16/08/19

MODULE: 02.

De Morgan's theorem.

I law: $\overline{A+B} = \bar{A}\cdot\bar{B}$.

The complement of sum of 2 variables is equal to product of complement of two individual variables.

A	B	A+B	$\overline{A+B}$	$\bar{A}$	$\bar{B}$	$\bar{A}\cdot\bar{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

II law: $\overline{A\cdot B} = \bar{A}+\bar{B}$.

The complement of product of 2 variables is equal to sum of complement of two individual variables

A	B	A·B	$\overline{A\cdot B}$	$\bar{B}$	$\bar{A}$	$\bar{A}+\bar{B}$
0	0	0	1	1	1	1
0	1	0	1	0	1	1
1	0	0	1	1	0	1
1	1	1	0	0	0	0

Imp **Positive and Negative logic code.**

- The binary number '0' is for low voltage and binary number '1' is for high voltage. and is called positive logic code.
- The binary number '0' is for high voltage and binary number '1' is for low voltage.

and is called negative logic code.

+ve logic.
0 → LOW
1 → HIGH.

-ve logic.
0 → HIGH.
1 → LOW.

① Prove +ve AND to -ve OR using truth table.

Soln:-

A	B	A·B
0	0	0 → LOW
0	1	0 → LOW
1	0	0 → LOW
1	1	1 → HIGH

⟹

A	B	A+B.
1	1	1 → LOW
1	0	1 → LOW
0	1	1 → LOW
0	0	0 → HIGH

② Prove +ve OR to -ve AND.

Soln:

A	B	A+B
0	0	0 → LOW
0	1	1 → HIGH
1	0	1 → HIGH
1	1	1 → HIGH

⟹

A	B	A·B.
1	1	1 → LOW
1	0	0 → HIGH
0	1	0 → HIGH
0	0	0 → HIGH

③ Prove +ve NOR to -ve NAND.

Soln:

A	B	$\overline{A+B}$
0	0	1 → HIGH
0	1	0 → LOW
1	0	0 → LOW
1	1	0 → LOW

⟹

A	B	$\overline{A \cdot B}$
1	1	0 → HIGH
1	0	1 → LOW
0	1	1 → LOW
0	0	1 → LOW

④ Prove +ve NAND to -ve NOR.

Soln:

A	B	$\overline{A \cdot B}$
0	0	1 → HIGH
0	1	1 → HIGH
1	0	1 → HIGH
1	1	0 → LOW

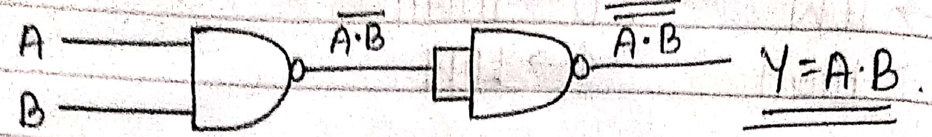
⟹

A	B	$\overline{A+B}$
1	1	0 → HIGH
1	0	0 → HIGH
0	1	0 → HIGH
0	0	1 → LOW.

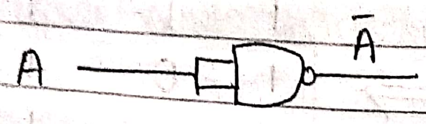
V. Imp.

Prove basic gates using only NAND gate

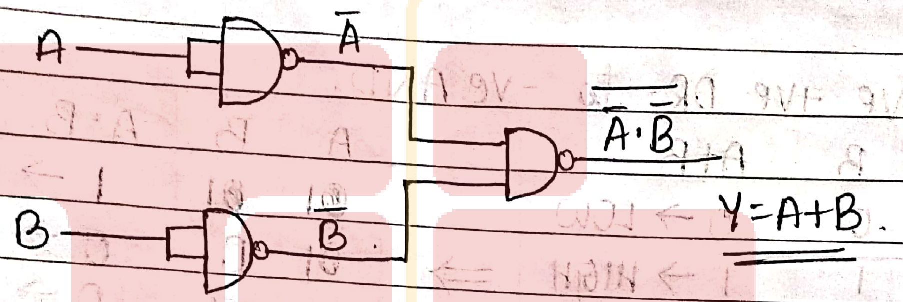
→ $A \cdot B$



→ $A \rightarrow \bar{A}$



→ $A + B$.



Representation of Combinational logic

	A (4)	B (2)	C (1)	
0	0	0	0	→ $\bar{A}\bar{B}\bar{C}$
1	0	0	1	→ $\bar{A}\bar{B}C$
2	0	1	0	→ $\bar{A}B\bar{C}$
3	0	1	1	→ $\bar{A}BC$
4	1	0	0	→ $A\bar{B}\bar{C}$
5	1	0	1	→ $A\bar{B}C$.
6	1	1	0	→ $AB\bar{C}$
7	1	1	1	→ ABC.

Note:

1. Commutative law.

$A+B = B+A$

$AB = BA$.

2. Associative law.

$(A+B)+C = A+(B+C)$.

$(AB)C = A(BC)$.

3. De Morgan's law.

$\overline{A+B} = \bar{A}\cdot\bar{B}$.

$\overline{A\cdot B} = \bar{A}+\bar{B}$.

4. OR operation.

$A+0 = A$.

$A+\bar{A} = 1$.

$A+A = A$.

$A+1 = 1$.

5. AND Operation.

$A\cdot 1 = A$

$A\cdot 0 = 0$.

$A\cdot\bar{A} = 0$.

$\bar{\bar{A}} = A$.

K-Map.

K-Map format.

① $F(A,B)$

A \ B	0	1
0	0	1
1	2	3

② $F(A,B,C)$.

AB \ C	0	1
00	0	1
01	2	3
11	6	7
10	4	5

③ F(A, B, C, D).

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

<u>Definition of summation. (Σ)</u>

Summation symbolises logical OR operation i.e perform on the corresponding min term.

<u>Problems.</u>

1) Plot a K-map. Given:

i) $F(A,B) = \Sigma m(0,1)$.

ii) $F(A,B,C) = \Sigma m(0,5,6,7)$.

iii) $F(A,B,C,D) = \Sigma m(0,4,5,6,8,10,12,13,14,15)$

i)

A \ B	0	1
0	1_0	1_1
1	$_2$	$_3$

ii)

AB \ C	0	1
00	1_0	1_1
01	$_2$	$_3$
11	1_6	1_7
10	$_4$	1_5

iii)

AB \ CD	00	01	11	10
00	1_0	$_1$	$_3$	$_2$
01	1_4	1_5	$_7$	1_6
11	1_{12}	1_{13}	1_{15}	1_{14}
10	1_8	$_9$	$_{11}$	1_{10}

① Find the minimum of the following boolean function using Kmap.

i) $F(A,B,C,D) = \sum m(14,15)$.

AB \ CD	00	01	11	10
00	0_0	0_1	0_3	0_2
01	0_4	0_5	0_7	0_6
11	0_{12}	0_{13}	1_{15}	1_{14}
10	0_8	0_9	0_{11}	0_{10}

$Y = ABCD + ABC\bar{D}$
$= ABC(D+\bar{D})$
$Y = ABC$.

ii) $F(A,B,C,D) = \sum m(3,7)$.

AB \ CD	00	01	11	10
00	0_0	0_1	1_3	0_2
01	0_4	0_5	1_7	0_6
11	0_{12}	0_{13}	0_{15}	0_{14}
10	0_8	0_9	0_{11}	0_{10}

$Y = \bar{A}CD$.

iii) $F(A,B,C,D) = \sum m(5,7,8,12)$.

AB \ CD	00	01	11	10
00	0_0	0_1	0_3	0_2
01	0_4	1_5	1_7	0_6
11	1_{12}	0_{13}	0_{15}	0_{14}
10	1_8	0_9	0_{11}	0_{10}

$Y = y_1 + y_2$
$Y = A\bar{C}\bar{D} + \bar{A}BD$.

iv) $F(A,B,C,D) = \sum m(5,7,13,15)$.

AB \ CD	00	01	11	10
00	0_0	0_1	0_3	0_2
01	0_4	1_5	1_7	0_6
11	0_{12}	1_{13}	1_{15}	0_{14}
10	0_8	0_9	0_{11}	0_{10}

$Y = BD$.

v) $F(A,B,C,D) = \sum m(4,5,6,7,12,13,14,15)$

AB \ CD	00	01	11	10
00	0_{0}	0_{1}	0_{3}	0_{2}
01	1_{4}	1_{5}	1_{7}	1_{6} → Y_1
11	1_{12}	1_{13}	1_{15}	1_{14} → Y_2
10	0_{8}	0_{9}	0_{11}	0_{10}

$$Y = \bar{A}B\bar{C}\bar{D} + AB\bar{C}D + \bar{A}BCD + \bar{A}BC\bar{D} + AB\bar{C}\bar{D} + AB\bar{C}D + ABCD + AB\bar{C}D$$

$$Y = \bar{A}B(\bar{C}\bar{D} + \bar{C}D + CD + C\bar{D}) + AB(\bar{C}\bar{D} + \bar{C}D + CD + C\bar{D})$$

$$Y = \bar{A}B(C \oplus D + \overline{C \oplus D}) + AB(C \oplus D + \overline{C \oplus D})$$

$$= \bar{A}B + AB$$

$$= B(\bar{A} + A)$$

$$\underline{\underline{Y = B}}$$

08/08/19

vi) $F(A,B,C,D) = \sum m(1,2,3,8,9,10,14,6,12,13)$.

AB \ CD	00	01	11	10
00	0_{0}	1_{1}	1_{3}	1_{2}
01	0_{4}	0_{5}	0_{7}	1_{6}
11	1_{12}	1_{13}	0_{15}	1_{14}
10	1_{8}	1_{9}	0_{11}	1_{10}

y_3, y_2, y_1

$$y = y_1 + y_2 + y_3$$

$$y = A\bar{C} + C\bar{D} + \bar{A}\bar{B}D.$$

vii) $F(A,B,C,D) = \sum m(0,1,2,4,5,6,12,13,10,8,9)$

AB \ CD	00	01	11	10
00	1 (0)	1 (1)	(3)	1 (2)
01	1 (4)	1 (5)	(7)	1 (6)
11	1 (12)	1 (13)	(15)	(14)
10	1 (8)	1 (9)	(11)	1 (10)

y_1, y_2, y_3

$Y = y_1 + y_2 + y_3$

$= \bar{C} + \bar{A}\bar{D} + \bar{B}\bar{D}$

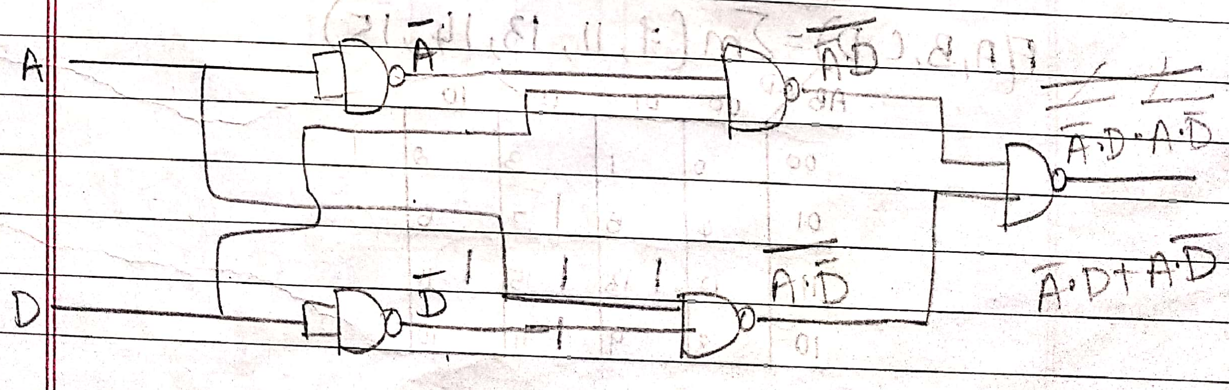
viii) Last year Question

Find the minimal of the SOP expression.
Given; $F(A,B,C,D) = \sum m(1,3,5,7,8,10,12,14)$.
Realise this function using NAND gate only.

AB \ CD	00	01	11	10
00	(0)	1 (1)	1 (3)	(2)
01	(4)	1 (5)	1 (7)	(6)
11	1 (12)	(13)	(15)	1 (14)
10	1 (8)	(9)	(11)	1 (10)

y_2, y_1

$Y = y_1 + y_2$.
$Y = A\bar{D} + \bar{A}D$.



★★ V.Imp (10m)

ix) A system has 4 inputs, the output is h[igh] only when the majority of the inputs are high. * Give truth table & simplify using Kmap. * Write the expression in summation minterm.
* Implement the simplified equation using o[nly] NAND gate.

Soln:-

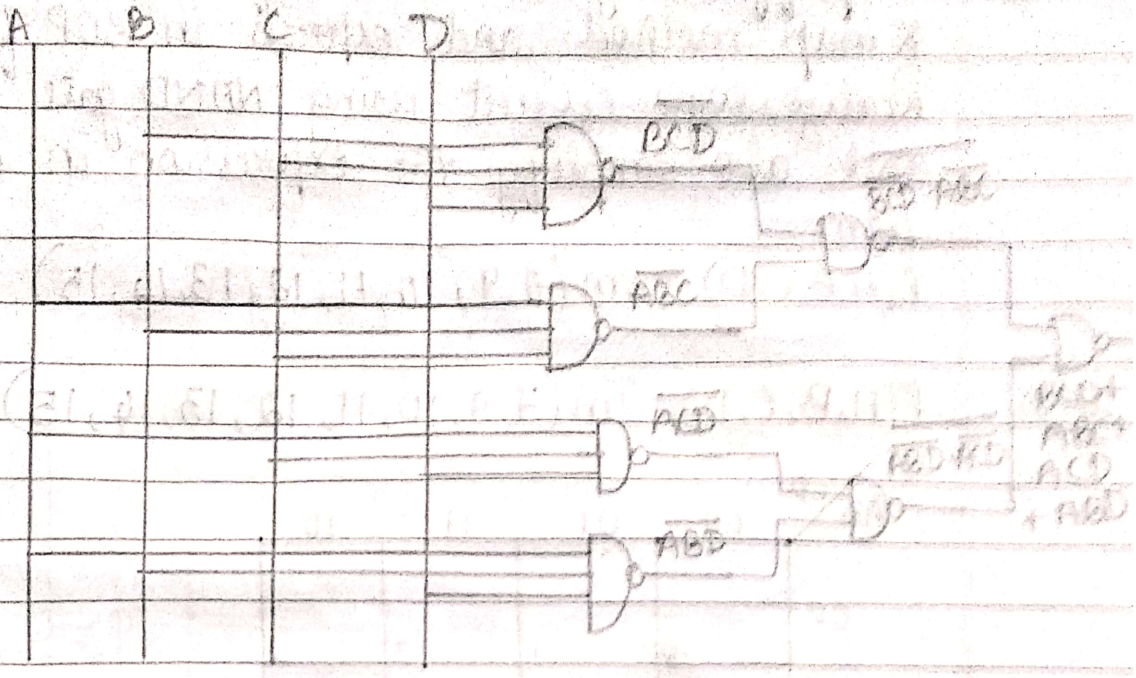
	A (8)	B (4)	C (2)	D (1)	Y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

$F(A,B,C,D) = \Sigma m(7, 11, 13, 14, 15)$

AB \ CD	00	01	11	10	
00	0	1	3	2	→ y_1
01	4	5	1 (7)	6	
11	12	1 (13)	1 (15)	1 (14)	→ y_2
10	8	9	1 (11)	10	
	↓ y_4		→ y_3		

$Y = y_1 + y_2 + y_3 + y_4$.

$Y = BCD + ABC + ACD + ABD$



Product of Sum (POS).

① Simplify the following boolean function using POS form:

$F(A,B,C,D) = \Sigma m(0,1,2,3,4,5,7)$

Solu:

$F(A,B,C,D) = \Pi M(6,8,9,10,11,12,13,14,15)$

AB \ CD	00	01	11	10
00	(0)	(1)	(3)	(2)
01	(4)	(5)	(7)	0 (6)
11	0 (12)	0 (13)	0 (15)	0 (14)
10	0 (8)	0 (9)	0 (11)	0 (10)

$\rightarrow y_1$

$\rightarrow y_2$

$Y = y_1 + y_2$

$= BC\overline{D} + A$ (SOP exp)

POS expression:

$y = \overline{A}(\overline{B} + \overline{C} + D)$.

13/08/19

(a) Simplify the given Boolean function using K map method and express in SOP form. Realize logic circuit using NAND gate only and also simplify the expression in POS form.

$F(A,B,C,D) = \Sigma m(7, 9, 10, 11, 12, 13, 14, 15)$.

Soln:

$F(A,B,C,D) = \Sigma m(7, 9, 10, 11, 12, 13, 14, 15)$

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	1 (7)	6
11	1 (12)	1 (13)	1 (15)	1 (14)
10	8	1 (9)	1 (11)	1 (10)

$\rightarrow y_4$, $\rightarrow y_1$, $\rightarrow y_2$, $\downarrow y_3$

$Y = y_1 + y_2 + y_3 + y_4$
$= AB + AC + AD + BCD$.

$F(A,B,C,D) = \pi M(0, 1, 2, 3, 4, 5, 6, 8)$

AB \ CD	00	01	11	10
00	0 (0)	0 (1)	0 (3)	0 (2)
01	0 (4)	0 (5)	7	0 (6)
11	12	13	15	14
10	0 (8)	9	11	10

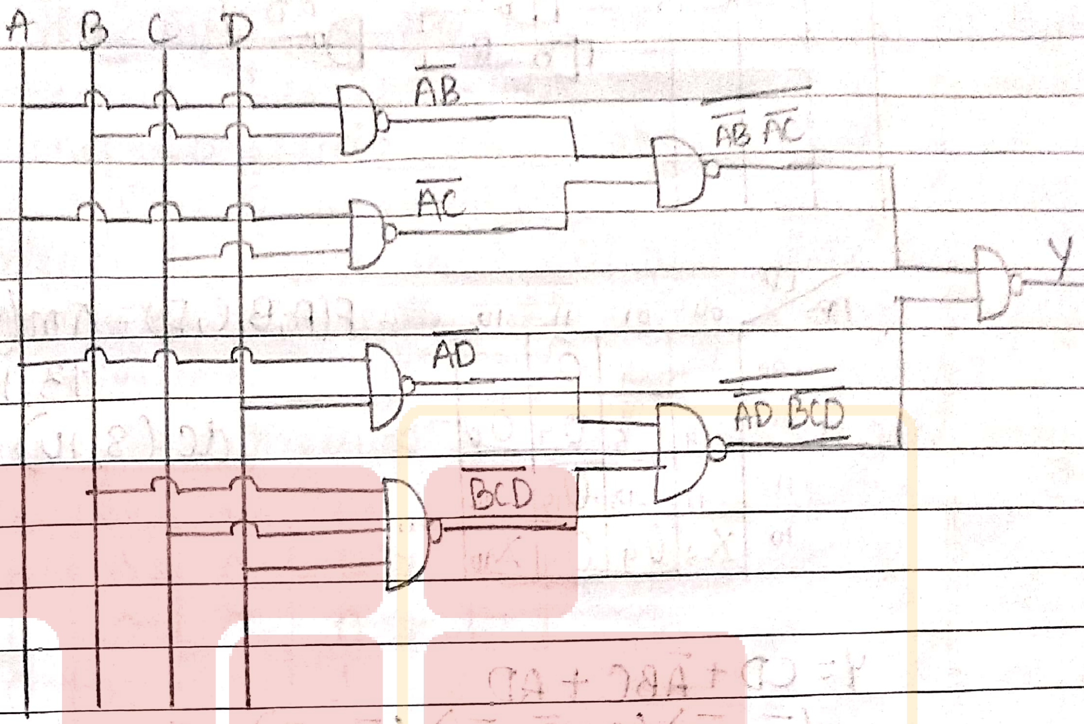
y_1, y_2, y_3, y_4

$Y = y_1 + y_2 + y_3 + y_4$

$Y = \bar{A}\,C + \bar{A}B + \bar{A}\bar{D} + B\bar{C}\bar{D}$.

Realizing using NAND gate :

$Y = AB + AC + AD + BCD$.



<u>Don't Care Condition</u>.

(1) Using K-map Simplify the following boolean expression and give implementation for the Same

1. SOP form - (NAND gate only).
2. POS form - (NOR gate only)

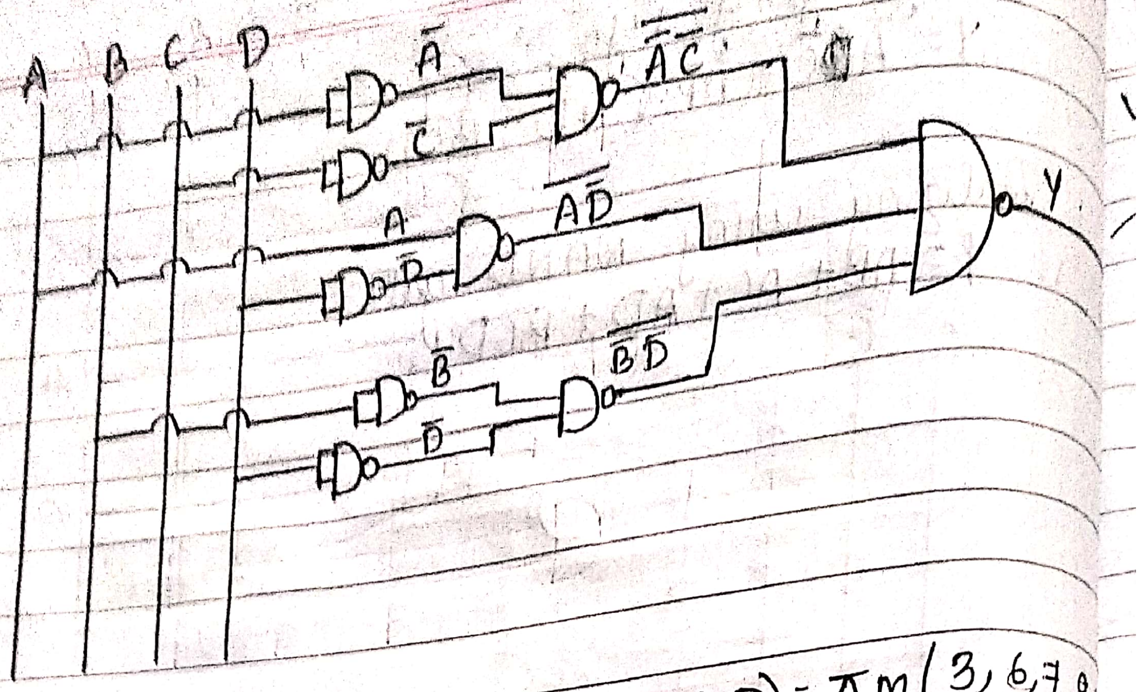
$F(A,B,C,D) = \Sigma m(0,1,2,4,5,12,14) + dc(8,10)$.

AB \ CD	00	01	11	10
00	1_0	1_1	$_3$	1_2
01	1_4	1_5	$_7$	$_6$
11	1_{12}	$_{13}$	$_{15}$	1_{14}
10	X_8	$_9$	$_{11}$	X_{10}

y_1, y_2, y_3

$Y = \bar{A}\bar{C} + A\bar{D} + \bar{B}\bar{D}$.

$\bar{A}\bar{C}$



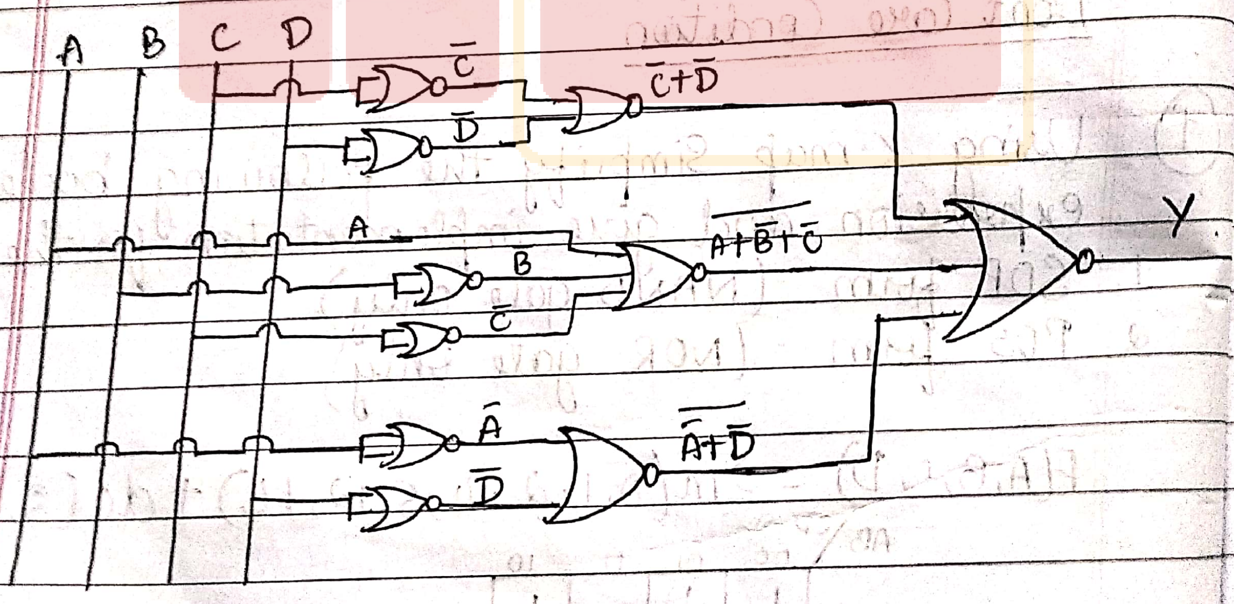
$F(A,B,C,D) = \pi m(3,6,7,9,13,15)$.

$dc(8,10)$.

AB \ CD	00	01	11	10	
00	0	1	0_3	2	$\rightarrow y_1$
01	4	5	0_7	0_6	$\rightarrow y_2$
11	12	0_{13}	0_{15}	14	$\rightarrow y_3$
10	X_8	0_9	0_{11}	X_{10}	

$Y = CD + \bar{A}BC + AD$.

$= (\bar{C}+\bar{D})(A+\bar{B}+\bar{C})(\bar{A}+\bar{D})$. $\boxed{POS}$



V.V. Imp

12m

② A digital system is to be designed in which the month of the year is given as input in 4-bit form. The month January is represented as "0 0 0 0" & February is "0 0 0 1" and so on. The output of the system should be '1' corresponding to the input of the month containing 31 days otherwise it is '0'. Consider the excess no. of input beyond "1 0 1 1" as dont care condition, for the system of 4 variable. Find the following boolean expression and ΣM and max term form. Write the truth table using K-map. Simplify the expression in min term form. Implement simplified expression in NAND-NAND gate only.

Soln:-

		8 A	4 B	2 C	1 D	Y
0	J	0	0	0	0	1
1	F	0	0	0	1	0
2	M	0	0	1	0	1
3	A	0	0	1	1	0
4	M	0	1	0	0	1
5	J	0	1	0	1	0
6	J	0	1	1	0	1
7	A	0	1	1	1	1
8	S	1	0	0	0	0
9	O	1	0	0	1	1
10	N	1	0	1	0	0
11	D	1	0	1	1	1
12		1	1	0	0	X
13		1	1	0	1	X
14		1	1	1	0	X
15		1	1	1	1	X

POS ⇒ $F(A,B,C,D) = \sum m(0,2,4,6,7,9,11) + dc(12,13,14,15)$

AB \ CD	00	01	11	10
00	1 (0)	(1)	(3)	1 (2)
01	1 (4)	(5)	1 (7)	1 (6)
11	X (12)	X (13)	X (15)	X (14)
10	(8)	1 (9)	1 (11)	(10)

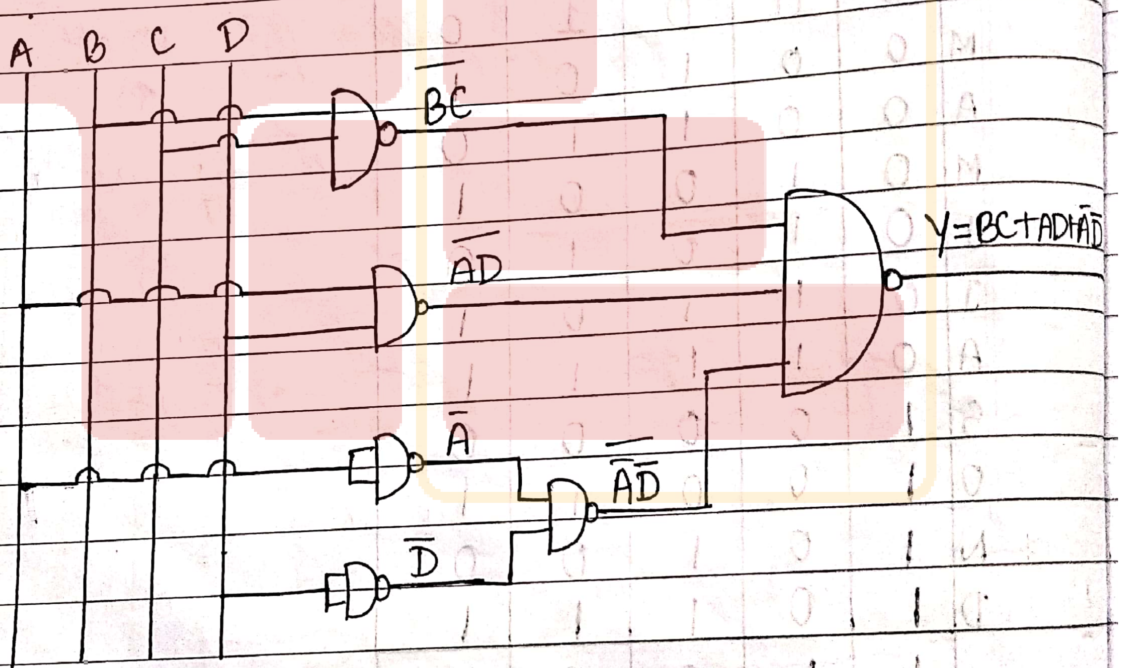
→ y_3

→ y_1

→ y_2

$Y = y_1 + y_2 + y_3$

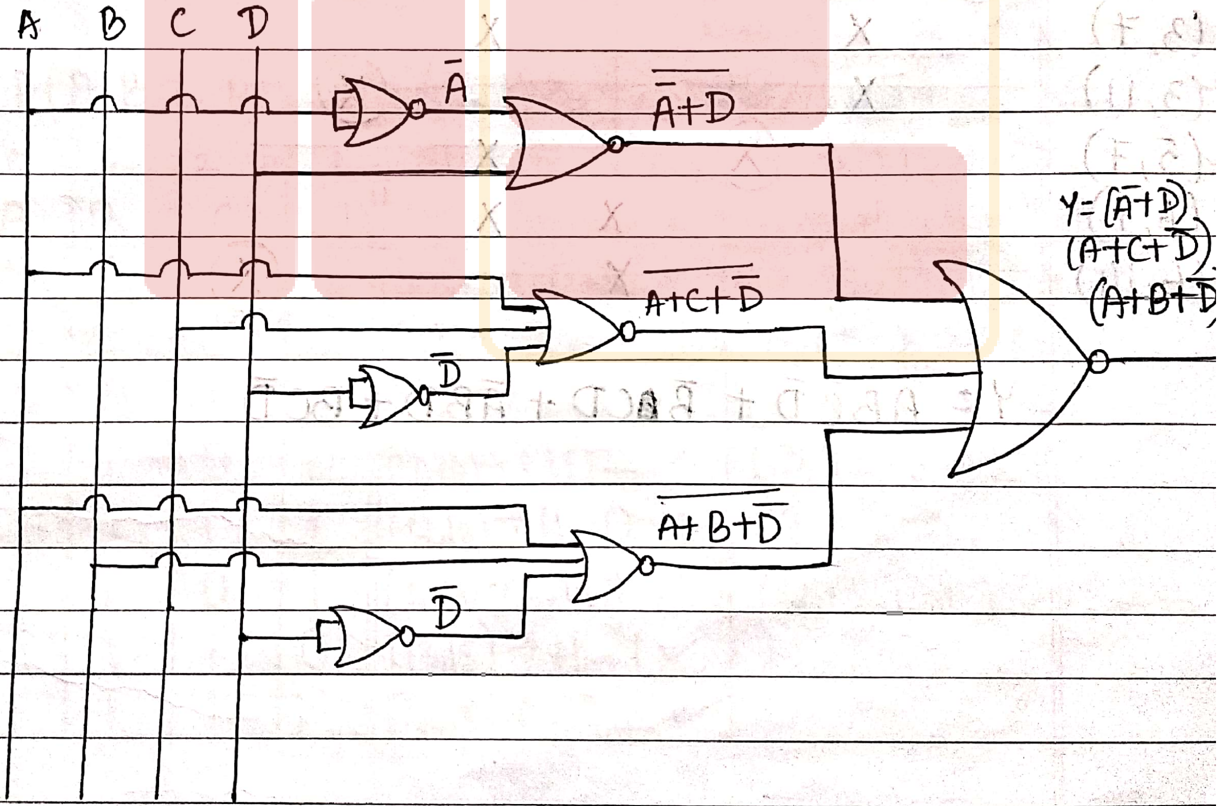
$Y = BC + AD + \bar{A}\bar{D}$



SOP ⇒ $F(A,B,C,D) = \Pi M(1,3,5,8,10) + dc(12,13,14,15)$.

AB \ CD	00	01	11	10	
00	0	0 (1)	0 (3)	2	→ y_3
01	4	0 (5)	7	6	→ y_2
11	X (12)	X (13)	X (15)	X (14)	
10	0 (8)	9	11	0 (10)	→ y_1

$$Y = y_1 + y_2 + y_3$$
$$= \bar{A}\bar{D} + \bar{A}\bar{C}D + \bar{A}\bar{B}D$$
$$= (\bar{A}+\bar{D})(A+C+\bar{D})(A+B+\bar{D})$$



Quine - Mcclusky.

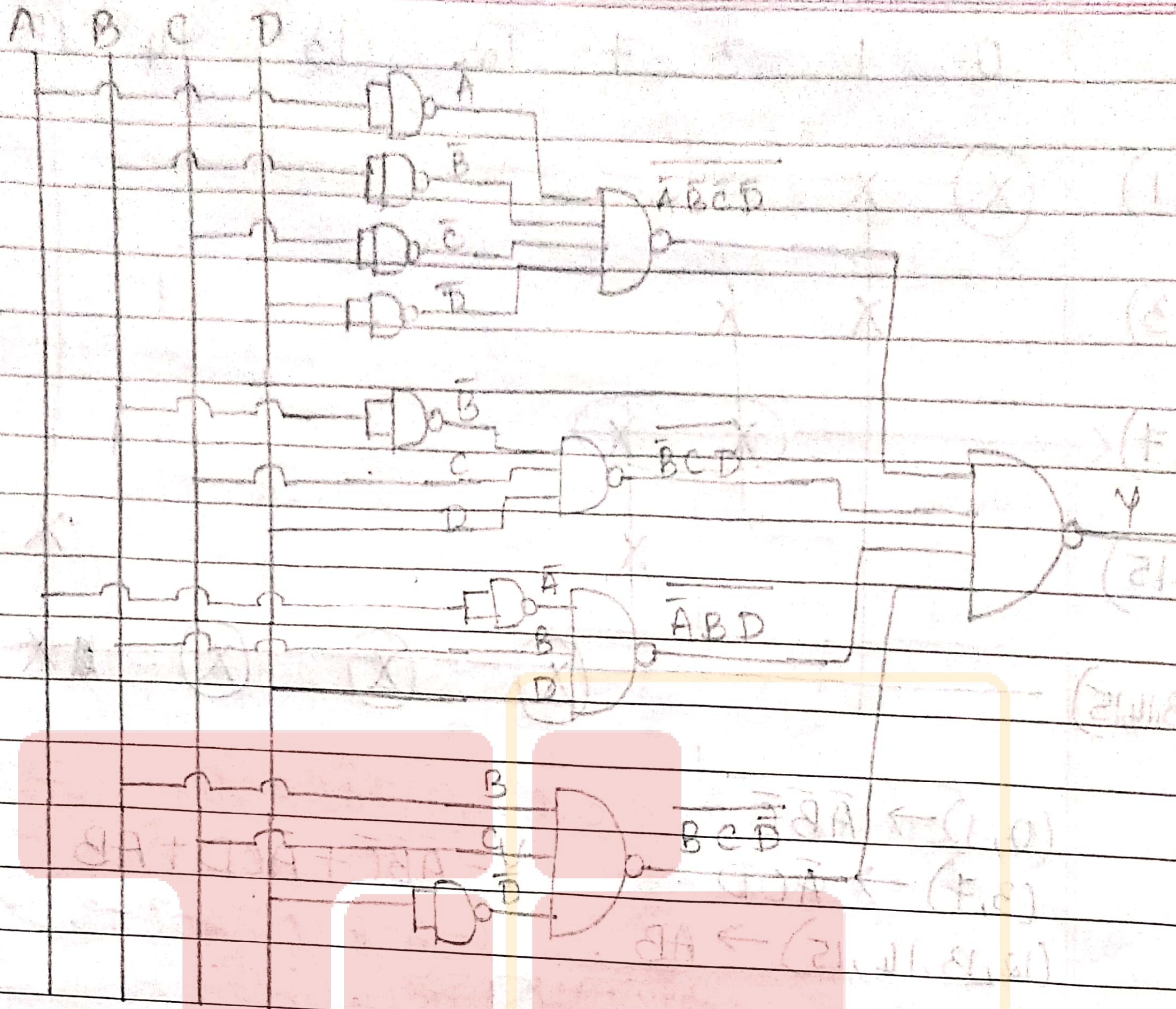
① $F(A,B,C,D) = \sum m(0,3,5,6,7,11,14)$

	STEP-I A	B	C	D	STEP-II
	A	B	C	D	
0	0	0	0	0	0 → 0 0 0 0
3	0	0	1	1	(3,7) → 0 – 1 1
5	0	1	0	1	(3,11) → – 0 1 1
6	0	1	1	0	(5,7) → 0 1 – 1
7	0	1	1	1	(6,7) → 0 1 1 –
11	1	0	1	1	(6,14) → – 1 1 0
14	1	1	1	0	

Prime Representation :

	0	3	5	6	7	11	14
✓ 0	X						
(3,7)		X			X		
✓(3,11)		X				X	
✓(5,7)			X		X		
(6,7)✓				X	X		
✓(6,14)				X			X

$Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{B}\,CD + \bar{A}BD + BC\bar{D}$



19/08/19

② $F(A,B,C,D) = \Sigma m(0,1,3,7,12,13,14,15)$

	A (8)	B (4)	C (2)	D (1)
0	0	0	0	0
1	0	0	0	1
3	0	0	1	1
12	1	1	0	0
7	0	1	1	1
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

STEP: 2

(0,1) → 000_
(1,3) → 00_1
(3,7) → 0_11
(12,13) → 110_ ✓
(12,14) → 11_0 ✓
(7,15) → _111
(13,15) → 11_1 ✓
(14,15) → 111_ ✓

STEP: 3

(12,13,14,15) → 11_ _
(12,14,13,15) → 11_ _
(0,1) → 000_
(1,3) → 00_1
(3,7) → 0_11
(7,15) → _111
(12,13,14,15) → 11_ _

	0	1	3	7	12	13	14	15
$P_1(0,1)$	X	X						
$P_2(1,3)$		X	X					
$P_3(3,7) \leftarrow$			X	X				
$P_4(7,15)$				X				X
$P_5(12,13,14,15)$					X	X	X	X

$(0,1) \rightarrow \bar{A}\bar{B}\bar{C}$.
$(3,7) \rightarrow \bar{A}CD$.
$(12,13,14,15) \rightarrow AB$.

$Y = \bar{A}\bar{B}\bar{C} + \bar{A}CD + AB$. $\rightarrow$ POS
$Y = (A+B+C)(A+\bar{C}+\bar{D})(\bar{A}+\bar{B})$ $\rightarrow$ SOP NOT

Petric method:

$\Rightarrow (x+y)(x+z) = (x+yz)$

$LHS = (x+y)(x+z)$
$x + xy + xz + yz$.
$x(1+y) + xz + yz$.
$x + xz + yz$
$x(1+z) + yz$
$RHS = (x+yz)$

$(P_1+P_2)(P_2+P_3)(P_3+P_4)(P_4+P_5) = 1$
$(P_2 + P_1P_3)(P_4 + P_3P_5) = 1$.
$P_2P_4 + \cancel{P_1P_3}P_4 + P_2P_3P_5 + \cancel{P_1P_3}P_5 = 1$
$P_2P_4 + P_2P_3P_5 = 1$.

③ $F(A,B,C,D) = \sum M(0,1,2,3,5,8,12,14,15)$

	STEP:1				STEP:2	STEP:3	
	8 A	4 B	2 C	1 D			
0	0	0	0	0	$(0,1) \rightarrow 000\_$	$(0,1,2,3) \rightarrow 00\_\_$	✓
1	0	0	0	1	$(0,2) \rightarrow 00\_0$	$(0,2,1,3) \rightarrow 00\_\_$	×
2	0	0	1	0	$(0,8) \rightarrow \_000$	$(0,8) \rightarrow \_000$	
8	1	0	0	0	$(1,3) \rightarrow 00\_1$	$(1,5) \rightarrow 0\_01$	
3	0	0	1	1	$(1,5) \rightarrow 0\_01$	$(8,12) \rightarrow 1\_00$	
5	0	1	0	1	$(2,3) \rightarrow 001\_$	$(12,14) \rightarrow 11\_0$	
12	1	1	0	0	$(8,12) \rightarrow 1\_00$	$(14,15) \rightarrow 111\_$	
14	1	1	1	0	$(12,14) \rightarrow 11\_0$		
15	1	1	1	1	$(14,15) \rightarrow 111\_$		

	0	1	2	3	5	8	12	14	15
(0,1,2,3) ✓ P_1	X	X	X	X					
(0,8) P_2	X					X			
(1,5) ✓ P_3		X			X				
(8,12) ✓ P_4						X	X		
(12,14) P_5							X	X	
(14,15) ✓ P_6								X	X

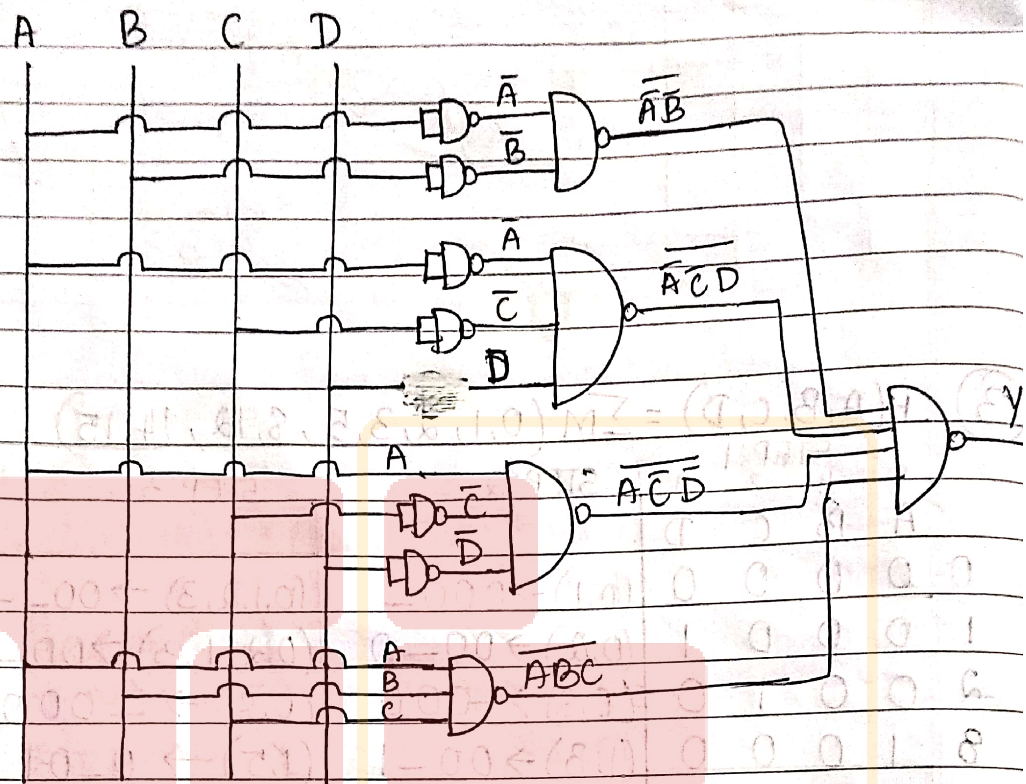
$(0,1,2,3) \rightarrow \bar{A}\bar{B}$

$(1,5) \rightarrow \bar{A}\bar{C}D$

$(8,12) \rightarrow A\bar{C}\bar{D}$

$(14,15) \rightarrow ABC$

$Y = \bar{A}\bar{B} + \bar{A}\bar{C}D + A\bar{C}\bar{D} + ABC$



Petric method:

$(P_1+P_2)(P_1+P_3)(P_2+P_4)(P_4+P_5)(P_5+P_6) = 1$

$\cancel{(P_1 + P_1P_3 + P_1P_2 + P_2P_3)}$

$(P_1 + P_2P_3)(P_4 + P_2P_5)(P_5 + P_6) = 1$

$= (P_1 + P_2P_3)(P_4P_5 + P_4P_6 + P_2P_5 + P_2P_6P_5) = 1$

$= P_1P_4P_5 + P_1P_4P_6 + P_1P_2P_5 + P_1P_2P_6P_5 + P_2P_3P_4P_5 +$
$P_2P_3P_4P_6 + P_2P_3P_2P_5 + P_2P_3P_6P_5$

$= (P_1P_4P_6) + (P_2P_3P_5) + (P_2P_3P_4P_5) + (P_1P_2P_5P_6)$

22/08/19

imp

X

12m

(4) $F = \sum m(0,1,2,5,6,7)$

Step: 1

	4 A	2 B	1 C
0	0	0	0
1	0	0	1
2	0	1	0
5	1	0	1
6	1	1	0
7	1	1	1

Step: 2

(0,1) → 0 0 –

(0,2) → 0 – 0

(1,5) → – 0 1

(2,6) → – 1 0

(5,7) → 1 – 1

(6,7) → 1 1 –

		0	1	2	5	6	7
P_1 ✓	(0,1)	X	X				
P_2	(0,2)	X		X			
P_3	(1,5)		X		X		
P_4 ✓	(2,6)			X		X	
P_5 ✓	(5,7)				X		X
P_6	(6,7)					X	X

$(0,1) \rightarrow \bar{A}\bar{B}$

$(2,6) \rightarrow B\bar{C}$

$(5,7) \rightarrow AC$

$Y = \bar{A}\bar{B} + B\bar{C} + AC.$

BC \ A	0	1
00	1 (0)	(4)
01	1 (1)	1 (5)
11	(3)	1 (7)
10	1 (2)	1 (6)

$Y = \bar{A}\bar{B} + B\bar{C} + AC.$

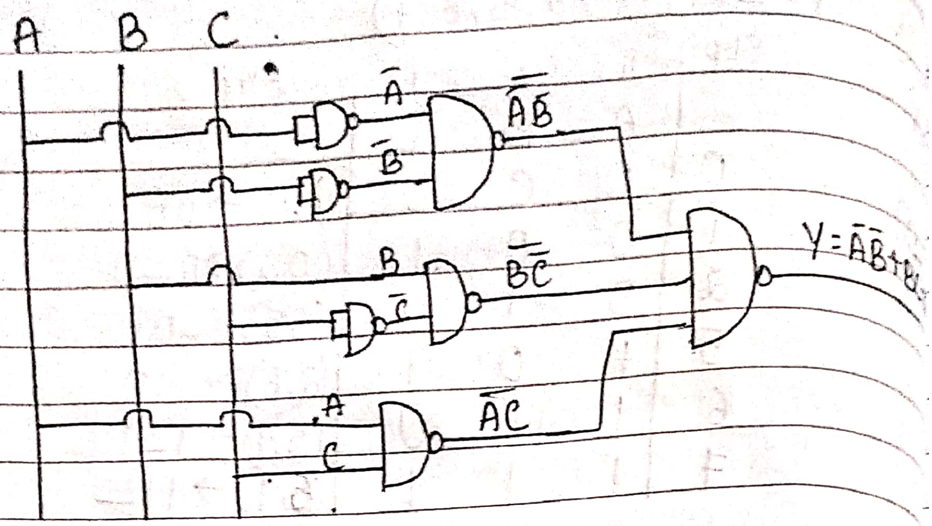
$(P_1+P_2)(P_1+P_3)(P_2+P_4)(P_3+P_5)(P_4+P_6)(P_5+P_6) = 1.$

$(P_1 + P_2P_3)(P_4 + P_2P_6)(P_5 + P_3P_6) = 1.$

$(P_1P_4 + P_1P_2P_6 + P_2P_3P_4 + P_2P_3P_6)(P_5 + P_3P_6) = 1$

$(P_1P_4P_5 + P_1P_2P_5P_6 + P_2P_3P_4P_5 + P_2\cancel{P_3}\cancel{P_5}P_6 + P_1P_4P_3P_6 +$
$P_1P_2\cancel{P_6}P_3 + P_2\cancel{P_3}P_4P_6 + P_2P_3P_6 = 1$

$P_1P_4P_5 + P_1P_2P_5P_6 + P_2P_3P_4P_5 + P_1P_3P_4P_6 + P_2P_3P_6 = 1.$



(5) $F = \sum M(1, 2, 5, 6, 7, 13, 15)$.

	A (8)	B (4)	C (2)	D (1)	Step 2	Step 3
1	0	0	0	1	$(1,5) \rightarrow 0{-}01$	$(5,7,13,15) \rightarrow {-}1{-}1$
2	0	0	1	0	$(2,6) \rightarrow 0{-}10$	$(5,13,7,15) \rightarrow {-}1{-}1$
5	0	1	0	1	$(5,7) \rightarrow 01{-}1$ ✓	$(1,5) \rightarrow 0{-}01$
6	0	1	1	0	$(5,13) \rightarrow {-}101$ ✓	$(2,6) \rightarrow 0{-}10$
7	0	1	1	1	$(6,7) \rightarrow 011{-}$	$(6,7) \rightarrow 011{-}$
13	1	1	0	1	$(7,15) \rightarrow {-}111$ ✓	
15	1	1	1	1	$(13,15) \rightarrow 11{-}1$ ✓	

	1	2	5	6	7	13	15	
(5,13,7,15) ✓			X		X	X	X	P_1
(1,5) ✓	X		X					P_2
(2,6) ✓		X		X				P_3
(6,7).				X	X			P_4

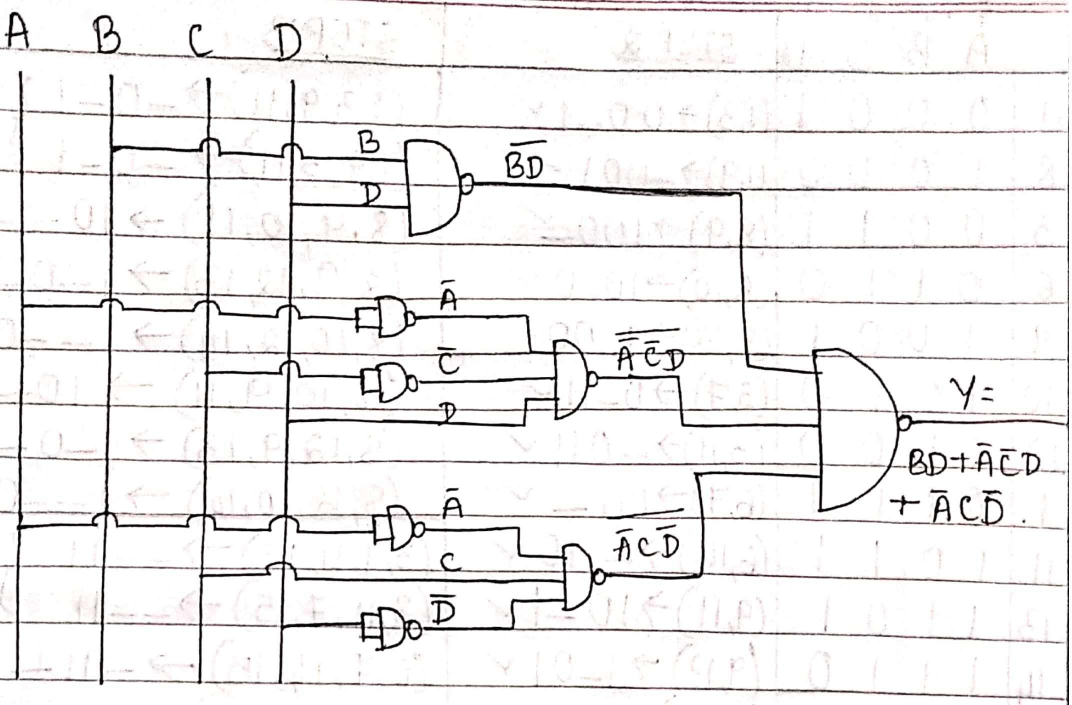
$$Y = BD + \bar{A}\bar{C}D + \bar{A}C\bar{D}$$

$$(P_1 + P_4)(P_3 + P_4)(P_1 + P_2) = 1$$

$$(P_4 + P_1 P_3)(P_1 + P_2) = 1$$

$$P_4 P_1 + P_4 P_2 + P_1 P_3 + P_1 P_2 P_3 = 1.$$

$$+ P_1 P_3 + P_2 P_3 P_4 = 1.$$



K-map Verification :

AB \ CD	00	01	11	10
00	0	1 (1)	3	1 (2)
01	4	1 (5)	1 (7)	1 (6)
11	12	1 (13)	1 (15)	14
10	8	9	11	10

y_3, y_2, y_1

$Y = BD + \bar{A}\bar{C}D + \bar{A}C\bar{D}$

24/08/14

(6) $F(A,B,C,D) = \sum m(1,3,6,7,8,9,10,12,14,15) + dc(11,13)$.

20-M

AB \ CD	00	01	11	10
00	0	1 (1)	1 (3)	2
01	4	5	1 (7)	1 (6)
11	1 (12)	X (13)	1 (15)	1 (14)
10	1 (8)	1 (9)	X (11)	1 (10)

$\bar{B}D$, BC, A

STEP-1.

	8 A	4 B	2 C	1 D
1	0	0	0	1
8	1	0	0	0
3	0	0	1	1
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0
7	0	1	1	1
11	1	0	1	1
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

STEP:2.

(1,3)→00_1 ✓
(1,9)→_001 ✓
(8,9)→100_ ✓
(8,10)→10_0 ✓
(8,12)→1_00 ✓
(3,7)→0_11 ✓
(3,11)→_011 ✓
(6,7)→011_ ✓
(6,14)→_110 ✓
(9,11)→10_1 ✓
(9,13)→1_01 ✓
(10,11)→101_ ✓
(10,14)→1_10 ✓
(12,13)→110_ ✓
(12,14)→11_0 ✓
(7,15)→_111 ✓
(11,15)→1_11 ✓
(13,15)→11_1 ✓
(14,15)→111_ ✓

STEP:3.

(1,3,9,11)→_0_1
(1,9,3,11)→_0_1
(8,9,10,11)→10__
(8,9,12,13)→1_0_
(8,10,12,14)→1__0
(8,10,9,11)→10__
(8,12,9,13)→1_0_
(8,12,10,14)→1__0
(3,7,11,15)→__11
(3,11,7,15)→__11
(6,7,14,15)→_11_
(6,14,7,15)→_11_
(9,11,13,15)→1__1 ✓
(9,13,11,15)→1__1 ✓
(10,11,14,15)→1_1_ ✓
(10,14,11,15)→1_1_ ✓
(12,13,14,15)→11__ ✓
(12,14,13,15)→11__ ✓

STEP:04 :

(8,9,10,11,12,13,14,15)→1___ ✓
(8,9,10,11,12,14,13,15)→1___
(8,9,12,13,10,14,11,15)→1___
(8,9,12,13,10,11,14,15)→1___
(8,10,12,14,9,11,13,15)→1___
(8,10,12,14,9,13,11,15)→1___
(8,10,9,11,12,14,13,15)→1___
(8,10,9,11,12,13,14,15)→1___
(8,12,9,13,10,11,14,15)→1___
(8,12,9,13,10,14,11,15)→1___
(8,12,10,14,9,11,13,15)→1___
(8,12,10,14,9,13,11,15)→1___

	1	3	6	7	8	9	10	12	14	15
(8,9,10,11,12,13,14,15) ✓					X	X	X	X	X	X
(1,3,9,11) ✓	X	X				X				
(3,7,11,15)		X		X						X
(6,7,14,15) ✓			X	X					X	X

$Y = A + \bar{B}D + BC$.

Petric method:

	1	3	6	7	8	9	10	12	14	15
P_1 (8,9,10,11,12,13,14,15)					X	X	X	X	X	X
P_2 (1,3,9,11)	X	X				X				
P_3 (3,7,11,15)		X		X						X
P_4 (6,7,14,15)		X	X	X					X	X

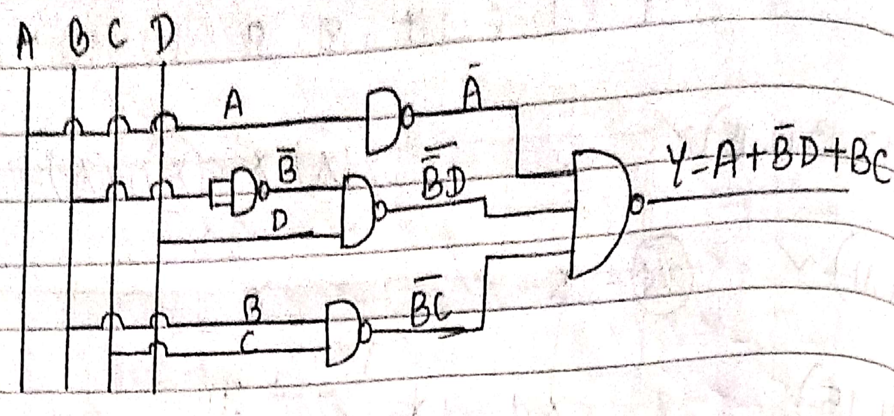
$(P_1+P_2)(P_2+P_3)(P_3+P_4)(P_1+P_4)(P_1+P_3) = 1$.

$(P_1+P_2P_4)(P_3+P_1P_2)(P_3+P_4) = 1$

$(P_1P_3 + P_1P_2 + P_2P_3 + P_1P_4)(P_3+P_4) = 1$

$P_1P_3 + P_1P_2P_3 + P_2P_3 + \cancel{P_1P_3} + \cancel{P_1P_3}P_4 + P_1P_2P_4 +$
$P_2P_3P_4 + P_1P_4 = 1$.

$P_1P_3 + P_1P_4 + P_1P_2P_3 + P_2P_3 + P_1P_2P_4 + P_2P_3P_4$.



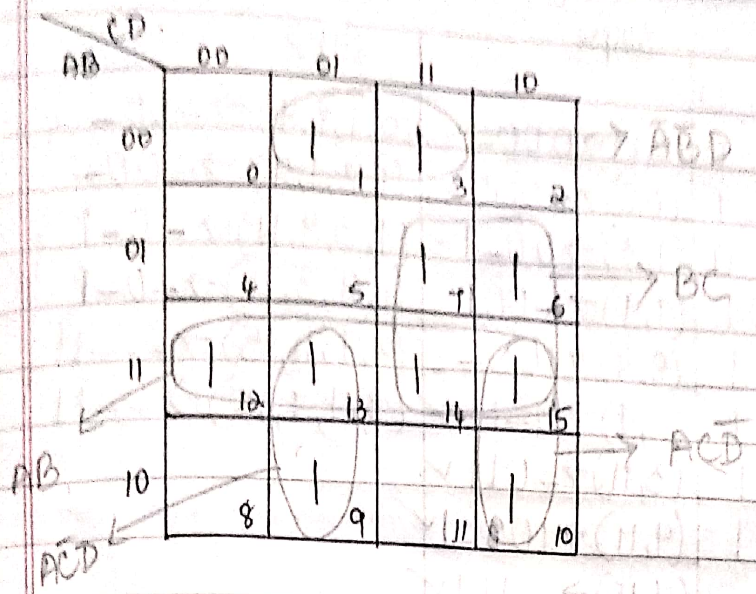
26/08/19

(7). $F(A,B,C,D) = \Sigma M(1,3,6,7,9,10,12,13,14,15)$

	A (8)	B (4)	C (2)	D (1)	Step 2	Step 3
1	0	0	0	1	(1,3) → 00-1	(6,7,14,15) → -11- ✓
3	0	0	1	1	(1,9) → -001	(6,14,7,15) → -11-
6	0	1	1	0	(3,7) → 0-11	(12,13,14,15) → 11-- ✓
9	1	0	0	1	(6,7) → 011- ✓	(12,14,13,15) → 11--
10	1	0	1	0	(6,14) → -110 ✓	Remaining:
12	1	1	0	0	(9,13) → 1-01	(1,3) → 00-1
7	0	1	1	1	(10,14) → 1-10	(1,9) → -001
13	1	1	0	1	(12,13) → 110- ✓	(3,7) → 0-11
14	1	1	1	0	(12,14) → 11-0 ✓	(9,13) → 1-01
15	1	1	1	1	(7,15) → -111 ✓	(10,14) → 1-10
					(13,15) → 11-1 ✓	
					(14,15) → 111- ✓	

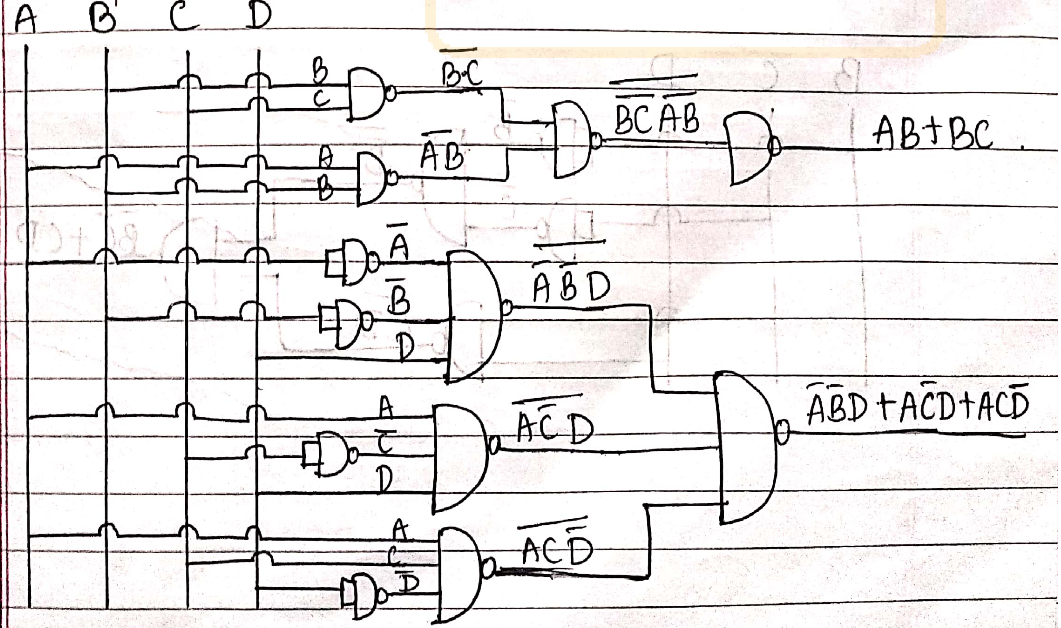
	1	3	6	7	9	10	12	13	14	15
(6,7,14,15) ✓			X	X					X	X
(12,13,14,15) ✓							X	X	X	X
(1,3) ✓	X	X								
(1,9)	X				X					
(3,7)		X		X						
(9,13) ✓					X			X		
(10,14) ✓						X			X	

$Y = BC + AB + \bar{A}\bar{B}D + A\bar{C}D + AC\bar{D}$



	1	3	6	7	9	10	12	13	14	15
P_1 (6,7,14,15) ✓			X	X					X	X
P_2 (12,13,14,15) ✓							X	X	X	X
P_3 (1,3) ✓	X	X								
P_4 (1,9)	X				X					
P_5 (3,7)		X		X						
P_6 (9,13) ✓					X			X		
P_7 (10,14) ✓						X			X	

(No petric)



(8) $F(A,B,C,D) = \Sigma m(0,1,3,7,8,9,11,15)$.

Step 1

	8 A	4 B	2 C	1 D
0	0	0	0	0
1	0	0	0	1
8	1	0	0	0
3	0	0	1	1
9	1	0	0	1
7	0	1	1	1
11	1	0	1	1
15	1	1	1	1

Step 2

(0,1) → 000– ✓
(0,8) → –000 ✓
(1,3) → 00–1 ✓
(1,9) → –001 ✓
(8,9) → 100– ✓
(3,7) → 0–11 ✓
(3,11) → –011 ✓
(9,11) → 10–1 ✓
(7,15) → –111 ✓
(11,15) → 1–11 ✓

Step 3.

(0,1,8,9) → –00–
(0,8,1,9) → –00–
(1,3,9,11) → –0–1
(1,9,3,11) → –0–1
(3,7,11,15) → ––11
(3,11,7,15) → ––11

	0	1	3	7	8	9	11	15
(0,1,8,9) ✓	X	X			X	X		
(1,3,9,11)		X	X			X	X	
(3,7,11,15) ✓			X	X			X	X

$Y = \bar{B}\bar{C} + CD$.