Example 8.10 : *The network of the Fig. 8.19 contains a current controlled current source. For this network, find y and z parameters.*

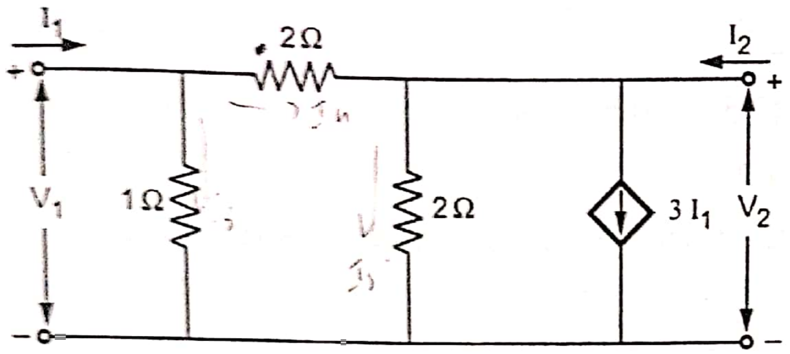


Fig. 8.19

Solution : Assuming two nodes 1 and 2 and branch currents I_3, I_4, I_5 as shown in Fig. 8.19 (a),

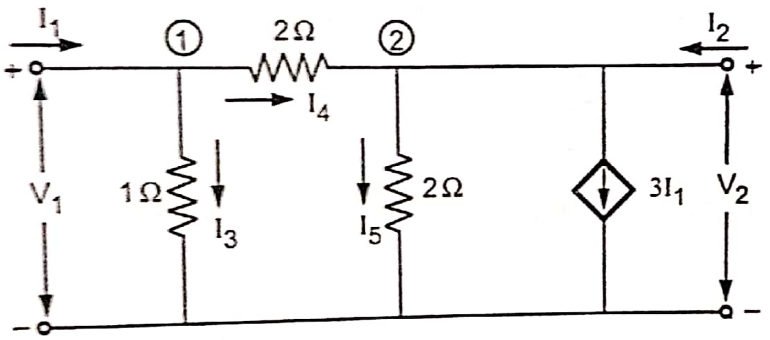


Fig. 8.19 (a)

Applying KCL at node 1,

$$I_1 = I_3 + I_4$$

$$\therefore \quad I_1 = \frac{V_1}{1} + \frac{V_1 - V_2}{2}$$

$$\therefore \quad 2\,I_1 = 3\,V_1 - V_2$$

$$\therefore \quad I_1 = 1.5\,V_1 - 0.5\,V_2 \qquad \ldots (1)$$

Applying KCL at node 2,

$$I_2 + I_4 = I_5 + 3I_1$$

$$I_2 + \frac{V_1 - V_2}{2} = \frac{V_2}{2} + 3I_1$$

$$\therefore \quad I_2 = \frac{V_2}{2} - \frac{V_1 - V_2}{2} + 3\,I_1$$

$$\therefore \quad I_2 = V_2 - 0.5\,V_1 + 3\,(1.5\,V_1 - 0.5\,V_2)$$

$$\therefore \quad I_2 = V_2 - 0.5\,V_1 + 4.5\,V_1 - 1.5\,V_2$$

$$\therefore \quad I_2 = 4\,V_1 - 0.5\,V_2 \qquad \ldots (2)$$

The y-parameters are,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1.5 & -0.5 \\ 4 & -0.5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} 1.5 & -0.5 \\ 4 & -0.5 \end{bmatrix} \mho$$

$$\therefore \quad \Delta_y = (1.5)(-0.5) - (4)(-0.5) = -0.75 + 2$$

$$= 1.25$$

Using the conversion formulae the z-parameters can be given as,

$$[z] = \begin{bmatrix} \dfrac{y_{22}}{\Delta y} & \dfrac{-y_{12}}{\Delta y} \\ \dfrac{-y_{21}}{\Delta y} & \dfrac{y_{11}}{\Delta y} \end{bmatrix}$$

$$[z] = \begin{bmatrix} \dfrac{-0.5}{1.25} & \dfrac{0.5}{1.25} \\ \dfrac{-4}{1.25} & \dfrac{1.5}{1.25} \end{bmatrix} = \begin{bmatrix} -0.4 & 0.4 \\ -3.2 & 1.2 \end{bmatrix} \Omega$$

Example 8.11 : *Find the z-parameters for the network shown in following Fig. 8.20.*

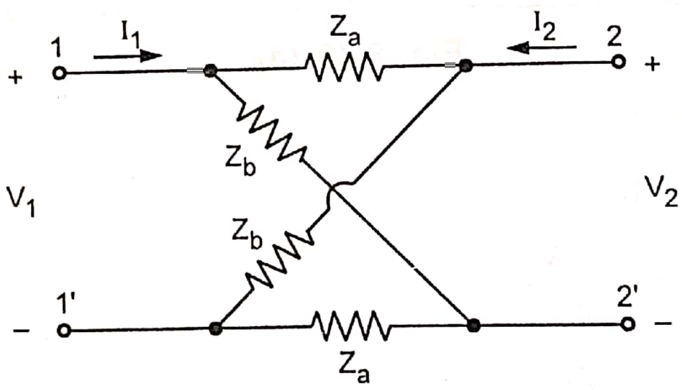


Fig. 8.20

Solution : By definition z-parameters are given as,

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

Hence, $$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0} ; \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} ; \quad z_{22} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0}$$

The given circuit can be redrawn by considering $I_2 = 0$,

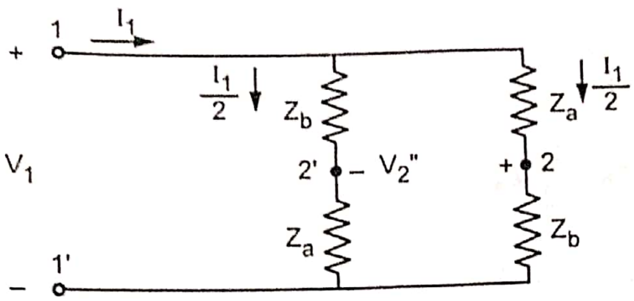


Fig. 8.20 (a)

$$V_1 = \frac{I_1}{2}[z_a + z_b]$$

$$\therefore \quad \frac{V_1}{I_1} = \frac{z_a + z_b}{2}$$

$$V_2'' = V_2 - V_2'$$

$$= \frac{I_1}{2} z_b - \frac{I_1}{2} z_a = I_1\left[\frac{z_b - z_a}{2}\right]$$

$$V_2'' = V_2 = I_1\left[\frac{z_b - z_a}{2}\right]$$

$$\therefore \quad \frac{V_2}{I_1} = \frac{z_b - z_a}{2}$$

Hence,

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0}$$

$$= \frac{z_a + z_b}{2} \ \Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0}$$

$$= \frac{z_b - z_a}{2} \ \Omega$$

The given circuit can be redrawn considering $I_1 = 0$.

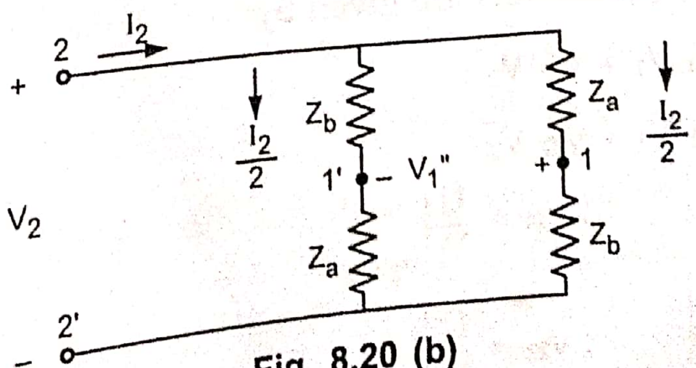


Fig. 8.20 (b)

$$V_2 = \frac{I_2}{2}[z_b + z_a]$$

$$\therefore \quad \frac{V_2}{I_2} = \frac{z_a + z_b}{2}$$

$$V_1'' = \frac{I_2}{2} z_b - \frac{I_2}{2} z_a$$

$$= I_2\left[\frac{z_b - z_a}{2}\right]$$

$$V_1'' = V_1 = I_2\left[\frac{z_b - z_a}{2}\right]$$

$$\therefore \quad \frac{V_1}{I_2} = \frac{z_b - z_a}{2}$$

Hence, $$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} = \frac{z_b - z_a}{2} \; \Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0} = \frac{z_a + z_b}{2} \; \Omega$$

Example 8.12 : *Let* $[y] = \begin{bmatrix} 0.1 & -0.0025 \\ -8 & +0.05 \end{bmatrix}$ *(s) for the two port network shown in following Fig. 8.21. Find the values for the ratios* $\frac{V_2}{V_1}, \frac{I_2}{I_1}$ *and* $\frac{V_1}{I_1}$.

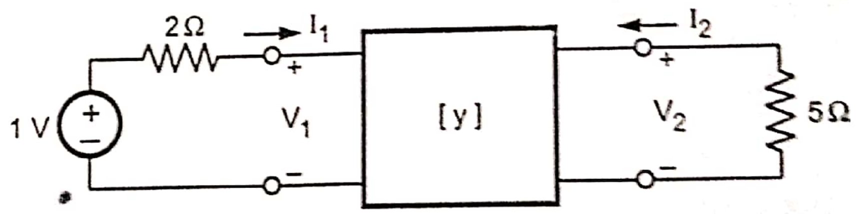


Fig. 8.21

Solution : By definition, y-parameters are given by,

$$I_1 = y_{11} V_1 + y_{12} V_2 \quad \text{... (i)}$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \quad \text{... (ii)}$$

Hence, $$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2 = 0} ; \; y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0} ; \; y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1 = 0}$$

$$[y] = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} 0.1 & -0.0025 \\ -8 & +0.05 \end{bmatrix}$$

$$y_{11} = 0.1 \ ; \ y_{12} = -0.0025$$

$$y_{21} = -8 \ ; \ y_{22} = +0.05$$

...ying KVL,

$$V_1 = 1 - 2 I_1 \quad \text{... (iii)}$$

$$V_2 = -5I_2 \quad \text{... (iv)}$$

...stituting equation (iv) in equation (ii),

$$-\frac{V_2}{5} = y_{21} V_1 + y_{22} V_2$$

$$-\frac{V_2}{5} - y_{22} V_2 = y_{21} V_1$$

$$\left[-\frac{1}{5} - y_{22}\right] V_2 = y_{21} V_1$$

$$\frac{V_2}{V_1} = \frac{y_{21}}{\left[-\frac{1}{5} - y_{22}\right]} = \frac{-8}{-\frac{1}{5} - (+0.05)}$$

$$= \frac{-8 \times 5}{-1 - 0.25} = 32$$

$$\frac{V_2}{V_1} = \mathbf{32}$$

$$V_2 = 32 V_1$$

...stituting above value in equation (i),

$$I_1 = y_{11} V_1 + y_{12} (32 V_1)$$

$$= [y_{11} + 32 y_{12}] V_1$$

$$\frac{V_1}{I_1} = \frac{1}{y_{11} + 32 y_{12}}$$

$$= \frac{1}{0.1 + 32(-0.0025)} = \frac{1}{0.02} = 50$$

$$\frac{V_1}{I_1} = \mathbf{50 \ \Omega}$$

...bstituting $V_2 = 32 V_1$ in both equation (i) and (ii) and taking the ratio of ...ons (i) and (ii),

$$\frac{I_2}{I_1} = \frac{(y_{21} + y_{22} \cdot 32) V_1}{(y_{11} + y_{12} \cdot 32) V_1} = \frac{y_{21} + 32 \cdot y_{22}}{y_{11} + 32 \cdot y_{12}}$$

Substituting equation (iv) in equation (iii),

$$I_2 = V_2 + 0.5\,(-3\,I_2)$$

$$(1 + 1.5)\,I_2 = V_2$$

$$\frac{I_2}{V_2} = \frac{1}{2.5} = 0.4$$

Hence, $$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \mathbf{0.4\ mho}$$

Substituting equation (iii) in equation (iv),

$$-V_1 = 3\,(V_2 + 0.5\,V_1)$$

$$\therefore \quad -V_1 = 3\,V_2 + 1.5\,V_1$$

$$\therefore \quad -V_1 - 1.5\,V_1 = 3\,V_2$$

$$\therefore \quad -2.5\,V_1 = 3\,V_2$$

$$\therefore \quad \frac{V_1}{V_2} = -1.2$$

Hence, $$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$= \mathbf{-1.2}$$

The overall h-parameters of network are given by,

$$[h] = \begin{bmatrix} -2.8 & -1.2 \\ -1.4 & 0.4 \end{bmatrix}$$

Example 8.14 : *Find the transmission parameters for the network shown in Fig. 8.23.*

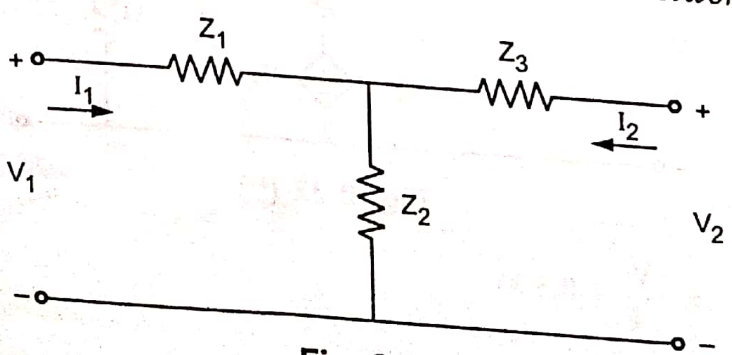


Fig. 8.23

Solution : By definition transmission parameters are given by,

$$V_1 = A\,V_2 - B\,I_2$$

$$I_1 = C\,V_2 - D\,I_2$$

Hence,
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \quad ; C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

$$B = \left.\frac{V_1}{(-I_2)}\right|_{V_2=0} \quad ; D = \left.\frac{I_1}{(-I_2)}\right|_{V_2=0}$$

We will make $I_2 = 0$;

Applying KVL, we get

$$V_1 = I_1 (z_1 + z_2) \quad \text{... (i)}$$

$\therefore$

$$V_2 = I_1 z_2 \quad \text{... (ii)}$$

$$\frac{V_2}{V_1} = \frac{I_1 z_2}{I_1 (z_1 + z_2)} = \frac{z_2}{z_1 + z_2}$$

$$\frac{V_1}{V_2} = \frac{z_1 + z_2}{z_2}$$

Hence,
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{z_1 + z_2}{z_2}$$

From (ii),
$$\frac{I_1}{V_2} = \frac{1}{z_2}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{z_2}\ \mho$$

Now, we will make $V_2 = 0$. Applying KVL to both loops, we have,

$$V_1 = I_1 z_1 + (I_1 + I_2) z_2 \quad \text{... (iii)}$$

$$I_2 z_3 + (I_2 + I_1) z_2 = 0 \quad \text{... (iv)}$$

From equation (iv),

$$I_1 z_2 = -I_2 z_3 - I_2 z_2$$

$$I_2 z_2 = -I_2 (z_2 + z_3)$$

$$\therefore \quad \frac{I_1}{(-I_2)} = \frac{z_2 + z_3}{z_2}$$

Hence,
$$D = \left.\frac{I_1}{(-I_2)}\right|_{V_2=0} = \frac{z_2 + z_3}{z_2}$$

From (iv),
$$I_1 = \frac{-I_2 (z_2 + z_3)}{z_2}$$

Substituting in equation (iii),

$$V_1 = (z_1 + z_2)\left[\frac{-I_2(z_2+z_3)}{z_2}\right] + I_2 z_2$$

$$= -I_2\left[\frac{(z_1+z_2)(z_2+z_3)}{z_2} - z_2\right]$$

$$= -I_2\left[\frac{z_1 z_2 + z_1 z_3 + z_2^2 + z_2 z_3 - z_2^2}{z_2}\right]$$

$$\therefore \quad \frac{V_1}{(-I_2)} = \frac{z_1 z_2 + z_2 z_3 + z_3 z_1}{z_2}$$

$$B = \left.\frac{V_1}{(-I_2)}\right|_{V_2=0} = \frac{z_1 z_2 + z_2 z_3 + z_3 z_1}{z_2}\ \Omega$$

Thus the total transmission parameters are given as,

$$[T] = \begin{bmatrix} \dfrac{z_1+z_2}{z_2} & \dfrac{z_1 z_2 + z_2 z_3 + z_3 z_1}{z_2} \\ \dfrac{1}{z_2} & \dfrac{z_2+z_3}{z_2} \end{bmatrix}$$

Example 8.15 : *Find the z-parameters of the overall network shown in following Fig. 8.24.*

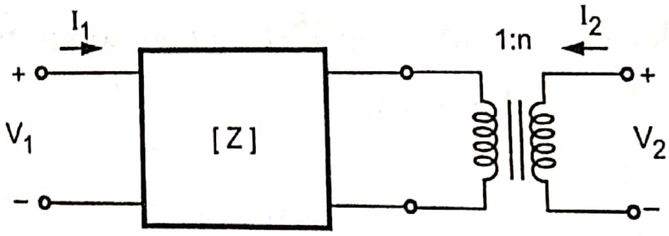


Fig. 8.24

Solution : Consider the following network first

We will find transmission parameters for this network

We will first make $I_2 = 0$

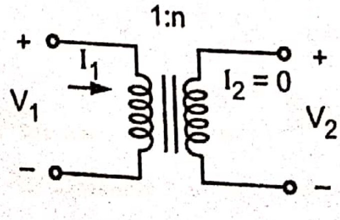


Fig. 8.24 (a)

$$\frac{V_1}{V_2} = \frac{1}{n}$$

$$V_2 = n\,V_1$$

$$\therefore \quad V_1 = \frac{1}{n} V_2 + 0\,(-I_2) \qquad \text{... (i)}$$

We will make $V_2 = 0$, we get

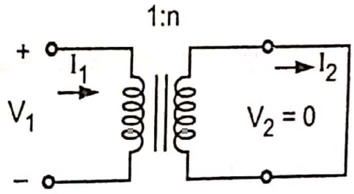


Fig. 8.24 (b)

$$\frac{I_2}{I_1} = \frac{1}{n}$$

$$I_1 = n\,I_2$$

$$\therefore \quad I_2 = 0\,V_2 + n\,(-I_2) \qquad \text{... (ii)}$$

The equations (i) and (ii) can also be written as,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

The ABCD parameters for the above network can be given as,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1/n & 0 \\ 0 & n \end{bmatrix}$$

The z-parameters for the network are

$$[z] = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

These parameters are given in terms of transmission parameters are given as,

$$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$$

Since the given two networks are in cascade the overall transmission parameters r the total network can be found by taking the matrix product of transmission rameters for individual networks.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{overall} = \begin{bmatrix} \dfrac{z_{11}}{z_{21}} & \dfrac{z_{11}z_{22} - z_{12}z_{21}}{z_{21}} \\ \dfrac{1}{z_{21}} & \dfrac{z_{22}}{z_{21}} \end{bmatrix} \begin{bmatrix} 1/n & 0 \\ 0 & n \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{1}{n}\dfrac{z_{11}}{z_{21}} & \dfrac{1}{n}\dfrac{z_{11}z_{22} - z_{12}z_{21}}{z_{21}} \\ \dfrac{1}{n}\dfrac{1}{z_{21}} & \dfrac{n\, z_{22}}{z_{21}} \end{bmatrix}$$

Now converting ABCD parameters into z-parameters.

$$z_{11} = \frac{A}{C}; \; z_{12} = \frac{\begin{vmatrix} A & B \\ C & D \end{vmatrix}}{C}; \; z_{21} = \frac{1}{C}; \; z_{22} = \frac{D}{C}$$

$$z_{11} = \frac{\left(\dfrac{z_{11}}{n\, z_{21}}\right)}{\left(\dfrac{1}{n\, z_{21}}\right)} = z_{11}$$

$$z_{21} = \frac{1}{\left(\dfrac{1}{n\, z_{21}}\right)} = n\, z_{21}$$

$$z_{22} = \frac{\left(\dfrac{n\, z_{22}}{z_{21}}\right)}{\left(\dfrac{1}{n\, z_{21}}\right)} = n^2 z_{22}$$

$$z_{12} = \frac{\left[\dfrac{z_{11}\, z_{22}}{z_{21}^2}\right] - \left[\dfrac{z_{11}\, z_{22} - z_{12}\, z_{21}}{z_{21}^2}\right]}{\dfrac{1}{n} \cdot \dfrac{1}{z_{21}}}$$

$$= \frac{\left[\dfrac{z_{11}\, z_{22} - z_{11}\, z_{22} + z_{12}\, z_{21}}{z_{21}^2}\right]}{\left[\dfrac{1}{n\, z_{21}}\right]} = \frac{z_{12}\, z_{21}}{z_{21}^2} \cdot n\, z_{21}$$

$$= n\, z_{12}$$

Hence z-parameters for overall network are given by,

$$[z] = \begin{bmatrix} z_{11} & n\, z_{12} \\ n\, z_{21} & n^2 z_{22} \end{bmatrix}$$

Example 8.16 : *For the network shown in Fig. 8.25 determine z and y parameters.*

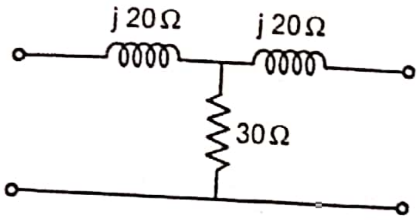


Fig. 8.25

Solution : z-parameters are calculated by open circuiting the ports. Let the terminals be 1- 1′ and 2 – 2′

[A] Let voltage V_1 is applied at port 1- 1′.

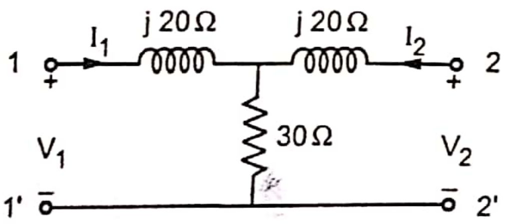


Fig. 8.25 (a)

$I_2 = 0$ because port 2-2′ is open circuited.

$$\therefore \; z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

I_1 is flowing from terminal 1 to 1′ through j20 Ω and 30 Ω only.

$$\therefore \qquad I_1 = \frac{V_1}{(j\,20 \; +30)}$$

Hence, $\quad z_{11} = (30 + j\,20)\ \Omega$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

Applying potential divider rule,

$$V_2 = V_1\left[\frac{30}{30 + j\,20}\right]$$

$$I_1 = \frac{V_1}{(30 + j20)}$$

Hence, $$z_{21} = \frac{V_2}{I_1} = \frac{\left[\dfrac{30}{30 + j\,20}\right]V_1}{\dfrac{V_1}{(30 + j\,20)}} = 30\ \Omega$$

[B] To find remaining two parameters,

Let port 1- 1′ be open circuited,

$$\therefore \quad I_1 = 0$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

I_2 is flowing through j 20 Ω and 30 Ω from terminal 2 to 2′.

$$\therefore \quad I_2 = \frac{V_2}{j\,20+30}$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \mathbf{(30 + j\ 20)\ \Omega}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{(30)(I_2)}{I_2} = \mathbf{30\ \Omega}$$

$$\text{Hence,} \quad [z] = \begin{bmatrix} (30+j\,20) & 30 \\ 30 & (30+j\,20) \end{bmatrix}$$

[C] To find y-parameters

$$y_{11} = \frac{z_{22}}{\Delta_z}$$

$$\therefore \quad \Delta_z = \begin{vmatrix} (30+j\,20) & 30 \\ 30 & (30+j\,20) \end{vmatrix}$$

$$= [\ (30 + j\ 20)\ (30 + j\ 20) - (30)\ (30)\ ]$$

$$= [\ 900 + j\ 1200 - 400 - 900\ ] = -400 + j\ 1200$$

$$\therefore \quad y_{11} = \frac{z_{22}}{\Delta_z} = \frac{(30+j\,20)}{(-400+j\,1200)} = \mathbf{(7.5 - j\ 27.5\ ) \times 10^{-3}\ \mho}$$

$$y_{12} = \frac{-z_{12}}{\Delta_z} = \frac{-30}{(-400+j\,1200)} = \mathbf{(-7.5 - j\ 22.5\ ) \times 10^{-3}\ \mho}$$

$$y_{21} = \frac{-z_{21}}{\Delta_z} = \frac{-30}{(-400+j\,1200)} = \mathbf{(-7.5 - j\ 22.5\ ) \times 10^{-3}\ \mho}$$

$$y_{22} = \frac{z_{11}}{\Delta z} = \frac{(30+j\,20)}{(-400+j\,1200)} = \mathbf{(7.5 - j\ 27.5\ ) \times 10^{-3}\ \mho}$$

Example 8.17 : *The network shown in the Fig. 8.26 contains both dependent current source and dependent voltage source. For the element values given, determine the y and z parameters.*

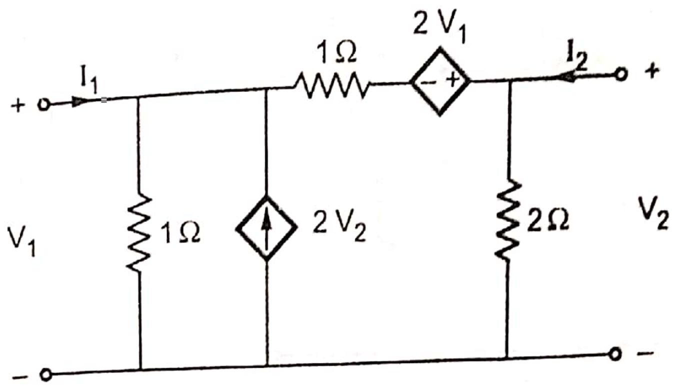


Fig. 8.26

Solution : 1) y-Parameters :

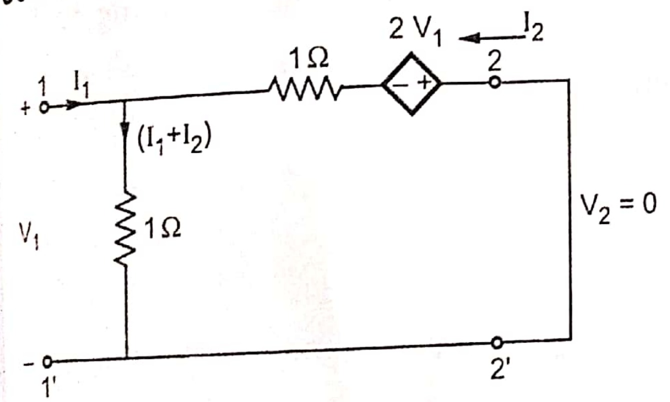


Fig. 8.26 (a)

i) To obtain y_{11} and y_{21}, short circuit terminals 2-2′ by making $V_2 = 0$. As $V_2 = 0$, $3V_2 = 0$ so we can neglect dependent current source. Also as terminals 2-2′ are shorted, 2 Ω resistor will be directly shorted. The simplified network is as shown in the Fig. 8.26 (a).

Here 1 Ω resistor connected in shunt, carries I_1 as well as I_2. So the total current flowing through 1 Ω resistor is $(I_1 + I_2)$.

$$\therefore \quad V_1 = 1\,(I_1 + I_2) = I_1 + I_2 \qquad \dots (1)$$

Applying KVL to outer loop,

$$-I_2 - 2V_1 = V_1$$

$$\therefore \quad -I_2 = 3V_1 \qquad \dots (2)$$

$$\therefore \quad y_{21} = \frac{I_2}{V_1} = -3\ \mho$$

From equation (1),

$$I_2 = V_1 - I_1$$

Substituting value of I_2 in equation (2)

$$\therefore \quad -(V_1 - I_1) = 3V_1$$

$$\therefore \quad -V_1 + I_1 = 3V_1$$

$$\therefore \quad I_1 = 4V_1$$

$\therefore \quad y_{11} = \frac{I_1}{V_1} = 4 \ ℧$

ii) To obtain y_{22} and y_{12}, short circuit terminals 1-1′ by making $V_1 = 0$. As terminals 1-1′ are short circuited, resistor 1 Ω connected in shunt will be short circuited. The simplified network is as shown in the Fig. 8.26 (b).

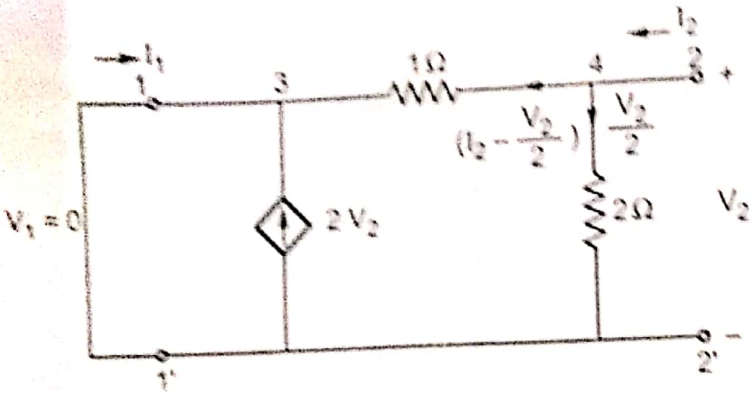


Fig. 8.26 (b)

The voltage across 2 Ω resistor is V_2. Thus current flowing through 2 Ω is given by $\frac{V_2}{2}$. At node 4, currents I_2 is entering while current $\frac{V_2}{2}$ is leaving. Hence according to KCL current leaving node 4 through 1 Ω is given as $\left(I_2 - \frac{V_2}{2}\right)$.

Applying KCL at node 3,

$$\therefore \quad I_1 + 2V_2 + I_2 - \frac{V_2}{2} = 0$$

$$\therefore \quad I_1 + I_2 = -2\,V_2 + \frac{V_2}{2}$$

$$\therefore \quad I_1 + I_2 = \frac{-3V_2}{2} \qquad \ldots (3)$$

Applying KVL to outer loop,

$$\therefore \quad 1 \cdot \left(I_2 - \frac{V_2}{2}\right) - V_2 = 0$$

$$\therefore \quad I_2 = V_2 + \frac{V_2}{2}$$

$$\therefore \quad I_2 = \frac{3V_2}{2} \qquad \ldots (4)$$

$$\therefore \quad y_{22} = \frac{I_2}{V_2} = \frac{3}{2} \ ℧$$

Substituting value of I_2 from equation (4) in equation (3),

$$\therefore \quad I_1 + \left(\frac{3V_2}{2}\right) = \frac{-3V_2}{2}$$

$$I_1 = \frac{-3V_2}{2} - \frac{3V_2}{2}$$

$\therefore$

$$I_1 = \frac{-6V_2}{2}$$

$\therefore$

$$y_{12} = \frac{I_1}{V_2} = \frac{-6}{2} = -3\ \mho$$

$\therefore$ For the given network, y-parameters are given by,

$$[y] = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -3 & 3/2 \end{bmatrix} \mho$$

$$\Delta y = \begin{bmatrix} 4 & -3 \\ -3 & 3/2 \end{bmatrix} = 4\left(\frac{3}{2}\right) - (-3)(-3)$$

$$= 6 - 9 = -3$$

By using conversion formula for y to z parameters conversion, z-parameters are given by,

$$z_{11} = \frac{y_{22}}{\Delta y} = \frac{3/2}{-3} = -\frac{1}{2}\ \Omega$$

$$z_{12} = \frac{-y_{12}}{\Delta y} = \frac{-(-3)}{-3} = -1\ \Omega$$

$$z_{21} = \frac{-y_{21}}{\Delta y} = \frac{-(-3)}{-3} = -1\ \Omega$$

$$z_{22} = \frac{y_{11}}{\Delta y} = \frac{4}{-3} = -\frac{4}{3}\ \Omega$$

Thus, for given network, z-parameters are given by,

$$[Z] = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & -1 \\ -1 & -\frac{4}{3} \end{bmatrix} \Omega$$

Example 8.18 : *Find the short circuit admittance parameters of the network shown in Fig. 8.27.*

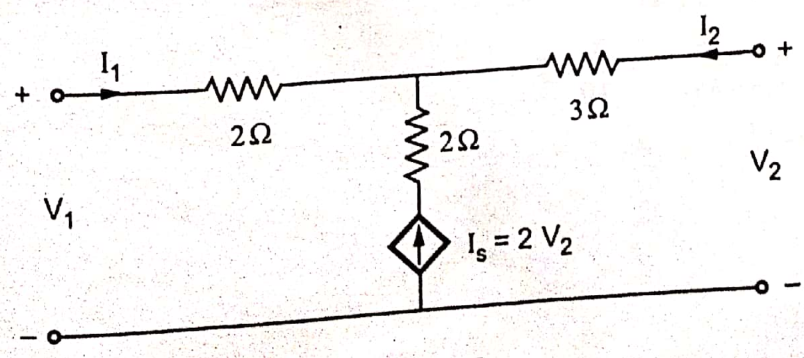


Fig. 8.27

Solution : The equations of y-parameters are,

$$I_1 = y_{11} V_1 + y_{12} V_2$$

$$I_2 = y_{21} V_1 + y_{22} V_2$$

By making $V_2 = 0$ we can get parameters as,

$$y_{11} = \frac{I_1}{V_1}, \quad y_{21} = \frac{I_2}{V_1}$$

By making $V_1 = 0$ we can get parameters as,

$$y_{22} = \frac{I_1}{V_2}, \quad y_{22} = \frac{I_2}{V_2}$$

[A] To find y_{11} and y_{21}

Short circuit 2 – 2′, i.e. $V_2 = 0$

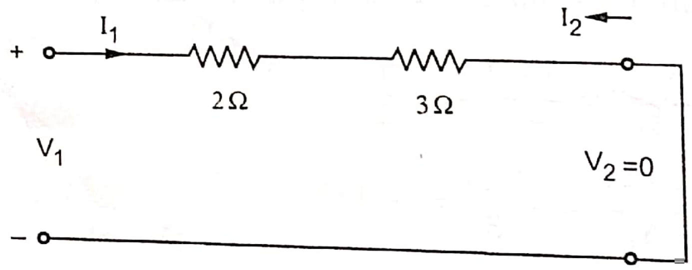


Fig. 8.27 (a)

$$V_1 = 5(I_1 - I_2) \quad \text{... (i)}$$

$$I_1 = - I_2 \quad \text{... (ii)}$$

$\therefore$

$$V_1 = 5 [I_1 - (- I_1)]$$

$$= 5 [2 I_1] = 10 I_1$$

$$y_{11} = \frac{I_1}{V_1} = \frac{1}{10} \text{℧}$$

Now,

$$V_1 = 5 (- I_2 - I_2)$$

$\therefore$

$$V_1 = - 10 I_2$$

$\therefore$

$$y_{21} = \frac{I_2}{V_1} = -\frac{1}{10} \text{℧}$$

B] To find y_{12} and y_{22}

Short circuit terminals 2 – 2′, i.e. $V_1 = 0$

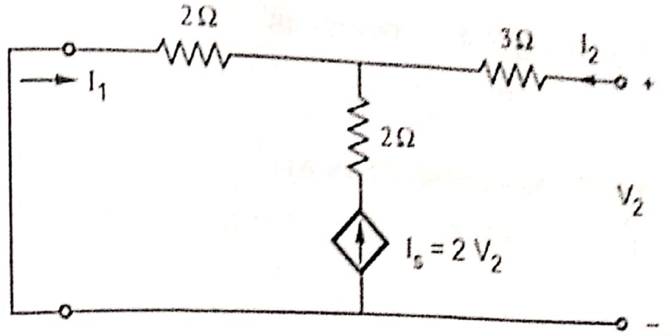


Fig. 8.27 (b)

$$V_2 = 3\,I_2 + 2\,(I_2 + 2\,V_2)$$

$$V_2 = 5\,I_2 + 4\,V_2$$

$$\therefore \quad -3V_2 = 5\,I_2$$

$$\therefore \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = -\frac{3}{5}\ \mho$$

$$\therefore \quad I_1 = -(I_2 + 2\,V_2)$$

$$I_1 = -\left[-\frac{3}{5}V_2 + 2\,V_2\right] = -\left[-\frac{3}{5}+2\right]V_2 = -\left[\frac{7}{5}\right]V_2$$

$$\therefore \quad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\frac{7}{5}\ \mho$$

The y-parameters of the given circuit are,

$$[y] = \begin{bmatrix} \frac{1}{10} & -\frac{7}{5} \\ -\frac{1}{10} & -\frac{3}{5} \end{bmatrix}$$

➡ **Example 8.19 :** *The model of a transistor in the CE mode is shown in the Fig. 8.28. Determine its h-parameters* **(March-2002)**

Solution : The equations of h-parameters are,

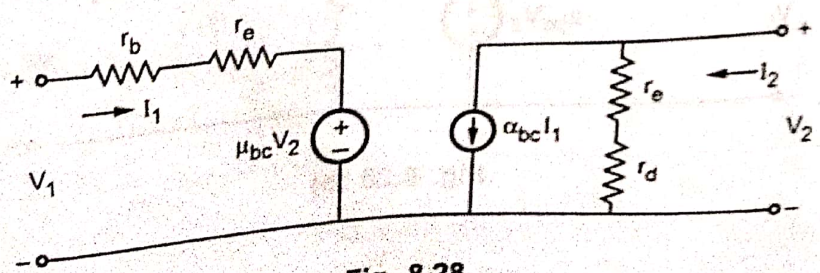


Fig. 8.28

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

By making $V_2 = 0$, we can get parameters as

$$h_{11} = \frac{V_1}{I_1} \quad ; h_{21} = \frac{I_2}{V_1}$$

By making $I_1 = 0$, we can get parameters as

$$h_{12} = \frac{V_1}{V_2} \quad ; h_{22} = \frac{I_2}{V_2}$$

[A] To find h_{11} and h_{21}

Making $V_2 = 0$. The circuit can be shown as

Applying KVL,

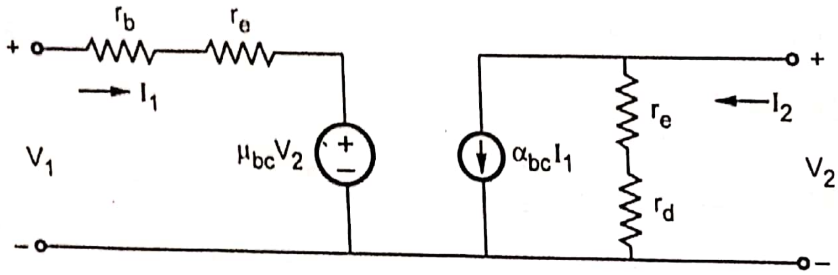


Fig. 8.28 (a)

$$V_1 = I_1 (r_b + r_e)$$

$$\therefore \quad h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0} = (r_b + r_e) \ \Omega$$

$$I_2 = \alpha_{cb} I_1$$

$$\therefore \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0} = \alpha_{cb}$$

[B] To find h_{12} and h_{22}

Making $I_1 = 0$. The circuit can be shown as,

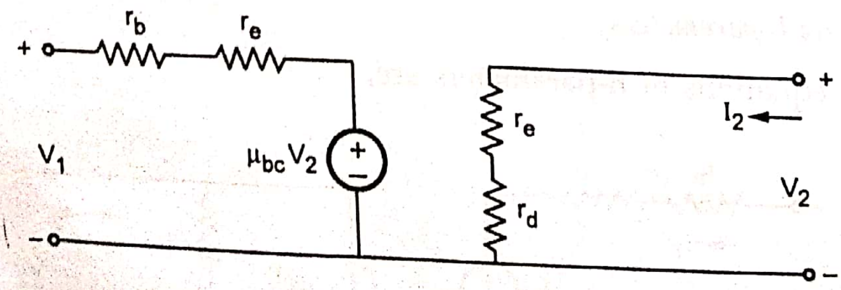


Fig. 8.28 (b)

$$V_1 = \mu_{bc} V_2$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0} = \mu_{bc}$$

$\therefore$

$$V_2 = I_2 (r_e + r_d)$$

Also,

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0} = \frac{1}{r_e + r_d}\ ℧$$

$\therefore$ The overall h-parameters for given circuit are,

$$[h] = \begin{bmatrix} r_b + r_e & \mu_{bc} \\ \alpha_{cb} & \dfrac{1}{r_e + r_d} \end{bmatrix}$$

Example 8.20 : *Find z and h-parameters for the network shown in Fig. 8.29 and check for symmetry and reciprocity in both the cases.* **(August-2000)**

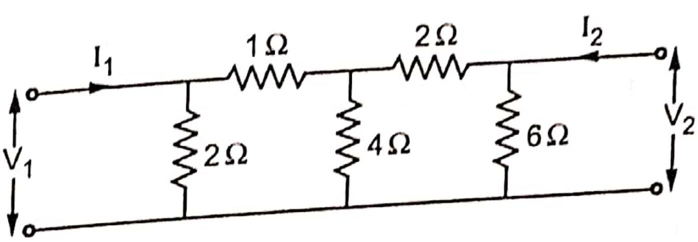


Fig. 8.29

Solution : By definition z-parameters are given as,

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0} \quad ; \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} \quad ; \quad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_2 = 0}$$

The given circuit can be simplified as follows.

Converting π formed by 4 Ω, 2 Ω, 6 Ω into equivalent 'T'.

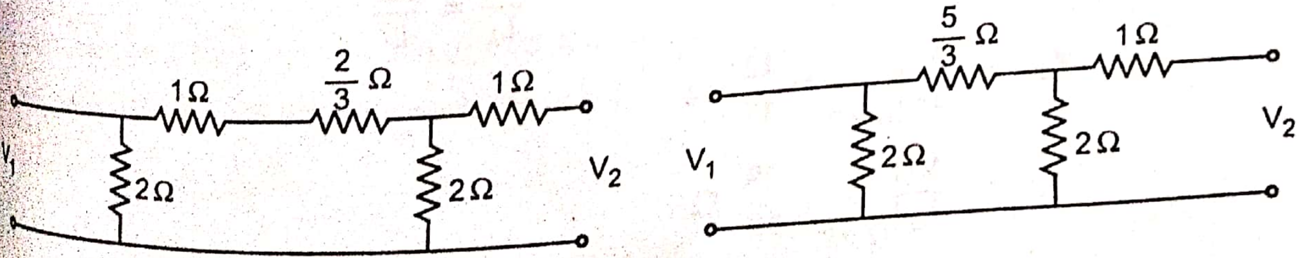


Fig. 8.29 (a)

Converting π formed by $\frac{5}{3}$ Ω, 2 Ω, 2 Ω into equivalent T,

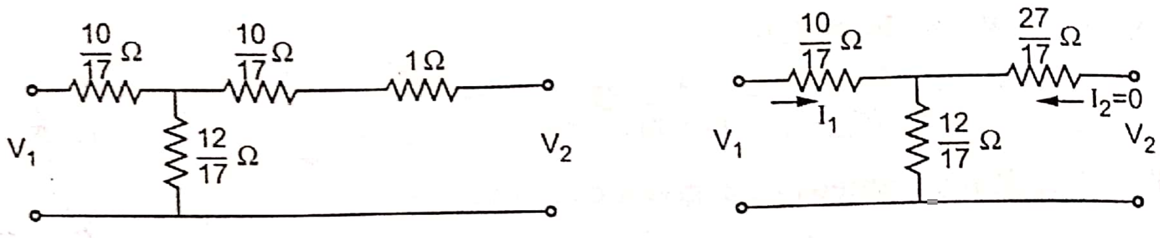


Fig. 8.29 (b)

Now first consider $I_2 = 0$;

$$V_1 = I_1\left(\frac{10}{17}+\frac{12}{17}\right) = I_1\left(\frac{22}{17}\right)$$

$$\therefore \quad \frac{V_1}{I_1} = \frac{22}{17}$$

$$V_2 = \frac{12}{17} I_1$$

$$\therefore \quad \frac{V_2}{I_1} = \frac{12}{17}$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{22}{17}\ \Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{12}{17}\ \Omega$$

Now, consider $I_1 = 0$

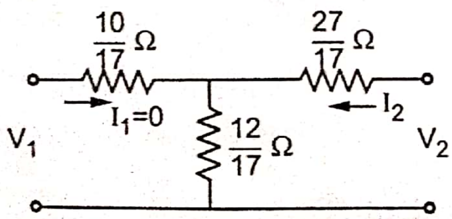


Fig. 8.29 (c)

$$V_2 = I_2\left(\frac{27}{17}+\frac{12}{17}\right) = \left(\frac{39}{17}\right) I_2$$

$$\frac{V_2}{I_2} = \frac{39}{17}$$

$$V_1 = I_2\left(\frac{12}{17}\right)$$

$$\frac{V_1}{I_2} = \frac{12}{17}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{12}{17}\ \Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_2=0} = \frac{39}{17}\ \Omega$$

The total z parameters are given by,

$$[z] = \begin{bmatrix} \frac{22}{17} & \frac{12}{17} \\ \frac{12}{17} & \frac{39}{17} \end{bmatrix}$$

For the given network it can be seen that $Z_{11} \neq Z_{22}$. Hence the network is not said be symmetrical.

Also it can be seen that $Z_{12} = Z_{21}$. Therefore the network is said to be reciprocal.

The h parameters in terms of z parameters are given by,

$$[h] = \begin{bmatrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ \frac{-Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix}$$

$$\Delta Z = Z_{11}.\ Z_{22} - Z_{12}\ .\ Z_{21} = \left[\left(\frac{22}{17}\right)\left(\frac{39}{17}\right)-\left(\frac{12}{17}\right)\left(\frac{12}{17}\right)\right]$$

$$= \frac{714}{289}$$

$$h_{11} = \frac{\Delta Z}{Z_{22}} = \frac{714}{289}\times\frac{17}{39} = \frac{42}{39}\ \Omega$$

$$h_{12} = \frac{Z_{12}}{Z_{22}} = \frac{12}{17}\times\frac{17}{39} = \frac{12}{39}$$

$$h_{21} = \frac{-Z_{21}}{Z_{22}} = -\frac{12}{17}\times\frac{17}{39} = -\frac{12}{39}$$

$$h_{22} = \frac{1}{Z_{22}} = \frac{17}{39}\ \mho$$

The overall h parameters are given by

$$[h] = \begin{bmatrix} \frac{42}{39} & \frac{12}{39} \\ \frac{-12}{39} & \frac{17}{39} \end{bmatrix}$$

It can be seen that $h_{12} = -h_{21}$ therefore the network is said to be reciprocal.

The network is said to be symmetrical if

$$h_{11}\ h_{22} - h_{12}\ h_{21} = 1$$

Now we will find $h_{11} \cdot h_{22} - h_{12} h_{21}$

$$= \left(\frac{42}{39} \cdot \frac{17}{39}\right) + \left(\frac{12}{39}\right)\left(\frac{12}{39}\right) = \frac{714}{39} + \frac{144}{39}$$

$$= \frac{858}{39} \neq 1$$

Hence the given network is not symmetrical.

Example 8.21 : *For a certain two port network V_1 and V_2 are given by*

$$V_1 = 60\, I_1 + 20\, I_2$$

$$V_2 = 20\, I_1 + 40\, I_2$$

Find y-parameters of the network **(March-2000)**

Solution : The equations for open circuit z-parametrs are given by

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \text{ and}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Comparing given equations with the basic equations of z-parameters we get, z parameters as follows,

$$Z_{11} = 60,\ Z_{12} = 20 \text{ and}$$

$$Z_{21} = 20,\ Z_{22} = 40$$

Hence z-parameter matrix can be written as,

$$[Z] = \begin{bmatrix} 60 & 20 \\ 20 & 40 \end{bmatrix}$$

The y-parameters of the networks from the z-parameters of the network can be written as follows,

$$y_{11} = \frac{Z_{22}}{\Delta Z},\ y_{12} = \frac{-Z_{12}}{\Delta Z}$$

$$y_{21} = \frac{-Z_{21}}{\Delta Z},\ y_{22} = \frac{Z_{11}}{\Delta Z}$$

$$\Delta Z = \begin{bmatrix} 60 & 20 \\ 20 & 40 \end{bmatrix}$$

$$= (60)(40) - (20)(20)$$

$$= 2000$$

$$\therefore \quad y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{40}{2000} = \mathbf{20\ m℧}$$

$$y_{12} = \frac{-Z_{12}}{\Delta Z} = \frac{-20}{2000} = -10 \text{ m } ℧$$

$$y_{21} = \frac{-Z_{21}}{\Delta Z} = \frac{-20}{2000} = -10 \text{ m } ℧$$

$$y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{60}{2000} = 30 \text{ m } ℧$$

Hence the y-parameters of the network are given as,

$$[y] = \begin{bmatrix} 20\times10^{-3} & -10\times10^{-3} \\ -10\times10^{-3} & 30\times10^{-3} \end{bmatrix}$$

Example 8.22 : *Find z and y parameters of the network shown in the Fig. 8.30.*

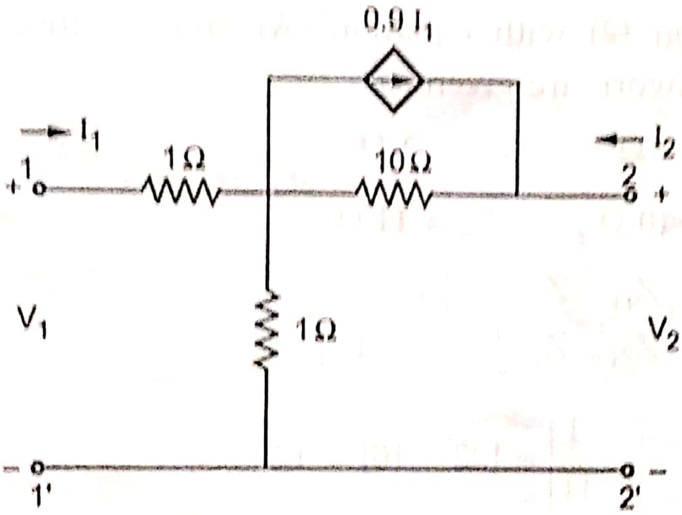


Fig. 8.30

Solution : Converting current source to its equivalent voltage source.

The value of voltage source is given by

$$V' = (0.9\ I_1)\ (10) = 9\ I_1\ V \qquad \dots (1)$$

The network can be drawn as shown in the Fig. 8.30 (a)

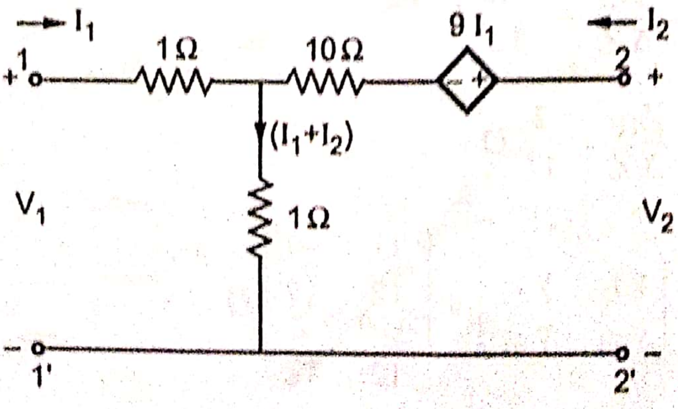


Fig. 8.30 (a)

Applying KVL to first loop,

$$1 \cdot I_1 + 1 \cdot (I_1 + I_2) = V_1$$

$$\therefore \quad 2I_1 + I_2 = V_1 \qquad \ldots (2)$$

Applying KVL to second loop,

$$\therefore \quad 9\, I_1 + 10\, I_2 + 1\, (I_1 + I_2) = V_2$$

$$\therefore \quad 9\, I_1 + 10\, I_2 + I_1 + I_2 = V_2$$

$$\therefore \quad 10\, I_1 + 11\, I_2 = V_2 \qquad \ldots (3)$$

But z-parameters are given by the equations,

$$Z_{11}\, I_1 + Z_{12}\, I_2 = V_1 \qquad \ldots (A)$$

$$Z_{21}\, I_1 + Z_{22}\, I_2 = V_2 \qquad \ldots (B)$$

Comparing equation (2) with equation (A) and equation (3) with equation (B), z-parameters of the network are given as,

$$Z_{11} = 2\,\Omega\,; \qquad Z_{12} = 1\,\Omega$$

$$Z_{21} = 10\,\Omega\,; \qquad Z_{22} = 11\,\Omega$$

$$\therefore \quad [Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 10 & 11 \end{bmatrix}$$

$$\therefore \quad \Delta Z = \begin{vmatrix} 2 & 1 \\ 10 & 11 \end{vmatrix} = [22 - 10] = 12$$

Thus, by using z to y parameters conversion formulae, y-parameters are given as follows,

$$y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{11}{12}\ \mho$$

$$y_{12} = \frac{-Z_{12}}{\Delta Z} = \frac{-1}{12}\ \mho$$

$$y_{21} = \frac{-Z_{21}}{\Delta Z} = \frac{-10}{12}\ \mho$$

$$y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{2}{12}\ \mho$$

$$\therefore \quad [y] = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} \frac{11}{12} & \frac{-1}{12} \\ \frac{-10}{12} & \frac{2}{12} \end{bmatrix} \mho$$

Example 8.23 : *The network shown in the Fig. 8.31 contains a voltage controlled source and current controlled source. For the element values specified determine y and z parameters.*

(Jan./Feb.-2006, July/Aug.-2006)

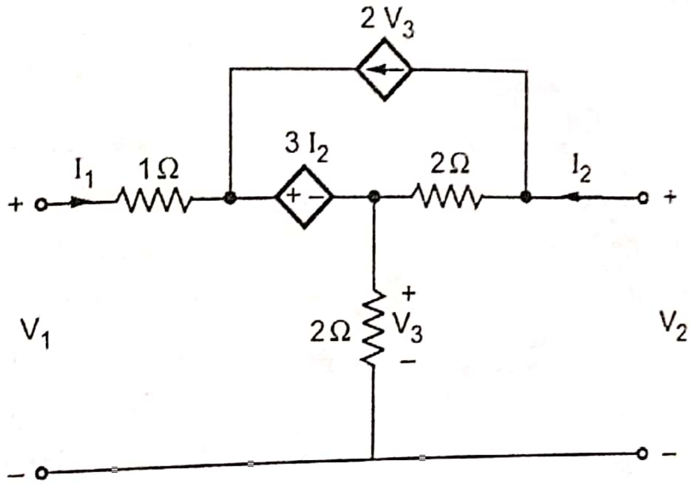


Fig. 8.31

Solution : 1) z-Parameters

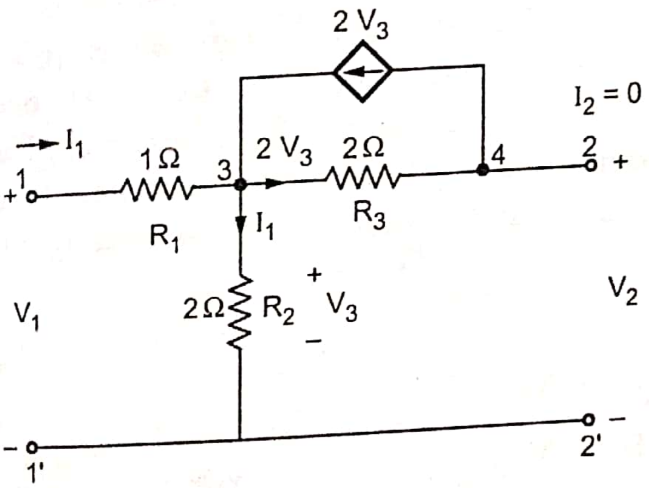


Fig. 8.31 (a)

i) To determine Z_{11}, Z_{21}, open circuit terminals 2-2′ by making $I_2 = 0$. With $I_2 = 0$, dependent voltage source $3I_2$ will be shorted. The network can be drawn as shown in the Fig. 8.31 (a).

As $I_2 = 0$, at node 4, current entering node 4 must be $2V_3$ as it is moving away from node 4 as dependent current source. So current of dependent source flows only through R_3 and the current I_1 flows through R_1 and R_2.

Applying KVL to loop 1-3-1′-1, ... (1)

$$I_1 \cdot R_1 + I_1 \cdot R_2 = V_1$$

$$\therefore \quad I_1 (1 + 2) = V_1$$

$$\therefore \quad Z_{11} = \frac{V_1}{I_1} = 3\ \Omega$$

Applying KVL to loop 2 - 4 - 3 - 2′ - 2, ... (2)

$$-R_3 \cdot 2V_3 + R_2 \cdot I_1 = V_2$$

But $$V_3 = I_1 \cdot R_2 = 2I_1$$

$$\therefore \quad -2 \cdot 2\,(2I_1) + 2\,I_1 = V_2$$

$$\therefore \quad -8I_1 + 2I_1 = V_2$$

$$\therefore \quad -6I_1 = V_2$$

$$\therefore \quad Z_{21} = \frac{V_2}{I_1} = -6\ \Omega$$

ii) To determine Z_{22} and Z_{12}, open circuit terminals 1-1′ by making $I_1 = 0$. As terminals 1-1′ are open, and resistor 1Ω is connected to open terminal, no current can flow through 1 Ω resistor. So we can neglect it. The network can be drawn as shown in the Fig. 8.31 (b).

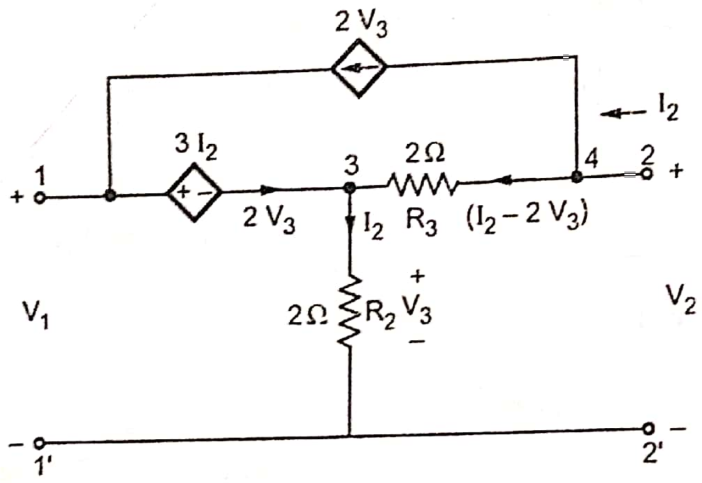


Fig. 8.31 (b)

At node 4, I_2 is entering while $2V_3$ is leaving node. Hence current flowing through R_3 is $(I_2 - 2V_3)$ according to KCL. At node 1, current 2 V_3 is entering. There is no other branch at node 1, so the current flowing through the dependent voltage source must be 2 V_3. Thus, the current flowing through R_2 is I_2 according to KCL applied at node 3.

Applying KVL to loop 2-4-3-2′-2,

$$\therefore \quad (I_2 - 2V_3) \cdot R_3 + R_2 \cdot I_2 = V_2 \quad \ldots (4)$$

$$\therefore \quad (I_2 - 2V_3) \cdot 2 + 2\,I_2 = V_2$$

But $$V_3 = 2I_2 \quad \ldots (5)$$

Substituting value of V_3 in above equation,

$$[I_2 - 2\,(2I_2)] \cdot 2 + 2\,I_2 = V_2$$

$$\therefore \quad 2\,(I_2 - 4I_2) + 2I_2 = V_2$$

$$\therefore \quad -6I_2 + 2I_2 = V_2$$

$$\therefore \quad -4\,I_2 = V_2$$

$$\therefore \quad Z_{22} = \frac{V_2}{I_2} = -4\ \Omega$$

Applying KVL to loop 1-3-1′-1,

$$3I_2 + 2I_2 = V_1$$

$$5I_2 = V_1 \quad \text{... (6)}$$

$$\therefore \quad Z_{12} = \frac{V_1}{I_2} = 5\ \Omega$$

Thus, z-parameters of the network are,

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ -6 & -4 \end{bmatrix} \Omega$$

y-Parameters :

For the given network, y-parameters in terms of z-parameters are given by,

$$y_{11} = \frac{Z_{22}}{\Delta Z}, \ y_{12} = \frac{-Z_{12}}{\Delta Z}$$

$$y_{21} = \frac{-Z_{21}}{\Delta Z}, \ y_{22} = \frac{Z_{11}}{\Delta Z}$$

The z-parameter matrix is given by

$$[Z] = \begin{bmatrix} 3 & 5 \\ -6 & -4 \end{bmatrix}$$

$$\therefore \quad \Delta Z = \begin{vmatrix} 3 & 5 \\ -6 & 4 \end{vmatrix} = -12 + 30 = 18$$

$$\therefore \quad y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{-4}{18} = \frac{-2}{9}\ \mho$$

$$y_{12} = \frac{-Z_{12}}{\Delta Z} = \frac{-5}{18}\ \mho$$

$$y_{21} = \frac{-Z_{21}}{\Delta Z} = \frac{-(-6)}{18} = \frac{1}{3}\ \mho$$

$$y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{3}{18} = \frac{1}{6}\ \mho$$

So y-parameters of given network are,

$$[y] = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} \frac{-2}{9} & \frac{-5}{18} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix} \mho$$

Example 8.24 : *Determine T parameters of the network shown in the Fig. 8.32.*

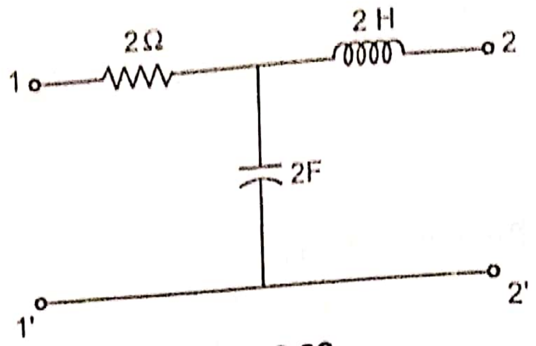


Fig. 8.32

Solution : Transforming the given network in s-domain as shown in the Fig. 8.32 (a).

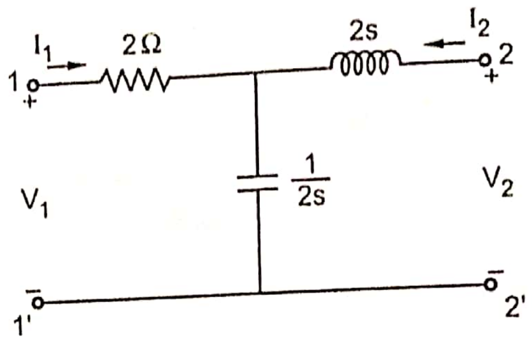


Fig. 8.32 (a)

1) Let $I_2 = 0$, i.e. output port is open circuited.

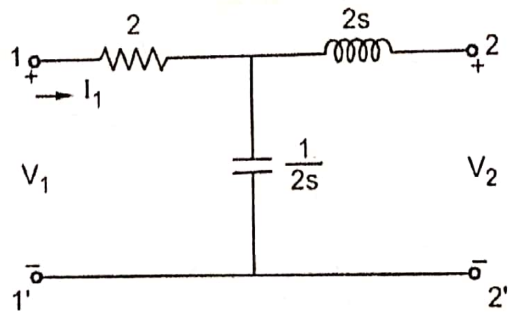


Fig. 8.32 (b)

Then the equations of the transmission parameter are given by

$V_1 = A\, V_2$ and

$I_1 = C\, V_2$

Consider two port network with $I_2 = 0$, as shown in the Fig. 8.32 (b).

Applying KVL at the input side, we get,

$$-(2)\, I_1 - \left(\frac{1}{2s}\right) I_1 + V_1 = 0$$

$$\therefore \quad 2\, I_1 + \frac{1}{2s}\, I_1 = V_1$$

$$\therefore \quad \frac{4s+1}{2s}\, I_1 = V_1 \qquad \text{... (1)}$$

$$\therefore \quad I_1 = \left(\frac{2s}{4s+1}\right) V_1 \qquad \text{... (2)}$$

But
$$V_2 = \left(\frac{1}{2s}\right) I_1$$

Putting value of I_1 from equation (2), we get,
$$V_2 = \left(\frac{1}{2s}\right)\left(\frac{2s}{4s+1}\right) V_1 = \frac{V_1}{(4s+1)}$$

$$\therefore \quad A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \mathbf{(4s + 1)}$$

From above equation we can write,
$$V_1 = (4s + 1)\, V_2$$

Putting this value in equation (2),
$$I_1 = \left(\frac{2s}{4s+1}\right) \cdot (4s + 1) \cdot V_2 = (2s)\, V_2$$

$$\therefore \quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \mathbf{(2s)\ \mho}$$

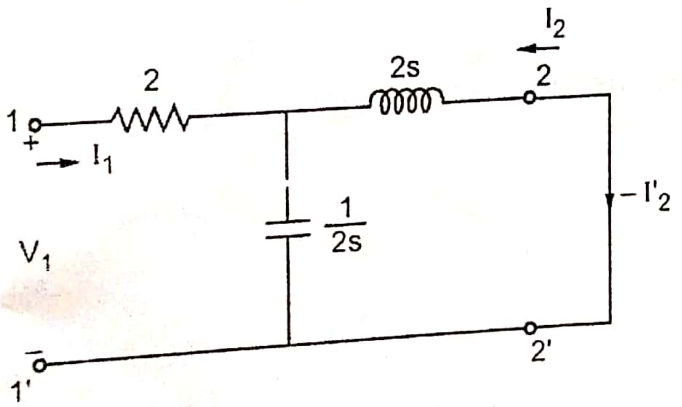


Fig. 8.32 (c)

2) Let $V_2 = 0$, i.e. output port short circuited as shown in the Fig. 8.32 (c).

Applying KVL to both the loops, we get two equations as follows,
$$V_1 = 2\, I_1 + \left(\frac{1}{2s}\right) I_1 + \left(\frac{1}{2s}\right) I_2$$

$$\therefore \quad V_1 = \left(2 + \frac{1}{2s}\right) I_1 + \left(\frac{1}{2s}\right) I_2$$

$$\therefore \quad V_1 = \left(\frac{4s+1}{2s}\right) I_1 + \left(\frac{1}{2s}\right) I_2 \qquad \ldots (3)$$

and
$$(2s)\, I_2 + \left(\frac{1}{2s}\right)(I_1 + I_2) = 0$$

$$\therefore \quad \left(2s + \frac{1}{2s}\right) I_2 + \left(\frac{1}{2s}\right) I_1 = 0$$

$$\therefore \quad \left(\frac{1}{2s}\right) I_1 = -\left(\frac{4s^2+1}{2s}\right) I_2$$

$$\therefore \quad I_1 = -(4s^2 + 1)\, I_2 \qquad \ldots \mathbf{(4)}$$

By definition,

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = (4s^2+1)$$

Putting value of I_1 in equation (3), we get,

$$V_1 = \frac{(4s+1)}{2s}\left[-(4s^2+1)\,I_2\right] + \frac{1}{2s}I_2$$

$$\therefore \quad V_1 = \left[\frac{-(16s^3+4s^2+4s+1)+1}{2s}\right]I_2$$

$$\therefore \quad V_1 = \frac{-16s^3-4s^2-4s-1+1}{2s}\cdot I_2$$

$$\therefore \quad V_1 = \frac{-16s^3-4s^2-4s}{2s}I_2$$

$$\therefore \quad V_1 = (8s^2+2s+2)(-I_2)$$

By definition,

$$B = \left.\frac{V_1}{(-I_2)}\right|_{V_2=0} = (8s^2+2s+2)\ \Omega$$

Thus, the transmission parameters for given network are

$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$= \begin{bmatrix} (4s+1) & (8s^2+2s+2) \\ (2s) & (4s^2+1) \end{bmatrix}$$

Example 8.25 : *Find z and y parameters of the network shown in the Fig. 8.33.*

(August-2002)

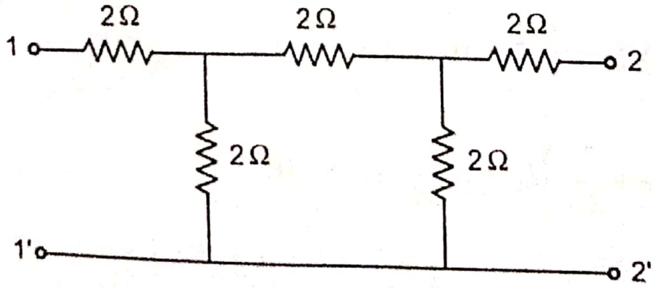


Fig. 8.33

Solution : i) To determine Z_{11} and Z_{21}, open circuit terminal 2-2[1]. Hence $I_2 = 0$. Assuming currents as shown in the Fig. 8.33 (a).

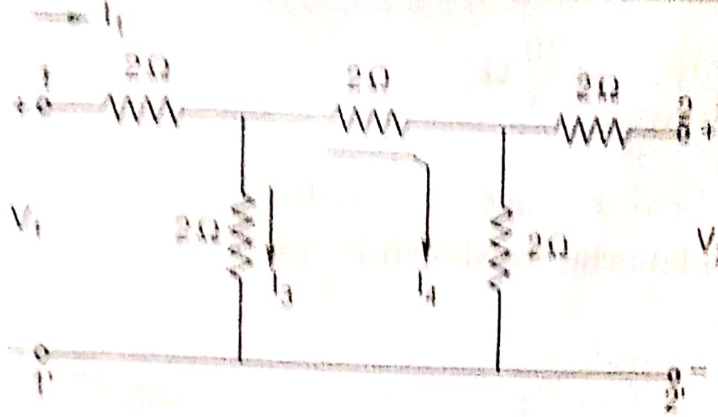


Fig. 8.33 (a)

By current divider rule, the currents I_3 and I_4 are given by,

$$I_3 = I_1\left[\frac{4}{4+2}\right] = \left(\frac{4}{6}\right)I_1 = \left(\frac{2}{3}\right)I_1 \qquad \dots (1)$$

and
$$I_4 = I_1\left[\frac{2}{4+2}\right] = \left(\frac{2}{6}\right)I_1 = \left(\frac{1}{3}\right)I_1 \qquad \dots (2)$$

But
$$I_4 = \frac{V_2}{2} \qquad \dots (3)$$

Putting value of I_4 from equation (2)

$$\left(\frac{1}{3}\right)I_1 = \frac{V_2}{2}$$

$$\therefore \quad \frac{V_2}{I_1} = \frac{2}{3}\,\Omega$$

Hence
$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{2}{3}\,\Omega$$

Applying KVL at input side,

$$-2I_1 - 2I_3 + V_1 = 0$$

$$\therefore \quad 2I_1 + 2I_3 = V_1$$

Putting value of I_3 from equation (1),

$$2I_1 + 2\left(\frac{2}{3}\right)I_1 = V_1$$

$$\therefore \quad I_1\left[2+\frac{4}{3}\right] = V_1$$

$$\therefore \quad \frac{V_1}{I_1} = \frac{10}{3}\,\Omega$$

Hence, $Z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0} = \dfrac{10}{3}\ \Omega$

ii) To determine Z_{12} and Z_{22}, open circuit terminals 1 – 1' i.e. $I_1 = 0$. Assuming currents through various branches as shown in the Fig. 8.33 (b).

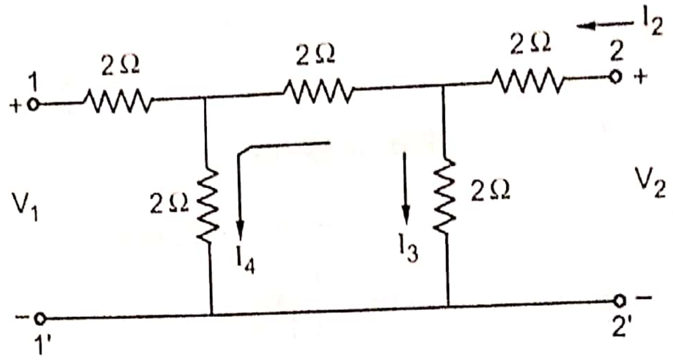


Fig. 8.33 (b)

By current divider rule,

$$I_3 = \left[\frac{4}{4+2}\right] I_2 = \left(\frac{4}{6}\right) I_2 = \left(\frac{2}{3}\right) I_2 \qquad \ldots (4)$$

and

$$I_4 = \left[\frac{2}{4+2}\right] I_2 = \left(\frac{2}{6}\right) I_2 = \left(\frac{1}{3}\right) I_2 \qquad \ldots (5)$$

But from the circuit diagram,

$$I_4 = \frac{V_1}{2} \qquad \ldots (6)$$

Comparing equations (5) and (6),

$$\frac{V_1}{2} = \frac{1}{3} I_2$$

$$\therefore \quad \frac{V_1}{I_2} = \frac{2}{3}$$

Hence $Z_{12} = \left.\dfrac{V_1}{I_2}\right|_{I_1=0} = \dfrac{2}{3}\ \Omega$

Applying KVL at output side,

$$-2I_2 - 2I_3 + V_2 = 0$$

$$\therefore \quad 2I_2 + 2I_3 = V_2$$

Putting value of I_3 from equation (4),

$$2I_2 + 2\left(\frac{2}{3}\right) I_2 = V_2$$

$$I_2\left[2+\frac{4}{3}\right] = V_2$$

$$\frac{V_2}{I_2} = \frac{10}{3}\ \Omega$$

...ence the z-parameter matrix is given by,

$$[Z] = \begin{bmatrix} \frac{10}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{10}{3} \end{bmatrix}$$

...or the calculation of y parameters, let us calculate ΔZ.

$$\Delta Z = \left(\frac{10}{3}\right)\left(\frac{10}{3}\right)-\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{100}{9}-\frac{4}{9} = \frac{96}{9} = \frac{32}{3}$$

Hence y-parameters are given by,

$$y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{\left(\frac{10}{3}\right)}{\left(\frac{32}{3}\right)} = \frac{10}{32} = \frac{5}{16}\ \mho$$

$$y_{12} = \frac{-Z_{12}}{\Delta Z} = \frac{-\left(\frac{2}{3}\right)}{\left(\frac{32}{3}\right)} = \frac{-2}{32} = \frac{-1}{16}\ \mho$$

$$y_{21} = \frac{-Z_{21}}{\Delta Z} = \frac{-\left(\frac{2}{3}\right)}{\left(\frac{32}{3}\right)} = \frac{-2}{32} = \frac{-1}{16}\ \mho$$

$$y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{\left(\frac{10}{3}\right)}{\left(\frac{32}{3}\right)} = \frac{10}{32} = \frac{5}{16}\ \mho$$

Hence y-parameter matrix for the given network is,

$$[y] = \begin{bmatrix} \frac{5}{16} & -\frac{1}{16} \\ -\frac{1}{16} & \frac{5}{16} \end{bmatrix}$$

Example 8.26 : *Find the transmission parameters for the network shown in the Fig. 8.34 and then obtain the z-parameters from them.* **(August-2001)**

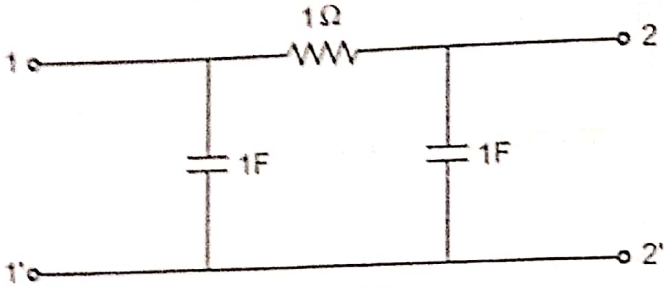


Fig. 8.34

Solution : Transforming given network in s-domain,

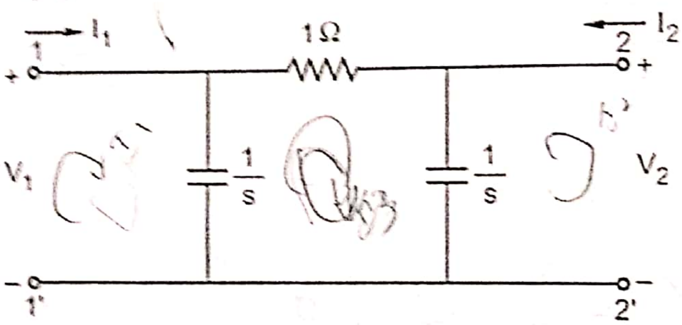


Fig. 8.34 (a)

By definitions, transmission parameters are given by,

$$V_1 = A\, V_2 + B\,(-I_2) \qquad \dots (1)$$

$$I_1 = C\, V_2 + D\,(-I_2) \qquad \dots (2)$$

A] Let $I_2 = 0$, i.e. open circuiting port 2.

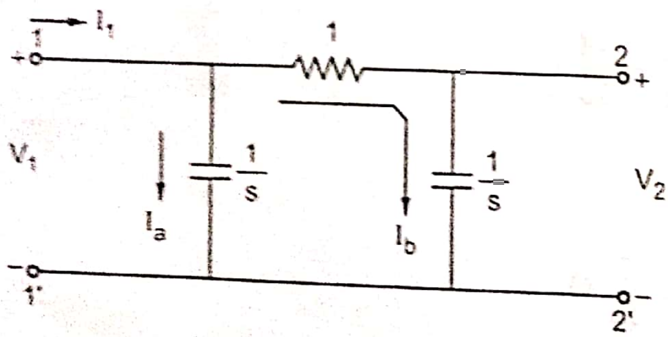


Fig. 8.34 (b)

Hence the network can be drawn with assumed branch currents as shown in the Fig. 8.34 (b).

Applying KCL,

$$I_1 = I_a + I_b$$

But $I_b = \dfrac{V_2}{\left(\frac{1}{s}\right)} = sV_2$

and

$$I_a = \frac{V_1}{\left(\frac{1}{s}\right)} = sV_1$$

Also

$$I_b = \frac{V_1 - V_2}{1}$$

Equating two equations,

$$\therefore \quad \frac{V_1 - V_2}{1} = sV_2$$

$$\therefore \quad V_1 = (s+1)\,V_2$$

$$\therefore \quad \frac{V_1}{V_2} = (s+1)$$

Hence $$A = \left.\frac{V_1}{V_2}\right|_{-I_2=0} = (s+1)$$

$$I_a + I_b = I_1$$

$$\therefore \quad sV_1 + sV_2 = I_1$$

$$\therefore \quad s\,(s+1)\,V_2 + sV_2 = I_1$$

$$\therefore \quad (s^2 + s + s)\,V_2 = I_1$$

$$\therefore \quad \frac{I_1}{V_2} = s\,(s+2)$$

Hence $$C = \left.\frac{I_1}{V_2}\right|_{-I_2=0} = s\,(s+1)\ \mho$$

B] Let $V_2 = 0$, i.e. short circuiting port 2. Hence capacitor at output side gets short ircuited. Hence the network can be drawn as shown in the Fig. 8.34 (c).

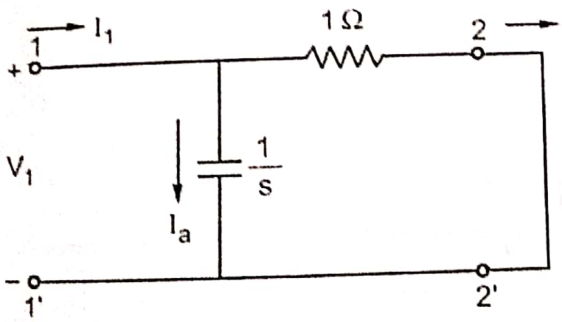


Fig. 8.34 (c)

By current divider rule,

$$-I_2 = I_1\left[\frac{\frac{1}{s}}{\frac{1}{s}+1}\right]$$

$$\therefore \quad -I_2 = I_1\left[\frac{1}{s+1}\right]$$

$$\therefore \quad \frac{I_1}{-I_2} = (s+1)$$

Hence $$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = (s+1)$$

By KCL $$I_a = I_1 - (-I_2) = I_1 + I_2$$

Also $$I_a = \frac{V_1}{\left(\frac{1}{s}\right)} = sV_1$$

Comparing equations and putting value of I_1 interms of $-I_2$,

$$-I_2(s+1)+I_2 = s V_1$$

$\therefore$
$$-I_2\{s+1-1\} = s V_1$$

Hence
$$\frac{V_1}{-I_2} = 1$$

Thus,
$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = 1\ \Omega$$

Hence transmission parameter matrix is given by

$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} (s+1) & 1 \\ s(s+2) & (s+1) \end{bmatrix}$$

$\therefore$
$$\Delta_T = (s+1)(s+1)-(1)(s)(s+2) = s^2+2s+1-s^2-2s$$
$$= 1$$

Hence Z-parameters are given by

$$Z_{11} = \frac{A}{C} = \frac{(s+1)}{s(s+2)}\ \Omega$$

$$Z_{12} = \frac{\Delta_T}{C} = \frac{1}{s(s+2)}\ \Omega$$

$$Z_{21} = \frac{1}{C} = \frac{1}{s(s+2)}\ \Omega$$

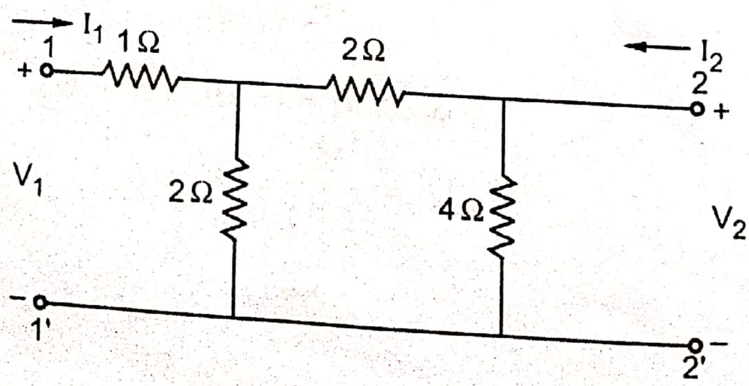
$$Z_{22} = \frac{D}{C} = \frac{(s+1)}{s(s+2)}\ \Omega$$

Hence Z-parameter matrix is given by

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{s+1}{s(s+2)} & \frac{1}{s(s+2)} \\ \frac{1}{s(s+2)} & \frac{s+1}{s(s+2)} \end{bmatrix}$$

Example 8.27 : *Find the h-parameters of the network shown in the Fig. 8.35.*

(Jan./Feb.-2004)



olution : By definition h-parameters are given by

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

(A) Let $V_2 = 0$ i.e. short circuiting port 2. Hence 4 Ω gets directly short circuited. he network can be redrawn as shown in the Fig. 8.35 (a).

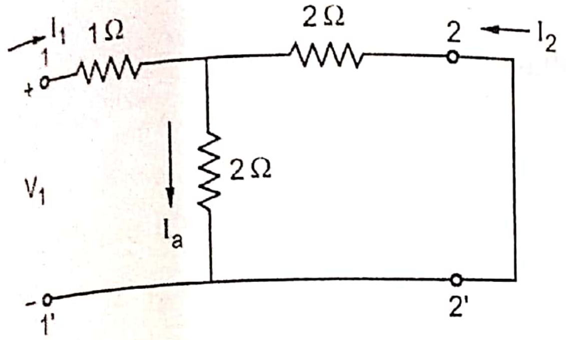


Fig. 8.35 (a)

By KCL $\quad I_a = I_1 + I_2$

By current divider rule,

$$I_2 = -I_1\left[\frac{2}{2+2}\right] = \frac{-I_1}{2}$$

$$\therefore \quad \frac{I_2}{I_1} = -\frac{1}{2}$$

Hence $\quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -\frac{1}{2}$

Applying KVL at the input side,

$$V_1 = I_1 + (I_a)(2) = I_1 + 2(I_1 + I_2)$$

$$\therefore \quad V_1 = 3I_1 + 2\left[-\frac{1}{2}I_1\right]$$

$$\therefore \quad V_1 = 3I_1 - I_1 = 2I_1$$

$$\therefore \quad h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = 2\ \Omega$$

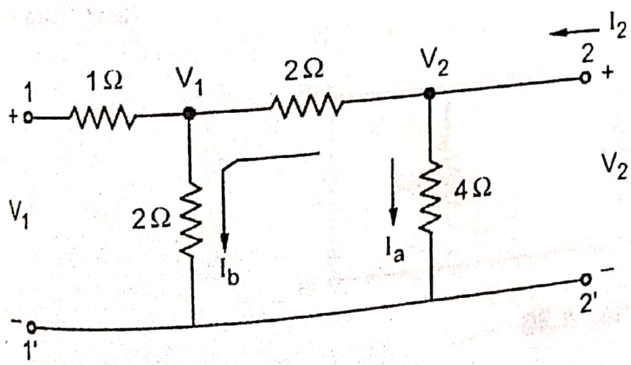


Fig. 8.35 (b)

(B) Let $I_1 = 0$, i.e. open circuiting port 1, we can draw network as shown in the Fig. 8.35 (b)

By KCL, $I_2 = I_a + I_b$

But $I_a = \frac{V_2}{4}$ and

$$I_b = \frac{V_1}{2}$$

But current I_b is also given by

$$I_b = \frac{V_2 - V_1}{2}$$

Equating equations for current I_b,

$$\frac{V_2 - V_1}{2} = \frac{V_1}{2}$$

$$\therefore \qquad V_2 = 2V_1$$

Hence $$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{1}{2}$$

Putting values of I_a and I_b in terms of voltages,

$$I_2 = \frac{V_2}{4} + \frac{V_1}{2}$$

But $$V_1 = 1/2\ V_2$$

$$\therefore \qquad I_2 = \frac{V_2}{4} + \frac{\left(\frac{V_2}{2}\right)}{2} = \frac{V_2}{4} + \frac{V_2}{4} = \frac{V_2}{2}$$

Hence $$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{2}\text{ mho}$$

Hence h-parameter matrix is given by,

$$[h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 2 & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Example 8.28 : *Find y-parameters for the network shown in Fig. 8.36.*

(July/Aug.-2004)

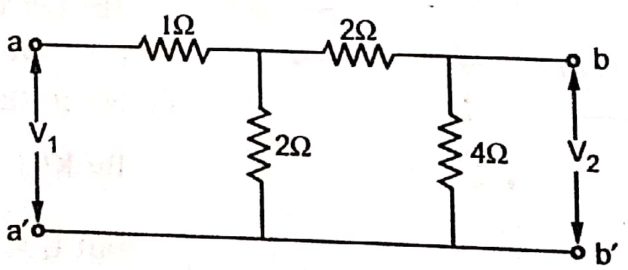


Fig. 8.36

Solution : By definition, y-parameters are given by,

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

(A) Let $V_2 = 0$ i.e. port 2 (b – b') short circuited. Hence 4 Ω gets directly short circuited as shown in the Fig. 8.36 (a).

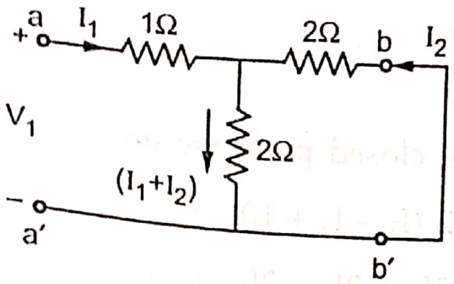


Fig. 8.36 (a)

By current divider rule,

$$I_2 = -I_1\left(\frac{2}{2+2}\right) = -\frac{I_1}{2} \quad \text{... (i)}$$

Applying KVL to the first closed path, we get,

$$-I_1 - 2(I_1 + I_2) + V_1 = 0$$

$$\therefore \quad -3I_1 - 2I_2 = -V_1$$

$$\therefore \quad 3I_1 + 2I_2 = V_1$$

But $\quad I_2 = -I_1/2,$

$$\therefore \quad 3I_1 + 2\left(-\frac{I_1}{2}\right) = V_1$$

$$\therefore \quad 3I_1 - I_1 = V_1$$

$$\therefore \quad 2I_1 = V_1 \quad \text{... (ii)}$$

$$\therefore \quad y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{2}\ \mho$$

Substituting value of I_1 interms of I_2 in equation (ii), we get,

$$2(-2I_2) = V_1$$

$$\therefore \quad -4I_2 = V_1$$

$$\therefore \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{1}{4}\ \mho$$

(B) Let $V_1 = 0$ i.e. port 1 (a − a′) short circuited assuming different currents as shown in the Fig. 8.36 (b).

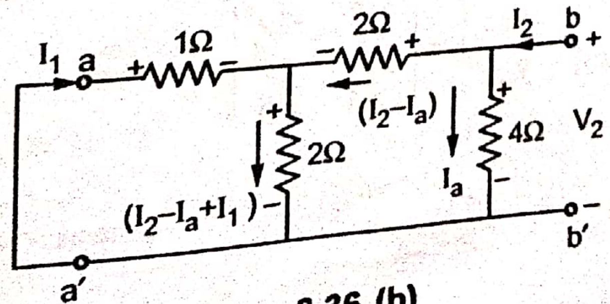


Fig. 8.36 (b)

From the circuit diagram,

$$V_2 = 4I_a \quad \text{... (iii)}$$

Applying KVL to the first closed path, we get,

$$-I_1 - 2(I_2 - I_a + I_1) = 0$$

$$\therefore \quad -I_1 - 2I_2 + 2I_a - 2I_1 = 0$$

$$\therefore \quad -3I_1 - 2I_2 + 2\left(\frac{V_2}{4}\right) = 0$$

$$\therefore \quad -3I_1 - 2I_2 = -\frac{V_2}{2}$$

$$\therefore \quad 6I_1 + 4I_2 = V_2 \quad \text{... (iv)}$$

Applying KVL to the middle closed path, we get

$$-2(I_2 - I_a) - 2(I_2 - I_a + I_1) + 4I_a = 0$$

$$\therefore \quad -2I_2 + 2I_a - 2I_2 + 2I_a - 2I_1 + 4I_a = 0$$

$$-2I_1 - 4I_2 + 8I_a = 0$$

$$\therefore \quad -2I_1 - 4I_2 = -8\left(\frac{V_2}{4}\right)$$

$$\therefore \quad 2I_1 + 4I_2 = 2V_2 \quad \text{... (v)}$$

Solving equations (iv) and (v) for I_1, we get

$$4I_1 = -V_2$$

$$\therefore \quad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0} = -\frac{1}{4}\ \mho$$

Similarly solving (iv) and (v) for I_2, we get,

$$I_2 = \frac{5}{8} V_2$$

$$\therefore \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1 = 0} = \frac{5}{8}\ \mho$$

Hence y-parameter matrix is given by,

$$[y] = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} 1/2 & -1/4 \\ -1/4 & 5/8 \end{bmatrix}$$

Example 8.29 : *Find the transmission or general parameters for the circuit shown in Fig. 8.37.* **(July/Aug.-2004)**

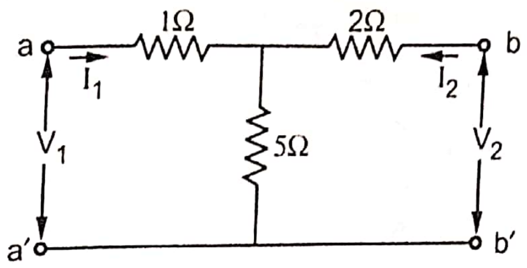


Fig. 8.37

Solution : By definition, transmission parameters are given by,

$$V_1 = A\,V_2 + B(-I_2)$$

$$I_1 = (V_2 + D\,(-I_2)$$

A) Let $-I_2 = 0$ i.e. open circuit terminals b – b′ (i.e. port 2) as shown in the Fig. 8.37 (a).

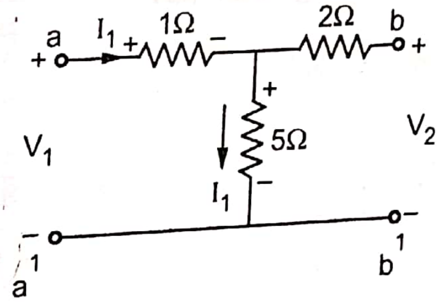


Fig. 8.37 (a)

From circuit drawn above, we can write,

$$V_2 = 5I_1 \quad \text{... (i)}$$

$$\therefore \quad C = \left.\frac{I_1}{V_2}\right|_{-I_2=0} = \frac{1}{5}\ \mho$$

Applying KVL at the input side, we get,

$$-I_1 - 5I_1 + V_1 = 0 \quad \text{... (ii)}$$

$$\therefore \quad 6I_1 = V_1$$

From equation (i), ... (iii)

$$I_1 = \frac{V_2}{5}$$

Substituting value of I_1 in equation (ii), we get,

$$\therefore \quad 6\left(\frac{V_2}{5}\right) = V_1$$

$$\therefore \quad V_2 = \frac{5}{6} V_1$$

$$\therefore \quad A = \left.\frac{V_1}{V_2}\right|_{-I_2 = 0} = \frac{6}{5}$$

B) Let $V_2 = 0$ i.e. short circuit port 2 (terminals b - b′) as shown in the Fig. 8.37(b).

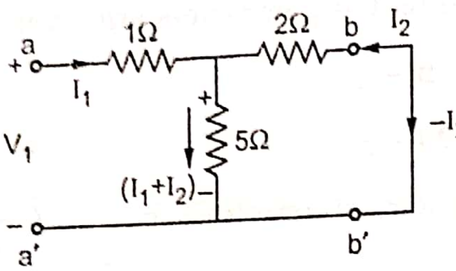


Fig. 8.37 (b)

Applying current divider rule, we get,

$$I_2 = -I_1\left[\frac{5}{2+5}\right]$$

$$\therefore \quad -I_2 = I_1\left(\frac{5}{7}\right)$$

$$\therefore \quad D = \left.\frac{I_1}{-I_2}\right|_{V_2 = 0} = \frac{7}{5}$$

Applying KVL at the input side, we get,

$$-I_1 - 5(I_1 + I_2) + V_1 = 0$$

$$\therefore \quad -6I_1 - 5I_2 = -V_1$$

$$\therefore \quad 6I_1 + 5I_2 = V_1$$

$$\therefore \quad 6\left(\frac{-7}{5} I_2\right) + 5I_2 = V_1$$

$$\therefore \quad \left(\frac{-42}{5} + 5\right) I_2 = V_1$$

$$\therefore \quad \frac{-17}{5} I_2 = V_1$$

$$\therefore \quad \left(\frac{17}{5}\right)(-I_2) = V_1$$

$$\therefore \quad B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = +\frac{17}{5}$$

Hence transmission parameters matrix is given by,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{17}{5} \\ \frac{1}{5} & \frac{7}{5} \end{bmatrix}$$

Example 8.30 : *Find [z] and [y] for the two port network shown in Fig. 8.38.*

(Jan./Feb.-2005, July/Aug.-2005)

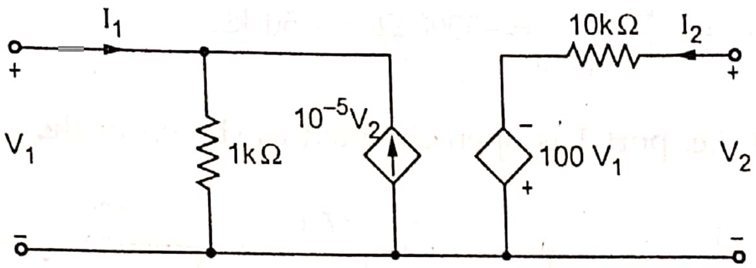


Fig. 8.38

Solution : By definition, z-parameters are given by,

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \text{...(1)}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{...(2)}$$

(A) Let $V_2 = 0$ i.e. port 2 open circuited as shown in the Fig. 8.38 (a).

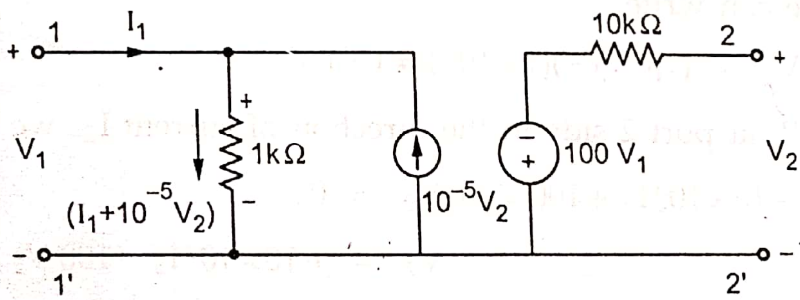


Fig. 8.38 (a)

From Fig. 8.38 (a), current through 1 kΩ is $(I_1 + 10^{-5} V_2)$. Hence we can write,

$$V_1 = 1 \times 10^3 [I_1 + 10^{-5} V_2] \quad \text{...(3)}$$

But at port 2, we can write,

$$V_2 = -100V_1 \quad \text{as no drop across 10 k}\Omega\text{ resistor} \quad \text{...(4)}$$

Substituting value of V_2 from equation (4) in equation (3), we get,

$$V_1 = 1\times10^3[I_1 + 10^{-5}(-100V_1)]$$

$$\therefore \quad V_1 = 1000\,I_1 + (-V_1)$$

$$\therefore \quad 2V_1 = 1000\,I_1$$

$$\therefore \quad V_1 = 500\,I_1 \qquad \ldots(5)$$

$$\therefore \quad Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 500\ \Omega$$

Substuting value of V_1 from equation (5), in equation (4), we get,

$$V_2 = -100(500\,I_1) = -50000 I_1$$

$$\therefore \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = -5000\ \Omega = -\ 50\ k\Omega$$

(B) Let $V_1 = 0$ i.e. port 1 is open circuited as shown in the Fig. 8.38 (b).

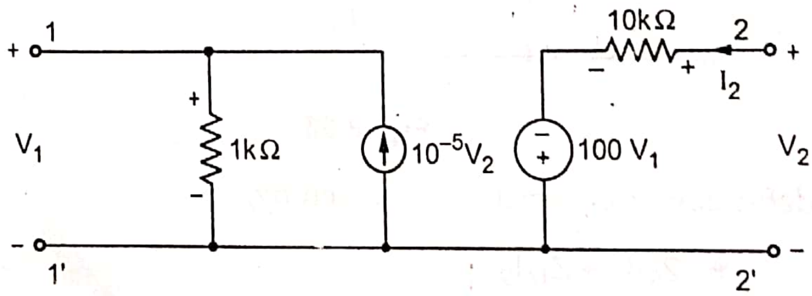


Fig. 8.38 (b)

At port 1, we can write,

$$V_1 = (10^{-5}\,V_2)(1\times10^3) = 0.01\,V_2 \qquad \ldots(6)$$

Applying KVL at port 2 side in the direction of current I_2, we get,

$$-10\times10^3 I_2 + 100\,V_1 + V_2 = 0$$

$$\therefore \quad V_2 = +10\times10^3 I_2 - 100\,V_1 \qquad \ldots(7)$$

Substituting value of V_1 from equation (6), we get,

$$V_2 = 10\times10^3 I_2 - 100(0.01)V_2$$

$$\therefore \quad 2V_2 = 10\times10^3 I_2$$

$$\therefore \quad V_2 = 5000 I_2 \qquad \ldots(8)$$

$$\therefore \quad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 5000\ \Omega$$

Substituting value of V_2 from equation (4) in equation (3), we get,

$$V_1 = 1\times10^3[I_1 + 10^{-5}(-100V_1)]$$

$$\therefore \quad V_1 = 1000\, I_1 + (-V_1)$$

$$\therefore \quad 2V_1 = 1000\, I_1$$

$$\therefore \quad V_1 = 500\, I_1 \quad \text{...(5)}$$

$$\therefore \quad Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 500\ \Omega$$

Substuting value of V_1 from equation (5), in equation (4), we get,

$$V_2 = -100(500\, I_1) = -50000 I_1$$

$$\therefore \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = -5000\ \Omega = -\ 50\ k\Omega$$

(B) Let $V_1 = 0$ i.e. port 1 is open circuited as shown in the Fig. 8.38 (b).

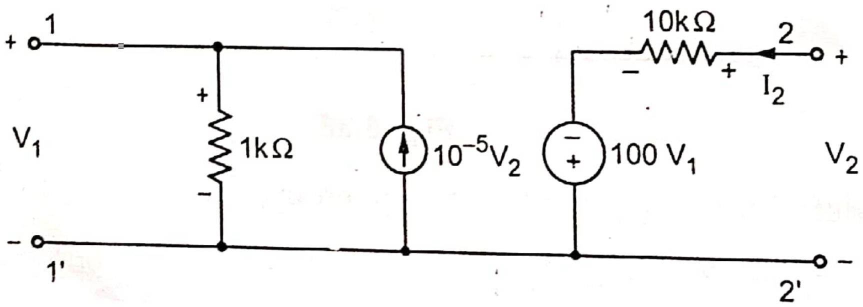


Fig. 8.38 (b)

At port 1, we can write,

$$V_1 = (10^{-5}\, V_2)(1\times10^3) = 0.01\, V_2 \quad \text{...(6)}$$

Applying KVL at port 2 side in the direction of current I_2, we get,

$$-10\times10^3 I_2 + 100\, V_1 + V_2 = 0$$

$$\therefore \quad V_2 = +10\times10^3 I_2 - 100\, V_1 \quad \text{...(7)}$$

Substituting value of V_1 from equation (6), we get,

$$V_2 = 10\times10^3 I_2 - 100(0.01)V_2$$

$$\therefore \quad 2V_2 = 10\times10^3 I_2$$

$$\therefore \quad V_2 = 5000 I_2 \quad \text{...(8)}$$

$$\therefore \quad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 5000\ \Omega$$

Substituting value of V_2 in equation (6), we get,

$$V_1 = 0.01(5000\, I_2) = 50 I_2$$

$$\therefore \quad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 50\ \Omega$$

Hence z-parameters for the two port network are as follows,

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 500 & 50 \\ -50000 & 5000 \end{bmatrix}$$

Example 8.31 : *Find the y-parameters for the circuit shown in Fig. 8.39. Then use the parameter relationship to find the ABCD parameters.* **(Jan./Feb.-2007)**

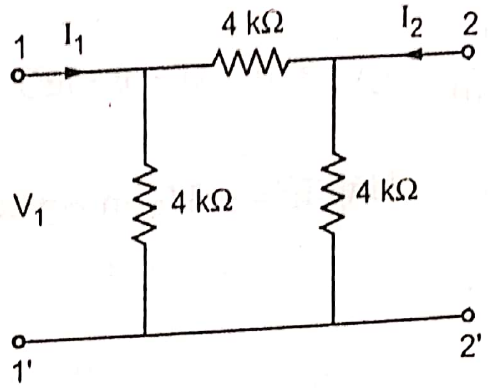


Fig. 8.39

Solution : By definition, y - parameters are given by,

$$I_1 = y_{11}\, v_1 + y_{12}\, V_2 \quad \text{... (A)}$$

$$I_2 = y_{21}\, v_1 + y_{22}\, V_2 \quad \text{... (B)}$$

[A] Let $v_2 = 0$ i.e. port - 2 short circuited. Due to short circuit 4 kΩ at port 2 gets shorted as shown in the Fig. 8.39 (b).

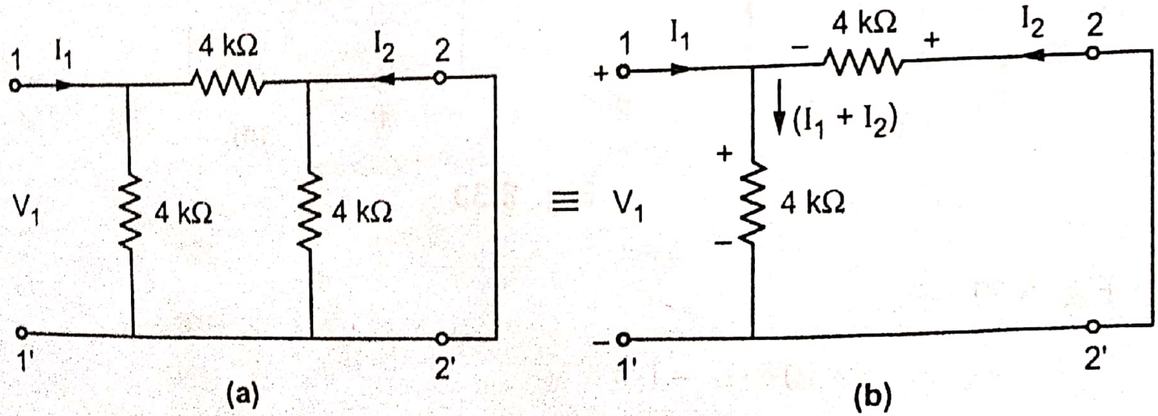


Fig. 8.39

From Fig. 8.39 (b)

$$v_1 = 4\times 10^3 \,(I_1 + I_2) \qquad \text{... (1)}$$

By current divider rule,

$$I_2 = -I_1 \left[\frac{4\times 10^3}{4\times 10^3 + 4\times 10^3}\right]$$

$$\therefore \quad I_2 = -\frac{1}{2} I_1 \qquad \text{... (2)}$$

Putting value of I_2 in equation (1), we get,

$$v_1 = 4\times 10^3 \left[I_1 - \frac{1}{2} I_1\right] = 4\times 10^3 \left[\frac{1}{2} I_1\right] = 2\times 10^3 \; I_1 \qquad \text{... (3)}$$

$$\therefore \; y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{2\times 10^3} = 0.5\times 10^{-3}\,\mho = 0.5 \text{ m}\mho$$

Similarly from euation (2) putting $I_1 = -2 I_2$ in equation (3) we get,

$$v_1 = 2\times 10^3 \,(-2 I_2) = -4\times 10^3 \; I_2$$

$$\therefore \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-1}{4\times 10^3} = -0.25\times 10^{-3}\mho = -0.25 \text{ m}\mho$$

[B] Let $v_1 = 0$ i.e. port - 1 short circuited. Due to short circuit, 4 kΩ at port - 1 gets shorted as shown in the Fig 8.39 (d).

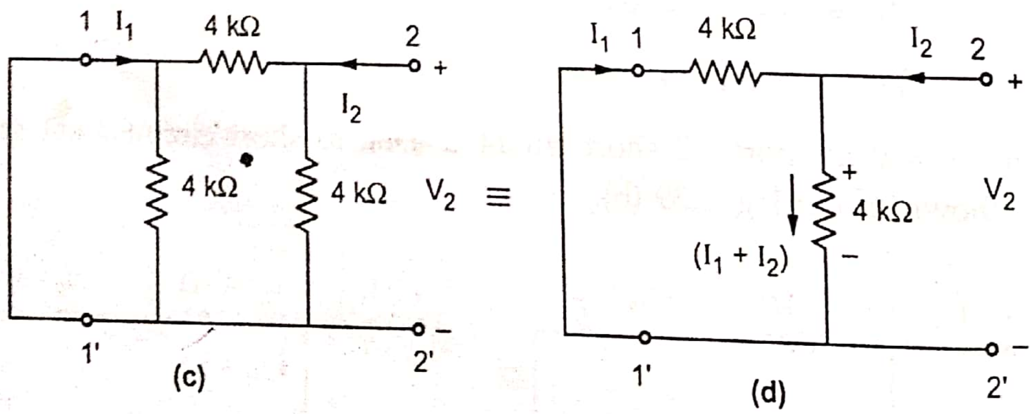


Fig. 8.39

From Fig. 8.39 (d),

$$v_2 = 4\times 10^3 \;(I_1 + I_2) \qquad \text{... (4)}$$

By current divider rule,

$$I_1 = -I_2 \left[\frac{4\times10^3}{4\times10^3 + 4\times10^3}\right] = -\frac{1}{2} I_2 \quad \text{... (5)}$$

Putting value of I_1 in equation (4), we get,

$$V_2 = 4\times10^3 \left(\frac{-1}{2} I_2 + I_2\right) = 4\times10^3 \left[\frac{1}{2} I_2\right] = 2\times10^3\ I_2 \quad \text{... (6)}$$

$$\therefore\ y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1}{2\times10^3} = 0.5\times10^{-3}\,\mho = -0.5\ \text{m}\mho$$

Putting $I_2 = -2 I_1$, from equation (5), in equation (6), we get,

$$V_2 = 2\times10^3 (-2 I_1) = -4\times10^3\ I_1$$

$$\therefore\ y_{12} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1}{4\times10^3} = -0.25\times10^{-3}\,\mho = -0.25\ \text{m}\mho$$

Thus y - parameter matrix is given by,

$$[y] = \begin{bmatrix} 0.5\times10^{-3} & -0.25\times10^{-3} \\ -0.25\times10^{-3} & 0.5\times10^{-3} \end{bmatrix}$$

Using conversion formulae, we can write,

$$A = \frac{-y_{22}}{y_{21}} = \frac{-0.5\times10^{-3}}{-0.25\times10^{-3}} = 2$$

$$B = \frac{-1}{y_{21}} = \frac{-1}{-0.25\times10^{-3}} = 4000$$

$$C = -\frac{\Delta y}{y_{21}} = \frac{-(y_{11}\ y_{22} - y_{21}\ y_{12})}{y_{21}}$$

$$= \frac{-[(0.5\times10^{-3}\times0.5\times10^{-3}) - (0.25\times10^{-3}\times0.25\times10^{-3})]}{-0.25\times10^{-3}}$$

$$= 0.75\times10^{-3}$$

$$D = \frac{-y_{11}}{y_{21}} = \frac{-0.5\times10^{-3}}{0.25\times10^{-3}} = 2$$

Hence A B C D parameter matrix is given by,

$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 2 & 4000 \\ 0.75\times10^{-3} & 2 \end{bmatrix}$$