MODULE-1 SEMICONDUCTORS

SYLLABUS

Semiconductors: Bonding forces in solids, Energy bands, Metals, Semiconductors and Insulators, Direct and Indirect semiconductors, Electrons and Holes, Intrinsic and Extrinsic materials, Conductivity and Mobility, Drift and Resistance, Effects of temperature and doping on mobility, Hall Effect. (Text 1: 3.1.1, 3.1.2, 3.1.3, 3.1.4, 3.2.1, 3.2.3, 3.2.4, 3.4.1, 3.4.2, 3.4.3, 3.4.5).

Contents

1. Bonding Forces in Solids

- Solids can be classified according to the nature of the bonding between their atomic or molecular components. The traditional classification distinguishes three kinds of bonding

1. Covalent bonding, which forms network covalent solids (sometimes called simply "covalent solids")

2. Ionic bonding, which forms ionic solids

3. Metallic bonding, which forms metallic solids

(a)
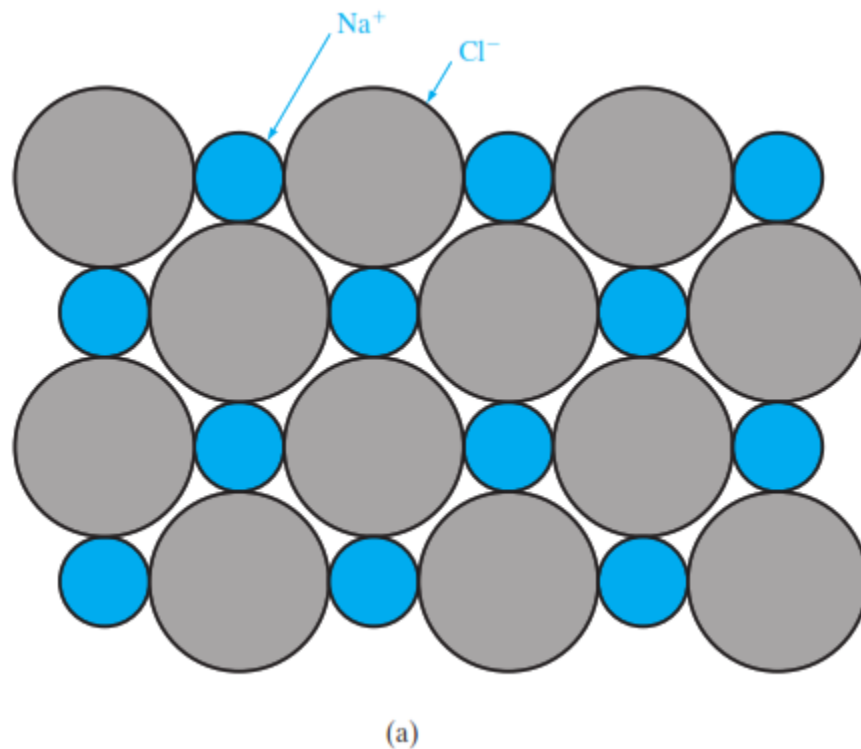
Figure 3–1
Different types of chemical bonding in solids: (a) an example of ionic bonding in NaCl; (b) covalent bonding in the Si crystal, viewed along a <100> direction (see also Figs. 1–8 and 1–9).

Two electrons per bond
(b)

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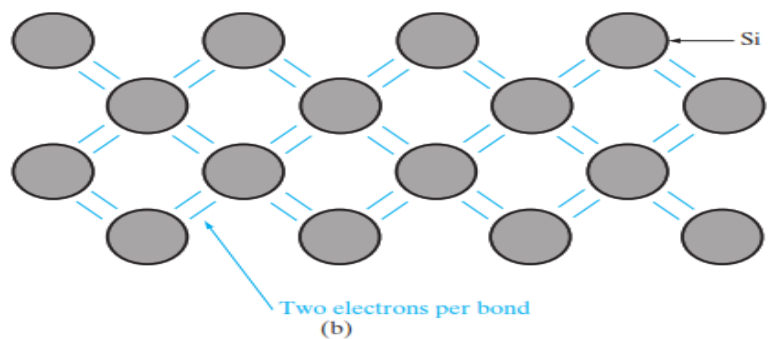
1.1   Ionic Bonding

- An ionic bond is the force of electrostatic attraction between positively charged ions (cations) and negatively charged ions (anions).

- Unlike charges attract, like charges repel.

- Many compounds of metals and non-metals are ionic and all Group I and Group II metal / non-metal compounds are ionic, e.g. NaCl, MgCl2, CaSO4, KNO3

Physical Properties of Ionic Compounds:

- **Hardness:** ionic bonds are strong and rigid and so ionic compounds tend to be hard, high melting point solids.

- **Brittleness:** unlike metallic bonds, ionic bonds are rigid and so break suddenly when enough force is applied – ionic solids tend to be brittle.

- **Electrical conductivity:** to conduct electricity we need mobile charge carriers. In metals these are the delocalised electrons. In ionic solids the ions are held in a rigid ionic lattice and are not free to move, so ionic solids are poor conductors of electricity. However, when molten (fused) ionic solids do conduct electricity.

Example: NaCl.

- Na (Z = 11) gives up its outermost shell electron to Cl (Z=17) atom, thus the crystal is made up of ions with the electronic structures of the inert atoms Ne and Ar.

- Note: the ions have net electric charges after the electron exchange$\rightarrow$ Na^+ion has a net positive charge, having lost an electron, and Cl^-ion has a net negative charge, having acquired an electron.

- Thus, an electrostatic attractive force is established, and the balance is reached when this equals the net repulsive force.

- Note: all the electrons are tightly bound to the atom.

- Since there are no loosely bound electrons to participate in current flow $\rightarrow$ NaCl is a good insulator.

1.2.   Metallic Bonding

- In metals, the outer shell is filled by no more than three electrons (loosely bound and given up easily)`=>`great chemical activity and high electrical conductivity.

- Outer electron(s) contributed to the crystal as a whole`=>` solid made up of ions with closed shells immersed in a sea of free electrons, which are free to move about the crystal under the influence of an electric field.

- Coulomb attraction force between the ions and the electrons hold the lattice together.

Physical Properties:

- **Hardness:** Depend on valance electron. If more hardness more

- **Brittleness:** can be bend without break.

- **Conductivity**: the delocalised electrons in metals are free to move within the metal and when electrodes are connected across a block of metal, these electrons move away from the negative electrode toward the positive electrode, carrying electric current with them. The electrons also conduct heat. Metals are *good conductors of electricity and heat*. Silver is the best electrically conducting metal.

1.3 Covalent Bonding

- that involves the sharing of electron pairs between atoms. These electron pairs are known as shared pairs or bonding pairs.

- Each atom shares its valence electrons with its four nearest neighbors.

- Bonding forces arise from a quantum mechanical interaction between the shared electrons.

Physical Properties of

- **Hardness**: covalent bonds are strong, in diamond each carbon makes 4 covalent bonds with neighbouring atoms, to break diamond all these bonds must be broken – diamond is *very hard and strong*.

- **Brittle:**, since the bonds are rigid and sufficient force will snap them suddenly. Diamond is one of the hardest known materials and makes excellent drills

- **Poor electrical conductivity**: the atoms are neutral and held together by rigid covalent bonds and the electrons are not free to move, so these materials are poor electrical conductors and *are insulators.*

Example diamond lattice silicon

- Each atom surrounded by four nearest neighbors, each having four electrons in the outermost orbit.

- Each atom shares its valence electrons with its four nearest neighbors.

- Bonding forces arise from a quantum mechanical interaction between the shared electrons.

- Both electrons belong to each bond, are indistinguishable, and have opposite spins.

- No free electrons available at 0 K, however, by thermal or optical excitation, electrons can be excited out of a covalent bond and can participate in current conduction => important feature of semiconductors.

Difference between each bond

Metallic Bonding	Ionic Bonding	Covalent Bonding
metal + metal	metal + non-metal	non-metal + non-metal
atoms release their electrons to become cations	atoms give off or accept electrons	atoms share electrons with other atoms
can be bended without breaking	easily breaks	fragility depends on state
can conduct electricity and heat	ability to conduct may depend on state	ability to conduct may depend on the atoms
high melting and boiling point	high melting and boiling point	low melting and boiling point

2. Energy Bands in Solids

- As isolated atoms are brought together to form a solid, the electron wave functions begin to overlap.

- Various interactions occur, and, at the proper interatomic spacing for the crystal, the forces of attraction and repulsion find a balance.

- Due to Pauli exclusion principle, the discrete energy levels of individual atoms split into bands belonging to the pair instead of to individual atoms.

- In a solid, due to large number of atoms, the split energy levels for essentially *continuous bands of energy*.

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- Imaginary formation of a diamond crystal from isolated carbon atoms $(1s^2 2s^2 2p^2)$.
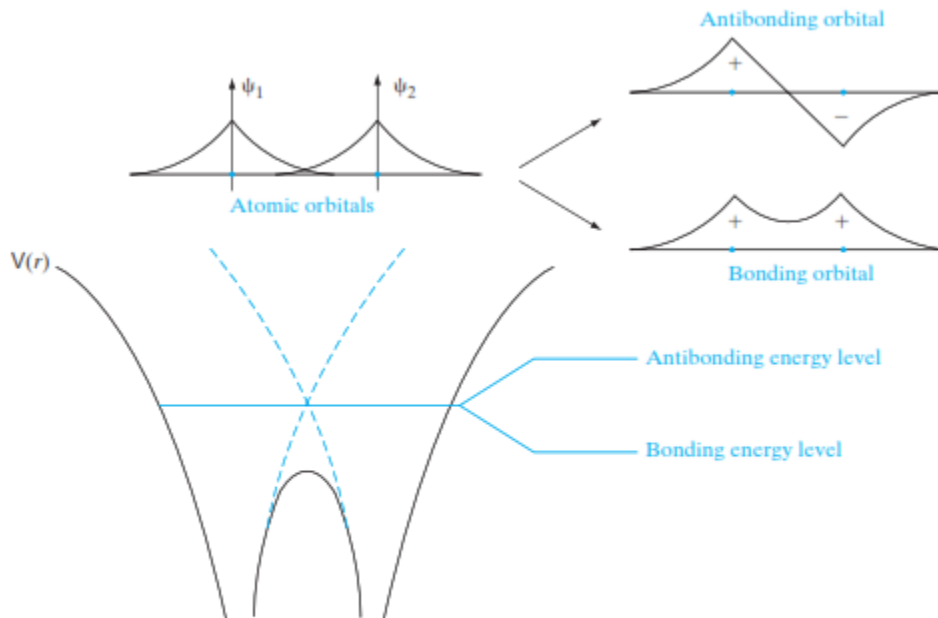
Figure 3–2
Linear combinations of atomic orbitals (LCAO): The LCAO when two atoms are brought together leads to two distinct "normal" modes—a higher energy antibonding orbital, and a lower energy bonding orbital. Note that the electron probability density is high in the region between the ion cores (covalent "bond"), leading to lowering of the bonding energy level and the cohesion of the crystal. If instead of two atoms, one brings together N atoms, there will be N distinct LCAO, and N closely spaced energy levels in a band.

- Each atom has two 1s states, two 2s states, six 2p states, and higher states.

- For N atoms, the numbers of states are 2N, 2N, and 6N of type 1s, 2s, and 2p respectively.

- With a reduction in the interatomic spacing, these energy levels split into bands, and the 2s and 2p bands merge into a single band having 8N available states.

- As the interatomic spacing approaches the equilibrium spacing of diamond crystal, this band splits into two bands separated by an energy gap E_g, where no allowed energy states for electrons exist => *forbidden gap*.

- The upper band (called the *conduction band*) and the lower band (called the *valence band*) contain 4N states each.
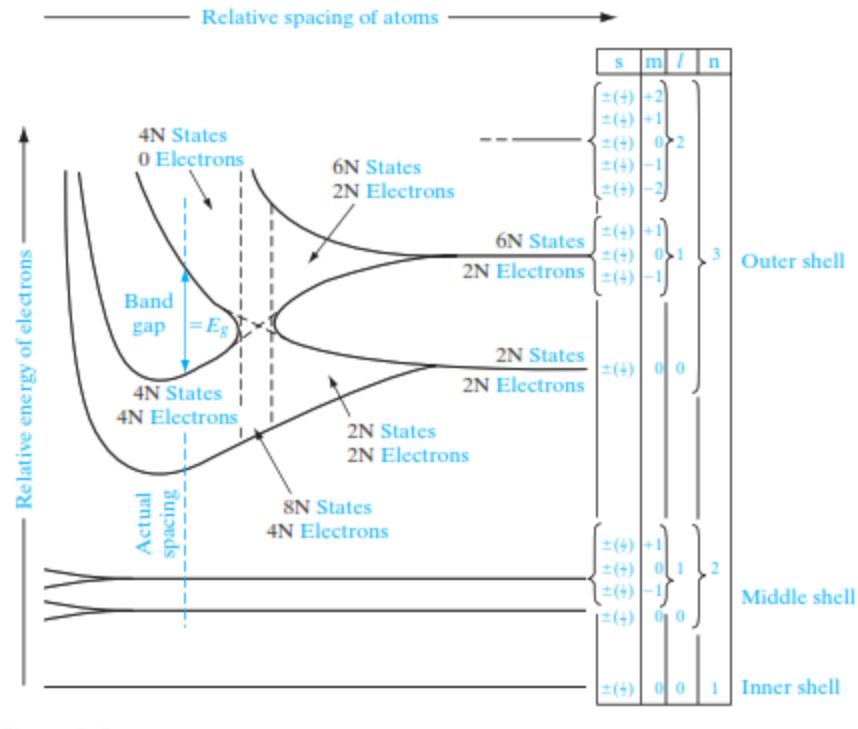
Fig.3.3 Splitting of individual energy levels to energy bands as atoms are brought closer together.

- The lower 1s band is filled with 2N electrons, however, the 4N electrons residing in the original n = 2 state will now occupy states either in the *valence band* or in the *conduction band*.

- At 0 K, the electrons will occupy the lowest energy states available to them => thus, the 4N states in the valence band will be completely filled, and the 4N states in the conduction band will be completely empty.

3. Metals, Semiconductors, and Insulators

- For electrons to move under an applied electric field, there must be states available to them.

- A completely filled band cannot contribute to current transport; neither can a completely empty band.

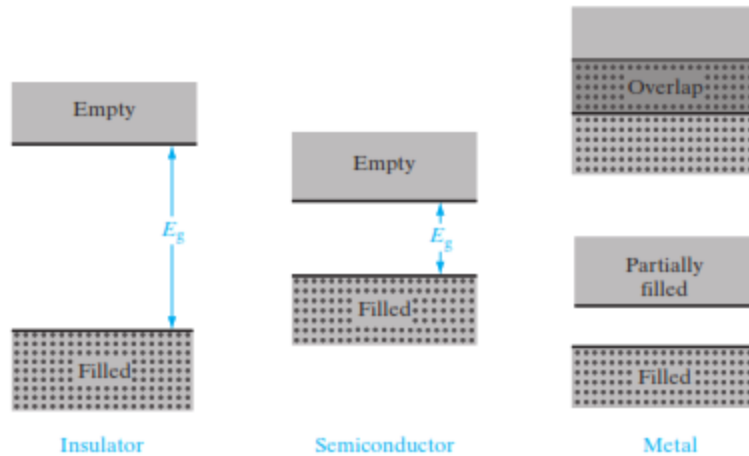- Thus, semiconductors at 0 K are perfect insulators.

Figure 3–4
Typical band
structures at 0 K.

- With thermal or optical excitation, some of these electrons can be excited from the valence band to the conduction band, and then they can contribute to the current transport process.

- At temperatures other than 0 K, the magnitude of the band gap separates an insulator from a semiconductor, e.g., at 300 K, E_g(diamond) = 5 eV (insulator), and E_g (Silicon) = 1.12 eV (semiconductor).

- Number of electrons available for conduction can be increased greatly in semiconductors by reasonable amount of thermal or optical energy.

- In metals, the bands are either partially filled or they overlap =>thus, electrons and empty states coexist=> great electrical conductivity.

4. Direct and Indirect Semiconductors

- In a typical quantitative calculation of band structures, the wave function of a single electron traveling through a perfectly periodic lattice is assumed to be in the form of a plane wave moving in the x-direction (say) with propagation constant k, also called a *wave vector*.

- In quantum mechanics, the electron momentum can be given by $p = \hbar k.$

- The space dependent wave function for the electron is
$$\psi_k(x) = U(k_x, x)e^{jk_x x} \quad (2.1)$$
where the function $U(k_x, x)$ modulates the wave function according to the periodicity of the lattice.

- Allowed values of energy, while plotted as a function of k, gives the E-k diagram.

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- Since the periodicity of most lattices is different in various directions, the E-k diagram is a complex surface, which is to be visualized in three dimensions.

Figure 3–5
Direct and indirect electron transitions in semiconductors: (a) direct transition with accompanying photon emission; (b) indirect transition via a defect level.
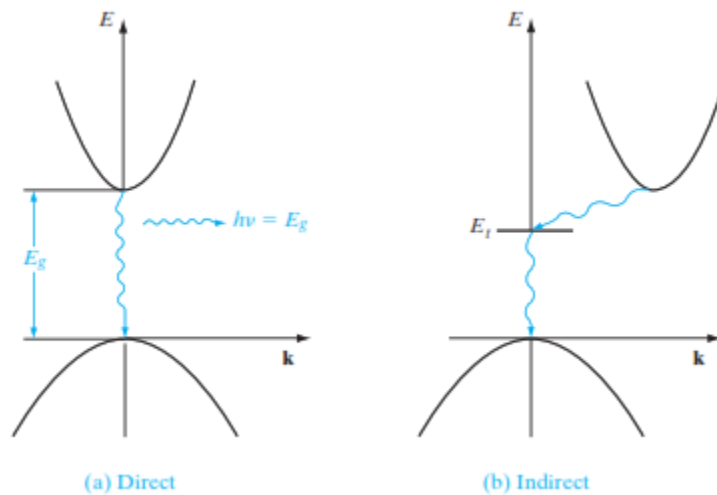
(a) Direct   (b) Indirect

- Direct band gap semiconductor: the minima of the conduction band and the maxima of the valence band occur at the same value of k =>an electron making the smallest energy transition from the conduction band to the valence band can do so without a change in k (and, the momentum).

- Indirect band gap semiconductor: the minima of the conduction band and the maxima of the valence band occur for different values of k, thus, the smallest energy transition for an electron requires a change in momentum.

- Electron falling from conduction band to an empty state in valence band => recombination.

- Recombination probability for direct band gap semiconductors is much higher than that for indirect band gap semiconductors.

- Direct band gap semiconductors give up the energy released during this transition (= E_g) in the form of light =>used for optoelectronic applications (e.g., LEDs and LASERs).

- Recombination in indirect band gap semiconductors occurs through some defect states within the band gap, and the energy is released in the form of heat given to the lattice.
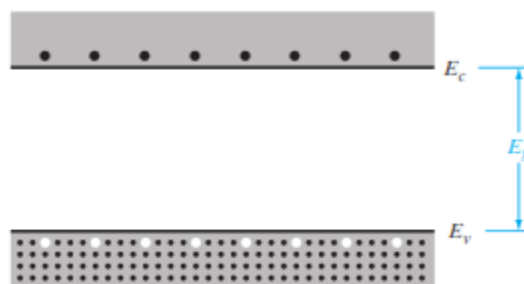
5. Charge Carriers in Semiconductors
- In a metal, the atoms are imbedded in a "sea" of free electrons, and these electrons can move as a group under the influence of an applied electric field.

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- In semiconductors at 0 K, all states in the valence band are full, and all states in the conduction band are empty.

- At T > 0 K, electrons get thermally excited from the valence band to the conduction band, and contribute to the conduction process in the conduction band.

- The empty states left in the valence band can also contribute to current conduction.

- Also, introduction of impurities has an important effect on the availability of the charge carriers.

- Considerable flexibility in controlling the electrical properties of semiconductors.

6. Electrons and Holes

- For T> 0 K, there would be some electrons in the otherwise empty conduction band, and some empty states in the otherwise filled valence band.

- The empty states in the valence band are referred to as holes.

- If the conduction band electron and the valence band hole are created by thermal excitation of a valence band electron to the conduction band, then they are called electron-hole pair (EHP).

- After excitation to the conduction band, an electron is surrounded by a large number of empty states, e.g., the equilibrium number of EHPs at 300 K in Si is $\sim 10^{10}/cm^3$, whereas the Si atom density is $\sim 10^{22}/cm^3$.

Figure 3–7
Electron–hole pairs in a semiconductor.

7. Energy Bands and Charge Carriers in Semiconductors

- Thus, the electrons in the conduction band are free to move about via the many available empty states.

- Corresponding problem of charge transport in the valence band is slightly more complex.

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- Current transport in the valence band can be accounted for by keeping track of the holes themselves.

- In a filled band, all available energy states are occupied.

- For every electron moving with a given velocity, there is an equal and opposite electron motion somewhere else in the band.

- Under an applied electric field, the net current is zero, since for every electron j moving with a velocity v_j, there is a corresponding electron j' moving with a velocity - v_j.

- In a unit volume, the current density J can be given by

$$J = (-q)\sum_{i}^{N} v_i = 0$$ (filled band) (2.2)

where N is the number of electrons/cm³ in the band, and q is the electronic charge.

- Now, if the jth electron is removed and a hole is created in the valence band, then the net current density

$$J = (-q)\sum_{i}^{N} v_i - (-q)v_j \qquad (j^{th}\ electron\ missing)$$

$$= (+q)v_j \qquad (since\ the\ first\ term\ is\ zero)$$

- Thus, the current contribution of the empty state (hole), obtained by removing the jth electron, is equivalent to that of a positively charged particle with velocity v_j.

- Note that actually this transport is accounted for by the motion of the uncompensated electron j' having a charge of q and moving with a velocity v_j.

- Its current contribution (- q)(- v_j) is equivalent to that of a positively charged particle with velocity + v_j.

- For simplicity, therefore, the empty states in the valence band are called holes, and they are assigned positive charge and positive mass.

- The electron energy increases as one moves up the conduction band, and electrons gravitate downward towards the bottom of the conduction band.

- On the other hand, hole energy increases as one moves down the valence band (since holes have positive charges), and holes gravitate upwards towards the top of the valence band.
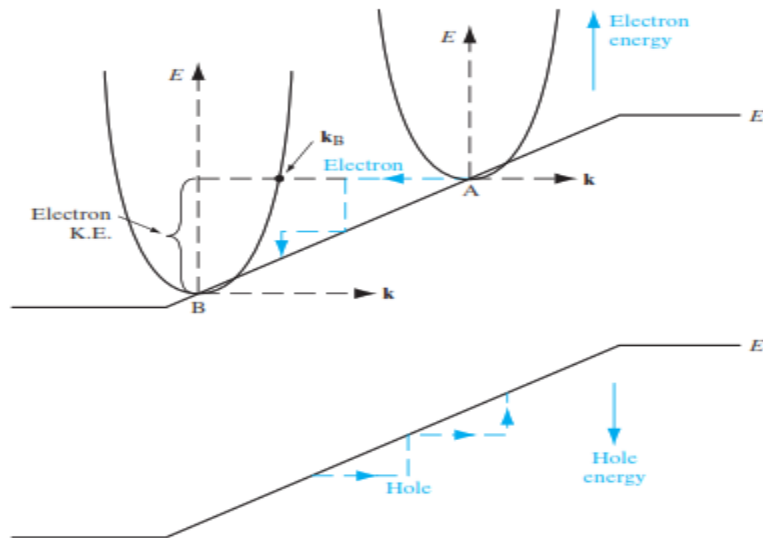

Figure 3–9 Superimposition of the (E, **k**) band structure on the E-versus-position simplified band diagram for a semiconductor in an electric field. Electron energies increase going up, while hole energies increase going down. Similarly, electron and hole wavevectors point in opposite directions and these charge carriers move opposite to each other, as shown.

EXAMPLE 3–1   In a long semiconductor bar ($E_G = 2\,eV$), conduction band electrons come in from the left in the positive x-direction with a kinetic energy of 3 eV. They move from location A to B to C to D. Between A and B, the electric field is zero; between locations B and C, there is a linearly varying voltage increase of 4 V; between C and D, the field is again zero. Assuming no scattering, sketch a *simplified* band diagram describing the motion of these electrons. Assuming that these electrons can be described as plane waves, with a free-electron mass, write down the wavefunction of the electrons at D. Leave your result in terms of an arbitrary normalization constant. *Draw a band diagram and give the wavefunction at D in terms of the normalization constant.*

SOLUTION
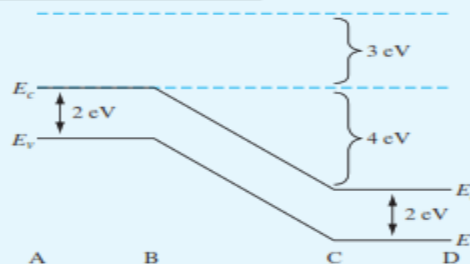
General wavefunction: $\Psi(x, t) = \alpha \times e^{j(kx - \omega t)}$

Energy at $D = \hbar \cdot \omega = \dfrac{\hbar^2 \cdot k^2}{2 \cdot m_o} = 3eV + 4eV = 7eV$

$$= 7eV \cdot 1.6 \cdot 10^{-19} \tfrac{J}{eV} = 1.12 \cdot 10^{-18}\,J$$

$$\omega = \frac{1.12 \cdot 10^{-18}\,J}{\hbar} = \frac{1.12 \cdot 10^{-18}\,J}{1.06 \cdot 10^{-34}\,J \cdot s} = 1.06 \cdot 10^{16}\,Hz$$

$$k = \sqrt{\frac{1.12 \cdot 10^{-18} J \cdot 2 \cdot m_o}{\hbar^2}} = \sqrt{\frac{1.12 \cdot 10^{-18} J \cdot 2 \cdot 9.11 \cdot 10^{-31} kg}{(1.06 \cdot 10^{-34} J \cdot s)^2}}$$

$$= 1.35 \cdot 10^{10} \tfrac{1}{m}$$

Wavefunction at D:

$\Psi(x,t) = \alpha \cdot e^{i(1.35 \cdot 10^{10}\frac{1}{m} \cdot x - 1.06 \cdot 10^{18} Hz \cdot t)}$ where α is the normalization constant

8. Intrinsic Material

- A perfect semiconductor crystal with no impurities or lattice defects.

- No carriers at 0 K, since the valence band is completely full and the conduction band is completely empty.

- For T > 0 K, electrons are thermally excited from the valence band to the conduction band (EHP generation).

- EHP generation takes place due to breaking of covalent bonds =>required energy = E_g.

- The excited electron becomes free and leaves behind an empty state (hole).

- Since these carriers are created in pairs, the electron concentration (n/cm^3) is always equal to the hole concentration (p/cm^3), and each of these is commonly referred to as the *intrinsic carrier concentration* (n_i).

- Thus, for intrinsic material n = p = n_i.

- These carriers are not localized in the lattice; instead they spread out over several lattice spacings, and are given by quantum mechanical probability distributions.

- Note: ni = f(T).

- To maintain a steady-state carrier concentration, the carriers must also recombine at the same rate at which they are generated.

- Recombination occurs when an electron from the conduction band makes a transition (direct or indirect) to an empty state in the valence band, thus annihilating the pair.

- At equilibrium, $r_i = g_i$, where g_i and r_i are the generation and recombination rates respectively, and both of these are temperature dependent.

- g_i(T) increases with temperature, and a new carrier concentration ni is established, such that the higher recombination rate r_i(T) just balances generation.

$$n = p = n_i$$
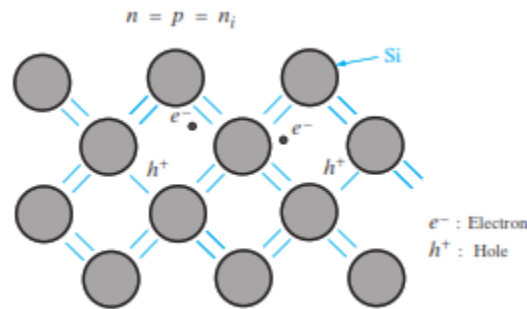
Figure 3–11
Electron–hole
pairs in the
covalent bonding
model of the
Si crystal.

e^- : Electron
h^+ : Hole

- At any temperature, the rate of recombination is proportional to the equilibrium concentration of electrons and holes, and can be given by

$$r_i = \alpha_r n_0 p_0 = \alpha_r n_i^2 = g_i \quad (2.5)$$

where α_r is a constant of proportionality (depends on the mechanism by which recombination takes place).

9. Extrinsic Material

➢ In addition to thermally generated carriers, it is possible to create carriers in the semiconductor by purposely introducing impurities into the crystal => *doping*.

➢ Most common technique for varying the conductivity of semiconductors.

➢ By doping, the crystal can be made to have predominantly electrons (n-type) or holes (p-type).

➢ When a crystal is doped such that the equilibrium concentrations of electrons (n0) and holes (p0) are different from the intrinsic carrier concentration (ni), the material is said to be *extrinsic*.

➢ Doping creates additional levels within the band gap.

➢ In Si, column V elements of the periodic table (e.g., P, As, Sb) introduce energy levels very near (typically 0.03-0.06 eV) the conduction band.

➢ At 0 K, these levels are filled with electrons, and very little thermal energy (50 K to 100 K) is required for these electrons to get excited to the conduction band.

➢ Since these levels donate electrons to the conduction band, they are referred to as the *donor* levels.

➢ Thus, Si doped with donor impurities can have a significant number of electrons in the conduction band even when the temperature is not sufficiently high enough for the intrinsic carriers to dominate, i.e., $n_0 >> n_i, \ p_0$

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=>n-type material, with electrons as *majority carriers* and holes as *minority carriers*.

➢ In Si, column III elements of the periodic table (e.g., B, Al, Ga, In) introduce energy levels very near (typically 0.03-0.06 eV) the valence band.

➢ At 0 K, these levels are empty, and very little thermal energy (50 K to 100 K) is required for electrons in the valence band to get excited to these levels, and leave behind holes in the valence band.

➢ Since these levels accept electrons from the valence band, they are referred to as the acceptor levels.

➢ Thus, Si doped with acceptor impurities can have a significant number of holes in the valence band even at a very low temperature, i.e., $p_0 >> n_i$, $n_0 =>$ p-type material, with holes as majority carriers and electrons as minority carriers.

10.   Conductivity and Mobility

• Even at thermal equilibrium, the carriers are in a constant motion within the lattice.

• At room temperature, the thermal motion of an individual electron may be visualized as random scattering from lattice atoms, impurities, other electrons, and defects.

• There is no net motion of the group of n $electrons/cm^3$ electrons/cm3 over any period of time, since the scattering is random, and there is no preferred direction of motion for the group of electrons and no net current flow.

• However, for an individual electron, this is not true the probability of an electron returning to its starting point after time t is negligibly small.

• Now, if an electric field E_x is applied in the x-direction, each electron experiences a net force qE_x from the field.

• This will create a net motion of group in the x-direction, even though the force may be insufficient to appreciably alter the random path of an individual electron.

• If p_x is the x-component of the total momentum of the group, then the force of the field on the n $electrons/cm^3$ is

$$-nqE_x = \left. \frac{dp_x}{dt} \right|_{field} \quad (2.18)$$

> **Note**: this expression indicates a constant acceleration in the x-direction, which realistically cannot happen.

- In steady state, this acceleration is just balanced by the deceleration due to the collisions.

- Thus, while the steady field E_x does produce a net momentum p_{-x}, for steady state current flow, the net rate of change of momentum must be zero when collisions are included.

- Note: the collision processes are totally random, thus, there is a constant probability of collision at any time for each electron.

- Consider a group of N_0 electrons at time t = 0, and define N(t) as the number of electrons that have not undergone a collision by time t

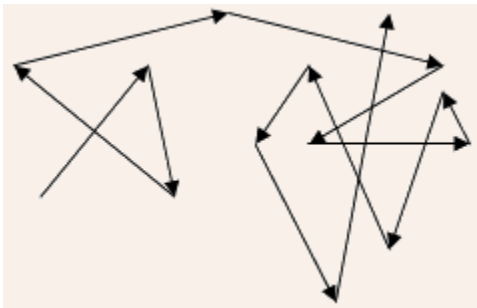Fig.2.7 The random thermal motion of an individual electron, undergoing random scattering.

- The rate of decrease of N(t) at any time t is proportional to the number left unscattered at t, i.e.

$$-\frac{dN(t)}{dt} = \frac{1}{\bar{t}}N(t) \quad (2.19)$$

where $\bar{t}^{-1}$ is the constant of proportionality.

- The solution is an exponential function
$$N(t) = N_0 e^{-t/\bar{t}} \quad (2.20)$$

and $\bar{t}$ represents the mean time between scattering events, called the mean free time.

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- The probability that any electron has a collision in time interval dt is $dt/\bar{t}$, thus, the differential change in p_x due to collisions in time dt is

$$dp_x = -p_x \frac{dt}{\bar{t}} \quad (2.21)$$

- Thus, the rate of change of p_x due to the decelerating effect of collisions is

$$\left. \frac{dp_x}{dt} \right|_{collisions} = -\frac{p_x}{\bar{t}} \quad (2.22)$$

- For steady state, the sum of acceleration and deceleration effects must be zero, thus,

$$-\frac{p_x}{\bar{t}} - nq\,E_x = 0 \quad (2.23)$$

- The average momentum per electron (averaged over the entire group of electrons) is

$$< p_x > = \frac{p_x}{n} = -q\bar{t}\,E_x \quad (2.24)$$

- Thus, as expected for steady state, the electrons would have on the average a constant net velocity in the -x-direction

$$< v_x > = \frac{< p_x >}{m_n^*} = -\frac{q\bar{t}}{m_n^*} E_x \quad (2.25)$$

- This speed is referred to as the drift speed, and, in general, it is usually much smaller than the random speed due to thermal motion $v_{th} \ (= \sqrt{3kT}/m_n^*)$.

- The current density resulting from this drift

$$J_x = -qn < v_x > = \frac{nq^2\bar{t}}{m_n^*} E_x = \sigma E_x \quad (2.26)$$

- This is the familiar Ohm's law with $\sigma \equiv \dfrac{nq^2\bar{t}}{m_n^*}$ being the conductivity of the sample, which can also be written as $\sigma = qn\mu_n$, with $\mu_n = \dfrac{q\bar{t}}{m_n^*}$ is defined as the *electron mobility*
(in $cm^2/V\text{-}sec$), and it describes the ease with which electrons drift in the material.

- The mobility can also be expressed as the average drift velocity per unit electric field, thus $\mu_n = -< v_x >/E_x$ with the negative sign denoting a positive value for mobility since electrons drift opposite to the direction of the electric field.

- The total current density can be given by $J_x = q(n\mu_n + p\mu_p)E_x$ (2.27) when both electrons and holes contribute to the current conduction; on the other hand, for predominantly n-type or p-type samples, respectively the first or the second term of the above equation dominates.

11.   Drift and Resistance

- Since GaAs has a strong curvature of the E-k diagram at the bottom of the conduction band, the electron effective mass in GaAs is very small $=>$ the electron mobility in GaAs is very high since μ_n is inversely proportional to m_n^*.

- The other parameter in the mobility expression, i.e., $\bar{t}$ (the mean free time between collisions) is a function of temperature and the impurity concentration in the semiconductor.

- both electron and hole drift currents are in the same direction, since holes (with positive charges) move along the direction of the electric field, and electrons (with negative charges) drift opposite to the direction of the electric field.

- For a uniformly doped semiconductor bar of length L, width w, and thickness t, the resistance R of the bar can be given by $L/(wt\sigma) = \rho L/(wt)$ where ρ is the resistivity.
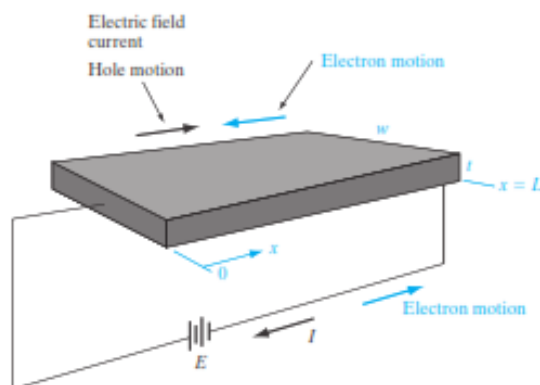

Figure 3–21
Drift of electrons and holes in a semiconductor bar.

12.   Effects of Temperature and Doping on Mobility

- The two main scattering events that influence electron and hole motion (and, thus, mobility) are the lattice scattering and the *impurity scattering*.

- All lattice atoms vibrate due to temperature and can scatter carriers due to collisions.

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- These collective vibrations are called phonons, thus *lattice scattering* is also known as *phonon scattering*.

- With increasing temperature, lattice vibrations increase, and the mean free time between collisions decreases => mobility decreases (typical dependence $\sim T^{-3/2}$).

- Scattering from crystal defects and ionized impurities dominate at low temperatures.

- Since carriers moving with low velocity (at low temperature) can get scattered more easily by ionized impurities, this kind of scattering causes a decrease in carrier mobility with decreasing temperature (typical dependence $\sim T^{3/2}$).

- Note: the scattering probability is inversely proportional to the mean free time (and to mobility), hence, the mobilities due to two or more scattering events add inversely:

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \dots \qquad (2.28)$$

Figure 3–22
Approximate temperature dependence of mobility with both lattice and impurity scattering.
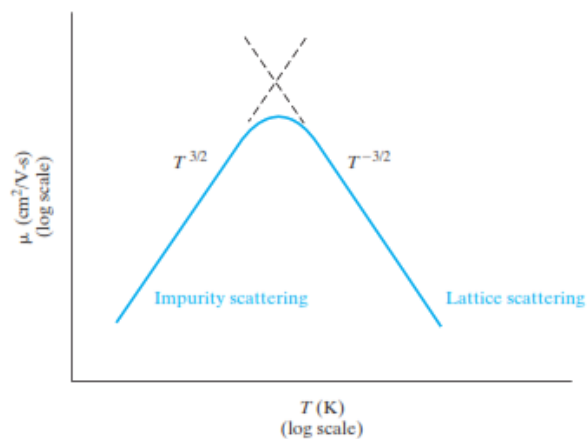
- Thus, the mechanism causing the lowest mobility value dominates.

- Mobility also decreases with increasing doping, since the ionized impurities scatter carriers more (e.g., μ_n for intrinsic Si is 1350 cm^2/V-sec at 300 K, whereas with a donor doping of 10^{17}/cm^3, n drops to 700 cm^2/V-sec).

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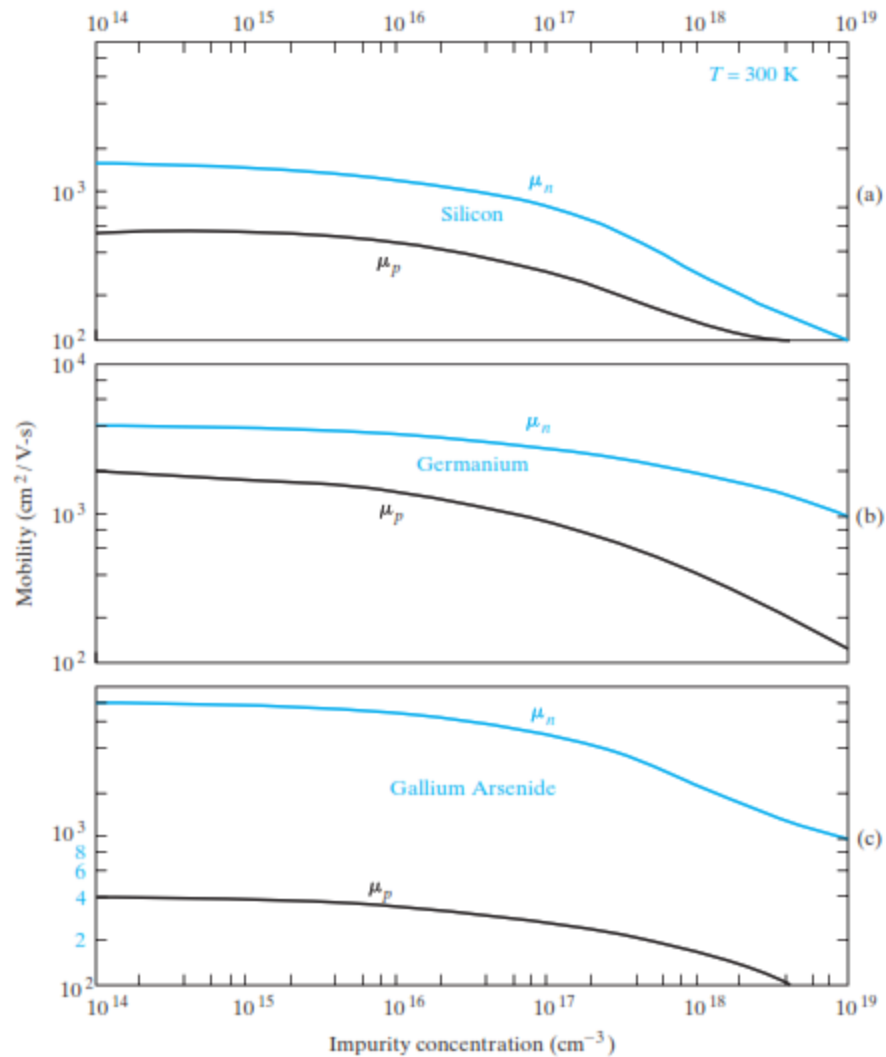
Figure 3–23
Variation of mobility with total doping impurity concentration ($N_a + N_d$) for Ge, Si, and GaAs at 300 K.

(a) *Find the current at 300 K with 10 V applied for a Si bar 1 μm long, 100 μm² in cross sectional area, and doped with $10^{17}cm^{-3}$ phosphorous.*

With $\mathscr{E} = \dfrac{10\ V}{10^{-4}\ cm} = 10^5 \frac{V}{cm}$ the sample is in the velocity saturation regime.

From Fig. 3–24, $v_s = 10^7 \frac{cm}{s}$.

$I = q \cdot A \cdot n \cdot v_s = 1.6 \cdot 10^{-19} C \cdot 10^{-6} cm^2 \cdot 10^{17} \frac{1}{cm^3} \cdot 10^7 \frac{cm}{s} = 0.16\ A$

13.   The Hall Effect

- An extremely important measurement procedure for determining the majority carrier concentration and mobility.
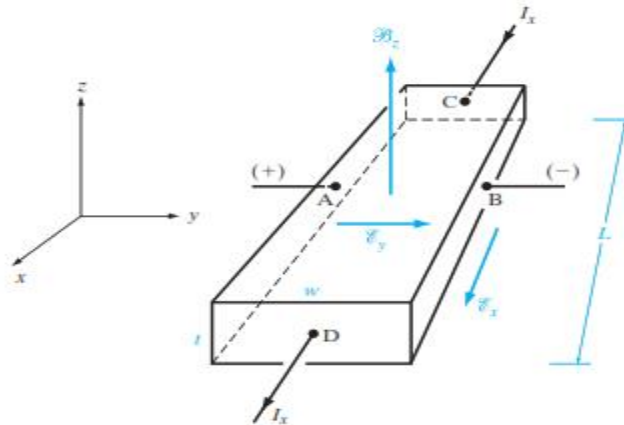

Figure 3–25
The Hall effect.

- If a magnetic field is applied perpendicular to the direction of carrier flow, the path of the carriers get deflected due to the Lorentz force experienced by the carriers, which can be given by

$$F = q(E + v \times B) \qquad (2.29)$$

- Thus, the holes will get deflected towards the -y-direction, and establish an electric field along the y-direction, such that in steady state $E_y = v_x B_z$.

- The establishment of this electric field is known as the *Hall effect*, and the resulting voltage $V_{AB} = E_y w$ is called the *Hall voltage*.

- Using the expression for the drift current, $E_y = R_H J_x B_z$, where $R_H \ (= [qp_0]^{-1})$ is called the Hall coefficient.

- A measurement of the Hall voltage along with the information for magnetic field and current density gives the majority carrier concentration $p_0 \ (= I_x B_z/[qtV_{AB}])$.

- Also, the majority carrier mobility $\mu_p \ (= R_H/\rho)$ can be obtained from a measurement of the resistivity $\rho \ (= [V_{CD}/I_x]/[L/(wt)])$.

- This experiment can be performed to obtain the variation of majority carrier concentration and mobility as a function of temperature.

- For n-type samples, the Hall voltage and the Hall coefficient are negative $=>$ a common diagnostic tool for obtaining the sample type.
  EXAMPLE 2.4: A sample of Si is doped with 10^{16} In $atoms/cm^3$. What will be

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the measured value of its resistivity? What is the expected Hall voltage in a 150 μm m thick sample if $I_x = 2$ mA and $B_z = 5$ kG (1 kG = 10^{-5} Wb/cm^2)?

SOLUTION:

The sample is p-type with $p_0 \gg n_0$, and the mobility $\mu_p \approx 400$ cm^2/V-sec.

Thus, the resistivity $\rho = (\sigma)^{-1} = (q\mu_p p_0)^{-1} = 1.5625$ Ω-cm.

The Hall coefficient $R_H = +(qp_0)^{-1} = +625$ cm^3/C (*note the unit of R_H*).

Note, alternately $R_H = \rho\mu_p = 625$ cm^3/C.

The Hall voltage $V_{AB} = (I_x B_z R_H)/t = (2 \times 10^{-3})(5 \times 10^{-5})(625)/(150 \times 10^{-4}) = 4.17$ mV.

Referring to Fig. 3–25, consider a semiconductor bar with $w = 0.1$ mm, $t = 10\,\mu m$, and $L = 5$ mm. For $\mathcal{B} = 10$ kG in the direction shown (1 kG = 10^{-5} Wb/cm^2) and a current of 1 mA, we have $V_{AB} = -2$ mV and $V_{CD} = 100$ mV. Find the type, concentration, and mobility of the majority carrier.   **EXAMPLE 3–8**

SOLUTION

$$\mathcal{B}_z = 10^{-4}\,\text{Wb/cm}^2$$

From the sign of V_{AB}, we can see that the majority carriers are electrons:

$$n_0 = \frac{I_x \mathcal{B}_z}{qt(-V_{AB})} = \frac{(10^{-3})(10^{-4})}{1.6 \times 10^{-19}(10^{-3})(2 \times 10^{-3})} = 3.125 \times 10^{17}\,\text{cm}^{-3}$$

$$\rho = \frac{R}{L/wt} = \frac{V_{CD}/I_x}{L/wt} = \frac{0.1/10^{-3}}{0.5/0.01 \times 10^{-3}} = 0.002\,\Omega \cdot \text{cm}$$

$$\mu_n = \frac{1}{\rho q n_0} = \frac{1}{(0.002)(1.6 \times 10^{-19})(3.125 \times 10^{17})} = 10,000\,\text{cm}^2(\text{V} \cdot \text{s})^{-1}$$