Similarly, we obtain y_{12} and y_{22} by connecting a current source I_2 to port 2 and short-circuiting port 1 as in Fig. 7.4, finding I_1 and V_2, and then calculating,

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

y_{12} is called the *short-circuit transfer admittance* and y_{22} is called the *short-circuit output admittance*. Collectively the parameters are referred to as short-circuit admittance parameters.

Please note that $y_{12} = y_{21}$ only when there are no dependent sources or Op-amps within the two-port network.

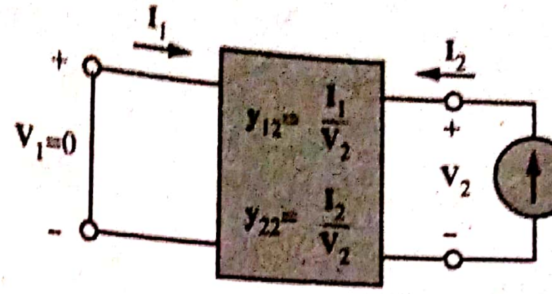

Figure 7.4 Determination of y_{12} and y_{22}

7.1

Determine the admittance parameters of the T network shown in Fig. 7.5.

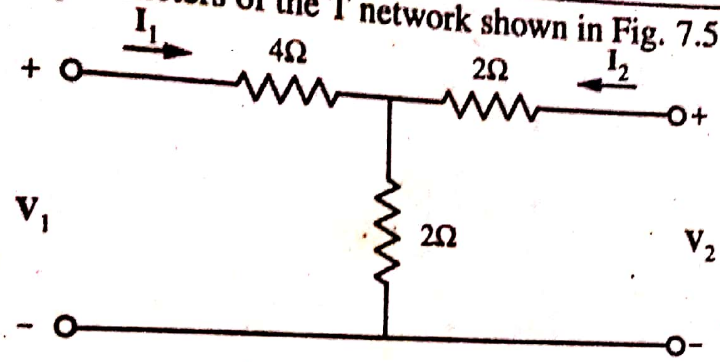

Figure 7.5

To find y_{11} and y_{21}, we have to short the output terminals and connect a current source I_1 to the input terminals. The circuit so obtained is shown in Fig. 7.6(a).

$$I_1 = \frac{V_1}{4 + \dfrac{2\times 2}{2+2}} = \frac{V_1}{5}$$

Hence, $$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{5}\,S$$

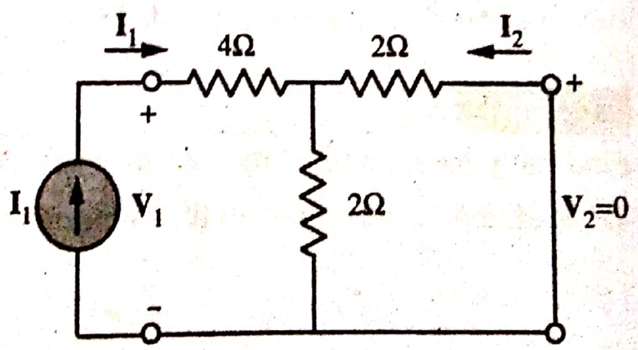

Figure 7.6(a)

Using the principle of current division,

$$-I_2 = \frac{I_1\times 2}{2+2} = \frac{I_1}{2}$$

$$\Rightarrow \quad -I_2 = \frac{1}{2}\left[\frac{V_1}{5}\right]$$

Hence, $$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-1}{10}\,S$$

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To find y_{12} and y_{22}, we have to short-circuit the input terminals and connect a current source I_2 to the output terminals. The circuit so obtained is shown in Fig. 7.6(b).

$$I_2 = \frac{V_2}{2 + \frac{4 \times 2}{4+2}}$$

$$= \frac{V_2}{2 + \frac{4}{3}}$$

$$= \frac{3V_2}{10}$$

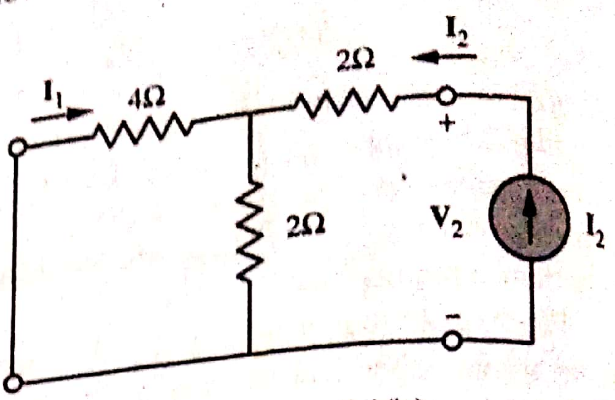


Figure 7.6(b)

Hence, $y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{3}{10} S$

Employing the principle of current division, we have

$$-I_1 = \frac{I_2 \times 2}{2+4}$$

$$\Rightarrow \quad -I_1 = \frac{2I_2}{6}$$

$$\Rightarrow \quad -I_1 = \frac{1}{3}\left[\frac{3V_2}{10}\right]$$

Hence, $y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{-1}{10} S$

It may be noted that, $y_{12} = y_{21}$. Thus, in matrix form we have

$$I = YV$$

$$\Rightarrow \quad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{-1}{10} \\ \frac{-1}{10} & \frac{3}{10} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

EXAMPLE 7.2

Find the y parameters of the two-port network shown in Fig. 7.7. Then determine the current in a 4Ω load, that is connected to the output port when a 2A source is applied at the input port.

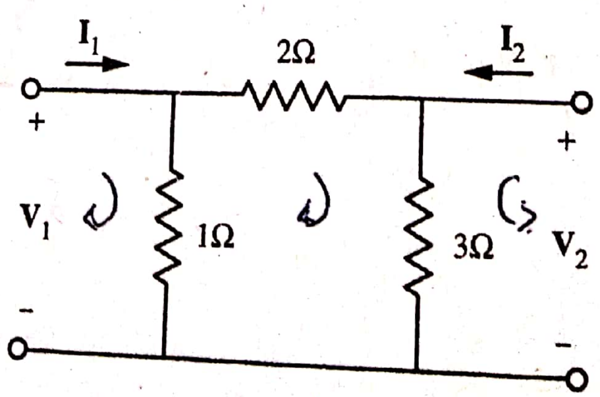


Figure 7.7

...d y_{11} and y_{21}, short-circuit the output terminals and connect a current source I_1 to the input ...als. The resulting circuit diagram is shown in Fig. 7.8(a).

$$I_1 = \frac{V_1}{1\Omega||2\Omega} = \frac{V_1}{\frac{1\times 2}{1+2}}$$

$$\Rightarrow \quad I_1 = \frac{3}{2}V_1$$

Hence,
$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{3}{2}S$$

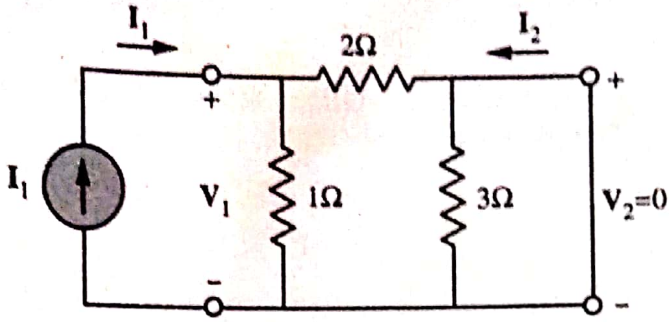


Figure 7.8(a)

Using the principle of current division,

$$-I_2 = \frac{I_1 \times 1}{1+2}$$

$$\Rightarrow \quad -I_2 = \frac{1}{3}I_1$$

$$\Rightarrow \quad -I_2 = \frac{1}{3}\left[\frac{3}{2}V_1\right]$$

Hence,
$$y_{21} = \frac{I_2}{V_1} = \frac{-1}{2}S$$

To find y_{12} and y_{22}, short the input terminals and connect a current source I_2 to the output ...inals. The resulting circuit diagram is shown in Fig. 7.8(b).

$$I_2 = \frac{V_2}{2\Omega||3\Omega}$$

$$= \frac{V_2}{\frac{2\times 3}{2+3}} = \frac{5V_2}{6}$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{5}{6}S$$

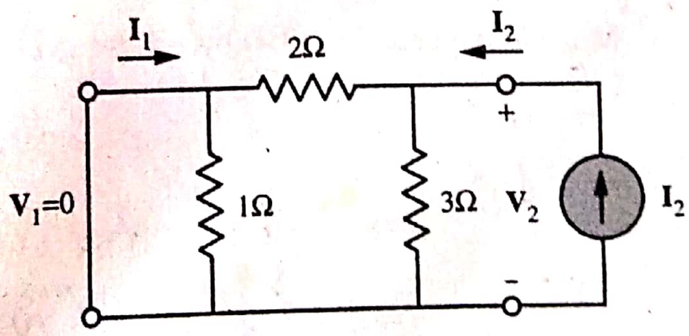


Figure 7.8(b)

Employing the current division principle,

$$-I_1 = \frac{I_2 \times 3}{2+3}$$

$$-I_1 = \frac{3}{5}I_2$$

$$\Rightarrow$$

$$\Rightarrow \qquad -I_1 = \frac{3}{5}\left[\frac{5V_2}{6}\right]$$

$$\Rightarrow \qquad I_1 = \frac{-1}{2}V_2$$

Hence,

$$y_{12} = \left.\frac{-I_1}{V_2}\right|_{V_1=0} = \frac{-1}{2}S$$

Therefore, the equations that describe the two-port network are

$$I_1 = \frac{3}{2}V_1 - \frac{1}{2}V_2 \tag{7.3}$$

$$I_2 = -\frac{1}{2}V_1 + \frac{5}{6}V_2 \tag{7.4}$$

Putting the above equations (7.3) and (7.4) in matrix form, we get

$$\begin{bmatrix} \frac{3}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{5}{6} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

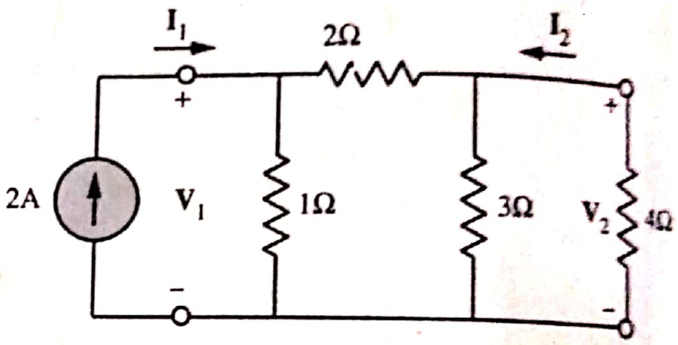


Figure 7.8(c)

Referring to Fig. 7.8(c), we find that $I_1 = 2A$ and $V_2 = -4I_2$

Substituting $I_1 = 2A$ in equation (7.3), we get

$$2 = \frac{3}{4}V_1 - \frac{1}{2}V_2 \tag{7.5}$$

Multiplying equation (7.4) by −4, we get

$$-4I_2 = 2V_1 - \frac{20}{6}V_2$$

$$\Rightarrow \qquad V_2 = 2V_1 - \frac{20}{6}V_2$$

$$\Rightarrow \qquad 0 = 2V_1 - \left(\frac{20}{6}+1\right)V_2$$

$$\Rightarrow \qquad 0 = \frac{-1}{2}V_1 + \frac{13}{12}V_2 \tag{7.6}$$

Putting equations (7.5) and (7.6) in matrix form, we get

$$\begin{bmatrix} \frac{3}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{13}{12} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

It may be noted that the above equations are simply the nodal equations for the circuit shown in Fig. 7.8(c). Solving these equations, we get

$$V_2 = \frac{3}{2}V$$

Hence,

$$I_2 = \frac{-1}{4}V_2 = \frac{-3}{8}A$$

EXAMPLE 7.3

For the network shown in the Fig. 7.9 containing a current-controlled current source. For this network, find the y parameters.

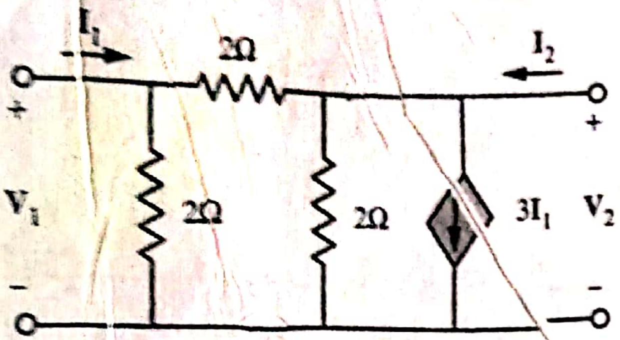


Figure 7.9

SOLUTION

To find y_{11} and y_{21} short the output terminals and connect a current source I_1 to the input terminals. The resulting circuit diagram is as shown in Fig. 7.10(a) and it is further reduced to Fig. 7.10(b).

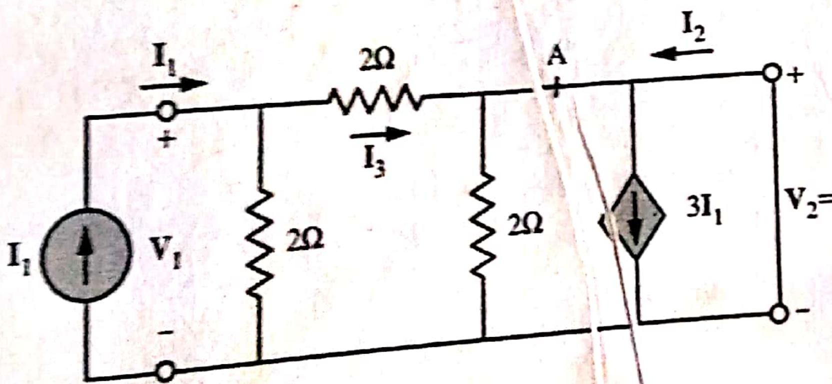


Figure 7.10(a)

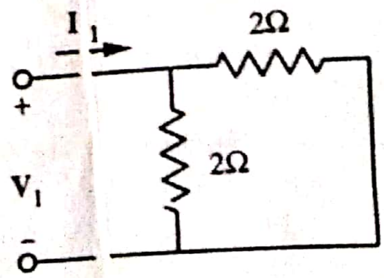


Figure 7.10(b)

$$I_1 = \frac{V_1}{\frac{2\times 2}{2+2}}$$

$$\Rightarrow \quad I_1 = V_1$$

Hence,

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 1S$$

Applying KCL at node A gives (Referring to Fig. 7.10(a)).

$$I_3 + I_2 = 3I_1$$

$$\Rightarrow \quad \frac{V_1}{2} + I_2 = 3I_1$$

$$\Rightarrow \quad \frac{V_1}{2} + I_2 = 3V_1$$

$$\Rightarrow \quad \frac{5V_1}{2} = I_2$$

Hence, $$y_{21} = \frac{I_2}{V_1} = \frac{5}{2}S$$

To find y_{22} and y_{12}, short the input terminals and connect a current source I_2 at the output terminals. The resulting circuit diagram is shown in Fig. 7.10(c) and further reduced to Fig. 7.10(d).

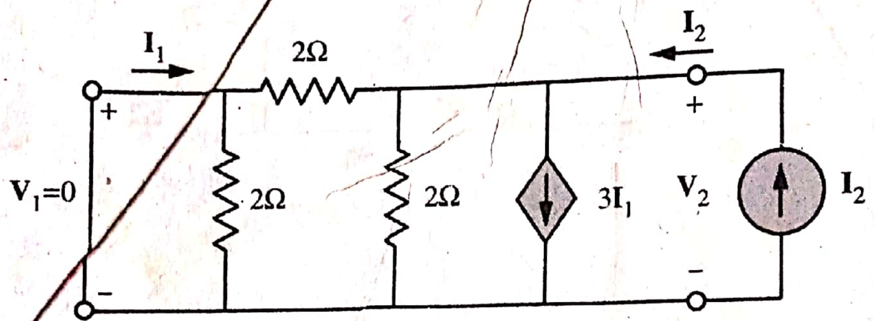


Figure 7.10(c)

$$I_2 = -I_1' = -\frac{V_2}{2}$$

$$\Rightarrow \quad -I_1 = \frac{V_2}{2}$$

Hence, $$y_{12} = \frac{I_1}{V_2} = \frac{-1}{2}S$$

Applying KCL at node B gives

$$I_2 = \frac{V_2}{2} + \frac{V_2}{2} + 3I_1$$

But $$I_1 = \frac{-V_2}{2}$$

Hence, $$I_2 = \frac{V_2}{2} + \frac{V_2}{2} - 3\frac{V_2}{2}$$

$$\Rightarrow \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = -0.5S$$

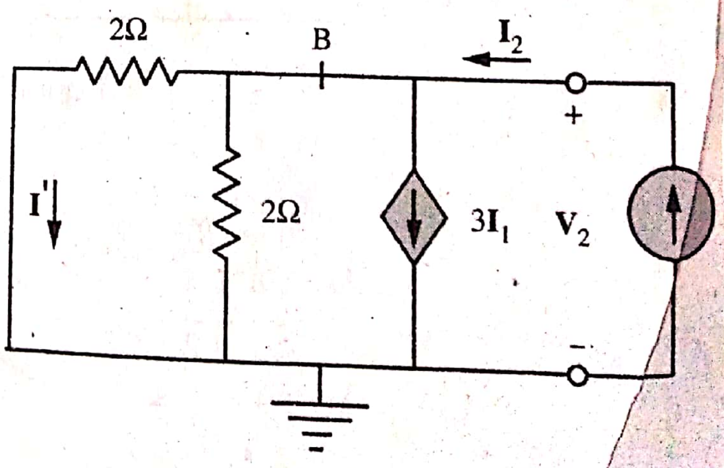


Figure 7.10(d)

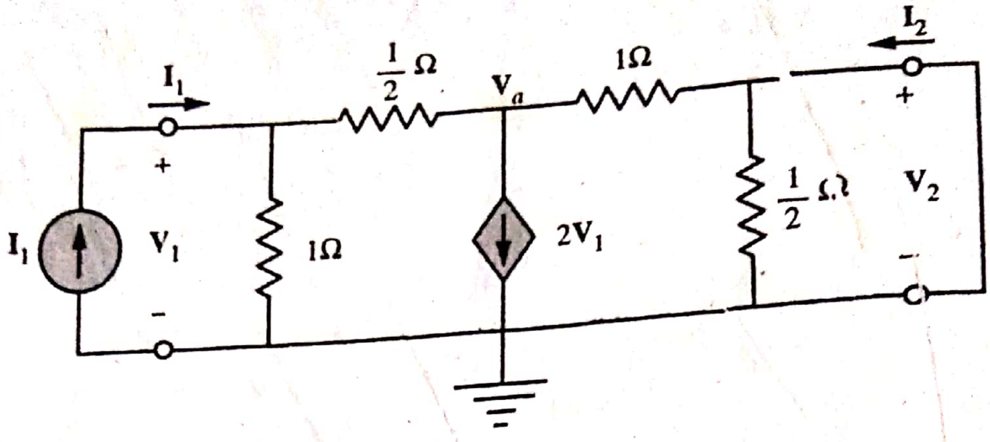


Figure 7.12(a)

KCL at node $\mathbf{V}_1$:

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{1} + \frac{\mathbf{V}_1 - \mathbf{V}_a}{\frac{1}{2}}$$

$$\Rightarrow \qquad 3\mathbf{V}_1 - 2\mathbf{V}_a = \mathbf{I}_1 \tag{7.7}$$

KCL at node $\mathbf{V}_a$:

$$\frac{\mathbf{V}_a - \mathbf{V}_1}{\frac{1}{2}} + \frac{\mathbf{V}_a - 0}{1} + 2\mathbf{V}_1 = 0$$

$$\Rightarrow \qquad 2\mathbf{V}_a - 2\mathbf{V}_1 + \mathbf{V}_a + 2\mathbf{V}_1 = 0$$

$$\Rightarrow \qquad \mathbf{V}_a = 0 \tag{7.8}$$

Making use of equation (7.8) in (7.7), we get

$$3\mathbf{V}_1 = \mathbf{I}_1$$

Hence,

$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} = 3\mathrm{S}$$

Since $\mathbf{V}_a = 0$, $\mathbf{I}_2 = 0$,

$$\Rightarrow \qquad \mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} = 0\mathrm{S}$$

To find $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$ short-circuit the input terminals and connect a current source $\mathbf{I}_2$ to the output terminals. The resulting circuit diagram is shown in Fig. 7.12(b).

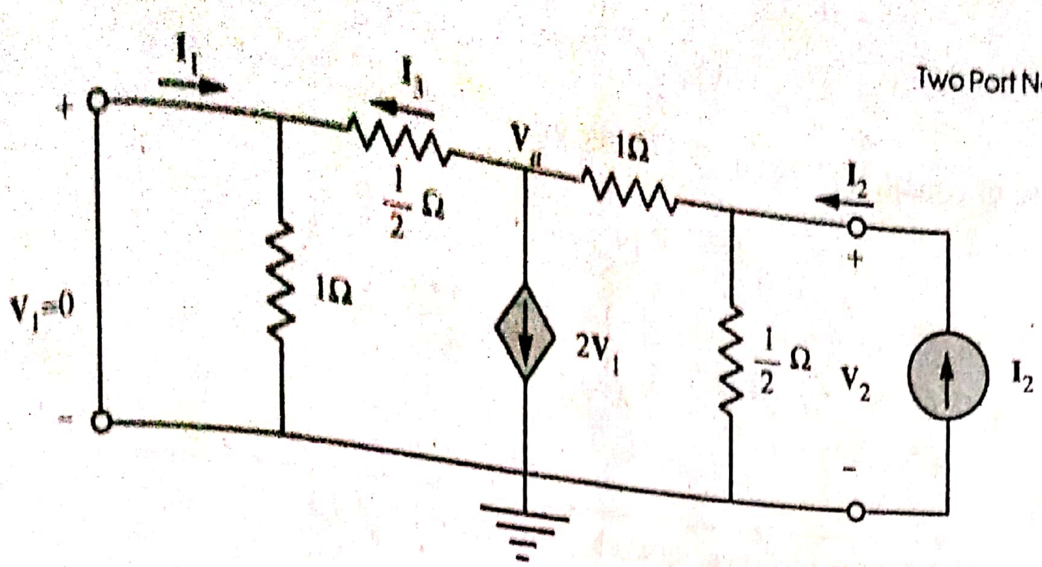


Figure 7.12(b)

CL at node $\mathbf{V}_2$:

$$\frac{\mathbf{V}_2}{\frac{1}{2}} + \frac{\mathbf{V}_2 - \mathbf{V}_a}{1} = \mathbf{I}_2$$

$$\Rightarrow \quad 3\mathbf{V}_2 - \mathbf{V}_a = \mathbf{I}_2 \tag{7.9}$$

CL at node $\mathbf{V}_a$:

$$\frac{\mathbf{V}_a - \mathbf{V}_2}{1} + \frac{\mathbf{V}_a - 0}{\frac{1}{2}} + 0 = 0$$

$$\Rightarrow \quad 3\mathbf{V}_a - \mathbf{V}_2 = 0$$

or

$$\mathbf{V}_a = \frac{1}{3}\mathbf{V}_2 \tag{7.10}$$

Substituting equation (7.10) in (7.9), we get

$$3\mathbf{V}_2 - \frac{1}{3}\mathbf{V}_2 = \mathbf{I}_2$$

$$\Rightarrow \quad \frac{8}{3}\mathbf{V}_2 = \mathbf{I}_2$$

Hence,

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{8}{3}\mathrm{S}$$

We have,

$$\mathbf{V}_a = \frac{1}{3}\mathbf{V}_2 \tag{7.11}$$

Also,

$$\mathbf{I}_1 + \mathbf{I}_3 = 0$$

$$\mathbf{I}_1 = -\mathbf{I}_3$$

$$\Rightarrow \quad = \frac{-\mathbf{V}_a}{\frac{1}{2}} = -2\mathbf{V}_a \tag{7.12}$$

Making use of equation (7.12) in (7.11), we get

$$-\frac{I_1}{2} = \frac{1}{3}V_2$$

Hence,

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{-2}{3}S$$

EXAMPLE 7.5

Find the y parameters for the resistive network shown in Fig. 7.13.

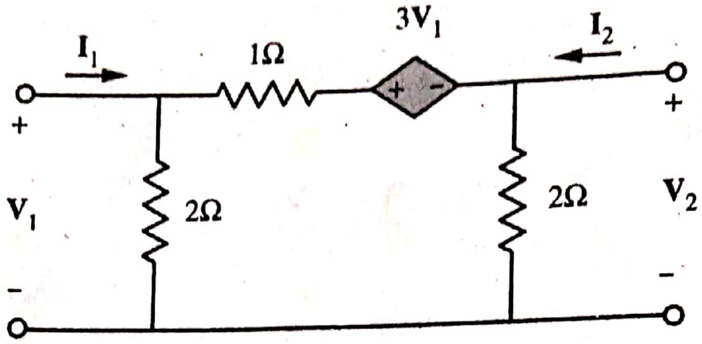


Figure 7.13

SOLUTION

Converting the voltage source into an equivalent current source, we get the circuit diagram shown in Fig. 7.14(a).

To find y_{11} and y_{21}, the output terminals of Fig. 7.14(a) are shorted and connect a current source I_1 to the input terminals. This results in a circuit diagram as shown in Fig. 7.14(b).

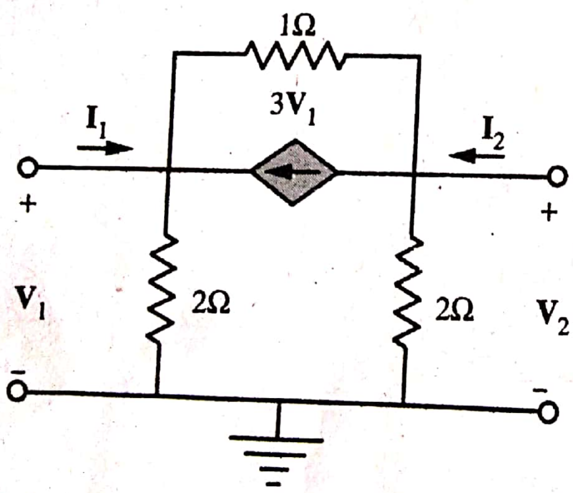


Figure 7.14(a)

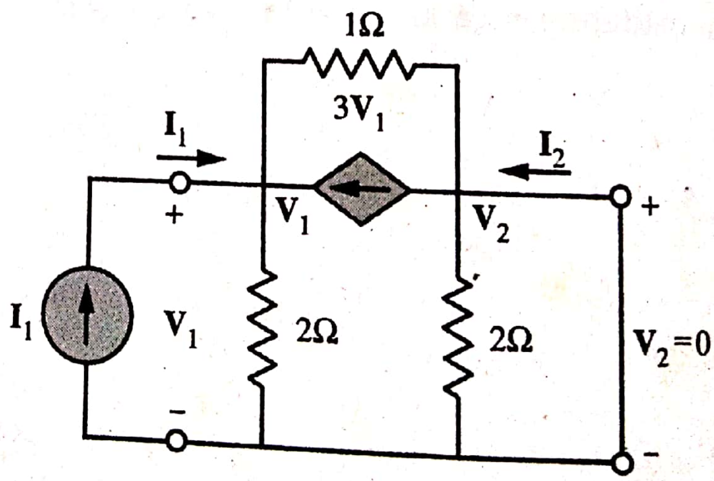


Figure 7.14(b)

KCL at node V_1:

$$\frac{V_1}{2} + \frac{V_1 - V_2}{1} = I_1 + 3V_1$$

Since $V_2 = 0$, we get

$$\frac{V_1}{2} + V_1 = I_1 + 3V_1$$

$$\Rightarrow \qquad I_1 = \frac{-3}{2}V_1$$

Hence,

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{-3}{2}S$$

KCL at node V_2:

$$\frac{V_2}{2} + 3V_1 + \frac{V_2 - V_1}{1} = I_2$$

Since $V_2 = 0$, we get

$$0 + 3V_1 - V_1 = I_2$$

$$\Rightarrow \qquad I_2 = 2V_1$$

Hence

$$y_{21} = \frac{I_1}{V_2} = 2S$$

To find y_{21} and y_{22}, the input terminals of Fig. 7.14(a) are shorted and connect a current source I_2 to the output terminals. This results in a circuit diagram as shown in Fig. 7.14(c).

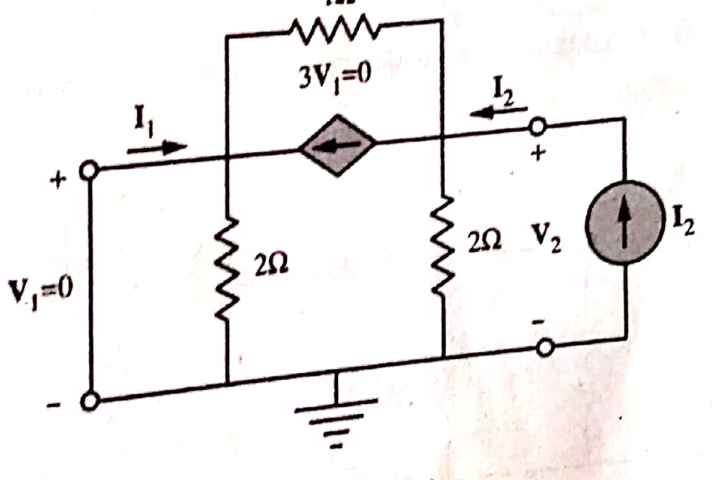


Figure 7.14(c)

$\Rightarrow$

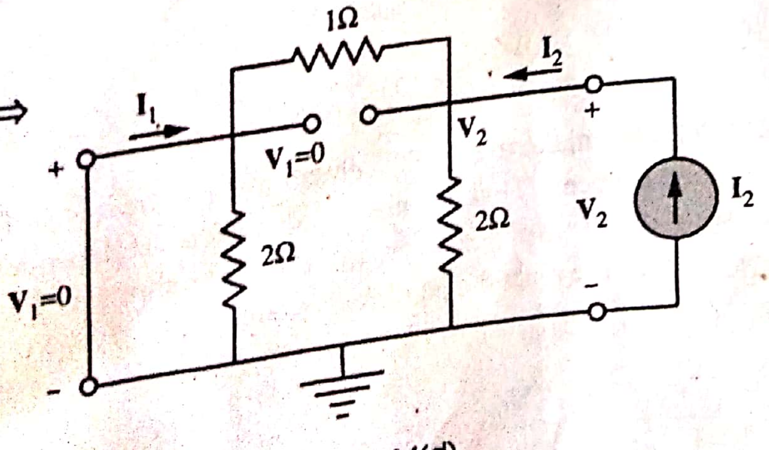


Figure 7.14(d)

KCL at node $\mathbf{V_2}$:

$$\frac{V_2}{2}+\frac{V_2-0}{1}=I_2$$

$$\Rightarrow \qquad \frac{3}{2}V_2=I_2$$

Hence,

$$y_{22}=\frac{I_2}{V_2}=\frac{3}{2}S$$

KCL at node $\mathbf{V_1}$:

$$I_1=\frac{V_1}{2}+\frac{V_1-V_2}{1}=0$$

Since $V_1=0$, we get

$$I_1=-V_2$$

Hence,

$$y_{12}=\frac{I_1}{V_2}=-1S$$

EXAMPLE 7.6

The network of Fig. 7.15 contains both a dependent current source and a dependent voltage source. Find the y parameters.

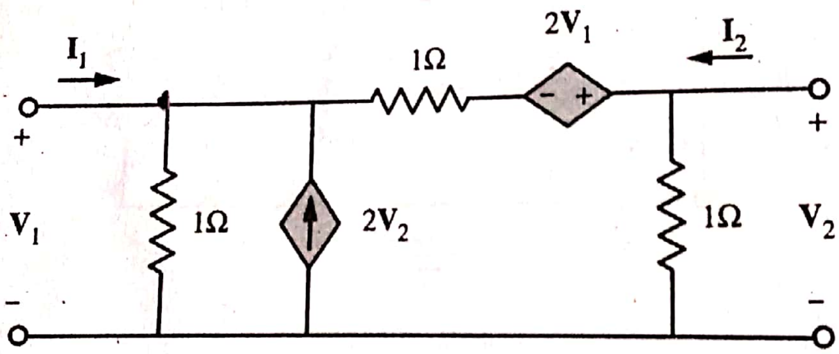


Figure 7.15

SOLUTION

While finding y parameters, we make use of *KCL* equations. Hence, it is preferable to have current sources rather than voltage sources. This prompts us to convert the dependent voltage source into an equivalent current source and results in a circuit diagram as shown in Fig. 7.16(a).

To find y_{11} and y_{21}, refer the circuit diagram as shown in Fig. 7.16(b).

KCL at node $\mathbf{V_1}$:

$$\frac{V_1}{1}+\frac{V_1-V_2}{1}+2V_1=2V_2+I_1$$

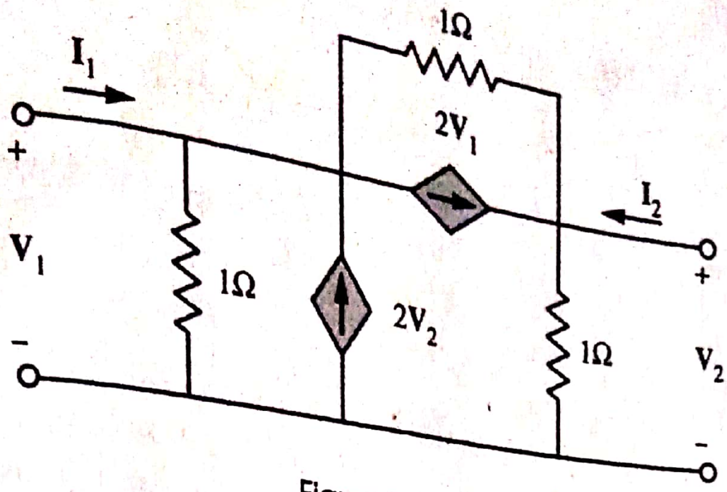


Figure 7.16(a)

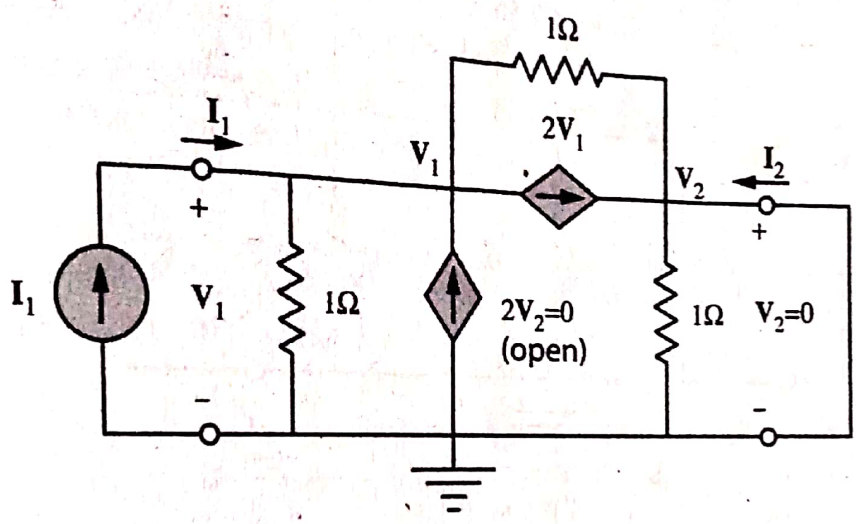


Figure 7.16(b)

Since $V_2 = 0$, we get

$$V_1 + V_1 + 2V_1 = I_1$$

$$\Rightarrow \quad 4V_1 = I_1$$

Hence,

$$y_{11} = \frac{I_1}{V_1} = 4S$$

KCL at node V_2:

$$\frac{V_2}{1} + \frac{V_2 - V_1}{1} = 2V_1 + I_2$$

Since $V_2 = 0$, we get

$$-V_1 = 2V_1 + I_2$$

$$\Rightarrow \quad -3V_1 = I_2$$

Hence,

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -3S$$

To find y_{22} and y_{12}, refer the circuit diagram shown in Fig. 7.16(c).

KCL at node V_1:

$$\frac{V_1}{1} + \frac{V_1 - V_2}{1} + 2V_1 = 2V_2 + I_1$$

Since $V_1 = 0$, we get

$$-V_2 = 2V_2 + I_1$$

$$\Rightarrow \qquad -3V_2 = I_1$$

Hence,

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -3S$$

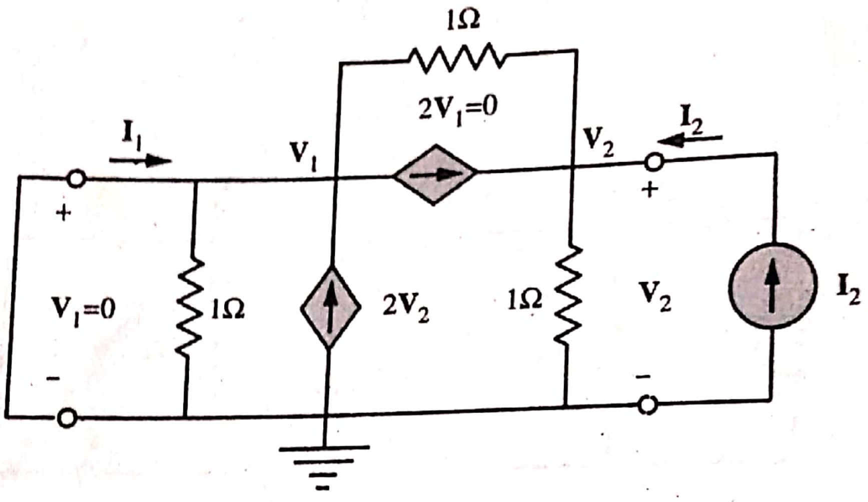


Figure 7.16(c)

KCL at node V_2:

$$\frac{V_2}{1} + \frac{V_2 - V_1}{1} = 2V_1 + I_2$$

Since $V_1 = 0$, we get

$$V_2 + V_2 = 0 + I_2$$

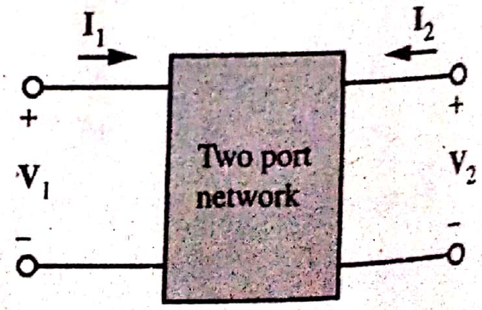
$$\Rightarrow \qquad -2V_2 = I_2$$

Hence,

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = 2S$$

7.3 Impedance parameters

Let us assume the two port network shown in Fig. 7.17 is a linear network that contains no independent sources. Then using superposition theorem, we can write the input and output voltages as the sum of two components, one due to I_1 and other due to I_2:



$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$