

Degree (A) = 3.

Degree (B) = 2.

Degree (C) = 1

Degree (E) = 0.

leaf nodes : E, F, G, H.

subnodes are called as siblings.

Ancestors of E are A & B.

The no of paths from deepest leaf is height of a tree.

Height: 2

Depth: how deep an element is from the root.

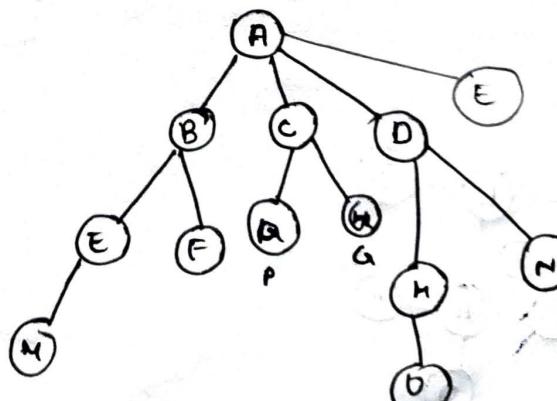
Representation of a Tree:

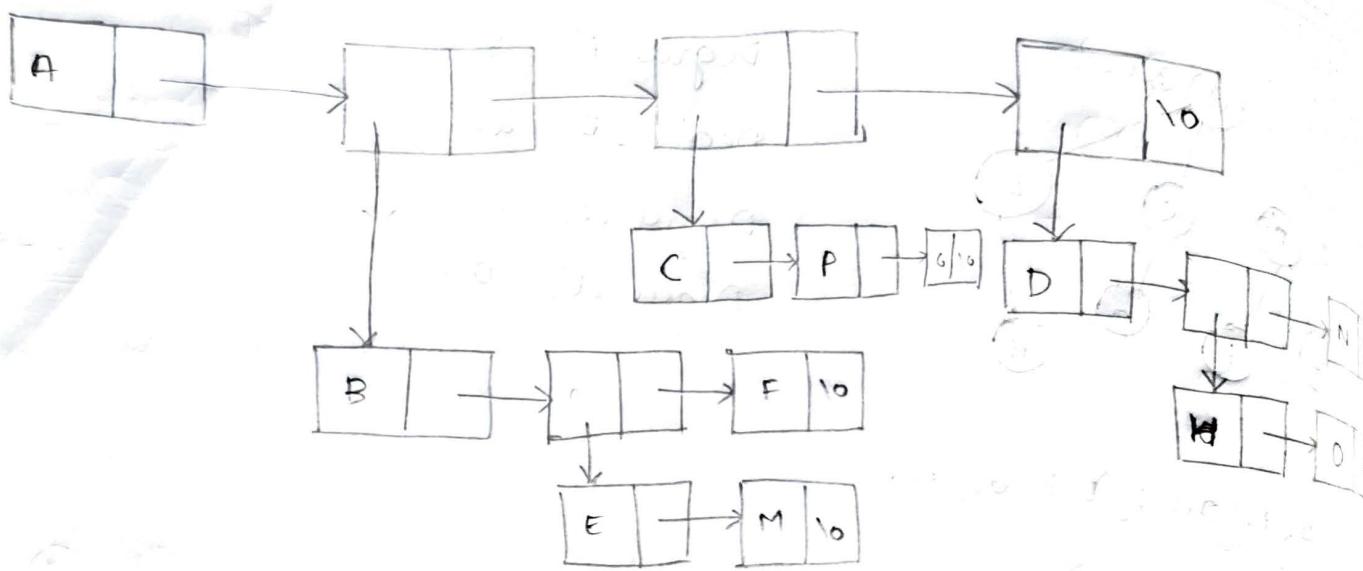
→ list

→ left child, Right sibling

→ Degree - 2.

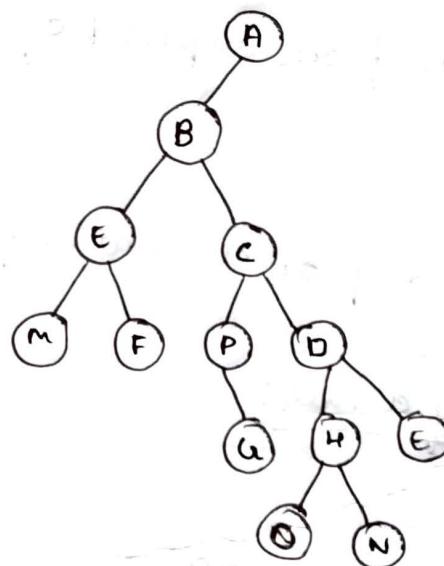
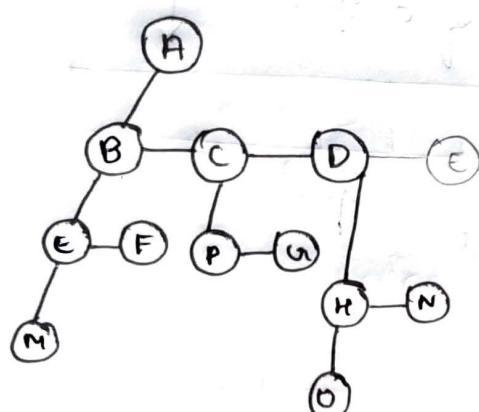
LIST REPRESENTATION:



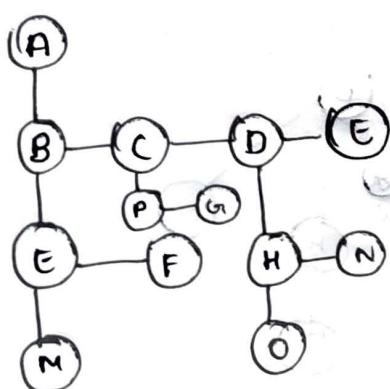


2) LCRS.

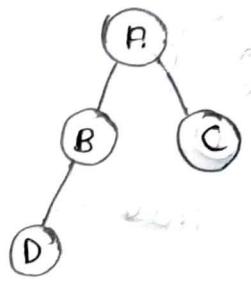
3) Degree - 2:



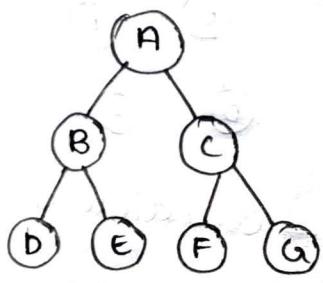
This should be straight



Binary tree:

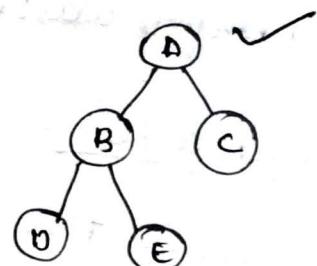
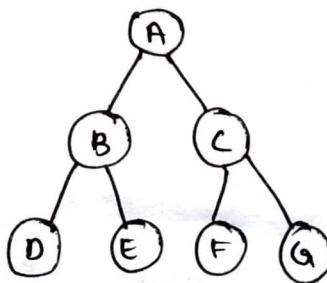
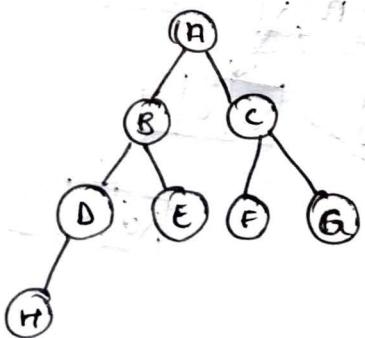


Full Binary Tree: Each node has either 0 or 2 children.



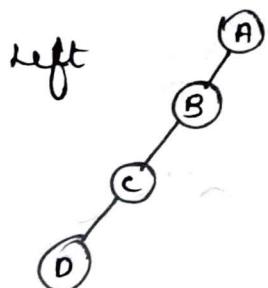
All full cannot be complete & vice versa is not possible.

Complete Binary Tree: (Except the last child, it should have 2 children). And the last leaf node, it should be first leaf i.e. it should be on the left most side. (Perfect Binary Tree).

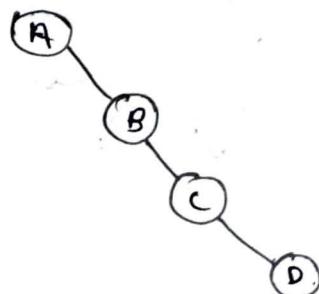


All full Binary tree are CBT but vice versa is not true.

Skewed Binary Tree:



Right:



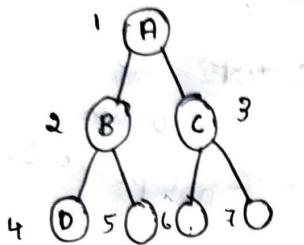
# Two Representations of Binary Tree

- array
- linked list.



## Array:

Max possible nodes:  $2^n - 1$ .



(Blanks are not defined by anything)

X	A	B	C	D		E
0	1	2	3	4	5	6

→ Memory wastage, therefore not efficient.

## linked list:

struct tree

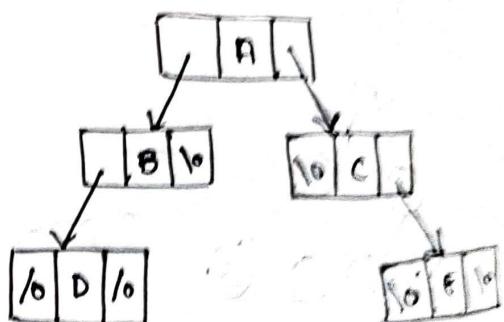
{

// Data char c;

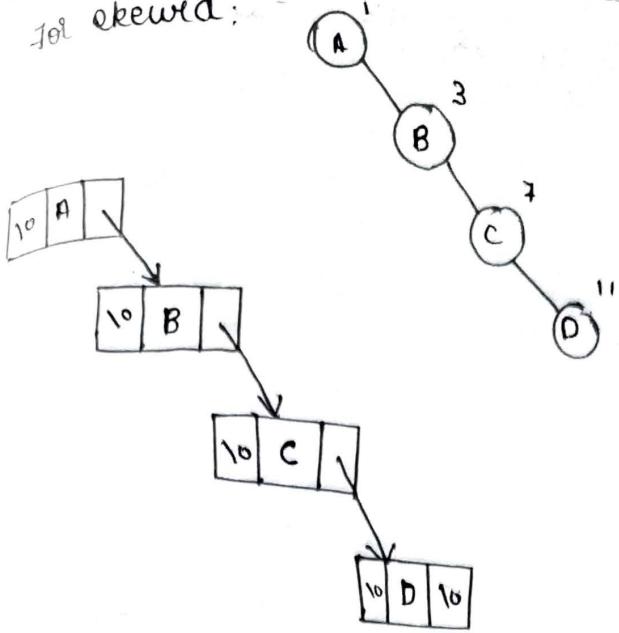
struct Tree \*left, \*right;

};

typedef struct tree Node;



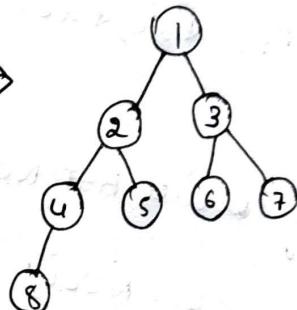
for skewed:



Traversal of a Tree: imp.

Construction of a tree!

Representation of 1, 2, 3, 4, 5, 6, 7, 8 →



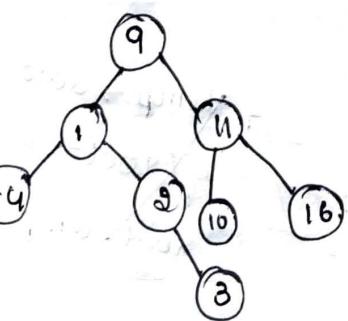
TRAVERSAL:

- Pre Order
- Post Order
- In-order.

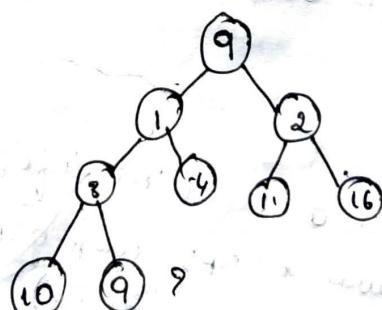
} lab programs.

Binary search tree: 9 1 2 3 -4 11 16 10 9

ignored.

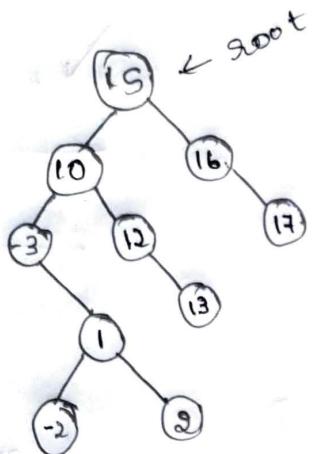


BST



BT

15 10 16 -3 1 a 17 12 13 -2 2



struct BST

```
{  
    int data;  
    struct BST *lc, *rc;
```

```
};  
typedef struct BST Node;
```

```
Node *root = NULL;
```

case 1: // read N

```
for(i=0; i<n; i++)
```

```
{  
    scanf(" %d", &info);
```

```
    create(& createinfo);
```

```
}
```

} create { in main }.

void create(int info)

```
{  
    Node *temp, temp = getnode();
```

```
    Node *getnode()
```

```
{  
    Node *ptr;
```

temp = data  
if root == null  
root = temp

```

ptr = (Node*) malloc(sizeof(Node))
if (ptr == NULL)
{
    exit(0);
}
ptr->lc = NULL;
ptr->rc = NULL;
return ptr;
}

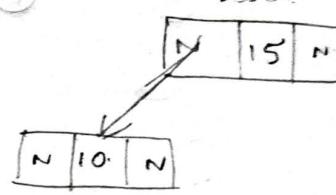
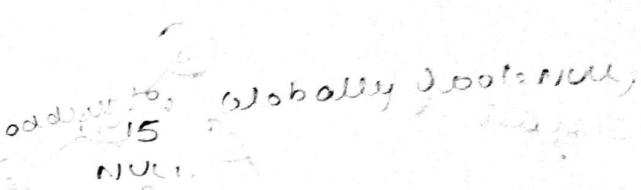
```

~~void creatree (int info, Node \*troot)~~

```

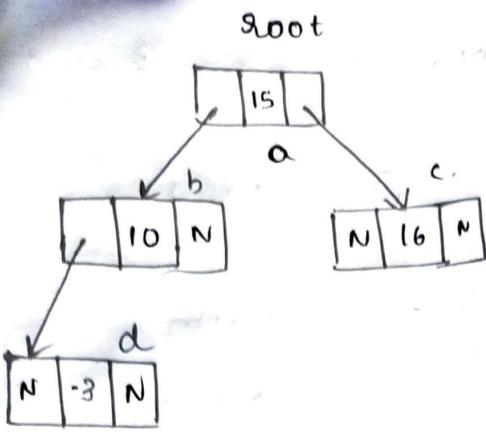
{
    Node *temp;
    temp = getnode();
    temp->data = info;
    if (troot == NULL)
        troot = temp;
    else if (troot->data > temp->data)
        creatree (info, troot->lc);
    else if (troot->data < temp->data)
        creatree (info, troot->rc);
    else
        printf("A duplicate element ", info);
}

```

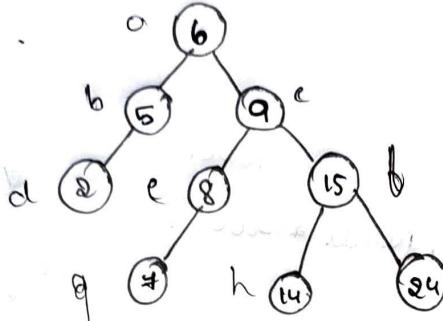


tmp.

troot



infix: a



2 5 6 4 8 9 14 15 24

LNR, RLR, RRL

check fd travel until

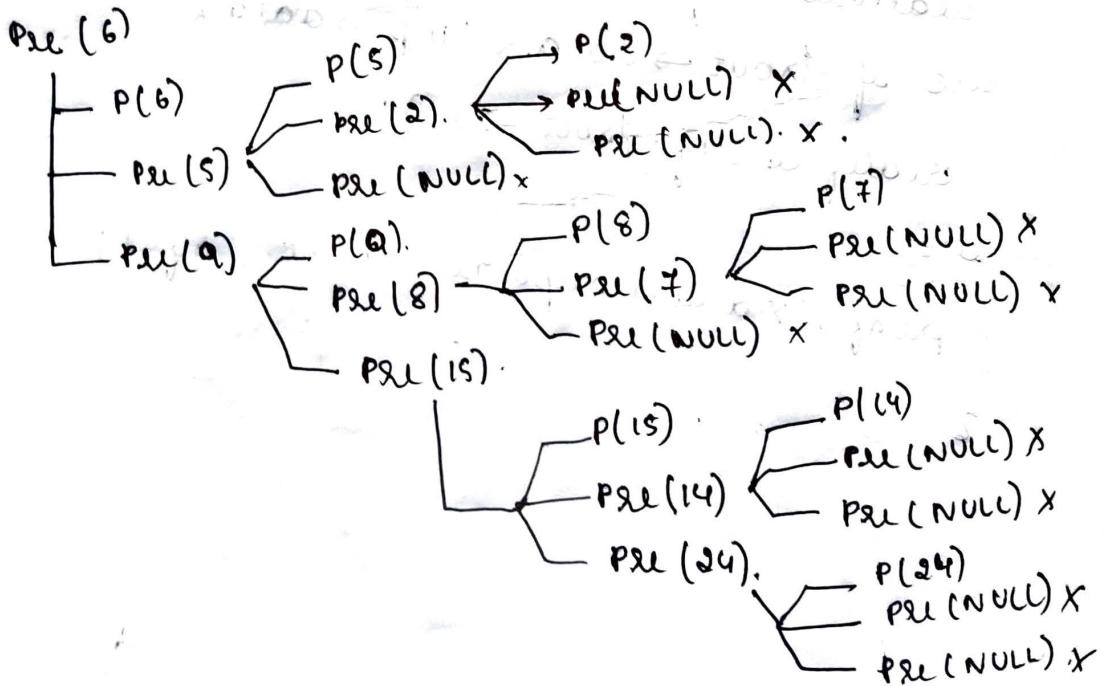
NULL.

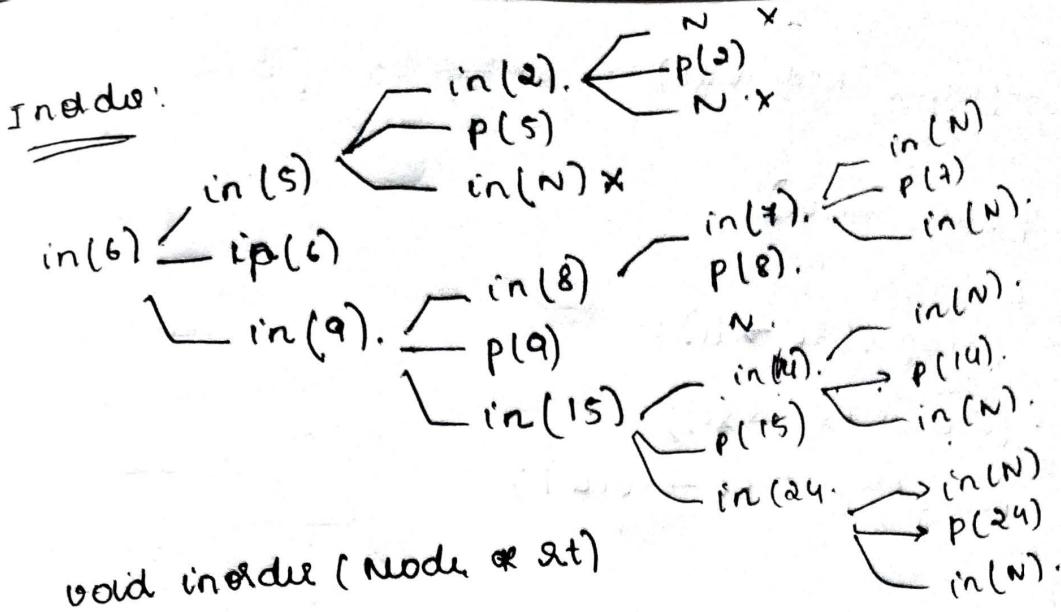
come one step back

pre order:

6. 5 2 9 8 7 15 14 24.

NLR.





void inorder (Node \*rt)

{  
if (rt != NULL)

{ inorder(rt->lc);  
printf ("%d", rt->data);  
inorder(rt->rc);

}

}

preorder: void preorder (Node \*rt),

{  
if (rt != NULL)

{ printf ("%d", rt->data);  
preorder(rt->lc);  
preorder(rt->rc);

}

}

```
void postfix(Node *rt)
```

```
{
```

```
    if(rt != NULL)
```

```
{
```

```
    postorder(rt->lc);
```

```
    postorder(rt->rc);
```

```
    printf("%d", rt->data);
```

```
}
```

```
}
```

```
struct BST
```

```
{
```

```
    int data;
```

```
    struct BST *lc, *rc;
```

```
};
```

```
main()
```

```
{
```

```
    Node *root = NULL;
```

```
c1: for(i=0 to N-1) read.
```

```
        Node *createtree(root, info);
```

```
c2: inorder(root);
```

```
c3: preorder(root);
```

```
c4: postorder(root);
```

```
c5: search(root, key);
```

```
}
```

without getne

```
Node * createtree (Node *root, int v)
{
    Node *temp;
    temp = (Node *) malloc (size of (Node))
    temp → lc = NULL;
    temp → rc = NULL;
    temp → data = v;
    if (root == NULL)
    {
        root = temp;
    }
    return root;
}

if (v < root → data)
{
    root → lc = createtree (root → lc, v);
}
else if (v > root → data)
{
    root → rc = createtree (root → rc, v);
}
else
{
    return root;
}
```

```
void inordre (Node *root)
```

Search (inorder)

```
void s (Node *root, int key)
{
    if (root)
    {
        if (root->data == key)
        {
            p.
            return;
        }
        s (root->lc, key);
        s (root->rc, key);
    }
}
```

void search (Node \*root, int key)

```
{
    if (!root)
        printf ("KNF");
    else if (key < root->data)
        search (root->lc, key);
    else if (key > root->data)
        search (root->rc, key);
    else
        printf ("KF");
}
```

node / stack:

void act ( Node \*root )

```
{  
    Node *R[MS];  
    for( ; ; ) { // while(1)  
        {  
            for( ; root; root = root->left )  
            {  
                push( root );  
            }  
            root = pop();  
            if(!root)  
                break;  
            printf("%d", root->data);  
            root = root->right;  
        }  
    }  
}
```

OR.

while( root )

{  
 push( root );  
 root = root->left;  
}

(16)

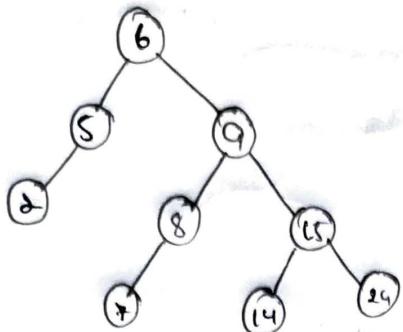
I	II	III	IV	V
root=2	root=5	root=6	push 9,8,7	root=8
print 2.	print 5	print 6.		print 8
root=N	root=N	root=9		root=N.

VI	VII	VIII	IX
root=9	root=14	root=15	root=7.
print 9	print 14	print 15	p(7).
a=15	a=N	a=24	root=N.



X	XI
breaks the outer for loop.	root=24. print 24. root=NULL

## Level order Travelling:

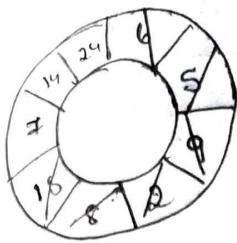


6 5 9 2 8 15 7 14 24



void CQLOT( Node \* &root )

```
{ node cq[MS];  
    insert( &root );  
    for( ; ; )  
    {  
        if( !root )  
            root = delete();  
        if( !root )  
            break;  
        printf("%d", root->data);  
        if( root->lchild )  
            insert( root->lchild );  
        if( root->rchild )  
            insert( root->rchild );  
    }  
}
```



$$\text{root} = 6$$

print 6.

root 5

print 5'

6 5 9 2 8

copy Tall!

node \* copy( node \* r1 )

{ Node & temp; } NULL.

四(2)

```
if (x1) {  
    temp = (node *) malloc (sizeof (node));
```

$\text{temp} \rightarrow \text{lc} = \text{copy} (\text{sl} \rightarrow \text{lc})$

$\text{temp} \rightarrow \text{sc} = \text{copy} (\text{sc} \rightarrow \text{sc})$

$\text{temp} \rightarrow \text{data} = x_1 \rightarrow \text{data};$

三

۱۳۶

{ return temp;

3

۳

## Complex Tree

```
int match (Node *n1, Node *n2);
```

۲

`if ((!(x1) && !(x2)) || ((x1 && x2) && (x1 → data  
= x2 → data)))`

$\& (x_1 \cdot \text{match}(x_1 \rightarrow l_C, x_2 \rightarrow l_C)) \& (\text{match}(x_1 \rightarrow x_C,$

$\lambda \lambda \rightarrow \lambda(\lambda)$ .

return 1;

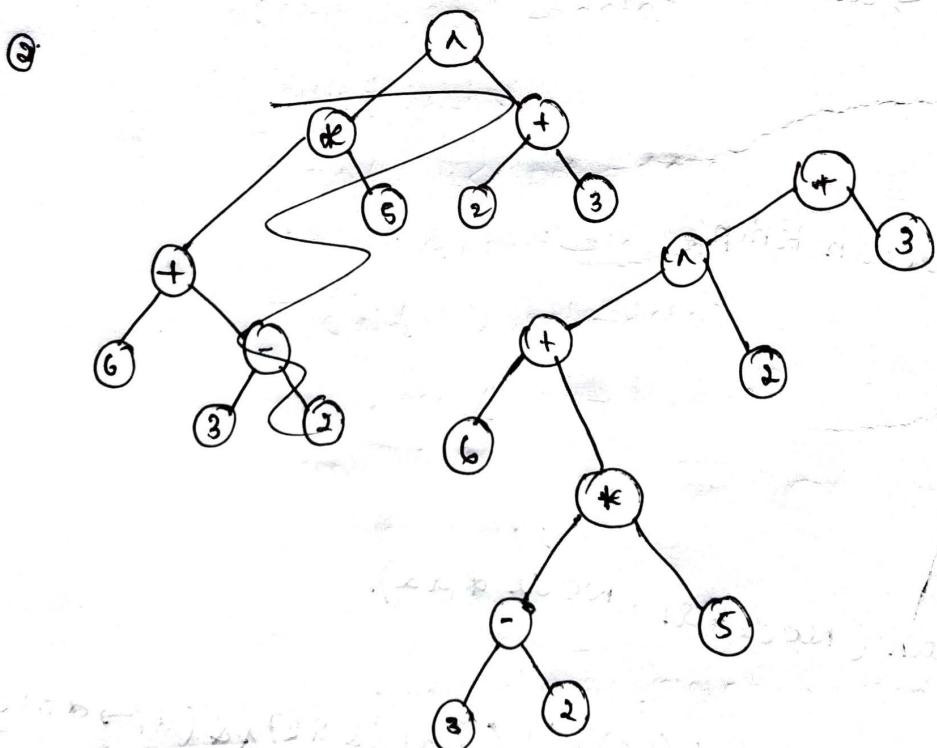
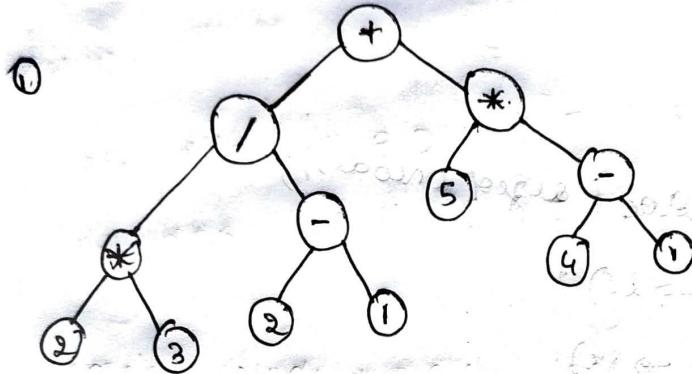
else

return 0;

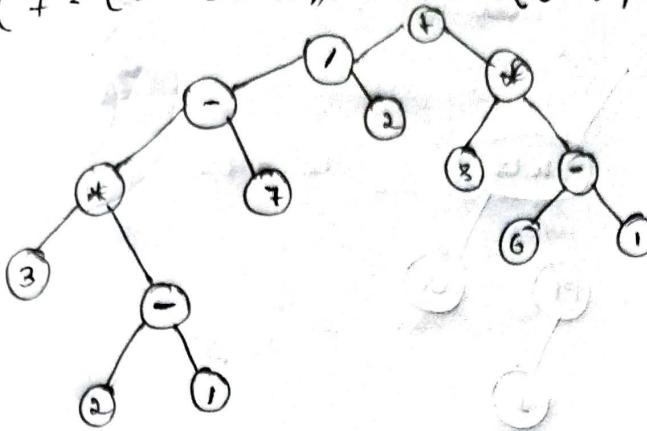
};

$$1) 2 * 3 / (2 - 1) + 5 * (4 - 1)$$

$$2) ((6 + (3 - 2) * 5)^2 + 3)$$



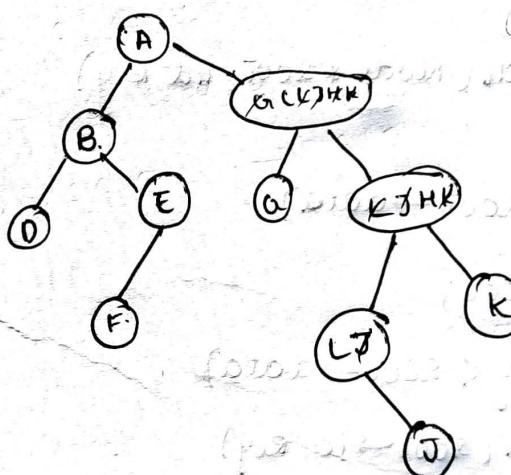
$$((7 - (3 * (2 - 7))) / 2 + 3 * (6 - 7))$$



\* pre-order = A B D E F C G H L J K

In-order = D B F E A G C L J H K

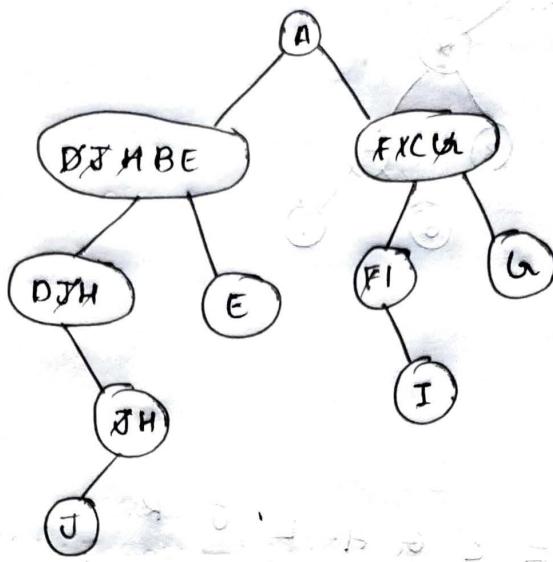
A B D E F C G H L J K



see from this side

\* post-order: J H D E B F G C A.

In-order: D J H B E A F I C G.



HW:

pre-order: A B D H J E C F I G

post-order: J H D E B I F G C ⑦

search: Node \*search (Node \*root, int key)

{

if (key == root->data)

return root;

else if (key < root->data)

root = search (root->lc, key)

else if (key > root->data)

root = search (root->rc, key)

else

return NULL;

}

without recursion:

Node \*searchIteration (Node \*root, int key)

{

while (root)

{

if (key == root->data)

return root;

else if (key < root->data)

root = root->lc;

else

root = root->rc;

}

return NULL;

}

\* on general tree

rem: if t is K-ary tree (a tree with degree k) with 'n' nodes each having a fixed size as shown in the figure:

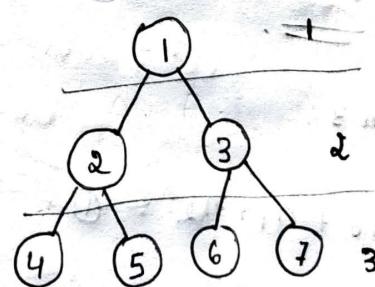
Data	$c_1$	$c_2$	$c_3$	...	$c_k$
------	-------	-------	-------	-----	-------

then  $n(k-1)+1$  of  $nK$  child fields are zero where  $n \geq 1$

from textbook you write:

Properties of BT.

1: The max no of nodes on level  $i$  of a binary tree is  $2^{i-1}$  where  $i \geq 1$ .



$$2^{i-1} = 2^0 = 1$$

$$2^{2-1} = 2^1 = 2$$

2: The max no of nodes in a binary tree of depth  $K$  is  $2^K - 1$ .

$$n_1 + n_2 + n_3 = 1 + 2 + 4 = 7 = 2^3 - 1$$

$$= 7$$

3. The relation between no of leaf nodes with respect to degree to nodes for any non empty binary tree 't' if no represents no of leaf nodes,  $n_0$ ,  $n_1$ ,  $n_2$ , no of nodes of degree 2. The relation is  $n_0 = n_2 + 1$ .

$$4 = 3 + 1$$

$$\underline{4 = 4}$$

Proof for Theorem 1: Base  $\rightarrow$  root = 1

Hypothesis  $\rightarrow$

$$\text{step } (i-1) \Rightarrow 2^{i-2}$$

$$i = 2 \& 2^{i-2}$$

$$i-2+1$$

$$2$$

$$i-1$$

$$2$$

Proof for Theorem 2:

$$2^k - 1$$

$$\sum_{i=1}^k 2^{i-1}$$

$$2^k - 1$$

$$\underline{\underline{}}$$

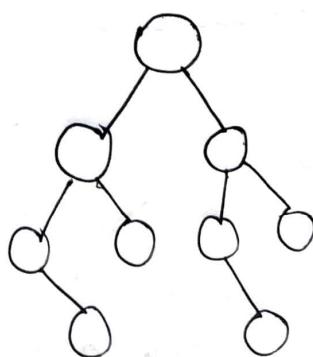
Proof for Theorem 3:

$$n = n_0 + n_1 + n_2 \quad \text{--- (1)}$$

$$= 4 + 2 + 3$$

$$n = B + 1 \quad \text{--- (2)}$$

$$B = n_1 + 2n_2 \quad \text{--- (3)}$$



$$\textcircled{1} \quad n = n_1 + 2n_2 + 1$$

$$n = n$$

$$n = n_0 + n_1 + n_2$$

$$n = n_1 + 2n_2 + 1$$

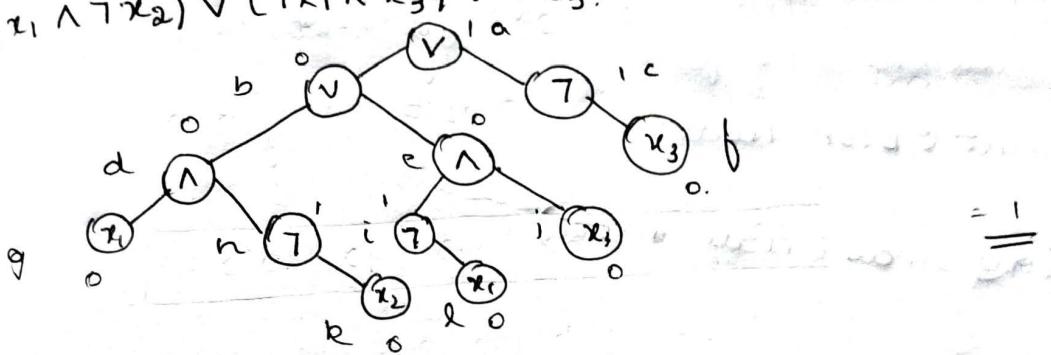
$$n_0 + n_1 + n_2 = n_1 + 2n_2 + 1$$

$$n_0 = n_2 + 1$$

(Satisfiability problem).

$$\neg(x_1 \wedge \neg x_2) \vee (\neg x_1 \wedge x_3) \vee \neg x_3.$$



Pseudocode:

for all  $2^n$  combinations

{ generate the next combination

    replace the value

    do postorder evaluation to get value of tree

    if (root  $\rightarrow$  value)

        { printf("satisfied problem"); }

    return;

}

}

```
point ("SCNA");
```

```
void postorderwalk(Node *root)
```

```
typedef enum {and, or, not, true, false} logical;
```

```
struct BTSP
```

```
{
```

```
logical data;
```

```
short int value;
```

```
struct BTSP *left, *right;
```

```
}
```

```
typedef struct BTSP Node;
```

```
void postorderwalk (Node *root)
```

```
{
```

```
if (root)
```

```
{
```

```
postorderwalk (root->left);
```

```
postorderwalk (root->right);
```

```
switch (root->data)
```

```
{ case 'not' : root->value = !(root->data);
```

```
break;
```

```
case 'a' : root->value =
```

```
(root->left->data) || root->right->data;
```

```
case 'and' :
```

```
case 'false' : root->value = 0;
```

```
case 'true' : root->value = 1;
```

```
}
```

posting

$$0. (\chi_1 \wedge \chi_2) \vee (\neg \chi_1 \wedge \chi_3) \vee \neg \chi_3.$$

$$\begin{matrix} 0 & 0 & 0 & 0 \\ x_3 & x_2 & x_1 & \end{matrix}$$

poec(a)  $\xrightarrow{\quad}$  poec(b)  $\xrightarrow{\quad}$  poec(c)

poec(d)  $\xrightarrow{\quad}$  poec(e)

poec(f)  $\xrightarrow{\quad}$  poec(g)

poec(h)  $\xrightarrow{\quad}$  poec(i)

poec(j)  $\xrightarrow{\quad}$  poec(k)

poec(l)  $\xrightarrow{\quad}$  poec(m)

poec(n)  $\xrightarrow{\quad}$  poec(o)

poec(p)  $\xrightarrow{\quad}$  poec(q)

poec(r)  $\xrightarrow{\quad}$  poec(s)

poec(t)  $\xrightarrow{\quad}$  poec(u)

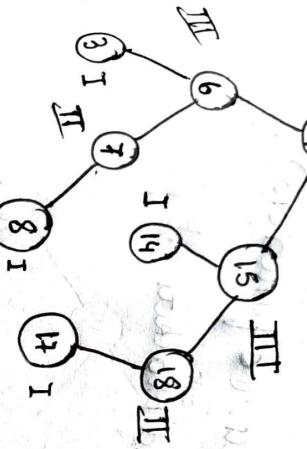
poec(v)  $\xrightarrow{\quad}$  poec(w)

3)

$\swarrow$  switch (v)

to adult  
III.

case 1 (if leaf node),  
d(3)

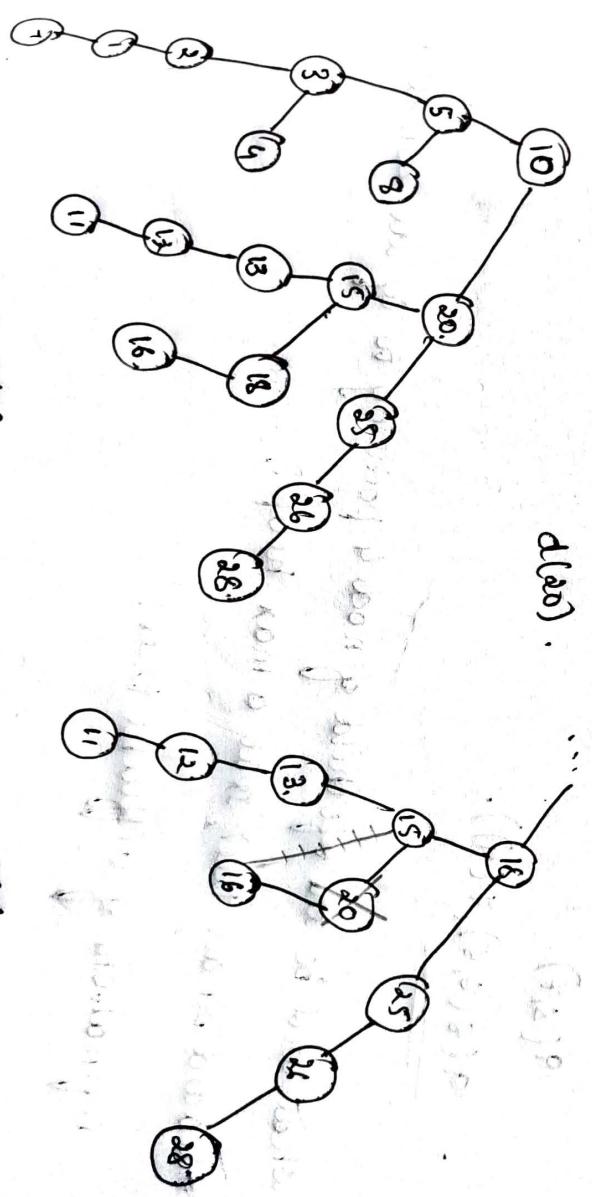


$\rightarrow$  search(3)  
 $\rightarrow$  if found  
adult mode having 3  
 $\rightarrow$  q node.

II (single child node)      III (two children  
node)

Inorder:  
3 6 1 8 9 14 15 17 18.

d(20).



LNR - in-order  
LRN - post-order

NLR - pre-order.  
level order

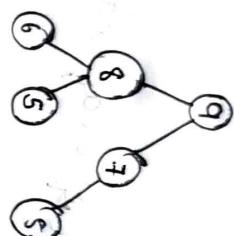
pre  
post  
level

Imp:

$$* ((6 + (3 - 2) * 5)^2 + 3).$$

Ans - A B O E F C G H I J K.

a) write a binary tree?

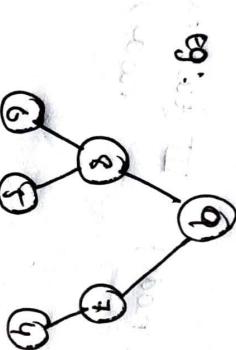


Explanation.

Explain

Ex write the pre-order, in-order, post-order traversals. (in functions)

\* what the tree notations using pairs of round brackets



9(8,7)

2 9(9(6,5),7(4))

what will be the left child of node 4 pointing to if we convert it to a threaded binary tree? Is it a max heap?

\* Definitions of different trees.