Project 1: Martingale

Samuel Wagner

swagner38@gatech.edu

Abstract —The Martingale strategy is a popular betting system employed in casino games, particularly in roulette. The strategy revolves around the concept of doubling one's bet after each loss, with the aim of recovering previous losses and making a profit equal to the initial bet. In the context of roulette, players using the Martingale strategy typically place bets on even-money options such as red or black, odd or even. While the strategy seems straightforward, it carries the risk of substantial losses, as an extended losing streak can lead to exponentially higher bets. Critics argue that the Martingale strategy is not foolproof and does not alter the fundamental odds of the game, making it a high-risk approach with no guaranteed success. The following report discusses results of two Martingale simulations: one without a bankroll and one with.

EXPERIMENT #1 (NO BANKROLL)

Question 1: In Experiment 1, based on the experiment results calculate and provide the estimated probability of winning $80 within 1000 sequential bets. Thoroughly explain your reasoning for the answer using the experiment output. Your explanation should NOT be based on estimates from visually inspecting your plots, but from analyzing any output from your simulation.

Based on the experiment results, the probability of winning $80 with this strategy is 100%. This value was calculated by counting the number of episodes in which the final winnings value was $80 (1000) and dividing by the total number of episodes (1000).

$$P(winnings = \$80) = \frac{episodes\ that\ reach\ \$80}{total\ episodes} = \frac{1000}{1000} = 1$$

Question 2: In Experiment 1, what is the estimated expected value of winnings after 1000 sequential bets? Thoroughly explain your reasoning for the answer.

Based on the experiment results, the expected value of winnings with this strategy is $80. This value can be calculated by taking the average of all 1000 episodes, or by the following formula:

$$E(winnings) = \sum_{x} xP(x) = [\$80 * P(\$80)] = [\$80 * 1] = \$80$$

Question 3: In Experiment 1, do the upper standard deviation line (mean + stdev) and lower standard deviation line (mean – stdev) reach a maximum (or minimum) value and then stabilize? Do the standard deviation lines converge with one another as the number of sequential bets increases? Thoroughly explain why it does or does not.

The experiment results, as illustrated in Figure 1 below, reveal a notable convergence of the upper and lower standard deviation lines shortly after 200 spins. This convergence is attributed to the simulation's infinite bankroll, allowing for increasingly larger bets after each loss. As a consequence, the maximum winnings of $80 are consistently attained, leading to the convergence of the standard deviation lines. The ability to continually escalate bets due to the unlimited bankroll influences the experimental outcome, highlighting a scenario where convergence is inevitable.

However, it is crucial to acknowledge that this particular experimental setting with an unlimited bankroll may not be representative of scenarios where real-world constraints, such as a finite bankroll, are introduced. In the next experiment, when a more realistic financial constraint is applied, it is anticipated that the standard deviation lines may exhibit different behavior. The convergence observed in the current experiment may not be replicated under conditions that mirror practical gambling scenarios, where financial limitations play a crucial role in determining betting patterns and outcomes.

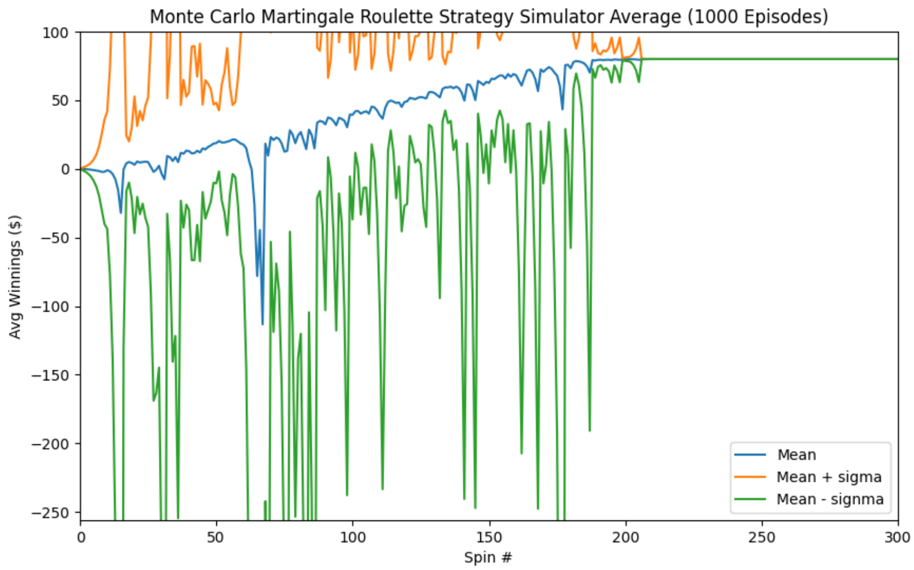

Figure 1 - Plot of Monte Carlo Martingale simulation results (without bankroll)

EXPERIMENT #2 (BANKROLL)

Question 4: In Experiment 2, based on the experiment results calculate and provide the estimated probability of winning $80 within 1000 sequential bets. Thoroughly explain your reasoning for the answer using the experiment output. Your explanation should NOT be based on estimates from visually inspecting your plots, but from analyzing any output from your simulation.

Based on the experiment results, the probability of winning $80 with this strategy is 63.2%. This value was calculated by counting the number of episodes in which the final winnings value was $80 and dividing by the total number of episodes.

$$P(win) = \frac{episodes\ that\ reach\ \$80}{total\ episodes} = \frac{632}{1000} = 0.632$$

Question 5: In Experiment 2, what is the estimated expected value of winnings after 1000 sequential bets? Thoroughly explain your reasoning for the answer.

Based on the experiment results, the expected value of winnings with this strategy is -$43.65. This value can be calculated by taking the average of all 1000 episodes, or by the following formula:

$$E(winnings) = \sum_x xP(x) = [\$80 * P(\$80)] + [-\$256 * P(-\$256)]$$
$$= [\$80 * 0.632] + [-\$256 * 0.368] = -\$43.65$$

Question 6: In Experiment 2, do the upper standard deviation line (mean + stdev) and lower standard deviation line (mean – stdev) reach a maximum (or minimum) value and then stabilize? Do the standard deviation lines converge with one another as the number of sequential bets increases? Thoroughly explain why it does or does not.

The analysis of the experiment results, as depicted in Figure 2 below, unveils an intriguing pattern where the upper and lower standard deviation lines refrain from converging. Instead, they exhibit a distinctive behavior by reaching respective maximum and minimum values before stabilizing. This unique trend is a consequence of introducing more realistic parameters into the simulation, specifically the incorporation of a bankroll. Unlike the previous scenario with an unlimited bankroll, the presence of a finite bankroll places constraints on both potential winnings and losses.

In this experiment, the application of a bankroll restricts any individual win to a maximum of $80 and limits losses to a maximum of $256. Consequently, the standard deviation lines encounter bounds within this range, leading to their stabilization after reaching the defined maximum and minimum values. The inclusion of a bankroll adds a layer of practicality to the simulation, aligning the experiment more closely with real-world gambling scenarios where financial constraints play a pivotal role in determining the dynamics of wins and losses.

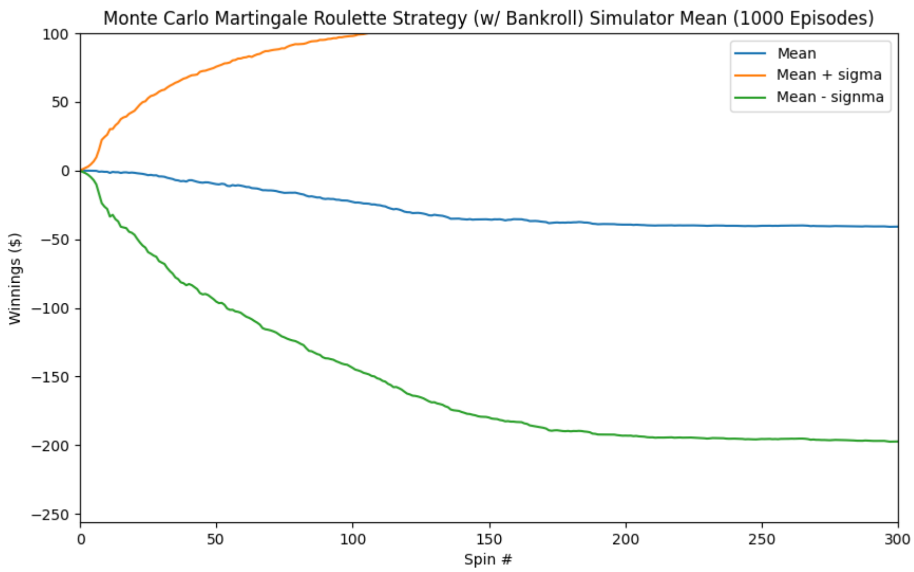

Figure 2 - Plot of Monte Carlo Martingale simulation results with bankroll

Question 7: What are some of the benefits of using expected values when conducting experiments instead of simply using the result of one specific random episode?

Using expected values in experiments offers several advantages over relying solely on the results of a single random episode. Firstly, expected values provide statistical stability by considering the average outcome over many trials, mitigating the impact of random fluctuations and outliers. This stability enhances the robustness of the analysis, ensuring that the conclusions drawn are more representative of the underlying probability distribution.

Secondly, expected values contribute to a comprehensive risk assessment. By taking into account the average outcome, researchers and decision-makers can better understand and manage risks associated with an experiment. This is particularly crucial in scenarios where one random episode might yield an extreme result. The expected value provides a balanced assessment of the probable outcomes, enabling more informed decision-making.

Thirdly, expected values offer consistency across trials and experiments. They provide a reliable measure that remains applicable across different scenarios, allowing for the comparison of strategies or interventions. This consistency is valuable in various domains, including finance, statistics, and decision sciences. Whether assessing risk in gambling or guiding decision-making in uncertain situations, expected values serve as a common metric that facilitates communication and comparison, contributing to a more nuanced understanding of experimental outcomes.