Laplace transform M-1

$f(t) \xrightarrow{L} Lf(t) = F(s)$

Formulae

$$L(f(t)) = \int_0^{\infty} e^{-st} f(t)\, dt = \bar{f}(s)$$

$L^{-1}(\bar{f}(s)) = f(t)$ (inverse laplace transform).

$a^t = \frac{1}{s - \log a}$

1. $a = a/s$
2) $e^{at} = \frac{1}{s-a}$
3) $\cosh at = \frac{s}{s^2 - a^2}$
4) $\cos at = \frac{s}{s^2 + a^2}$
5) $\sinh at = \frac{a}{s^2 - a^2}$
6) $\sin at = \frac{a}{s^2 + a^2}$
7) $t^n = \frac{\Gamma(n+1)}{s^{n+1}}$

$t^n = \frac{n!}{s^{n+1}}$, $n = 1, 2, 3, \ldots$

$t = 1/s^2$

Properties of laplace transform

$L(f(t)) = \bar{f}(s)$

1) $L(e^{at} f(t)) = \bar{f}(s-a)$
2) $L(t^n f(t)) = (-1)^n \frac{d^n}{ds^n}(\bar{f}(s))$
3. $L\left(\frac{f(t)}{t}\right) = \int_s^{\infty} \bar{f}(s)\, ds$

$\Gamma n = n-1\, \Gamma n-1$

$\Gamma \tfrac{1}{2} = \sqrt{\pi}$

$\Gamma \tfrac{7}{2} = \tfrac{5}{2} \cdot \tfrac{3}{2} \cdot \tfrac{1}{2} \sqrt{\pi}$

$$\int \frac{1}{s^2 + a^2}\, ds = \frac{1}{a} \tan^{-1}\left(\frac{s}{a}\right)$$

$$\int \frac{s}{s^2 + a^2}\, ds = \frac{1}{2} \log(s^2 + a^2)$$

$$\int \frac{1}{s-a} = \log(s-a)$$

$\log\left(\frac{a}{b}\right) = \log a - \log b$

$\log(a \times b) = \log a + \log b$

$\log a^b = b \log a$

$\log_b a = \frac{1}{\log_a b}$

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$\sin A \sin B = -\frac{1}{2}[\cos(A+B) - \cos(A-B)]$

$\cosh at = \frac{e^{at} + e^{-at}}{2}$ $\qquad$ $\sinh at = \frac{e^{at} - e^{-at}}{2}$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$ $\qquad$ $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$L(e^{at} f(t)) =$

$L(f(t)) = f(s)$

$L(e^{at} f(t)) = F(s-a)$

$L(t^n f(t)) = (-1)^n \frac{d^n(f(s))}{ds^n}$

$L(f(t)) = f(s)$

$(uv)' = uv' + vu'$

$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$

$L\left(\frac{f(t)}{t}\right) = \int_s^\infty f(s)\, ds$

$L(f(t)) = f(s)$

$\int(u\,dv) = uv - \int v\,du$

$\int uv = u\int v - u'\iint v + u''\iiint v$

$L(f(t) + g(t)) = Lf(t) + L(g(t))$

$L(af(t)) = aLf(t)$

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$$L\left(2^t + \frac{\cos 2t - \cos 3t}{t} + t^5 e^{3t} \sinh t\right]$$

$$L(2^t) + L\left(\frac{\cos 2t - \cos 3t}{t}\right) + L(t^5 e^{3t} \sinh t]$$

$$L(2^t) = \frac{1}{s - \log 2}$$

$$L\left(\frac{\cos 2t - \cos 3t}{t}\right) = L\left(\frac{f(t)}{t}\right) = \int_s^\infty f(s)\, ds$$

$$L(f(t)) = \frac{s}{s^2+4} - \frac{s}{s^2+9}$$

$$= \int_s^\infty \left(\frac{s}{s^2+4} - \frac{s}{s^2+9}\right) ds$$

$$= \frac{1}{2}\log(s^2+4) - \frac{1}{2}\log(s^2+9)\Big]_s^\infty$$

$$= \frac{1}{2}\Big[\log(s^2+4) - \log(s^2+9)\Big]_s^\infty$$

$$= \frac{1}{2}\left(\log\left(\frac{s^2+4}{s^2+9}\right)\right|_s^\infty =$$

$$= \lim_{s\to\infty}\left[\frac{1}{2}\log\left(\frac{\cancel{s^2}[1+4/s^2]}{\cancel{s^2}[1+9/s^2]}\right) - \frac{1}{2}\log\left(\frac{s^2+4}{s^2+9}\right)\right]$$

$$-\frac{1}{2}\left[\log\left(\frac{s^2+4}{s^2+9}\right)\right]$$

$-\log m + \log n$

$$\frac{1}{2}\left[\log\left(\frac{s^2+9}{s^2+4}\right)\right]$$

$$\log\left(\frac{s^2+9}{s^2+4}\right)^{1/2}$$

$$L(t^5 e^{3t} \sinh t)$$

$$L\left(t^5 e^{3t}\left[\frac{e^{t}-e^{-t}}{2}\right]\right)$$

$$\frac{1}{2}L\left[t^5 e^{4t} - t^5 e^{2t}\right]$$

$$\frac{1}{2}\left[L(t^5 e^{4t}) - L(t^5 . e^{2t})\right]$$

$$L(t^5) = \frac{5!}{s^6}$$

$$L(t^5 e^{4t}) = \frac{5!}{(s-4)^6}$$

$$L(t^5 e^{2t}) = \frac{5!}{(s-2)^6}$$

$$\frac{1}{2}\left[\frac{5!}{(s-4)^6} + \frac{5!}{(s-2)^6}\right]$$

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$$e^{2t}\cos^2 t = e^{2t}\left[\frac{1+\cos 2t}{2}\right]$$

$$= \frac{1}{2}\left[e^{2t}\left[1+\cos 2t\right]\right]$$

$$f(t) = 1+\cos 2t$$

$$f(s) = \frac{1}{s} + \frac{s}{s^2+4}$$

$$L\left(e^{2t}\left[1+\cos 2t\right]\right) = \frac{1}{s-2} + \frac{s-2}{(s-2)^2+4}$$

$$L\left(\frac{e^{at}-e^{bt}}{t}\right) =$$

$$L\left(e^{at}-e^{bt}\right) = \frac{1}{s-a} - \frac{1}{s-b}$$

$$= \int_s^\infty \left(\frac{1}{s-a} - \frac{1}{s-b}\right) ds$$

$$= \log(s-a) - \log(s-b) \Big|_s^\infty$$

$$= \log\left(\frac{s-a}{s-b}\right)\Big|_s^\infty$$

$$= \log\left(\frac{s-b}{s-a}\right)$$

$$L\left(e^{-3t}\sin 5t \sin 3t\right)$$

$$\sin 5t \sin 3t = -\tfrac{1}{2}\left[\cos 8t - \cos 2t)\right]$$

$$L\left(e^{-3t}\, -\tfrac{1}{2}\left((\cos 8t - \cos 2t)\right]\right.$$

$$\tfrac{1}{2}\, L\left[\cos 2t\, e^{-3t} - e^{-3t}\cos 8t)\right]$$

$$L(\cos 2t) = \frac{s}{s^2+4}$$

$$L\left(e^{-3t}\cos 2t\right) = \frac{s+3}{(s+3)^2+4}$$

$$L(\cos 8t) = \frac{s}{s^2+64}$$

$$L\left(e^{-3t}\cos 8t\right) = \frac{s+3}{(s+3)^2+64}$$

$$\tfrac{1}{2}\left[\frac{s+3}{(s+3)^2+4} + \frac{s+3}{(s+3)^2+64}\right]$$

$$L\left(\frac{\cos at - \cos bt}{t} + t\sin at\right)$$

$$L\left(\frac{\cos at - \cos bt}{t}\right) + L(t\sin at)$$

$$L\left(\frac{\cos at - \cos bt}{t}\right) = \int_s^{\infty} f(s)\,ds$$

$$L(\cos at - \cos bt) = \frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}$$

$$\int_s^{\infty}\left(\frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}\right)ds$$

$$= \frac{1}{2}\log(s^2+a^2) - \frac{1}{2}\left[\log(s^2+b^2)\right]\Big|_s^{\infty}$$

$$= \frac{1}{2}\log\left(\frac{s^2+a^2}{s^2+b^2}\right)\Big|_s^{\infty}$$

$$= \frac{1}{2}\log\left(\frac{s^2+b^2}{s^2+a^2}\right)$$

$$= \sqrt{\log\left(\frac{s^2+b^2}{s^2+a^2}\right)}$$

$L(t \sin at)$

$L(\sin at) = \frac{a}{s^2+a^2}$

$L(t \sin at) = (-1)^1 \frac{d}{ds}\left(\frac{a}{s^2+a^2}\right)$

$= (-1) \frac{-(a)(2s)}{(s^2+a^2)^2}$

$= \frac{2as}{(s^2+a^2)^2}$

$L(t e^{2t}) = \cancel{\frac{1}{s^2}}$

$L(t) = 1/s^2$

$L(e^{2t}\, t) = \frac{1}{(s-2)^2}$

$L\left(\frac{2 \sin 3t}{t}\right) = 2 L\left(\frac{\sin 3t}{t}\right)$

$L(\sin 3t) = \frac{3}{s^2+9}$

$\int_s^\infty \frac{3}{s^2+9}\, ds$

$= \tan^{-1}\left(\frac{s}{3}\right)\Big|_s^\infty$

$= \tan^{-1}(\infty) - \tan^{-1}\left(\frac{s}{3}\right)$

$= \pi/2 - \tan^{-1}\left(\frac{s}{3}\right)$

$L\left(\sqrt{e^{4(t+3)}}\right)$

$= e^{4(t+3)/2} = e^{2(t+3)}$

$= e^{2t+6}$

$= e^{2t} \cdot e^{6}$

$= e^{6} L(e^{2t})$

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Periodic

$f(t+T) = f(t)$

T period

$L(f(t)) = \frac{1}{1-e^{-sT}} \int_0^T f(t) e^{-st} dt$

$$f(t) = \begin{cases} t & 0<t<a \\ 2a-t & a<t<2a \end{cases}$$

$$f(t+2a) = f(t)$$

$$T = 2a$$

$$L(f(t)) = \frac{1}{1-e^{-s2a}} \int_0^{2a} f(t)\, e^{-st}\, dt$$

$uv = uv' + vu'$

$$= \frac{1}{1-e^{-2as}} \int_0^a \underset{u}{t}\, \underset{dv}{e^{-st} dt} + \int_a^{2a} 2a-t\, e^{-st} dt$$

$\int u\,dv = uv - \int v\,du$

$$dv = e^{-st}$$

$$v = \frac{e^{-st}}{-s}$$

$\int (uv) = u\int v - u' \int\int v$

$$= \frac{1}{\cancel{1-e^{-2as}}} \left[\cancel{t \frac{e^{-st}}{-s}} - \int \cancel{e^{-st}} \right.$$

$$= \frac{1}{1-e^{-2as}} \left[t \frac{e^{-st}}{-s} - 1 \frac{e^{-st}}{s^2} \right]_0^a + \left[(2a-t) \frac{e^{-st}}{-s} - \frac{(-1)e^{-st}}{s^2} \right]_a^{2a}$$

$$= \frac{1}{-s(1-e^{-2as})} \left[+a\cancel{e^{-as}} + \frac{\cancel{e^{-sa}}}{\cancel{s}} - \frac{1}{s} \right] + \left[-\frac{e^{-2as}}{s} - \frac{ae^{-as}}{s} + \frac{e^{-as}}{s} \right.$$

$$\left. + \frac{e^{-2as}}{s} - \cancel{ae^{-as}} + \frac{\cancel{e^{-as}}}{\cancel{s}} \right]$$

$$= \cancel{\frac{1}{-s[1-e^{-2as}]}\left[2a\,e^{-as} - \frac{1}{s} - \frac{e^{-2as}}{s}\right]}$$

$$= \cancel{\frac{1}{[1-e^{-2as}]}}\left[2\right.$$

$$= \frac{1}{-s[1-e^{-2as}]}\left[\frac{2e^{-as}}{s} - \frac{1}{s} - \frac{e^{-2as}}{s}\right]$$

$$= \frac{1}{s^2[1-e^{-2as}]}\left[e^{-2as} + 1 - 2e^{-as}\right]$$

$a^2 + b^2 - 2ab$

$a^2 - b^2$

$$= \frac{1\,[1-e^{-as}]^2}{s^2[1-e^{-as}][1+e^{-as}]}$$

$$= \frac{1}{s^2}\frac{[1-e^{-as}]}{[1+e^{-as}]}\frac{e^{as/2}}{e^{as}}$$

$$= \frac{1}{s^2}\tanh(as/2)$$

$$f(t) = \begin{cases} E & 0 < t < a/2 \\ -E & a/2 < t < a \end{cases}$$

$$f(t+a) = f(t)$$

$$T = a$$

$$L(f(t)) = \frac{1}{1-e^{-sT}} \int_0^T f(t)\, e^{-st}\, dt$$

$$= \frac{1}{1-e^{-as}} \int_0^a f(t)\, e^{-st}\, dt$$

$$= \frac{1}{1-e^{-as}} \int_0^{a/2} E\, e^{-st}\, dt + \int_{a/2}^{a} -E\, e^{-st}\, dt$$

$$= \frac{E}{1-e^{-as}} \left[\left. \frac{e^{-st}}{-s} \right|_0^{a/2} - \left[\frac{e^{-st}}{-s} \right]_{a/2}^{a} \right]$$

$$= \frac{E}{1-e^{-as}} \left[-\frac{e^{-sa/2}}{s} + 1/s \right] + \left(\frac{e^{-as}}{s} - \frac{e^{-as/2}}{s} \right)$$

$$= \frac{E}{s[1-e^{-as}]} \left[1 + e^{-as} - 2e^{-as/2} \right]$$

$$[a^2 - b^2]$$

$$= (a+b)(a-b)$$

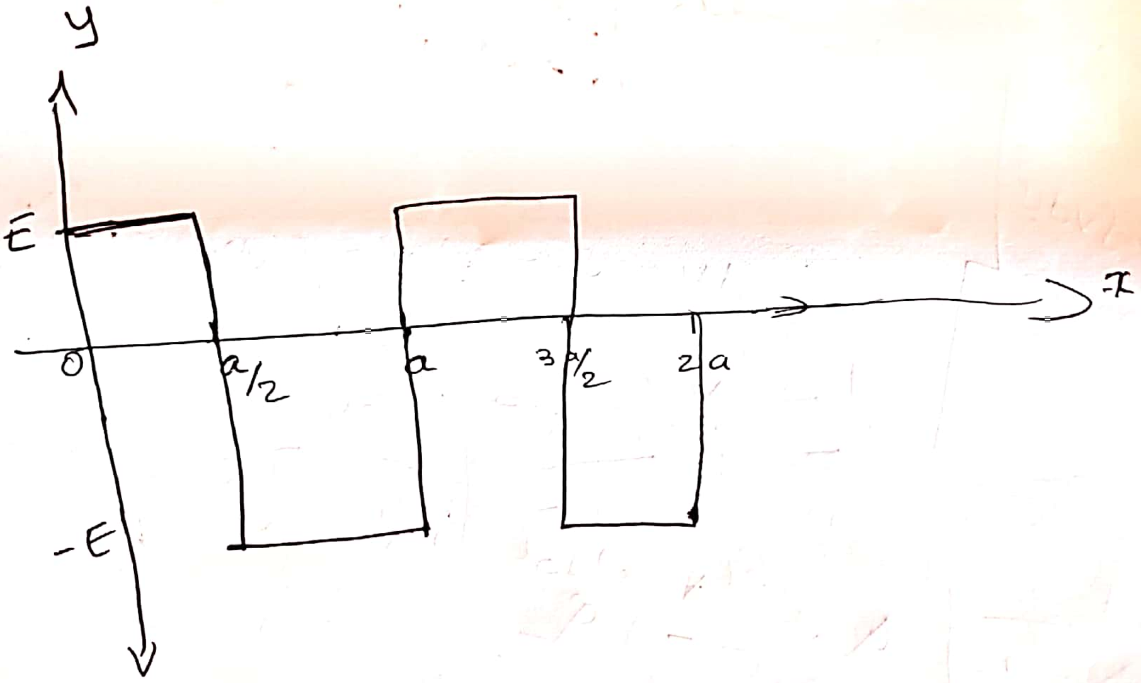
$$a^2 + b^2 - 2ab = (a-b)^2$$

$$[1 - e^{-as/2}]^2$$

$$= \frac{E \left[1 - e^{-as/2}\right]^{\cancel{2}}}{s\left[1 + e^{-as/2}\right]\cancel{\left[1 - e^{-as/2}\right]}}$$

$$= \frac{E\left[1 - e^{-as/2}\right]\left[e^{as/4}\right]}{s\left[1 + e^{-as/2}\right] e^{as/4}} = \frac{E}{s} \frac{\left[e^{as/4} - e^{-\frac{as}{4}}\right]}{\left[e^{+as/4} + e^{-as/4}\right]}$$

$$= \frac{E}{s} \frac{\sinh \frac{as}{4}}{\cosh \frac{as}{4}} = \frac{E}{s} \tanh \frac{as}{4}$$



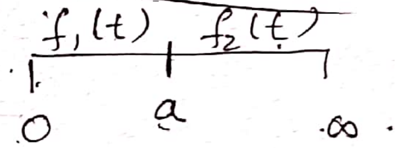
Unit step

$$L(f(t-a)\, u(t-a)) = e^{-as} f(s)$$

$$L(f(t-a) \to L f(t)$$

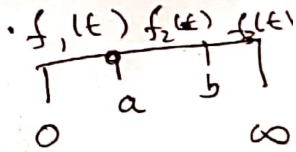
$$L(u(t-a)) = \frac{e^{-as}}{s}$$

$$f(t) = \begin{cases} f_1(t) & 0 < t < a \\ f_2(t) & t \geq a \end{cases}$$

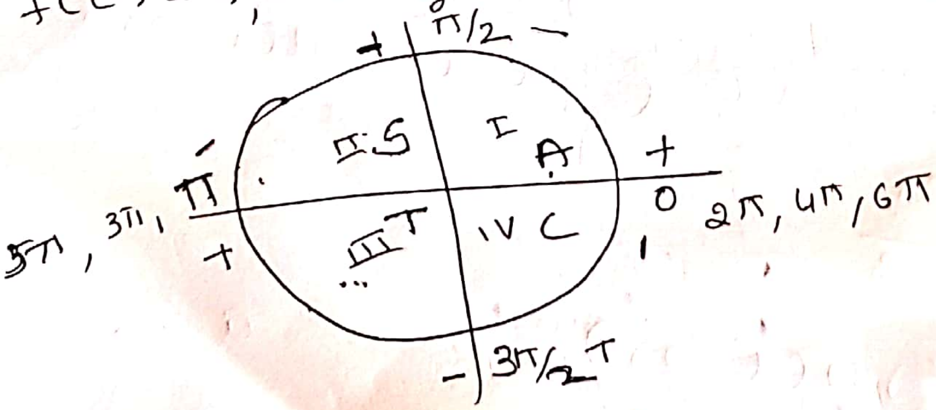


$$f(t) = f_1(t) + (f_2(t) - f_1(t))\, u(t-a)$$

$$f(t) = \begin{cases} f_1(t) & 0 < t < a \\ f_2(t) & a < t < b \\ f_3(t) & t \geq b \end{cases}$$



$$f(t) = f_1(t) + (f_2(t) - f_1(t))\, u(t-a) + [f_3(t) - f_2(t))\, u(t-b)$$



$\pi/2$

$\sin \leftrightarrow \cos$

$\pi, 2\pi, 3\pi$

$\sin \leftrightarrow \sin$

$\cos \leftrightarrow \cos$

$\sin(\pi/2 + \theta) = \cos\theta$

$\cos(\pi/2 + \theta) = -\sin\theta$

$\sin(\pi + \theta) = -\sin\theta$

$\cos(\pi + \theta) = -\cos\theta$

$\cos(4\pi + \theta) = +\cos\theta$

$$f(t) = \begin{cases} t^2 \quad f_1(t) & 0 < t \le 2 \\ 4t \quad f_2(t) & t \ge 2 \end{cases}$$

$f(t) = f_1(t) + f_2(t) - f_1(t) \quad u(t-a)$

$= t^2 + (4t - t^2)\, u(t-2)$

$= L(t^2) + L(4t - t^2)\, u(t-2))$

$L(t^2) = \dfrac{2!}{s^3}$

$L(\underline{(4t - t^2)}\, u(t-2))$ $\quad a = 2$

$f(t-a) \quad u(t-a)$

$f(t-2) = 4t - t^2$

$f(t) = 4(t+2) - (t+2)^2$

$= \cancel{4t} + 8 - [t^2 + \cancel{4t} + 4]$

$f(t) = 4 - t^2$

$f(s) = 4/s - \frac{2!}{s^3}$

$L(4t - t^2)\ u(t-2)) = e^{-2s}\left[4/s - \frac{2!}{s^3}\right]$

$f(t) = \begin{cases} \cos t & 0 < t < \pi \\ \cos 2t & \pi < t < 2\pi \\ \cos 3t & t \geq 2\pi \end{cases}$

$f(t) = \cos t + (\cos 2t - \cos t)\, u(t-\pi) + (\cos 3t - \cos 2t)\, u(t-2\pi)$

$= L(\cos t) + L(\cos 2t - \cos t)\, u(t-\pi) + L((\cos 3t - \cos 2t)\, u(t-2\pi)$

$L(\cos t) = \frac{s}{s^2+1}$

$L(f(t-a)\, u(t-a)) = L(\cos 2t - \cos t \quad u(t-\pi)) = e^{-\pi s} f(s)$

$a = \pi$

$f(t) = \cos 2(t+\pi) - \cos(t+\pi)$

$= \cos(2t+2\pi) - \cos(t+\pi)$

$= \cos 2t + \cos t$

$L(\cos 2t + \cos t) = \frac{s}{s^2+4} + \frac{s}{s^2+1}$

$$= e^{-\pi s}\left[\frac{s}{s^2+4} + \frac{s}{s^2+1}\right]$$

Q $L((\cos 3t - \cos 2t)\, u(t-2\pi)) = e^{-2\pi s} f(s)$

$f(t-a) = \cos 3t - \cos 2t \qquad a = 2\pi$

$f(t) = \cos 3(t+2\pi) - \cos 2(t+2\pi)$

$= \cos(3t+6\pi) - \cos(2t+4\pi)$

$= \cos 3t - \cos 2t$

$$L(f(t)) = \frac{s}{s^2+9} - \frac{s}{s^2+4}$$

$$= e^{-2\pi s}\left[\frac{s}{s^2+9} - \frac{s}{s^2+4}\right]$$

$$f(t) = \begin{cases} \cos t & 0 \le t \le \pi \\ 1 & \pi \le t \le 2\pi \\ \sin t & t > 2\pi \end{cases}$$

$f(t) = \cos t + (1 - \cos t)\, u(t-\pi) + (\sin t - 1)\, u(t - 2\pi)$

$L(f(t)) = L(\cos t) + L(1 - \cos t)\, u(t-\pi) + L(\sin t - 1)\, u(t - 2\pi)$

$$L(\cos t) = \frac{s}{s^2+1}$$

$L(1-\cos t)\, u(t-\pi) = e^{-\pi s} \bar{f}(s)$

$\cancel{f(t)} \Rightarrow f(t-a)\, u(t-a) = (1-\cos t)\, u(t-\pi)$

$a = \pi$

$L(f(t-\pi+\pi)) = (1 - \cos(t+\pi))$

$= 1 + \cos t$

$$L(f(t)) = \frac{1}{s} + \frac{s}{s^2+1}$$

$$= e^{-\pi s}\left(\frac{1}{s} + \frac{s}{s^2+1}\right).$$

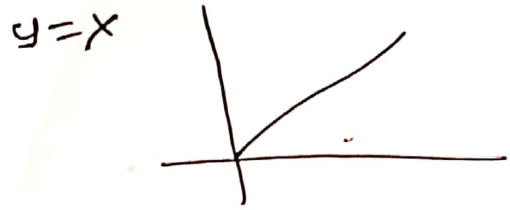
$L(\sin t - 1)\, u(t - 2\pi) = e^{-2\pi s} \bar{f}(s)$

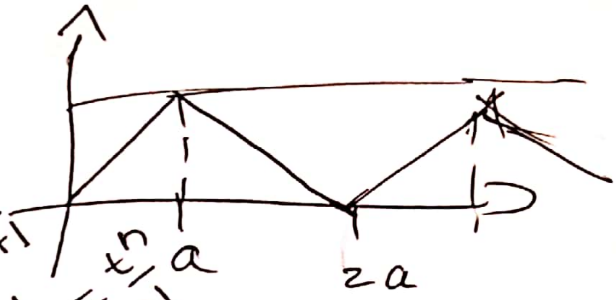
$L \quad \sin(t + 2\pi) - 1$

$\sin t - 1$

$$\frac{1}{s^2+1} - \frac{1}{s}$$

$$\Rightarrow e^{-2\pi s}\left(\frac{1}{s^2+1} - \frac{1}{s}\right)$$





$t^n = \frac{n!}{s^{n+1}}$

$\frac{1}{s^{n+1}} = \frac{t^n}{n!}$

f(s)	f(t)
a/s	a
$1/s$	1
$\frac{1}{s-a}$	e^{at}
$\frac{1}{s+a}$	e^{-at}
$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$
$1/s^2$	t
$\frac{1}{s^2+a^2}$	$\frac{1}{a}\sin at$
$\frac{s}{s^2+a^2}$	$\cos at$
$\frac{1}{s^2-a^2}$	$1/a \sinh at$

Convolution

$$L^{-1}\left(\overline{f(s)}\,\overline{g(s)}\right) = \int_0^t f(u)\, g(t-u)\, du$$

$f(u) = f(t)$

$L^{-1}(f(s)) = f(t) \to f(u)$

$L^{-1}(g(s)) = g(t) \to g(t-u)$

$$\frac{1}{s(s^2+a^2)} = \frac{1}{s} \quad \frac{1}{s^2+a^2}$$

$f(s) \quad G(s)$

$L^{-1}(F(s)) = 1 = f(t)$

$L^{-1}(G(s)) = \frac{1}{a}\sin at = g(t)$

$f(u) = 1$

$g(t-u) = \frac{1}{a}\sin(a(t-u))\, du$

$$= \int_0^t 1 \quad \sin(at - au))\, du$$

$$= \frac{-\cos(at-au)}{-a^2}\Bigg|_0^t = \left[\frac{1}{a^2} - \frac{\cos at}{a^2}\right]$$

$$\frac{s}{(s^2+a^2)^2} = \frac{s}{(s^2+a^2)(s^2+a^2)}$$

$$= \frac{s}{s^2+a^2} \quad \frac{1}{s^2+a^2}$$

$$F(s) \qquad G(s)$$

$$L^{-1}(F(s)) = \cos at = f(t)$$

$$L^{-1}(G(s)) = \frac{1}{a}\sin at = g(t)$$

$$f(u) = \cos au$$

$$g(t-u) = \frac{1}{a}\sin(a(t-u)) = \frac{1}{a}\sin(at-au))$$

$$\frac{1}{a}\int_0^t \cos au \ \sin(at-au)\,du$$

$$\frac{1}{a}\int_0^t \frac{1}{2}\left[\sin(\cancel{au}+at-\cancel{au}] - \sin(au-(at-au))du\right.$$

$$\frac{1}{2a}\int_0^t [\sin at - \sin(au-at+au)du$$

$$\frac{1}{2a}\int_0^t [\sin at - \sin(2au-at)du$$

$$= \frac{1}{2a}\left[\sin at\cdot u + \frac{\cos(2au-at)}{2a}\right]_0^t$$

$$= \frac{1}{2a}\left[\sin at\,[t] + \frac{1}{2a}\left[\cos \cancel{a}t - \cos \cancel{a}t\right)\right]$$

$$= \frac{t \sin at}{2a}$$

$$\frac{s^2}{(s^2+a^2)(s^2+b^2)} = \frac{s}{s^2+a^2} \cdot \frac{s}{s^2+b^2}$$

$\underset{\cancel{\cos at}}{f(t)}$ $\qquad$ $\underset{\cancel{\cos bt}}{g(t)}$.

$f(s) = \cos at = f(t)$

$f(u) = \cos au$

$g(s) = \cos bt = g(t)$

$g(u) = \cancel{\cos bu}\ \cos b(t-u) \Rightarrow \cos(bt - bu)$

$\cancel{L^{-1}(f(t)) = \int_0^t \cos at\, du}$

$$L^{-1}(f(t)) = \int_0^t f(u)\, g(t-u)\, du$$

$$= \int_0^t \cos au \cdot \cos(bt - bu)\, du$$

$$= \cancel{\left[\sin au \cdot u \cdot \frac{\sin(bt-bu)}{-u}\right]_0^t}$$

$$\int_0^t \frac{1}{2}\left(\sin(au + bt - bu) - \cos(\cancel{au}\ \cancel{bt} + bu)\right)$$

$$\frac{s}{(s-1)(s^2+4)} = \frac{1}{(s-1)} \quad \frac{s}{s^2+4}$$

$$f(t) = e^t$$

$$g(t) = \cos 2t$$

$$f(u) = e^u$$

$$g(t-u) = \cos 2(t-u)$$

$$\int_0^t e^u \cos(\underset{\to b}{2t} - \underset{\to bt}{2u})\,du$$

$$\boxed{\begin{aligned} \int e^{at} \cos bt\,dt &= \frac{e^{at}}{a^2+b^2}\left[a\cos bt + b\sin bt\right] \\ \int e^{at} \sin bt\,dt &= \frac{e^{at}}{a^2+b^2}\left(a\sin bt - b\cos bt\right] \end{aligned}}$$

$$\left[\frac{e^u}{1+4}\left[\cos(2t-2u) - 2\sin(2t-2u)\right]\right]_0^t$$

$$= \left[\frac{e^t}{5}(1\right] - \frac{1}{5}\left[\cos 2t - 2\sin 2t\right]$$

$$L^{-1}\left[\frac{s}{(s^2-a^2)^2}\right] = \frac{1}{s^2-a^2} \quad \frac{s}{s^2-a^2}$$

$$f(u) = \frac{1}{a}\sinh au$$

$$g(t-u) = \cosh(a(t-u))$$

$((a+b)c\, c+d)$

$$\frac{1}{a}\int_0^t \sinh au \cosh(at-au)\,du$$

$$\frac{1}{a}\int_0^t \left[\frac{e^{au}-e^{-au}}{2}\right]\left[\frac{e^{at-au}+e^{-at+au}}{2}\right]$$

$$\frac{1}{4a}\int_0^t \left[e^{at}+e^{2au-at}-e^{at-2au}-e^{-at}\right]du$$

$$=\frac{1}{4a}\left[(e^{at}-e^{-at})u + \frac{e^{2au-at}}{2a} + \frac{e^{at-2au}}{2a}\right]_0^t$$

$$=\frac{1}{4a}\left[2\sinh at\,[t]\right] + \frac{e^{at}}{2a} - \frac{e^{-at}}{2a} + \frac{e^{-at}}{2a} - \frac{e^{at}}{2a}$$

$$\frac{s^2}{(s^2+a^2)(s^2+b^2)} = \frac{s}{s^2+a^2} \quad \frac{s}{s^2+b^2}$$

$f(t) = \cos at$

$g(t) = \cos bt$

$f(u) = \cos au$

$\cancel{g(u) = \cos bu}$ +

$g(t-u) = \cos(bt - bu)$

$$\int_0^t f(u)\, g(t-u)\, du = \int_0^t \cos au \cos(bt-bu)\, du$$

$$= \frac{1}{2}\int_0^t [\cos[au + bt - bu] + \cos(au - bt + bu))\, du$$

$$= \frac{1}{2}\int_0^t \cos[bt + (a-b)u) + \cos((a+b)u - bt)\, du$$

$$= \frac{1}{2}\left[\frac{\sin[bt + (a-b)u]}{a-b} + \frac{\sin(a+b)u - bt]}{a+b}\right]_0^t$$

$$= \frac{1}{2}\left(\frac{\sin(\cancel{bt} + at - \cancel{bt})}{a-b} + \frac{\sin(a+b)t - \cancel{bt}}{a+b}\right]$$

$$= \frac{1}{2}\left(\frac{\sin at}{a-b} + \frac{\sin at}{a+b}\right] = \frac{\sin at}{\cancel{2}}\left(\frac{a+\cancel{b}+a-\cancel{b}}{a^2-b^2}\right)$$

$$= \frac{a \sin at}{a^2-b^2}$$