Rakshitha

CV

CBCS Scheme

USN | | | | | | | | | | |

15MAT31

Third Semester B.E. Degree Examination, June/July 2018

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Obtain the Fourier series for the function :

$$f(x)=\begin{cases}-\pi, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$$

Hence deduce that $\dfrac{\pi^2}{8}=\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+----$. (08 Marks)

b. Obtain the half–range cosine series for the function $f(x)=(x-1)^2$, $0\le x\le 1$. Hence deduce that $\dfrac{\pi^2}{6}=\dfrac{1}{1^2}+\dfrac{1}{2^2}+\dfrac{1}{3^2}+-----$. (08 Marks)

OR

2 a. Find the Fourier series of the periodic function defined by $f(x)=2x-x^2$, $0<x<3$. (06 Marks)

b. Show that the half range sine series for the function $f(x)=\ell x-x^2$ in $0<x<\ell$ is $\dfrac{8\ell^2}{\pi^3}\sum_0^{\infty}\dfrac{1}{(2n+1)^3}\sin\left(\dfrac{2n+1}{\ell}\right)\pi x$. (05 Marks)

c. Express y as a Fourier series upto 1st harmonic given :

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(05 Marks)

Module-2

3 a. Find the Fourier transform of

$$f(x)=\begin{cases}1-|x|, & |x|\le 1\\ 0, & |x|>1\end{cases}$$

and hence deduce that $\displaystyle\int_0^{\infty}\frac{\sin^2 t}{t^2}dt=\frac{\pi}{2}$. (06 Marks)

b. Find the Fourier Sine and Cosine transforms of $f(x)=e^{-\alpha x}$, $\alpha>0$. (05 Marks)

c. Solve by using z – transforms $y_{n+1}+\frac{1}{4}y_n=\left(\frac{1}{4}\right)^n$ $(n\ge 0)$, $y_0=0$. (05 Marks)

OR

4 a. Find the Fourier transform of $f(x) = e^{-|x|}$. (06 Marks)
b. Find the Z – transform of $\sin(3n + 5)$. (05 Marks)
c. Find the inverse Z – transform of : $\frac{z}{(z-1)(z-2)}$. (05 Marks)

Module-3

5 a. Find the correlation coefficient and the equation of the line of regression for the following values of x and y. (06 Marks)

x	1	2	3	4	5
y	2	5	3	8	7

b. Find the equation of the best fitting straight line for the data : (05 Marks)

x	0	1	2	3	4	5
y	9	8	24	28	26	20

c. Use Newton – Raphson method to find a real root of the equation $x \log_{10} x = 1.2$ (carry out 3 iterations). (05 Marks)

OR

6 a. Obtain the lines of regression and hence find the coefficient of correlation for the data : (06 Marks)

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

b. Fit a second degree parabola to the following data : (05 Marks)

x	1	2	3	4	5
y	10	12	13	16	19

c. Use the Regula–Falsi method to find a real root of the equation $x^3 - 2x - 5 = 0$, correct to 3 decimal places. (05 Marks)

Module-4

7 a. Given Sin45° = 0.7071, Sin50° = 0.7660, Sin55° = 0.8192, Sin60° = 0.8660 find Sin57° using an appropriate interpolation formula. (06 Marks)
b. Construct the interpolation polynomial for the data given below using Newton's divided difference formula : (05 Marks)

x	2	4	5	6	8	10
y	10	96	196	350	868	1746

c. Use Simpson's $\frac{1}{3}$rd rule with 7 ordinates to evaluate $\int_2^8 \frac{dx}{\log_{10} x}$. (05 Marks)

OR

8 a. Given f(40) = 184, f(50) = 204, f(60) = 226, f(70) = 250, f(80) = 276, f(90) = 304, find f(38) using Newton's forward interpolation formula. **(06 Marks)**

b. Use Lagrange's interpolation formula to fit a polynomial for the data :

x	0	1	3	4
y	–12	0	6	12

Hence estimate y at x = 2. **(05 Marks)**

c. Evaluate $\int_0^1 \frac{x}{1+x^2}dx$ by Weddle's rule taking seven ordinates and hence find $\log_e 2$. **(05 Marks)**

Module-5

9 a. Find the area between the parabolas $y^2 = 4x$ and $x^2 = 4y$ using Green's theorem in a plane. **(06 Marks)**

b. Verify Stoke's theorem for the vector $\vec{F} = (x^2 + y^2)i - 2xyj$ taken round the rectangle bounded by x = 0, x = a, y = 0, y = b. **(05 Marks)**

c. Find the extremal of the functional : $\int_{x_1}^{x_2} [y' + x^2(y')^2]\,dx$. **(05 Marks)**

OR

10 a. Verify Green's theorem in a plane for $\oint_c (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$ where c is the boundary of the region enclosed by $y = \sqrt{x}$ and $y = x^2$. **(06 Marks)**

b. If $\vec{F} = 2xyi + yz^2j + xzk$ and S is the rectangular parallelopiped bounded by x = 0, y = 0, z = 0, x = 2, y = 1, z = 3 evaluate $\iint_S \vec{F}\cdot\hat{n}\,ds$. **(05 Marks)**

c. Find the geodesics on a surface given that the arc length on the surface is $S = \int_{x_1}^{x_2} \sqrt{x[1+(y')^2]}\,dx$. **(05 Marks)**

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1. a. Obtain the fourier series for the function :

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\cdots$

Solⁿ: Given :

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

$2l = \pi - (-\pi)$

$2l = 2\pi$

$l = \pi$

The fourier series of $f(x)$ is given by,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

Here $l = \pi$,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \longrightarrow \text{(I)}$$

$\phi(x) = -\pi$

$\phi(-x) = -\pi \neq \psi(x)$

$\neq -\psi(x)$

The function is neither even nor odd

$$a_0 = \frac{1}{l}\int_{c}^{c+2l} f(x).dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x).dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x).dx + \int_{0}^{\pi} f(x).dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi.dx + \int_{0}^{\pi} x.dx\right]$$

$$= \frac{1}{\pi}\left\{\left[-\pi x\right]_{-\pi}^{0} + \left[\frac{x^2}{2}\right]_0^{\pi}\right\}$$

$$= \frac{1}{\pi}\left[-(0-\pi^2) + \left(\frac{\pi^2}{2} - 0\right)\right]$$

$$= \frac{1}{\pi}\left[-\pi^2 + \frac{\pi^2}{2}\right]$$

$$= \frac{-1}{\cancel{\pi}} \times \frac{\pi^{\cancel{2}}}{2}$$

$$a_0 = \frac{-\pi}{2}$$

$$\frac{a_0}{2} = \frac{-\pi}{4}$$

$$a_n = \frac{1}{l}\int_c^{c+2l} f(x).\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x).\cos nx.dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x).\cos nx.dx + \int_0^{\pi} f(x).\cos nx.dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi\cos nx.dx + \int_0^{\pi} x\cos nx.dx\right]$$

$$= \frac{1}{\pi}\left[-\pi\left(\frac{\cancel{\sin nx}}{n}\right)_{-\pi}^{0} + \left[\left(x.\frac{\cancel{\sin nx}}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$= \frac{1}{n^2\pi}\left[\cos nx\right]_0^{\pi}$$

$$\frac{1}{n^2\pi}\left[\cos n\pi-\cos 0\right]$$

$$a_n=\frac{1}{n^2\pi}\left[(-1)^n-1\right]$$

$$b_n=\frac{1}{l}\int_c^{c+2l} f(x).\ \sin nx.\ dx$$

$$=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x).\ \sin nx.\ dx$$

$$=\frac{1}{\pi}\left[\int_{-\pi}^{0} f(x).\sin nx.\ dx+\int_0^{\pi} f(x).\ \sin nx.\ dx\right]$$

$$=\frac{1}{\pi}\left[\int_{-\pi}^{0}-\pi\sin nx.\ dx+\int_0^{\pi} x\sin nx.\ dx\right]$$

$$=\frac{1}{\pi}\left[-\pi\left[\frac{-\cos nx}{n}\right]_{-\pi}^{0}+\left\{x\left[\frac{-\cos nx}{n}\right]-\left[-\cancel{\frac{\sin nx}{n^2}}^{\,0}\right]\right\}_0^{\pi}\right]$$

$$=\frac{1}{\pi}\left[\frac{\pi}{n}\left[\cos nx\right]_{-\pi}^{0}+\left[-x\ \frac{\cos nx}{n}\right]_0^{\pi}\right]$$

$$=\frac{1}{\pi}\left[\frac{\pi}{n}\left[\cos 0-\cos n\pi\right]-\frac{1}{n}\left[\pi\cos n\pi-0\right]\right]$$

$$=\frac{1}{n\pi}\left[\pi(1-(-1)^n)-\pi(-1)^n\right]$$

$$=\frac{1}{n\pi}\left[\pi\left[1-(-1)^n-(-1)^n\right]\right]$$

$$b_n=\frac{1}{n}\left[1-2(-1)^n\right]$$

Substituting this in (I) we get

$$f(x) = -\frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{1}{n^2\pi}\left[(-1)^n - 1\right]\cos nx + \sum_{n=1}^{\infty} \frac{\left[1-2(-1)^n\right]}{n}\sin nx \quad \text{— (II)}$$

Put $x = 0$ in equation (II) we get,

$$f(0) = -\frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{1}{n^2\pi}\left[(-1)^n - 1\right]$$

$$\frac{-\pi}{2} = -\frac{\pi}{4} - \frac{1}{\pi}\left\{\frac{2}{1^2} + \frac{2}{3^2} + \frac{2}{5^2}\right\}$$

$$\therefore \frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\cdots$$

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16) Obtain the half-range cosine series for the function $f(x) = (x-1)^2$, $0 \leq x \leq 1$. Hence deduce that,

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\cdots$$

Solⁿ:

Half range fourier Cosine series is given by,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) \quad \text{— (1)}$$

Here $l = 1$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x$$

$$a_0 = 2\int_0^l f(x)\,.dx$$

$$= 2\int_0^1 (x-1)^2 . dx$$

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$$= 2\left[\frac{(x-1)^3}{3}\right]_0^1$$

$$= \frac{2}{3}\left[(1-1)^3 - (0-1)^3\right]$$

$$a_0 = \frac{2}{3}$$

$$\frac{a_0}{2} = \frac{2}{6} = \frac{1}{3}$$

$$a_n = 2\int_0^l f(x).\cos\left(\frac{n\pi x}{l}\right).dx$$

Here $l = 1$

$$a_n = 2\int_0^1 (x-1)^2 . \cos(n\pi x) . dx$$

$$a_n = 2\left[(x-1)^2\left(\frac{\cancel{\sin n\pi x}}{n\pi}\right)^{0} - 2(x-1)\left(-\frac{\cos n\pi x}{n^2\pi^2}\right) + 2\left(-\frac{\cancel{\sin n\pi x}}{n^3\pi^3}\right)^{0}\right]_0^1$$

$$a_n = 2\left\{2(-1)\left(-\frac{1}{n^2\pi^2}\right)\right\}$$

$$a_n = \frac{4}{n^2\pi^2}$$

Equ<u>n</u> (1) becomes

$$f(x) = \frac{1}{3} + \sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}\cos n\pi x \quad \text{———} \quad (2)$$

Put $x = 0$ in Equ<u>n</u> (2) and noting that $f(0) = 1$, we get

$$1 = \frac{1}{3} + \sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}$$

$$\therefore \frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \text{-------}$$

2a. Find the fourier series of the period function defined by $f(x) = 2x - x^2$, $0 \leq x \leq 3$

$f(x) = 2x - x^2$

$2l = 3 - 0$

$l = \frac{3}{2}$

The fourier series of $f(x)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$l = 3/2$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{2n\pi x}{3} + \sum_{n=1}^{\infty} b_n \sin\frac{2n\pi x}{3} \rightarrow \text{(I)}$$

$f(x) = x(2-x)$

$f(3/2 - x) = 3 - x\,(2 - (3-x))$

$= (3-x)(1+x)$

The function is neither even nor odd.

$$a_0 = \frac{1}{l}\int_c^{c+2l} f(x)\,dx$$

$$= \frac{1}{3/2}\int_0^3 (2x - x^2)\,dx$$

$$= \frac{2}{3}\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^3$$

$$= \frac{2}{3}\left[9 - 0 - \frac{1}{3}(27 - 0)\right]$$

$$= \frac{2}{3}[9 - 9]$$

$a_0 = 0$

$\frac{a_0}{2} = 0$

$$a_n = \frac{1}{l}\int_c^{c+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{1}{3/2}\int_0^3 (2x-x^2)\cos\left(\frac{2n\pi x}{3}\right)dx$$

$$= \frac{2}{3}\left[(2x-x^2)\frac{\cancel{\sin\left(\frac{2n\pi x}{3}\right)}}{2n\pi/3} - (2-2x)\times\frac{-\cos\frac{2n\pi x}{3}}{\frac{4n^2\pi^2}{9}} + (-2)\frac{\cancel{-\sin\left(\frac{2n\pi x}{3}\right)}}{\frac{8n^3\pi^3}{27}}\right]$$

$$= \frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{9}\,3}{\cancel{4}n^2\pi^2}\left[(2-2x)\cos\left(\frac{2n\pi x}{3}\right)\right]_0^3$$

$$= \frac{3}{2n^2\pi^2}\left[-4\cos 2n\pi - 2\cos 0\right]$$

$$= \frac{3}{2n^2\pi^2}\times -6 = \frac{-9}{n^2\pi^2}$$

$$b_n = \frac{1}{l}\int_c^{c+2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{1}{3/2}\int_0^3 (2x-x^2)\sin\left(\frac{2n\pi x}{3}\right)dx$$

$$= \frac{2}{3}\left[(2x-x^2)\times\frac{\cos\left(\frac{2n\pi x}{3}\right)}{\frac{2n\pi}{3}} - (2-2x)\frac{-\sin\left(\frac{2n\pi x}{3}\right)}{\frac{4\pi^2n^2}{9}} + (-2)\frac{\cos\left(2n\pi x/3\right)}{\frac{8n^3\pi^3}{27}}\right]_0^3$$

$$= \frac{2}{3}\left[\frac{-3}{2n\pi}\{-3\cos 2n\pi - 0\} - \frac{27}{4n^3\pi^3}\cancel{\{\cos 2n\pi - \cos 0\}}\right]$$

$$= \frac{2}{3}\times\frac{-3}{2n\pi}\times -3$$

$$b_n = \frac{3}{n\pi}$$

substituting this in ①

$$2x - x^2 = 0 + \sum_{n=1}^{\infty}\frac{-9}{n^2\pi^2}\times\cos\frac{2n\pi x}{3} + \sum_{n=1}^{\infty}\frac{3}{n\pi}\times\sin\left(\frac{2n\pi x}{3}\right)$$

26. Show that half range sine series for the function $f(x) = lx - x^2$ in $0 < x < l$ is $\frac{8l^2}{\pi^3}\sum_{0}^{\infty}\frac{1}{(2n+1)^3}\sin\left(\frac{2n+1}{l}\right)\pi x$.

Given:

$f(x) = lx - x^2$ in $(0, l)$

$l = l - 0$

$l = l$

The half range sine series is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right) \rightarrow (1)$$

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{2}{l}\int_0^l (lx - x^2)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{2}{l}\left[\cancel{(lx - x^2)\frac{-\cos\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}}}^{\,0} - (l-2x)\cancel{\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}}^{\,0} + (-2)\frac{\cos\frac{n\pi x}{l}}{\left(\frac{n\pi}{l}\right)^3}\right]_0^l$$

$$= \frac{2}{l}\left[0 - \frac{2l^3}{n^3\pi^3}\{\cos n\pi - \cos 0\}\right]$$

$$= \frac{2}{l}\left[\frac{-2l^3}{n^3\pi^3}\{(-1)^n - 1\}\right]$$

$$b_n = \frac{4l^2}{n^3\pi^3}\left[1 - (-1)^n\right]$$

Thus the required half sine series is

$$f(x) = lx - x^2 = \sum_{n=1}^{\infty}\frac{4l^2}{n^3\pi^3}\left[1 - (-1)^n\right] \times \sin\left(\frac{n\pi x}{l}\right)$$

$$= \frac{4l^2}{\pi^3}\sum_{n=1}^{\infty}\frac{\{1 - (-1)^n\}}{n^3}\sin\left(\frac{n\pi x}{l}\right)$$

$$= \frac{4l^2}{\pi^3}\sum_{n=1,3,5}\frac{2}{n^3}\sin\left(\frac{n\pi x}{l}\right)$$

$$= \frac{8l^2}{\pi^3}\sum_{n=1,3,5}\frac{1}{n^3}\sin\left(\frac{n\pi x}{l}\right)$$

$$= \frac{8l^2}{\pi^3}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^3}\sin\left(\frac{2n+1}{l}\right)\pi x \,//.$$

2c. Express y as a fourier series upto 1st harmonics given

x	0	1	2	3	4	5
y	4	8	15	7	6	2

$2l = 6$

$l = 3$

$N = 6$

$a_0 = \frac{2}{N} \Sigma y$

$a_n = \frac{2}{N} \Sigma y \cos n\theta$

$b_n = \frac{2}{N} \Sigma y \sin n\theta$

$\theta = \frac{\pi x}{l}$

x	y	$\theta = \frac{\pi x}{l} = \frac{\pi x}{3}$	$\cos\theta$	$y\cos\theta$	$\sin\theta$	$y\sin\theta$
0	4	0	1	4	0	0
1	8	$\pi/3 = 60$	0.5	4	0.866	6.928
2	15	$2\pi/3 = 120°$	-0.5	-7.5	0.866	12.99
3	7	$\pi = 180°$	-1	-7	0	0
4	6	$4\pi/3 = 240°$	-0.5	-3	-0.866	-5.196
5	2	$5\pi/3 = 300°$	+0.5	1	-0.866	-1.732
	$\Sigma y = 42$			$\Sigma y\cos\theta = -8.5$		$\Sigma y\sin\theta = -2.598$

$a_0 = \frac{2}{6} \times 42$

$= 14$

$\frac{a_0}{2} = 7$

$a_1 = \frac{2}{6} \Sigma y \cos\theta = \frac{2}{6}(-8.5) = -2.833$

$b_1 = \frac{2}{6} \Sigma y \sin\theta = \frac{2}{6}(12.99) = 4.33$

$a_2 = \frac{2}{6} \Sigma y \cos 2\theta = \frac{2}{6}(-4.5) = -1.5$

$b_2 = \frac{2}{6} \Sigma y \sin 2\theta = \frac{2}{6}(-2.598) = -0.866$

Thus, fourier series of y upto second harmonics

$$y = \frac{a_0}{2} + a_1 \cos\theta + b_1 \sin\theta + a_2 \cos 2\theta + b_2 \sin 2\theta$$

$$= 7 - 2.833 \cos\theta + 4.33 \sin\theta - 1.5 \cos 2\theta - 0.866 \sin 2\theta$$

3a. Find the fourier transform $f(x) = \begin{cases} 1-|x| & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ & hence deduce that

$$\int_0^\infty \frac{\sin^2 t}{t^2}\,dt = \frac{\pi}{2}$$

by definition,

$$F[f(x)] = \int_{-\infty}^{\infty} f(x)\, e^{iux}\,dx$$

$$= \int_{-\infty}^{-1} f(x)\, e^{iux}\,dx + \int_{-1}^{0} f(x)\, e^{iux}\,dx + \int_{0}^{1} f(x)\, e^{iux}\,dx + \int_{1}^{\infty} f(x)\, e^{iux}\,dx$$

$$= 0 + \int_{-1}^{0} (1+x)\, e^{iux}\,dx + \int_{0}^{1} (1-x)\, e^{iux}\,dx + 0$$

$$= \left[(1+x)\frac{e^{iux}}{iu} + \frac{e^{iux}}{u^2}\right]_{-1}^{0} + \left[(1-x)\frac{e^{iux}}{iu} - \frac{e^{iux}}{u^2}\right]_0^1$$

$$= \left[(1+x)\frac{e^{iux}}{iu}\right]_{-1}^{0} + \frac{1}{u^2}\left[e^{iux}\right]_{-1}^{0} + \frac{1}{iu}\left[(1-x)e^{iux}\right]_0^1 - \frac{1}{u^2}\left[e^{iux}\right]_0^1$$

$$= \frac{1}{iu}[1-0] + \frac{1}{u^2}[e^0 - e^{-iu}] + \frac{1}{iu}[0 - e^0] - \frac{1}{u^2}[e^{iu} - e^0]$$

$$= \frac{1}{iu} + \cancel{\frac{1}{u^2}} - \frac{e^{-iu}}{u^2} + \frac{e^0}{iu} - \frac{e^{iu}}{u^2} - \cancel{\frac{1}{u^2}}$$

$$= \frac{1-e^{-iu}}{u^2} - \frac{e^{iu}-1}{u^2}$$

$$= \frac{1-e^{-iu} - e^{iu} + 1}{u^2}$$

$$= \frac{2-(e^{iu}+e^{-iu})}{u^2}$$

$$= \frac{2-2\cos u}{u^2}$$

$$= \frac{2(1-\cos u)}{u^2}$$

$$= \frac{2(2\sin^2 u/2)}{u^2}$$

$$\hat{f}(u) = \frac{4\sin^2 u/2}{u^2}$$

By inverse fourier transform

$$F^{-1}[\hat{f}(u)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(u)\, e^{-iux}\, du$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{4\sin^2 u/2}{u^2}\, e^{-iux}\, du$$

put $x=0$

$$f(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\sin^2 u/2}{(u/2)^2}\, du$$

$$1 = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\sin^2 u/2}{(u/2)^2}\, du$$

$$2\pi = \int_{-\infty}^{\infty} \frac{\sin^2 (u/2)}{(u/2)^2}\, du$$

put $u/2 = t$

$\frac{du}{2} = dt \Rightarrow du = 2dt$

$t \Rightarrow -\infty$ to ∞

$$2\pi = \int_{-\infty}^{\infty} \frac{\sin^2 t}{t^2}\, dt$$

$$2\pi = 2\int_{0}^{\infty} \frac{\sin^2 t}{t^2}\, dt$$

$$\pi = 2\int_{0}^{\infty} \frac{\sin^2 t}{t^2}\, dt$$

$$\int_{0}^{\infty} \frac{\sin^2 t}{t^2}\, dt = \frac{\pi}{2}$$

//.

3b. Find the fourier sine and cosine transforms of $f(x) = e^{-\alpha x}$, $\alpha > 0$

$$F_S[f(x)] = \int_0^\infty f(x) \sin ux \, dx \qquad \left[\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx)\right]$$

$$= \int_0^\infty e^{-\alpha x} \sin ux \, dx$$

$$= \left[\frac{e^{-\alpha x}}{(-\alpha)^2+u^2}(-\alpha \sin ux - u \cos ux)\right]_0^\infty$$

$$= \left[0 - \frac{1}{\alpha^2+u^2}(-u)\right]$$

$$F_S[f(x)] = \frac{u}{\alpha^2+u^2}$$

Fourier cosine of $f(x)$ is given by,

$$F_C[f(x)] = \int_0^\infty f(x) \cos ux \, dx$$

$$\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2}(a \cos bx + b \sin bx)$$

$$= \int_0^\infty e^{-\alpha x} \cos ux \, dx$$

$$= \left[\frac{e^{-\alpha u}}{(-\alpha)^2+u^2}(-\alpha \cos ux + u \sin ux)\right]_0^\infty$$

$$= \left[0 - \frac{1}{\alpha^2+u^2}(-\alpha(1)+0)\right]$$

$$F_C[f(x)] = \frac{\alpha}{\alpha^2+u^2}$$

3c. Solve by using z-transforms $y_{n+1} + \frac{1}{4} y_n = \left(\frac{1}{4}\right)^n$ $(n \geq 0)$ $y_0 = 0$

taking Z_T on both side

$$Z_T[y_{n+1}] + Z_T\left[\frac{1}{4} y_n\right] = Z_T\left(\frac{1}{4}\right)^n$$

$$z[\bar{y}(z) - y_0] + \frac{1}{4}[\bar{y}(z)] = \frac{z}{z - 1/4}$$

$$z\bar{y}(z) + \frac{1}{4}\bar{y}(z) = \frac{z}{z-\frac{1}{4}}$$

$$\bar{y}(z)\left[z + \frac{1}{4}\right] = \frac{z}{z-1/4}$$

$$\bar{y}(z) = \frac{z}{(z+1/4)(z-\frac{1}{4})}$$

$$Z_T^{-1}[\bar{y}(z)] = Z_T^{-1}\left[\frac{z}{(z+\frac{1}{4})(z-\frac{1}{4})}\right]$$

$$Z_T^{-1}\left[\frac{z}{(z+\frac{1}{4})(z-\frac{1}{4})}\right] = A\frac{z}{z+\frac{1}{4}} + B\frac{z}{z-1/4}$$

$$\frac{z}{\cancel{(z+1/4)(z-\frac{1}{4})}} = \frac{Az(z-1/4) + Bz(z+1/4)}{\cancel{(z+1/4)(z-1/4)}}$$

$$z = Az(z-1/4) + Bz(z+1/4)$$

$$z = z[A(z-1/4) + B(z+1/4)]$$

put $z = 1/4$

$$1 = B(1/4 + 1/4)$$

$$B(1/2) = 1$$

$$B = 2$$

put $z = 0$

$$1 = A(-1/4) + B(1/4)$$

$$1 = -A/4 + 2/4$$

$$1 - \frac{1}{2} = -A/4$$

$$1/2 \times 4 = -A$$

$$\boxed{A = -2}$$

$$\frac{z}{(z+1/4)(z-1/4)} = \frac{2z}{z+1/4} + \frac{2z}{z-1/4}$$

Taking Z_T^{-1} on both side

$$Z_T^{-1}\left[\frac{z}{(z+1/4)(z-1/4)}\right] = 2Z_T^{-1}\left[\frac{z}{z+1/4}\right] + 2 \cdot Z_T^{-1}\left[\frac{z}{z-1/4}\right]$$

$$= -2\left(-\tfrac{1}{4}\right)^n + 2\left(\tfrac{1}{4}\right)^n$$

Eq^n ① becomes

$$y_n = -2\left[\left(-\tfrac{1}{4}\right)^n - \left(\tfrac{1}{4}\right)^n\right]$$

(or)

4a. Find the fourier tranform of $f(x) = e^{-|x|}$

$$|x| = \begin{cases} -x & \text{if } x<0 \\ x & \text{if } x>0 \end{cases}$$

$$e^{-|x|} = \begin{cases} e^{-(-x)} & \text{if } x<0 \\ e^{-(x)} & \text{if } x>0 \end{cases}$$

by definition

$$F[f(x)] = \int_{-\infty}^{\infty} f(x)\, e^{iux}\, dx$$

$$= \int_{-\infty}^{0} f(x)\, e^{iux}\, dx + \int_{0}^{\infty} f(x)\, e^{iux}\, dx$$

$$= \int_{-\infty}^{0} e^{x}\, e^{iux}\, dx + \int_{0}^{\infty} e^{-x} \cdot e^{iux}\, dx$$

$$= \int_{-\infty}^{0} e^{(1+iu)x}\, dx + \int_{0}^{\infty} e^{-(1-iu)x}\, dx$$

$$= \left[\frac{e^{(1+iu)x}}{1+iu}\right]_{-\infty}^{0} + \left[\frac{e^{-(1-iu)x}}{-(1-iu)}\right]_{0}^{\infty}$$

$$= \frac{1}{1+iu}\left[e^0 - e^{-\infty}\right] - \frac{1}{1-iu}\left[e^{-\infty} - e^0\right]$$

$$= \frac{1}{1+iu} + \frac{1}{1-iu}$$

$$F\left[e^{-|x|}\right] = \frac{1-iu+1+iu}{(1+iu)(1-iu)}$$

$$= \frac{2}{1^2-(iu)^2}$$

$$F\left[e^{-|x|}\right] = \frac{2}{1+u^2}$$

4b. Find the z-transform of $\sin(3n+5)$

Given: $\sin(3n+5)$

$\sin(a+b) = \sin A \cos B - \cos A \sin B$.

$\sin(3n+5) = \sin 3n \cos 5 - \cos 3n \sin 5$

$\sin(3n+5) = 0.9961 \sin 3n - 0.0871 \cos 3n$

take Z_T on both side

$Z_T[\sin(3n+5)] = 0.9961\, Z_T[\sin 3n] - 0.0871\, Z_T[\cos 3n]$

$= 0.9661 \left[\dfrac{z \sin 3}{z^2 - 2z\cos 3 + 1}\right] + 0.0871 \left[\dfrac{z(z-\cos 3)}{z^2 - 2z\cos 3 + 1}\right]$

4C: $\frac{z}{(z-1)(z-2)}$: Find the inverse z-transform.

Given

$$\frac{z}{(z-1)(z-2)} = A\frac{z}{z-1} + B\frac{z}{z-2}$$

$$\frac{z}{\cancel{(z-1)(z-2)}} = \frac{Az(z-2) + Bz(z-1)}{\cancel{(z-1)(z-2)}}$$

$$\cancel{z} = \cancel{z}[A(z-2) + B(z-1)]$$

$$1 = A(z-2) + B(z-1)$$

put $z=1 \Rightarrow 1 = A(1-2) + 0 \Rightarrow A = -1$

$z = 2 \Rightarrow 1 = 0 + B(2-1) \Rightarrow B = 1$

$$\frac{z}{(z-1)(z-2)} = \frac{-z}{z-1} + \frac{z}{z-2}$$

$$z_T^{-1}\left[\frac{z}{(z-1)(z-2)}\right] = -1 \times z_T^{-1}\left[\frac{z}{z-1}\right] + z_T^{-1}\left[\frac{z}{z-2}\right]$$

$$= (-1) + 2^n$$

$$= 2^n - 1 \quad //$$

5. a. Find the correlation co-efficient and the equation of the line of regression for the following values of x and y.

x	1	2	3	4	5
y	2	5	3	8	7

x	y	$X = x - \bar{x}$	$Y = y - \bar{y}$	X^2	Y^2	XY
1	2	-2	-3	4	9	6
2	5	-1	0	1	0	0
3	3	0	-2	0	4	0
4	8	1	3	1	9	3
5	7	2	2	4	4	4
$\Sigma x = 15$	$\Sigma y = 25$			$\Sigma X^2 = 10$	$\Sigma Y^2 = 26$	$\Sigma XY = 13$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{15}{5} = 3$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{25}{5} = 5$$

$$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2} \cdot \sqrt{\Sigma Y^2}} = \frac{13}{\sqrt{10}\sqrt{26}} = 0.8062$$

The regression line y on x is given by,

$$Y = \frac{\Sigma XY}{\Sigma X^2} X$$

$$y - 5 = \frac{13}{10}(x - 3)$$

$$y - 5 = 1.3x - 3.9$$

$$y = 1.3x + 1.1$$

The regression line x on y is given by,

$$X = \frac{\Sigma XY}{\Sigma Y^2} Y$$

$$(x - 3) = \frac{13}{26}(y - 5)$$

$$x - 3 = 0.5y - 2.5$$

$$x = 0.5y + 0.5$$

5b. Find the equation of the best fitting straight line for the data:

x	0	1	2	3	4	5
y	9	8	24	28	26	20

Let $y = ax + b$

here $n = 6$

The normal eq^n are

$nb + a\Sigma x = \Sigma y$

$b\Sigma x + a\Sigma x^2 = \Sigma xy$

x	y	x^2	xy
0	9	0	0
1	8	1	8
2	24	4	48
3	28	9	84
4	26	16	104
5	20	25	100
$\Sigma x = 15$	$\Sigma y = 115$	$\Sigma x^2 = 55$	$\Sigma xy = 344$

$6b + 15a = 115$

$15b + 55a = 344$

$a = 3.22$

$b = 11.09$

$y = 3.22x + 11.09$

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5c. Use Newton-Raphson method to find a real root of the equation $x \log_{10} x = 1.2$ [Carry out 3 iterations].

Sol^n: $x \log_{10} = 1.2$

$x \log_{10} - 1.2 = 0$

Let $f(x) = x \log_{10} - 1.2$

$f(1) = 1 \log_{10} 1 - 1.2 = -1.2$

$f(2) = 2 \log_{10} 2 - 1.2 = -0.597 < 0$

$f(3) = 3 \log_{10} 3 - 1.2 = 0.23 > 0$.

Root lies in $(2, 3)$

Let $x_0 = 3$

$f'(x) = x \cdot \frac{0.4343}{x} + \log x$

$f'(x) = 0.4343 + \log_{10} x$

By Newton's raphson's method.

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 3 - \frac{f(3)}{f'(3)}$

$= 3 \left[\frac{3 \log_{10} 3 - 1.2}{0.4343 + \log_{10} 3} \right]$

$= 2.7461$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.7461 - \left[\frac{2.7461 \log 2.7461 - 1.2}{0.4343 + \log 2.7461} \right] = 2.7406$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.7406 - \left[\frac{2.7406 \log 2.7406 - 1.2}{0.4343 + \log 2.7406} \right] = 2.7406$

Thus the real root is 2.7406.

6a. Obtain the lines of regression and hence find the co-efficient of correlation for the data:

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

Sol^n:

$n = 7$

$\bar{X} = \frac{\Sigma x}{n} = \frac{28}{7} = 4$

$\bar{Y} = \frac{\Sigma y}{n} = \frac{77}{7} = 11$

x	y	$X = x - \bar{x}$	$Y = y - \bar{y}$	X^2	Y^2	XY
1	9	-3	-2	9	4	6
2	8	-2	-3	4	9	6
3	10	-1	-1	1	1	1
4	12	0	1	0	1	0
5	11	1	0	1	0	0
6	13	2	2	4	4	4
7	14	3	3	9	9	9
$\Sigma x = 28$	$\Sigma y = 77$			$\Sigma X^2 = 28$	$\Sigma Y^2 = 28$	$\Sigma XY = 26$

$$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2}\cdot\sqrt{\Sigma Y^2}} = \frac{26}{\sqrt{28}\sqrt{28}} = 0.9285 \approx 0.93$$

The regression line y on x is given by, $Y = \frac{\Sigma XY}{\Sigma X^2} X \Rightarrow y - \bar{y} = \frac{\Sigma XY}{\Sigma X^2}(x - \bar{x})$

$(y - \bar{y}) = \frac{26}{28}(x - \bar{x})$

$y - 11 = 0.93(x - 4)$

$y = 0.93x + 7.28$

The regression line x on y is given by, $X = \frac{\Sigma XY}{\Sigma Y^2} \cdot Y \Rightarrow (x - \bar{x}) = \frac{\Sigma XY}{\Sigma X^2}(y - \bar{y})$

$(x - 4) = \frac{26}{28}(y - 11)$

$x - 4 = 0.93y - 10.23$

~~x =~~ $x = 0.3y - 6.23$

6b] Fit a second degree parabola to the following data.

x	1	2	3	4	5
y	10	12	13	16	19

Sol^n:- $nc + b\Sigma x + a\Sigma x^2 = \Sigma y$

$c\Sigma x + b\Sigma x^2 + a\Sigma x^3 = \Sigma xy$

$c\Sigma x^2 + b\Sigma x^3 + a\Sigma x^4 = \Sigma x^2 y$.

x	y	x^2	x^3	x^4	xy	x^2y
1	10	1	1	1	10	10
2	12	4	8	16	24	48
3	13	9	27	81	39	117
4	16	16	64	256	64	256
5	19	25	125	625	95	475
$\Sigma x = 15$	$\Sigma y = 70$	$\Sigma x^2 = 55$	$\Sigma x^3 = 225$	$\Sigma x^4 = 979$	$\Sigma xy = 232$	$\Sigma x^2 y = 906$

$5c + 15b + 55a = 70$

$15c + 55b + 225a = 232$

$55c + 225b + 979a = 867$

$a = 0.29$, $b = 0.49$, $c = 9.4$

Thus $y = ax^2 + bx + c$

$y = 0.29x^2 + 0.49x + 9.4$ is the required parabola.

At $x = 6$.

$y = 0.29(6)^2 + 0.49(6) + 9.4$

$\boxed{y = 22.78}$

6c. Use the Regula-Falsi method to find a real root of the equation $x^3-2x-5=0$, correct to 3 decimal places.

Sol^n:- $x^3-2x-5=0$

Let $f(x)=x^3-2x-5$

$f(0)=0-0-5=-5<0$
$f(1)=(1)^3-2(1)-5=-6<0$
$f(2)=(2)^3-2(2)-5=-1<0$
$f(3)=(3)^3-2(3)-5=16>0$

$\therefore$ Root lies b/w 2 & 3

Here $f(2)<0$ & $f(3)>0$

$f(2.1)=(2.1)^3-2(2.1)-5=0.061>0$

Thus the root lies b/w $(2, 2.1)$

$a=2$, $b=2.1$

By regula falsi method we have, $x=\dfrac{af(b)-bf(a)}{f(b)-f(a)}$

$$x_1=\frac{2(f(2.1))-2.1f(2)}{f(2.1)-f(2)}=\frac{2(0.061)-2.1(-1)}{0.061-(-1)}=2.0942$$

$f(2.0942)=(2.0942)^3-2\times2.0942-5=-0.0039<0$

$f(2.0942)<0$, $f(2.1)>0$

$\therefore$ Root is $(2.0942, 2.1)$

$a=2.0942$ $b=2.1$

$$\therefore x_2=\frac{2.0942\times f(2.1)-2.1\times f(2.0942)}{f(2.1)-f(2.0942)}$$

$$=\frac{2.0942\times0.061-2.1\times(-0.0039)}{0.061-(-0.0039)}$$

$$=2.0945$$

Thus a real root of given equation is $\boxed{x=2.0945}$

7a. Given $\sin 45° = 0.7071$, $\sin 50° = 0.7660$ $\sin 55° = 0.8192$, $\sin 60° = 0.8660$. Find $\sin 57°$ using an appropriate interpolation formula.

x	$y = \sin x$	∇y	$\nabla^2 y$	$\nabla^3 y$
45°	0.7071			
		0.0589		
50°	0.7660		-5.7×10^{-3}	
		0.0532		-7×10^{-4}
55°	0.8192		-6.4×10^{-3}	
		0.0468		
60°	0.8660			

$$r = \frac{x - x_n}{h} = \frac{57-60}{5} = \frac{-3}{5} = -0.6$$

Newton's Backward interpolation formula is given by,

$$y_r = y_n + r \nabla y_n + \frac{r(r+1)}{2!} \nabla^2 y_n + \frac{r(r+1)(r+2)}{3!} \nabla^3 y_n$$

$$y_r = 0.8660 + (-0.6)(0.0468) + \frac{(-0.6)(-0.6+1)}{2!}(-6.4 \times 10^{-3}) + \cancel{(-0.6) \times (-0.}$$
$$+ \frac{(-0.6)(-0.6+1)(-0.6+2)}{3!}(-7 \times 10^{-4})$$

$f(57) = 0.8387$

Thus $\sin 57° = 0.8387$

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76. Construct the interpolation polynomial for the data given below using Newton's divided difference formula.

x	2	4	5	6	8	10
y	10	96	196	350	868	1746

Soln:

x	y	I. DD	II D.D	III D.D	IV D.D
2	10				
		$\frac{96-10}{4-2}=43$			
			$\frac{100-43}{5-(2)}=19$		
4	96			$\frac{27-19}{6-2}=2$	
		$\frac{196-96}{5-4}=100$			0
			$\frac{154-100}{6-4}=27$		
5	196			$\frac{35-27}{8-4}=2$	
		$\frac{350-196}{6-5}=154$			0
			$\frac{259-154}{8-5}=35$		
6	350			$\frac{45-35}{10-5}=2$	
		$\frac{868-350}{8-6}=259$			
			$\frac{439-259}{10-6}=45$		
8	868				
		$\frac{1746-868}{10-8}=439$			
10	1746				

Newton's divided difference formula is given by,

$$y = f(x) = f(x_0)+(x-x_0)f(x_0x_1)+(x-x_1)f(x_0x_1x_2)+(x-x_0)(x-x_1)(x-x_2)f(x_0x_1x_2x_2)$$

$$= 10+43x-86+(x^2-4x-2x+8)19+(x^2-4x-2x+8)(2x-10)$$

$$= 43x-76+19x^2-114x+152+2x^3-12x+16x-10x^2+60x-80$$

$$f(x) = 2x^3-3x^2+5x-4$$

7C. Use Simpson's $\frac{1}{3}$rd rule with 7 ordinates to evaluate $\int_2^8 \frac{dx}{\log_{10} x}$

Sol^n:- Here $a=2$, $b=8$ and $n=6$

$$h = \frac{b-a}{n}$$

$$h = \frac{8-2}{6}$$

$$\boxed{h=1}$$

x	$x_0=2$	$x_1=3$	$x_2=4$	$x_3=5$	$x_4=6$	$x_5=7$	$x_6=8$
y	$y_0=3.3219$	$y_1=2.0959$	$y_2=1.6610$	$y_3=1.4307$	$y_4=1.2851$	$y_5=1.1833$	$y_6=1.1073$

~~By~~ Simpson's $\frac{1}{3}$rd rule,

$$I = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+y_5+\cdots y_{n-1}) + 2(y_2+y_4+y_6+\cdots y_{n-2})\right]$$

For $n=6$

$$I = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$= \frac{1}{3}\left[(3.3219+1.1073)+4(2.0959+1.4307+1.1833)+2(1.6610+1.2851)\right]$$

$$I = 9.7203$$

8a. Given $f(40)=184$, $f(50)=204$, $f(60)=226$ $f(70)=250$, $f(80)=276$, $f(90)=304$, find $f(38)$ using Newton's forward interpolation formula.

Sol^n:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
40	184			
		20		
50	204		2	
		22		0
60	226		2	
		24		0
70	250		2	
		26		0
80	276	~~28~~	2	
		28		
90	304			

$$r = \frac{x - x_0}{h}$$

$$r = \frac{38-40}{10}$$

$$\boxed{h = -0.2}$$

By Newton's forward interpolation formula,

$$y_r = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0$$

$$= 184 + (-0.2)(20) + \frac{(-0.2)(-0.2-1)}{2} \times 2$$

$$y_r = f(38) = 180.24$$

8b. Use Lagrange's interpolation formula to fit a polynomial for the data:

x	0	1	3	4
y	-12	0	6	12

Hence estimate y at $x=2$

Solⁿ:- $x_0 = 0 \quad x_1 = 1 \quad x_2 = 3 \quad x_3 = 4$

$y_0 = -12 \quad y_1 = 0 \quad y_2 = 6 \quad y_3 = 12$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$= \frac{(x-1)(x-3)(x-4)}{(0-1)(0-3)(0-4)}(-12) + \cancel{\frac{(x-0)(x-3)(x-4)}{(1-0)(1-3)(1-4)}}^{\,0} \times 0 + \frac{(x-0)(x-1)(x-4)}{(3-0)(3-1)(3-4)} \times 6$$

$$+ \frac{(x-0)(x-1)(x-3)}{(4-0)(4-1)(4-3)}(12)$$

$$= \frac{(x-1)(x-3)(x-4)}{(-1)(-3)\cancel{(-4)}}\cancel{(-12)} + \frac{x(x-1)(x-4)}{(3)\cancel{(2)}(-1)}\cancel{(6)} + \frac{x(x-1)(x-3)}{\cancel{4}\cancel{(3)}(1)}\cancel{12}$$

$$= +(x-1)(x-3)(x-4) - x(x-1)(x-4) + x(x-1)(x-3)$$

$$= +(x^2-3x-x+3)(x-4) - (x^2-x)(x-4) + (x^2-x)(x-3)$$

$$= x^3-4x^2+3x-4x^2+16x-12-[x^3-4x^2-x^2+4x]+[x^3-x^2-3x^2+3x]$$

$\cancel{= x^3 - 8x^2 +}$

$$f(x) = x^3 - 7x^2 + 18x - 12$$

At $x=2$

$y \; \cancel{f(2)} = (2)^3 - 7(2)^2 + 18(2) - 12$

$\boxed{y=4}$

8C. Evaluate $\int_0^1 \frac{x}{1+x^2} \cdot dx$ by weddle's rule taking seven ordinates and hence find $\log_e 2$

Soln:

$a_0 = 0$, $b = 1$ $\quad y = \frac{x}{1+x^2}$

$h = \frac{1-0}{6} = \frac{1}{6}$

x	$x_0 = 0$	$x_1 = 1/6$	$x_2 = \frac{2}{6}$	$x_3 = \frac{3}{6}$	$x_4 = \frac{4}{6}$	$x_5 = \frac{5}{6}$	$x_6 = \frac{6}{6}$
y	$y_0 = 0$	$y_1 = 0.1621$	$y_2 = 0.3$	$y_3 = 0.4$	$y_4 = 0.4615$	$y_5 = 0.4918$	$y_6 = 0.5$

For $n = 6$

$$I = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\right]$$

$$I = \cancel{3h}\ \frac{3 \times 1/6}{10}\left[0 + (5 \times 0.1621) + 0.3 + 6(0.4) + 0.4615 + (5 \times 0.4918) + 0.5\right]$$

$$I = 0.3466$$

June / July-2017

MODULE - 1

1 a. Obtain the Fourier Series expansion of

$$f(x) = \begin{cases} \pi x & 0 \le x \le 1 \\ \pi(2-x) & 1 \le x \le 2 \end{cases}$$ and deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\cdots = \frac{\pi^2}{8}$

Soln:- Here $2l = 2-0 \Rightarrow 2(l) = 2$

$\boxed{l=\frac{2}{2}=1}$, $\boxed{l=1}$

Fourier Series of $f(x)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

w.r.t $\boxed{l=1}$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x + \sum_{n=1}^{\infty} b_n \sin n\pi x \longrightarrow \text{(I)}$$

Here $\phi(x) = \pi x$, $\psi(x) = \pi(2-x)$

$\phi(2-x) = \pi(2\pi - x) = \psi(x)$

$f(x)$ is even , $b_n = 0$

we have

$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$

$= \frac{2}{1}\int_0^1 \pi x\,dx$

$= 2\left[\pi \cdot \frac{x^2}{2}\right]_0^1$

$= 2\left[\pi\left(\frac{1^2}{2}\right)\right]$

$= \frac{\cancel{2}\pi}{\cancel{2}}$

$\boxed{a_0 = \pi}$

$\boxed{\frac{a_0}{2} = \frac{\pi}{2}}$

$a_n = \frac{2}{l}\int_0^l f(x)\cdot\cos\left(\frac{n\pi x}{l}\right)dx$

$= \frac{2}{1}\int_0^1 \pi x \cdot \cos n\pi x\,dx$

$= 2\left[\cancel{\pi}x \cdot \frac{\sin(n\pi x)}{n\pi} - \cancel{\pi}\left(-\frac{\cos(n\pi x)}{n^2\pi^{\cancel{2}}}\right) + 0\right]_0^1$

$= \frac{2}{n^2\pi}\left[\cos n\pi x\right]_0^1$

$a_n = \frac{2}{n^2\pi}\left[\cos n\pi - 0\right] = \frac{2}{n^2\pi}\left[(-1)^n - 1\right]$

Substitute a_0 & a_n in eqn (I)

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{n^2\pi}\left[(-1)^n - 1\right]\cos n\pi x + 0 \rightarrow \text{(II)}$$

To deduce Put $x=0$ in equ (II)

$$f(x)=\frac{\pi}{2}+\frac{2}{\pi}\left[-\frac{2}{1}\cos\pi x+0+\frac{-2}{3^2}\cos 3\pi x+0+\frac{-2}{5^2}\cos 5\pi x+0\right]$$

$$=\frac{\pi}{2}-\frac{4}{\pi}\left[\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\right]$$

$$-\frac{\pi}{2}=\frac{-4}{\pi}\left[\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\right]$$

$$\frac{\pi^2}{8}=\left[\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\right]$$

1 b. obtain the constant term & first sine & cosine terms in the Fourier expansion of y from the following table

x	0	1	2	3	4	5
y	9	18	24	28	26	20

Soln:- $2l=6$, $l=\frac{6}{2}=3$, $\boxed{l=3}$ $\boxed{N=6}$

x	y	$\theta=\frac{\pi x}{l}$	$\cos\theta$	$y\cos\theta$	$\sin\theta$	$y\sin\theta$
0	9	$\frac{\pi(0)}{3}=0°$	1	9	0	0
1	18	$\frac{\pi(1)}{3}=60°$	0.5	9	0.866	15.588
2	24	$\frac{\pi(2)}{3}=120°$	-0.5	-12	0.866	20.784
3	28	$\frac{\pi(3)}{3}=180°$	-1	-28	0	0
4	26	$\frac{\pi(4)}{3}=240$	-0.5	-13	-0.866	-22.516
5	20	$\frac{\pi(5)}{3}=300$	0.5	10	-0.866	-17.32
	$\Sigma y = 125$		$\Sigma y\cos\theta = -25$		$\Sigma y\sin\theta = -3.464$	

$a_0=\frac{2}{N}\Sigma y$ $\quad a_n=\frac{2}{N}\Sigma y\cos nx$ $\quad b_n=\frac{2}{N}\Sigma y\sin nx$

$a_0=\frac{2}{6}\times125=\underline{41.66}$ $\quad a_1=\frac{2}{6}\times-25=\underline{-8.33}$ $\quad b_1=\frac{2}{6}\times-3.464=\underline{-1.1546}$

$\frac{a_0}{2}=\frac{41.66}{2}$

$\boxed{\frac{a_0}{2}=20.83}$

we have $y=f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}b_n\sin nx$

$$\boxed{y=f(x)=20.83-8.33\cos x-1.1546\sin x}$$

2 | a. Expand $f(x) = |x|$ as a Fourier Series in $-\pi \le x \le \pi$ & deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots = \frac{\pi^2}{8}$ (07)

Soln :- Given :- i.e

$$f(x) = \begin{cases} -x, & -\pi < x \le 0 \\ x, & 0 \le x < \pi \end{cases}$$

$2l = \pi - (-\pi) = 2\pi$, $\boxed{l=\pi}$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$l=\pi$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \rightarrow \text{(I)}$$

$\phi(x) = -x \qquad \psi(x) = x$

$\phi(-x) = -(-x) = x = \psi(x) \quad \therefore f(x)$ is even, $b_n = 0$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx \qquad a_n = \frac{2}{l}\int_0^l f(x)\cdot\cos\frac{n\pi x}{l}\,dx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} x\,dx \qquad a_n = \frac{2}{\pi}\int_0^{\pi} \underset{u}{x}\cdot \underset{v}{\cos nx}\,dx$$

$$= \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi} \qquad a_n = \frac{2}{\pi}\left[x\cdot\cancel{\frac{\sin nx}{n}}^{=0} - 1\cdot\left(-\frac{\cos nx}{n^2}\right)+0\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{\pi^2}{2}\right] \qquad = \frac{2}{n^2\pi}\left[+\cos nx\right]_0^{\pi}$$

$$\boxed{a_0 = \pi} \qquad = \frac{2}{n^2\pi}\left[\cos n\pi - \cos 0\right]$$

$$\boxed{\frac{a_0}{2} = \frac{\pi}{2}} \qquad a_n = \frac{2}{n^2\pi}\left[(-1)^n - 1\right]$$

Substitute $\frac{a_0}{2}$ & a_n in equ (I) we get

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{n^2\pi}\left[(-1)^n - 1\right]\cos nx$$

To deduce put $x = 0$.

$$= \frac{\pi}{2} + \frac{2}{\pi}\left[-\frac{2}{1^2}\cos 0 + 0 - \frac{2\cos 3(0)}{3^2} + 0 - \frac{2\cos 5(0)}{5^2} + \cdots\right]$$

$$-\frac{\pi}{2} = \cancel{0} - \frac{4}{\pi}\left[\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\right]$$

$$\frac{\pi^2}{8} = \frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$$

Q 2(b) Obtain half range Cosine Series of $f(x) = x \sin x$ in $0 < x < \pi$

Soln :- $l = \pi$

Half range cosine series is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right).$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \quad \text{--- (I)}$$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx.$$

$$a_0 = \frac{2}{\pi}\int_0^\pi \underset{u}{x}\, \underset{v}{\sin x}\,dx.$$

$$a_0 = \frac{2}{\pi}\left[x\cdot(-\cos x) - 1\cdot\cancel{(-\sin x)}^{=0}\right]_0^\pi$$

$$= \frac{2}{\pi}\left[-x\cos x\right]_0^\pi$$

$$= \frac{2}{\pi}\left[-\pi\cos\pi - 0\cdot\cos 0\right]$$

$$= \frac{2}{\pi}\left[-\pi(-1) - 0\right]$$

$$a_0 = \frac{2}{\pi}(\pi)$$

$$\boxed{\frac{a_0}{2} = 1}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cdot\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{2}{\pi}\int_0^\pi x\cdot\sin x\cdot\cos nx\,dx.$$

$\cos nx \sin x$
$\cos A \sin B$

$$= \frac{\cancel{2}}{\pi}\int_0^\pi x \times \frac{1}{\cancel{2}}\left[\sin(1+n)x + \sin(1-n)x\right]dx$$

$$= \frac{1}{\pi}\left(\int_0^\pi x\sin(1+n)x\,dx + \int_0^\pi x\sin(1-n)x\,dx\right)$$

$$= \frac{1}{\pi}\left[x\cdot\frac{-\cos(n+1)x}{n+1} - 1\times-\frac{\sin(n+1)x}{(n+1)^2}^{\to 0} - x\cdot\frac{-\cos(n-1)x}{n-1} - 1\times-\frac{\sin(n-1)x}{(n-1)^2}^{\to 0}\right]_0^\pi$$

$$= \frac{\pi}{\pi}\left[\frac{-\cos(n+1)\pi}{n+1} + \frac{\cos(n-1)\pi}{n-1}\right]$$

$$a_n = (-1)^n\left[\frac{-2}{n^2-1}\right] \Rightarrow \boxed{\frac{2(-1)^{n+1}}{n^2-1}} \quad n \neq 1$$

$$a_1 = \frac{2}{\pi}\int_0^l f(x)\cdot\cos\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{\pi}\int_0^\pi x\sin x\cdot\cos x\,dx \quad \boxed{n=1}$$

$$= \frac{\cancel{2}}{\pi}\int_0^\pi x\cdot\frac{\sin 2x}{\cancel{2}}$$

$$= \frac{1}{\pi}\left[x\cdot\frac{-\cos 2x}{2} - 1\left(-\frac{\sin 2x}{4}\right)^{=0}\right]_0^\pi$$

$$= \frac{1}{2\pi}\left[-\pi\cdot\cos\pi - 0\right]$$

$$\boxed{a_1 = -\frac{1}{2}}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx).$$

$$f(x) = 1 + \left(-\frac{1}{2}\right)\cos x + \sum_{n=1}^{\infty}\frac{2(-1)^{n+1}}{n^2-1}\cdot\cos nx$$

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2. [C] The following table gives variation of Periodic current over a period T. Show that there is a direct current part of 0.75 amp in the variable current & obtain amplitude of first harmonic

t (sec)	0	$\frac{T}{6}$	$\frac{T}{3}$	$T/2$	$\frac{2T}{3}$	$\frac{5T}{6}$
A (amp)	1.98	1.3	1.05	1.3	-0.88	-0.25

$\boxed{t = T/6}$

$\frac{2\pi (T)}{6T} = 60°$

Soln:- $2l = T$, $l = T/2$, $\boxed{N = 6}$ $\theta = \frac{\pi t}{l} = \frac{\pi \times t}{T/2} = \frac{2\pi t}{T}$

t(sec)	y_0 A (amp)	$\theta = \frac{2\pi t}{T}$	$\cos\theta$	$A\cos\theta$	$\sin\theta$	$A\sin\theta$
0	1.98	0	1	1.98	0	0
T/6	1.3	60°	0.5	0.65	0.866	1.1258
T/3	1.05	120°	-0.5	-0.525	0.866	0.9093
T/2	1.3	180°	-1	-1.3	0	0
2T/3	-0.88	240°	-0.5	0.44	-0.866	0.76208
5T/6	-0.25	300°	0.5	-0.125	-0.866	0.2165
	A(amp) = 4.5			$\Sigma A\cos\theta = 1.12$		$\Sigma A\sin\theta = 3.0136$

$y = f(x) = \frac{a_0}{2} + a_1 \cos\theta + b_1 \sin\theta$

$a_0 = \frac{2}{N} \times \Sigma A = \frac{2}{6} \times 4.5 = 1.5$ $\frac{a_0}{2} = \frac{1.5}{2} = 0.75$

$a_1 = \frac{2}{N} \times \Sigma A\cos\theta = \frac{2}{6} \times 1.12 = 0.3733$

$b_1 = \frac{2}{N} \times \Sigma A\sin\theta = \frac{2}{6} \times 3.0136 = 1.0045$

$$\boxed{y = f(x) = 0.75 + 0.3733\cos\theta + 1.0045\sin\theta}$$

Amplitude of 1st harmonics $= \sqrt{a_1^2 + b_1^2}$

$= \sqrt{0.3733^2 + 1.0045^2} = \underline{1.0716}$

Direct part of the current is equal to $= \frac{a_0}{2} = \frac{1.5}{2} = \underline{0.75}$ amps

MODULE - 2

3. a. Find the fourier transform of

$f(x) = \begin{cases} 1-x^2 & |x| \le 1 \\ 0 & |x| > 1 \end{cases}$ Hence evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos \frac{x}{2}\, dx$.

Soln:- By defn

$$F\{f(x)\} = \int_{-\infty}^{\infty} f(x)\, e^{iux}\, dx$$

$$= \int_{-\infty}^{-1} f(x)\, e^{iux}\, dx + \int_{-1}^{1} f(x)\, e^{iux}\, dx + \int_{1}^{\infty} f(x)\, e^{iux}\, dx$$

$$= 0 + \int_{-1}^{1} (1-x^2)\, e^{iux}\, dx + 0$$

By Bernoulli's method of integration

$$= \left[(1-x^2) \cdot \frac{e^{iux}}{iu} - (-2x)\frac{e^{iux}}{i^2u^2} + (-2)\frac{e^{iux}}{i^3u^3} \right]_{-1}^{1}$$

$$= \left[0 + 2\frac{e^{iu}}{i^2u^2} - (-2)\frac{e^{iu}}{i^2u^2} - \frac{2}{i^3u^3}\left[e^{iu} - e^{-iu}\right]\right]$$

$$= \frac{2}{i^2u^2}\left[e^{iu} + e^{-iu}\right] - \frac{2}{i^3u^3}\left[e^{iu} - e^{-iu}\right]$$

$$= \frac{2}{i^2u^2}\left[2\cos u\right] - \frac{2}{i^3u^3}\left[2i\sin u\right]$$

$$= \frac{4}{i^2u^2}\left[\cos u - \frac{i\sin u}{i^3 u}\right]$$

$e^{iu} = \cos u + i\sin u$
$e^{-iu} = \cos u - i\sin u$
$e^{iu} + e^{-iu} = 2\cos u$
$\cos u = \frac{e^{iu} + e^{-iu}}{2}$
$i^2 = -1$
$i^3 = -i$

$$F[f(x)] = \frac{4}{i^2u^2}\left[\frac{u\cos u - \sin u}{-u}\right] = \frac{-4u\cos u + 4\sin u}{i^2u^3} = -4\left[\frac{u\cos u - \sin u}{u^3}\right] = \hat{f}(u)$$

$$\left\langle F^{-1}[\hat{f}(x)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(u)\, e^{-iux}\, dx \right\rangle$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} -4\left(\frac{u\cos u - \sin u}{u^3}\right) e^{-iux}\, dx$$

$$= -\frac{2}{\pi}\int_{-\infty}^{\infty} \left(\frac{u\cos u - \sin u}{u^3}\right) e^{-iux}\, dx \qquad \therefore \text{Put } x = \tfrac{1}{2}$$

$$f(\tfrac{1}{2}) = -\frac{2}{\pi}\int_{-\infty}^{\infty} \frac{u\cos u - \sin u}{u^3}\, e^{-\frac{iu}{2}}\, du.$$

$$f(x) = 1-x^2 \Rightarrow f(\tfrac{1}{2}) = 1 - (\tfrac{1}{2})^2 = 1 - \tfrac{1}{4} = \tfrac{3}{4}$$

$$\frac{3}{4} = -\frac{2}{\pi}\int_{-\infty}^{\infty} \frac{u\cos u - \sin u}{u^3} \times \left(\cos\frac{u}{2} - i\sin\frac{u}{2}\right) du.$$

$$-\frac{3\pi}{8} = \int_{-\infty}^{\infty} \left(\frac{u\cos u - \sin u}{u^3}\right)\cos\frac{u}{2} - i\left(\frac{u\cos u - \sin u}{u^3}\right)\sin u/2 \, du$$

By equating real part B.S

$$-\frac{3\pi}{8} = \int_{-\infty}^{\infty} \frac{u\cos u - \sin u}{u^3} \times \cos\frac{u}{2}\, du$$

for $\int_{-\infty}^{\infty}$ have to replace u by $(-u)$

$$f(-u) = \frac{-u\cos(-u) - \sin(-u)}{(-u)^3} \times \cos\left(-\frac{u}{2}\right)$$

$$= \frac{-(u\cos u - \sin u)}{-u^3} \times \cos u/2$$

$$f(-u) = \frac{u\cos u - \sin u}{u^3} \times \cos u/2$$

$f(-u) = f(u)$ is even. [Since it is an even

so $f(x) = 2\int_0^{\infty} f(x)\,dx$

$$-\frac{3\pi}{8} = 2\int_0^{\infty} \frac{u\cos u - \sin u}{u^3} \cdot \cos\left(\frac{u}{2}\right) du$$

$$-\frac{3\pi}{16} = \int_0^{\infty} \frac{u\cos u - \sin u}{u^3} \times \cos\left(\frac{u}{2}\right) du$$

$$\boxed{-\frac{3\pi}{16} = \int_0^{\infty} \frac{x\cos x - \sin x}{x^3} \times \cos\left(\frac{x}{2}\right) dx}$$

3. $\boxed{b}$ Find the fourier Cosine transform of

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2-x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$$

$$F\{f(x)\} = \int_0^\infty f(x) \cdot \cos ux \, dx$$

$$= \int_0^1 \underset{u}{x} \underset{v}{\cos ux} \, dx + \int_1^2 \underset{u}{2-x} \underset{v}{\cos ux} + \int_2^\infty 0 \cos ux \, dx$$

$$= \left[x \cdot \frac{\sin ux}{u} - 1\left(-\frac{\cos ux}{u^2}\right) \right]_0^1 + \left[2-x \frac{\sin ux}{u} - (-1)\left(-\frac{\cos ux}{u^2}\right) \right]_1^2$$

$$= \left[\frac{1}{u}[\sin u - 0] + \frac{1}{u^2}[\cos u - \cos 0] \right] + \left[\frac{1}{u}[0 - \sin u] - \frac{1}{u^2}[\cos 2u - \cos u] \right]$$

$$= \cancel{\frac{\sin u}{u}} + \frac{\cos u}{u^2} - \frac{1}{u^2} - \cancel{\frac{\sin u}{u}} - \frac{\cos 2u}{u^2} + \frac{\cos u}{u^2}$$

$$F\{f(x)\} = \frac{2\cos u}{u^2} - \frac{1}{u^2} - \frac{\cos 2u}{u^2}$$

3. [C] Find the inverse Z-transform of $\dfrac{3z^2+2z}{(5z-1)(5z+2)}$

Soln:- $\dfrac{3z^2+2z}{25\left(z-\frac{1}{5}\right)\left(z+\frac{2}{5}\right)} = \dfrac{1}{25}\left[\dfrac{3z^2+2z}{\left(z-\frac{1}{5}\right)\left(z+\frac{2}{5}\right)}\right]$

Consider

$$\frac{3z^2+2z}{\left(z-\frac{1}{5}\right)\left(z+\frac{2}{5}\right)} = A\frac{z}{z-\frac{1}{5}} + B\frac{z}{z+\frac{2}{5}}$$

$$\frac{3z^2+2z}{\cancel{\left(z-\frac{1}{5}\right)\left(z+\frac{2}{5}\right)}} = \frac{A z\left(z+\frac{2}{5}\right) + Bz\left(z-\frac{1}{5}\right)}{\cancel{\left(z-\frac{1}{5}\right)\left(z+\frac{2}{5}\right)}}$$

$$\cancel{z}(3z+2) = \cancel{z}\left[A\left(z+\tfrac{2}{5}\right) + B\left(z-\tfrac{1}{5}\right)\right]$$

Put $z=\frac{1}{5} \Rightarrow 3\left(\frac{1}{5}\right)+2 = A\left(\frac{1}{5}+\frac{2}{5}\right)+0$

$$\frac{3+10}{5} = A\left(\tfrac{3}{5}\right)$$

$$\frac{13}{\cancel{5}} = A\left(\frac{3}{\cancel{5}}\right)$$

$$\boxed{A = \frac{13}{3}}$$

Put $z=-\frac{2}{5} \Rightarrow 3\left(-\frac{2}{5}\right)+2 = A(0)+B\left(-\frac{2}{5}-\frac{1}{5}\right)$

$$\frac{-6+10}{\cancel{5}} = -\frac{3}{\cancel{5}}B$$

$$\boxed{B = -\tfrac{4}{3}}, \quad 4 = -3B$$

$$\frac{3z^2+2z}{(z-\frac{1}{5})(z+\frac{2}{5})} = \frac{13}{3}\frac{z}{z-\frac{1}{5}} - \frac{4}{3}\frac{z}{z+\frac{2}{5}}$$

$$\frac{3z^2+2z}{(5z-1)(5z+2)} = \frac{1}{25}\left[\frac{3z^2+2z}{(z-\frac{1}{5})(z+\frac{2}{5})}\right]$$

$$= \frac{1}{25}\left[\frac{13}{3}\frac{z}{(z-\frac{1}{5})} - \frac{4}{3}\frac{z}{(z+\frac{2}{5})}\right]$$

Inverse Z-transform on B.S

$$Z_T^{-1}\left[\frac{3z^2+2z}{(5z-1)(5z+2)}\right] = \frac{13}{75} Z_T^{-1}\left[\frac{z}{z-\frac{1}{5}}\right] - \frac{4}{75} Z_T^{-1}\left[\frac{z}{z+\frac{2}{5}}\right]$$

$$= \frac{13}{75}\left(\frac{1}{5}\right)^n - \frac{4}{75}\left(-\frac{2}{5}\right)^n$$

$\boxed{07}$

4. $\boxed{b}$ Find the Z-transform of

i) $\cosh n\theta$

w.k.t. $\cosh n\theta = \frac{e^{n\theta}+e^{-n\theta}}{2}$

$$Z_T[\cosh n\theta] = Z_T\left[\frac{(e^{\theta})^n + (e^{-\theta})^n}{2}\right]$$

$$= \frac{1}{2}\left\{Z_T\{(e^{\theta})^n\} + Z_T\{(e^{-\theta})^n\}\right\}$$

$$= \frac{1}{2}\left[\frac{z}{z-e^{\theta}} + \frac{z}{z-e^{-\theta}}\right]$$

$$= \frac{z}{2}\left[\frac{1}{z-e^{\theta}} + \frac{1}{z-e^{-\theta}}\right]$$

$$= \frac{z}{2}\left[\frac{z-e^{-\theta}+z-e^{\theta}}{(z-e^{\theta})(z-e^{-\theta})}\right]$$

$$= \frac{z}{2}\left[\frac{2z-(e^{-\theta}+e^{\theta})}{z^2-ze^{-\theta}-ze^{\theta}+e^{\theta}e^{-\theta}}\right]$$

$$= \frac{z}{2}\left[\frac{2z-2\cosh x}{z^2-z2\cosh hx+1}\right]$$

$$Z_T[\cosh n\theta] = \frac{\cancel{2}}{\cancel{2}}\left[\frac{z^2-z\cosh hx}{z^2-2z\cosh hx+1}\right] \text{ or } \frac{z(z-\cosh\theta)}{z^2-2z\cosh\theta+1}$$

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2 ii) n^2.

$$Z_T(n^2) = \frac{z^2+z}{(z-1)^3}$$

w.k.T

$$Z_T(n^k) = -z\frac{d}{dz} Z_T(n^{k-1})$$

Put $k=2$

$$Z_T(n^2) = -z\frac{d}{dz} Z_T(n^{2-1})$$

$$= -z\frac{d}{dz} Z_T(n)$$

$$= -z\frac{d}{dz}\frac{z}{(z-1)^2}$$

$$= -z\left[\frac{(z-1)^2(1) - z\,2(z-1)}{(z-1)^4}\right]$$

$$= -z\,(\cancel{z-1})\left[\frac{(z-1)-2z}{(z-1)^4}\right]$$

$$= -z\left[\frac{(z-1)-2z}{(z-1)^3}\right]$$

$$= -z\left[\frac{-z-1}{(z-1)^3}\right] = \frac{z^2+z}{(z-1)^3}$$

4. c. Solve the difference equation

$y_{n+2} + 4y_{n+1} + 3y_n = 3^n$ with $y_0 = 0,\ y_1 = 1$.

Soln :- By taking Z-transform on B.S

$$Z_T[y_{n+2}] + 4Z_T[y_{n+1}] + 3Z_T[y_n] = Z_T[3^n]$$

$$z^2\left[\bar{y}(z) - y_0 - \frac{y_1}{z}\right] + 4z\left[\bar{y}(z) - y_0\right] + 3\bar{y}(z) = \frac{z}{z-3}$$

$$z^2\bar{y}(z) - z + 4z\bar{y}(z) - 0 + 3\bar{y}(z) = \frac{z}{z-3}$$

$$\bar{y}(z)\,(z^2+4z+3) - z = \frac{z}{z-3}$$

$$\bar{y}(z)\,(z^2+4z+3) = \frac{z}{z-3} + z = \frac{z+z^2-3z}{z-3} = \frac{z^2-2z}{z-3}$$

$z^2+3z+z+3$

$$\bar{y}(z)\,(z+3)(z+1) = \frac{z^2-2z}{z-3}$$

$$\boxed{\bar{y}(z) = \frac{z^2-2z}{(z-3)(z+3)(z+1)}}$$

Taking Z_T^{-1} on B.S

$$Z_T^{-1}\left[\bar{y}(z)\right] = Z_T^{-1}\left[\frac{z^2-2z}{(z-3)(z+3)(z+1)}\right] \rightarrow \text{equ } 1$$

Consider

$$\frac{z^2-2z}{(z-3)(z+3)(z+1)} = A\frac{z}{z-3} + B\frac{z}{z+3} + C\frac{z}{z+1},$$

$$\frac{z^2-2z}{\cancel{(z-3)(z+3)(z+1)}} = \frac{Az(z+3)(z+1) + Bz(z-3)(z+1) + Cz(z-3)(z+3)}{\cancel{(z-3)(z+3)(z+1)}}$$

$$\cancel{z}(z-2) = \cancel{z}\left[A(z+3)(z+1) + B(z-3)(z+1) + C(z-3)(z+3)\right]$$

Put $z=3$

~~$1 = 24A$.~~

$$(3-2) = \left[A(3+3)(3+1) + B\underbrace{(3-3)}_{=0}(3+1) + C\underbrace{(3-3)}_{=0}(3+3)\right]$$

$$1 = 24A, \quad \boxed{A = \frac{1}{24}}$$

Put $z=-3$

$$-5 = 0 + 12B$$

$$\boxed{B = -\frac{5}{12}}$$

Put $z=-1$

$$-3 = 0 + 0 - 8C$$

$$\boxed{C = \frac{3}{8}}$$

$$Z_T^{-1}\left[\frac{z^2-2z}{(z-3)(z+3)(z+1)}\right] = \frac{1}{24} Z_T^{-1}\left[\frac{z}{z-3}\right] - \frac{5}{12} Z_T^{-1}\left[\frac{z}{z+3}\right] + \frac{3}{8} Z_T^{-1}\left[\frac{z}{z+1}\right]$$

$$Z_T^{-1}\left[\frac{z^2-2z}{(z-3)(z+3)(z+1)}\right] = \frac{1}{24}\left[3^n\right] - \frac{5}{12}(-3)^n + \frac{3}{8}(-1)^n.$$

$$y_n = \frac{3^n}{24} - \frac{5(-3)^n}{12} + \frac{3(-1)^n}{8}$$

4. a) Find the fourier sine transform of $\frac{e^{-ax}}{x}$, $a>0$

Soln:- $F_S[f(x)] = \int_0^\infty f(x)\sin ux\,dx$

$$= \int_0^\infty \frac{e^{-ax}}{x}\sin ux\,dx.$$

$$= \int_0^\infty \frac{1}{x}\cdot e^{-ax}\sin ux\,dx$$

Method of

∴ This problem is [illegible] out of the syllabus

5 a) Find the coefficient of Correlation & two regression lines for the following data

x	1	2	3	4	5	6	7	8	9	10
y	10	12	16	28	25	36	41	49	40	50

x	y	$z=x-y$	x^2	y^2	z^2
1	10	-9	1	100	81
2	12	-10	4	144	100
3	16	-13	9	256	169
4	28	-24	16	784	576
5	25	-20	25	625	400
6	36	-30	36	1296	900
7	41	-34	49	1681	1156
8	49	-41	64	2401	1681
9	40	-31	81	1600	961
10	50	-40	100	2500	1600
$\Sigma x = 55$	$\Sigma y = 307$	$\Sigma z = -252$	$\Sigma x^2 = 385$	$\Sigma y^2 = 11387$	$\Sigma z^2 = 7624$

$n = 10$

$\bar{x} = \frac{\Sigma x}{n} = \frac{55}{10} = 5.5$

$\bar{y} = \frac{\Sigma y}{n} = \frac{307}{10} = 30.7$

$\bar{z} = \frac{\Sigma z}{n} = \frac{-252}{10} = -25.2$

$r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\sigma_x \sigma_y}$

$y - \bar{y} = r \cdot \frac{\sigma_y}{\sigma_x}(x - \bar{x})$

$x - \bar{x} = r \cdot \frac{\sigma_x}{\sigma_y}(y - \bar{y})$

$\sigma_x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2 = \frac{385}{10} - (5.5)^2 = 8.25$, $\sigma_x = \sqrt{8.25} = 2.8722$

$\sigma_y^2 = \frac{\Sigma y^2}{n} - (\bar{y})^2 = \frac{11387}{10} - (30.7)^2 = 196.21$, $\sigma_y = \sqrt{196.21} = 14.007$

$\sigma_{x-y}^2 = \sigma_z^2 = \frac{\Sigma z^2}{n} - (\bar{z})^2 = \frac{7624}{10} - (-25.2)^2 = 127.36$

$r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\sigma_x \cdot \sigma_y} = \frac{8.25 + 196.21 - 127.36}{2 \times 2.8722 \times 14.007} = 0.9582$

Regression line of y on x

$y - \bar{y} = r \cdot \frac{\sigma_y}{\sigma_x}(x - \bar{x})$

$y - 30.7 = 0.9582 \cdot \left(\frac{14.007}{2.8722}\right)(x - 5.5)$

$y - 30.7 = 4.672x - 25.696$

$\therefore y = 4.672x - 25.696 + 30.7$

$\boxed{y = 4.672x + 5.004}$

Regression line of x on y

$$x-\bar{x} = r \cdot \frac{\sigma_x}{\sigma_y}(y-\bar{y})$$

$$x - 5.5 = 0.9582 \cdot \left(\frac{2.8722}{14.007}\right)(y-30.7)$$

$$x - 5.5 = 0.19644 - 6.0320$$

$$x = 0.1964(y) - 6.0320 + 5.5$$

$$\boxed{x = 0.1964(y) - 0.5320}$$

5.) b) Fit a curve of the form $y = ae^{bx}$ for following data

x	5	6	7	8	9	10
y	133	55	23	7	2	2

Soln :- Given :- $y = ae^{bx}$

taking $\log_e$ on B.S

$\log_e y = \log_e(a \cdot e^{bx})$

$\log_e y = \log_e a + \log_e e^{bx}$

$\log_e y = \log_e a + bx$

$Y = \log_e y$; $A = \log_e a$, $B = bx$

$Y = A + Bx$

x	y	$Y = \log_e y$	x^2	xy
5	133	4.8903	25	24.4515
6	55	4.0073	36	24.0438
7	23	3.1354	49	21.9478
8	7	1.9459	64	15.5672
9	2	0.6931	81	6.2379
10	2	0.6931	100	6.931
Σx = 45	222	Σy = 15.3651	Σx^2 = 355	Σxy = 99.1792

Normal equations are

$nA + B\Sigma x = \Sigma Y$

$A\Sigma x + B\Sigma x^2 = \Sigma xY \Rightarrow$

$6A + 45B = 15.3651$

$45A + 355B = 99.1792$ $\Rightarrow$ $A = 9.4433$, $B = -0.91786$

$\log_e a = A$

$\log_e a = 9.4433$

$a = e^{9.443}$

$$\boxed{a = 12623.305}$$

$B = b = -0.91786$

Thus required curve is

$$y = (12623.305)\, e^{(-0.91786)x}$$

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5. c. Use Newton-Raphson method to find a real root of the equation $x \sin x + \cos x = 0$ near $x = \pi$.

Soln:- $f(x) = x \sin x + \cos x$

$f'(x) = \cancel{\sin x} + x\cos x - \cancel{\sin x} = + x \cos x.$

Here $x_0 = \pi$

By Newton Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = \pi - \frac{f(\pi)}{f'(\pi)} = \pi - \left[\frac{\pi \sin \pi + \cos \pi}{\pi \cos \pi}\right]$$

$$= \pi - 0.3183 = \underline{2.8232}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.832 - \frac{f(2.823)}{f'(2.823)} = 2.823 - \left[\frac{2.823 \sin 2.8 + \cos 2.823}{2.823 \cos 2.823}\right]$$

$$= \underline{2.7985}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.7985 - \left[\frac{2.798 \sin 2.798 + \cos 2.798}{2.798 \cos 2.798}\right] = \underline{2.7983}$$

$\therefore$ The real root of the equation is $x = \underline{2.798}$

-or-

6. a) In a partially destroyed lab record, only the lines of regression of y on x & x on y are available as $4x - 5y + 33 = 0$ & $20x - 9y = 107$ respectively. Calculate $\bar{x}, \bar{y}$ & co-efficient of correlation b/w x & y.

Soln:- $4x - 5y = -33$ — (1)

$20x - 9y = 107$ — (2).

We know that the lines of regression always pass through the point $\bar{x}, \bar{y}$

$4\bar{x} - 5\bar{y} = -33$

$20\bar{x} - 9\bar{y} = 107$

$$\boxed{\bar{x} = 13, \quad \bar{y} = 17}$$

Solving eqn (1) for y

$\Rightarrow 4x - 5y = -33$

$-5y = -33 - 4x$

$33 - 5y = -4x$

$33 + 4x = 5y$

$$y = \frac{33 + 4x}{5}$$

$$\boxed{y = 0.8x + 6.6}$$

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$\Rightarrow 20x - 9y = 107$

$x = \dfrac{107 + 9y}{20}$

$\boxed{x = 0.45y + 5.35}$

Now. co-efficient of correlation is given by

$r = \pm\sqrt{(\text{co-effi } x) \times (\text{co-eff } y)} = \pm\sqrt{0.8 \times 0.45} = \pm 0.6$

$\boxed{r = \pm 0.6}$

6 $\boxed{b}$. Fit a second degree parabola to the following data.

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1.1	1.3	1.6	2.0	2.7	3.4	4.1

Soln :- we have the Parabola of the form $\Rightarrow y = ax^2 + bx + c$

x	y	x^2	x^3	xy	x^4	x^2y
1.0	1.1	1	1	1.1	1	1.1
1.5	1.3	2.25	3.375	1.95	5.0625	2.925
2.0	1.6	4	8	3.2	16	6.4
2.5	2.0	6.25	15.625	5	39.0625	12.5
3.0	2.7	9	27	8.1	81	24.3
3.5	3.4	12.25	42.875	11.9	150.0625	41.65
4.0	4.1	16	64	16.4	256	65.6
$\Sigma x =$ 17.5	$\Sigma y =$ 16.2	$\Sigma x^2 =$ 50.75	$\Sigma x^3 =$ 161.875	$\Sigma xy =$ 47.65	$\Sigma x^4 =$ 548.1875	$\Sigma x^2y =$ 154.475

$n = 7$

$nc + b\Sigma x + a\Sigma x^2 = \Sigma y$

$c\Sigma x + b\Sigma x^2 + a\Sigma x^3 = \Sigma xy$

$c\Sigma x^2 + b\Sigma x^3 + a\Sigma x^4 = \Sigma x^2 y$

$7c + b\,17.5 + a\,50.75 = 16.2$

$17.5c + 50.75b + a\,161.875 = 47.65$

$50.75c + 161.87b + a\,548.187 = 154.475$

$a = 0.24269$, $b = -0.19205$, $c = 1.0348$

6 $\boxed{c}$ use the regula-falsi method to obtain a root of the equation $2x - \log_{10} x = 7$ which lies b/w 3.5 & 4.

Carry out 2 iterations

Soln: $f(x) = 2x - \log_{10} x - 7 = 0$

$a = 3.5$, $b = 4$

$f(3.5) = 2 \times 3.5 - \log_{10}(3.5) - 7 = -0.544 < 0$

$f(4) = 2 \times 4 - \log_{10}(4) - 7 = 0.3979 > 0$

$f(3.5) < 0$, $f(4) > 0$

Root lies b/w $(3.5, 4)$
$\quad\quad\quad\quad\quad\quad a \quad b$

By Regular falsi,

$$x = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$x_1 = \frac{3.5 \times f(4) - 4 f(3.5)}{f(4) - f(3.5)} = \frac{3.5 \times 0.3979 - 4 \times -0.5441}{0.3979 - (-0.5441)} = \underline{3.788}$$

$f(3.788) = -0.002409 < 0$

Root lies in $(3.788, 4)$

$a = 3.788$, $b = 4$

$$x_2 = \frac{3.788 \times 0.3979 - 4 \times -0.002409}{0.3979 - (-0.002409)} = \underline{3.789}$$

$x_2 = 3.789$

Thus, the real root of the given equation is

$x = \underline{3.789}$

MODULE – 4

7. a. The population of a town is given by the table.

Year	1951	1961	1971	1981	1991
Population in thousands	19.96	39.65	58.81	77.21	94.61

using Newton's forward & backward interpolation formula, Calculate the increase in the population from the year 1955 to 1985

Soln :-

Years x	Population in thousands	I^{st} differ	II^{nd} diff	III^{rd} differen	IV^{th} diff
1951	19.96				
		19.69			
1961	39.65		-0.53		
		19.16		-0.23	
1971	58.81		-0.76		-0.01
		18.4		-0.24	
1981	77.21		-1		
		17.4			
1991	94.61				

Case 1 :- Newton's forward Interpolation formula to find 1955

$$r = \frac{x - x_0}{h} = \frac{1955 - 1951}{10} = \underline{\underline{0.4}}$$

$$f(x) = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \frac{r(r-1)(r-2)(r-3)}{4!} \times \Delta^4 y_0$$

$$f(1955) = 19.96 + (0.4 \times 19.96) + \frac{0.4(0.4-1)}{2 \times 1} \times 0.53 + \frac{0.4(0.4-1)(0.4-2)}{3 \times 2 \times 1} - 0.23$$
$$+ \frac{0.4(0.4-1)(0.4-2)(0.4-3)}{4 \times 3 \times 2 \times 1} \times -0.01$$

$$f(1955) = 27.8996 - 0.01472 + 4.16 \times 10^{-4}$$

$$\boxed{f(1955) = 27.885296}$$

Case 2 :- Newton's Backward Interpolation formula to find 1985

$$r = \frac{x - x_n}{h} = \frac{1985 - 1991}{10} = \frac{-6}{10} = -0.6$$

$$y_r = y_n + r\nabla y_n + \frac{r(r+1)}{2!}\nabla^2 y_n + \frac{r(r+1)(r+2)}{3!} \times \nabla^3 y_n + \frac{r(r+1)(r+2)(r+3)}{4!} \times \nabla^4 y_n$$

$$= 94.61 - 0.6 \times 17.4 + \frac{(-0.6)(-0.6+1)}{2 \times 1} \times -1 + \left[\frac{-0.6(-0.6+1)(-0.6+2)}{3 \times 2 \times 1} \times -0.24\right] + \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)}{4 \times 3 \times 2 \times 1} \times -0.01$$

$$f(1985) = \underline{\underline{84.30}}$$

$$\Rightarrow f(1985) - f(1955)$$
$$= 84.30 - 27.89 = 56.41 \text{ thousand}$$

Thus, the increase in population from the year 1985 - 1955 = $\underline{56.41}$ thousands

7. [c] Given the values

x	2	4	5	6	8	10
y	10	96	196	350	868	1746

Construct the interpolating polynomial using Newton's divided difference interpolation formula.

Soln:-

x	$f(x)=y$	1st D.D	2nd D.D	3rd D.D
$x_0=2$	$f(x_0)=10$	$f(x_0,x_1)=\frac{96-10}{4-2}=43$	$f(x_0,x_1,x_2)=\frac{100-43}{5-2}=19$	$f(x_0,x_1,x_2,x_3)=2$
$x_1=4$	$f(x_1)=96$	$f(x_1,x_2)=\frac{196-96}{5-4}=100$	$f(x_1,x_2,x_3)=\frac{154-100}{6-4}=27$	$f(x_1,x_2,x_3,x_4)=2$
$x_2=5$	$f(x_2)=196$	$f(x_2,x_3)=\frac{350-196}{6-5}=154$	$f(x_2,x_3,x_4)=\frac{259-154}{8-5}=35$	$f(x_2,x_3,x_4,x_5)=2$
$x_3=6$	$f(x_3)=350$	$f(x_3,x_4)=\frac{868-350}{8-6}=259$	$f(x_3,x_4,x_5)=\frac{439-259}{10-6}=45$	
$x_4=8$	$f(x_4)=868$	$f(x_4,x_5)=\frac{1746-868}{10-8}=439$		
$x_5=10$	$f(x_5)=1746$			

$$f(x)=f(x_0)+(x-x_0)f(x_0,x_1)+(x-x_0)(x-x_1)f(x_0,x_1,x_2)+(x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3)$$

$$=10+(x-2)43+(x-2)(x-4)19+(x-2)(x-4)(x-5)2$$

$$=10+43x-86+19x^2-114x+152+2x^3-2x^2+16x-10x^2+60x-80$$

$$f(x)=-4+5x-9x^2+2x^3$$

$$\boxed{f(x)=2x^3-3x^2+5x-4}$$

$\boxed{\text{or}}$

8 [a] From the following table, estimate the number of students who obtained marks b/w 40 & 45.

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

Soln :- Less than 40 marks = 31

Less than 50 marks = 31+42 = 73

-11- 60 -11- = 73+51 = 124

-11- 70 -11- = 124+35 = 159

-11- 80 -11- = 159+31 = 190

Marks = x	Students = $f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$x_0 = 40$	31				
		42			
$x_1 = 50$	73		9		
		51		-25	
$x_2 = 60$	124		-16		37
		35		12	
$x_3 = 70$	159		-4		
		31			
$x_4 = 80$	190				

$$r = \frac{x - x_0}{h} = \frac{45-40}{10} = \frac{5}{10} = 0.5$$

$$f(x) = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \frac{r(r-1)(r-2)(r-3)}{4!}\Delta^4 y_0$$

$$f(45) = 31 + 0.5\times 42 + \frac{0.5(0.5-1)}{2\times 1}\times 9 + \frac{0.5(0.5-1)(0.5-2)}{3\times 2\times 1}\times -25 + \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{4\times 3\times 2\times 1}\times 37$$

$$f(45) = 47.8671 \simeq 48$$

$$f(45) - f(40) = 47.8671 - 31$$

$$= 48 - 31 = \underline{17} \text{ Students}$$

8. b Apply Lagrange's formula inversely to obtain the root of the equation $f(x) = 0$, given $f(30) = -30$, $f(34) = -13$, $f(38) = 3$, $f(42) = 18$.

Soln :- $x_0 = 30$ $x_1 = 34$ $x_2 = 38$ $x_3 = 42$

$y_0 = -30$ $y_1 = -13$ $y_2 = 3$ $y_3 = 18$

$y = f(x) = 0$

$y = 0$

$x = ?$

using Inverse Lagrange's formula

$$x = \frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)}\times x_0 + \frac{(y-y_0)(y-y_2)(y-y_3)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)}\times x_1 + \frac{(y-y_0)(y-y_1)(y-y_3)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)}\times x_2 + \frac{(y-y_0)(y-y_1)(y-y_2)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)}\times x_3$$

$$x = \frac{(0-(-13))(0-13)(0-18)}{(-30-(-13))(-30-3)(-30-18)} \times 30 + \frac{(0-(-30))(0-3)(0-18)}{(-13+30)(-13-3)(-13-18)} \times 34 + \frac{(0+30)(0+13)(0-18)}{(3+30)(3+13)(3-18)} \times 38$$

$$+ \frac{(0+30)(0+13)(0-3)}{(18+30)(18+13)(18-3)} \times 42$$

$$= -0.78208 + 6.5322 + 33.6818 - 2.201612$$

$$\boxed{x = 37.23}$$

7 b use Lagrange's interpolation formula to find y of $x=10$, given

x	5	6	9	11
y	12	13	14	16

$x_0 = 5$, $x_1 = 6$, $x_2 = 9$, $x_3 = 11$ $\quad x = 10$

$y_0 = 12$, $y_1 = 13$, $y_2 = 14$, $y_3 = 16$ $\quad y = ?$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$= \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)} \times 12 + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)} \times 13 + \frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)} \times 14$$

$$+ \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)} \times 16$$

$$= 2 - 4.333 + 11.666 + 5.33$$

$$\boxed{y = f(10) = 14.666}$$

8 c use Simpson's $\frac{1}{3}$ rule to find $\int_0^{0.6} e^{-x^2} dx$ by taking 7 ordinates

Soln :- $a = 0$, $b = 0.6$, $n = 6$.

$$h = \frac{b-a}{n} = \frac{0.6-0}{6} = 0.1$$

$$y = e^{-x^2}$$

x	$x_0=0$	$x_1=0.1$	$x_2=0.2$	$x_3=0.3$	$x_4=0.4$	$x_5=0.6$	$x_6=0.6$
y	$y_0=1$	$y_1=0.99$	$y_2=0.9608$	$y_3=0.9139$	$y_4=0.8521$	$y_5=0.7788$	$y_6=0.6976$

Simpson's $\frac{1}{3}$rd rule is given by

$$I = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+y_5+\cdots y_{n-1}) + 2(y_2+y_4+\cdots y_{n-2})\right]$$

for $n=6$

$$I = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{0.1}{3}\left[(1+0.6976) + 4(0.99+0.9139+0.7788) + 2(0.9608+0.8521)\right]$$

$$\boxed{I = 0.535}$$

Vasumathi

CBCS Scheme

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Third Semester B.E. Degree Examination, Dec.2017/Jan.2018
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Module-1

1 a. Express $f(x) = (\pi - x)^2$ as a Fourier series of period 2π in the interval $0 < x < 2\pi$. Hence deduce the sum of the series $1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots$ **(08 Marks)**

b. The turning moment T units of the Crank shaft of a steam engine is a series of values of the crank angle θ in degrees. Find the first four terms in a series of sines to represent T. Also calculate T when θ = 75°. **(08 Marks)**

θ:	0°	30°	60°	90°	120°	150°	180°
T:	0	5224	8097	7850	5499	2626	0

OR

2 a. Find the Fourier Series expansion of the periodic function,

$$f(x) = \begin{cases} l + x, & -l \le x \le 0 \\ l - x, & 0 \le x \le l \end{cases}$$ **(06 Marks)**

b. Obtain a half-range cosine series for $f(x) = x^2$ in $(0, \pi)$. **(05 Marks)**

c. The following table gives the variations of periodic current over a period:

t sec:	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$
A amp:	1.98	1.30	1.05	1.30	-0.88	-0.25

Show that there is a direct current part 0.75 amp in the variable current and obtain the amplitude of the first harmonic. **(05 Marks)**

Module-2

3 a. Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ and evaluate $\int_0^\infty \left(\frac{\sin x}{x}\right) dx$ **(06 Marks)**

b. Find the Fourier cosine transform of. $f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$. **(05 Marks)**

c. Obtain the inverse Z-transform of the following function, $\frac{z}{(z-2)(z-3)}$. **(05 Marks)**

OR

4 a. Find the Z-transform of $\cos\left(\frac{n\pi}{2} + \alpha\right)$. **(06 Marks)**

b. Solve $u_{n+2} - 5u_{n+1} + 6u_n = 36$ with $u_0 = u_1 = 0$, using Z-transforms. **(05 Marks)**

c. If Fourier sine transform of f(x) is $\frac{e^{-n\alpha}}{\alpha}$, $\alpha \neq 0$. Find f(x) and hence obtain the inverse Fourier sine transform of $\frac{1}{\alpha}$. **(05 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

Module-3

5 a. Calculate the Karl Pearson's co-efficient for the following ages of husbands and wives: **(06 Marks)**

Husband's age x:	23	27	28	28	29	30	31	33	35	36
Wife's age y:	18	20	22	27	21	29	27	29	28	29

b. By the method of least square, find the parabola $y = ax^2 + bx + c$ that best fits the following data: **(05 Marks)**

x:	10	12	15	23	20
y:	14	17	23	25	21

c. Using Newton-Raphson method, find the real root that lies near x = 4.5 of the equation $\tan x = x$ correct to four decimal places. (Here x is in radians). **(05 Marks)**

OR

6 a. In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as $4x - 5y + 33 = 0$ and $20x - 9y = 107$ respectively. Calculate $\overline{x}$, $\overline{y}$ and the coefficient of correlation between x and y. **(06 Marks)**

b. Find the curve of best fit of the type $y = ae^{bx}$ to the following data by the method of least squares: **(05 Marks)**

x:	1	5	7	9	12
y:	10	15	12	15	21

c. Find the real root of the equation $xe^x - 3 = 0$ by Regula Falsi method, correct to three decimal places. **(05 Marks)**

Module-4

7 a. From the following table of half-yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age of 46: **(06 Marks)**

Age:	45	50	55	60	65
Premium (in Rupees):	114.84	96.16	83.32	74.48	68.48

b. Using Newton's divided difference interpolation, find the polynomial of the given data: **(05 Marks)**

x	3	7	9	10
f(x)	168	120	72	63

c. Using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule to find $\int_0^{0.6} e^{-x^2}dx$ by taking seven ordinates. **(05 Marks)**

OR

8 a. Find the number of men getting wages below ₹ 35 from the following data: **(06 Marks)**

Wages in ₹ :	0 – 10	10 – 20	20 – 30	30 – 40
Frequency :	9	30	35	42

b. Find the polynomial f(x) by using Lagrange's formula from the following data: **(05 Marks)**

x:	0	1	2	5
f(x):	2	3	12	147

c. Compute the value of $\int_{0.2}^{1.4}(\sin x - \log_e x + e^x)dx$ using Simpson's $\left(\frac{3}{8}\right)^{th}$ rule. **(05 Marks)**

Module-5

9 a. A vector field is given by $\vec{F} = \sin y\,\hat{i} + x(1+\cos y)\hat{j}$. Evaluate the line integral over a circular path given by $x^2 + y^2 = a^2$, $z = 0$. **(06 Marks)**

b. If C is a simple closed curve in the xy-plane not enclosing the origin. Show that $\int_C \vec{F}.d\vec{R} = 0$, where $\vec{F} = \dfrac{y\hat{i} - x\hat{j}}{x^2 + y^2}$. **(05 Marks)**

c. Derive Euler's equation in the standard form viz., $\dfrac{\partial f}{\partial y} - \dfrac{d}{dx}\left[\dfrac{\partial f}{\partial y'}\right] = 0$. **(05 Marks)**

OR

10 a. Use Stoke's theorem to evaluate $\int_C \vec{F}.d\vec{R}$ where $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy-plane. **(06 Marks)**

b. Show that the geodesics on a plane are straight lines. **(05 Marks)**

c. Find the curves on which the functional $\int_0^1 \left((y')^2 + 12xy\right)dx$ with $y(0) = 0$ and $y(1) = 1$ can be extremized. **(05 Marks)**

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1 (a) Express $f(x) = (\pi - x)^2$ as a fourier series of period 2π in the interval $0 < x < 2\pi$. hence deduce the sum of the series $1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$

Given

$f(x) = (\pi - x)^2$

$2l = 2\pi - 0 = 2\pi \qquad l = \pi$

The fourier series of $f(x)$ is given by.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\cancel{\pi} x}{\cancel{\pi}}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\cancel{\pi} x}{\cancel{\pi}}\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \longrightarrow ①$$

Here $f(2\pi - x) = (\pi - (2\pi - x))^2$

$= (\pi - 2\pi + x)^2$

$= (-\pi + x)^2$

$= (-(\pi - x))^2 \Rightarrow (\pi - x)^2$

$f(x)$ is even $\therefore b_n = 0$

let $a_0 = \frac{2}{l} \int_0^l f(x)\, dx$

$a_0 = \frac{2}{\pi} \int_0^{\pi} (\pi - x)^2\, dx$

$a_0 = \frac{2}{\pi} \int_0^{\pi} \pi^2 + x^2 - 2\pi x\, dx$

$a_0 = \frac{2}{\pi} \left[\pi^2 x + \frac{x^3}{3} - 2\pi\left(\frac{x^2}{2}\right) \right]_0^{\pi}$

$a_0 = \frac{2}{\pi} \left[\pi^2(\pi) + \frac{\pi^3}{3} - 2\pi\left(\frac{\pi^2}{2}\right) - 0 \right]$

$a_0 = \frac{2}{\pi} \left[\frac{\pi^3}{3} \right]$

$$a_0 = \frac{2\pi^2}{3}$$

$$\boxed{\frac{a_0}{2} = \frac{\pi^2}{3}}$$

$$a_n = \frac{2}{l}\int_0^l f(x) \cdot \cos\left(\frac{n\pi x}{l}\right) dx$$

$$a_n = \frac{2}{\pi}\int_0^\pi (\pi - x)^2 \cos\left(\frac{n\pi x}{\pi}\right) dx$$

$$a_n = \frac{2}{\pi}\int_0^\pi (\pi - x)^2 \cos nx \, dx$$

$$a_n = \frac{2}{\pi}\left[\cancel{(\pi - x)^2 \frac{\sin nx}{n}}^{\to 0} + \frac{2(\pi - x)(-\cos nx)}{n^2} + \cancel{2(-1)\frac{(-\sin nx)}{n^3}}^{\to 0}\right]_0^\pi$$

$$\cancel{a_n = \frac{2}{\pi}\left[\frac{1}{n}\{(\pi - \pi)^2 \sin n\pi - \pi - \right.}$$

$$a_n = \frac{2}{\pi}\left[\frac{2}{n^2}\left\{-(\pi - \pi)\cos n\pi - (\pi)(-\cos 0)\right\}\right]$$

$$a_n = \frac{2}{\pi n^2}\left[2\pi \cos 0\right]$$

$$a_n = \frac{2}{n^2 \pi}\left[2\pi\right]$$

$$\boxed{a_n = \frac{4}{n^2}}$$

substituting in equation (1)

$$f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2} \cos nx .$$

$$= \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{\cos nx}{n^2} \qquad \therefore n = 1, 2, 3 - - -$$

$$= \frac{\pi^2}{3} + 4\left[\frac{\cos x}{1^2} + \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} + - - - - -\right]$$

put $x = 0$

$$f(0) = \frac{\pi^2}{3} + 4\left[\frac{\cos 0}{1^2} + \frac{\cos 0}{2^2} + \frac{\cos 0}{3^2} + - - -\right]$$

$$\pi^2 = \frac{\pi^2}{3} + 4\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$$

$$\pi^2 - \frac{\pi^2}{3} = 4\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$$

$$\frac{2\pi^2}{3} = 4\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$$

$$\frac{\pi^2}{8} = \left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$$

1 (b) The turning moment T units of the crank shaft of a stream engine is a series of values of the crank angle θ in degree. Find the first four terms in a series to represent T. Also calculate T when θ = 75°

θ	0°	30°	60°	90°	120°	150°	180°
T	0	5224	8097	7850	5499	2626	0

θ	T	T sinθ	T sin2θ	T sin3θ	T sin4θ
0	0	0	0	0	0
30	5224	2612.0	4524.116	5224.0	4524.116
60	8097	7012.207	7012.207	0	7012.207
90	7850	7850	0	-7850	0
120	5499	4762.273	-4762.273	0	4762.273
150	2626	1313.0	-2274.182	2626.0	-2274.182
	ΣT = 29296	23549.48	4499.868	0	0

$b_1 = \frac{2}{N} \Sigma T \sin\theta$

$b_1 = \frac{2}{6}(23549.48)$

$\boxed{b_1 = 7849.827}$

$b_2 = \frac{2}{N} \Sigma T \sin 2\theta$

$b_2 = \frac{2}{6}(4499.868)$

$\boxed{b_2 = 1499.956}$

The fourier series upto the first four terms is given by

$T = b_1 \sin\theta + b_2 \sin 2\theta + b_3 \sin 3\theta + b_4 \sin 4\theta$

$T = 7849.827 \sin\theta + 1499.965 \sin 2\theta + 0 + 0$

~~value~~ $T = 7849.827 \sin\theta + 1499.965 \sin 2\theta$

When $\theta = 75°$

$T = 7849.827 \sin(75) + 1499.965 \sin 2(75)$

$\boxed{T = 8332.328}$

2. (a) Find the fourier series expansion of the periodic function $f(x) = \begin{cases} l+x & -l \le x \le 0 \\ l-x & 0 \le x \le l \end{cases}$

Given

$$f(x) = \begin{cases} l+x & -l \le x \le 0 \\ l-x & 0 \le x \le l \end{cases}$$

$2l = l - (-l) = 2l \qquad \therefore l = l$

The fourier series of $f(x)$ is given by.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right) \longrightarrow (1)$$

here $\phi(x) = l + x \qquad \psi(x) = l - x$

$\phi(-x) = l + (-x) = l - x = \psi(x)$.

Hence it is even $\qquad \therefore b_n = 0$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$$a_0 = \frac{2}{l}\int_0^l (l-x)\,dx$$

$$a_0 = \frac{2}{l}\left[lx - \frac{x^2}{2}\right]_0^l$$

$$a_0 = \frac{2}{l}\left[l(l) - \frac{l^2}{2}\right]$$

$$a_0 = \frac{2}{l}\left[l^2 - \frac{l^2}{2}\right]$$

$$a_0 = \frac{\cancel{2}}{\cancel{l}}\left[\frac{l^{\cancel{2}}}{\cancel{2}}\right]$$

$$a_0 = l$$

$$\boxed{\frac{a_0}{2} = \frac{l}{2}}$$

$$a_n = \frac{2}{l}\int_0^l f(x).\cos\left(\frac{n\pi x}{l}\right)dx$$

$$a_n = \frac{2}{l}\int_0^l (l-x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$a_n = \frac{2}{l}\left[\cancel{(l-x)\frac{\sin\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}}}^{\,0} - \frac{(-1)\left(-\cos\left(\frac{n\pi x}{l}\right)\right)}{\frac{n^2\pi^2}{l^2}}\right]_0^l$$

$$a_n = \frac{2}{l}\left[-\frac{\cos\left(\frac{n\pi x}{l}\right)}{\frac{n^2\pi^2}{l^2}}\right]_0^l$$

$$a_n = \frac{2\times l^{\cancel{2}}}{n^2\pi^2\cancel{l}}\left[-\cos\left(\frac{n\pi x}{l}\right)\right]_0^l$$

$$a_n = \frac{2l}{n^2\pi^2}\left[-\cos\left(\frac{n\pi l}{l}\right) + \cos 0\right]$$

$$a_n = \frac{2l}{n^2\pi^2}\left[-\cos n\pi + \cos 0\right]$$

$$a_n = \frac{2l}{n^2\pi^2}\left[-(-1)^n + 1\right]$$

$$\boxed{a_n = \frac{2l}{n^2\pi^2}\left[1 - (-1)^n\right]}$$

substituting in eqn 1

$$f(x) = \frac{l}{2} + \sum_{n=1}^{\infty}\frac{2l}{n^2\pi^2}\left[1-(-1)^n\right]\cos\left(\frac{n\pi x}{l}\right)$$

2\. (b) obtain a half - range cosine series for $f(x) = x^2$ in $(0, \pi)$

Given

$f(x) = x^2$

$l = \pi$.

The half range fourier series is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx \rightarrow (1)$$

let $b_n = \frac{2}{l}\int_0^l f(x) \cdot \sin\left(\frac{n\pi x}{l}\right) dx$

2\. $b_n = \frac{2}{l}\int_0^l x^2 \sin\left(\frac{n\pi x}{l}\right) dx$

$$b_n = \frac{2}{\pi}\int_0^\pi x^2 \sin nx \, dx$$

$$b_n = \frac{2}{\pi}\left[x^2 - \frac{\cos nx}{n} - \frac{2x(-\sin nx)}{n^2} + \frac{2\cos nx}{n^3}\right]_0^\pi$$

$$b_n = \frac{2}{\pi}\left[-\frac{1}{n}\{\pi^2 \cos n\pi - 0\} + \frac{2}{n^3}\{\cos n\pi - \cos 0\}\right]$$

$$b_n = \frac{2}{\pi}\left[-\frac{1}{n}\pi^2 \cos n\pi + \frac{2}{n^3}\cos n\pi - 1\right]$$

$$b_n = \frac{2}{\pi}\left[-\frac{1}{n}\pi^2(-1)^n + \frac{2}{n^3}\{(-1)^n - 1\}\right]$$

$$b_n = \frac{2}{\pi}\left[\frac{\pi^2(-1)^{n+1}}{n} + \frac{2}{n^3}\{(-1)^n - 1\}\right]$$

Equation (1) becomes.

$$f(x) = \sum_{n=1}^{\infty} \frac{2}{\pi}\left[\frac{\pi^2(-1)^{n+1}}{n} + \frac{2}{n^3}\{(-1)^n - 1\}\right] \sin nx$$

2\. (c) The following table is given the variations of periodic current over a period.

t sec	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$
A amp :	1.98	1.30	1.05	1.30	-0.88	-0.25

Show that there is a direct part 0.75 amp in the variable current & obtain the amplitude of the first harmonic

put $\theta = \frac{\pi x}{l}$ $\Rightarrow 2l = T$

$l = T/2$

$\theta = \frac{\pi t}{T/2}$

$\theta = \frac{2\pi t}{T}$

t	A	$\theta = \frac{2\pi t}{T}$	$A\cos\theta$	$A\sin\theta$
0	1.98	0	1.98	0
$T/6$	1.3	60°	0.65	1.1258
$T/3$	1.05	120°	-0.5250	0.9093
$T/2$	1.3	180°	-1.3000	0
$2T/3$	-0.88	240	0.44	0.7621
$5T/6$	-0.25	300	-0.1250	0.2165
	$\Sigma A = 4.5$		$\Sigma A\cos\theta = 1.120$	$\Sigma A\sin\theta = 3.0137$

$a_0 = \frac{2}{N} \Sigma A$

$a_0 = \frac{2}{6} \times 4.5$

$a_0 = 1.5$

$\boxed{\frac{a_0}{2} = 0.75}$

$a_1 = \frac{2}{N} \Sigma A\cos\theta$

$a_1 = \frac{2}{6} \times 1.12 = 0.3733$

$a_2 = \frac{2}{N} \Sigma A\sin\theta$

$a_2 = \frac{2}{6} \times 3.0137$

$\boxed{a_2 = 1.0046}$

The fourier series upto 1^{st} harmonics is given by

$A = \frac{a_0}{2} + (a_1 \cos A + b_1 \sin A)$

$A = 0.75 + (0.3733)\cos\theta + 1.0046 \sin\theta$

Since $\frac{A_0}{2} = 0.75$

The direct current part in variable part is 0.75A

Amplitude of I harmonic $= \sqrt{a_1^2 + b_1^2}$

$= \sqrt{(0.3733)^2 + (1.0046)^2}$

$= 1.0717$

Module - 2

3. (a) Find the fourier transform of $f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$

and evaluate $\int_0^\infty \left(\frac{\sin x}{x}\right) dx$

By defination

$F(f(x)) = \int_{-\infty}^{\infty} f(x) e^{iux} dx$

$= \int_{-\infty}^{-1} f(x) e^{iux} dx + \int_{-1}^{1} f(x) dx + \int_{1}^{\infty} f(x) e^{iux} dx$

$= 0 + \int_{-1}^{1} 1 e^{iux} dx + 0$

$= \left[\frac{e^{iux}}{iu}\right]_{-1}^{1}$

$F(f(x)) = \frac{e^{iu} - e^{-iu}}{iu}$

$e^{iu} = \cos u + i\sin u$

$e^{-iu} = \cos u - i\sin u$

$(-) \quad (-) \quad (+)$

$e^{iu} - e^{-iu} = 2i\sin u$

$F(f(x)) = \frac{1}{u}\left[\frac{e^{iu} - e^{-iu}}{i}\right]$

$F(f(x)) = \frac{2}{u}\left[\frac{e^{iu} - e^{-iu}}{2i}\right]$

$F(f(x)) = \frac{2\sin u}{u} = \hat{f}(u)$

By inverse transform

$$\bar{F}^{-1}(\hat{f}(x)) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(u)\, e^{iux}\, du$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\frac{\sin u}{u}\, e^{iux}\, du$$

put $x = 0$

$$f(0) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sin u}{u}\, e^{0}\, du$$

$$1 = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sin u}{u}\, du$$

[since $\frac{\sin u}{u}$ is an even function of u]

$$1 = \frac{2}{\pi}\int_{0}^{\infty} \frac{\sin u}{u}\, du$$

$$\frac{\pi}{2} = \int_{0}^{\infty} \frac{\sin u}{u}\, du$$

Thus by changing u to x we have

$$\int_{0}^{\infty} \frac{\sin x}{x}\, dx = \frac{\pi}{2}$$

3. (b) The fourier cosine transform of $f(x) = \begin{cases} x & \text{for } 0<x<1 \\ 2-x & \text{for } 1<x<2 \\ 0 & \text{for } x>2 \end{cases}$

Given

$$f(x) = \begin{cases} x & \text{for } 0<x<1 \\ 2-x & \text{for } 1<x<2 \\ 0 & \text{for } x>2 \end{cases}$$

By definations

$$F_c(f(x)) = \int_{0}^{\infty} f(x)\cdot\cos ux\, dx$$

$$= \int_{0}^{1} f(x)\cdot\cos ux + \int_{1}^{2} f(x)\cos ux + \int_{2}^{\infty} f(x)\cos ux\, dx$$

$$= \int_{0}^{1} x\cos ux\, dx + \int_{1}^{2} (2-x)\cos ux + 0$$

$$= \left[x\frac{\sin ux}{u} - \frac{(-\cos ux)}{u^2}\right]_0^1 + \left[(2-x)\frac{\sin ux}{u} - \frac{(-1)(-\cos ux)}{u^2}\right]_1^2$$

$$= \left[\frac{1}{u}\{\sin u - 0\} + \frac{1}{u^2}\{\cos u - 1\}\right] + \left[\frac{1}{u}\{0 - \sin u\} - \frac{1}{u^2}\{\cos 2u - \cos u\}\right]$$

$$= \frac{\sin u}{u} + \frac{1}{u^2}[\cos u - 1] - \frac{\sin u}{u} - \frac{1}{u^2}[\cos 2u - \cos u]$$

$$= \cancel{\frac{\sin u}{u}} + \frac{\cos u}{u^2} - \frac{1}{u^2} - \cancel{\frac{\sin u}{u}} - \frac{\cos 2u}{u^2} + \frac{\cos u}{u^2}$$

$$= \frac{2\cos u - 1 - \cos 2u}{u^2}$$

$\therefore 1 + \cos 2u = 2\cos^2 u$

$$= \frac{2\cos u - (1 + \cos 2u)}{u^2}$$

$$\boxed{F_c(f(x)) = \frac{2\cos u - 2\cos^2 u}{u^2}}$$

3. (c). obtain the inverse z-transform of the following function, $\frac{z}{(z-2)(z-3)}$

given

$$\frac{z}{(z-2)(z-3)} = A\frac{z}{(z-2)} + B\frac{z}{(z-3)}$$

$$\frac{z}{\cancel{(z-2)(z-3)}} = \frac{Az(z-3) + Bz(z-2)}{\cancel{(z-2)(z-3)}}$$

$$\cancel{z} = \cancel{z}[A(z-3) + B(z-2)]$$

$$1 = A(z-3) + B(z-2)$$

put $z = 2$

$1 = A(2-3) + 0$

$1 = -A$

$\boxed{A = -1}$

put $z = 3$

$1 = 0 + B(3-2)$

$1 = B(1)$

$\boxed{B = 1}$

$$\frac{z}{(z-2)(z-3)} = \frac{-z}{(z-2)} + \frac{z}{(z-3)}$$

$$z_1^{-1}\left[\frac{z}{(z-2)(z-3)}\right] = -1\, z^{-1}\left[\frac{z}{z-2}\right] + z_1^{-1}\left[\frac{z}{z-3}\right]$$

$$Z_T^{-1}\left[\frac{z}{(z-2)(z-3)}\right] = -2^n + 3^n$$

$$\boxed{Z_T^{-1}\left[\frac{z}{(z-2)(z-3)}\right] = 3^n - 2^n}$$

4 (a) find the z-transform of $\cos\left[\frac{n\pi}{2}+\alpha\right]$

We have

$$U_n = \cos\left(\frac{n\pi}{2}+\alpha\right)$$

$$U_n = \cos\left(\frac{n\pi}{2}\right)\cos\alpha - \sin\left(\frac{n\pi}{2}\right)\sin\alpha$$

$$U_n = \cos\alpha\, Z_T\left[\cos\left(\frac{n\pi}{2}\right)\right] - \sin\alpha\, Z_T\left[\sin\left(\frac{n\pi}{2}\right)\right] \rightarrow (1)$$

WKT

$$Z_T[\cos n\theta] = \frac{z^2 - z\cos\theta}{z^2 - 2z\cos\theta + 1}$$

put $\theta = \pi/2$

$$Z_T\left[\cos\frac{n\pi}{2}\right] = \frac{z^2 - z\cos(\pi/2)}{z^2 - 2z\cos(\pi/2) + 1}$$

$$= \frac{z^2 - 0}{z^2 - 0 + 1}$$

$$\boxed{Z_T\left[\cos\left(\frac{n\pi}{2}\right)\right] = \frac{z^2}{z^2+1}}$$

WKT $Z_T[\sin n\theta] = \dfrac{z\sin\theta}{z^2 - 2z\cos\theta + 1}$

put $\theta = \pi/2$

$$Z_T\left[\sin\left(\frac{n\pi}{2}\right)\right] = \frac{z\sin(\pi/2)}{z^2 - 2z\cos(\pi/2) + 1}$$

$$\boxed{Z_T\left[\sin\left(\frac{n\pi}{2}\right)\right] = \frac{z}{z^2+1}}$$

From equation (1)

$$Z_T\left[\cos\left(\frac{n\pi}{2}+\alpha\right)\right] = \cos\alpha\left[\frac{z^2}{z^2+1}\right] - \sin\alpha\left[\frac{z}{z^2+1}\right]$$

$$\boxed{Z_T\left[\cos\left(\frac{n\pi}{2}\right)+\alpha\right] = z\left[\frac{z\cos\alpha - \sin\alpha}{z^2+1}\right]}$$

4. (b) Solve $u_{n+2} - 5u_{n+1} + 6u_n = 36$ with $u_0 = u_1 = 0$ using z-transforms

Taking Z_T on both side

$$Z_T[u_{n+2}] - 5Z_T[u_{n+1}] + 6Z_T[u_n] = 36\,Z_T(1)$$

$$z^2\left[\bar{u}(z) - u_0 - \frac{u_1}{z}\right] - 5z[\bar{u}(z) - u_0] + 6[\bar{u}(z)] = 36\left[\frac{z}{z-1}\right]$$

$$z^2\bar{u}(z) - z^2u_0 - zu_1 - 5z\bar{u}(z) + 5zu_0 + 6\bar{u}(z) = 36\left[\frac{z}{z-1}\right]$$

$$\bar{u}(z)[z^2-5z+6] - u_0[z^2-5z] - zu_1 = 36\left[\frac{z}{z-1}\right]$$

$$(z-6)(z+1)\bar{u}(z) = u_0[z^2-5z] + u_1z + 36\left[\frac{z}{z-1}\right]$$

$$\bar{u}(z) = u_0\left[\frac{z^2-5z}{(z-6)(z+1)}\right] + u_1\left[\frac{z}{(z-6)(z+1)}\right] + 36\left[\frac{z}{(z-6)(z-1)(z+1)}\right]$$

$$\bar{u}(z) = u_0\,p(z) + u_1\,q(z) + 36\,r(z) \longrightarrow (1)$$

Consider

$$p(z) = \frac{z^2-5z}{(z-6)(z+1)} = \frac{Az}{(z-6)} + \frac{Bz}{(z+1)}$$

$$\frac{z^2-5z}{(z-6)(z+1)} = \frac{zA}{(z-6)} + \frac{Bz}{(z+1)}$$

$$\frac{z(z-5)}{\cancel{(z-6)(z+1)}} = \frac{Az(z+1)+zB(z-6)}{\cancel{(z-6)(z+1)}}$$

$$\cancel{z}(z-5) = \cancel{z}[A(z+1) + B(z-6)]$$

$$(z-5) = A(z+1) + B(z-6)$$

put $z = 6$

$1 = A(7) + 0$

$\boxed{A = 1/7}$

put $z = -1$

$-6 = B(-7)$

$\boxed{B = 6/7}$

hence

$$Z_T^{-1}(p(z)) = \frac{1}{7} Z_T^{-1}\left[\frac{z}{z-6}\right] + \frac{6}{7} Z_T^{-1}\left[\frac{z}{z+1}\right]$$

$$Z_T^{-1}(p(z)) = \frac{1}{7}(6)^n + \frac{6}{7}(-1)^n \longrightarrow (2)$$

Consider

$$q(z) = \frac{z}{(z-6)(z+1)} = \frac{cz}{(z-6)} + \frac{dz}{(z+1)}$$

$$\frac{\cancel{z}}{(z-6)\cancel{(z+1)}} = \frac{\cancel{z}[c(z+1) + d(z-6)]}{(z-6)\cancel{(z+1)}}$$

$$1 = c(z+1) + d(z-6)$$

put $z = 6 \Rightarrow 1 = c(7)$ $\therefore \boxed{c = 1/7}$

put $z=-1$ $1 = D(-7)$ $\therefore \boxed{D = -1/7}$

Hence $Z_T^{-1}[q(z)] = \frac{1}{7} Z_T^{-1}\left[\frac{z}{z-6}\right] - \frac{1}{7} Z_T^{-1}\left[\frac{z}{z+1}\right]$

$$Z_T^{-1}[q(z)] = \frac{1}{7}(6)^n - \frac{1}{7}(-1)^n \rightarrow (3)$$

consider

$$r(z) = \frac{z}{(z-6)(z-1)(z+1)} = \frac{Az}{(z-6)} + \frac{Bz}{(z-1)} + \frac{Cz}{(z+1)}$$

$$\frac{\cancel{z}}{(z-6)\cancel{(z-1)}(z+1)} = \cancel{z}\left[\frac{A(z-1)(z+1) + B(z-6)(z+1) + C(z-6)(z-1)}{(z-6)\cancel{(z-1)}(z+1)}\right]$$

$$1 = A(z-1)(z+1) + B(z-6)(z+1) + C(z-6)(z-1)$$

put $z = 6$
$1 = A(5)(7)$
$1 = 35A$
$\boxed{A = 1/35}$

put $z = 1$
$1 = B(-5)(2)$
$1 = -10B$
$\boxed{B = -1/10}$

put $z = -1$
$1 = C(-7)(-2)$
$1 = 14C$
$\boxed{C = 1/14}$

hence

$$Z_T^{-1}[r(z)] = \frac{1}{35} Z_T^{-1}\left[\frac{z}{z-6}\right] - \frac{1}{10} Z_T^{-1}\left[\frac{z}{z-1}\right] + \frac{1}{14} Z_T^{-1}\left[\frac{z}{z+1}\right]$$

$$Z_T^{-1}[r(z)] = \frac{1}{35}(6)^n - \frac{1}{10}(1) + \frac{1}{14}(-1)^n \rightarrow (4)$$

using ②, ③ & ④ in equation ①

$\bar{U}(z) = U_0 p(z) + U_1 q(z) + 36 r(z)$

$Z_T^{-1}[\bar{U}(z)] = U_0 Z_T^{-1}[p(z)] + U_1 Z_T^{-1}[q(z)] + 36 Z_T^{-1}[r(z)]$

$Z_T^{-1}[\bar{U}(z)] = U_0\left[\frac{1}{7}(6)^n + \frac{6}{7}(-1)^n\right] + U_1\left[\frac{1}{7}(6)^n - \frac{1}{7}(-1)^n\right] +$

$36\left[\frac{1}{35}(6)^n - \frac{1}{10}(1) + \frac{1}{14}(-1)^n\right]$

$Z_T^{-1}[\bar{U}(z)] = \left[\frac{U_0}{7} + \frac{U_1}{7} + \frac{36}{35}\right](6)^n + \left[\frac{6U_0}{7} - \frac{U_1}{7} + \frac{36}{14}\right](-1)^n - \frac{36}{10}(1)$

let $C_1 = \frac{U_0}{7} + \frac{U_1}{7} + \frac{36}{35}$ and $C_2 = \frac{6U_0}{7} - \frac{U_1}{7} + \frac{36}{14}$

where C_1 & C_2 are the arbitrary constants

Thus $U_n = C_1(6)^n + C_2(-1)^n - \frac{36}{10}$ is the required solution.

4 (c) If the fourier sine transform of $f(x)$ is $\frac{e^{-a\alpha}}{\alpha}$ $\alpha \neq 0$ find $f(x)$ & hence obtain the inverse fourier the sine transform of $\frac{1}{\alpha}$

we have

$F_s(u) = \int_0^\infty f(x) \sin ux \, dx$

let

$f(x) = \frac{e^{-a\alpha}}{\alpha}$

$F_s(u) = \int_0^\infty \frac{e^{-a\alpha}}{\alpha} \sin u\alpha \, d\alpha \rightarrow ①$

we cannot evaluate this integral directly and hence we employ the rule of differentiation under the integral sign.

$\frac{d}{du}[f_s(u)] = \int_0^\infty \frac{e^{-a\alpha}}{\alpha} \frac{\partial}{\partial u}(\sin u\alpha) \, d\alpha$

$= \int_0^\infty \frac{e^{-a\alpha}}{\alpha} \alpha \cos u\alpha \, d\alpha.$

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$$\frac{d}{du}[F_s(u)] = \int_0^\infty e^{-a\alpha} \cos u\alpha \, d\alpha$$

$$= \left[\frac{e^{-a\alpha}}{a^2+u^2}(-a\cos u\alpha + u\sin u\alpha)\right]_{\alpha=0}^{\infty}$$

$$= \frac{1}{a^2+u^2}(0+a)$$

$$\frac{d}{du}[F_s(u)] = \frac{a}{a^2+u^2}$$

Hence $\frac{d}{du}[F_s(u)] = \frac{a}{a^2+u^2}$ and by integrating wrt u we get

$$F_s(u) = \tan^{-1}(u/a) + c$$

To evaluate c, let us put $u=0$

$$F_s(0) = \tan^{-1}(0) + c$$

But $F_s(0) = 0$ from ① & hence $c = 0$

Thus $\boxed{F_s(u) = \tan^{-1}(u/a)}$

5. (a) Calculate the Karl Pearson's co-efficient for the following ages of husbands & wives.

Husband's age (x)	23	27	28	28	29	30	31	33	35	36
Wife's age (y)	18	20	22	27	21	29	27	29	28	29

here $N = 10$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{300}{10} = 30$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{250}{10} = 25$$

$$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2}\sqrt{\Sigma Y^2}} = \frac{129}{\sqrt{138}\sqrt{159}}$$

$$\boxed{r = 0.870}$$

x	y	$x-\bar{x}$	$y-\bar{y}$	x^2	Y^2	XY
23	18	-7	-7	49	49	49
27	20	-3	-5	9	25	15
28	22	-2	-2	4	4	4
28	27	-2	2	4	4	4
29	21	-1	-4	1	16	4
30	29	0	4	0	16	0
31	27	1	2	1	4	2
33	29	3	4	9	16	12
35	28	5	3	25	9	15
36	29	6	4	36	16	24
$\Sigma x=300$	$\Sigma y=250$			$\Sigma X^2=138$	$\Sigma Y^2=159$	$\Sigma XY=129$

Regression of line y on x is

$$y-\bar{y} = \frac{\Sigma XY}{\Sigma X^2}(x-\bar{x})$$

$$y-25 = \frac{129}{138}(x-30)$$

$$y-25 = 0.934x - 28.29$$

$$y = 0.934x - 28.29 + 25$$

$$y = 0.934x - 3.29$$

Regression line of x on y is

$$x-\bar{x} = \frac{\Sigma XY}{\Sigma Y^2}(y-\bar{y})$$

$$x-30 = \frac{129}{159}(y-25)$$

$$x-30 = 0.81134y - 20.283$$

$$x = 0.8113y - 20.283 + 30$$

$$x = 0.8113y + 9.716$$

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5. (b) By method of least square find the parabola $y = ax^2 + bx + c$ the best fits the following data

x	10	12	15	23	30
y	14	17	23	25	21

x	y	x^2	xy	x^3	x^2y	x^4
10	14	100	140	1000	1400	10,000
12	17	144	204	1728	2448	20,736
15	23	225	345	3375	5175	50,625
23	25	529	575	12167	13225	279841
30	21	900	630	27000	18000	810000
$\Sigma x = 90$	$\Sigma y = 100$	$\Sigma x^2 = 1898$	$\Sigma xy = 1894$	$\Sigma x^3 = 45270$	40248	1171202

here $n = 5$ the required parabola eqn $y = ax^2 + bx + c$

Normal equation is.

$$nc + b\Sigma x + a\Sigma x^2 = \Sigma y$$

$$c\Sigma x + b\Sigma x^2 + a\Sigma x^3 = \Sigma xy$$

$$c\Sigma x^2 + b\Sigma x^3 + a\Sigma x^4 = \Sigma x^2 y$$

$$5c + 90b + 1898a = 100$$

$$90c + 1898b + 45270a = 1894$$

$$1898c + 45270b + 1171202 = 40248$$

$\therefore$ $c = 20.756$

$b = -0.0474$

$a = 0.00255$

$$y = ax^2 + bx + c$$

$$y = 0.00255x^2 - 0.0474x + 20.756$$

5. (c) Using newtons - raphson method find the real root that lies near $x = 4.5$ of the equation $\tan x = x$ correct to four decimal places.

given

$\tan x = x$

$f(x) = \tan x - x$ $\qquad$ $f'(x) = \sec^2 x - 1 = \tan^2 x$

here $x = 4.5$

By newton's raphson method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 4.5 - \frac{f(4.5)}{f'(4.5)} = \frac{4.5 - (\tan 4.5 - 4.5)}{\tan^2(4.5)} = 4.493$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 4.493 - \frac{f(4.493)}{f'(4.493)} = 4.493$$

Thus real root of this equation is $\boxed{x = 4.493}$

6. (a) In a partially destroyed laboratory record, only the lines of regression of y on x & x on y are available as $4x - 5y + 33 = 0$ & $20x - 9y = 107$ respectively. Calculate $\bar{x}$ & $\bar{y}$ & the co-efficient of correlation between x & y.

given

$4x - 5y + 33 = 0$

$20x - 9y - 107 = 0$

here

$4x - 5y = -33 \rightarrow (1)$

$20x - 9y = 107 \rightarrow (2)$

WKT the line of regression always pass through the point $(\bar{x}, \bar{y})$

$4\bar{x} - 5\bar{y} = -33$

$20\bar{x} - 9\bar{y} = 107$

$\bar{x} = 13$ $\qquad$ $\bar{y} = 17$

solving eqn (1) for y

$$5y = 4x + 66$$

$$y = 0.8x + 6.6 \rightarrow (3)$$

solving eqn (2) for x

$$20x = 9y + 107$$

$$x = 0.45y + 5.35 \rightarrow (4)$$

the co-efficient of correlation

$$r = \pm\sqrt{(0.8) \times (0.45)}$$

$$\boxed{r = \pm 0.6}$$

Here both the co-efficient are +ve

$$\boxed{r = 0.6}$$

6. (b) Find the curve of best fit of the type $y = ae^{bx}$ to the following data by the method of least squares.

x	1	5	7	9	12
y	10	15	12	15	21

here $N = 5$ let $y = ae^{bx}$

$$\log y = \log ae^{bx}$$

$$\log y = \log a + \log e^{bx}$$

$$\log y = \log a + bx \log e$$

$$\log y = \log a + bx$$

let $\log y = Y$, $\log a = A$ $b = B$ $x = X$

then normal equation are

$$nA + B\Sigma X = \Sigma Y$$

$$A\Sigma X + B\Sigma X^2 = \Sigma XY$$

$X = x$	y	$Y = \log y$	XY	X^2
1	10	2.3025	2.3025	1
5	15	2.7080	13.54	25
7	12	2.4849	17.3943	49
9	15	2.7080	24.372	81
12	21	3.0445	36.534	144
$\Sigma X = 34$		$\Sigma Y = 13.2479$	$\Sigma XY = 94.1428$	$\Sigma X^2 = 300$

$$5A + 34B = 13.2479$$

$$34A + 300B = 94.1428$$

$$A = 2.2485$$

$$B = 0.05896$$

Thus $y = (9.4735)\, e^{(0.05896)x}$

$$\log a = 2.2485$$

$$a = e^{2.2485}$$

$$\boxed{a = 9.4735}$$

6 (c) Find the real root of the equation $xe^x - 3 = 0$ by regular falsi method, correct to three decimal places.

given

$xe^x - 3 = 0$

$f(x) = xe^x - 3$

let $f(0) = -3$

$f(1) = -0.281$

$f(2) = 11.778.$

The real root lies b/w $(1, 2)$

$f(1.1) = 0.304$

Now the root lies b/w $(1, 1.1)$ $\quad \therefore a = 1, \quad b = 1.1$

By applying regular falsi method.

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1f(1.1) - 1.1f(1)}{f(1.1) - f(1)} = \frac{0.304 - 1.1(-0.281)}{(0.304 + 0.281)}$$

$x_1 = 1.0480$

$f(1.0480) = -0.0111$

the root lies b/w $(1.0480, 1.1)$

$$x_2 = \frac{1.0480f(1.1) - 1.1f(1.0480)}{f(1.1) - f(1.0480)} = \frac{1.0480 \times 0.304 + 1.1(0.0111)}{0.304 + 0.0111}$$

$x_2 = 1.0498$

$f(1.0498) = -0.000637$

the root lies b/w $(1.0498, 1.1)$

$$x_3 = \frac{1.0498 \cdot f(1.1) - 1.1f(1.0498)}{f(1.1) - f(1.0498)} = \frac{1.0498 \times 0.304 + 1.1 \times 0.000637}{0.304 + 0.000637}$$

$x_3 = 1.0498$

The real root is $\boxed{x = 1.0498}$

7. (a) Form the following table of half yearly premium for policies maturing at different ages, estimate the premium for periodic maturing at age of 46

Age	45	50	55	60	65
premium (in Rs)	114.84	96.16	83.32	74.48	68.48

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
45	114.84				
		-18.68			
50	96.16		5.84		
		-12.84		-1.84	
55	83.32		4		0.68
		-8.84		-1.16	
60	74.48		2.84		
		-6			
65	68.48				

$$r = \frac{x - x_0}{h} = \frac{46-45}{5} = 0.2$$

$$y_r = f(x_0 + rh) = f(45 + 0.2\times 5) = f(46)$$

By Newton's forward interpolation formula

$$y_r = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \frac{r(r-1)(r-2)(r-3)}{3!}\Delta^4 y_0 + \cdots$$

$$y_r = 114.84 + 0.2(-18.68) + \frac{0.2(0.2-1)}{2!}\times 5.84 + \frac{0.2(0.2-1)(0.2-2)}{3!}\times(-1.84) + \frac{0.2(0.2-1)(0.2-2)(0.2-3)}{4!}\times 0.68$$

$$y_r = 114.84 + (-3.736) + 0.08(5.84) + 0.048(-1.84) + (-0.02248)$$

$$\boxed{y_r = 110.52}$$

(b) Using newton's divided difference interpolation, find the polynomial of the given data.

x	3	7	9	10
$f(x)$	168	120	72	63

	x	y	1st DD	2nd DD	3rd DD
x_0	3	168			
			$f(x_0 x_1) = -12$		
x_1	7	120		$f(x_0 x_1, x_2) = -2$	
			$f(x_1 x_2) = -24$		$f(x_0 x_1, x_2 x_3) = 1$
x_2	9	72		$f(x_1 x_2 x_3) = 5$	
			$f(x_2 x_3) = -9$		
x_3	10	63			

Newton's divided difference formula is given by

$$y = f(x) = f(x_0) + (x - x_0) f(x_0 x_1) + (x-x_0)(x-x_1) f(x_0 x_1, x_2) + \cdots$$

$$y = 3 + (x-3)(-12) + (x-3)(x-7)(-2) + (x-3)(x-7)(x-9)(1)$$

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$y = 3 - 12x + 36 + (x^2 - 7x - 3x + 21)(-2) + (x^2 - 7x - 3x + 21)(x - 9)$

$y = 39 - 12x + 10x - 2x^2 - 42 + x^3 - 9x^2 - 7x^2 + 63x - 3x^2 + 18x + 21x - 189$

$y = x^3 - 21x^2 + 119x - 192$

7 (c) using simpson's ($\frac{1}{3rd}$) rule to find $\int_0^{0.6} e^{-x^2} dx$ by taking seven ordinates.

$a = 0, \quad b = 0.6, \quad n = 6 \quad y = e^{-x^2}$

$h = \frac{b-a}{n} = \frac{0.6-0}{6} = 0.1$

x	$x_0 = 0$	$x_1 = 0.1$	$x_2 = 0.2$	$x_3 = 0.3$	$x_4 = 0.4$	$x_5 = 0.5$	$x_6 = 0.6$
y	$y_0 = 1$	$y_1 = 0.99$	$y_2 = 0.9608$	$y_3 = 0.9139$	$y_4 = 0.8521$	$y_5 = 0.7788$	$y_6 = 0.6976$

Simpson's ($\frac{1}{3rd}$) rule

$I = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$

$I = \frac{0.1}{3}[(1 + 0.6976) + 4(0.99 + 0.9139 + 0.7788) + 2(0.9608 + 0.8521)]$

$\boxed{I = 0.5351}$

8 (a) find the number of men getting wages below ₹ 35 from the following data

wages in ₹	0-10	10-20	20-30	30-40
frequency	9	30	35	42

less than 10 ₹ = 9

less than 20 ₹ = 9 + 30 = 39

less than 30 ₹ = 39 + 35 = 74

less than 40 ₹ = 74 + 42 = 116.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
10	9			
		30		
20	39		5	
		35		2
30	74		7	
		42		
40	116			

$r = \frac{x - x_n}{h} = \frac{35-40}{10} = -0.5$

$$y_r = y_n + r\nabla y_n + \frac{r(r+1)}{2!}\nabla^2 y_n + \frac{r(r+1)(r+2)}{3!}\nabla^3 y_n$$

$$y(35) + 116 + (-0.5)(42) + \frac{(-0.5)(-0.5+1)}{2!} \times 7 + \frac{(0.5)(-0.5+1)(-0.5+2)}{3!} \times 2$$

$y(35) = 94$

8 (b) find the polynomial $f(x)$ using lagrange's formula from the following.

x	0	1	2	5
$f(x)$	2	3	12	147

let $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 5$

$y_0 = 2$ $\quad y_1 = 3$ $\quad y_2 = 12$ $\quad y_3 = 147$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}(y_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x-x_3)} \times (y_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times (y_3)$$

$$y = \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)}(2) + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)}(3) +$$

$$\frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)}(12) + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)} \times (147)$$

$$y = -\frac{1}{5}\left[(x^2-3x+2)(x-5)\right] + \frac{3}{4}\left[(x^2-2x)(x-5)\right] - 2\left[(x^2-x)(x-5)\right]$$

$+ \frac{147}{60}\left[(x^2-x)(x-2)\right]$

$$y = -\frac{1}{5}\left[x^3-5x^2-3x^2+15x+2x-10\right] + \frac{3}{4}\left[x^3-5x^2-2x^2+10x\right] - 2\left[x^3-5x^2-x^2+5x\right] + \frac{147}{60}\left[x^3-2x^2-x^2+2x\right]$$

$$y = -\frac{1}{5}\left[x^3\right] + \frac{8x^2}{5} - \frac{17x}{5} + 2 + \frac{3}{4}x^3 - \frac{21}{4}x^2 + \frac{30x}{4} - 2x^3 + 12x^2 - 10x + \frac{147x^3}{60} - \frac{441}{60}x^2 + \frac{294}{60}x$$

$$y = \left[\frac{-1}{5} + \frac{3}{4} - 2 + \frac{147}{60}\right]x^3 + \left[\frac{8}{5} - \frac{21}{4} + 12 - \frac{441}{6}\right]x^2 + \left[\frac{-17}{5} + \frac{30}{4} - 10 + \frac{294}{60}\right]x + 2$$

$$\underline{y = x^3 + x^2 - x + 2}$$

8 (c). compute the value of $\int_{0.2}^{1.4}(\sin x - \log_e x + e^x)$ using simpson's $\left(\frac{3}{8}\right)^{th}$ rule.

$a = 0.2 \qquad b = 1.4 \qquad n = 6 \qquad y = \sin x - \log_e x + e^x$

$$h = \frac{b-a}{n} = \frac{1.4-0.2}{6} = 0.2$$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y	3.029	2.797	2.897	3.166	3.559	4.069	4.7042

For $n = 6$

simpson's $\left(\frac{3}{8}th\right)$ rule

$$I = \frac{3h}{8}\left[(y_0+y_6) + 3(y_1+y_2+y_4+y_5) + 2(y_3)\right]$$

$$I = \frac{3\times 0.2}{8}\left[(3.029+4.7042) + 3(2.797+2.897+3.559+4.069) + 2(3.166)\right]$$

$$\boxed{I = 4.0530}$$

9 (a) A vector field is given by $\vec{F} = \sin y\,\hat{i} + x(1+\cos y)\,\hat{j}$ evaluate the line integral over a circular path is given by $x^2+y^2=a^2$, $z=0$.

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Module - 1

1a. Expand $f(x) = x - x^2$ as a fourier series in the interval $(-\pi, \pi)$

Given: $f(x) = x - x^2$.

$2l = 2\pi$

$\therefore l = \pi$

Fourier series of $f(x)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) dx + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right) dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \rightarrow ①.$$

$f(x) = x - x^2$

$f(-x) = -x - (-x^2)$

$= -x - x^2$

$f(x)$ is neither even nor odd.

$$a_0 = \frac{1}{l}\int_{c}^{c+2l} f(x)\, dx.$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} (x - x^2)\, dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{\pi} x\, dx - \int_{-\pi}^{\pi} x^2\, dx\right]$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\left(\frac{\pi^2}{2} - \frac{\pi^3}{3}\right) - \left(\frac{(-\pi)^2}{2} - \frac{(-\pi)^3}{3}\right)\right]$$

$$= \frac{1}{\pi}\left[\cancel{\frac{\pi^2}{2}} - \frac{\pi^3}{3} - \cancel{\frac{\pi^2}{2}} - \frac{\pi^3}{3}\right]$$

$$= \frac{1}{\cancel{\pi}}\left[-\frac{2\pi^{\cancel{3}}}{3}\right]$$

$$a_0 = \cancel{\frac{1}{\pi}}\left[-\frac{2\pi^2}{3}\right] \Rightarrow \frac{a_0}{2} = -\frac{\cancel{2}\pi^2}{3} \times \frac{1}{\cancel{2}}$$

$$\boxed{\frac{a_0}{2} = -\frac{\pi^2}{3}}$$

$$a_n = \frac{1}{l}\int_c^{c+2l} f(x)\cos\frac{(n\pi x)}{l}\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}(x-x^2)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[(x-x^2)\cancel{\frac{\sin nx}{n}}^{\to 0} - (1-2x)\frac{(-\cos nx)}{n^2} + (-2)\cancel{\frac{(-\sin nx)}{n^3}}^{\to 0}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[(1-2x)\frac{\cos nx}{n^2}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[(1-2\pi)\frac{\cos n\pi}{n^2} - (1+2\pi)\frac{\cos(-n\pi)}{n^2}\right]$$

$$= \frac{1}{n^2\pi}\left[(1-2\pi)\cos n\pi - (1+2\pi)\cos n\pi\right]$$

$$= \frac{1}{n^2\pi}\left[(1-2\pi)(-1)^n - (1+2\pi)(-1)^n\right]$$

$$= \frac{(-1)^n}{n^2\pi}\left[\cancel{1}-2\pi-\cancel{1}-2\pi\right]$$

$$\boxed{a_n = \frac{-4\cancel{\pi}}{n^2\cancel{\pi}}(-1)^n} \Rightarrow \boxed{a_n = \frac{-4}{n^2}(-1)^n.}$$

$$b_n = \frac{1}{l}\int_c^{c+2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}(x-x^2)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[(x-x^2)\frac{(-\cos nx)}{n} - (1-2x)\cancel{\frac{(-\sin nx)}{n^2}}^{\to 0} + (-2)\frac{\cos nx}{n^3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[(x-x^2)\frac{(-\cos nx)}{n} - 2\frac{\cos nx}{n^3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{n\pi}\left[(x-x^2)(-\cos nx) - 2\frac{\cos nx}{n^2}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{n\pi}\left[\left((\pi-\pi^2)(-\cos n\pi) - \frac{2\cos n\pi}{n^2}\right) - \left((-\pi-\pi^2)(-\cos n\pi) - \frac{2\cos n\pi}{n^2}\right)\right]$$

$$= \frac{1}{n\pi}\left[\pi-\cancel{\pi^2}(-(-1)^n) - \cancel{\frac{2(-1)^n}{n^2}} + \pi + \cancel{\pi^2}(+(-1)^n) + \cancel{\frac{2(-1)^n}{n^2}}\right]$$

$$= \frac{(-1)^n}{n\cancel{\pi}}(2\cancel{\pi})$$

$$\boxed{b_n = \frac{2(-1)^n}{n}}$$

Substituting in ① the required fourier series is

$$x - x^2 = -\frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{-4}{n^2}(-1)^n \cos nx + \sum_{n=1}^{\infty} \frac{2(-1)^n}{n} \sin nx$$

1b. Obtain the fourier half range cosine series for the function $f(x) = x(l-x)$ in the interval $0 \le x \le l$.

Given: $f(x) = x(l-x)$

$\therefore l = l.$

The half range cosine series is given by.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l} \rightarrow ①$$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$$= \frac{2}{l}\int_0^l xl - x^2\,dx$$

$$= \frac{2}{l}\left[l\left(\frac{x^2}{2}\right) - \frac{x^3}{3}\right]_0^l$$

$$= \frac{2}{l}\left[\frac{l(l^2)}{2} - \frac{l^3}{3}\right]$$

$$= \frac{2}{l}\left[\frac{l^3}{2} - \frac{l^3}{3}\right]$$

$$= \frac{2}{l}\left[\frac{3l^3 - 2l^3}{6}\right]$$

$$= \frac{\cancel{2}}{\cancel{l}}\left[\frac{l^{\cancel{3}}}{\cancel{6}_3}\right] \Rightarrow a_0 = \frac{l^2}{3}$$

$$\boxed{\frac{a_0}{2} = \frac{l^2}{6}}$$

$$a_n = \frac{2}{1}\int_0^1 f(x)\cos\frac{n\pi x}{1}\,dx$$

$$= \frac{2}{l}\int_0^l (lx - x^2)\cos\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l}\left[\cancel{(lx-x^2)\frac{\sin\frac{n\pi x}{l}}{\frac{n\pi}{l}}} - (l-2x)\frac{\left(-\cos\frac{n\pi x}{l}\right)}{\frac{n^2\pi^2}{l^2}} + \cancelto{0}{(-2)\frac{\left(-\sin\frac{n\pi x}{l}\right)}{\frac{n^3\pi^3}{l^3}}}\right]_0^l$$

$$= \frac{2}{l}\left[\frac{l^2(l-2x)}{n^2\pi^2}\cos\frac{n\pi x}{l}\right]_0^l$$

$$= \frac{2}{l}\left[\frac{l^2(-l)}{n^2\pi^2}\cos\frac{n\pi l}{l} - \frac{l^3}{n^2\pi^2}\cos\frac{n\pi(0)}{l}\right]$$

$$= \frac{2}{l}\left[-\frac{l^2}{n^2\pi^2}\cos n\pi l - \frac{l^2}{n^2\pi^2}\right]$$

$$a_n = \frac{-2l}{n^2\pi^2}\left[\cos n\pi l + 1\right]$$

Substituting in (1)

$$f(x) = \frac{l^2}{6} + \sum_{n=1}^{\infty}\left(\frac{-2l}{n^2\pi^2}\right)\left[\cos n\pi l + 1\right]\left(\cos\frac{n\pi x}{l}\right)$$

2a. Obtain the fourier series of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2\pi$. Hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$

Given: $f(x) = \frac{\pi - x}{2}$.

$2l = 2\pi$

$l = \pi$

The fourier series of $f(x)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \rightarrow (1)$$

Here.

$$f(2\pi - x) = \frac{\pi - (2\pi - x)}{2}$$

$$= \frac{\pi - 2\pi + x}{2}$$

$$= \frac{-\pi + x}{2}.$$

$$= -\left(\frac{\pi - x}{2}\right)$$

$$= -f(x)$$

$\therefore$ $f(x)$ is odd

$a_0 = 0 \qquad a_n = 0.$

$$b_n = \frac{2}{l}\int_0^l f(x) \sin\left(\frac{n\pi x}{l}\right) dx.$$

$$= \frac{2}{\pi}\int_0^\pi \left(\frac{\pi - x}{2}\right) \sin nx \, dx.$$

$$= \frac{2}{\pi}\int_0^\pi \frac{\pi}{2} - \frac{x}{2} \sin nx \, dx$$

$$= \frac{2}{\pi} \times \frac{1}{2}\left[(\pi - x)\left(\frac{-\cos nx}{n}\right) - 1\cancel{\left(\frac{-\sin nx}{n^2}\right)}^{0}\right]_0^\pi$$

$$= \frac{1}{n\pi}\left[(\pi - x)(-\cos nx)\right]$$

$$= -\frac{1}{n\pi}\left[(\pi - x)\cos nx\right]_0^{\pi}$$

$$= \frac{-1}{n\pi}\left[0 - \pi\cos 0\right]$$

$$= -\frac{1}{n\pi}\left[-\pi\right]$$

$$\boxed{b_n = \frac{1}{n}}$$

Substituting in (1)

$$\frac{\pi - x}{2} = 0 + 0 + \sum_{n=1}^{\infty} \frac{1}{n}\sin nx$$

$$\frac{\pi - x}{2} = \sum_{n=1}^{\infty} \frac{\sin nx}{n}$$

2b Find the half-range sine series for the function

$$f(x) = \begin{cases} \frac{1}{4} - x & \text{in } 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & \text{in } \frac{1}{2} < x < 1 \end{cases}$$

Here $l = 1$

The sine half range series is given by.

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} -$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin n\pi x \longrightarrow (1)$$

$$b_n = \frac{2}{l}\int_0^{l} f(x)\sin\frac{n\pi x}{l}\,dx.$$

$$b_n = 2\left[\int_0^{1/2}\left(\tfrac{1}{4} - x\right)\sin n\pi x\,dx + \int_{1/2}^{1}\left(x - \tfrac{3}{4}\right)\sin n\pi x\,dx\right]$$

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$$= 2\left[-\frac{1}{n\pi}\left\{-\frac{1}{4}\cos\frac{n\pi}{2} - \frac{1}{4}\cos 0\right\} - \frac{1}{n^2\pi^2}\left\{\sin n\pi/2 - 0\right\} - \right.$$

$$\left.\frac{1}{n\pi}\left\{\frac{1}{4}\cos n\pi - (-1/4)\cos\frac{n\pi}{2}\right\} + \frac{1}{n^2\pi^2}\left\{0 - \sin\frac{n\pi}{2}\right\}\right]$$

$$= 2\left[\frac{1}{4n\pi}\cancel{\cos}\frac{n\pi}{2} - \frac{1}{4n\pi} - \frac{1}{n^2\pi^2}\sin\frac{n\pi}{2} - \frac{1}{4n\pi}\cos n\pi - \frac{1}{4n\pi}\cancel{\cos}\frac{n\pi}{2} - \frac{1}{n^2\pi^2}\sin\frac{n\pi}{2}\right]$$

$$= 2\left[\frac{1}{4n\pi} - \frac{1}{n^2\pi^2}\sin\frac{n\pi}{2} - \frac{(-1)^n}{4n\pi} - \frac{1}{n^2\pi^2}\sin n\pi/2\right]$$

$$b_n = 2\left[\frac{1}{4n\pi}\left\{1 - (-1)^n\right\} - \frac{2}{n^2\pi^2}\sin n\pi/2\right]$$

Substitute in (1)

$$f(x) = \sum_{n=1}^{\infty} 2\left[\frac{1}{4n\pi}\left\{1 - (-1)^n\right\} - \frac{2}{n^2\pi^2}\sin n\pi/2\right]\sin(n\pi x)$$

2c. Compute the constant term and the coefficient of the 1^{st} sine and cosine terms in the fourier series of y as given in the following table:

x	0	1	2	3	4	5
y	4	8	15	7	6	2

Here $2l = 6$

$l = 3$

$N = 6.$

x	y	$\theta = \frac{\pi x}{l}$	$\cos\theta$	$y\cos\theta$	$\sin\theta$	$y\sin\theta$
0	4	0	1	4	0	0
1	8	$\pi/3$	0.5	4	0.866	6.928
2	15	$2\pi/3$	-0.5	-7.5	0.866	12.99
3	7	π	-1	-7	0	0
4	6	$4\pi/3$	-0.5	-3	-0.866	-5.196
5	2	$5\pi/3$	0.5	1	-0.866	-1.732
	$\Sigma y = 42$			$\Sigma y\cos\theta = -8.5$		$\Sigma y\sin\theta = 12.99$

$a_0 = \frac{2}{N}\Sigma y$

$a_0 = \frac{2}{6} \times 42$

$a_0 = 14$

$\boxed{\frac{a_0}{2} = 7}$

$a_1 = \frac{2}{N}\Sigma y\cos\theta$

$= \frac{2}{6} \times -8.5$

$\boxed{a_1 = -2.833}$

$b_1 = \frac{2}{N}\Sigma y\sin\theta$

$= \frac{2}{6} \times 12.99$

$\boxed{b_1 = 4.33}$

Fourier Series of y upto 1^{st} harmonic is given by

$y = \frac{a_0}{2} + a_1\cos\theta + b_1\sin\theta$

$y = 7 - 2.833\cos\theta + 4.33\sin\theta$

Module - 2

3a. If $f(x) = \begin{cases} 1-x^2 ; & |x|<1 \\ 0 ; & |x| \geq 1 \end{cases}$ Find the fourier transform of $f(x)$ and hence find the value of $\int_0^\infty \frac{x\cos x - \sin x}{x^3}\,dx$.

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By def^n

$$F[f(x)] = \int_{-\infty}^{\infty} f(u) e^{inx} dx$$

$$= \int_{-\infty}^{-1} f(u) e^{inx} dx + \int_{-1}^{1} f(x) e^{inx} dx + \int_{1}^{\infty} f(x) e^{inx} dx$$

$$= 0 + \int_{-1}^{1} (1-x^2) e^{inx} dx + 0.$$

By Bernoulli's method of integration

$$= \left[(1-x^2)\frac{e^{inx}}{iu} - (-2x)\frac{e^{inx}}{(iu)^2} + (-2)\frac{e^{inx}}{(iu)^3}\right]_{-1}^{1}$$

$$= (0-0) + \frac{2}{i^2u^2}\left[e^{iu} - (-e^{iu})\right] - \frac{2}{i^3u^3}\left[e^{iu} - e^{-iu}\right]$$

$$= -\frac{2}{u^2}\left[e^{iu} + e^{-iu}\right] + \frac{2}{iu^3}\left[e^{iu} - e^{-iu}\right] \quad \times 4 \div \text{by } 2$$

$$= -\frac{4}{u^2}\left(\frac{e^{iu} + e^{-iu}}{2}\right) + \frac{4}{u^3}\left[\frac{e^{iu} - e^{-iu}}{2}\right]$$

$$\cancel{-\frac{4}{u^2}\left[e^{iu}\right.} = -4\frac{\cos u}{u^2} + 4\frac{\sin u}{u^3}$$

$$= -4\left(\frac{\cos u}{u^2} - \frac{\sin u}{u^3}\right)$$

$$F[f(x)] = -4\left[\frac{u\cos u - \sin u}{u^3}\right] = \hat{f}(u)$$

By inverse fourier transform

$$F^{-1}[\hat{f}(u)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(u) e^{-iux} du$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} -4\left(\frac{u\cos u - \sin u}{u^3}\right) e^{-iux} dx$$

$$f(x) = -\frac{2}{\pi}\int_{-\infty}^{\infty} \frac{u\cos u - \sin u}{u^3} e^{-iux} dx$$

Put $x = 1/2$

$$f(1/2) = -\frac{2}{\pi}\int_{-\infty}^{\infty} \frac{u\cos u - \sin u}{u^3} e^{iu/2}\, du.$$

$$3/4 = -\frac{2}{\pi}\int_{-\infty}^{\infty} \frac{u\cos u - \sin u}{u^3}\left(\cos\frac{u}{2} - i\sin\frac{u}{2}\right) du$$

$$-\frac{3\pi}{8} = \int_{-\infty}^{\infty} \frac{u\cos u - \sin u}{u^3}\cos\frac{u}{2} - i\left(\frac{u\cos u - \sin u}{u^3}\right)\sin u/2\, du$$

By Equating the real part B.S

$$-3\pi/8 = \int_{-\infty}^{\infty} \frac{u\cos u - \sin u}{u^3}\cos\frac{u}{2}\, du.$$

$$-\frac{3\pi}{8} = 2\int_{0}^{\infty} \frac{u\cos u - \sin u}{u^3}\cos(u/2)\,du \quad \because f(x) \text{ is even}$$

$$-\frac{3\pi}{16} = \int_{0}^{\infty} \frac{u\cos u - \sin u}{u^3}\cos(u/2)\,du$$

$$-\frac{3\pi}{16} = \int_{0}^{\infty} \frac{x\cos x - \sin x}{x^3}\cos(x/2)\,dx.$$

3b. Find the fourier Sine and cosine transforms

$$f(x) = \begin{cases} x, & 0 < x < 2 \\ 0, & \text{elsewhere.} \end{cases}$$

By def^n

$$F_c[f(x)] = \int_0^{\infty} f(x)\cos ux\, dx$$

$$= \int_0^2 f(x)\cos ux\, dx + \int_2^{\infty} f(x)\cos ux\, dx$$

$$= \int_0^2 x\cos ux\, dx + \cancel{\int_2^{\infty} 0\cos ux\, dx}^{\,0}$$

By Bernoulli's method of integration

$$= \left[x\frac{\sin ux}{u} - (1)\left(-\frac{\cos ux}{u^2}\right)\right]_0^2$$

$$= \left[\frac{2\sin 2u}{u} + \frac{\cos 2u}{u^2} - \left(0 + \frac{1}{u^2}\right) \right]$$

$$\boxed{F_c[f(x)] = \frac{2u\sin 2u + \cos 2u - 1}{u^2}}$$

$$F_s[f(x)] = \int_0^\infty f(x) \sin ux \, dx$$

$$= \int_0^2 x \sin ux \, dx + \int_2^\infty 0 \sin ux \, dx$$

By bernoullis method of integration

$$= \left[x\left(\frac{-\cos ux}{u}\right) - (1)\left(-\frac{\sin ux}{u^2}\right) \right]_0^2$$

$$= \frac{-2\cos 2u}{u} + \frac{\sin 2u}{u} - 0$$

$$\boxed{F_s[f(x)] = \frac{\sin 2u - 2u\cos 2u}{u^2}}$$

3c. Solve by using z- transform $y_{n+2} - 4y_n = 0$ given that $y_0 = 0$, $y_1 = 2$.

Given: $y_{n+2} - 4y_n = 0$.

Taking z-transform on B. S

$$Z_T[y_{n+2}] - 4Z_T[y_n] = Z_T[0]$$

$$z^2\left[\bar{y}(z) - y_0 - \frac{y_1}{z}\right] - 4\bar{y}(z) = 0$$

$$z^2\bar{y}(z) - 0 - 2z - 4\bar{y}(z) = 0$$

$$(z^2-4)\bar{y}(z) = 2z.$$

$$\bar{y}(z) = \frac{2z}{z^2-4}$$

$$Z_T^{-1}[\bar{y}(z)] = Z_T^{-1}\left[\frac{2z}{z^2-4}\right] = Z_T^{-1}\left[\frac{2z}{(z+2)(z-2)}\right]$$

$$\frac{2z}{(z+2)(z-2)} = A\frac{z}{z+2} + B\frac{z}{z-2}$$

$$\frac{2z}{\cancel{(z+2)(z-2)}} = \frac{Az(z-2) + Bz(z+2)}{\cancel{(z+2)(z-2)}}$$

$$2\cancel{z} = \cancel{z}\,[A(z-2) + B(z-2)]$$

$$2 = A(z-2) + B(z+2)$$

Put $z = 2$ $z = -2$

$2 = B(4)$ $2 = A(-4)$

$\boxed{B = 1/2}$ $\boxed{A = -1/2}$

$$\frac{2z}{(z+2)(z-2)} = -\frac{1}{2}\,\frac{z}{z+2} + \frac{1}{2}\,\frac{z}{(z-2)}$$

Taking Inverse Z-T on B.S

$$Z_T^{-1}\left[\frac{2z}{(z+2)(z-2)}\right] = -\frac{1}{2}\,Z_T^{-1}\left[\frac{z}{z+2}\right) + \frac{1}{2}\,Z_T^{-1}\left[\frac{z}{z-2}\right)$$

$$Z_T^{-1}\left[\frac{2z}{(z+2)(z-2)}\right] = -\frac{1}{2}(-2)^n + \frac{1}{2}(2)^n$$

(or)

49. Obtain the inverse fourier sine transform of $F_S(\alpha) = \dfrac{e^{-a\alpha}}{\alpha}$, $a > 0$

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4b. Find the z-transform of $2n + \sin\left(\frac{n\pi}{4}\right) + 1$

Let $u_n = 2n + \sin\left(\frac{n\pi}{4}\right) + 1$

$$Z_T(u_n) = Z_T\left[2n + \sin\left(\frac{n\pi}{4}\right) + 1\right]$$

$$= 2Z_T(n) + Z_T \sin\frac{n\pi}{4} + Z_T(1)$$

$$Z_T(\sin n\theta) = \frac{z\sin\theta}{z^2 - 2z\cos\theta + 1}$$

$$Z_T u_n = \frac{2z}{(z-1)^2} + \frac{z\sin(\pi/4)}{z^2 - 2z\cos\pi/4 + 1} + \frac{z}{z-1}$$

$$Z_T(u_n) = \frac{2z}{(z-1)^2} + \frac{z(1/\sqrt{2})}{z^2 - 2z(1/\sqrt{2}) + 1} + \frac{z}{z-1}$$

$$\boxed{Z_T(u_n) = \frac{2z}{(z-1)^2} + \frac{z}{\sqrt{2}(z^2 - \sqrt{2}z + 1)} + \frac{z}{z-1}}$$

4c. If $u(z) = \dfrac{z}{z^2+7z+10}$, find the inverse z-transform

$$u(z) = \frac{z}{z^2+7z+10} \Rightarrow \frac{z}{(z+5)(z+2)}$$

$$\frac{z}{(z+5)(z+2)} = A\frac{z}{z+5} + B\frac{z}{z+2}$$

$$\frac{z}{\cancel{(z+5)(z+2)}} = \frac{Az(z+2) + Bz(z+5)}{\cancel{(z+5)(z+2)}}$$

$$\cancel{z} = \cancel{z}[A(z+2) + B(z+5)]$$

$$1 = A(z+2) + B(z+5)$$

Put $z=-2$ $\qquad z=-5$

$1 = B(3)$ $\qquad 1 = A(-3)$

$\boxed{B = 1/3}$ $\qquad \boxed{A = -1/3}$

$$\frac{z}{(z+5)(z+2)} = -\frac{1}{3}\frac{z}{z+5} + \frac{1}{3}\frac{z}{z+2}$$

Inverse z-Transform on B.S.

$$z_T^{-1}\left[\frac{z}{(z+5)(z+2)}\right] = -1/3\, z_T\left[\frac{z}{z+5}\right] + 1/3\, z_T\left[\frac{z}{z+2}\right]$$

$$\bar{u}(z) = -\frac{1}{3}(-5)^n + 1/3\,(-2)^n$$

$$\boxed{\bar{u}(z) = 1/3\left[(-2)^n - (-5)^n\right]}$$

Module-3.

5a. Obtain the coefficient of correlation for the following data

x	10	14	18	22	26	30
y	18	12	24	6	30	36

$n = 6$.

x	y	$X = x-\bar{x}$	$Y = y-\bar{y}$	X^2	Y^2	XY
10	18	-10	-3	100	9	30
14	12	-6	-9	36	81	54
18	24	-2	3	4	9	-6.
22	6	2	-15	4	225	-30
26	30	6	9	36	81	54
30	36	10	15	100	225	150
$x=120$	$y=126$			$\Sigma X^2=280$	$\Sigma Y^2=630$	$\Sigma XY=252$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{120}{6} = 20$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{126}{6} = 21$$

$$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2}\sqrt{\Sigma Y^2}} = \frac{252}{\sqrt{280}\sqrt{630}}$$

$$\boxed{r = 0.6}$$

5b. By the method of least square find the Straight line that best fits the following data:

x	1	2	3	4	5
y	14	27	40	55	68

Let $y = ax+b$ be the required Straight line

The normal Equations are,

$$nb + a\Sigma x = \Sigma y.$$

$$b\Sigma x + a\Sigma x^2 = \Sigma xy.$$

$$n = 5$$

x	y	x^2	xy
1	14	1	14
2	27	4	54
3	40	9	120
4	55	16	220
5	68	25	340
$\Sigma x=15$	$\Sigma y=204$	$\Sigma x^2=55$	$\Sigma xy=748$

Thus

$5b+15a=204$

$15b+55b=748$

$a=13.6$

$b=0$

1

5c. use Newton-Raphson method to find a root of the equation $\tan x - x=0$ near $x=4.5$. carry out two iterations

Given: $\tan x = x=0$

$f(x)=\tan x - x$

$f'(x)=\sec^2 x - 1$

$f'(x)=\tan^2 x$

$x_0=4.5$

By Newton Raphson method

$$x_{n+1} = x_0 - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 4.5 - \frac{\tan(4.5)-4.5}{\tan^2(4.5)}$$

$$x_1 = 4.4936.$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 4.4936 - \frac{\tan(4.4936) - 4.4936}{\tan^2(4.4936)}$$

$x_2 = 4.4934$

Thus real root is $x = 4.4934$

(01)

69. Find the regression line of y on x for the following data:

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

x	y	$X = x - \bar{x}$	$Y = y - \bar{y}$	X^2	Y^2	XY
1	1	-6	-4	36	16	24
3	2	-4	-3	16	9	12
4	4	-3	-1	9	1	3
6	4	-1	-1	1	1	1
8	5	1	0	1	0	0
9	7	2	2	4	4	4
11	8	4	3	16	9	12
14	9	7	4	7	16	28
$\Sigma x = 56$	$\Sigma y = 40$			$\Sigma X^2 = 90$	$\Sigma Y^2 = 56$	$\Sigma XY = 84$

$\bar{x} = \frac{\Sigma x}{n} = \frac{56}{8} = 7$ $\qquad$ $\bar{y} = \frac{\Sigma y}{n} = \frac{40}{8} = 5$

$\cancel{r = \frac{\Sigma XY}{\sqrt{\Sigma X^2}\sqrt{\Sigma Y^2}}} = \cancel{\frac{84}{\sqrt{90}\sqrt{56}}}$

Regression line of y on x

$$y = \frac{\Sigma xy}{\Sigma x^2} x$$

$$y-\bar{y} = \frac{\Sigma xy}{\Sigma x^2}(x-\bar{x})$$

$$y-5 = \frac{84}{90}(x-7)$$

$$y-5 = 0.933(x-7)$$

$$y-5 = 0.93x - 6.53$$

$$y = 0.93x - 6.53 + 5$$

$$y = 0.93x - 1.53$$

when $x=10$

$$y = 0.93(10) - 1.53$$

$$\boxed{y = 7.7}$$

6b. Fit a second degree parabola to the following data.

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

Let $y=ax^2+bx+c$ be the required parabola

The normal eqn is

$$nc + b\Sigma x + a\Sigma x^2 = \Sigma y$$

$$c\Sigma x + b\Sigma x^2 + a\Sigma x^3 = \Sigma xy$$

$$c\Sigma x^2 + b\Sigma x^3 + a\Sigma x^4 = \Sigma x^2 y$$

$$n=5$$

7a. From the data given in the following table. Find the number of students who obtained less than 70 marks.

Marks	0-19	20-39	40-59	60-79	80-99
Number of students	41	62	65	50	17

Less than 19 $= 41$

Less than 39 $= 41+62=103$

Less than 59 $= 103+65=168$

Less than 79 $= 168+50=218$

Less than 99 $= 218+17=235$

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
19	41				
		62			
39	103		3		
		65		-18	
59	168		-15		0
		50		-18	
79	218		-33		
		17			
99	235				

By Newton's Backward interpolation formula

$$y_r = y_n + r\nabla y_n + \frac{r(r+1)}{2!}\nabla^2 y_n + \frac{r(r+1)(r+2)}{3!}\nabla^3 y_n + \frac{r(r+1)(r+2)(r+3)}{4!}\nabla^4 y_n + \cdots$$

7a.

$$r = \frac{x - x_n}{h} = \frac{70 - 99}{20} = -1.45$$

$f(x_n + rh) = f(99 + (-1.45)20) = f(70)$

$= 235 + (-1.45) \times 17 + \frac{(-1.45)(-1.45+1)}{2}(-33) + \frac{(-1.45)(-1.45+1)(-1.45+2)}{3 \times 2}(-18)$

$= 200.32 \approx 200$

$y(70) = 200$

7b. Find the equation of the polynomial which passes through the points (4, -43); (7, 83), (9, 327) and (12, 1053) using Newton's divided difference interpolation.

x	y	1st DD	2nd DD	3rd DD
4	-43	42		
7	83		16	
		122		1
9	327		24	
12	1053	242		

$f(x) = f(x_0) + (x - x_0) f(x_0 x_1) + (x - x_0)(x - x_1) f(x_0 x_1 x_2) + (x - x_0)(x - x_1)(x - x_2) f(x_0 x_1 x_2 x_3)$

$= -43 + (x-4)(42) + (x-4)(x-7)(16) + (x-4)(x-7)(x-9)\,1$

$= -43 + 42x - 168 + (x^2 - 7x - 4x + 28)(16) + (x^2 - 7x - 4x + 28)(x-9)$

x	y	x^2	x^3	x^4	xy	x^2y
0	1	0	0	0	0	0
1	1.8	1	1	1	1.8	1.8
2	1.3	4	8	16	2.6	5.2
3	2.5	9	27	81	7.5	22.5
4	6.3	16	64	256	25.2	100.8
$\Sigma x=10$	$\Sigma y=12.9$	$\Sigma x^2=30$	$\Sigma x^3=100$	$\Sigma x^4=354$	$\Sigma xy=37.1$	$\Sigma x^2y=130.3$

$$5c + 10b + 30a = 12.9$$

$$10c + 30b + 100a = 37.1$$

$$30c + 100b + 354a = 130.3$$

$$a = 1.42$$

$$b = -1.07$$

$$c = 0.55$$

6c. Solve $xe^x - 2 = 0$ using Regula-Falsi method.

Given: $xe^x - 2 = 0$

$$f(x) = xe^x - 2.$$

$$f(0) = -2 < 0$$

$$f(1) = 0.718 > 0.$$

Here $f(0) < 0$ & $f(1) = > 0$

$\therefore$ A real root lies b/w 0 & 1 in $(0, 1)$

$$f(0.8) = -0.219 < 0$$

$$f(0.9) = 0.213 > 0$$

Thus root lies b/w $(\overset{a}{0.8}, \underset{b}{0.9})$

By Regula-falsi method, we have.

$$x = \frac{a.f(b) - b.f(a)}{f(b) - f(a)}$$

$$= \frac{0.8f(0.9) - 0.9f(0.8)}{f(0.9) - f(0.8)}$$

$$= \frac{0.8 \times 0.213 - 0.9(-0.219)}{0.213 - (-0.219)}$$

$x = 0.8506$

$f(0.8506) = -0.0087 < 0$

$f(0.8506) < 0$, $f(0.9) > 0$

$\therefore$ Root is $(0.8506, 0.9)$

$$x_2 = \frac{0.8506f(0.9) - 0.9f(0.8506)}{f(0.9) - f(0.8506)}$$

$$= \frac{0.8506 \times 0.213 - 0.9(-0.0087)}{0.213 - (-0.0087)}$$

$x_2 = 0.852$

$f(0.852) = -0.0026 < 0$

$\therefore$ Root is $(0.852, 0.9)$

$$x_3 = \frac{0.852f(0.9) - 0.9f(0.852)}{f(0.9) - f(0.852)}$$

$$= \frac{0.852 \times 0.213 - 0.9 \times (-0.0026)}{0.213 - (-0.0026)}$$

$x_3 = 0.852$.

Thus root is 0.852.

$$= -43 + 42x - 168 + 16x^2 - 112x - 64x + 488 + x^3 - 7x^2$$
$$- 4x^2 + 28x - 9x^2 + 63x + 36x - 252$$

$$f(x) = x^3 - 4x^2 - 7x - 15$$

(or)

8a. Using Newton's backward interpolation formula find the interpolating polynomial for the function given by the following table Hence find $f(12.5)$

x	10	11	12	13
$f(x)$	22	24	28	34

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
10	22			
		2		
11	24		2	
		4		0
12	28		2	
		6		
13	34			

$$r = \frac{x - x_n}{h}$$
$$= \frac{x - 13}{1}$$
$$r = x - 13$$

$$y_r = y_n + r\nabla y_n + \frac{r(r+1)}{2!}\nabla^2 y_n + \frac{r(r+1)(r+2)}{3}\nabla^3 y_n$$

$$= 34 + (x-13)6 + \frac{(x-13)(x+13+1)}{2!} \times 2 + 0$$

$$f(x) = 34 - 6x - 18 + x^2 - 12x - 13x + 156$$

$$f(x) = x^2 - 19x + 112.$$

$$f(12.5) = \underline{\underline{30.75}}$$