MODULE - 3

Two port network.

Parameter	Dependent variable	Independent variable	Equations
Z-parameter	$V_1 V_2$	I_1, I_2	$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$
Y parameter	$I_1 I_2$	$V_1 V_2$	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$
h parameter	$V_1 I_2$	$I_1 V_2$	$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$
T parameter	$V_1\ I_1$	$V_2\ I_2$	$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$

---

1. Impedance parameter or Z or open circuit impedance parameter

$$V_1 = f(I_1, I_2) = Z_{11} I_1 + Z_{12} I_2 \quad - (1)$$

$$V_2 = f(I_1, I_2) = Z_{21} I_1 + Z_{22} I_2 \quad - (2)$$

In matrix form

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[V] = [Z]\,[I]$$

The independent Z-parameter are defined as follows.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

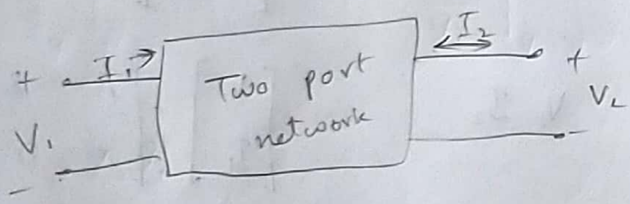
$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$ driving impedance

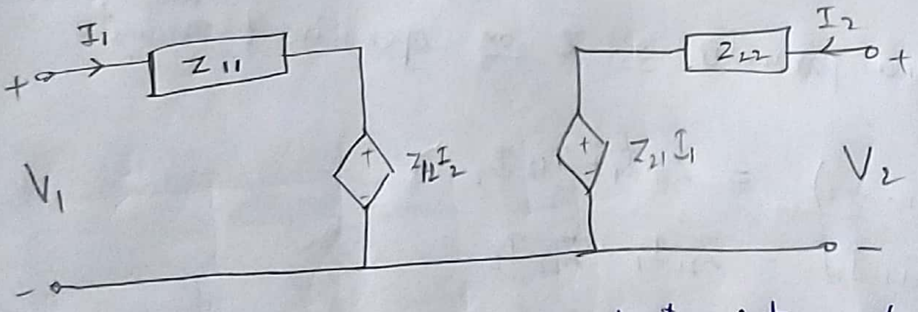
$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$ reverse transfer impedance

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$ forward transfer impedance

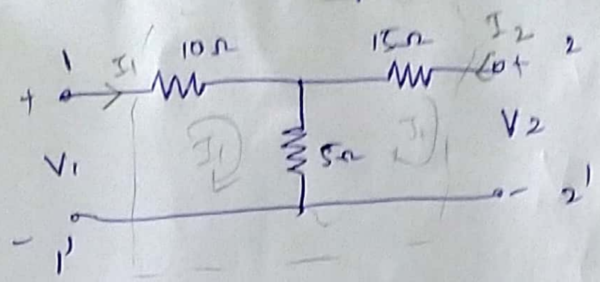
$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$ output impedance





Equivalent network of a 2 port network in terms of Z parameter.

① Find Z parameters for network shown in figure.

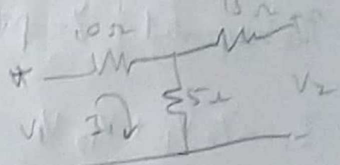


$\rightarrow V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

Let $I_2 = 0$ ie port 2 is open circuit.

As port 2 is open current flowing through 5 Ω is only I_1.

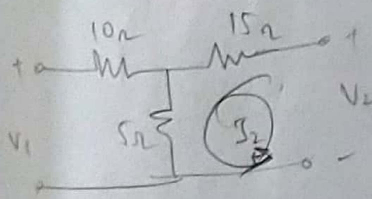


$-10 I_1 - 5 I_1 + V_1 = 0 \Rightarrow \frac{V_1}{I_1} = 15\,\Omega = Z_{11}$

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{5 I_1}{I_1} = Z_{21} = 5\,\Omega$

Let $I_1 = 0$ ie port 1 is open circuit

Current through 5 Ω is only I_2.



$V_2 - 5 I_2 - 15 I_2 = 0$

$V_2 - 20 I_2 = 0$

$Z_{22} = \frac{V_2}{I_2} = 20\,\Omega$

$Z = \begin{bmatrix} 15 & 5 \\ 5 & 20 \end{bmatrix} \Omega$

$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{5 I_2}{I_2} = 5\,\Omega$

$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ 5 & 20 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

(2) Y-parameter or Admittance parameter or

$I_1 = f(V_1, V_2) = Y_{11}V_1 + Y_{12}V_2$

$I_2 = f(V_1, V_2) = Y_{21}V_1 + Y_{22}V_2$

In matrix for

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [Y][V]$$

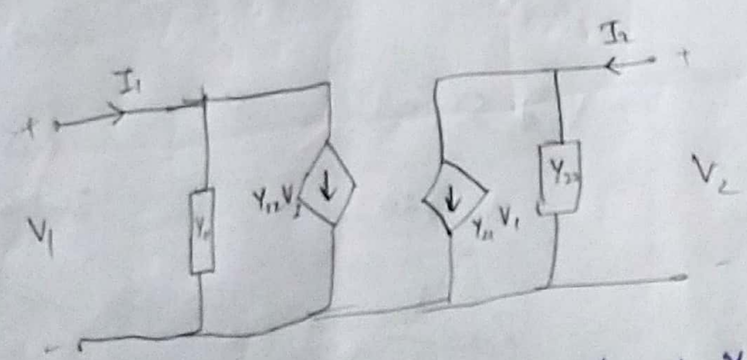
The independent Y-parameter are defined as follows

$I_1 = Y_{11}V_1 + Y_{12}V_2$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

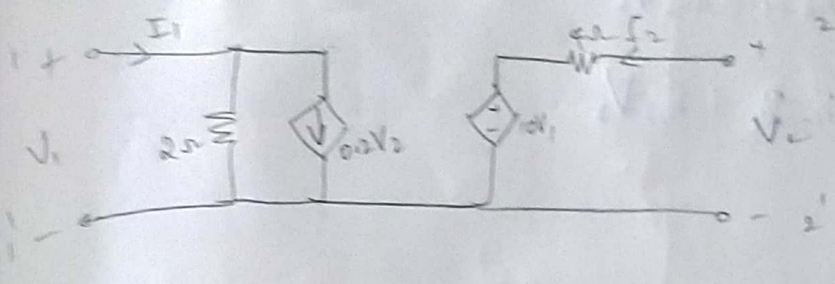
$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$



Equivalent n/w of a 2 port n/w of Y parameter

Find Y parameter for circuit shown.

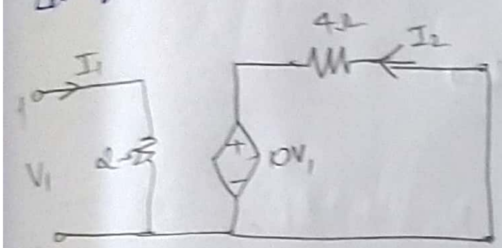
$$[Y] = \begin{bmatrix} 0.5 & 0.2 \\ -2.5 & 0.25 \end{bmatrix} ℧$$



$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

Let $V_2 = 0$



$$V_1 = 2 I_1$$

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = \frac{I_1}{2I_1} = 0.5 ℧$$

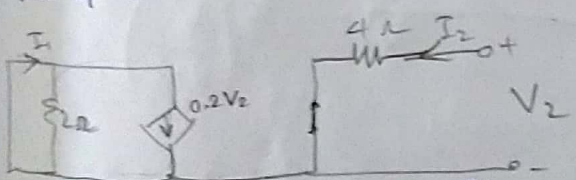
KVL at output side

$$4I_2 + 10V_1 = 0$$

$$-10V_1 = +4I_2$$

$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} = \frac{\frac{-10V_1}{4}}{V_1} = -\frac{10}{4} ℧ = -2.5 ℧$$

Let $V_1 = 0$

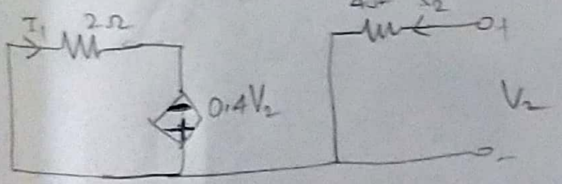


$$V_2 = 4 I_2$$

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} = \frac{I_2}{4I_2} = 0.25 ℧$$

convert current source into voltage source

KVL at input $2I_1 - 0.4V_2 = 0 \Rightarrow 2I_1 = 0.4V_2$



$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$$= \frac{0.2 V_2}{V_2}$$

$$Y_{12} = 0.2 ℧$$

(3) h parameter or hybrid parameter

$$V_1 = f(I_1, V_2) = h_{11} I_1 + h_{12} V_2$$

$$I_2 = f(I_1, V_2) = h_{21} I_1 + h_{22} V_2$$

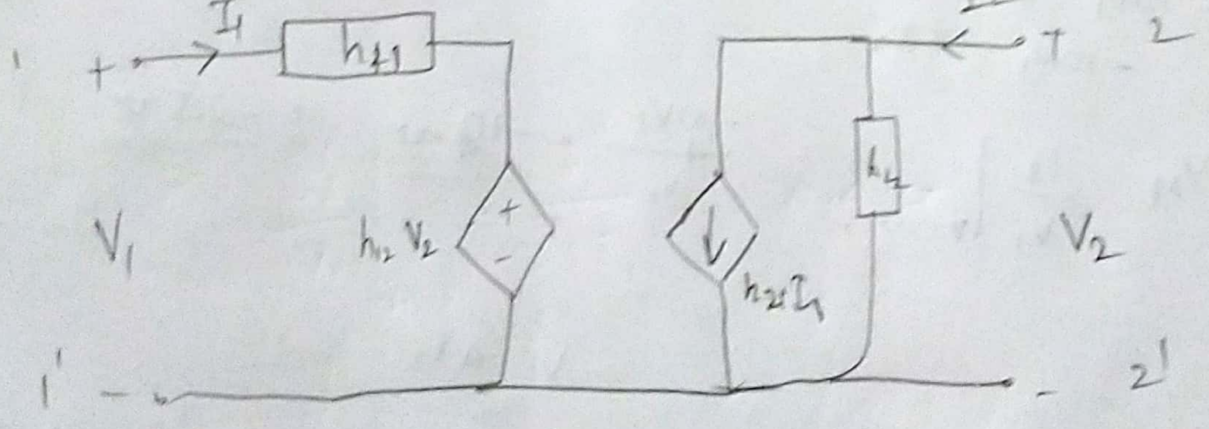
In matrix form

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

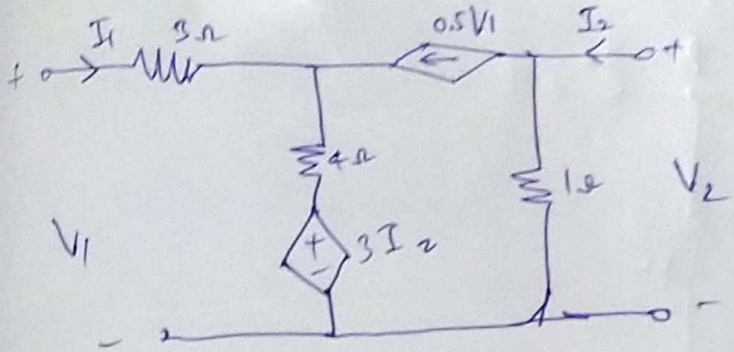
Individual h parameters are defined as follows.

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



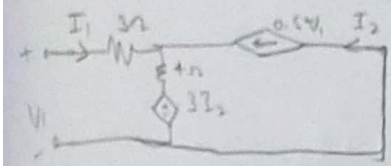
Find h parameter for 2 port network shown.



→ $V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

Let $V_2 = 0$



KVL at ~~o~~/p

$-V_1 + 3I_1 + 4(I_1 + I_2) + 3I_2 = 0$

$V_1 = 7I_1 + 7I_2$ ← $I_2 = 0.5V_1$

$V_1 - 7(0.5)V_1 = 7I_1$

$-\frac{5}{2}V_1 = 7I_1$

$-\frac{5}{14}V_1 = I_1$

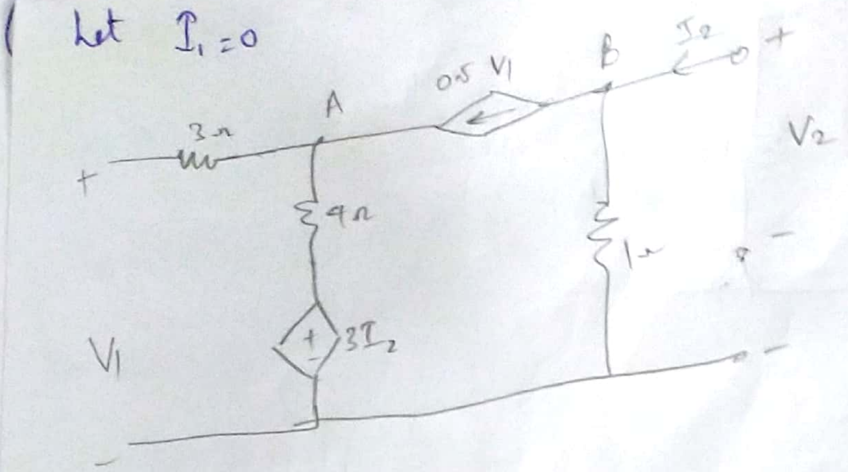
$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0} = \frac{\cancel{V_1}}{-\frac{5}{14}\cancel{V_1}} = -2.8\ \Omega$

KVL at i/p $V_1 = -2.8 I_1$

$I_2 = 0.5(-2.8)I_1 = -1.4 I_1$

$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0} = \frac{-1.4 I_1}{I_1} = -1.4$

Let $I_1 = 0$



KCL at B

$$0.5V_1 + \frac{V_B}{1} - I_2 = 0 \qquad I_2 = V_B + 0.5V_1 \quad \text{—(1)}$$

also

$$V_1 = 4(0.5V_1) + 3I_2$$

$$V_1 = 2V_1 + 3I_2$$

$$V_1 = -3I_2 \quad \text{—(2)}$$

$$I_2 = V_B + (0.5)(-3I_2) \Rightarrow I_2 + 1.5I_2 = V_B$$

$$V_2 = V_B = 2.5I_2$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{I_2}{2.5I_2} = \underline{0.4\ \mho}$$

$$\text{(2)} \quad V_1 = -3\left(\frac{V_2}{2.5}\right) = \frac{-3V_2}{2.5}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$= \frac{-3/2.5\ V_2}{\cancel{V_2}}$$

$$\underline{h_{12} = -1.2}$$

$$[h] = \begin{bmatrix} -2.8 & -1.2 \\ -1.4 & 0.4 \end{bmatrix}$$

(4) T parameter or [illegible] parameter

$$V_1 = f(V_2, -I_2) = AV_2 + B(-I_2)$$

$$I_1 = f(V_2, -I_2) = CV_2 + D(-I_2)$$

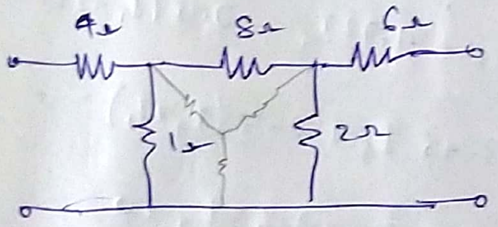
Matrix

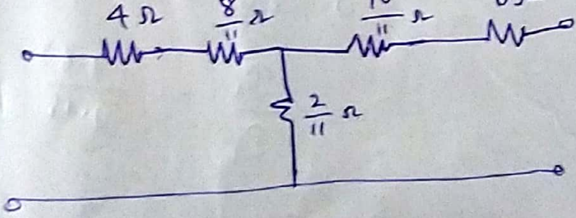
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

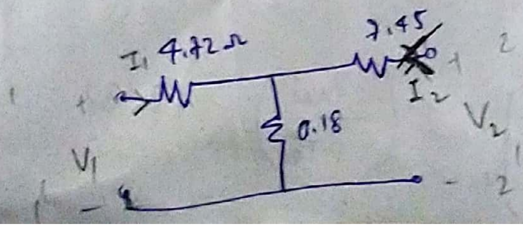
$$A = \left.\frac{V_1}{V_2}\right|_{-I_2=0} \qquad B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{-I_2=0} \qquad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

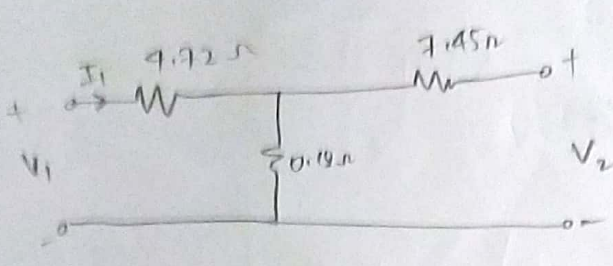
Find T parameter







Let $I_2 = 0$



$V_2 = 0.18\, I_1$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{I_1}{0.18 I_1} = \underline{5.55\ \mho}$$

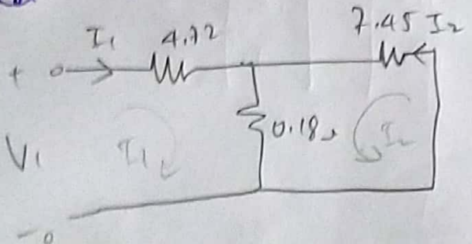
KVL

$$-V_1 + 4.72 I_1 + 0.18\, I_1 = 0$$

$$-V_1 + 4.9\, I_1 = 0 \Rightarrow V_1 = 4.9\, I_1$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{4.9\, I_1}{0.18\, I_1} = \underline{27.22}$$

Let $V_2 = 0$



$$-V_1 + 4.72 I_1 + 0.18 (I_1 + I_2) = 0$$

$$7.45\, I_2 + 0.18 (I_2 + I_1) = 0$$

$$0.18\, I_1 + 7.63\, I_2 = 0$$

$$0.18 I_1 = -7.63\, I_2$$

$$\frac{I_1}{I_2} = \frac{-7.63}{0.18}$$

~~$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{I_1}{\frac{7.63}{0.18}} =$~~

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = -\left(\frac{-7.63}{0.18}\right) = \underline{42.38}$$

Scanned by CamScanner

$$V_1 = 4.9 I_1 + 0.18 I_2$$

$$V_1 = 4.9 \left(-\frac{7.63}{0.18} I_2 \right) + 0.18 I_2$$

$$V_1 = -207.52 I_2$$

$$B = \left. \frac{V_1}{-I_2} \right|_{V_2 = 0} = \frac{-207.52 \cancel{I_2}}{-\cancel{I_2}} = \underline{207.52\ \Omega}$$

$$\Rightarrow [T] = \begin{bmatrix} 27.22 & 207.52 \\ 5.55 & 42.38 \end{bmatrix}$$

Parameters	Condition for	
	Reciprocity	Symmetry
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
h	$h_{12} = -h_{21}$	$h_{11}h_{22} - h_{12}h_{21} = 1$
T	$AD - BC = 1$	$A = D$

Relation between 2 port parameters.

(1) Relation between Z and Y parameters

$$[V] = [Z][I]$$

then $$[I] = [Z]^{-1}[V]$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$Z^{-1} = \frac{1}{\Delta Z} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$ where $\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

||ly $$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{\Delta Y} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

② Relation between Y and h parameters

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

comparing

$$Y_{11} V_1 = I_1 - Y_{12} V_2$$

$$Y_{21} V_1 = I_2 - Y_{22} V_2$$

$$\begin{bmatrix} Y_{11} & 0 \\ Y_{21} & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 & -Y_{12} \\ 0 & -Y_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & 0 \\ Y_{21} & -1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -Y_{12} \\ 0 & -Y_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$= \frac{-1}{Y_{11}} \begin{bmatrix} -1 & 0 \\ -Y_{21} & Y_{11} \end{bmatrix} \begin{bmatrix} 1 & -Y_{12} \\ 0 & -Y_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$= \frac{-1}{Y_{11}} \begin{bmatrix} -1 & Y_{12} \\ -Y_{21} & Y_{12}Y_{21} - Y_{11}Y_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}.$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 1/Y_{11} & \frac{-Y_{12}}{Y_{11}} \\ \frac{+Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{bmatrix}$$ where $\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21}$

③ Relation between T & h parameter

$$\left.\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}\right\} ①$$

$$\left.\begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}\right\} ②$$

rearrange ②

$$V_1 - h_{11} I_1 = h_{12} V_2$$

$$-h_{21} I_1 = h_{22} V_2 - I_2$$

matrix

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & -h_{11} \\ 0 & -h_{21} \end{bmatrix}^{-1} \begin{bmatrix} h_{12} & 0 \\ h_{22} & -1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$[T] = \frac{1}{-h_{21}} \begin{bmatrix} -h_{21} & h_{11} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_{12} & 0 \\ h_{22} & -1 \end{bmatrix}$$

take note might be wrong

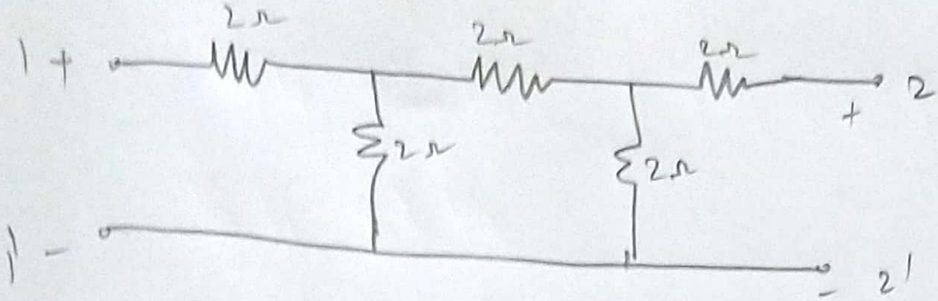
$$= \begin{bmatrix} 1 & -\frac{h_{11}}{h_{21}} \\ 0 & \frac{1}{-h_{21}} \end{bmatrix} \begin{bmatrix} h_{12} & 0 \\ h_{22} & -1 \end{bmatrix}$$

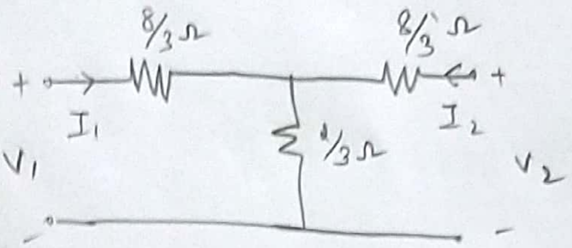
$$-\frac{\Delta h}{h_{21}}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cancel{h_{12} - \frac{h_{11} h_{22}}{h_{21}}} & \frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{11}} & -\frac{1}{h_{21}} \end{bmatrix}$$

D

Find Z and Y parameters of the network shown in figure.

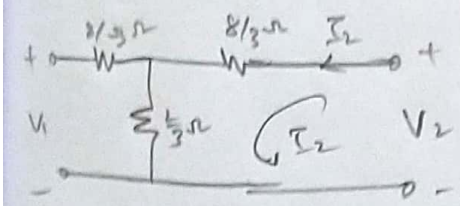




$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

let $I_1 = 0$



$V_1 = \frac{2}{3} I_2$ $\Rightarrow$ $\boxed{Z_{12} = \frac{2}{3} \ \Omega}$