Chapter 9

Inverters

LEARNING OBJECTIVES:

- To describe the need and function of an inverter.
- To consider the operation of single-phase half-bridge and full-bridge transistorised inverters.
- To introduce the performance parameters of inverters.
- To examine the operation of unipolar and bipolar pwm inverters.
- To consider the operation and design of series inverter with different circuit arrangements.
- To examine the operation of a three-phase series inverter.
- To consider the operation of a high frequency series inverter.
- To consider the operation and design considerations of self-commutated inverters.
- To consider the operation and design details of parallel inverter with different circuit arrangements.
- To examine the operation and detailed design aspects of various voltage source bridge inverter circuits.
- To examine the operation of three-phase bridge inverters with different conduction modes.
- To consider the means of controlling a variable frequency inverter output-voltage.
- To introduce various schemes of pulse width modulated inverters.
- To examine the basic techniques of harmonic filtering and to introduce filter types.
- To examine the operation of current source inverters as a means of producing a variable frequency supply.

9.1 INTRODUCTION

The d.c. to a.c. power converters are known as inverters. In other words, an inverter is a circuit which converts a d.c. power into an a.c. power at desired output voltage and frequency. The a.c. output voltage could be fixed at a fixed or variable frequency. This conversion can be achieved either by controlled turn-on and turn-off devices (e.g. BJTs, MOSFETs, IGBTs, MCTs, SITs, GTOs, SITHs) or by forced commutated thyristors, depending on applications. For low and medium power outputs, the above-mentioned power devices are suitable but for high power outputs, thyristors should be used. The output voltage waveforms of an ideal inverter should be sinusoidal. The voltage waveforms of practical inverters are, however, nonsinusoidal and contain certain harmonics. Square wave or quasi-square wave voltages may be acceptable for low and medium power applications, and for high power applications low-distorted, sinusoidal waveforms are required. The output frequency of an inverter is determined by the rate at which the semiconductor devices are switched *on* and *off* by the inverter control circuitry and consequently, an adjustable frequency a.c. output is readily provided. The harmonic contents of output voltage can be minimized or reduced significantly by switching techniques of available high speed power semiconductor devices. The filtering of harmonics is not feasible when the output frequency varies over a wide range, and the generation of a.c. waveforms with low harmonic content is important. When the a.c. output voltage of an inverter is given to a transformer or a.c. motor, this output voltage must be varied in conjunction with frequency to maintain the proper magnetic conditions. Therefore, the output voltage control is an essential feature of an adjustable frequency system, and various techniques for achieving voltage control are discussed in this chapter.

The d.c. power input to the inverter may be battery, fuel cell, solar cells or other d.c. source. But in most industrial applications, it is fed by a rectifier. This configuration of a.c. to d.c. converter and d.c. to a.c. inverter is called a d.c. link converter because it is a two-stage static frequency converter in which a.c. power at network frequency is rectified and then filtered in the d.c. link before being inverted to a.c. at an adjustable frequency. Rectification is achieved by standard diode or thyristor converter circuits, and inversion is achieved by the circuit techniques described in this chapter.

Inverters are mainly classified as voltage source inverters and current source inverters. A voltage fed inverter (VFI), or voltage source inverter (VSI), is one in which the d.c. source has small or negligible impedance. In other words, a voltage source inverter has stiff d.c. voltage source at its input terminals. Because of a low internal impedance, the terminal voltage of a voltage source inverter remains substantially constant with variations in load. It is, therefore, equally suitable to single motor and multi-motor drives. Any short-circuit across its terminals causes current to rise very fast, due to the low time constant of its internal impedance. The fault current cannot be regulated by current control and must be cleared by fast-acting fuse links. On the other hand, the current fed, or current source, inverter (CSI) is supplied with a controlled current from a d.c. source of high impedance. Typically, a phase controlled thyristor rectifier feeds the inverter with a regulated current through a large series inductor. Thus, load current rather than load voltage is controlled, and the inverter output voltage is dependent upon the load impedance. Because of a large internal impedance, the terminal voltage of a current source inverter changes substantially with a change in load. Therefore, if used in a multi-motor drive, a change in load on any motor affects other motors. Hence, current source inverters are not suitable for multi-motor drives. Since the inverter current is independent of load impedance, it has inherent protection against short-circuits across its terminals. Some of the important industrial applications of inverters are:

- Variable speed a.c. motor drives.
- Induction heating.
- Aircraft power supplies.
- Uninterruptible power supplies (UPS).
- High voltage d.c. transmission lines.
- Battery-vehicle drives.
- Regulated voltage and frequency power supplies, etc.

9.2 CLASSIFICATION OF INVERTERS

Inverters can be classified on the basis of a number of factors:

(a) Classification According to the Nature of Input Source Based on the nature of input power source, inverters are classified as

(i) Voltage source inverters (VSI)
(ii) Current source inverters (CSI)

In case of VSI, the input to the inverter is provided by a ripple free dc voltage source whereas in CSI, the voltage source is first converted into a current source and then used to supply the power to the inverter.

(b) Classification According to the Waveshape of the Output Voltage The inverters can be classified according to the nature of output voltage waveform as:

(i) Square-wave inverter
(ii) Quasi-square wave inverter
(iii) Pulse-width modulated (PWM) inverters

A square-wave inverter produces a square-wave ac voltage of a constant magnitude. The output voltage of this type of inverter can only be varied by controlling the input dc voltage. Square-wave ac-output voltage of an inverter is adequate for low and medium power applications. However, the sine-wave output voltage is the ideal waveform for many high-power applications. Two methods can be used to make the output closer to a sinusoid. One is to use a filter circuit on the output side of the inverter. This filter must be capable of handling the large power output of the inverter, so it must be large and will therefore add to the cost and weight of the inverter. Moreover, the efficiency will be reduced due to the additional power-losses in the filter.

The second method, pulse-width modulation (PWM) uses a switching scheme within the inverter to modify the shape of the output voltage waveform.

9.2.1 Thyristor Inverter Classification

The thyristor inverters can be classified in the following categories:

1. According to the method of commutation.
2. According to the connections.

(a) Classification According to the Method of Commutation According to the method of commutation, the SCR inverters can mainly be categorised in two types, viz.

1. Line commutated inverters
2. Forced commutated inverters.

1. Line Commutated Inverters In case of a.c. circuits, a.c. line voltage is available across the device. When the current in the SCR goes through a natural zero, the device is turned-off. This process is known as natural commutation process and the inverters based on this principle are known as line commutated inverters.

2. Forced Commutated Inverters In case of d.c. circuits, since the supply voltage does not go through the zero point, some external source is required to commutate the device. This process is known as the forced commutation process and the inverters based on this principle are called as forced commutated inverters. As the device is to be commutated forcefully, these types of inverters require complicated commutation circuitries. These inverters are further classified as: (i) Auxiliary commutated inverters and (ii) Complementary commutated inverters.

(b) Classifications According to Connections According to the connections of the thyristors and commutating components, the inverters can be classified mainly in three groups. These are:

1. Series inverters.
2. Parallel inverters.
3. Bridgė inverters.: Bridge inverters are further classifed as: (i) Half-bridge and (ii) Full-Bridge.

9.3 SINGLE-PHASE HALF-BRIDGE VOLTAGE-SOURCE INVERTERS

Figure 9.1 shows the basic configuration of single-phase half-bridge inverter. Switches S_1 and S_2 are the gate-commutated devices such as power BJTs, MOSFETs, GTO, IGBT, MCT, etc. When closed, these switches conducts and current flows in the direction of arrow. The operation of this inverter for different types of load is explained in the following sections:

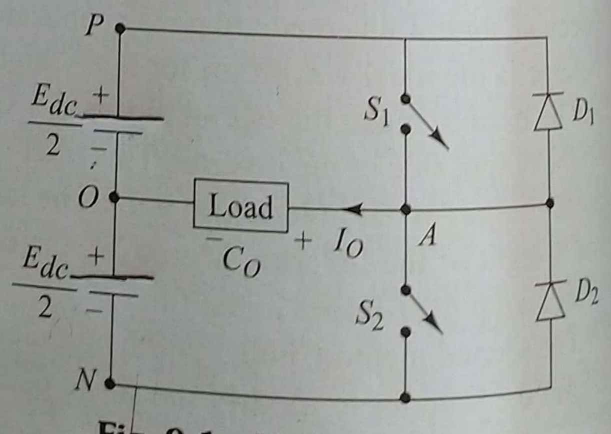


Fig. 9.1 *Half-bridge inverter*

9.3.1 Operation with Resistive Load

The operation of the circuit can be divided into two periods:

(i) Period-I, where switch S_1 is conducting from $0 \leq 1 \leq T/2$ and

(ii) Period-II, where switch S_2 is conducting from $T/2 \leq t \leq T$,

where $T = 1/f$ and f is the frequency of the output voltage waveform. Figure 9.2 shows the waveforms for the output voltage and switch currents for a resistive-load.

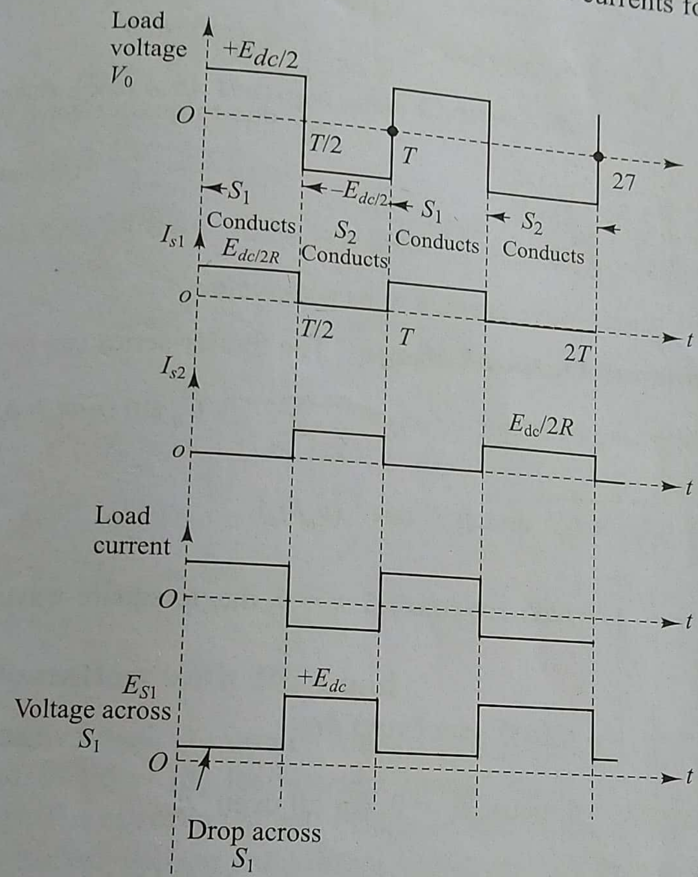


Fig. 9.2 *Voltage and current waveforms*

Switch S_1 is closed for half-time period ($T/2$) of the desired ac output. It connects point p of the dc source to point A and the output voltage e_0 becomes equal to $+E_{dc}/2$.

At $t = T/2$, gating signal is removed from S_1 and it turns-off. For the next half-time period ($T/2 < t < T$), the gating signal is given to S_2. It connects point N of the dc source to point A and the output voltage reverses. Thus, by closing S_1 and S_2 alternately, for half-time periods, a square-wave ac voltage is obtained at the output. With resistive load, waveshape of load current is identical to that of output voltage. Simply by controlling the time periods of the gate-drive signals, the frequency can be varied. Here diodes D_1 and D_2 do not play any role. The voltage across the switch when it is OFF is E_{dc}. Gating circuit should be designed such that switches S_1 and S_2 should not turn-on at the same time.

The n^{th} harmonic-component is given by

$$e_0(n) = \frac{c_n}{\sqrt{2}} = \frac{2\,E_{dc}}{n\pi \cdot \sqrt{2}} = \frac{\sqrt{2}}{n}\frac{E_{dc}}{n} \quad \text{for } n = 1, 3, 5, \ldots \tag{9.3}$$

RMS value of fundamental components is obtained by substituting $n = 1$ in Eq. (9.3) $\therefore$ $E_{1(rms)} = \frac{\sqrt{2}}{\pi} E_{dc} = 0.45\, E_{dc}$ (9.4)

(iii) Switch (Device) Voltage and Current Ratings From Fig. 9.2,

$$V_{CEO(transistor)} \geq 2\,\frac{E_{dc}}{2} \geq E_{dc} \tag{9.5}$$

The current waveform for switch is a square-wave with a peak value of $E_{dc}/2R$.

$$\therefore \quad I_{Tavg} = \frac{1}{T}\int_0^{T/2} \frac{E_{dc}}{2R}\, dt = \frac{E_{dc}}{4R} \tag{9.6}$$

$$\therefore \quad I_{T_{rms}} = \sqrt{\frac{1}{T}\cdot\int_0^{T/2}\left(\frac{E_{dc}}{2R}\right)^2 dt} = \frac{E_{dc}}{2\sqrt{2}R} \tag{9.7}$$

and

$$I_{T_{peak}} = \frac{E_{dc}}{2R} \tag{9.8}$$

9.3.2 Operation with RL Load

With an inductive-load, the output voltage waveform is similar to that with a resistive-load, however the load-current cannot change immediately with the output voltage. The operation of half-bridge inverter with RL load is divided into four distinct modes. Voltage and current waveforms are shown in Fig. 9.3. D_1 and D_2 are known as the feedback diodes.

Mode I ($t_1 < t < t_2$): S_1 is turned-on at instant t_1, the load voltage is equal to $+E_{dc}/2$ and the positive load current increases gradually. At instant t_2, the load-current reaches the peak value. Switch S_1 is turned-off at this instant. Due to same-polarity of load voltage and load current, the energy is stored by the load [Fig. 9.4(a)].

Mode II ($t_2 < t < t_3$): Due to inductive-load, the load current direction will be maintained even-after S_1 is turned-off. The self-induced voltage in the load will be negative. The load current flows through lower half of the supply and D_2 as shown in Fig. 9.4(b). In this mode, the stored energy in load is fedback to the lower half of the source and the load voltage is clamped to $-E_{dc}/2$.

Mode III ($t_3 < t < t_4$): At instant t_3, the load-current goes to zero, indicating that a_1, the stored energy, has been returned back to the lower half of supply. At

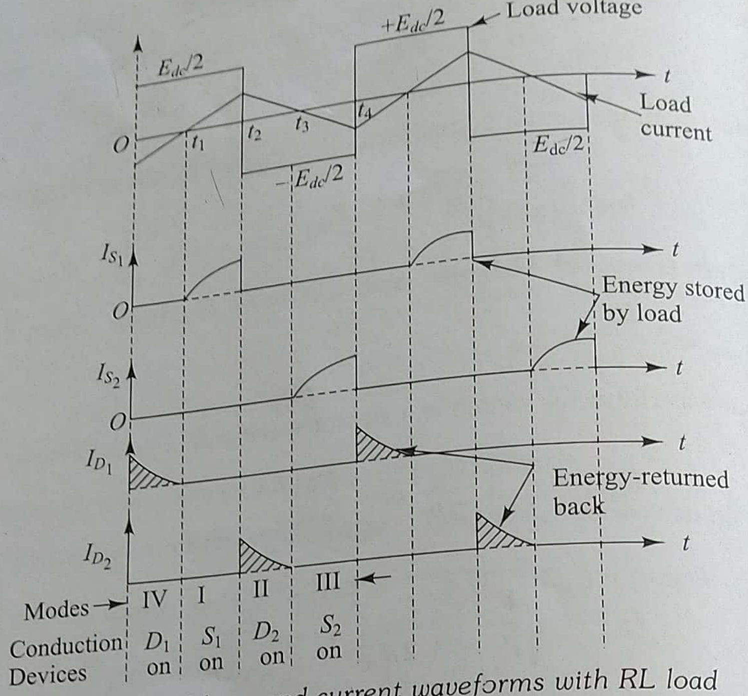


Fig. 9.3 *Voltage and current waveforms with RL load*

instant t_3, S_2 is turned-on. This will produce a negative load voltage $e_0 = -E_{dc}/2$ and a negative load current. Load current reaches a negative peak at the end of this interval (Fig. 9.4(c)].

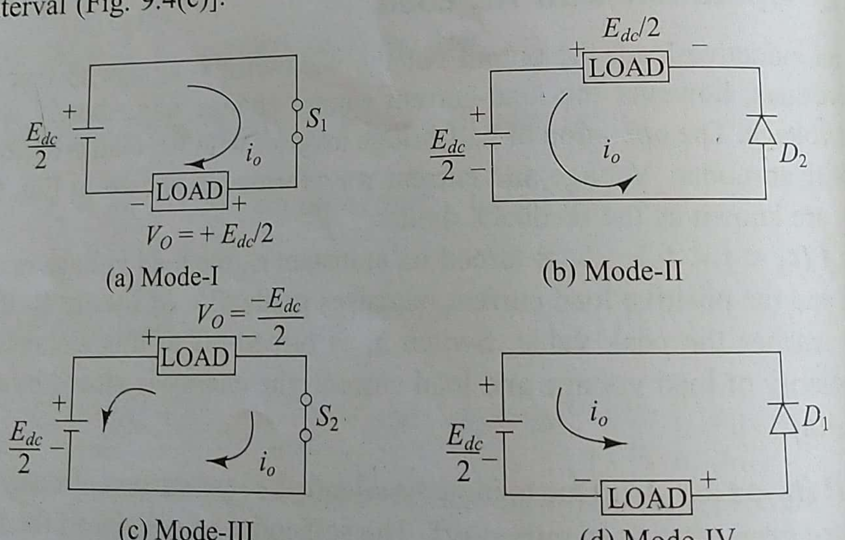


Fig. 9.4 *Operating modes*

Mode IV ($t_0 < t < t_1$): Switch S_2 is turned-off at instant t_4. The self induced voltage in the inductive load will maintain the load current. The load voltage changes its polarity to become positive $E_{dc}/2$, load current remains negative and the stored energy in the load is returned back to the upper half of the dc source (Fig. 9.4(d)).

At t_5, the load current goes to 0 and S_1 can be turned-on again. This cycle of operation repeats.

Circuit-Equations

(i) Instantaneous Current i_0:- With RL load, the equation for instantaneous current i_0 can be obtained from Eq. (9.2), as

$$i_0(t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{2\,E_{dc}}{n\,\pi\,\sqrt{R^2 + (n\,\omega L)^2}} \sin\,(n\omega t - \theta_n) \qquad (9.9)$$

Here, $Z_n = \sqrt{R^2 + (n\,\omega L)^2}$ is the impedance offered by the load to the n^{th} harmonic component, $\dfrac{2\,E_{dc}}{n\pi}$ is the peak amplitude of n^{th} harmonic voltage, and

$$\theta_n = \tan^{-1}\left(\frac{n\,\omega L}{R}\right) \qquad (9.10)$$

(ii) Fundamental Output Power: The output power at fundamental frequency ($n = 1$) is given by

$$P_{1_{rms}} = E_{1_{rms}} \cdot I_{1_{rms}} \cdot \cos\,\theta_1 = I_{1_{rms}}^2 \cdot R \qquad (9.11)$$

where $E_{1_{rms}}$ = RMS value of fundamental output voltage.

$I_{1_{rms}}$ = RMS value of fundamental output current

$\theta_1 = \tan^{-1}\,(\omega L/R)$

But

$$I_{1_{rms}} = \frac{2\,E_{dc}}{\sqrt{2}\cdot\pi\cdot\sqrt{R^2 + (\omega L)^2}} \qquad (9.12)$$

$$P_{1_{rms}} = I_{1_{rms}}^2 \cdot R = \left[\frac{2\,E_{dc}}{\pi\cdot\sqrt{2}\,\sqrt{R^2 + (\omega L)^2}}\right]^2 \cdot R \qquad (9.13)$$

$$= \left[\frac{4\,E_{dc}^2 \cdot R}{2\pi^2\,(R^2 + \omega^2\,L^2)}\right] = \left[\frac{2\,E_{dc}^2 \cdot R}{\pi^2\,(R^2 + \omega^2\,L^2)}\right] \qquad (9.14)$$

Importance of fundamental power is that in many applications such as electric motor drives, the output power due to fundamental current is generally the useful power and the power due to harmonic current is dissipated as heat and increases the load dissipation.

9.3.3 Cross Conduction or Shoot through Fault

In the half-bridge inverter circuit, each switch conducts for a period of $T/2$ secs. At any particular instant, one switch is turned-on and the other is turned-off. However, the outgoing switch does not turn-off instantaneously due to its finite turn-off delay. Due to this, both switches (incoming and outgoing) conduct simultaneously for a short-time. This is known as cross-conduction or shoot-through-fault.

When both switches conducts simultaneously, the input dc supply is short-circuited and with this switches get damaged. Cross conduction can be avoided by allowing the outgoing switch to turn-off completely first and then applying the gate-drive to the incoming device. A dead-band or delay is introduced between the trailing-edge of the base-drive of outgoing device and the leading-edge of the base-drive of the incoming device. Therefore, during the dead-band interval, no device receives base-drive. Hence, the dead-band should be longer than the turn-off time of the power-devices used in the inverter circuit.

SOLVED EXAMPLES

Example 9.1 The single-phase half-bridge inverter has a resistive load of 10 Ω and the center-tap dc input voltage is 96 V. Compute:

(i) RMS value of the output voltage.
(ii) Fundamental component of the output voltage waveform.
(iii) First five harmonics of the output-voltage waveform.
(iv) Fundamental power consumed by the load.
(v) RMS power consumed by the load.
(vi) Verify that the rms value determined by harmonic summation method is nearly equal to the value determined by integration method.

Solution:

(i) From Eq. (9.1), $E_{0(\text{rms})} = \dfrac{E_{\text{dc}}}{2} = 96$ volts

(ii) From Eq. (9.4), $E_{1(\text{fund})} = \dfrac{\sqrt{2}}{\pi} E_{\text{dc}} = 0.9 \times 96 = 86.40$ volts

(iii) From Eq. (9.3), first five harmonics are given by

$$E_{0(3)} = \frac{E_1}{3} = \frac{86.4}{3} = 28.8\ \text{V}, \quad E_{0(5)} = \frac{E_1}{5} = \frac{86.4}{5} = 17.28\ \text{V}$$

$$E_{0(7)} = \frac{E_1}{7} = \frac{86.4}{7} = 12.34\ \text{V}, \quad E_{0(9)} = \frac{E_1}{9} = \frac{86.4}{9} = 9.6\ \text{V}$$

$$E_{0(11)} = \frac{E_1}{11} = \frac{86.4}{11} = 7.85\ \text{V}$$

(iv) Fundamental power, $P_{0(\text{fund})} = \dfrac{E_{1(\text{fund})}^2}{R} = \dfrac{(86.0)^2}{10} = 746.5$ W

(v) RMS power, $P_{0(\text{rms})} = \dfrac{E_{0(\text{rms})}^2}{R} = \dfrac{(96)^2}{10} = 921.6$ W

(vi) RMS value by harmonic summation method is

$$E_{0(\text{rms})} = \sqrt{E_1^2 + E_3^2 + E_5^2 + E_7^2 + E_9^2 + E_{11}^2} = 94.34 \text{ V}$$

Thus, the two values are equal. The value obtained by harmonic summation method is always less than that found by direct integration method.

9.4 SINGLE-PHASE FULL-BRIDGE INVERTERS

Figure 9.5 shows the power-diagram of the single-phase bridge inverter. The inverter uses two pairs of controlled switches (S_1S_2 and S_3S_4) and two pairs of diodes (D_1D_2 and D_3D_4). The devices of one pair operate simultaneously. In order to develop a positive voltage ($+E_0$) across the load, switches S_1 and S_2 are turned-on simultaneously whereas to have a negative voltage ($-E_0$) across the load, we need to turn-on the switches S_3 and S_4. Diodes D_1, D_2, D_3 and D_4 are known as the feedback diodes.

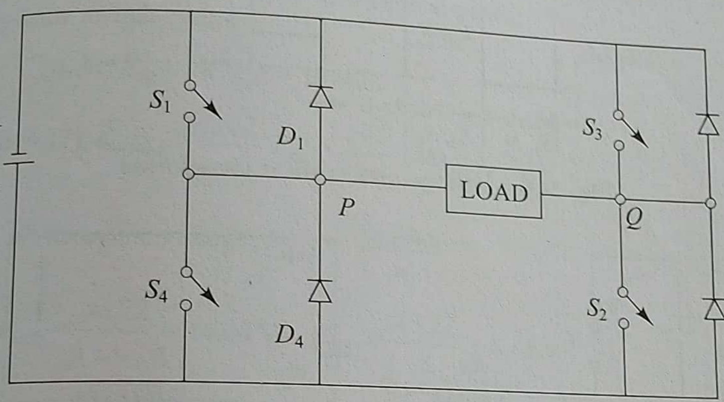


Fig. 9.5 *Single-phase full-bridge inverter*

9.4.1 Operation with Resistive Load

Voltage and current waveforms with resistive-load are shown in Fig. 9.6. The bridge-inverter operates in two-modes in one-cycle of the output.

(i) Mode-I ($0 < t < T/2$): In this mode, switches S_1 and S_2 conducts simultanecusly. The load voltage is $+E_{dc}$ and load current flows from P to Q. The equivalent circuit for mode-I is shown in Fig. 9.7(a). At $t = T/2$, S_1 and S_2 are turned-off and S_3 and S_4 are turned-on.

(ii) Mode-II ($T/2 < t < T$): At $t = T/2$, switches S_3 and S_4 are turned-on and S_1 and S_2 are turned-off. The load voltage is $-E_{dc}$ and load current flows from Q to P. The equivalent circuit for mode-II is shown in Fig. 9.7 (b). At $t = T$, S_3 and S_4 are turned-off and S_1 and S_2 are turned-on again.

As the load is resistive, it does not store any energy. Therefore, feedback diodes are not effective here.

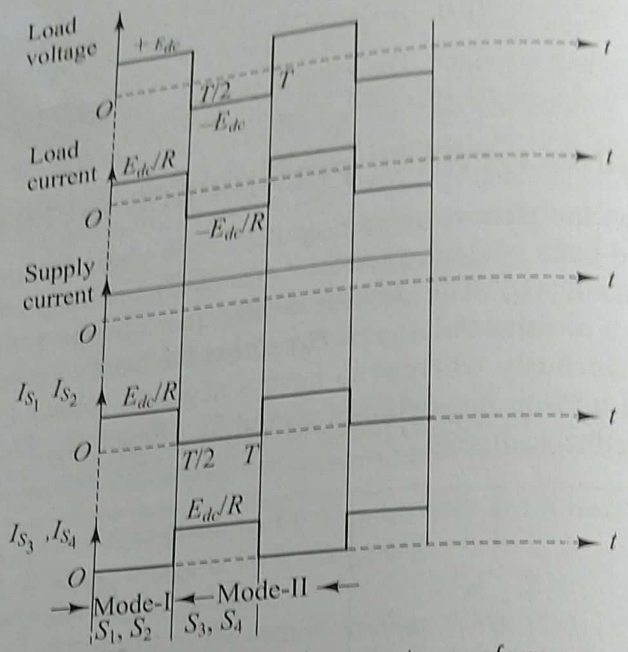
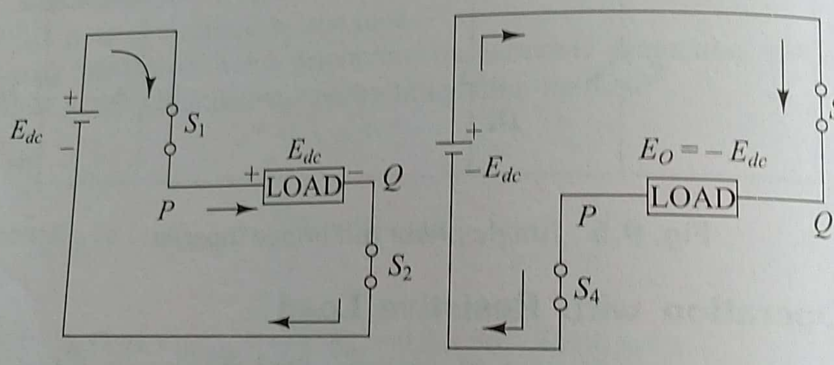


Fig. 9.6 *Voltage and current waveforms*



(a) Mode-I (b) Mode-II

Fig. 9.7 *Equivalent circuit*

Circuit-Analysis The analysis of the full-bridge inverter with resistive-load can be carried-out on similar lines of half-bridge inverter with resistive-load. Hence all equations of half-bridge are valid with $E_{dc}/2$ replaced by E_{dc}.

(i) RMS output voltage, $E_{0(\text{rms})} = E_{dc}$ (9.15)

(ii) Fourier series, $$e_{0(\omega t)} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4\,E_{dc}}{n\pi} \sin(n\,\omega t) \tag{9.16}$$

(iii) Fundamental output voltage, $$E_{0(\text{fund})} = \frac{2\sqrt{2}}{\pi} \cdot E_{dc} \tag{9.17}$$

(iv) n^{th} harmonic voltage $$E_0(n) = \frac{E_{0(\text{fund})}}{n} \tag{9.18}$$

9.4.2 Operation with RL Load

Voltage and current waveforms for single-phase bridge inverter with RL load are shown in Fig. 9.8. The operation of the circuit is explained in four-modes.

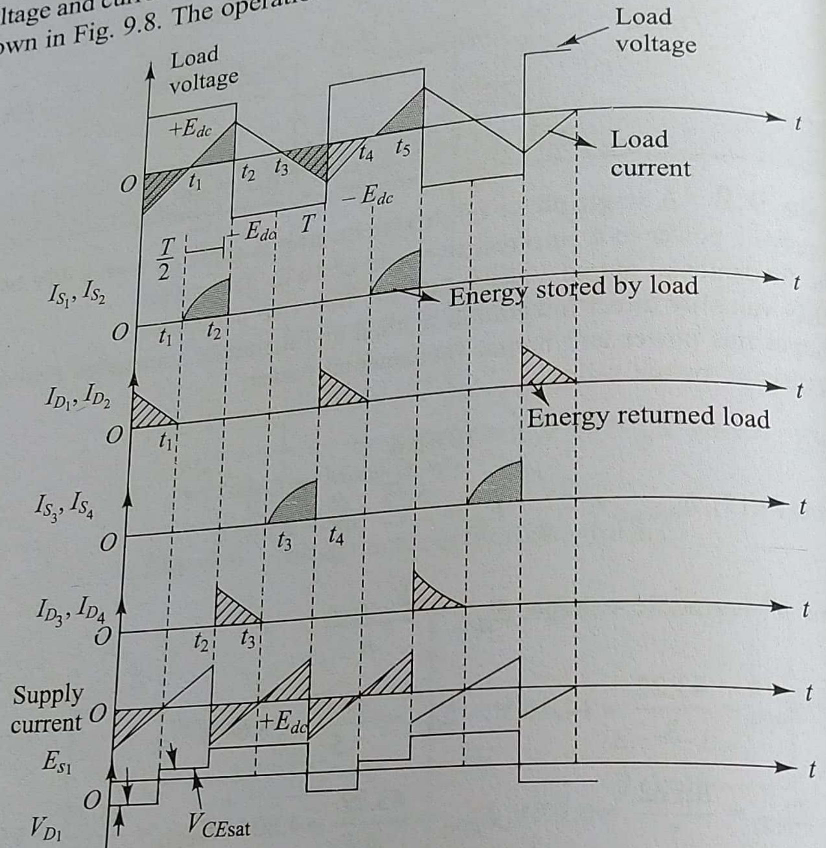
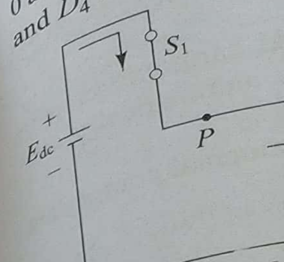


Fig. 9.8 *Voltage and current waveforms*

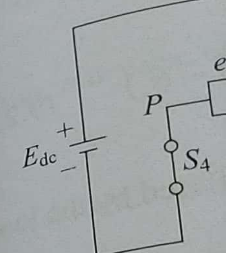
(i) Mode-I ($t_1 < t < t_2$): At instant t_1, the switch S_1 and S_2 are turned-on. Switches are assumed to be ideal switches. Point P gets connected to positive point of d.c. Source E_{dc} through S_1 and point Q gets connected to negative point of input supply. The output voltage, $e_0 = +E_{dc}$, Fig. 9.9(a). The load current starts increasing exponentially due to the inductive nature of the load. The instantaneous current through S_1 and S_2 is equal to the instantaneous load current. During this interval, energy is stored in inductive load.

(ii) Mode-II ($t_2 < t < t_3$): Both the switches Q_1 and Q_2 are turned-off at instant t_2. Due to the inductive nature of the load, the load current does not reduce to zero instantaneously. There is a self-induced voltage across the load which maintains the flow of current in the same-direction. The polarity of this voltage is exactly opposite to that in mode-1, The output voltage becomes $-E_{dc}$, but the load current continues to flow in the same direction, through D_3 and D_4 as shown in Fig. 9.9(b). Thus, in this mode, the stored energy in the load inductance

is returned back to the
0 at instant t_3 when all
and D_4 are turned-of



(a) M



(c) M

(iii) Mode II
at instant t_3.
current will r
and the load

(iv) Mode I
t_4). The load
inducing the
The load ene
$e_0 = +E_{dc}$ b
towards 0.
be turned-o
be $T/4$ or 9
switches w
factor.

Circuit A

(i) RM

is returned back to the source. Load current decreases exponentially and goes to 0 at instant t_3 when all the energy stored in the load is returned back to supply. D_3 and D_4 are turned-off at t_3.

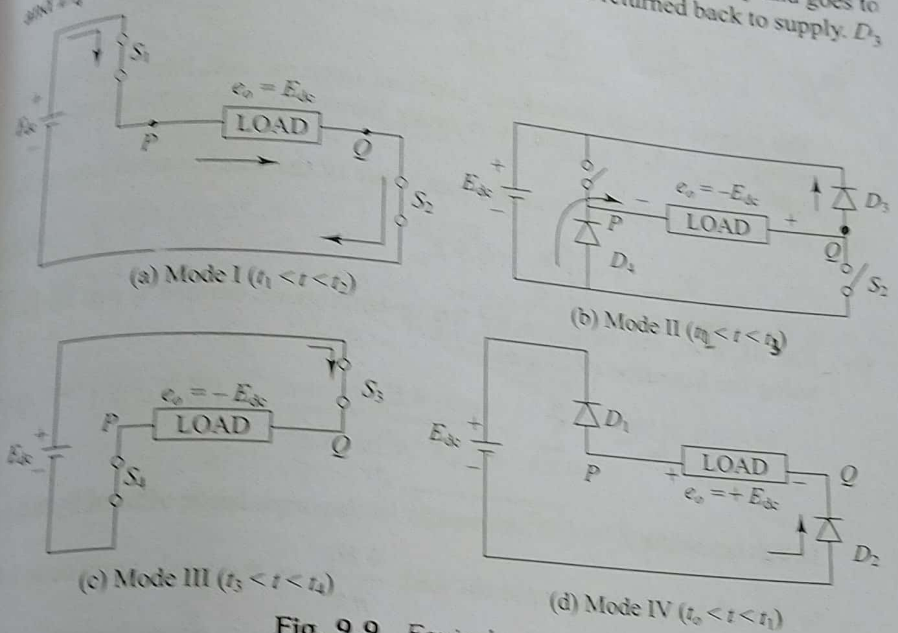


Fig. 9.9 *Equivalent circuits*

(iii) Mode III ($t_3 < t < t_4$): Switches S_3 and S_4 are turned-on simultaneously at instant t_3. Load voltage remains negative ($-E_{dc}$) but the direction of load current will reverse. The current increases exponentially in the other direction and the load again stores the energy.

(iv) Mode IV ($t_0 < t < t_1$): Switches S_3 and S_4 are turned-off at instant t_0 (or t_4). The load inductance tries to maintain the load current in the same direction by inducing the positive-load voltage. This will forward-bias the diodes D_1 and D_2. The load energy is returned back to the input dc supply. The load voltage becomes $e_0 = +E_{dc}$ but the load current remains negative and decreases exponentially towards 0. At t_1 (or t_5), the load current goes to zero and switches S_1 and S_2 can be turned-on again. The conduction period with a very highly inductive load, will be $T/4$ or 90° for all the switches as well as the diodes. The conduction period of switches will increase towards $T/2$ or 180° with increase in the load power-factor.

Circuit Analysis

(i) RMS output voltage can be obtained from

$$E_{0rms} = \left[\frac{2}{T/2}\int_0^{T/2} E^2\, dt\right]^{1/2} \quad \therefore \quad E_{0rms} = E_{dc} \tag{9.20}$$