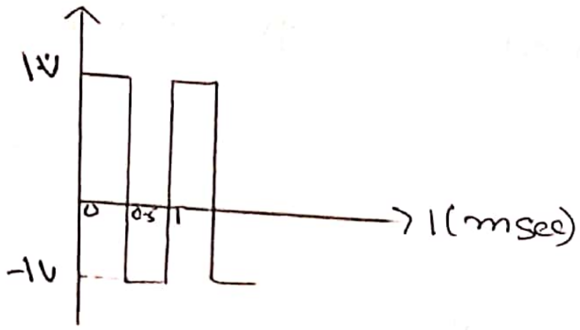
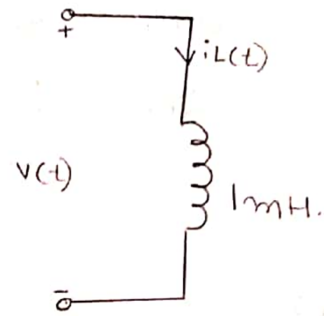
1 A square waveform as shown in figure is applied across 1mH. ideal inductor The current through inductor is a 1

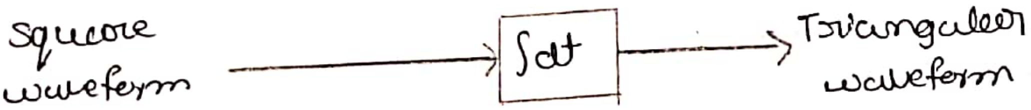




current through an inductor is given by,

$$i_L(t) = \frac{1}{L}\int_0^1 v(t)\,dt$$

So current through inductor is the integration of the applied voltage across the inductor.

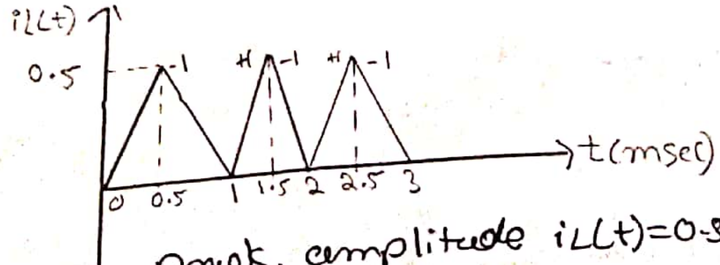


The square wave can be written as,

$$V(t) = u(t) - 2u(t-0.5) + 2u(t-1) + \cdots$$

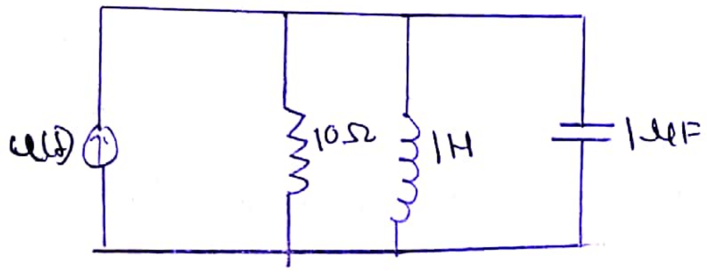
Integrating the voltage to get current through the inductor,

$$i_L(t) = r(t) - 2r(t-0.5) + 2r(t-1) - \cdots$$

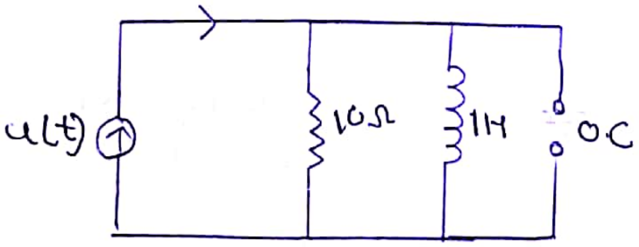


Peak. amplitude $i_L(t) = 0.5$ Amp.

2 A 10Ω resistor, a 1H inductor and $1\mu F$ capacitor are connected in parallel. The combination is driven by a unit step current. Under the steady state condition, the source current flows through.....



Under steady state condition, inductor behaves as a short circuit. and capacitor, behaves as an open circuit.



Hence, the source current flow through the inductor only.

3 If the Laplace transform of the voltage across a capacitor of value of $\frac{1}{2}$ F is $V_C(S) = \frac{S+1}{S^3+S^2+S+1}$, the value of current through the capacitor at $t=0^+$ is

given: $C = \frac{1}{2}F$, $V_C(S) = \frac{S+1}{S^3+S^2+S+1}$

For capacitor, $X_C = \frac{1}{CS} = \frac{1}{\frac{1}{2}S} = \frac{2}{S}$

Transform voltage of capacitor is given by,

$V_C(S) = I_C(S) X_C$.

$$I_C(S) = \frac{V_C(S)}{X_C} = \frac{(S+1)S}{(S^3+S^2+S+1)2}$$

The value of current at t=0, means it is the initial value.

By initial theorem,

$$i_C(0^+) = \lim_{S \to \infty} S I_C(S)$$

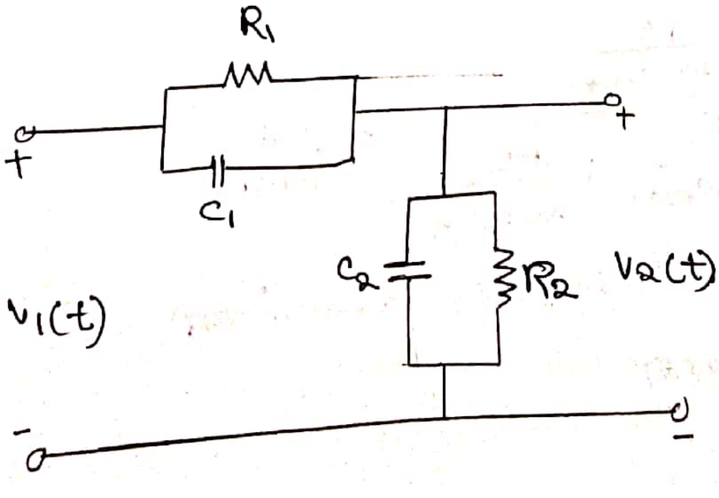
$$i_C(0^+) = \lim_{S \to \infty} \frac{S \times S(S+1)}{2(S^3+S^2+S+1)}$$

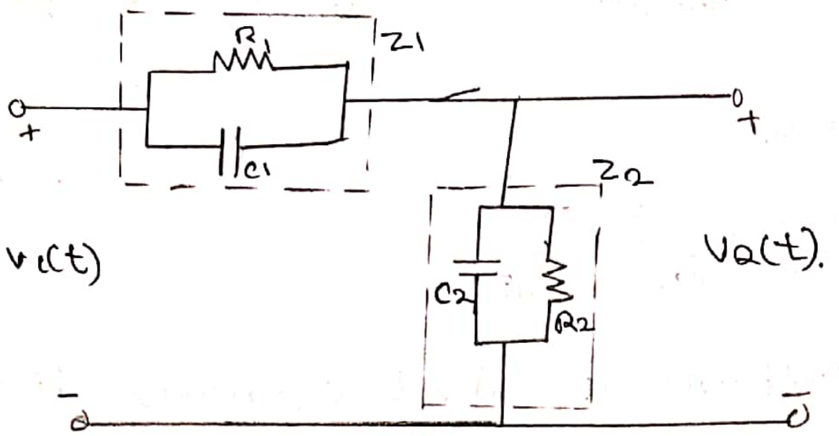
$$i_C(0^+) = \lim_{S \to \infty} \frac{S^2(S+1)}{2(S^2+1)(S+1)}$$

$$i_C(0^+) = \lim_{S \to \infty} \frac{S^3(1+\frac{1}{S})}{2S^2(1+\frac{1}{S^2})S(1+\frac{1}{S})}$$

$$i_C(0^+) = \frac{1}{2+0} = \frac{1}{2} A$$

4 For the compensated attenuator of figure the impulse response under the condition $R_1C_1 = R_2C_2$ is



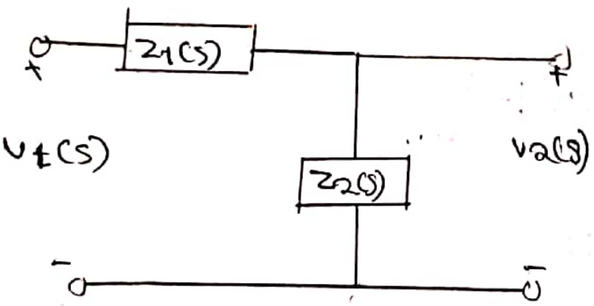


$$Z_1(s) = \frac{R \times \frac{1}{C_1 s}}{\frac{1}{C_1 s}}$$

$$Z_1(s) = \frac{R_1}{R_1C_1s+1} = \frac{R_1}{R_2C_2s+1}$$

[Given: $R_1C_1 = R_2C_2$]

$$Z_2(s) = \frac{R_2 \times \frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s}} = \frac{R_2}{R_2C_2s+1}$$



$$\frac{V_2(s)}{V_1(s)} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{\frac{R_2}{R_2C_2s+1}}{\frac{R_1}{R_2C_2s+1} + \frac{R_2}{R_2C_2s+1}} = \frac{R_2}{R_1+R_2}$$

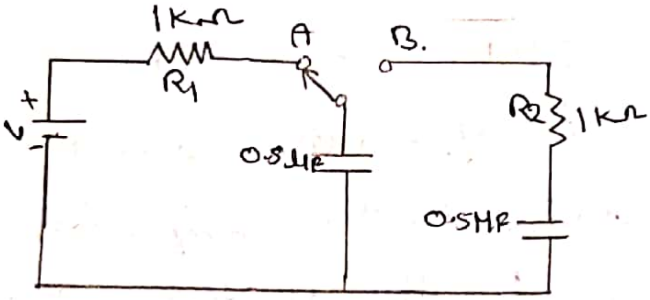
taking inverse Laplace transform,

$$v_2(t) = \frac{R_2}{R_1+R_2} v_1(t)$$

$$v_1(t) = \delta(t)$$

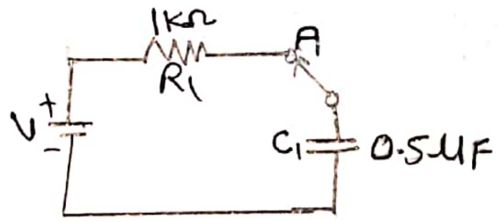
$$V_2(t) = \frac{R_2}{R_1+R_2}\, \partial(t)$$

5 For the circuit shown below different time constant are given.



1) 0.5×10^{-3} sec, ② 2×10^{-3} sec, ③ 0.25×10^{-3} sec, ④ 10^{-3} sec

i) Calculation of charging time constant when a switch is at position A,



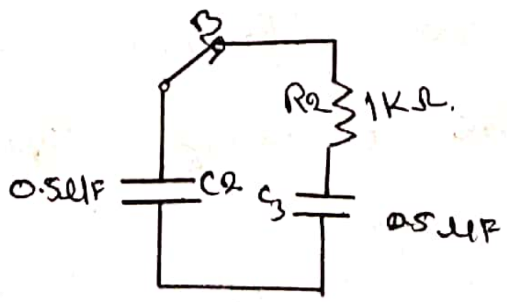
Above figure is charging circuit, hence for a RC network, charging time constant is given by

$\tau_1 = R_1 C_1 = 1 \times 10^3 \times 0.5 \times 10^{-6}$

$\tau_1 = 0.5 \times 10^{-3}$ sec

ii) calculation of discharging time constant.

when switch is at position B,



Above figure is a discharging circuit, because it does not have any independent source

From a R-C network, discharging time constant is given by,

$T_2 = R_2C = 10^3 \times 0.25 \times 10^{-6}$

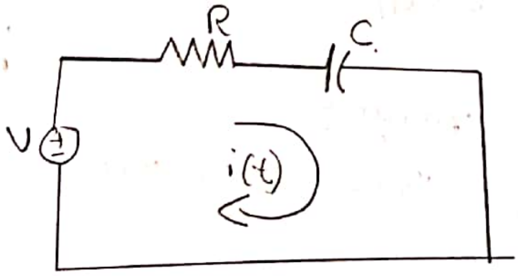
$T_2 = 0.25 \times 10^{-3}$ sec

6 Consider a DC voltage source connected to a series R-C circuit, when the steady-state reaches, the ratio of the energy stored in the capacitor to the total energy supplied by the voltage source, is equal to.

current through capacitor is given by

$i_C[t] = i_C(\infty) + [i_C(0^+) - i_C(\infty)]e^{-1/t}$ ... (1)

(i) At ≥ 0 (transient):



For a R-C network

Time constant, $T = RC$ sec

ii) At $t = 0^+$;

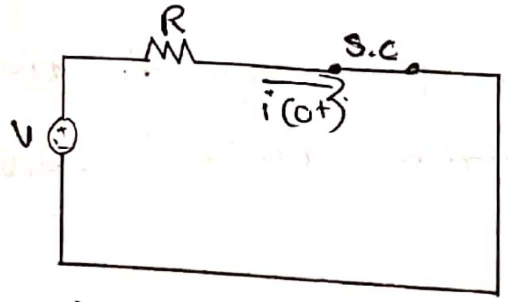
If there is no information about capacitor then we assume uncharged capacitor i.e

$V_C(0^-) = V_C(0^+) = 0V$.

At $t = 0^+$, capacitor is replaced by voltage source with initial value, i.e.

$\cancel{v_c(0)} = 0$

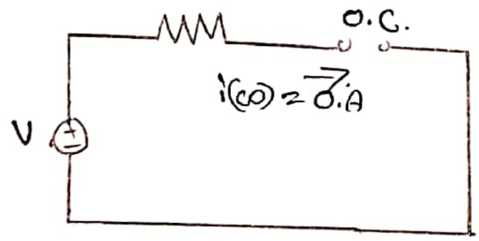
$V_C(0^-) = 0V$ (short circuit)



$i(0^+) = \frac{V}{R}.$

iii) At t = ∞ / steady state:

In steady state, capacitor behaves as an open circuit



Put the values of $i(0^+)$, $i(\infty)$ and τ in equation (i)

$$i(t) = \frac{V}{R} e^{\frac{-1}{RC}} u(t)$$

Total energy Ws supplied by the voltage source for $0 < t < \infty$ is given by,

$$W_s = \int V i(t)\, dt$$

$$W_s = \int_0^\infty \frac{V^2}{R} e^{\frac{-1}{RC}} dt = \frac{V^2}{R} (-RC) \left[e^{\frac{-1}{RC}} \right]_0^\infty$$

$\cancel{W_s}$ $W_s = CV^2 J$ ——— (2)

$V_C(\infty) = V.$

Energy stored in the capacitor at $t = \infty$,

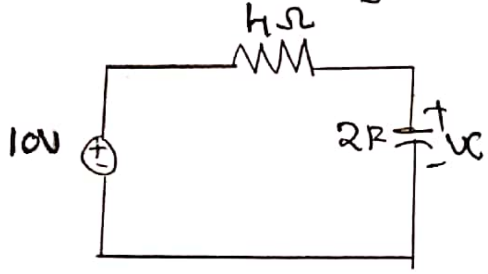
$W_C = \frac{1}{2} CV^2 J$ ———→ (3)

8

From equation (2) and (3)

$$\frac{W_C}{W_S} = \frac{\frac{1}{2}CV^2}{CV^2} = \frac{1}{2}$$

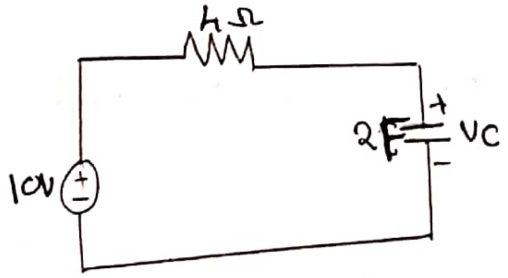
7 In the circuit of figure the energy absorbed by the 4Ω resistor in time interval (0, ∞) is [Given: $V_C(0) = 6V$]



Voltage across the capacitor given by,

$V_C(t) = V_C(0^+) = 6V$ [Given]

(i) At $t \geq 0$ (Transient):



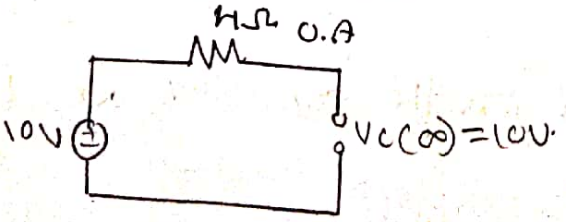
For a R-C network

Time constant, $\tau = RC = 4 \times 2 = 8 sec$

ii) At $t = \infty$ / steady state:

In steady state, capacitor behaves as an open circuit.

The circuit will becomes as shown below



Put the values of $V_C(0^+)$, $V_C(\infty)$ and τ in equation (i)

$$v_C(t) = 10 + [6-10]\, e^{t/8}$$

$$v_C(t) = 10 - 4e^{-t/8}\,V.$$

current through capacitor is given by,

$$i_C(t) = C\frac{d}{dt}v_C(t)$$

$$i_C(t) = 2\frac{d}{dt}\left(10 - 4e^{-t/8}\right)$$

$$i_C(t) = 2\left[-4e^{-t/8} \times \left(\frac{-1}{8}\right)\right]$$

$$i_C(t) = e^{-t/8}\,A$$

Power ~~abso~~ absorbed by resistor R is given by,

$$P = i_C^2(t)\,R$$

~~Energy of~~

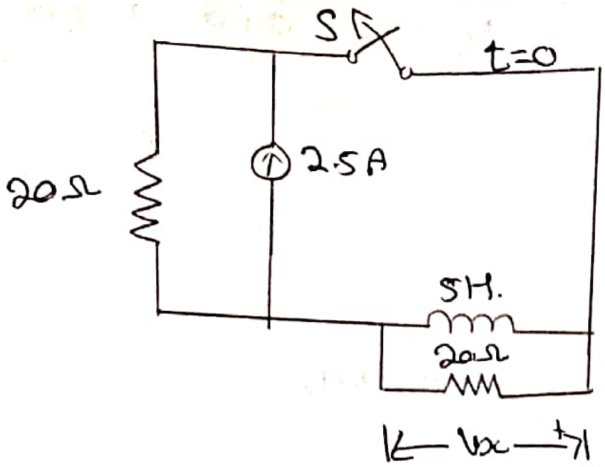
Energy absorbed by Resistor R is given by,

$$E_R = \int_0^\infty P\,dt = \int_0^\infty i_C^2(t)\,R\,dt = \int_0^\infty \left(e^{-t/8}\right)^2 \times 4\,dt$$

$$E_R = \int_0^\infty 4e^{-t/4}\,dt = 4\times(-4)\left[e^{-t/4}\right]_{t=0}^{\infty}$$

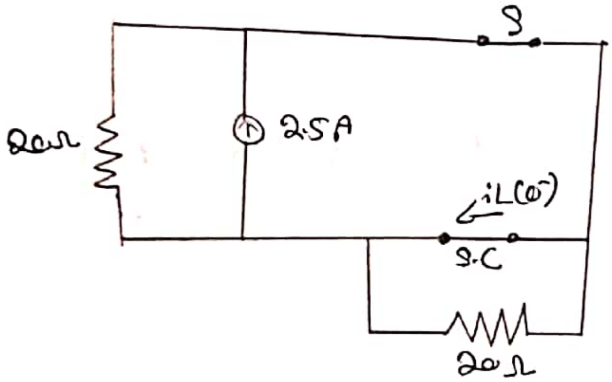
$$E_R = -16\,[0-1] = 16 \text{ Joules}$$

8 In figure the switch was closed for a long time before opening at $t=0$. The voltage V_x at $t=0^+$ will be.



(i) At t=0⁻ /t<0/ steady state:

In steady state, inductor behaves as a short circuit



$i_L(0^-) = 2.5A$

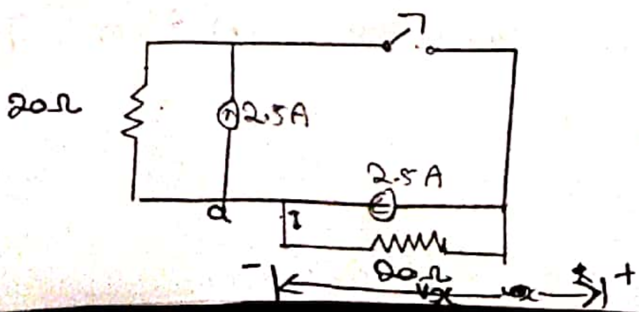
From property of inductor.

$i_L(0^-) = i_L(0^+) = 2.5A$

ii) At t=0+

inductor is replaced by current source with initial value i.e.

$i_L(0^-) = i_L(0^+) = 2.5A$



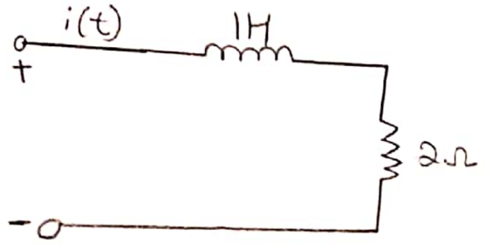
2.5A a current will flow through 20Ω resistor.

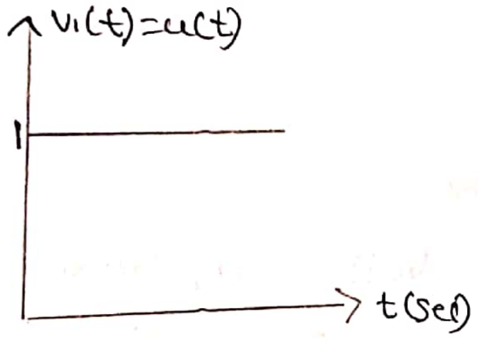
Hence, $I = 2.5A$

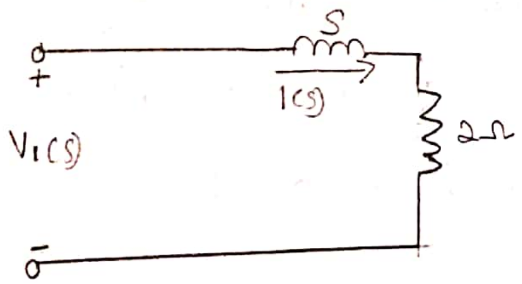
From ohm's law.

$$V_x = -2.5 \times 20 = -50 \text{ volt}$$

11Q For the R-L circuit shown in figure the input voltage. $v_1(t) = u(t)$. The current $i(t)$ is







Applying KVL in loop shown.

$$V_i(s) = sI(s) + 2I(s)$$

$$V_i(s) = I(s)[s+2]$$

$$\text{input} = v_i(t) = u(t) \text{ [Given]}$$

$$V_i(s) = \frac{1}{s}$$

$$\frac{1}{s} = I(s)[s+2]$$

$$I(s) = \frac{1}{s(s+2)} = \frac{1}{2}\left[\frac{1}{s} - \frac{1}{s+2}\right]$$

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Taking Inverse Laplace transform,

$$i(t) = \frac{1}{2}\,(1-e^{-2t})\,u(t)$$

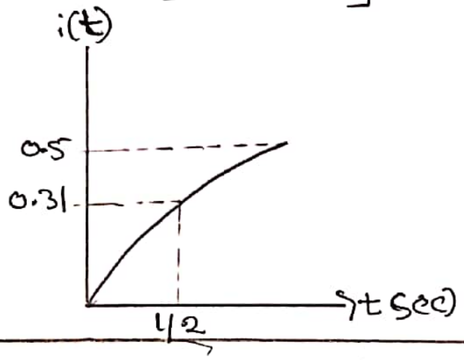
Time constant, $\tau = \frac{1}{2}$ sec

At $t = \tau = \frac{1}{2}$ sec

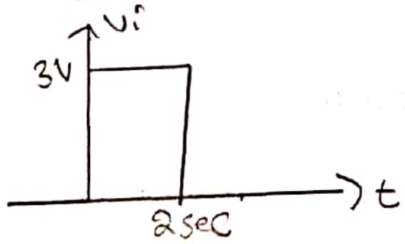
$$i(t) = 0.5[1-e^{-1}] = 0.31A$$

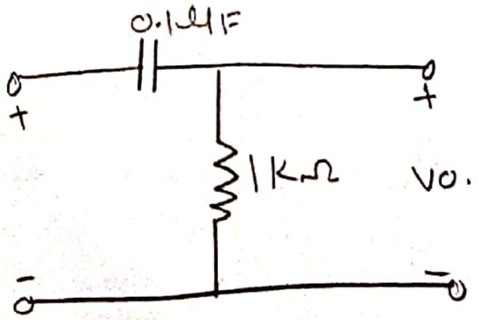
At $t = \infty$,

$$i(t) = 0.5\,[1-e^{-2\times\infty}] = 0.5A$$



13 A square pulse of 3 volt amplitude is applied to C-R circuit shown in the figure. The capacitor is initially uncharged. The output voltage Vo at time t=2 sec is.





Given: R kΩ and C = 0.1 μF

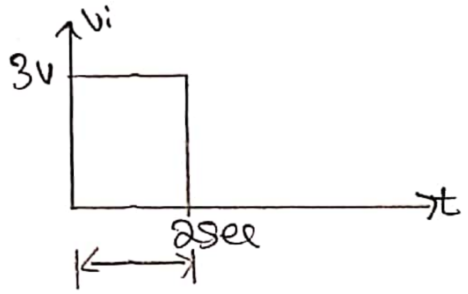
For a R-C network, time constant

$\tau = RC = 1000 \times 10^{-7} = 100\,\mu s$

Time duration pulse $T_p = 2sec$

settling time is given by,

$t_s = 5\tau = 5 \times 100 = ~~500 \mu sec~~\ 500\,\mu sec$



At ~~settling~~ settling time, practically circuit will be in steady state

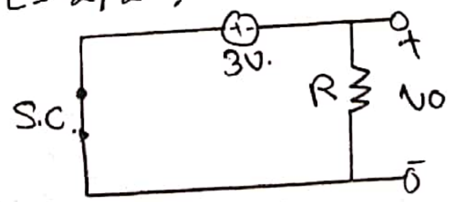
As $T_p >> t_s$, so steady state is reached much before the end of pulse and C behaves as open circuit

So; $V_C = V_1 = 3V$.

$V_C(1.99\ sec) = 3V$.

$V_C(2^-) = V_C(2^+) = 3V$.

At $t = 2/2^+$;



~~From figure~~ From figure

$0 + 3 + V_0 = 0$,

$V_0 = -3V$