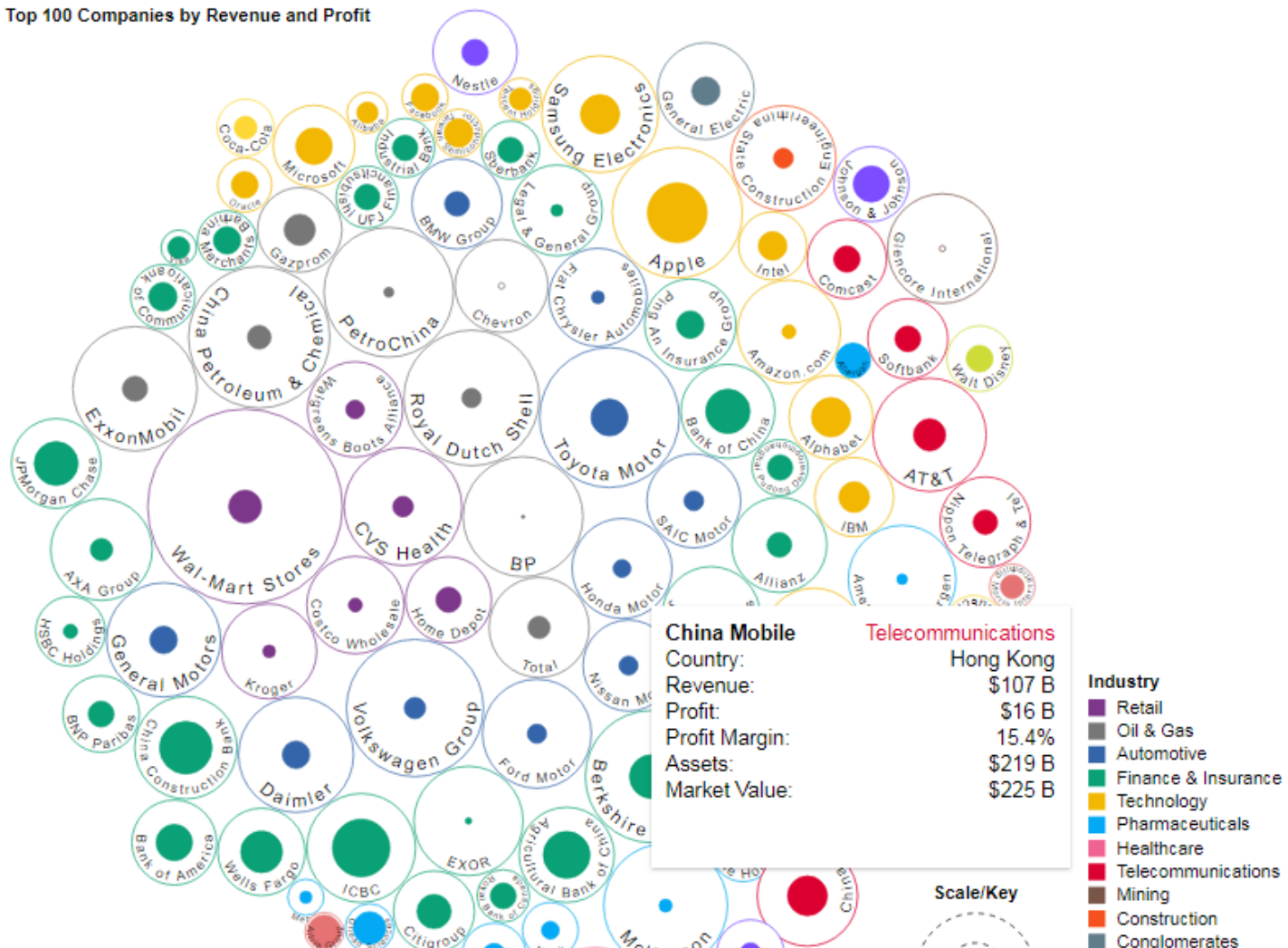
    Estimates the effect of the treatment (T_i) on the outcome (Y_i).
  - **Model 2: Propensity Score Model**

$$E_i = \tau_0 + \tau_1 C_i$$

    Matches individuals based on their covariates (C_i).
- Both models must ideally be correctly specified; however:
  - Correct specification is rarely achieved in practice.
  - Robust versions of Propensity Score Matching allow one model to be misspecified.
- **Key Point:** The fewer assumptions we make, the better. Robust methods provide more reliable causal effect estimates.

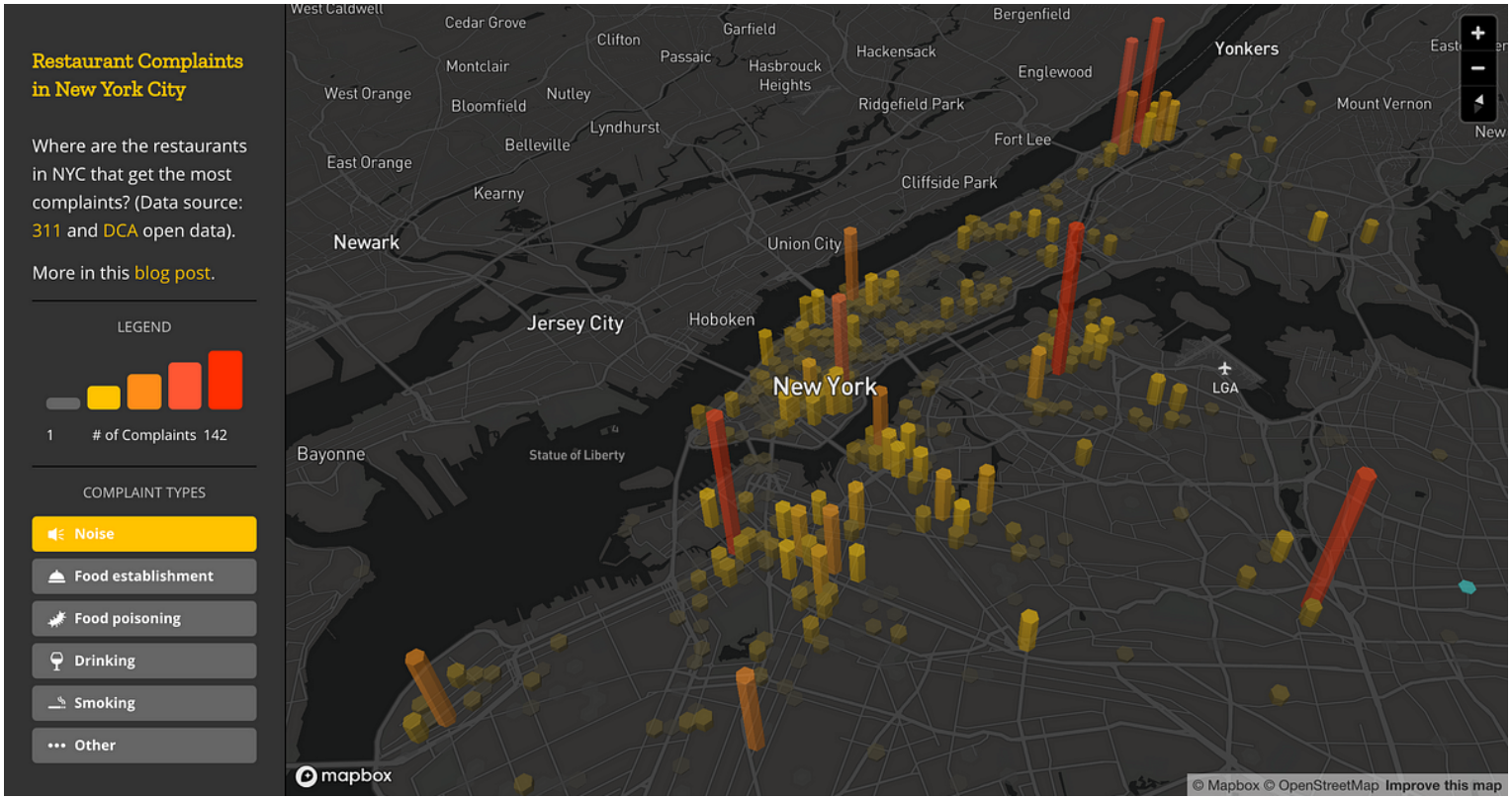
08: Exploratory Data Analysis (EDA)

- Emphasizes visualization and insights over intense theory and computation.
- Useful in understanding the relation between data, fitted model, and predictions.



Connection to NYC Open Data

- Apply statistical methods to NYC datasets.
- Example: Visualize and analyze restaurant complaints using robust inference and EDA.



https://labs.mapbox.com/bites/00304/

What does it mean for an idea to be important?

- Gelman emphasized avoiding citation counts of papers where concepts originated.
- Instead, he assessed principles that influenced the development of ideas in modern statistical practice.
- Example from the literature: The paper "The Importance of Being Clustered" (Anderluci et al., 2019) analyzed trends in statistics from 1970–2015 using clustering techniques to identify influential statistical ideas and themes.
- **My Remarks:**
  - Many of these ideas seem to have arisen from the analysis of very large datasets and the challenges inherent in combining sets of data.
  - The availability of computational power has also increased the tendency to model these statistical problems, pushing forward the development of modern methods.

The Importance of Human Oversight in Statistical Innovations

- As computational power advances, machine learning and statistical algorithms can model complex systems.
- However, these models are only as good as the assumptions and data they are based on.
- Example: Self-driving cars can use machine learning to navigate, but human oversight is needed to determine:
  - Are the outcomes (e.g., accident rates) statistically significant?
  - Are the algorithms operating ethically and equitably?
- **Key Point**: Computational tools are powerful, but without human observation and ethical guidance, they can lead to unintended consequences.

Reflection from Gelman

"On one hand, you have all these amazing things that machine learning can do, like self-driving cars, but you'll need a statistician to tell you if the number of people being killed by the self-driving cars is statistically significant." – Paraphrased from Andrew Gelman in a separate interview

Questions?

Thank you! Any questions?

References

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Images:

- bootstrap_populations.png
- neural_networks.png
- meta_analysis.png
- propensity_score_matching.png
- bubble-chart.png