Lecture 2:
n-gram Language Models

Instructor: Swabha Swayamdipta
USC CSCI 544 Applied NLP
Aug 29, Fall 2024

Some slides adapted from Dan Jurafsky and Chris Manning

Announcements

+

Recap

Logistics and Announcements

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- Syllabus changes (see website) based on requests
    - e.g. Quiz 4 date changed to accommodate Grace Hopper Conference attendance
    - Project Dates have changed to give you more time for the status report and presentations
    - Add / drop dates for class: Sep 6 and project team formation deadline: Sep 10

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        - Change: Will allow some groups of 6, but higher expectations from these groups
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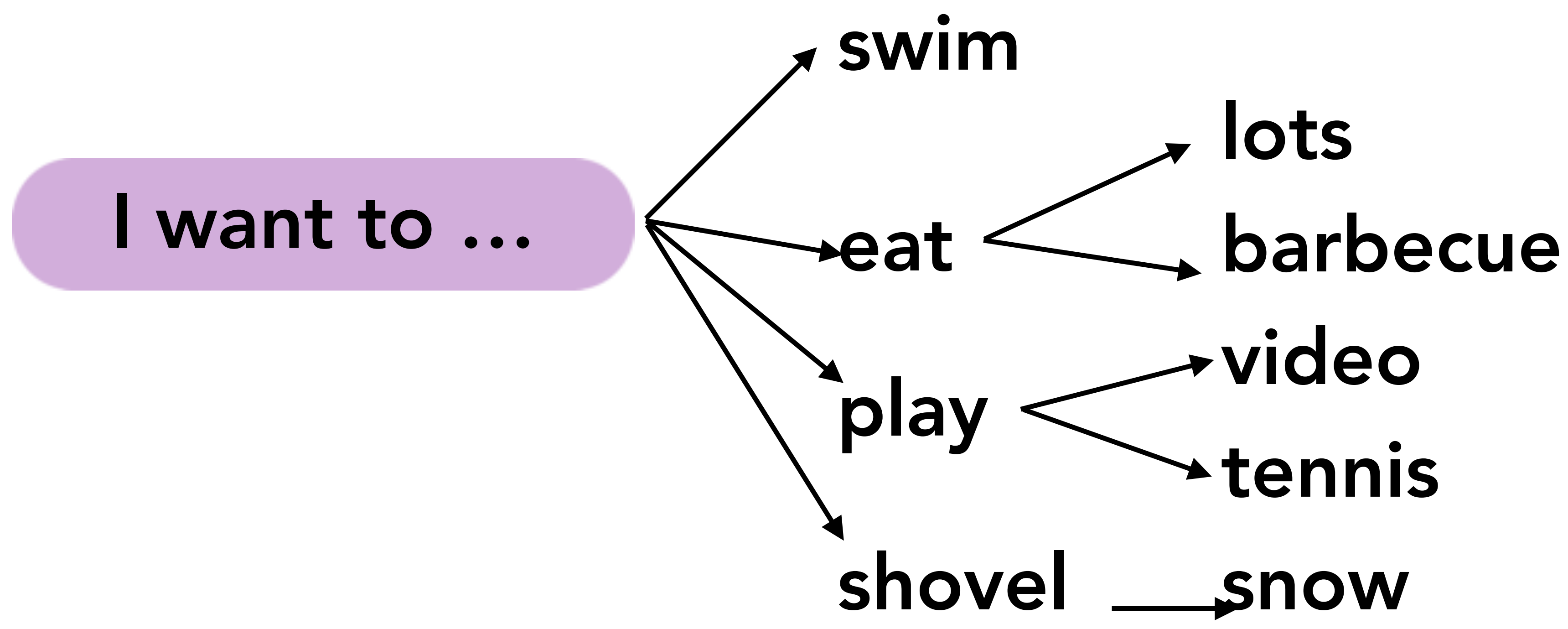
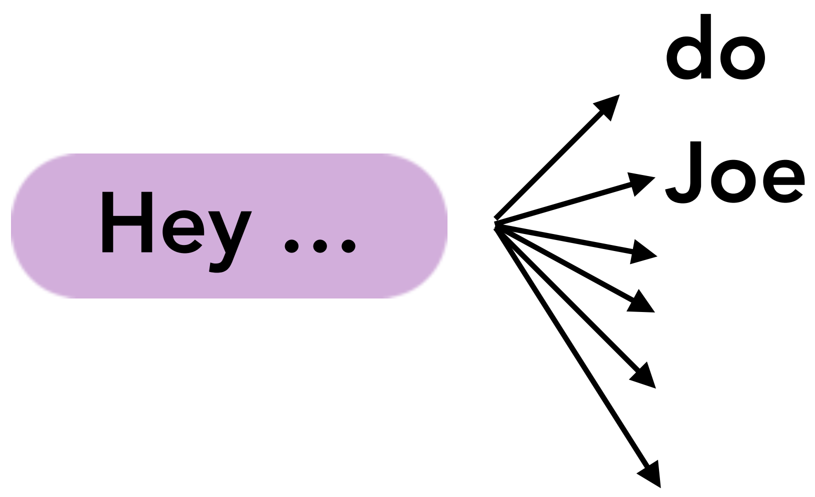
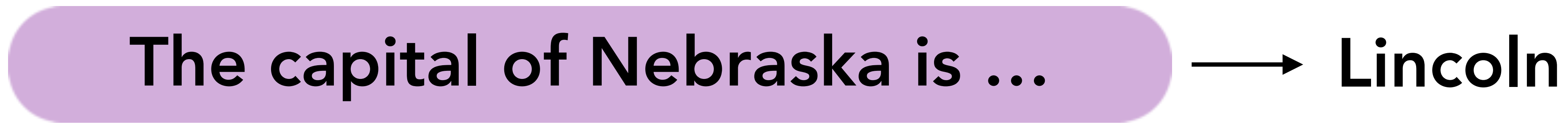
Building a Language Model


I want to …
swim
eat
lots
barbecue
play
video
tennis
shovel
snow


Hey …
do
Joe


The capital of Nebraska is …
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- Task: Given a sequence of words so far (the **context**), predict what comes next

- We never know for sure what comes next, but we can still make good guesses!

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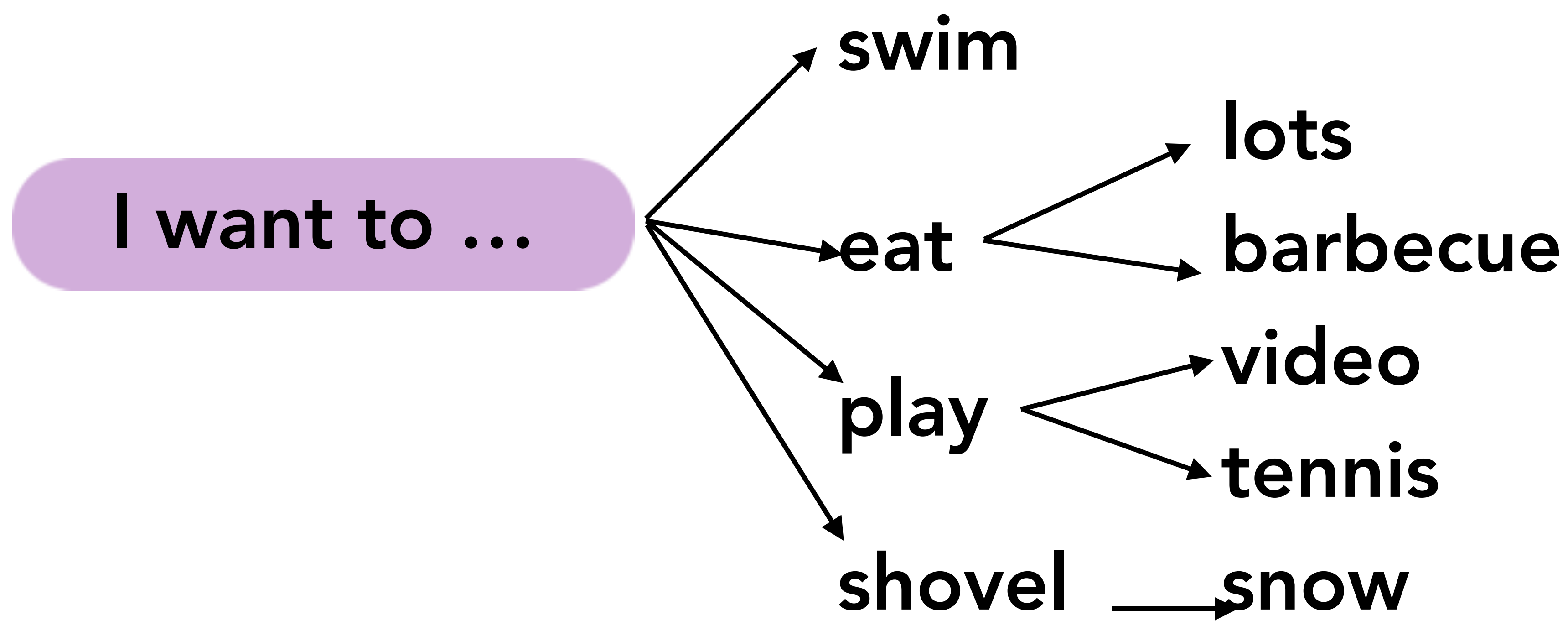

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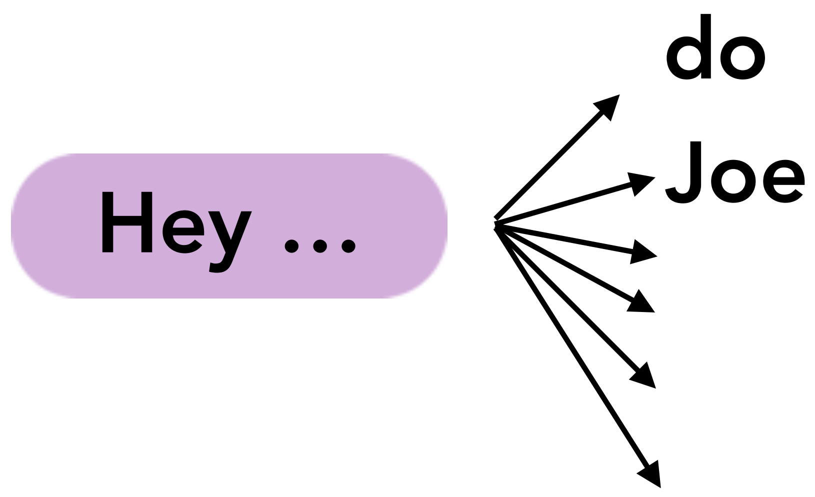

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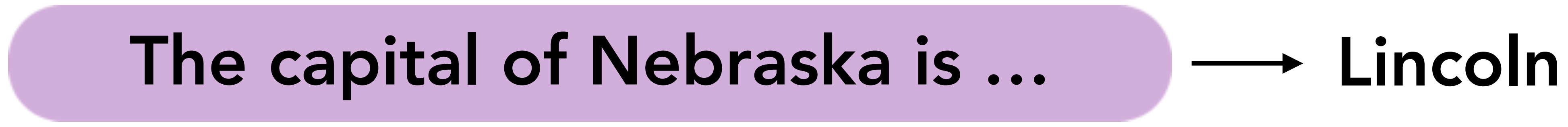

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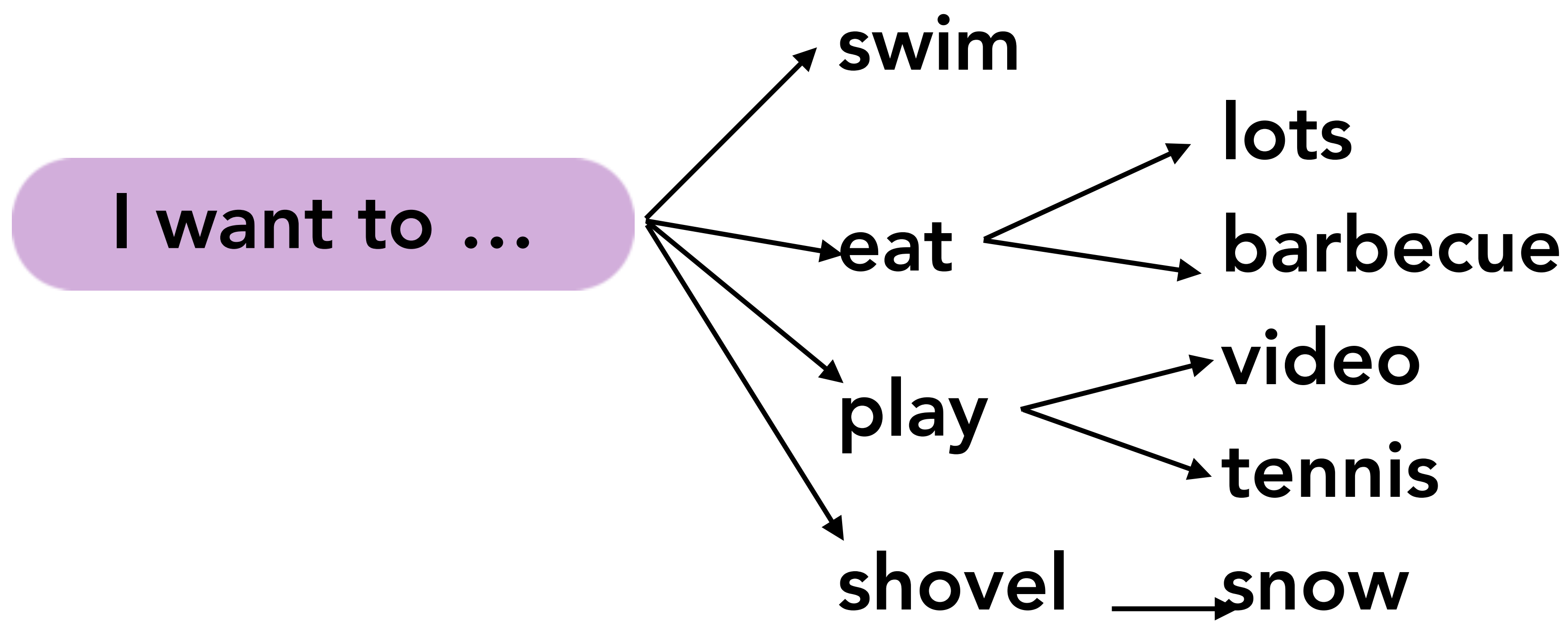

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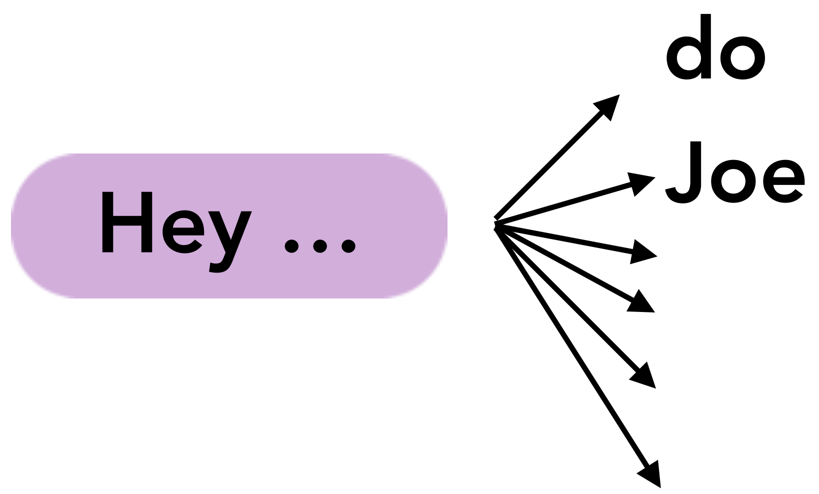

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Certain sentence constructions are more likely than others, due to grammaticality, obscurity or commonness

Sentences have different probabilities!

Lecture Outline

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2. Probabilistic Language Models
3. n-gram Language Models
4. Evaluation and Perplexity
5. Generating from an n-gram Language Model
   i. Zeroes
6. Smoothing

Probabilistic Language Models!

Assign a probability to a sentence

Probabilistic Language Modeling

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Goal: compute the probability of a sentence or sequence of words:

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Related task: probability of an upcoming word: $P(w_n \mid w_1, w_2, w_3, w_4, \ldots w_{n-1})$

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A model that assigns probabilities to sequences of words (e.g., either of these: $P(\mathbf{w})$ or $P(w_n \mid w_1, w_2, \ldots w_{n-1})$) is called a language model

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Goal: compute the probability of a sentence or sequence of words:

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Difference

Related task: probability of an upcoming word: $P(w_n \mid w_1, w_2, w_3, w_4, \ldots w_{n-1})$

A model that assigns probabilities to sequences of words (e.g., either of these: $P(\mathbf{w})$ or $P(w_n \mid w_1, w_2, \ldots w_{n-1})$) is called a language model "its water is so transparent that you can see the bottom" "its water is so transparent that you can see the bottom"

$$P(\text{its water is so transparent that you can see the bottom})$$

“its water is so transparent that you can see the bottom”

$$P(\text{its water is so transparent that you can see the bottom})$$

$$P(\text{its, water, is, so, transparent, that, you, can, see, the, bottom})$$

How to compute $P(W)$?

“its water is so transparent that you can see the bottom”

P(its water is so transparent that you can see the bottom)

P(its, water, is, so, transparent, that, you, can, see, the, bottom)

How to compute this joint probability, $P(\mathbf{w}) = P(w_1, w_2, w_3, w_4, w_5, \ldots w_n)$?

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Intuition: let's rely on the Chain Rule of Probability

Chain Rule for words in a sentence

$P(\text{its water is so transparent}) =$

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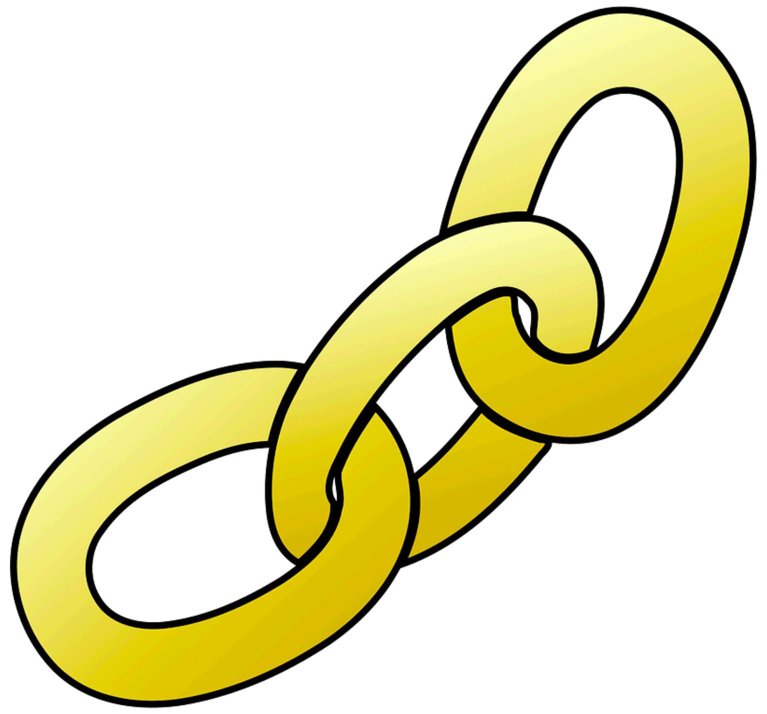
$$P(w_1, w_2, \ldots w_n) = \prod_{i=1}^{n} P(w_i \mid w_{i-1} \ldots w_1)$$



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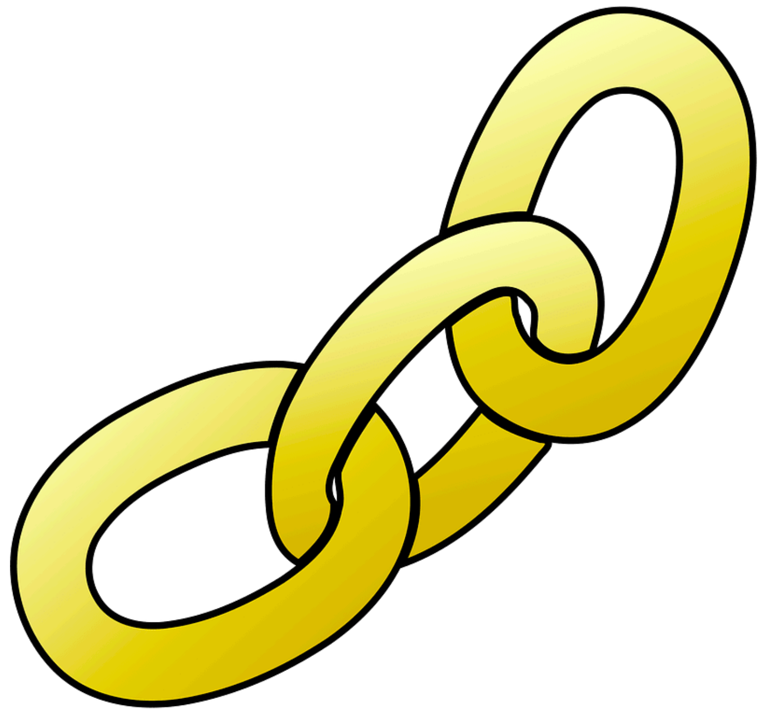

$$\begin{aligned} P(\text{its water is so transparent}) = \; & P(\text{its}) \times \\ & P(\text{water} \mid \text{its}) \times \\ & P(\text{is} \mid \text{its water}) \times \\ & P(\text{so} \mid \text{its water is}) \times \\ & P(\text{transparent} \mid \text{its water is so}) \end{aligned}$$

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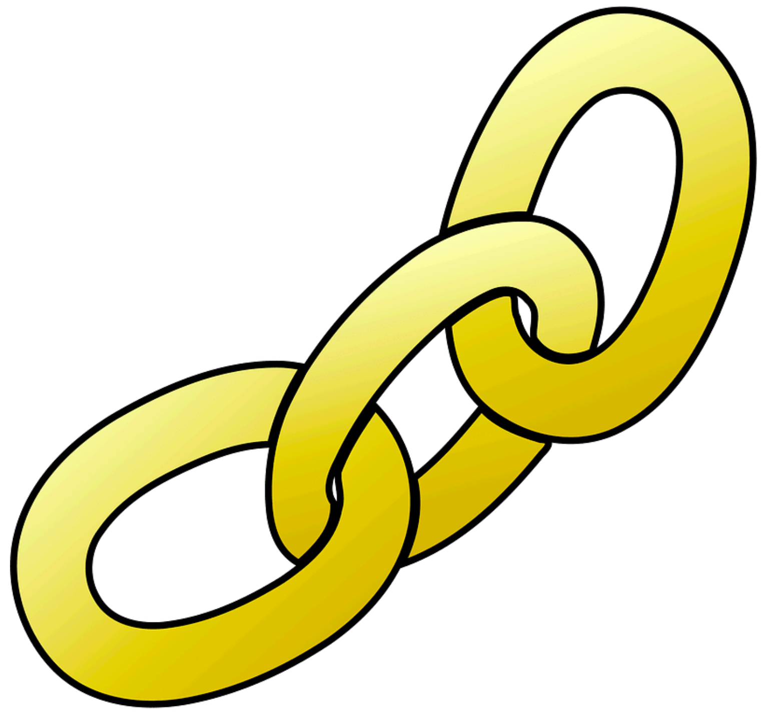

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Ordering matters in language!

Why Probabilistic Models?

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I will be back soonish

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- Probabilities are essential for language generation
- Any task in which we have to identify words in noisy, ambiguous input, like speech recognition
- For writing tools like spelling correction or grammatical error correction

I will be back soonish

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Your so silly

You’re so silly

Everything has improve

Everything has improved

Probabilistic Language Models

Machine Translation:

- P(high winds tonight) $>$ P(large winds tonight)

Spell Correction:

- P(I'm about fifteen minuets away) $<$ P(I'm about fifteen minutes away)

Speech Recognition:

- P(I saw a van) $>>$ P(eyes awe of an)

Summarization, question-answering, etc., etc.!!

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But how to learn these probabilities?

CoolClips.com

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Probability Estimation via Statistical Modeling

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The probability of the data is $ppp(1-p)$: if you take the derivative and set it equal to zero and find $p = 0.75$

n-gram Language Model

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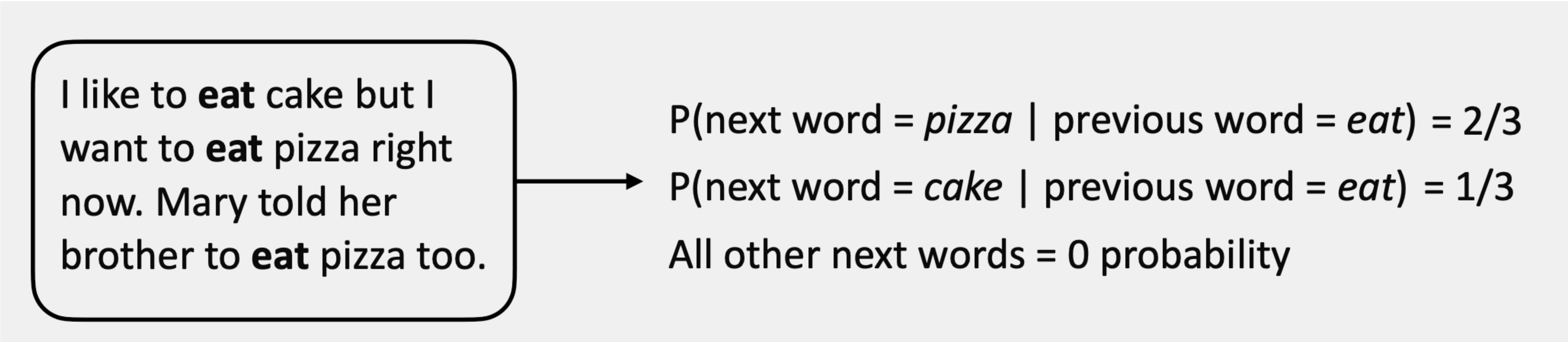

I like to **eat** cake but I want to **eat** pizza right now. Mary told her brother to **eat** pizza too.

P(next word = *pizza* | previous word = *eat*) = 2/3

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All other next words = 0 probability

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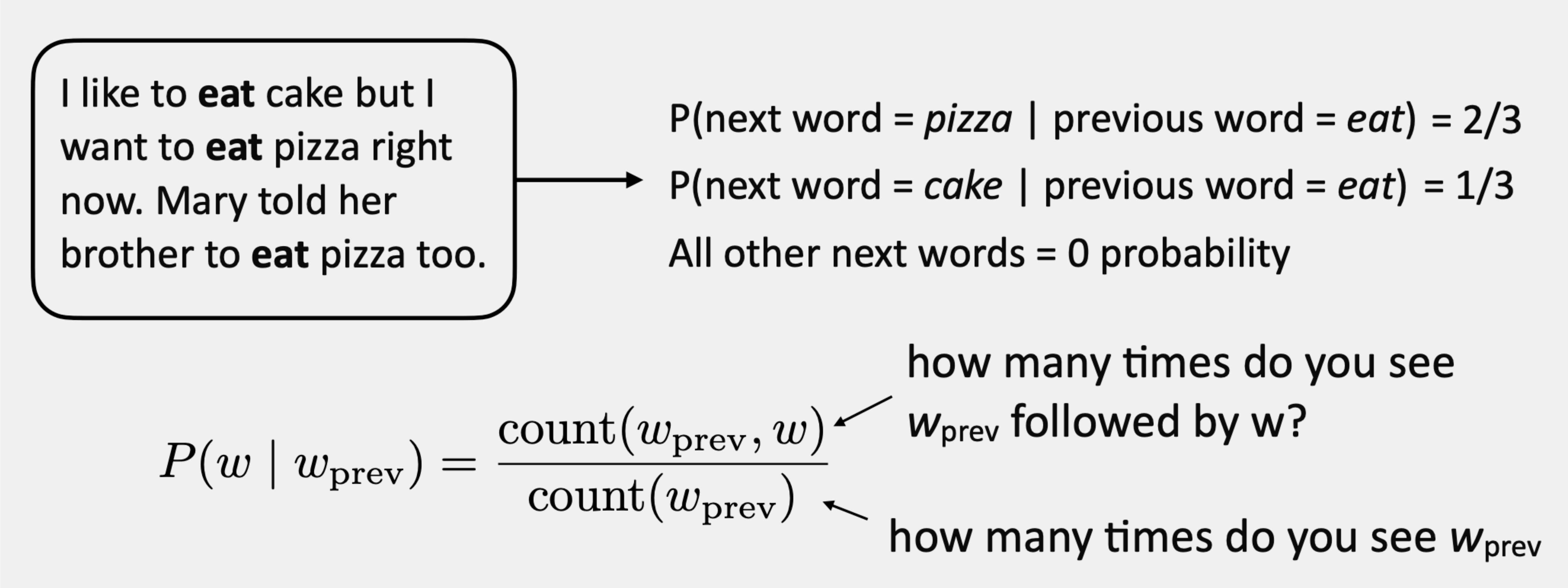

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how many times do you see w_{prev} followed by w?

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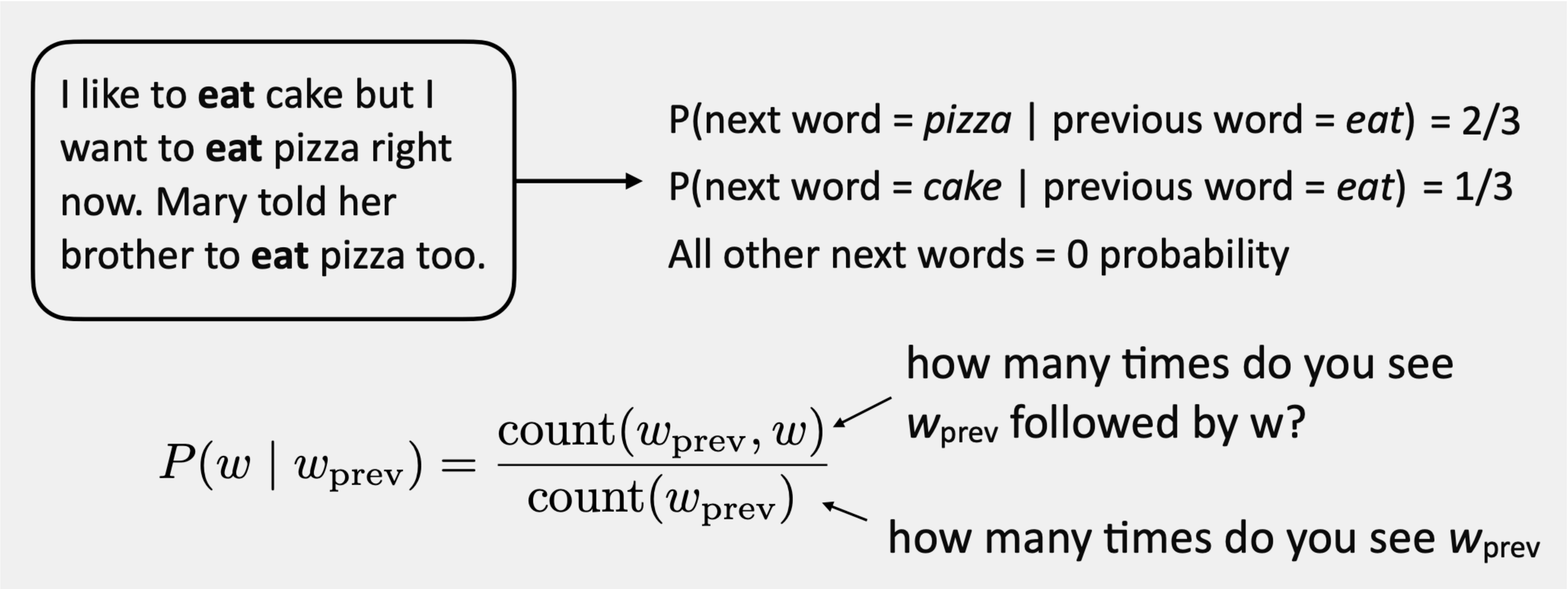

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Vocabulary

How to estimate the probability of the next word?

$$P(\text{that} \mid \text{its water is so transparent}) = \frac{Count(\text{its water is so transparent that})}{Count(\text{its water is so transparent})}$$

How to estimate the probability of the next word?

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Could we just count and divide?

No! Too many possible sentences!

We’ll never see enough data for estimating these

Simplifying Assumption:

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$$P(\text{that} \,|\, \text{its water is so transparent }) \approx P(\text{that} \,|\, \text{transparent})$$

Markov Assumption

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Andrei Markov

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Or maybe…

$$P(\text{that} \mid \text{its water is so transparent}) \approx P(\text{that} \mid \text{so transparent})$$

Markov Assumption contd.

$$P(w_1, w_2, \ldots w_n) = \prod_i P(w_i \,|\, w_{i-k} \ldots w_{i-1})$$

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$(k+1)$-th order Markov assumption

Mini Recap: Probabilistic Modeling

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- What is a probabilistic language model?

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Mini Recap: Probabilistic Modeling

- What is a probabilistic language model?
- Why would we need one?
- How do we estimate one?
- How do we simplify the estimation problem?
- Next: a simple probabilistic language model

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n-gram Language Models

simplest probabilistic model

Simplest Case: Unigram model

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Simplest Case: Unigram model

$$P(w_1, w_2, \dots w_n) \approx \prod_i P(w_i)$$

Some automatically generated sentences from a unigram model

- fifth, an, of, futures, the, an, incorporated, a, a, the, inflation, most, dollars, quarter, in, is, mass
- thrift, did, eighty, said, hard, 'm, july, bullish
- that, or, limited, the

Bigram Model

Condition on the previous word:

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Some automatically generated sentences from a bigram model

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Some automatically generated sentences from a bigram model

- texaco, rose, one, in, this, issue, is, pursuing, growth, in, a, boiler, house, said, mr., gurria, mexico, 's, motion, control, proposal, without, permission, from, five, hundred, fifty, five, yen
- outside, new, car, parking, lot, of, the, agreement, reached
- this, would, be, a, record, november

n-gram Language Models

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In general this is an insufficient model of language

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Long-distance / Long-range dependencies

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> “The computer which I had just put into the machine room on the fifth floor crashed.”

Long-distance / Long-range dependencies

But we can often get away with n-gram models, where n is a small number

Estimating bigram probabilities

The maximum likelihood estimate

$$P(w_i \mid w_{i-1}) = \frac{count(w_{i-1}, w_i)}{count(w_{i-1})}$$

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

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What happens when $i = 1$?

Estimating bigram probabilities

The maximum likelihood estimate

$$P(w_i \mid w_{i-1}) = \frac{count(w_{i-1}, w_i)}{count(w_{i-1})}$$

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

What happens when $i = 1$?

Special edge case tokens: <s> and </s> for beginning of sentence and end of sentence, respectively

An example

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

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<s> I am Sam </s>

<s> Sam I am </s>

<s> I do not like green eggs and ham </s>

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<s> I am Sam </s>
<s> Sam I am </s>
<s> I do not like green eggs and ham </s>

$$P(\texttt{I} \mid \texttt{<s>}) = \tfrac{2}{3} = .67 \qquad P(\texttt{Sam} \mid \texttt{<s>}) = \tfrac{1}{3} = .33 \qquad P(\texttt{am} \mid \texttt{I}) = \tfrac{2}{3} = .67$$

$$P(\texttt{</s>} \mid \texttt{Sam}) = \tfrac{1}{2} = 0.5 \qquad P(\texttt{Sam} \mid \texttt{am}) = \tfrac{1}{2} = .5 \qquad P(\texttt{do} \mid \texttt{I}) = \tfrac{1}{3} = .33$$

Larger Example:
Berkeley Restaurant Project (BRP)

- can you tell me about any good cantonese restaurants close by
- mid priced thai food is what i'm looking for
- tell me about chez panisse
- can you give me a listing of the kinds of food that are available
- i'm looking for a good place to eat breakfast
- when is caffe venezia open during the day

Total: 9222 similar sentences

BRP: Raw Counts

Out of 9222 sentences

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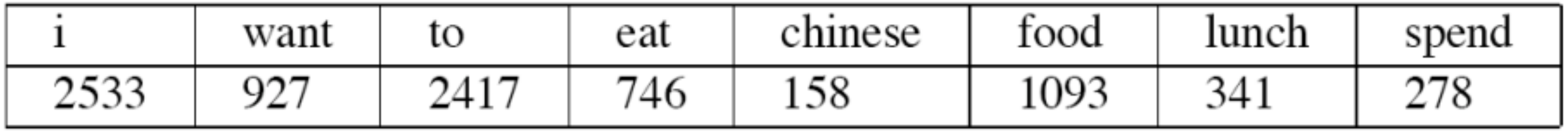
Out of 9222 sentences

Unigrams

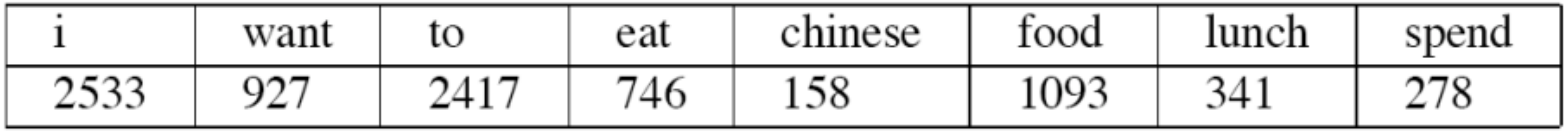


i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

BRP: Raw Counts

Out of 9222 sentences

Unigrams



i	want	to	eat	chinese	food	lunch	spend
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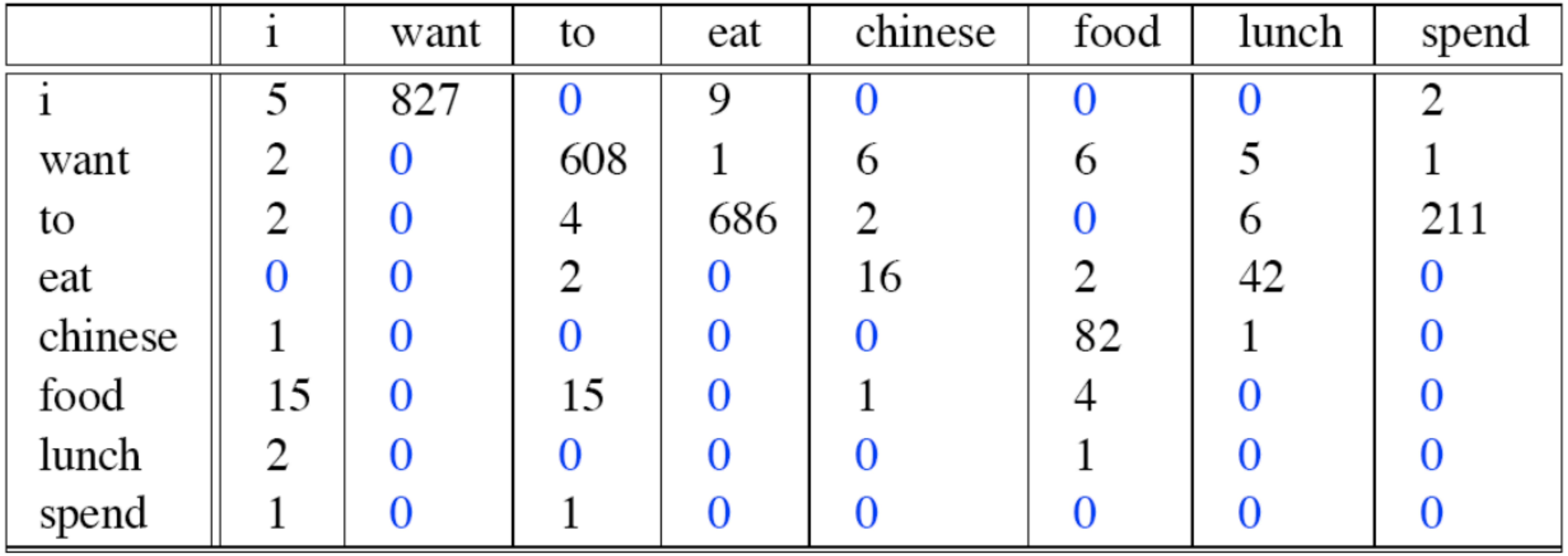
Next Word

Bigrams

History



	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

BRP: Bigram Probabilities

Bigram Probabilities: Raw bigram counts normalized by unigram counts

$$P(w_i \,|\, w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

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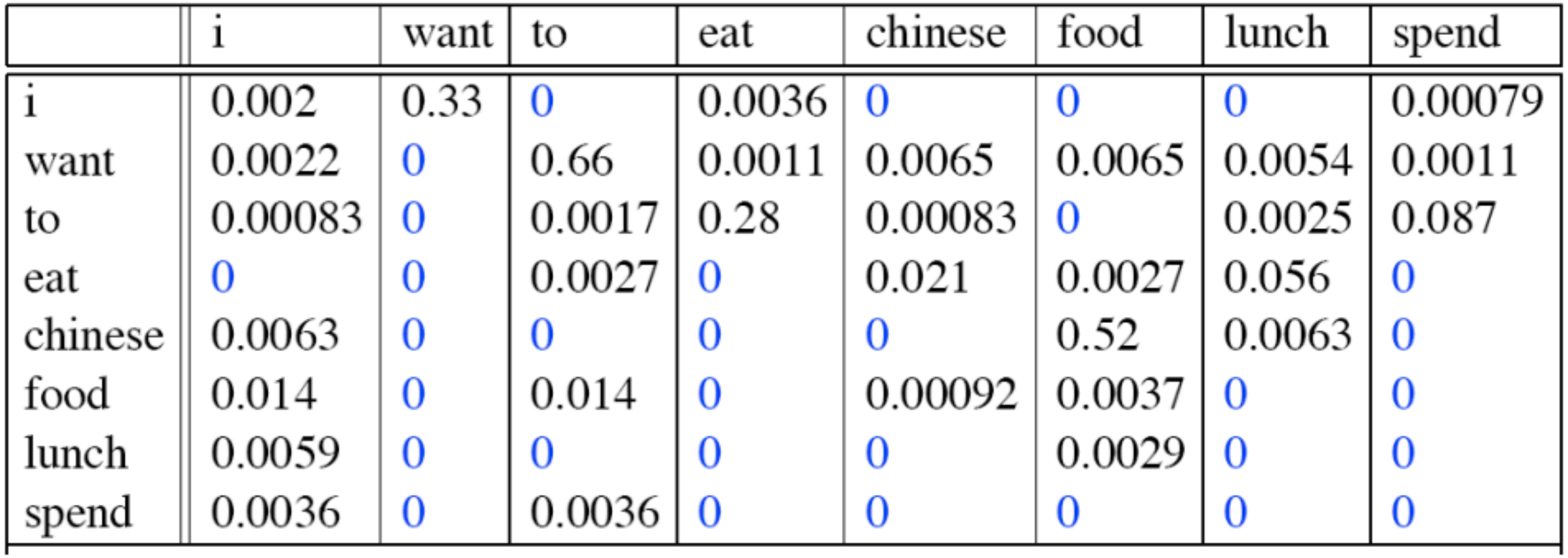
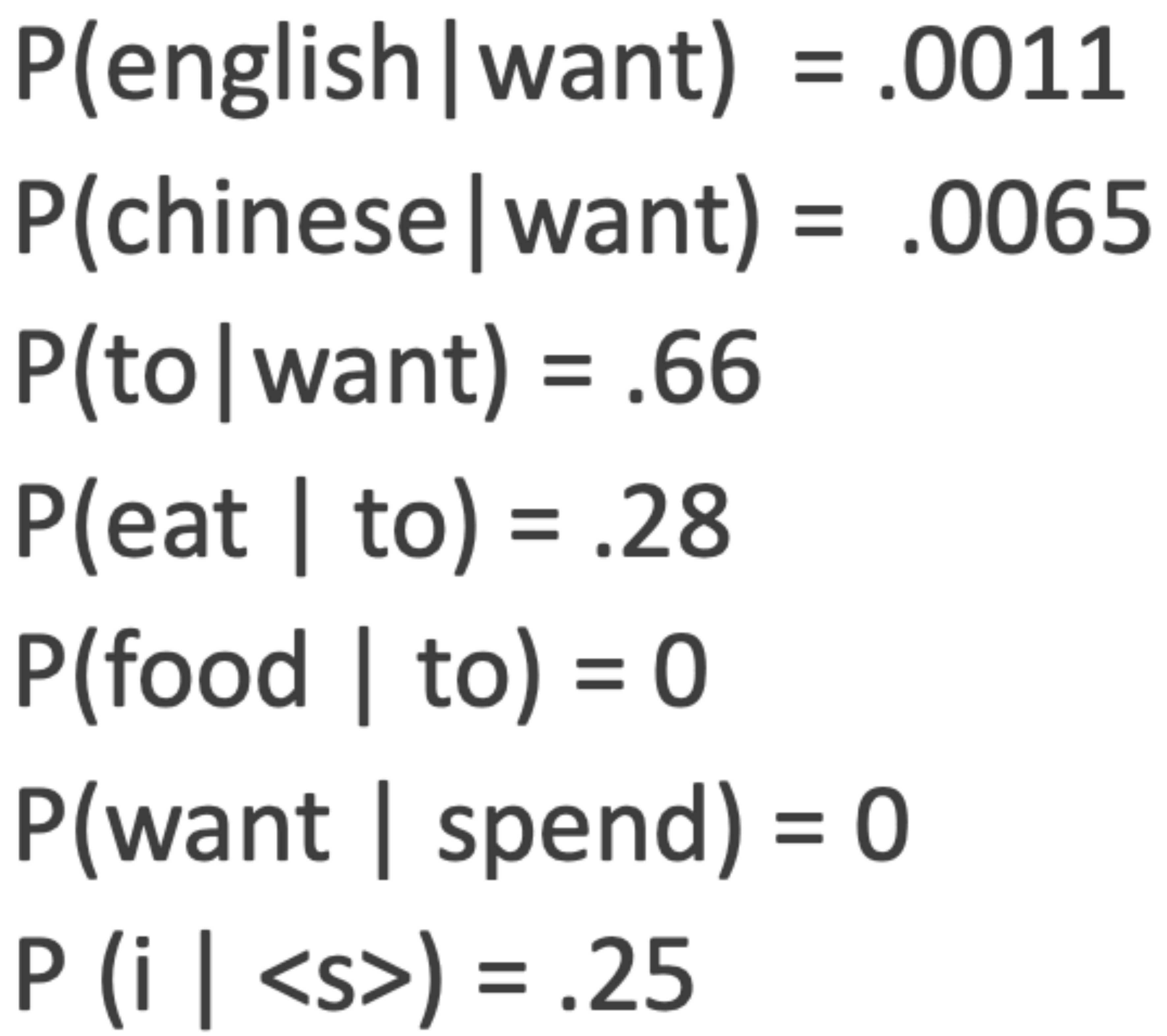
w_i

w_{i-1}



	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

What kinds of knowledge?



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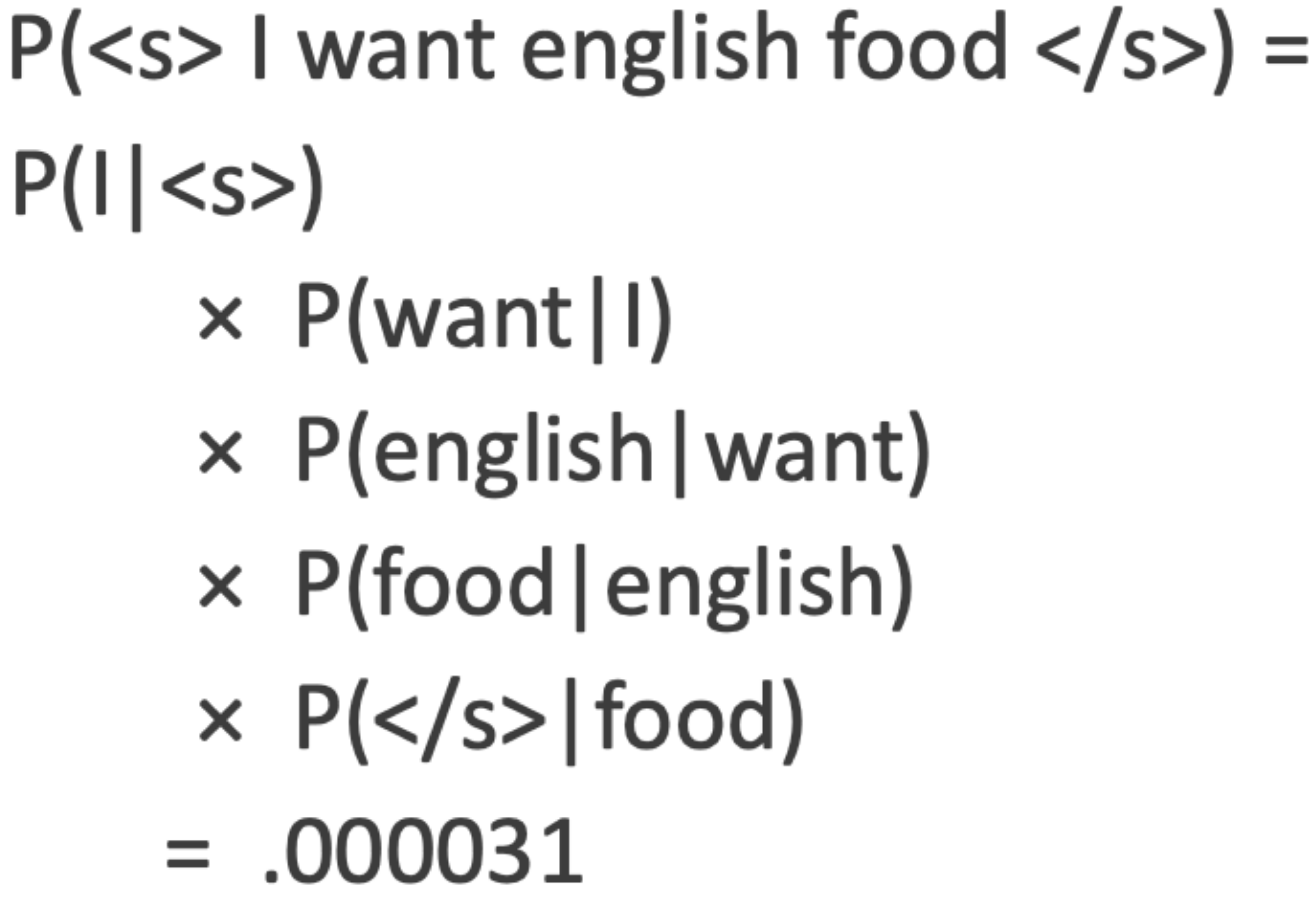
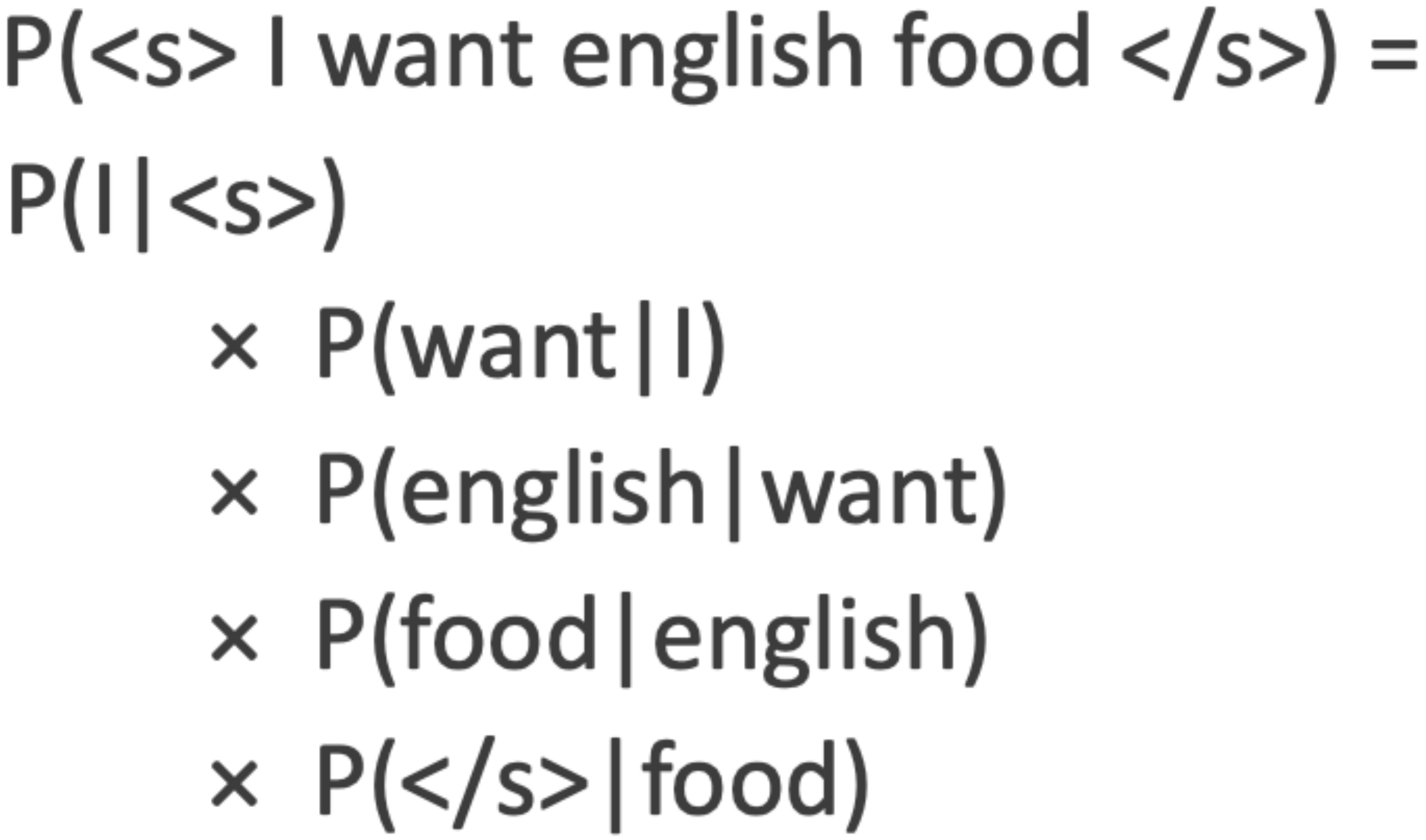
Bigram estimates of sentence probabilities



$$
\begin{aligned}
&P(\text{<s> I want english food </s>}) = \\
&P(\text{I}|\text{<s>}) \\
&\quad \times P(\text{want}|\text{I}) \\
&\quad \times P(\text{english}|\text{want}) \\
&\quad \times P(\text{food}|\text{english}) \\
&\quad \times P(\text{</s>}|\text{food}) \\
&= .000031
\end{aligned}
$$

Bigram estimates of sentence probabilities



$$
\begin{aligned}
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&\quad \times P(\text{food}|\text{english}) \\
&\quad \times P(\text{</s>}|\text{food})
\end{aligned}
$$

$$\boxed{= .000031}$$

Quite low…

Underflow Issues

We do everything in log space

- Avoid underflow
- Adding is faster than multiplying

$$\log(p_1 \times p_2 \times p_3 \times p_4) = \log p_1 + \log p_2 + \log p_3 + \log p_4$$

Lecture Outline

1. Announcements + Recap
2. Probabilistic Language Models
3. n-gram Language Models
4. Evaluation and Perplexity
5. Generating from an n-gram Language Model
   i. Zeroes
6. Smoothing

Evaluation and Perplexity

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- An **evaluation metric** tells us how well our model does on the test set.

Intuition of Perplexity

The **Shannon Game**: How well can we predict the next word?

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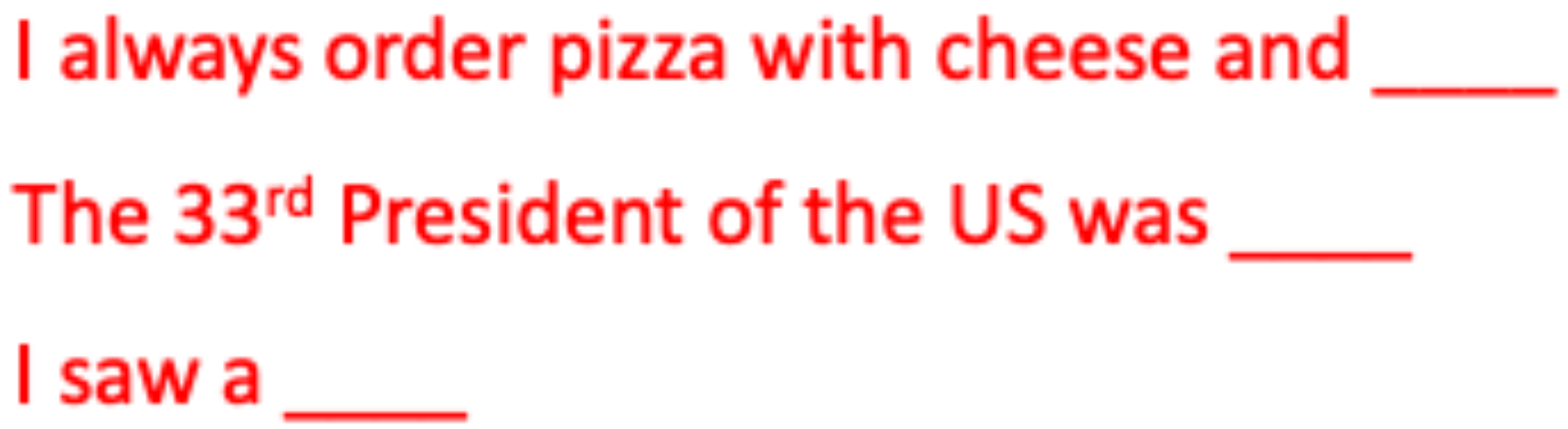

I always order pizza with cheese and ____

The 33rd President of the US was ____

I saw a ____

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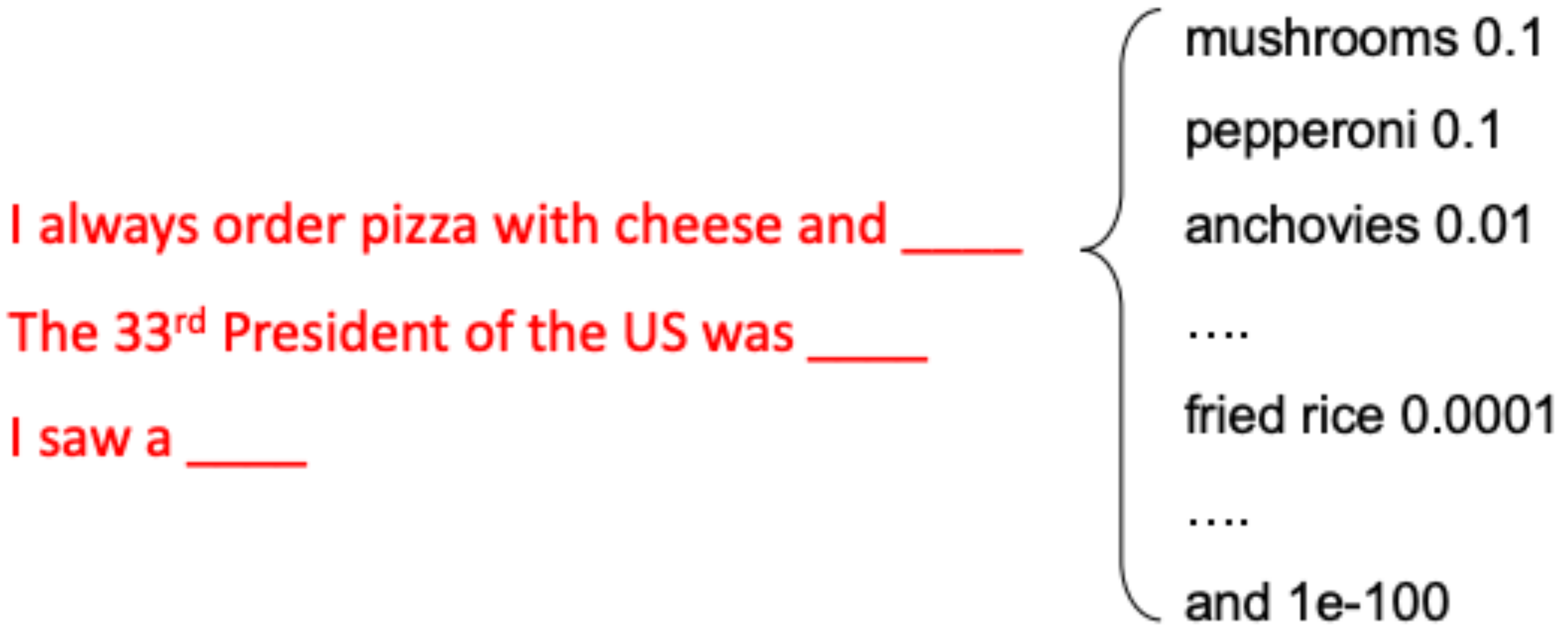

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mushrooms 0.1

pepperoni 0.1

anchovies 0.01

….

fried rice 0.0001

….

and 1e-100

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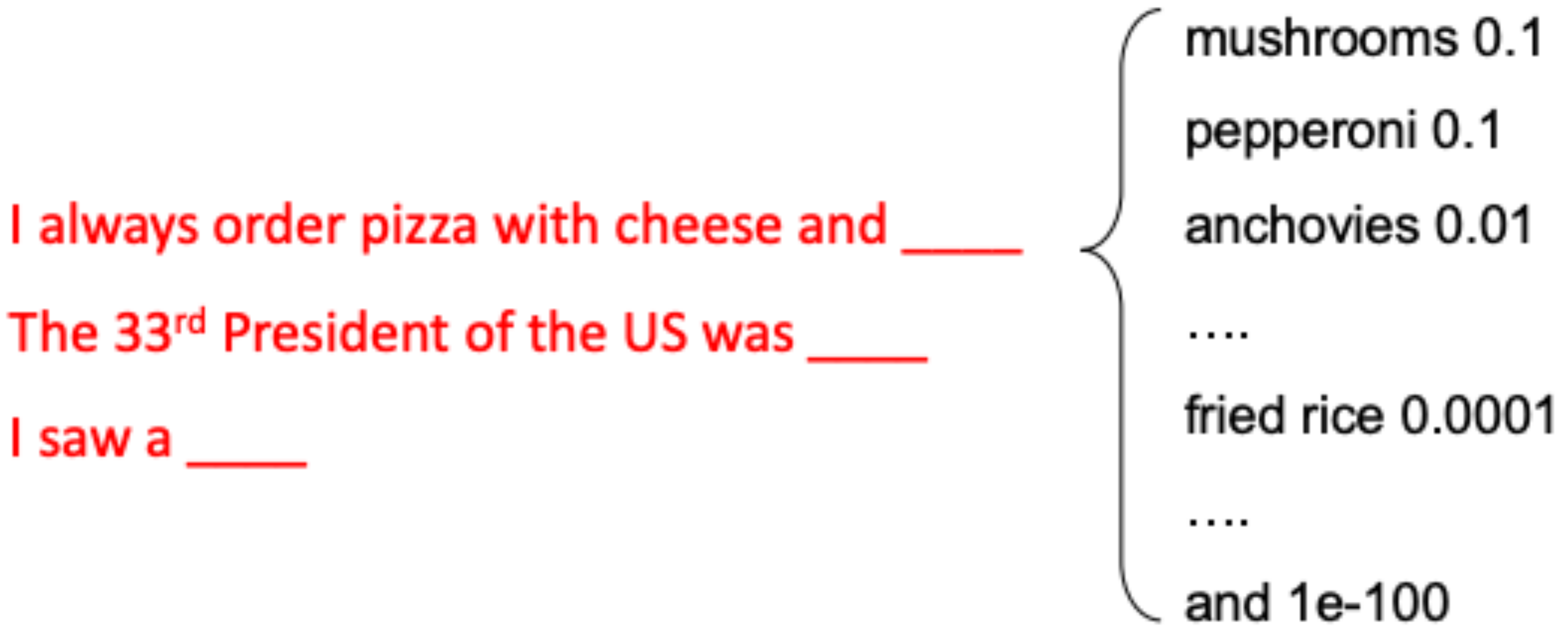

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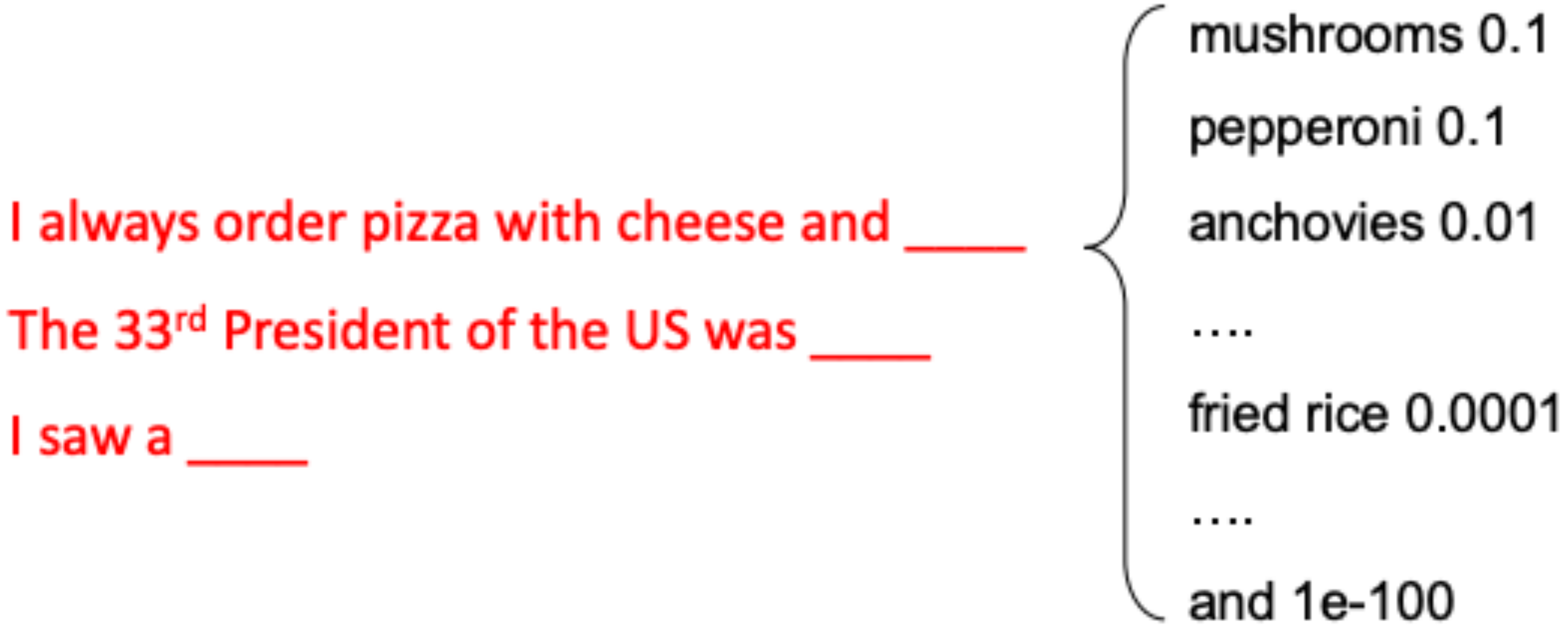

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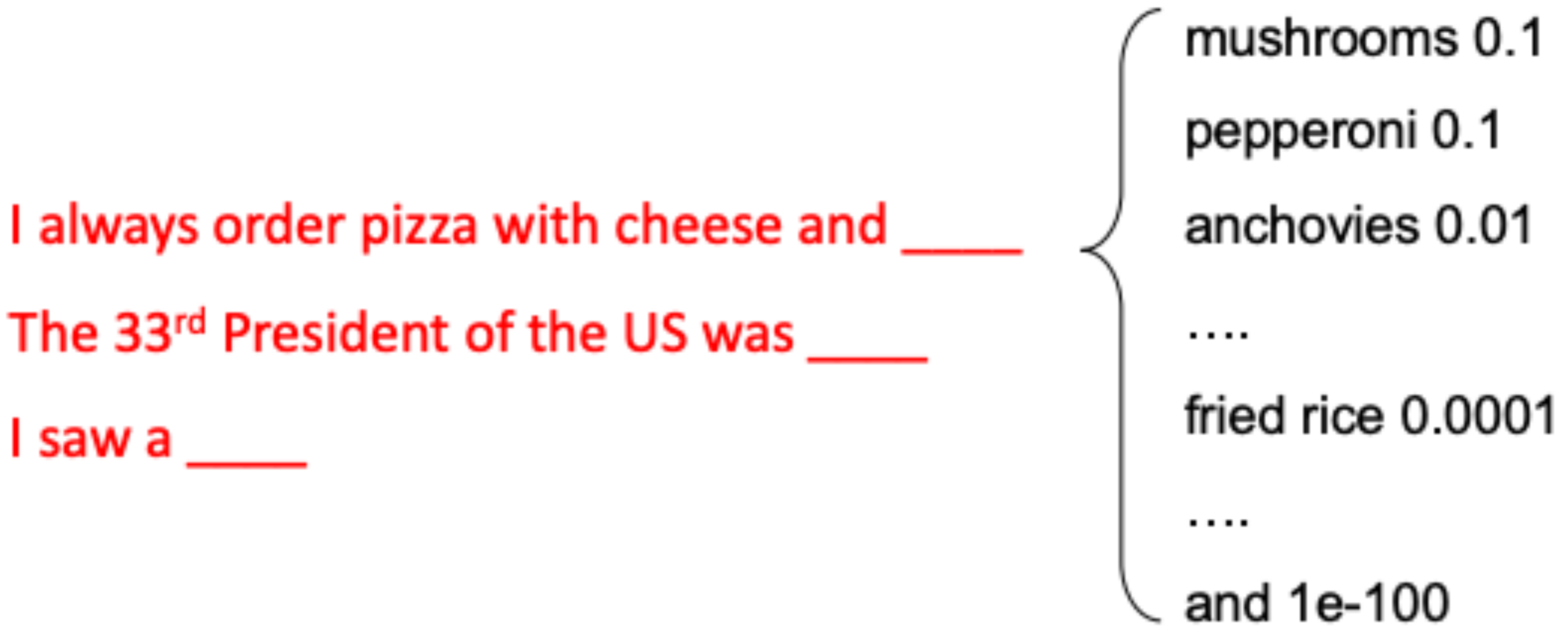

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A better model of a text is one which assigns a higher probability to the word that actually occurs

Perplexity

The best language model is one that best predicts an unseen test set

- Gives the highest P(sentence), for most sentences acceptable to humans

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Minimizing perplexity is the same as maximizing probability

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$$= 10$$

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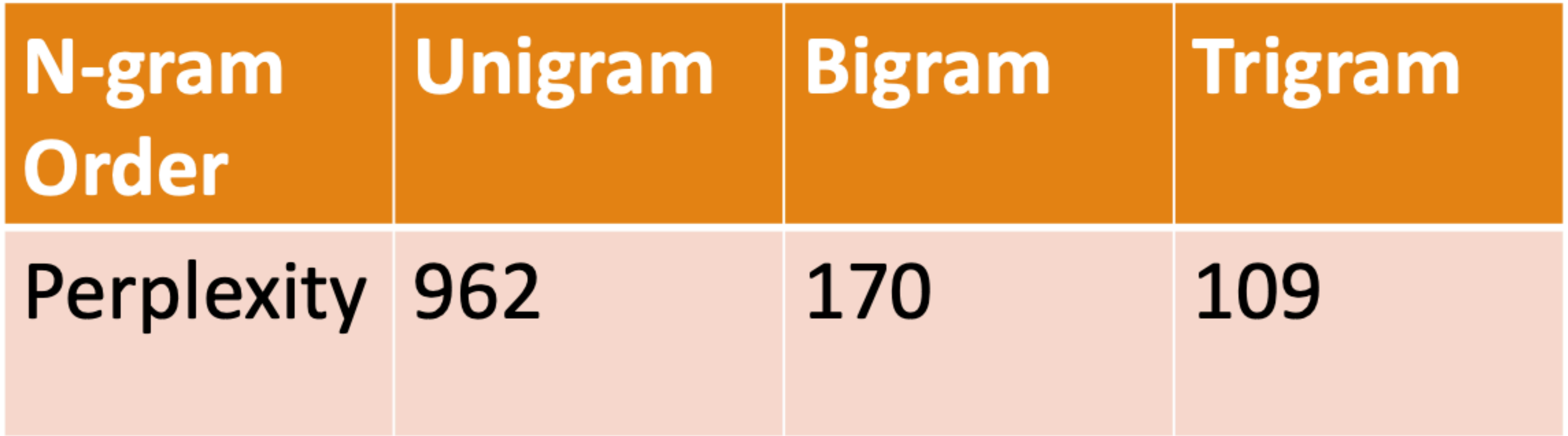

N-gram Order	Unigram	Bigram	Trigram
Perplexity	962	170	109

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What are the two things that might affect perplexity?

Lecture Outline

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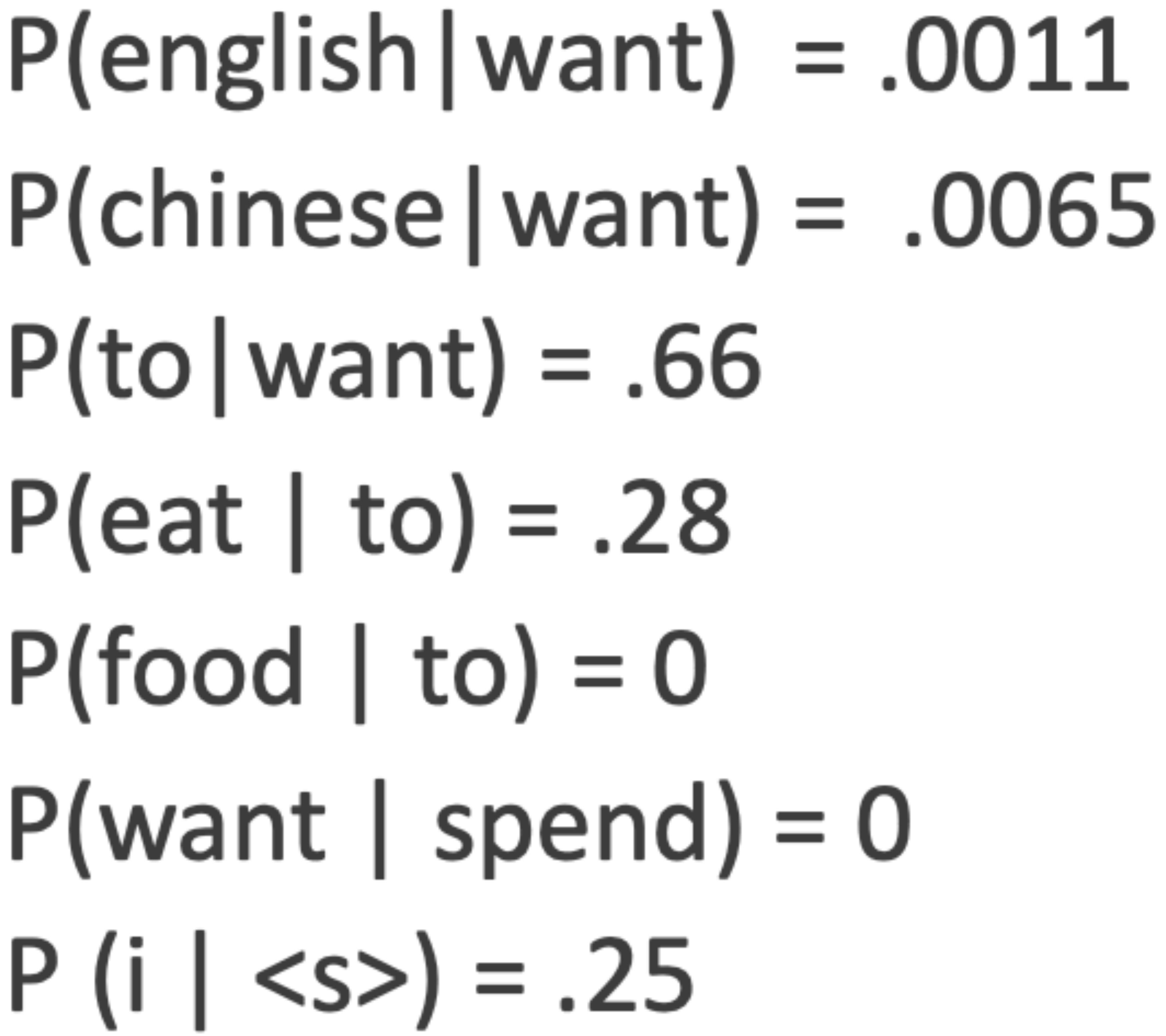
Generating from an n-gram model and Zeros

Recall: BRP



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P (i | <s>) = .25

How can we generate sentences from this bigram model?

Generating from a bigram model

Generating from a bigram model

- Choose a random bigram (<s>, w) according to its probability

Generating from a bigram model

`<s> I`

- Choose a random bigram (<s>, w) according to its probability

Generating from a bigram model

```
<s> I
    I want
```

- Choose a random bigram (<s>, w) according to its probability
- Now choose a random bigram (w, x) according to its probability

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- And so on until we choose </s>

```
<s> I
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           to eat
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                  Chinese food
                          food  </s>
```

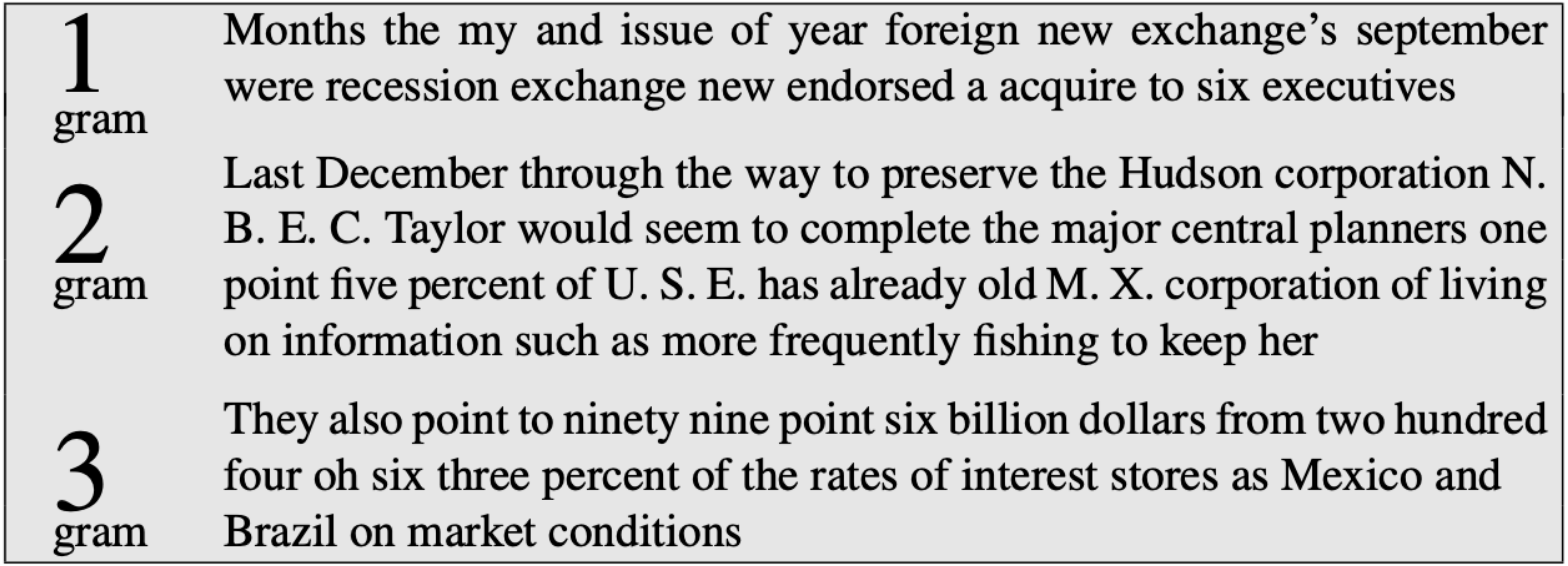
Generating from a bigram model

- Choose a random bigram (<s>, w) according to its probability
- Now choose a random bigram (w, x) according to its probability
- And so on until we choose </s>
- Then string the words together

```
<s> I
    I want
      want to
           to eat
              eat Chinese
                  Chinese food
                          food  </s>
```

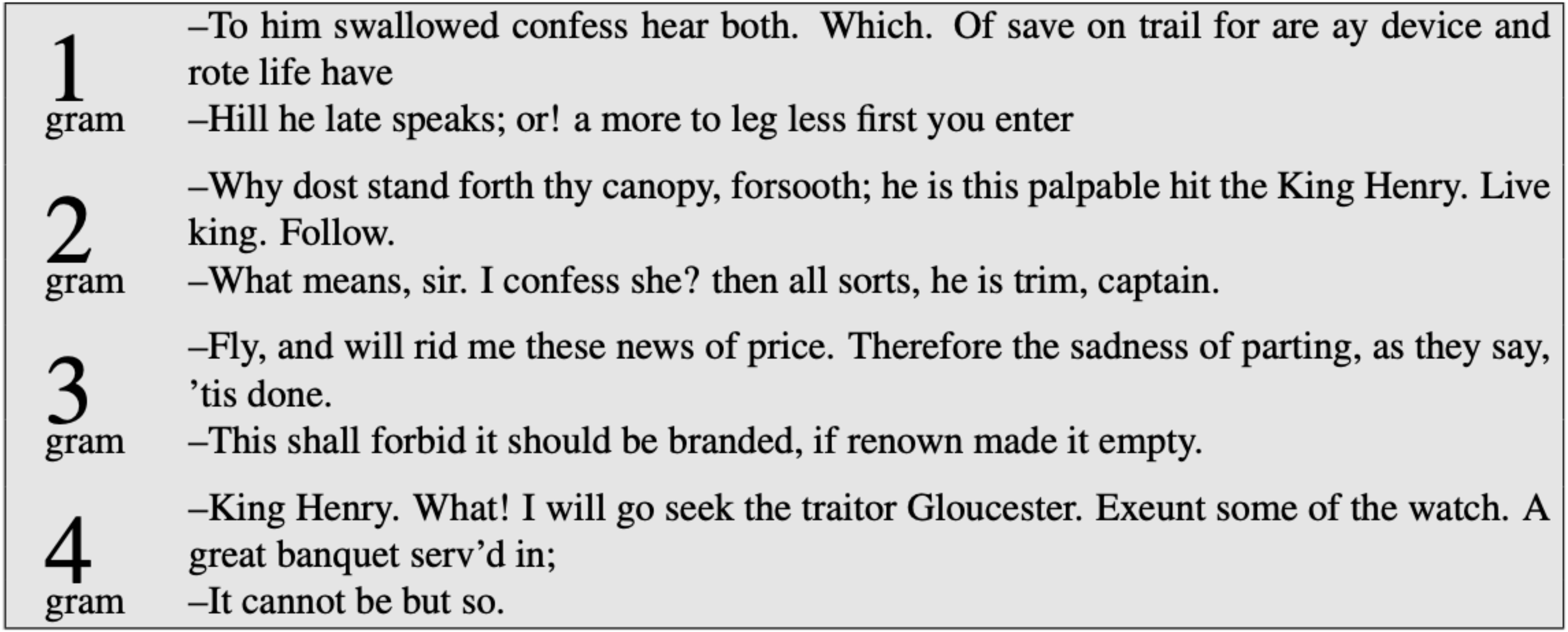
I want to eat Chinese food

The WSJ is no Shakespeare!



1 gram	Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives
2 gram	Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her
3 gram	They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions

Shakespearean n-grams



1 gram	–To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have –Hill he late speaks; or! a more to leg less first you enter
2 gram	–Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow. –What means, sir. I confess she? then all sorts, he is trim, captain.
3 gram	–Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, ’tis done. –This shall forbid it should be branded, if renown made it empty.
4 gram	–King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv’d in; –It cannot be but so.

Shakespeare as a corpus

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N=884,647 tokens, V=29,066

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What's coming out looks like Shakespeare because it is Shakespeare!

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So why not just sample from very high order n-gram models? Do we even need GPT-style LLMs?

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The successes we are seeing here is a phenomena commonly known as overfitting

Overfitting bad!

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  - Technical terms for “doing well on the test data” or “doing well on any test data”

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- In real life, it often doesn’t
- We need to train **robust** models that **generalize**!
  - Technical terms for “doing well on the test data” or “doing well on any test data”
- One kind of generalization: Zeros!
  - Things that don’t ever occur in the training set
    - But occur in the test set

Training set:

… denied the allegations

… denied the reports

… denied the claims

… denied the request

Training set:

… denied the allegations

… denied the reports

… denied the claims

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Test set

… denied the offer

… denied the loan

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$P\,(\textbf{offer} \mid \textbf{denied the}) =$

Zeros

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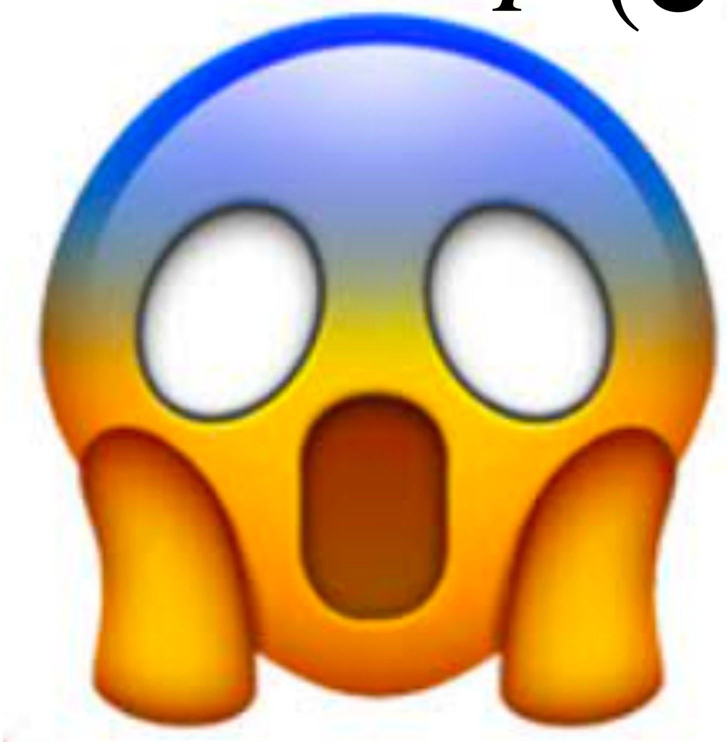
$P\,(\textbf{offer}\,|\,\textbf{denied the}) = \;\; 0$



will assign 0 probability to the test set!

Zeros

Training set:

… denied the allegations

… denied the reports

… denied the claims

… denied the request

Test set

… denied the offer

… denied the loan

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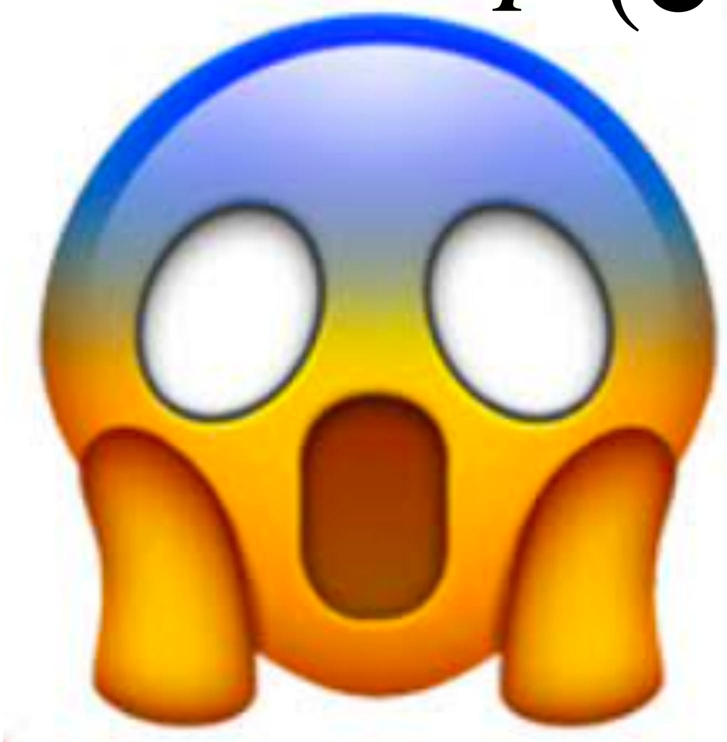

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What happens to perplexity??

One solution: the UNK token

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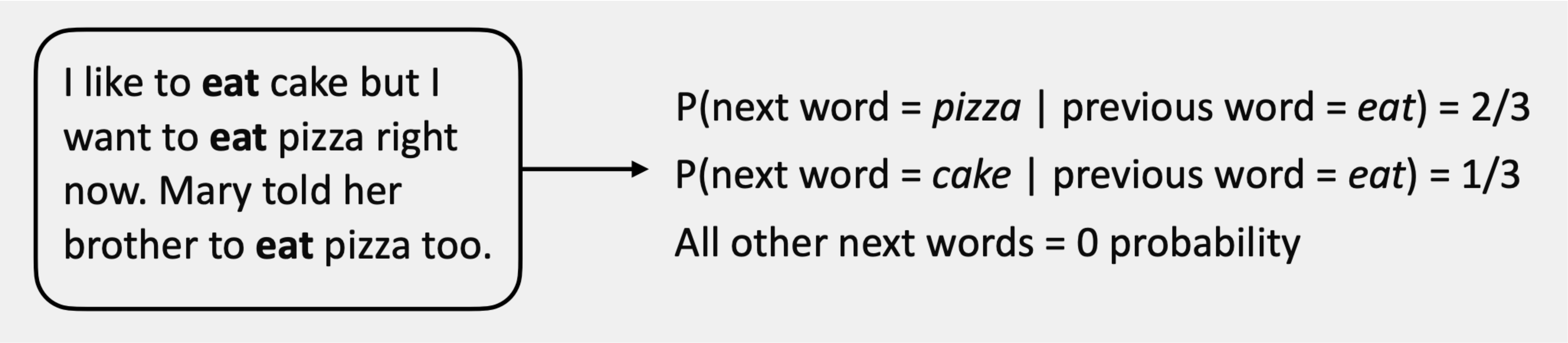
Lecture Outline

1. Announcements + Recap
2. Probabilistic Language Models
3. *n*-gram Language Models
4. Evaluation and Perplexity
5. Generating from an n-gram Language Model
    i. Zeroes
6. Smoothing
    i. Add-one / Laplace
    ii. Interpolation

Intuition for Smoothing



I like to **eat** cake but I want to **eat** pizza right now. Mary told her brother to **eat** pizza too.

P(next word = *pizza* | previous word = *eat*) = 2/3

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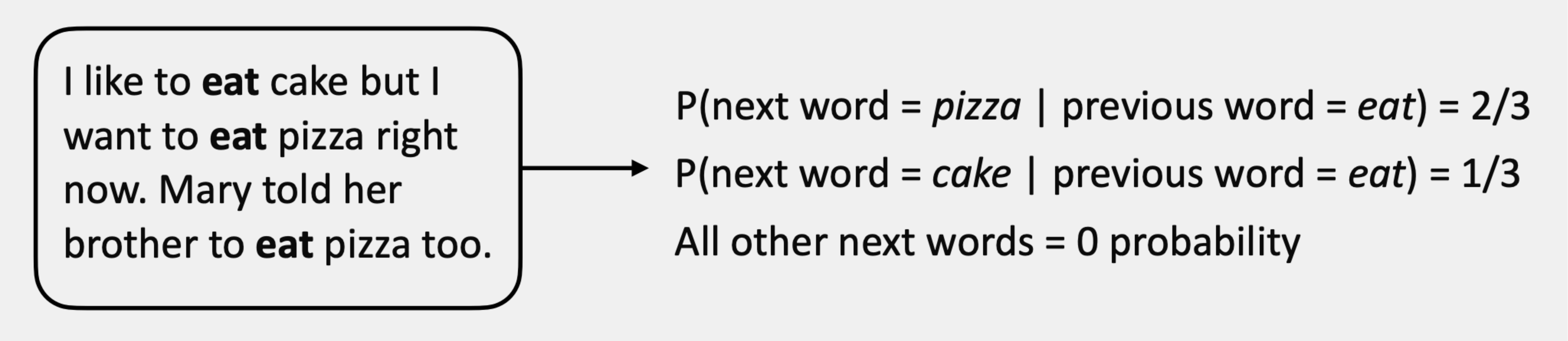

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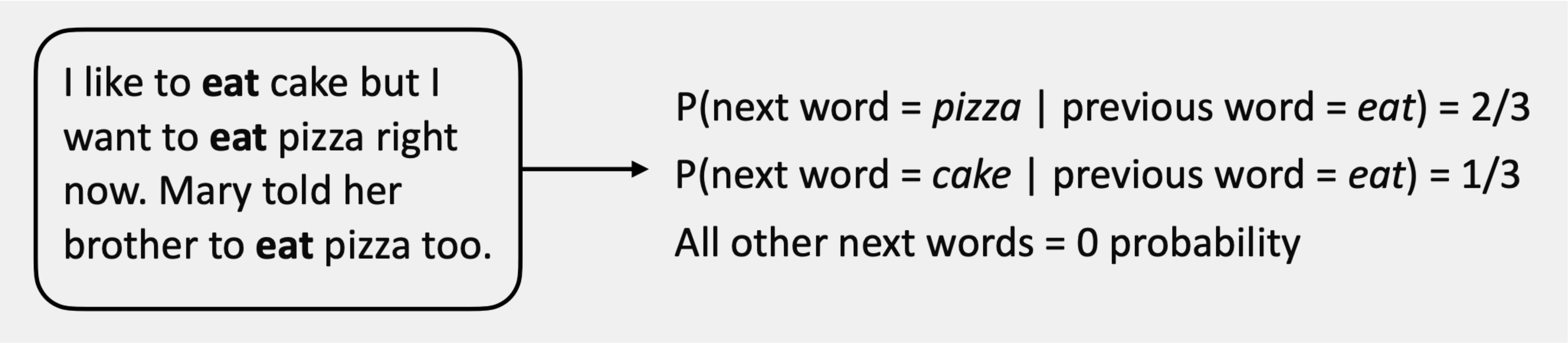

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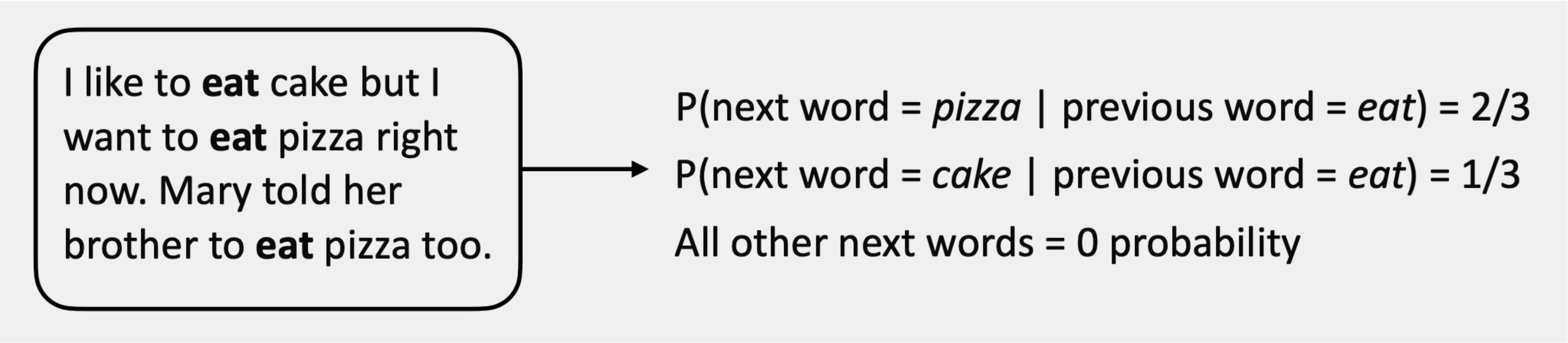

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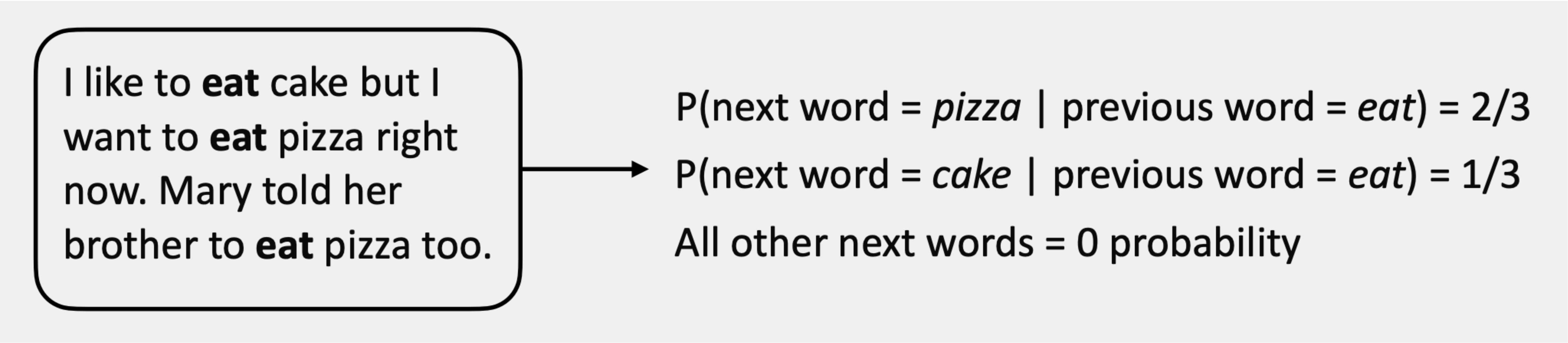

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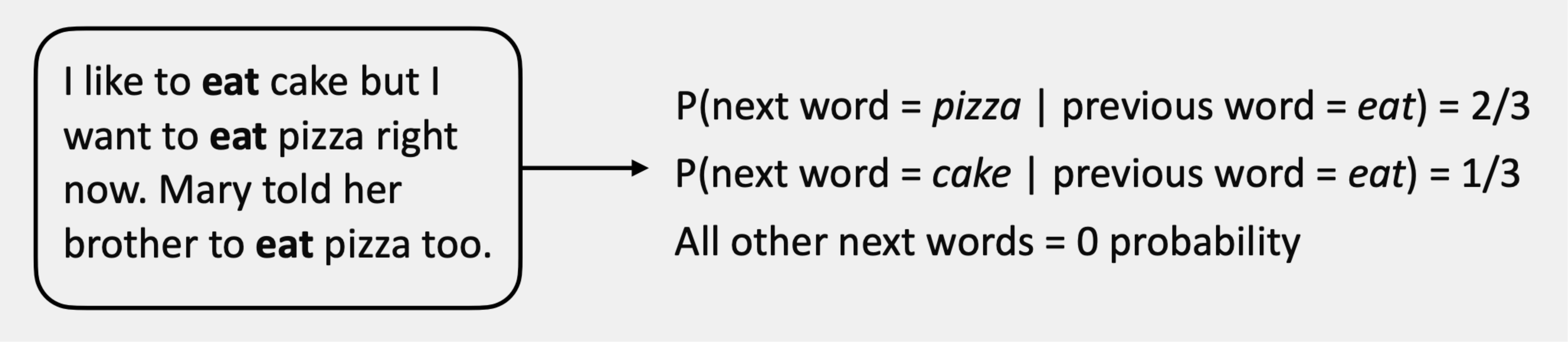

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What does a count distribution look like?

Zipf's Law

Zipf, G. K. (1949). Human behavior and the principle of least effort.

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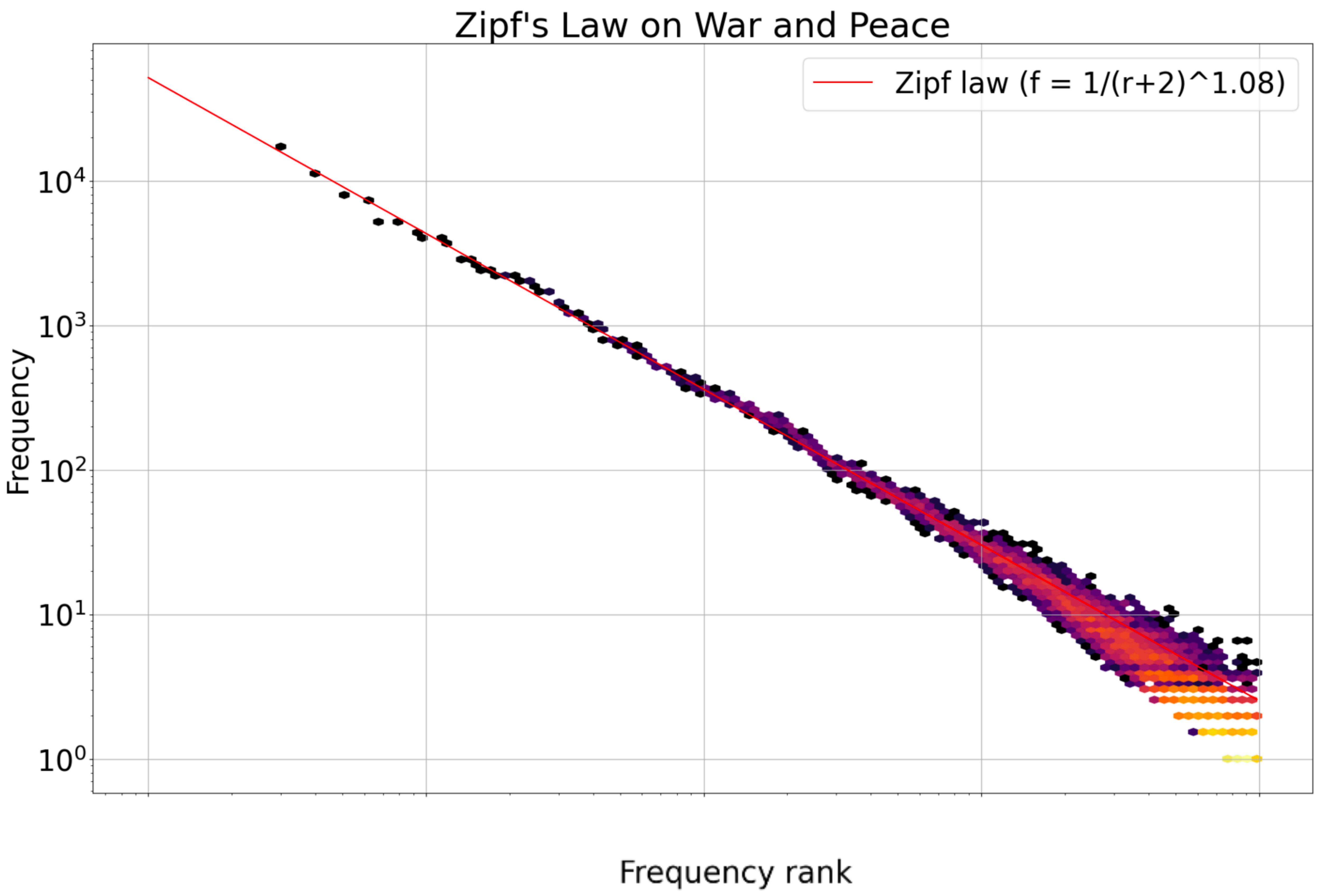

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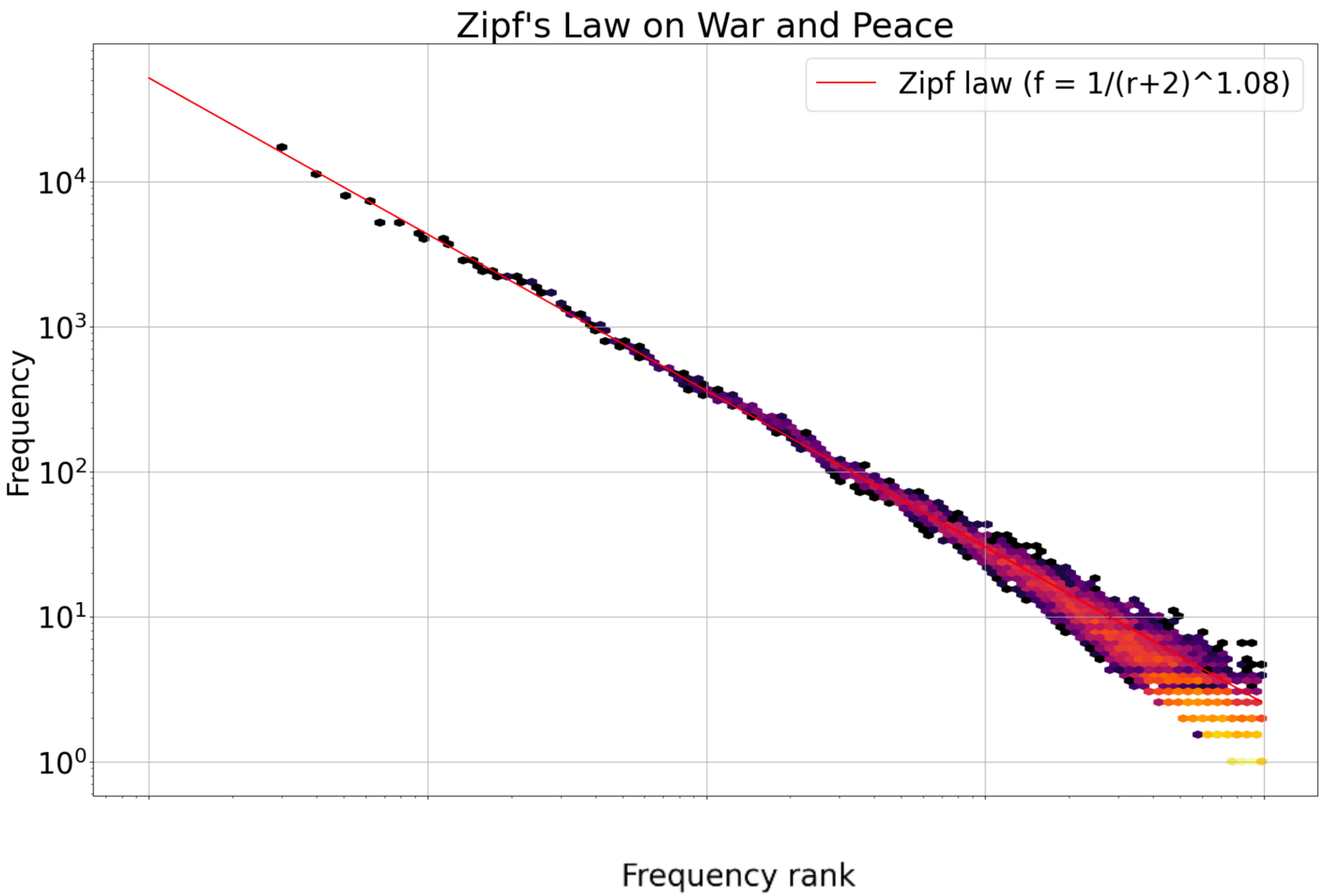

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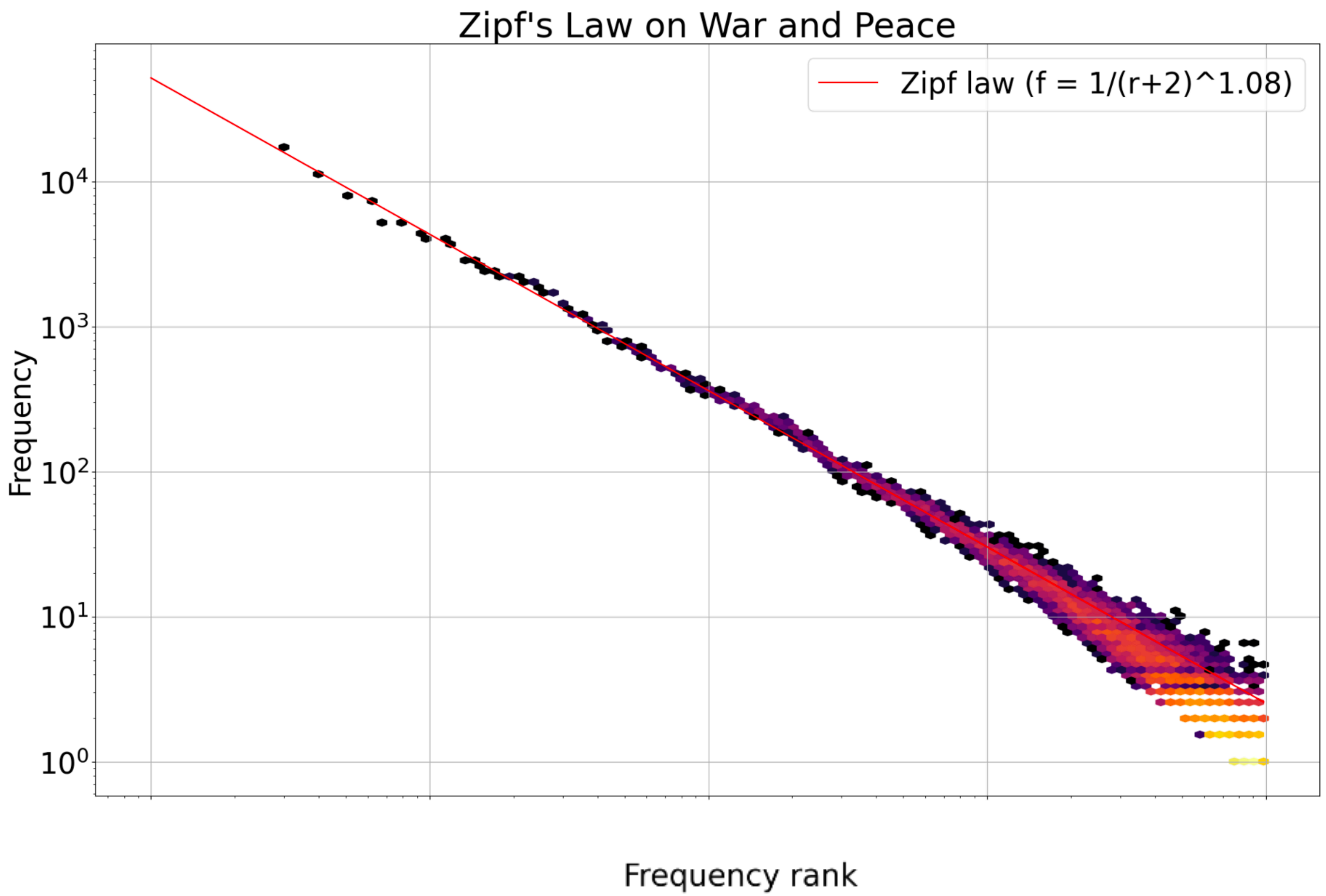
NLP algorithms must be especially robust to observations that do not occur or rarely occur in the training data



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Smoothing ~ Massaging Probability Masses

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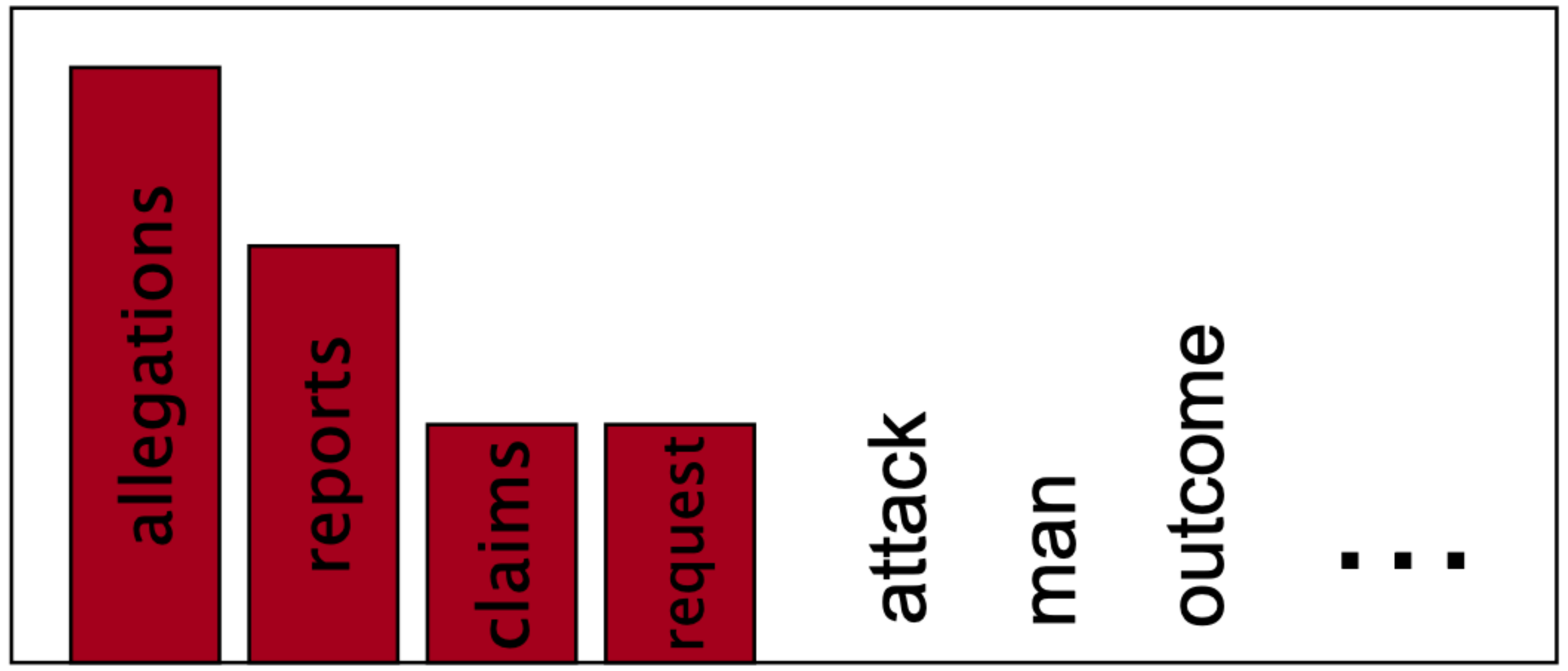
When we have sparse statistics: $Count(w \mid \text{denied the})$

3 allegations
2 reports
1 claims
1 request
7 total



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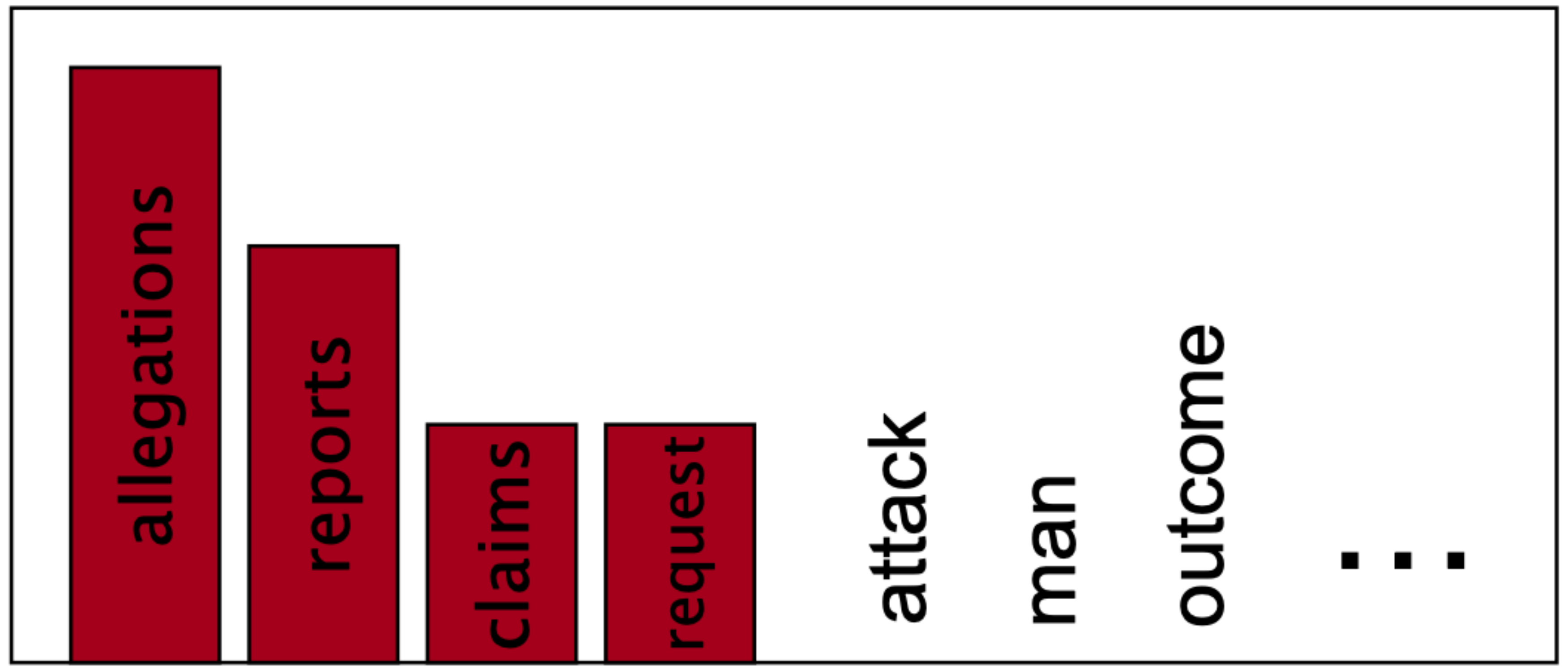
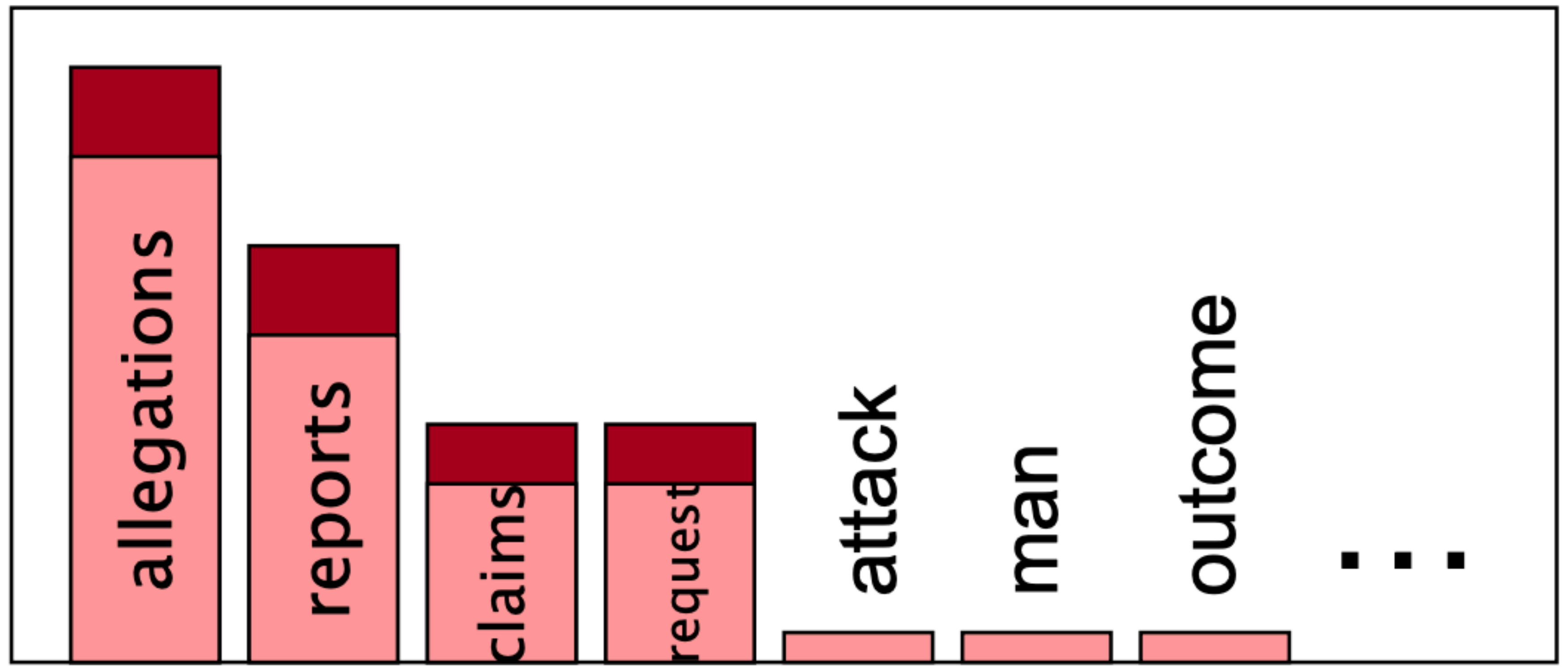


Steal probability mass to generalize better: $Count(w \mid \text{denied the})$

2.5 allegations
1.5 reports
0.5 claims
0.5 request
2 other
7 total


allegations
reports
claims
request
attack
man
outcome
…