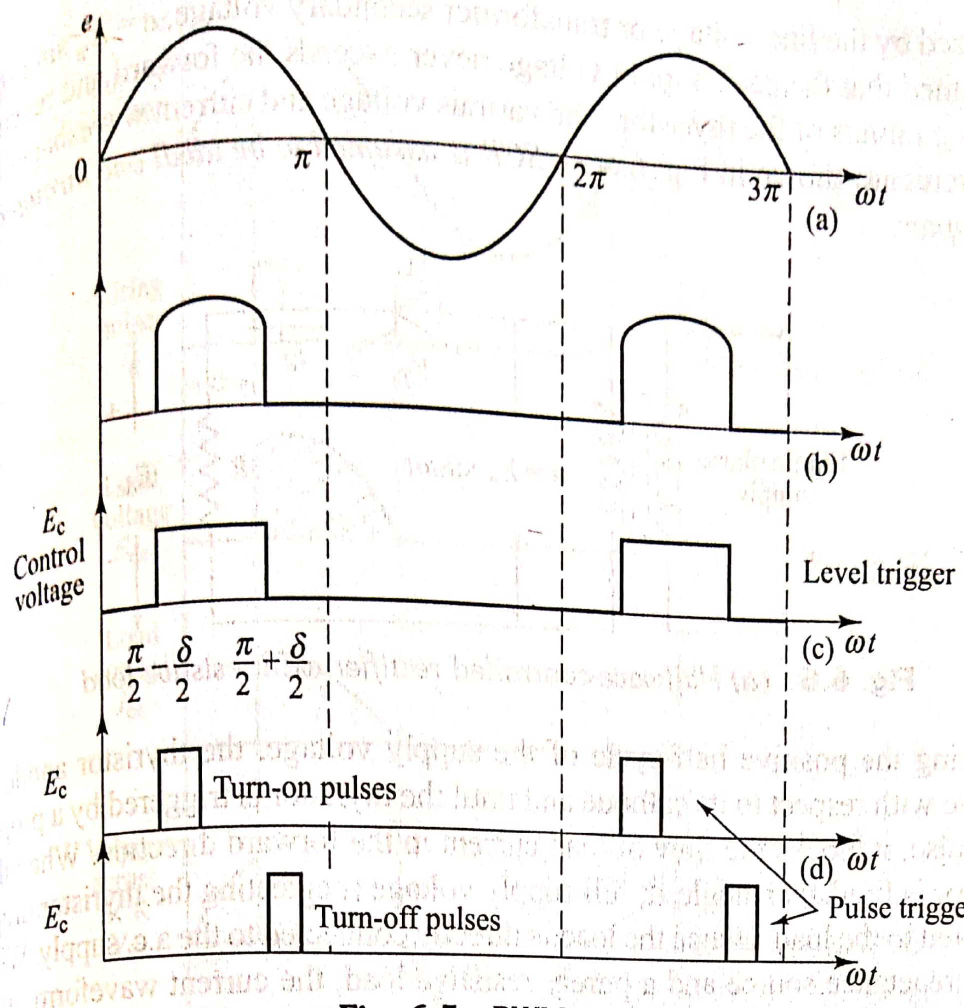


**Fig. 6.5** *PWM control*

In multiple pulse width modulation (MPWM) control, control signals consists of $p$-pulses per half-period of the input voltage waveform. Both the pulse-width $\delta$ and the number of pulses $p$ can be used as the control parameters. The two widely used methods are:

(a) Uniform multiple PWM [$p$-pulses of uniform pulse-width] and

(b) Sine-PWM [$p$-pulses with sinusoidal variation of pulse-width]

This type of control is suited for level triggered switches.

## 6.3 SINGLE PHASE HALF-WAVE CONTROLLED RECTIFIER

In its simplest form, phase control can be described by considering the half-wave thyristor circuit. In a half-wave single-phase controlled rectifier only one SCR is employed in the circuit. It is included in between the a.c. source and the load. The performance of the controlled rectifier very much depends upon the type and parameters of the output (load) circuit.

### 6.3.1 With Resistive Load

Figure 6.6(a) shows the circuit-diagram of a single-phase half-wave converter with resistive load. Triggering circuit is not shown in the figure. The circuit is energized by the line voltage or transformer secondary voltage, $e = E_m \sin \omega t$. It is assumed that the peak supply voltage never exceeds the forward and reverse-blocking ratings of the thyristor. The various voltage and current waveshapes for this circuit are shown in Fig. 6.6(b). *SCR is assumed to be ideal one throughout the chapter.*

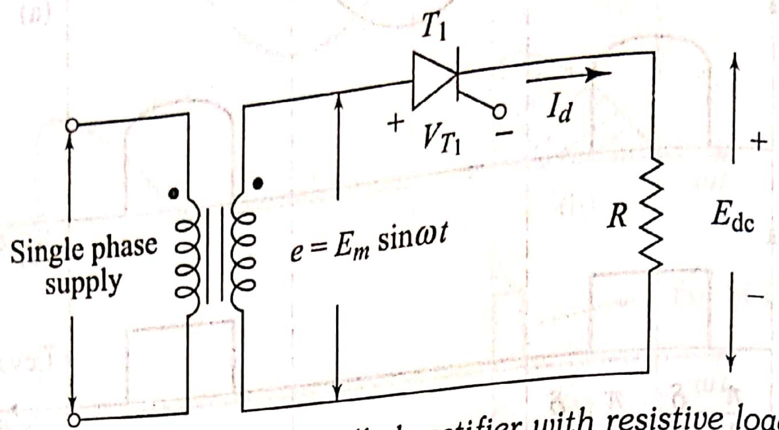


**Fig. 6.6** **(a)** *Halfwave-controlled rectifier with resistive load*

During the positive half-cycle of the supply voltage, the thyristor anode is positive with respect to its cathode and until the thyristor is triggered by a proper gate-pulse, it blocks the flow of load current in the forward direction. When the thyristor is fired at an angle $\alpha$, full supply voltage (neglecting the thyristor drop) is applied to the load. Hence the load is directly connected to the a.c. supply. With a zero reactance source and a purely resistive load, the current waveform after the thyristor is triggered will be identical to the applied voltage wave, and of a magnitude dependent on the amplitude of the voltage and the value of load resistance $R$. As shown in Fig. 6.6(b), the load current will flow until it is commutated by reversal of supply voltage at $\omega t = \pi$. The angle $(\pi - \alpha = \beta)$ during which the thyristor conducts is called the conduction angle. By varying the firing angle $\alpha$, the output voltage can be controlled. During the period of conduction, voltage drop across the device is of the order of one volt.

During the negative half-cycle of the supply voltage, the thyristor blocks the flow of load current and no voltage is applied to the load $R$.

The voltage and current relations are derived as follows:

*(a)* ***Average Load Voltage*** The average value of the load-voltage can be derived as

$$E_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\pi} E_m \cdot \sin \omega t \, d(\omega t)$$

where $E_m$ is the peak value of the a.c. input voltage

$$= \frac{1}{2\pi} E_m [-\cos \omega t]_{\alpha}^{\pi} \quad E_{dc} = \frac{E_m}{2\pi}[1 + \cos \alpha]. \tag{6.1}$$

The maximum output voltage is obtained when $\alpha = 0$.

$$\therefore \quad E_{dc\max} = \frac{E_m}{\pi} \tag{6.2}$$

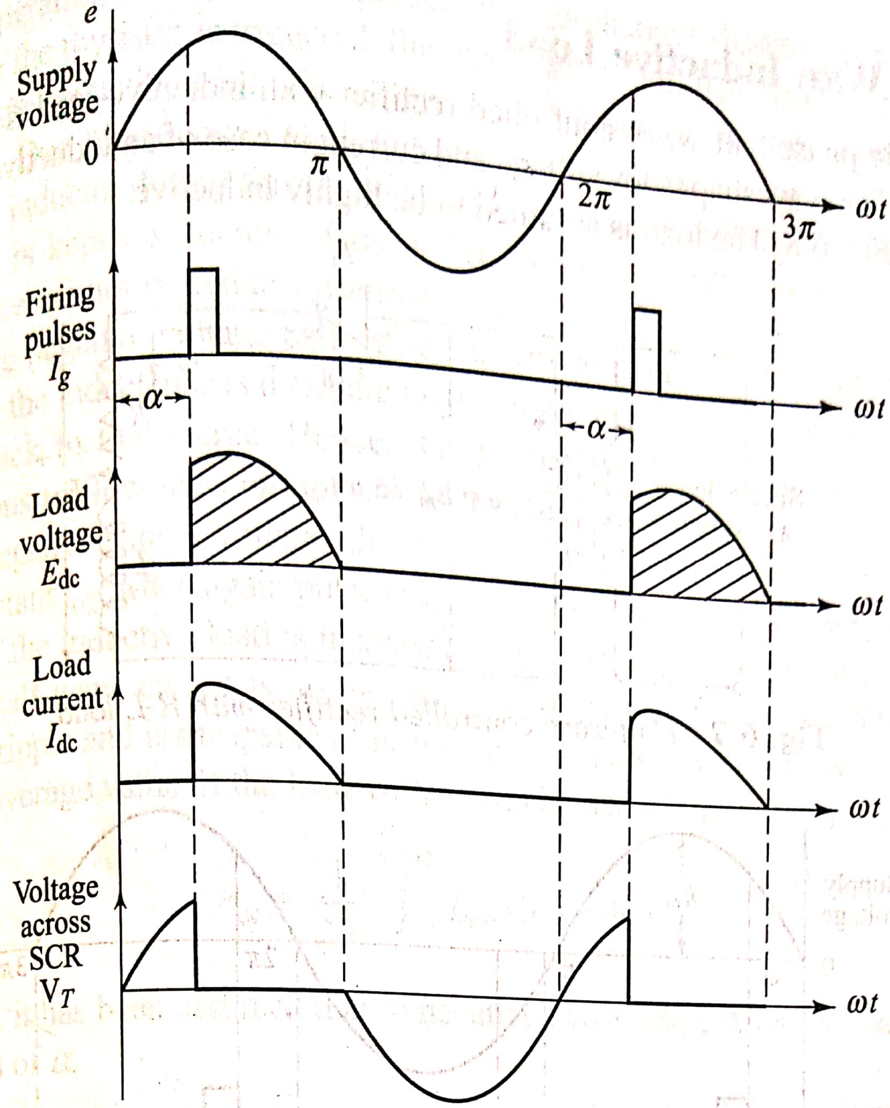


**Fig. 6.6** ***(b)*** *Waveforms for a half-wave circuit*

(b) ***Average load current*** With resistive load, the average load current is directly proportional to the average load voltage divided by the load resistance:

$$\therefore \qquad I_d = \frac{E_m}{2\pi R}[1 + \cos\alpha] \tag{6.3}$$

(c) ***RMS load voltage*** The RMS load voltage for a given firing angle $\alpha$ is given by

$$E_{rms} = \left[\frac{1}{2\pi}\int_{\alpha}^{\pi}(E_m \sin\omega t)^2\, d(\omega t)\right]^{1/2} = \left[\frac{E_m^2}{2\pi}\int_{\alpha}^{\pi}\sin^2\omega t\, d(\omega t)\right]^{1/2}$$

$$= E_m\left[\frac{1}{2\pi}\int_{\alpha}^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right) d(\omega t)\right]^{1/2} = E_m\left[\frac{1}{4\pi}\left(\omega t - \frac{\sin 2\omega t}{2}\right)_{\alpha}^{\pi}\right]^{1/2}$$

$$E_{rms} = E_m\left[\frac{\pi-\alpha}{4\pi} + \frac{\sin 2\alpha}{8\pi}\right]^{1/2} \tag{6.4}$$

For firing angle $\alpha = 0$, $E_{rms} = \dfrac{E_m}{2}$ (6.5)

### 6.3.2 With Inductive Load

The single phase half-wave controlled rectifier with inductive-load is shown in Fig. 6.7. The waveshapes for voltage and current in case of an inductive load are given in Fig. 6.8. The load is assumed to be highly inductive.

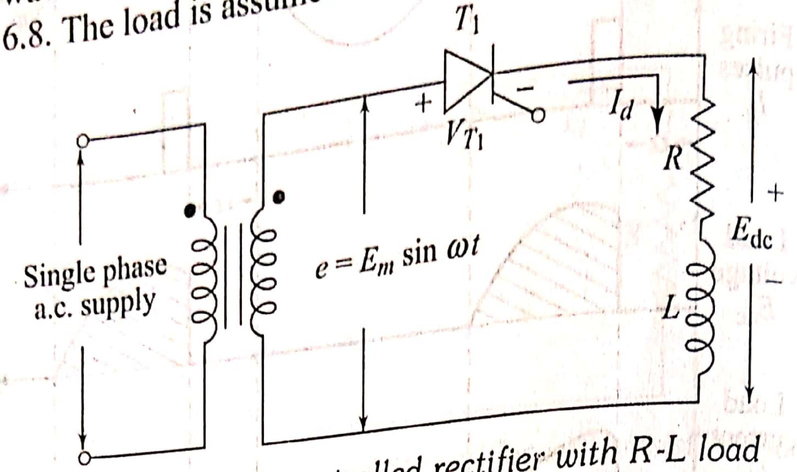


**Fig. 6.7** *Half-wave controlled rectifier with R-L load*

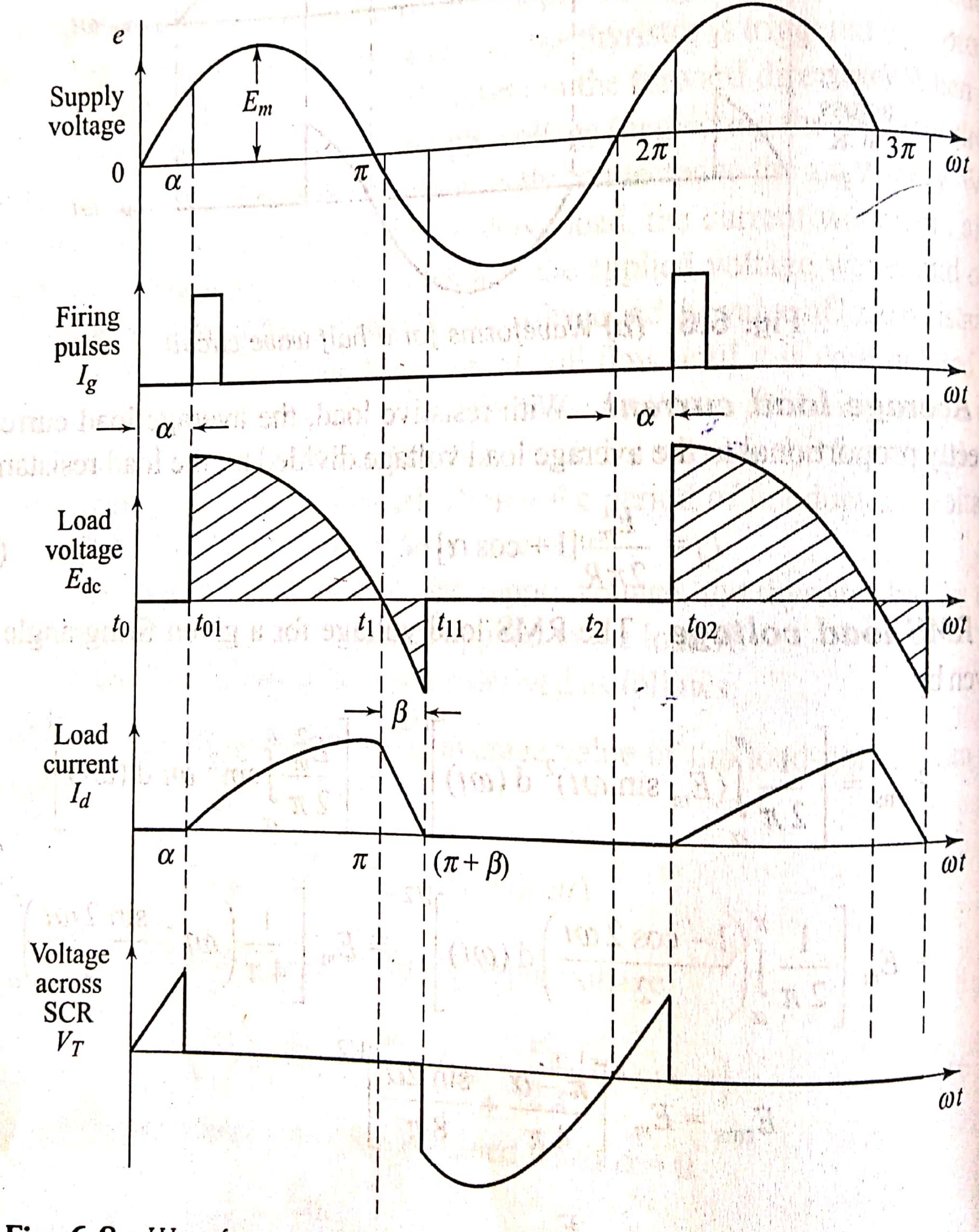


**Fig. 6.8** *Waveforms for a half-...*

The operation of the circuit on inductive loads changes slightly. Now at instant $t_{01}$, when the thyristor is triggered, the load-current will increase in a finite-time through the inductive load. The supply voltage from this instant appears across the load. Due to inductive load, the increase in current is gradual. Energy is stored in inductor during time $t_{01}$ to $t_1$. At $t_1$, the supply voltage reverses, but the thyristor is kept conducting. This is due to the fact that current through the inductance cannot be reduced to zero.

During negative-voltage half-cycle, current continues to flow till the energy stored in the inductance is dissipated in the load-resistor and a part of the energy is fed-back to the source. Hence, due to energy stored in inductor, current continuous to flow upto instant $t_{11}$. At instant, $t_{11}$, the load-current is zero and due to negative supply voltage, thyristor turns-off.

At instant $t_{02}$, when again pulse is applied, the above cycle repeats. Hence the effect of the inductive load is increased in the conduction period of the SCR.

The half-wave circuit is not normally used since it produces a large output voltage ripple and is incapable of providing continuous load-current.

The average value of the load-voltage can be derived as:

$$E_{dc} = \frac{1}{2\pi} \int_{\alpha}^{\pi+\alpha} E_m . \sin \omega t \, d(\omega t)$$

Here, it has been assumed that in negative half-cycles, the SCR conducts for a period of $\alpha$.

$$\therefore \; E_{dc} = \frac{E_m}{2\pi} [-\cos \omega t]_{\alpha}^{\pi+\alpha} \quad \text{Or,} \quad E_{dc} = \frac{E_m}{\pi} \cos \alpha \tag{6.6}$$

From Eqs (6.1) and (6.6), it is clear that the average load-voltage is reduced in case of inductive load. This is due to the conduction of SCR in negative cycle.

### 6.3.3 Effect of Freewheeling Diode

Many circuits, particularly those which are half or uncontrolled, include a diode across the load as shown in Fig. 6.9. This diode is variously described as a commutating diode, flywheel diode or by-pass diode. This diode is commonly described as a commutating diode as its function is to commutate or transfer load current away from the rectifier whenever the load-voltage goes into a reverse-state.

This diode serves two main functions:

(i) It prevents reversal of load voltage except for small diode voltage-drop.
(ii) It transfers the load current away from the main rectifier, thereby allowing all of its thyristors to regain their blocking states.

Figure 6.10 shows a half-wave controlled rectifier with a freewheeling diode $D_f$ connected across $R$–$L$ load. The load-voltage and current waveforms are also shown in Fig. 6.11.

With diode $D_f$, thyristor will not be able to conduct beyond 180°.

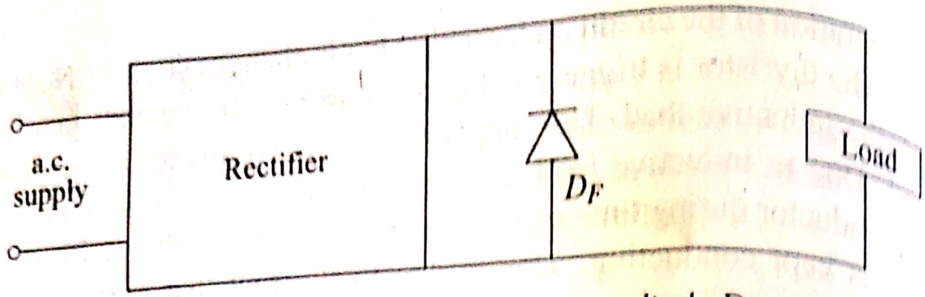


Fig. 6.9 *Position of commutating diode $D_F$*

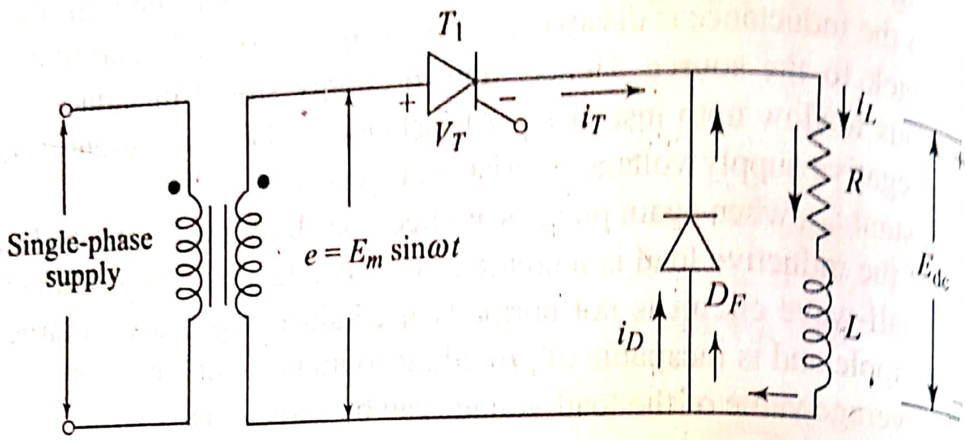


Fig. 6.10 *Half-wave rectifier with a freewheeling diode*

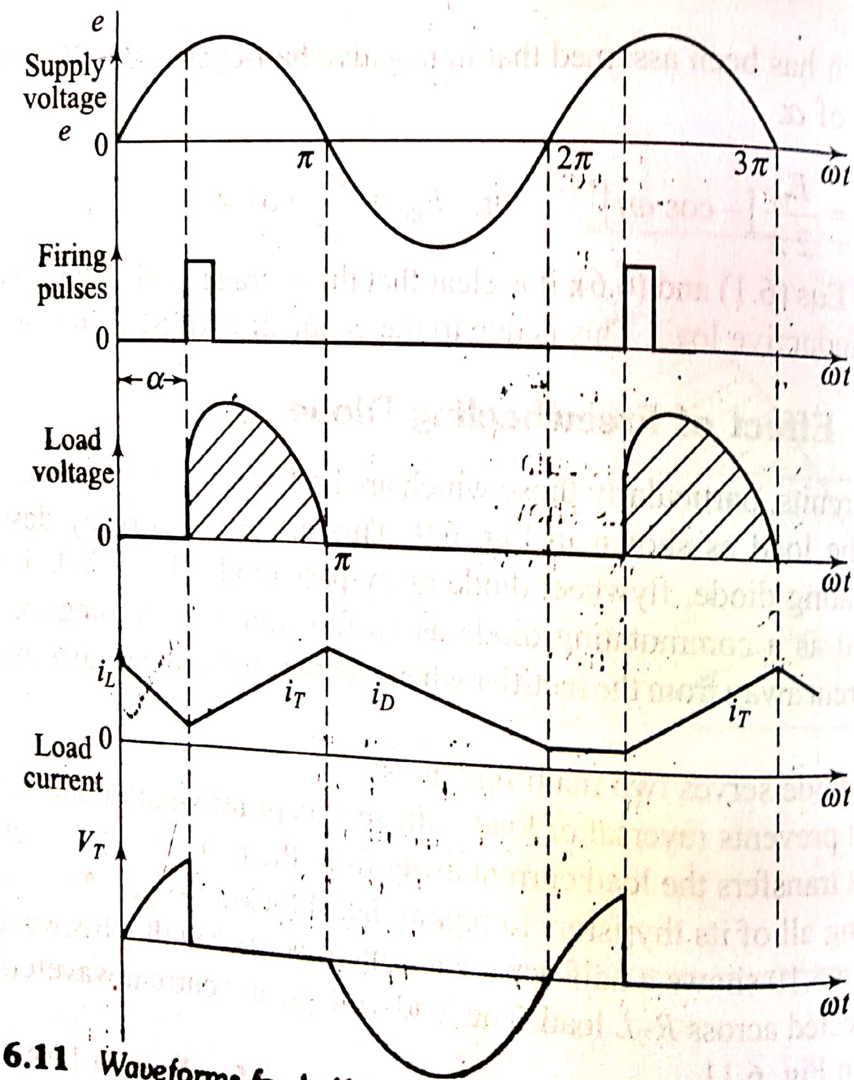


Fig. 6.11 *Waveforms for half-wave con...*

We know that during the positive half-cycle, voltage is induced in the inductance. Now, this induced voltage in inductance will change its polarity as the $di/dt$ changes its sign and diode $D_f$ will start conducting as soon as the induced voltage is of sufficient magnitude, thereby enabling the inductance to discharge its stored energy into the resistance.

Hence, after 180°, the load current will freewheel through the diode and a reverse-voltage will appear across the thyristor. The power flow from the input takes place only when the thyristor is conducting. If there is no freewheeling diode, during the negative portion of the supply voltage, thyristor returns the energy stored in the load inductance to the supply line. With diode $D_f$, the freewheeling action takes place and no power will be returned to the source. Hence, the ratio of the reactive power flow from the input to the total power consumed in the load is less for the phase-control circuit with a freewheeling diode. In other words, the freewheeling diode improves the input power-factor.

Mathematically:

$$\frac{EI\sin\phi}{EI} = \text{less} \quad \therefore \sin\phi = \text{less} \tag{6.7}$$

Since, $\phi$ = less $\therefore$ Power-factor $\cos\phi$ = more (6.8)

Hence it is clear that the freewheeling diode helps in improvement of power-factor of the system.

## SOLVED EXAMPLES

**Example 6.1** If the half-wave controlled rectifier has a purely resistive load of $R$ and the delay angle is $\alpha = \pi/3$. Determine:

(a) Rectification efficiency (b) Form factor (c) Ripple factor
(d) Transformer utilization factor (e) Peak inverse voltage for SCR $T_1$

*Solution:*

(a) Rectification efficiency $\eta = \dfrac{p_{dc}}{p_{ac}}$ where, $p_{dc}$ = dc load power = $E_{dc}^2/R$ and

$p_{ac}$ = rms load power = $\dfrac{E_{rms}^2}{k}$. We have the relation, from Eq. (6.1),

$$E_{dc} = \frac{E_m}{2\pi}(1+\cos\alpha), \text{ Since, } \alpha = \pi/3 \quad \therefore E_{dc} = 0.239\,E_m$$

Also, from equation (6.4), we have

$$E_{rms} = E_m\left[\frac{\pi-\alpha}{4\pi} + \frac{\sin 2\alpha}{8\pi}\right]^{1/2}$$

For firing angle $\alpha = \pi/3$, $E_{rms} = 0.485\,E_m$

$\because$ Rectification efficiency $\eta = \dfrac{(0.239\,E_m)^2}{(0.485\,E_m)^2} \quad \therefore \eta = 24.28\%$

(c) Supply power factor $= \dfrac{I_{rms}^2 \cdot R + I_d \cdot E_b}{E_{rms} \cdot I_{rms}}$

$$= \frac{1096.68 + 1194}{230 \times 14.81} = 0.672 \text{ lagging.}$$

## 6.4 SINGLE-PHASE FULL-WAVE CONTROLLED RECTIFIER (TWO-QUADRANT CONVERTERS)

There are two basic configurations of full wave controlled rectifiers. Their classification is based on the type of SCR configuration employed. They are—

(1) Mid-point converters. (2) Bridge converters.

### 6.4.1 Mid-point Converters (M-2 Connection)

In a single phase full-wave controlled-rectifier circuit with mid-point configuration two SCRs (M-2) and a single-phase-transformer with centre-tapped secondary windings are employed. These converters are also referred to as two pulse converters as two triggering pulses or two sets of triggering pulses are to be generated during every cycle of the supply to trigger the various SCRs. Single-phase full-wave circuit with transformer mid-point configuration are generally used for rectifiers of low ratings.

***1. With Resistive Load*** Figure 6.12 illustrates a 2-pulse mid-point converter circuit with resistive-load. This type of full-wave rectifier circuit uses two SCRs connected to the centre-tapped secondary of a transformer, as shown in Fig. 6.12. The input signal is coupled through the transformer to the centre-tapped secondary.

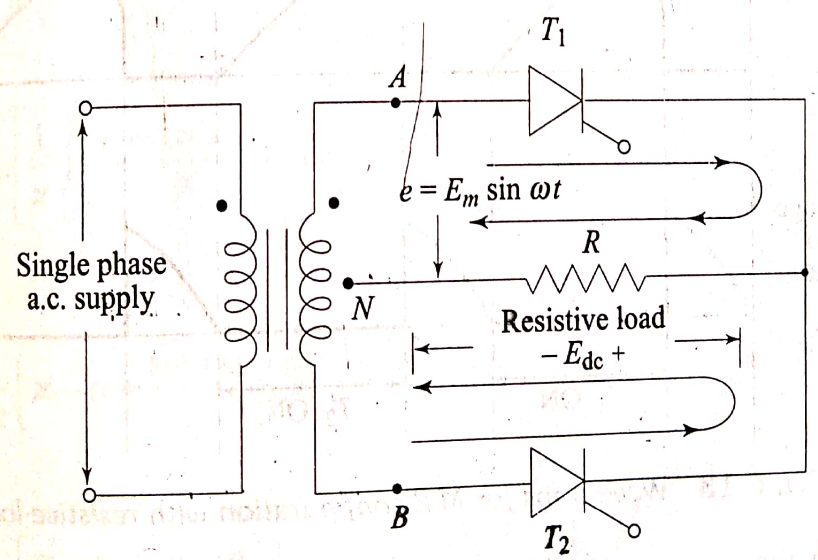


**Fig. 6.12** *Full-wave mid-point circuit with resistive load*

During the positive half-cycle of the a.c. supply, i.e. when terminal $A$ of the transformer is positive with respect to terminal $B$, or the secondary-winding terminal $A$ is positive with respect to $N$, $SCR_1$ ($T_1$) is forward-biased and $SCR_2$

($T_2$) is reverse-biased. Since no triggering pulses are given to the gates of the SCRs, initially they are in off-state. When $SCR_1$ is triggered at a firing-angle $\alpha$, current would flow from terminal $A$ through $SCR_1$, the resistive load $R$ and back to the centre-tap of the transformer (i.e. terminal $N$). This current path is also shown in Fig. 6.12. This current continuous to flow up to angle $\pi$ when the line voltage reverses its polarity and $SCR_1$ is turned-off. Depending upon the value of $\alpha$ and the load circuit parameters, the conduction angle of $SCR_1$ may be any value between 0 and $\pi$.

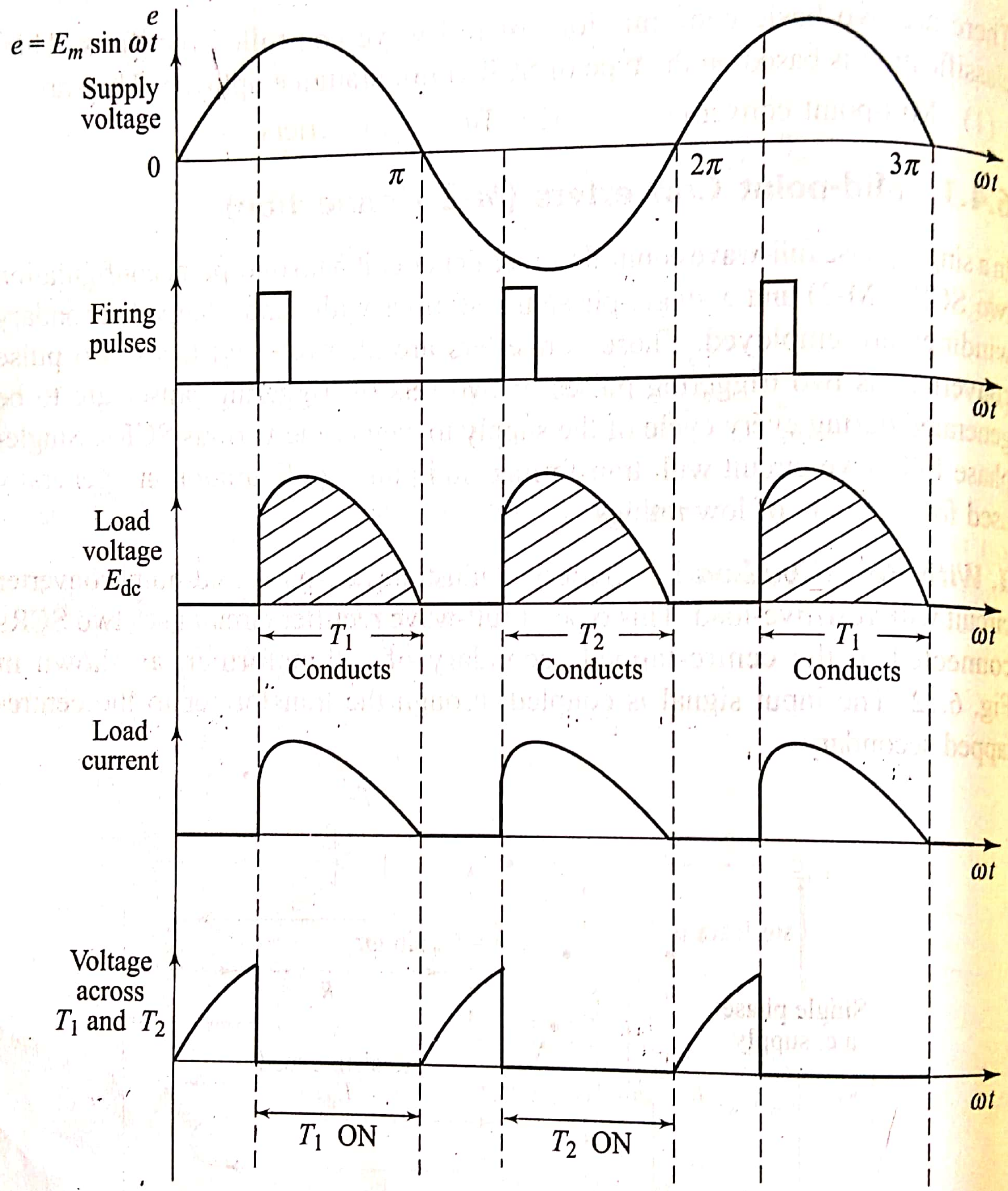


**Fig. 6.13** *Waveforms for M-2 configuration with resistive-load*

During the negative half-cycle of the a.c. supply, the terminal $B$ of the transformer is positive with respect to $N$. $SCR_2$ is forward-biased. When $SCR_2$ is triggered at an angle ($\pi + \alpha$), current would flow from terminal $B$, through $SCR_2$, the resistive load and back to centre-tap of the transformer. This current continues till angle $2\pi$, then $SCR_2$ is turned off. Here it is assumed that both thyristors are triggered with the same firing angle, hence they share the load current equally.

Each half of the input-wave is applied across the load. Thus, across the load, there are two pulses of current in the same direction. Hence the ripple frequency across the load is twice that of the input supply frequency. The voltage and current waveforms of this configuration is shown in Fig. 6.13. It is clear from Fig. 6.13 that with purely resistive load, the load current is always discontinuous.

The voltage and current relations are derived as follows:

***(a) Average d.c. Output Voltage*** The output d.c. voltage, $E_{dc}$, across the resistive load is given by

$$E_{dc} = \frac{1}{\pi}\int_{\alpha}^{\pi} E_m . \sin \omega t \, d(\omega t) = \frac{E_m}{\pi}[-\cos \omega t]_{\alpha}^{\pi} = \frac{E_m}{\pi}[1+\cos\alpha] \qquad (6.9)$$

***(b) Average-load Current*** The average-load current is given by

$$I_{dc} = \frac{E_m}{\pi \cdot R}[1+\cos\alpha] \qquad (6.10)$$

***(c) RMS Load-voltage*** The RMS load-voltage for a given firing angle $\alpha$ is given by

$$E_{rms} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi} E_m^2 \sin^2 \omega t \, d\,\omega t\right]^{\frac{1}{2}} = E_m \cdot \left[\frac{1}{\pi}\int_{\alpha}^{\pi} \sin^2 \omega t \, d\,\omega t\right]^{\frac{1}{2}}$$

$$= E_m \cdot \left[\frac{1}{\pi}\int_{\alpha}^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right) d\,\omega t\right]^{\frac{1}{2}} = E_m \cdot \left[\frac{1}{2\pi}\left(\omega t - \frac{\sin 2\omega t}{2}\right)_{\alpha}^{\pi}\right]^{\frac{1}{2}}$$

$$= E_m \cdot \left[\frac{1}{2\pi}\left(\pi - \alpha + \frac{\sin 2\alpha - \sin 2\pi}{2}\right)\right]^{\frac{1}{2}} = E_m \cdot \left[\frac{1}{2\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2} - 0\right)\right]^{\frac{1}{2}}$$

$$E_{rms} = E_m \cdot \left[\frac{\pi - \alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}\right]^{\frac{1}{2}} \qquad (6.11)$$

**2. *With inductive Load*** The circuit diagram of the single-phase full wave, or bi-phase half-wave controlled rectifier with $R_L$ load is shown in Fig. 6.14. The ... in Fig. 6.15.

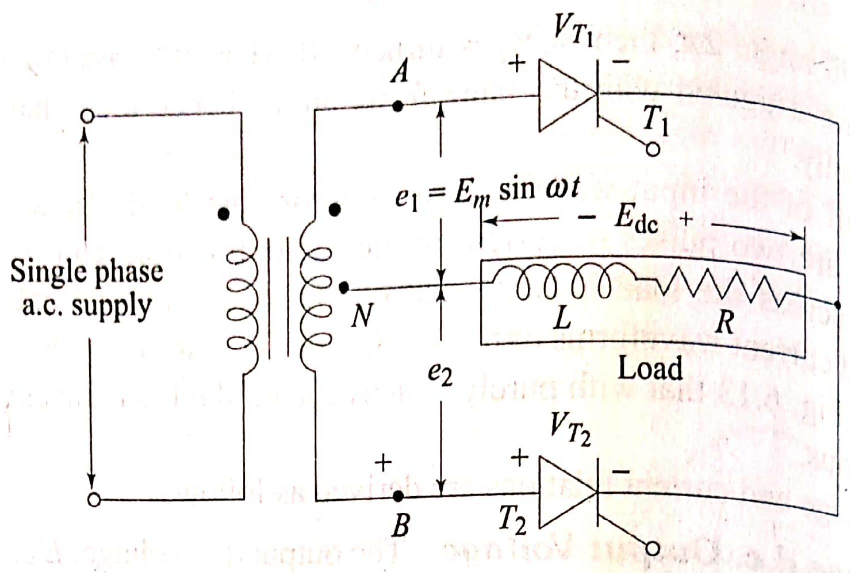


**Fig. 6.14** *Bi-phase half-wave circuit*

With reference to Fig. 6.14, thyristor $T_1$ can be fired into the on-state at any time after $e_1$ goes positive. Once thyristor $T_1$ is turned-on, current builds up in the inductive load, maintaining thyristor $T_1$ in the on-state up to the period when $e_1$ goes negative. However, once $e_1$ goes negative, $e_2$ becomes positive, and the firing of thyristor $T_2$ immediately turns on thyristor $T_2$ which takes up the load current, placing a reverse voltage on thyristor $T_1$, its current being commutated (transferred) to thyristor $T_2$. The thyristor voltage, $V_T$, waveform in Fig. 6.15 shows that it can be fired into conduction at anytime when $V_T$ is positive. The peak reverse (and forward) voltage that appears across the thyristor is $2\,E_m$, that is, the maximum value of the complete transformer secondary voltage.

The load-current may be continuous or discontinuous, depending on the inductance value. The load current is continuous if inductance value is greater than its critical value. It is discontinuous if inductance value is less than its critical value. The analysis given here assumes that the inductance is sufficiently large, so that each thyristor conducts for a period of 180° (conduction of current is continuous). Also, both thyristors are triggered with the same delay angle, hence they share the load current equally. As shown in Fig. 6.15, due to large inductance in the circuit and continuous current conduction, the thyristors continue to conduct even when their anode voltages are negative with respect to the cathode. The load current is shown to be constant d.c.

Now, the output d.c. voltage $E_{dc}$ can be obtained as

$$E_{dc} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} E_m . \sin \omega t \, d(\omega t) = \frac{E_m}{\pi}\left[\cos\alpha - \cos(\pi+\alpha)\right]$$

$$E_{dc} = \frac{2\,E_m}{\pi}\cos\alpha. \tag{6.12}$$

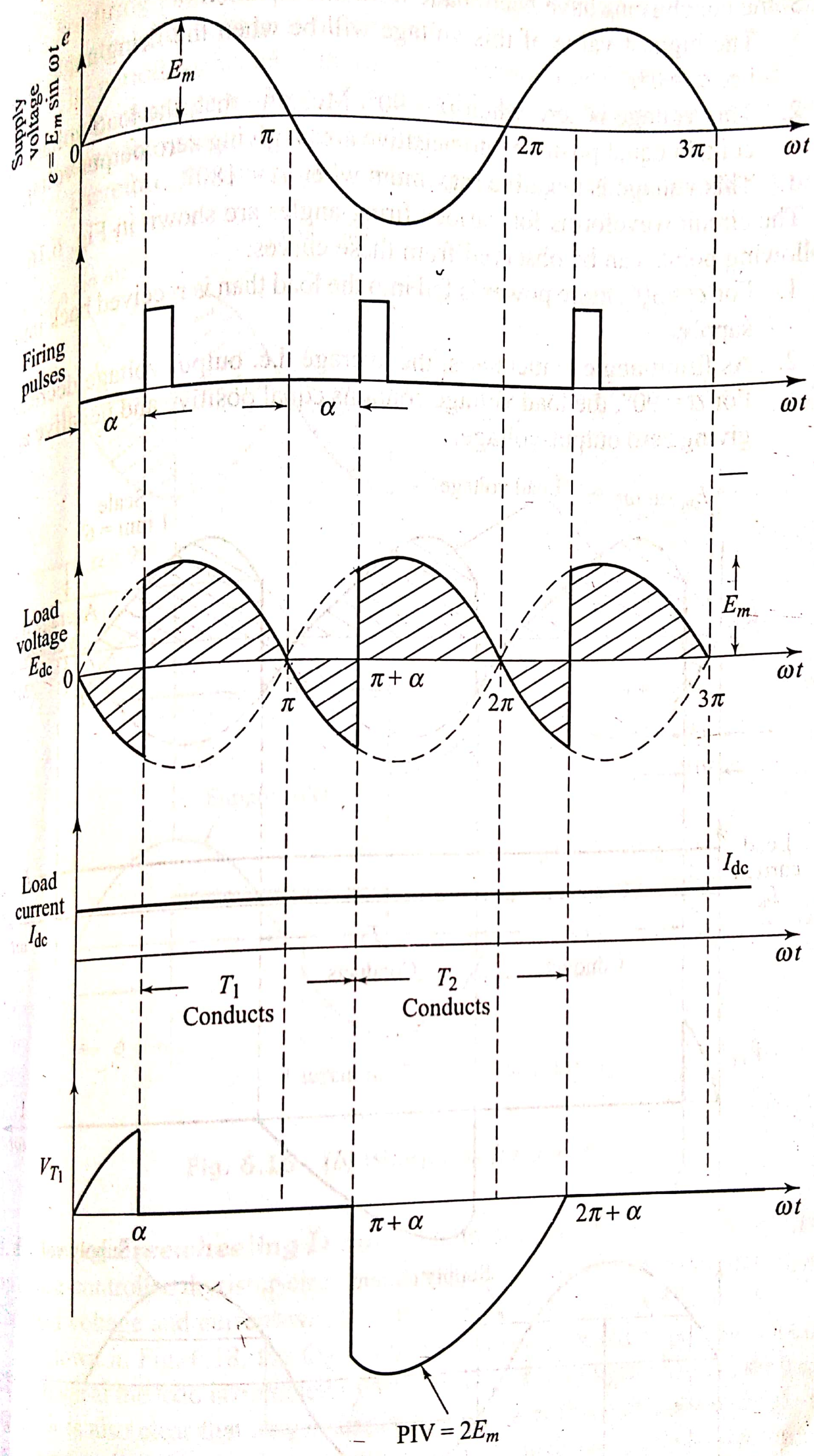


**Fig. 6.15** *Waveforms for M-2 connection with R-L load*